

Heterogeneous difference-in-differences estimation

StataCorp LLC

September 08, 2023

Context: Treatment-effects estimation in Stata

The effect of a treatment or exposure on an outcome

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The effect of a treatment or exposure on an outcome

- Average treatment effect (ATE) and average treatment effect on the treated (ATET)
- `teffects`: cross-sectional data selection on observables
- `didregress` and `xtdidregress`: repeated-measures selection on unobservables
- Estimation, inference, visualization, diagnostics, and tests

Context: Treatment-effects estimation in Stata

The effect of a treatment or exposure on an outcome

- Average treatment effect (ATE) and average treatment effect on the treated (ATET)
- `teffects`: cross-sectional data selection on observables
- `didregress` and `xtdidregress`: repeated-measures selection on unobservables
- Estimation, inference, visualization, diagnostics, and tests
- One treatment effect. That assumes that the treatment does not change over groups or time. Homogeneous treatment effects
- New: Heterogeneous treatment effects for selection on unobservables. Heterogeneous Difference in differences (DID). Multiple ATETs

Starting point: Why heterogeneous treatment effects?

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- DID with different groups treated at different times
- With multiple treatment times the ATET for DID was obtained via

$$y_{it} = \beta_0 + D_{it}\beta_1 + \gamma_t + \gamma_g + e_{it}$$

- β_1 is the ATET
- A generalization of the well understood 2 by 2 model.

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$$y_{it} = \beta_0 + D_{it}\beta_1 + \gamma_t + \gamma_g + e_{it}$$

- β_1 is the ATET
- A generalization of the well understood 2 by 2 model.
- Two-way fixed-effects (TWFE) was criticized in the last six years
- Why:
 - 1 Homogeneity (well understood)
 - 2 Hidden cost of generalizing of 2 by 2

What happens when we go beyond the 2 by 2 model?

- ATET might be different for groups treated at different times
- Within groups, ATET might change over time
- If this is the case:

$$y_{it} = \beta_0 + D_{it}\beta_1 + \gamma_t + \gamma_g + e_{it}$$

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- Weighted average
 - Borusyak, Jaravel, and Spiess (2018)
 - de Chaisemartin and D'Haultfoeuille (2020)
 - Goodman-Bacon (2021) (estat bdecomp)

Bacon decomposition

- β_1 is a weighted average of (2 by 2) estimates

$$y_{it} = \beta_0 + D_{it}\beta_1 + \gamma_t + \gamma_g + e_{it}$$

- 2 by 2 estimates using:
 - 1 Never treated groups
 - 2 Early treated groups
 - 3 Later treated groups

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$$y_{it} = \beta_0 + D_{it}\beta_1 + \gamma_t + \gamma_g + e_{it}$$

- 2 by 2 estimates using:
 - 1 Never treated groups
 - 2 Early treated groups
 - 3 Later treated groups
- Some of the comparisons are valid comparisons. Some are not.
- Validity: satisfying a parallel-trends assumption.

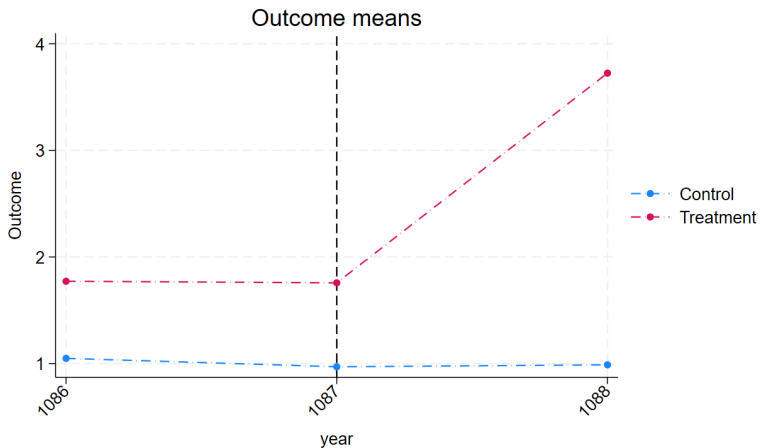
Let's see it work

- What a 2 by 2 model looks like
- What a DID model with multiple treatment times looks like graphically
 - homogeneous treatment
 - heterogeneous treatment
- What the Bacon decomposition tells us
 - homogeneous treatment
 - heterogeneous treatment

