A SIMPLE BUSINESS-CYCLE MODEL WITH SCHUMPETERIAN FEATURES

Luís F. Costa

(ISEG, Technical University of Lisbon and UECE)

Huw D. Dixon

(University of York)

ABSTRACT

We develop a dynamic general equilibrium model of imperfect competition where a sunk cost of creating a new product regulates the type of entry that dominates in the economy: new products or more competition in existing industries. Considering the process of product innovation is irreversible, introduces hysteresis in the business cycle. Expansionary shocks may lead the economy to a new 'prosperity plateau,' but contractionary shocks only affect the market power of mature industries.

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Instituto Superior de Economia e Gestão
Universidade Técnica de Lisboa
Rua do Quelhas 6
1200-781 LISBOA
Portugal
Tel.: +351-213 925 976
Fax: +351-213 922 808
E-mail: lukosta@iseg.utl.pt
http://www.iseg.utl.pt/~lukosta

Department of Economics and Related Studies
University of York
Heslington
YO10 5DD
United Kingdom
Tel.: +44-(0)1904-433 782
Fax: +44-(0)1904-433 759
E-mail: hdd1@york.ac.uk
http://www.york.ac.uk/depts/econ/res/indiv/dixon.htm

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A SIMPLE BUSINESS-CYCLE MODEL WITH SHUMPETERIAN FEATURES*

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We develop a dynamic general equilibrium model of imperfect competition where a sunk cost of creating a new product regulates the type of entry that dominates in the economy: new products or more competition in existing industries. Considering the process of product innovation is irreversible, introduces hysteresis in the business cycle. Expansionary shocks may lead the economy to a new 'prosperity plateau,' but contractionary shocks only affect the market power of mature industries.

1. Introduction

(…) On the one hand, then, change that comes from within the system, as well as change that comes from without it, impinges on situations, induces short-time adaptations and produces short-time equilibria, which in many cases conform well to the picture drawn by the authors of the theory of monopolistic competition. On the other hand, new firms producing new commodities or old commodities by new methods will, as a rule, try to behave according to it, for that is the obvious method of exploiting to the full, and of keeping alive, the temporary advantages they enjoy.

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Schumpeter (1964), p. 42

Most existing models of the business cycle or growth either assume that there is a fixed range of industries and what happens over time is that new firms enter the existing industries – e.g. Costa (2004), Portier (1995). The alternative approach is to assume a fixed industrial structure (for example monopolistic) with the number of industries varying over time – e.g. Devereux et al. (1996), Heijdra (1998). What is new in this paper is that we allow for both types of change. Firms can either be set up in an existing industry or create an entirely new industry. Setting up a new industry has a one-off sunk cost: this is set against the fact that the firm that undertakes this will enjoy a monopoly profit for a limited period. Joining an existing industry means that the firm does not incur a sunk cost, but has to share the market with the existing firms in that industry. Our approach allows for a two-dimensional industrial structure described by the number of industries and the number of firms per industry. The industrial structure of the economy represents the technology by which the economy transforms labour into output for consumption. There is no capital as such; the only state variables are the number of firms per industry and the number of industries. There is free entry which drives the profits down to zero in both dimensions: effectively, this also acts as an arbitrage condition equating the returns between entering and existing industry and setting up a new one.

These two different ways that new firms can be set up behave differently. Once a new industry is set up, it is irreversible: the new technology or product it represents will always be available for free. This means that over time the number of industries can only grow, which creates a form of hysteresis in the economy. The entry of firms into an existing industry is reversible: firms can come and go over the business cycle.

Furthermore, as the number of firms in the industry varies, so will the mark-up in the product market (we assume Cournot competition). If we look at the whole economy, the average mark-up is determined by two things: the number of firms in mature industries, and the number of new industries (the new industries are all monopolies for one period). The effect of technological shocks in this setting is to increase the mark-up. A permanent positive technological shock leads to an increase in new industries: since these are all monopolists for one period, the average mark-up in the economy increases for one period following the technological shock. A negative technological shock leads to a permanent increase in the mark-up: since the creation of a new industry is irreversible, the adverse change in technology leads to exit from existing industries and an increase in the mark-up. The same argument holds for demand shocks.

Perhaps the most interesting result relates to a temporary positive shock: it can have a permanent adverse effect on efficiency. The mechanism is easy to understand. The temporary shock leads to an increase in the number of industries (even though it is temporary, it is worth the set-up costs to gain the temporary monopoly profit). When the shock dies away, this leads to exit from all industries (irreversibility again). Hence mark-ups increase and efficiency declines.

In section 2, we describe the dominant market structure and the business creation and destruction process. On section 3, we build the macroeconomic model from its micro-foundations. On section 4, we analyse the equilibrium existence, features, and model dynamics. Section 5 describes the business-cycles features generated by this model. Section 6 concludes.

2. Business creation and destruction

Let us assume the production sector is constituted by n_t (a very large number of) industries in period t and each industry j is composed by m_{jt} firms. We also assume firms compete over quantities within the same industry and they compete over prices with other industries. This type of market structure corresponds to Cournotian Monopolistic Competition (CMC), according to d'Aspremont et al. (1997). If we consider all the firms and industries are identical, we have $m_{jt} = m_t$.