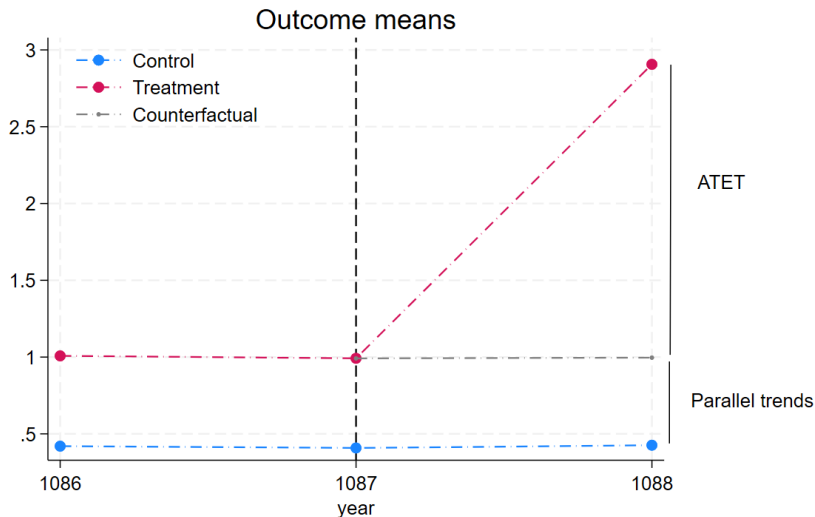
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- What a 2 by 2 model looks like
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 - homogeneous treatment
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- What the Bacon decomposition tells us
 - homogeneous treatment
 - heterogeneous treatment
- Build our understanding of heterogeneous DID

2 by 2 framework: estat trendplots



2 by 2 framework: Parallel(Common)-trends assumption



Homogeneous treatment with multiple treatment times I

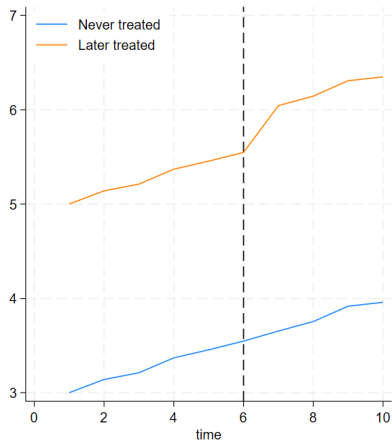
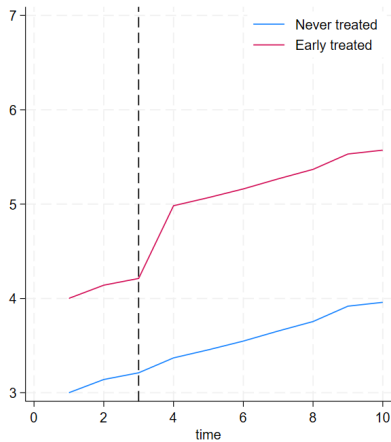
- 10 time periods
- Treatment occurs for some groups at time 3 (Earlier treated)
- For others at time 6 (Later treated)
- Some units remain untreated

Homogeneous treatment with multiple treatment times I

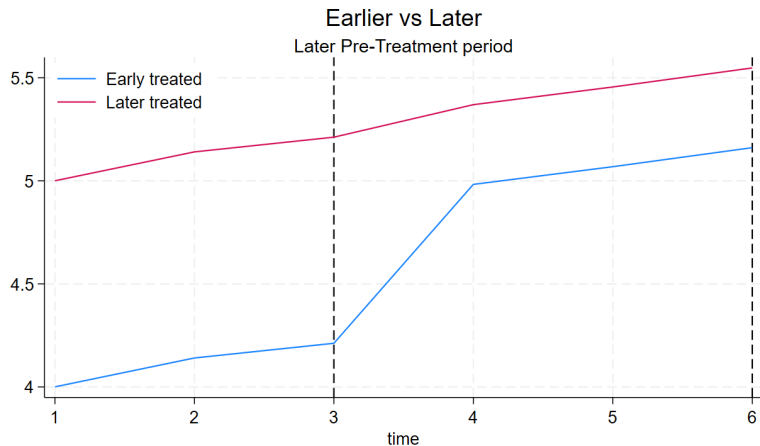
- 10 time periods
- Treatment occurs for some groups at time 3 (Earlier treated)
- For others at time 6 (Later treated)
- Some units remain untreated
- What are the comparisons used by TWFE