The usual way of endogenising the number of firms is to assume firms/industries are created or destroyed following profit opportunities, i.e. the well-known zero-profit condition regulates the number of firms. In a monopolistically competitive model with instantaneous free entry, $m_t = 1$ and the zero-profit condition determines n_t.[1] If the number of goods/industries is fixed ($n_t = \overline{n}$), the free-entry condition controls m_t, and induces a (counter-cyclical) endogenous desired mark-up model, using the classification in Rotemberg and Woodford (1999).[2]

However, the arbitrage equation given by the zero-profit condition is not able to simultaneously determine both n_t and m_t. Furthermore, in the absence of additional costs, investors would always prefer to create a new industry where they can act as monopolists to entering a 'mature' industry where they would have to share market

[1] See, *inter alia*, Blanchard and Kiyotaki (1987), Heijdra (1998), or Startz (1989).

[2] See, *inter alia*, Chatterjee et al. (1993), Costa (2001), Portier (1995).

power. Thus, the monopolistic competition type of entry appears to be dominant in a frictionless world.

Of course further investigation is needed in order to solve the entry indeterminacy. In real economies, profit opportunities are taken by producing a close substitute to an existing product or by creating a new one. The relative cost of these two forms of entry and its nature (flow and/or sunk costs) must be compared with the relative profitability of the alternatives in terms of expected discounted profits. Furthermore, the type of costs involved (R&D, marketing and advertising, etc. in the first case and intellectual property protection, royalties, etc. in the second), vary across time and space.

Hence, we assume the number of industries at a given period is governed by:

1. a sunk cost of creating a new industry that is fixed and represented by Ω_t;
2. a one-period protection for the creators of new industries, i.e. $m_{kt} = 1$, when k is a new industry created in period t;
3. the irreversibility of product innovation, i.e. $n_t \geq n_{t-1}$.

In the first feature, we implicitly assume there are no costs of creating a perfect substitute to a mature product, but the costs of product innovation may be substantial. Of course, we can notice that if $\Omega_t = 0$, we have the usual free-entry monopolistic competition (MC) model. However, if Ω_t is very large, we have a free-entry Cournot competition (CC) model in each industry.

Without the second element (intellectual property protection or a simple time-to-learn effect), there would be no incentive to create new products, as new firms would immediately enter the new industry free-riding on the innovators expense.

The last characteristic may look strange if we face it from a very micro-focused point of view: where are the Betamax VCRs or the stone-made tools, after all? We can find three counter-arguments for this remark: i) the technology to produce them is still available and these industries can be revived at any point in time, provided there is no civilisation catastrophe (e.g. the Dark Ages); ii) the home-video device industry or the mechanical tools industry is well alive and kicking; iii) from the macroeconomic point of view there is strong evidence of an ever-increasing number of products, but not of the total number of firms which appears to be pro-cyclical.

Therefore, a new industry k is created if the expected present value of profits exceeds the sunk cost associated with the new product:

$$\Omega_t \leq E_t \sum_{s=t}^{\infty} a_s . \pi_{ks} \quad , \tag{1}$$

where π_{ks} represents the operational profits of firm k in period $s, a_s = \prod_{q=t+1}^{s}(1+i_q)^{-1}$, for $s \geq t + 1$, and $a_t = 1$, is the appropriate discount factor, and i_s represents the interest rate.

When investors expect future profit opportunities, the creation of new industries is only limited by i) the size of Ω_t and ii) the existing stock of viable ideas. On the latter, we assume this stock is very large and always increasing, so that it is not a binding constraint. In this case, considering the irreversibility assumed, the number of products existing in period t is given by

$$n_t = \max\left\{n_{t-1}, N_t\right\} \ , \tag{2}$$

where N_t represents the 'optimal' number of products in the period given the information available in period t and the value of Ω_t.

Once this protection period is over, the absence of barriers to entry mean no pure profits can persist in any industry.

3. The model

We use a simplified intertemporal general equilibrium model to assess the importance of the assumptions made in the previous section in the macroeconomic equilibrium. Here, labour is assumed to be the only input, so we can insulate the dynamics of entry from capital accumulation.

3.1. Households

Population is stable in this economy and there is a large number of identical households. Therefore, we can use a representative agent to study consumption and labour supply. This infinitely living representative household maximises the discounted value of its utility given by

$$\max_{\{C_t, L_t\}} \sum_{s=t}^{\infty} \left(1+\rho\right)^{-s} . \left(\frac{C_s^{\,1-\theta} - 1}{1-\theta} - b.\frac{L_s^{\,1+\chi}}{1+\chi} \right) \ , \tag{3}$$

where C_t is a consumption basket of the n_t goods, $L_t \geq 0$ is the labour effort supply, $\rho \in [0,1]$ stands for the discount rate, $b > 0$, and $1/\theta > 0$ and $1/\chi > 0$ represent the elasticities of marginal utility of consumption and in labour supply.