Homogeneous treatment with multiple treatment times I

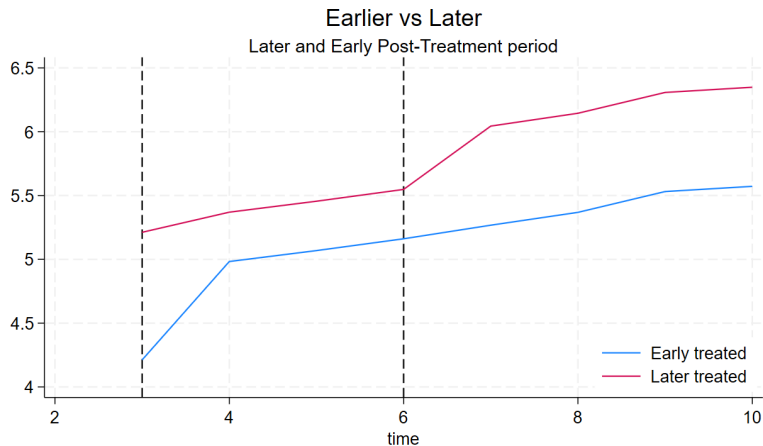
Never treated vs. Later and Earlier



Homogeneous treatment with multiple treatment times II



Homogeneous treatment with multiple treatment times III



Homogeneous treatment: estat bdecomp

```
. estat bdecomp, summaryonly
DID treatment-effect decomposition
ATET = 4.041694
```

```
Number of obs   = 100,000
Number of groups = 10,000
Number of cohorts = 3
```

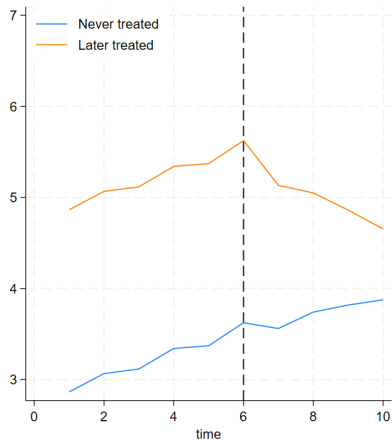
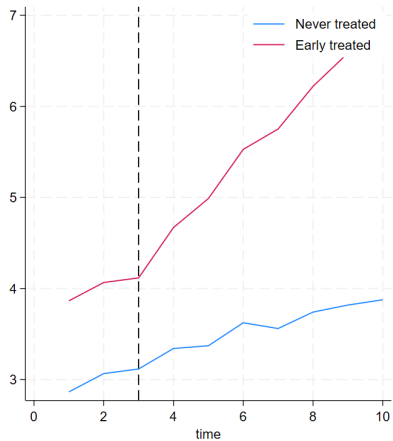
ATET decomposition summary	ATET component	Weight
Treated vs never treated	4.0435917	0.865605
Treated earlier vs later	4.0245287	0.057598
Treated later vs earlier	4.0331799	0.076797

Note: Number of cohorts includes never treated.

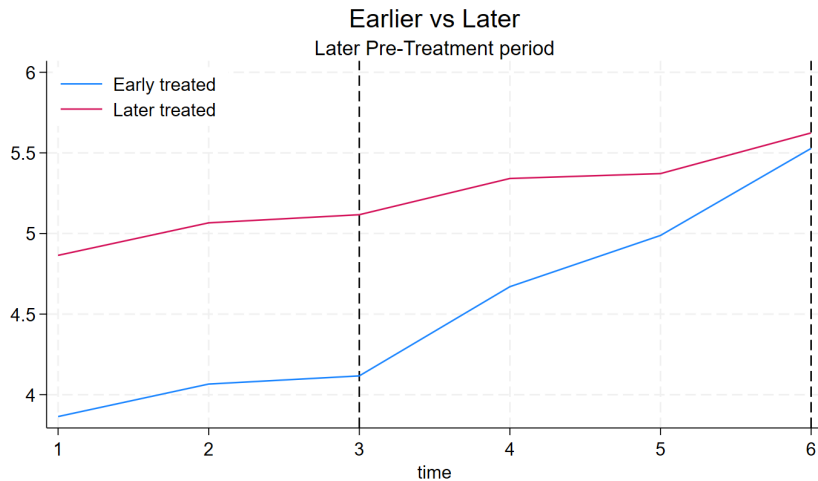
Note: The ATET reported by `xtddidregress` is a weighted average of the ATET components. If any component is substantially different from the ATET reported by `xtddidregress` and the weight is large, consider accounting for treatment-effect heterogeneity by using `xthdidregress`.

Heterogeneous treatment effect I

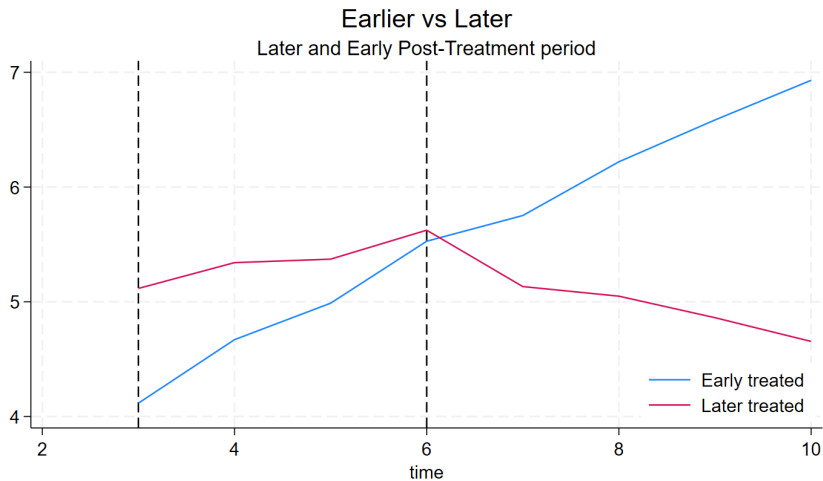
Never treated vs. Later and Earlier



Heterogeneous treatment effect II



Heterogeneous treatment effect III



Heterogeneous treatment: estat bdecomp

```
. estat bdecomp, summaryonly  
DID treatment-effect decomposition  
ATET = 1.389688
```

```
Number of obs   = 50,000  
Number of groups = 5,000  
Number of cohorts = 3
```

ATET decomposition summary	ATET component	Weight
Treated vs never treated	1.9728088	0.860939
Treated earlier vs later	4.3441312	0.059597
Treated later vs earlier	-7.1439268	0.079463

Note: Number of cohorts includes never treated.

Note: The ATET reported by `xtdidregress` is a weighted average of the ATET components. If any component is substantially different from the ATET reported by `xtdidregress` and the weight is large, consider accounting for treatment-effect heterogeneity by using `xthdidregress`.

What would the aggregated effect be with “good” comparison groups?

```
. estat aggregation
```

```
Overall ATET
```

```
Number of obs = 50,000
```

```
(Std. err. adjusted for 50 clusters in state)
```

y	ATET	Robust std. err.	z	P> z	[95% conf. interval]	
tr (1 vs 0)	3.654175	.962046	3.80	0.000	1.768599	5.53975

What have we learned?

- With multiple treatment times, traditional DID assumes homogeneity
- 2 by 2 comparisons are well defined, but not all are useful
 - Comparison group matters
 - Time of comparison matters
- With multiple treatment times, it is important to assess heterogeneity
- If we suspect heterogeneity or do not want to assume homogeneity,

What have we learned?

- With multiple treatment times, traditional DID assumes homogeneity
- 2 by 2 comparisons are well defined, but not all are useful
 - Comparison group matters
 - Time of comparison matters
- With multiple treatment times, it is important to assess heterogeneity
- If we suspect heterogeneity or do not want to assume homogeneity, use `hdidregress` and `xthdidregress`

Heterogeneous treatment effects approaches

- Solution 1: Callaway and Sant'Anna (2021)
 - Use valid comparison groups
 - Split the problem into 2 by 2 comparisons
 - Compute $ATE_T(g, t)$, where g is the cohort and t is the time
- Solution 2: Wooldridge (2021)
 - (Do not blame the messenger) Include heterogeneity in your regression
 - Add the adequate interactions with cohort and time
 - Compute $ATE_T(g, t)$, where g is the cohort and t is the time
 - Computing $ATE_T(g)$ or $ATE_T(t)$ is also possible
- Other solutions exist. Good surveys are Roth et al. (2022) and de Chaisemartin and D'Haultfoeuille (forthcoming)

Framework I

$$Y_{it} = Y_{it}(0) + \sum_{g=2}^T [Y_{it}(g) - Y_{it}(0)] G_{ig}$$

- Y_{it} observed outcome
- $Y_{it}(0)$ potential outcome of not being treated
- G_{ig} is an indicator for treatment group
- g is the time at which a group of individuals is treated (cohort)
- $Y_{it}(g)$ potential outcome for cohort g

Framework II

- Treatment is staggered
- Parallel trends
- No anticipation
- Overlap

Callaway and Sant'Anna

- Regression adjustment (RA)
- Inverse-probability weighting (IPW)
- Augmented inverse-probability weighting (AIPW)

$$ATE_T(g, t) = E \left\{ \frac{G_g}{E(G_g)} [Y_t - Y_{g-1} - m_{gt}(X)] \right\}$$

- $m_{gt}(X) = E(Y_t - Y_{g-1} | X, C = 1)$
- $C = 1$ is the never treated group ($G_g = \infty$)

$$ATE_T(g, t) = E \left\{ \frac{G_g}{E(G_g)} [Y_t - Y_{g-1} - m_{gt}(X)] \right\}$$

- $ATE_T(g, t)$ is calculated using two groups: g and $C = 1$, never treated
- Outcomes are computed for two points in time

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- $ATE_T(g, t)$ is calculated using two groups: g and $C = 1$, never treated
- Outcomes are computed for two points in time
- 2 by 2 idea
- This is done for all g and all t
- We could have other 2 by 2 comparisons, i.e, using the not yet treated
- Identification assumptions are the same but need to hold for each 2 by 2

Heuristically

- 1 keep if time is t or $g - 1$
- 2 keep if cohort is g or cohort is $C = 1$
- 3 generate $Y_t - Y_{g-1} \equiv \Delta Y$
- 4 regress ΔY on X for $C = 1$ and predict, $\hat{m}_{gt}(X)$
- 5 generate $\Delta Y - \hat{m}_{gt}(X) = \widehat{TE}$
- 6 summarize \widehat{TE} if cohort is g
 - This is done for each g and t
 - Doing it for all is a GMM problem, Callaway and Sant'Anna use influence functions.

$$ATE_T(g, t) = E \left\{ \left(\frac{G_g}{E(G_g)} - \frac{\frac{p_g(X)}{1-p_g(X)}}{E \left[\frac{p_g(X)}{1-p_g(X)} \right]} \right) [Y_t - Y_{g-1}] \right\}$$

- $p_g(X) = P(G_g = 1|X, G_g + C = 1)$, i.e., conditional on the sample we keep
- Steps are similar to before with the additional computation of $\hat{p}_g(X)$

and the quotient $\frac{\frac{\hat{p}_g(X)}{1-\hat{p}_g(X)}}{\hat{E} \left[\frac{\hat{p}_g(X)}{1-\hat{p}_g(X)} \right]}$

$$ATET(g, t) = E \left\{ \left(\frac{G_g}{E(G_g)} - \frac{\frac{p_g(X_1)}{1-p_g(X_1)}}{E \left[\frac{p_g(X_1)}{1-p_g(X_1)} \right]} \right) [Y_t - Y_{g-1} - m_{gt}(X_2)] \right\}$$

- Notice $m_{gt}(X_2)$. Emphasizes we could have different covariates
- AIPW is doubly robust. You may incorrectly specify at least one of $m_{gt}(X_2)$ or $p_g(X_1)$ and still recover $ATET(g, t)$

$$Y_{it} = \eta + \sum_{g=q}^T G_{ig} \alpha_g + \sum_{s=q}^T f_s \alpha_s + \sum_{g=q}^T \sum_{s=g}^T D_{it} G_{ig} f_s \tau_{gs}$$

- q is the first treatment time and $q \dots T$ the post period
- f_s is 1 if the time period is s and 0 otherwise
- We have group and time effects α_g and α_s

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- Heterogeneity is captured by interacting group and time effects
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- If I have covariates, they enter fully interacted

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- $\tau_{gs} \equiv ATET(g, t)$
- We use all our data and do not partition them
- If I have covariates, they enter fully interacted
- Extended two-way fixed effects

Let's see it work

```
. webuse akc, clear
(Fictional dog breed and AKC registration data)
. describe
Contains data from https://www.stata-press.com/data/r18/akc.dta
Observations:      1,410      Fictional dog breed and AKC registration data
Variables:         5          1 Feb 2023 14:23
```

Variable name	Storage type	Display format	Value label	Variable label
year	int	%10.0g		Year
breed	int	%34.0g	Breed	Dog breed
movie	byte	%9.0g		Was a movie protagonist
best	byte	%9.0g		Won best in show in past 10 years
registered	int	%9.0g		Number of AKC registrations

Sorted by: breed

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Sorted by: breed

- Out of the 113 contests, the Terrier group has won 47 times

Staggered treatment

```
. tabulate year movie
```

Year	Was a movie protagonist		Total
	0	1	
2031	141	0	141
2032	141	0	141
2033	141	0	141
2034	137	4	141
2035	137	4	141
2036	134	7	141
2037	119	22	141
2038	119	22	141
2039	119	22	141
2040	119	22	141
Total	1,307	103	1,410

Specification: Panel/Longitudinal

```
xthdidregress estimator (registered best) (movie), group(breed)
```

Specification: Panel/Longitudinal

```
xthdidregress estimator (registered best) (movie), group(breed)
```

- You need to `xtset` with panel ID and time variable
- *estimator* is one of:
 - `ra`
 - `twfe`
 - `ipw`
 - `aipw`

Output 1

```
. xtset breed year
. xthdidregress ra (registered best) (movie), group(breed)
note: variable _did_cohort, containing cohort indicators formed by treatment variable
      movie and group variable breed, was added to the dataset.
Computing ATET for each cohort and time:
Cohort 2034 (9): ..... done
Cohort 2036 (9): ..... done
Cohort 2037 (9): ..... done
Treatment and time information
Time variable: year
Time interval: 2031 to 2040
Control:      _did_cohort = 0
Treatment:    _did_cohort > 0
```

	_did_cohort
Number of cohorts	4
Number of obs	
Never treated	1190
2034	40
2036	30
2037	150

Output II

```
. xthdidregress ra (registered best) (movie), group(breed)
(output omitted)
```

```
Heterogeneous-treatment-effects regression          Number of obs   = 1,410
                                                    Number of panels = 141
```

```
Estimator:      Regression adjustment
Panel variable:  breed
Treatment level: breed
Control group:   Never treated
```

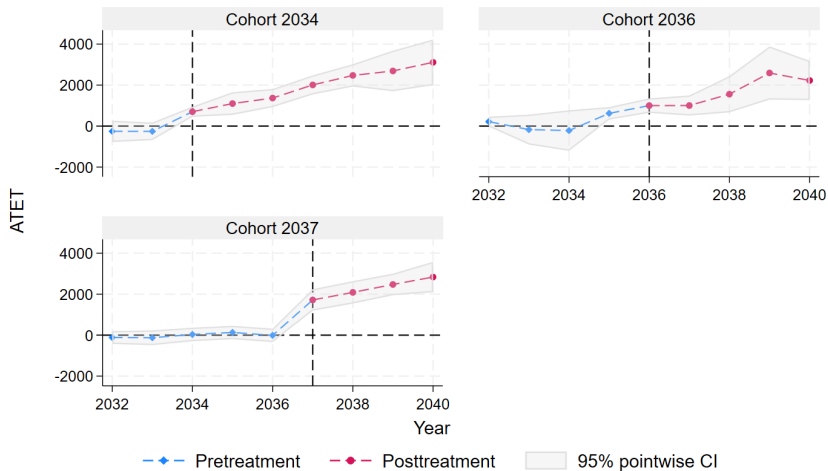
(Std. err. adjusted for 141 clusters in breed)

Cohort	ATET	Robust std. err.	z	P> z	[95% conf. interval]	
2034						
year						
2032	-254.8927	266.1024	-0.96	0.338	-776.4439	266.6584
2033	-257.5329	217.9389	-1.18	0.237	-684.6852	169.6194
2034	701.1318	127.0935	5.52	0.000	452.0331	950.2304
2035	1099.044	282.0704	3.90	0.000	546.196	1651.892
2036	1367.632	225.8702	6.05	0.000	924.9343	1810.329
2037	2008.294	237.2396	8.47	0.000	1543.313	2473.275
2038	2472.624	278.2949	8.88	0.000	1927.176	3018.072
2039	2689.615	504.3324	5.33	0.000	1701.142	3678.088
2040	3110.97	568.916	5.47	0.000	1995.915	4226.025

(output omitted)

Note: ATET computed using covariates.

Graphical representation: estat atepplot



How to interpret my results

- I might want an average over all the $ATET(g, t)$
- I might want to know the effect of treatment within each group
- I might want to know the effect of treatment within a particular year
- I might want to see how the effect evolves with the duration of treatment

Overall

```
. estat aggregation, overall
```

```
Overall ATET
```

```
Number of obs = 1,410
```

```
(Std. err. adjusted for 141 clusters in breed)
```

registered	ATET	Robust std. err.	z	P> z	[95% conf. interval]	
movie (1 vs 0)	2093.318	122.5752	17.08	0.000	1853.075	2333.561

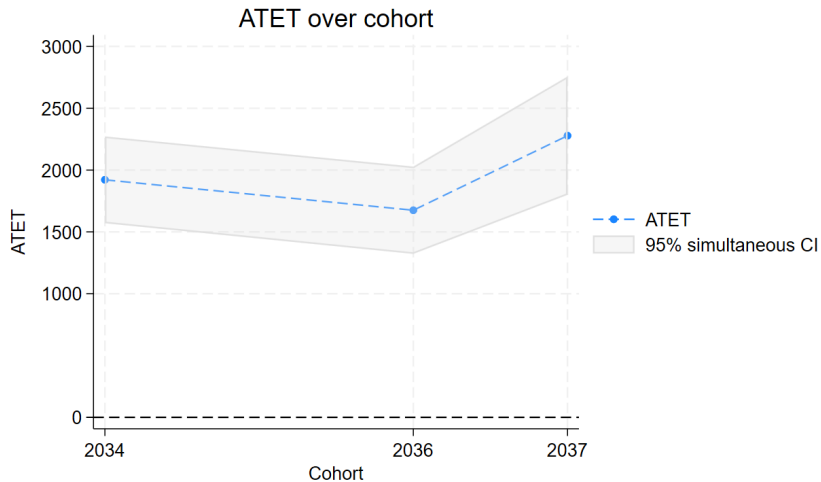
Cohort Tabular

```
. estat aggregation, cohort graph sci
ATET over cohort                               Number of obs = 1,410
                                                Replications = 999
                                                (Std. err. adjusted for 141 clusters in breed)
```

Cohort	Observed ATET	Bootstrap std. err.	Simultaneous [95% conf. interval]	
2034	1921.33	126.9843	1570.576	2272.084
2036	1675.093	127.749	1322.227	2027.959
2037	2278.136	173.1852	1799.767	2756.504

Note: Simultaneous confidence intervals provide inference across all aggregations simultaneously.

Cohort Graphical



Cohort list

```
. estat aggregation, cohort(2036 2034)
ATET over cohort                                Number of obs = 1,410
                                                (Std. err. adjusted for 141 clusters in breed)
```

Cohort	ATET	Robust std. err.	z	P> z	[95% conf. interval]	
2034	1921.33	187.2787	10.26	0.000	1554.271	2288.389
2036	1675.093	130.4929	12.84	0.000	1419.332	1930.855

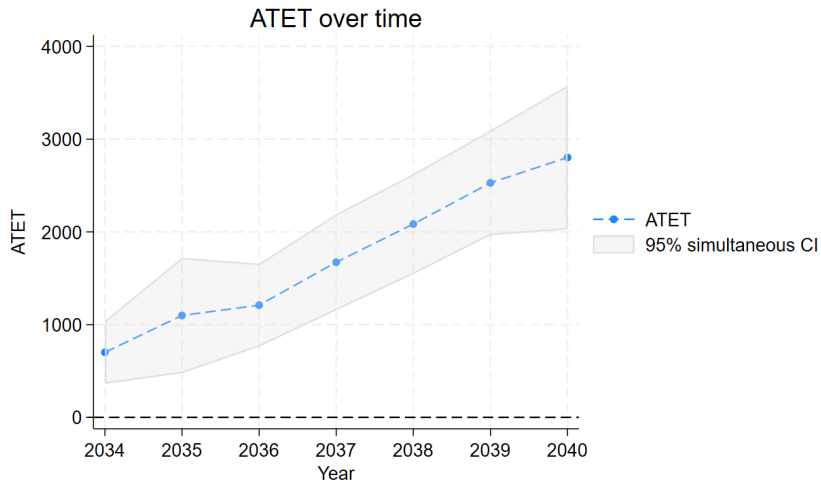
Time Tabular

```
. estat aggregation, time graph sci
ATET over time                               Number of obs = 1,410
                                                Replications = 999
                                                (Std. err. adjusted for 141 clusters in breed)
```

Time	Observed ATET	Bootstrap std. err.	Simultaneous [95% conf. interval]	
2034	701.1318	135.4942	360.2741	1041.989
2035	1099.044	247.3894	476.6955	1721.392
2036	1209.68	177.7654	762.4823	1656.878
2037	1672.655	206.0035	1154.42	2190.891
2038	2084.658	214.6857	1544.581	2624.735
2039	2528.847	225.004	1962.813	3094.882
2040	2802.171	308.5624	2025.932	3578.41

Note: Simultaneous confidence intervals provide inference across all aggregations simultaneously.

Time Graphical



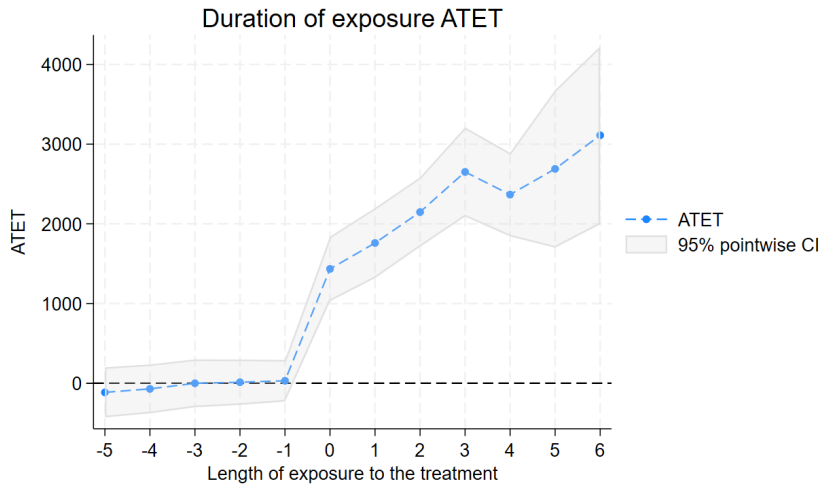
Duration Tabular

```
. estat aggregation, dynamic graph
Duration of exposure ATET                                     Number of obs = 1,410
                                                              (Std. err. adjusted for 141 clusters in breed)
```

Exposure	ATET	Robust std. err.	z	P> z	[95% conf. interval]	
-5	-114.582	160.0972	-0.72	0.474	-428.3668	199.2028
-4	-70.65034	156.3185	-0.45	0.651	-377.029	235.7283
-3	-.9117242	153.0999	-0.01	0.995	-300.982	299.1585
-2	12.79653	144.8216	0.09	0.930	-271.0486	296.6417
-1	30.71473	132.8508	0.23	0.817	-229.668	291.0975
0	1434.409	206.3277	6.95	0.000	1030.014	1838.804
1	1759.461	224.0229	7.85	0.000	1320.385	2198.538
2	2147.486	221.903	9.68	0.000	1712.564	2582.408
3	2651.452	284.8928	9.31	0.000	2093.073	3209.832
4	2366.805	267.4253	8.85	0.000	1842.661	2890.949
5	2689.615	504.3324	5.33	0.000	1701.142	3678.088
6	3110.97	568.916	5.47	0.000	1995.915	4226.025

Note: Exposure is the number of periods since the first treatment time.

Duration Graphical



$$Y_{it} = \eta + \sum_{g=q}^T G_{ig} \alpha_g + \sum_{s=q}^T f_s \alpha_s + \sum_{g=q}^T \sum_{s=g}^T D_{it} G_{ig} f_s \tau_{gs}$$

- Number of parameters increases with t , g , and covariates
- This is true for Callaway and Sant'Anna for each 2 by 2 subset

$$Y_{it} = \eta + \sum_{g=q}^T G_{ig} \alpha_g + \sum_{s=q}^T f_s \alpha_s + \sum_{g=q}^T \sum_{s=g}^T D_{it} G_{ig} f_s \tau_{gs}$$

- Number of parameters increases with t , g , and covariates
- This is true for Callaway and Sant'Anna for each 2 by 2 subset
- `twfe` gives us a chance to address this at estimation with `hettype()`

Cohort heterogeneity

```
. xthdidregress twfe (registered best) (movie), group(breed) hettype(cohort)
(output omitted)
Computing ATETs using margins ...
(output omitted)
Heterogeneous-treatment-effects regression          Number of obs   = 1,410
                                                    Number of panels =   141

Estimator:          Two-way fixed effects
Panel variable:     breed
Treatment level:    breed
Control group:      Never treated
Heterogeneity:      Cohort
```

(Std. err. adjusted for 141 clusters in breed)

Cohort	ATET	Robust std. err.	t	P> t	[95% conf. interval]	
2034	1662.492	108.002	15.39	0.000	1448.966	1876.017
2036	1978.645	54.21043	36.50	0.000	1871.468	2085.822
2037	2276.223	70.63244	32.23	0.000	2136.579	2415.867

Note: ATET computed using covariates.

Time heterogeneity

```
. xthdidregress twfe (registered best) (movie), group(breed) hettype(time)
      (output omitted)
Computing ATETs using margins ...
      (output omitted)
Heterogeneous-treatment-effects regression                Number of obs   = 1,410
                                                           Number of panels = 141

Estimator:                Two-way fixed effects
Panel variable:           breed
Treatment level:          breed
Control group:            Never treated
Heterogeneity:            Time

                               (Std. err. adjusted for 141 clusters in breed)
```

Time	ATET	Robust std. err.	t	P> t	[95% conf. interval]	
year						
2034	368.7667	177.8305	2.07	0.040	17.18625	720.3472
2035	825.1479	269.4845	3.06	0.003	292.3625	1357.933
2036	1262.367	85.71962	14.73	0.000	1092.895	1431.839
2037	1692.904	165.318	10.24	0.000	1366.061	2019.746
2038	2110.763	162.5539	12.99	0.000	1789.386	2432.141
2039	2554.188	241.3463	10.58	0.000	2077.033	3031.342
2040	2829.94	243.2751	11.63	0.000	2348.972	3310.908

Note: ATET computed using covariates.

Parting thoughts

- Continued interest and development in causal inference/treatment effects
- Heterogeneous DID
- Estimation and postestimation tools displayed in tabular and graphical forms
- Different ways of thinking about heterogeneity