The consumption basket is assumed to be CES:

$$C_t = n_t^{\frac{1-\lambda}{1-\sigma}} . \left(\sum_{j=1}^{n_t} c_{jt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} , \tag{4}$$

where c_{jt} represents the consumption of variety j and $\sigma \geq 1$ stands for the elasticity of substitution between varieties. The parameter $\lambda \geq 0$ represents *love for variety*: λ=0 corresponds to 'no love,' and $\lambda = 1$ corresponds to the Dixit and Stiglitz (1977) assumption.

ASSUMPTION 1: *We suppose $\lambda < 1$, i.e., the sharing effect is always stronger than the love-for-variety effect.*

In this case, and using duality theory in the usual two-step maximisation procedure, the appropriate cost-of-living index is given by

$$P_t = \left(\frac{1}{n_t^{1-\lambda}} . \sum_{j=1}^{n_t} p_{jt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} , \tag{5}$$

where p_{jt} represents the price of variety j, so that $P_t.C_t = \sum_{j=1}^{n_t} p_{jt}.c_{jt}$. Thus, considering the aggregate consumption level as given, the demand function for good j is represented by

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\sigma} . \frac{C_t}{n_t^{1-\lambda}} \ . \tag{6}$$

The budget constraint is given by:

$$w_t.L_t + \pi_t - T_t = \Delta n_t.\Omega_t + P_t.C_t \ , \tag{7}$$

where w_t is the wage rate, π_t stands for non-wage income, and T_t represents a lump-sum tax. Since the economy is closed, there is no fixed capital accumulation, and it is not plausible to assume government would lend/borrow as much resources as households require, these agents are in fact liquidity constrained. The only way households can shift resources across time is by creating new industries, facing a sunk cost to obtain short-run abnormal profits. However, this is not a symmetric option as Ω_t is non-refundable, i.e. the option of liquidating industries (even if we allowed for reversibility of product innovation) adds nothing to the consumption possibility frontier.

Thus, we can define net profit income as $\Pi_t = \pi_t - \Omega_t.\Delta n_t$ (total operational profits of new industries minus the corresponding set-up costs), and the intertemporal decision becomes equivalent to a static problem in consumption and labour supply, where the only dynamic decision is given by (1). The first-order conditions of the maximisation problem lead to the following behavioural functions

$$C_t = \frac{w_t.L_t + \Pi_t - T_t}{P_t} \ , \tag{8}$$

$$L_t = \left(\frac{1}{b}.\frac{w_t}{P_t}\right)^{\frac{1}{\chi}} .C_t^{-\frac{\theta}{\chi}} \ , \tag{9}$$

where (8) stands for the consumption function and (9) represents the labour supply.

3.2. Government

We ignore positive externalities from government consumption in both household utility and production technologies. Government is assumed to have preferences similar to those of the household.[3] Furthermore, since Ricardian equivalence holds in this model, not much is lost if we ignore government borrowing and impose a balanced-budget constraint in each period. Therefore, we have

$$g_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\sigma} \cdot \frac{G_t}{n_t^{1-\lambda}} \, , \tag{10}$$

where $G_t = n_t^{\frac{1-\lambda}{1-\sigma}} . (\sum_{j=1}^{n_t} g_{jt}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ represents the aggregate government consumption index, and g_{jt} stands for consumption of good j; and also $P_t.G_t = T_t$.

3.3. Firms

We assume firms are price takers in the labour market and compete strategically in product markets (inter-industrial price competition and intra-industrial quantity competition). Firm i in industry j maximises the present discounted value of its real profits

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[3] The existence of love for variety in government preferences may be criticised. However, as Heijdra and van der Ploeg (1996) noticed, using different aggregators may influence the mark-up by changing the composition of aggregate demand. Galí (1994a) showed this additional source of endogenous mark-ups may introduce further complications in the model. Hence, we use this assumption for sake of simplicity.

$$\max_{\{y_{it},L_{it}\}} \sum_{s=t}^{\infty} a_s.\Pi_{is} \quad , \tag{11}$$

where y_{it} represents its output, L_{it} denotes its labour input, and profits are given by

$$\Pi_{it} = p_{jt}.y_{it} - w_t.L_{it} \quad . \tag{12}$$

Considering labour is the only input, the production technology is represented by

$$y_{it} = \begin{cases} A_t.\left(L_{it} - \Phi - H_{jt}.\varpi\right) & \Leftarrow L_{it} \geq \Phi + H_{jt}.\varpi \\ 0 & \Leftarrow 0 \leq L_{it} < \Phi + H_{jt}.\varpi \end{cases} , \quad H_{jt} = \begin{cases} 1 \Leftarrow m_{jt-1} = 0 \\ 0 \Leftarrow m_{jt-1} \geq 1 \end{cases} , \tag{13}$$

where $A_t > 0$ is the marginal product of labour, $\Phi > 0$ stands for overhead labour and it generates a flow fixed cost ($w_t.\Phi$), H_{jt} is equal to 1 if industry j is being created in period t and 0 otherwise, and $\varpi \geq 0$ represents the labour effort necessary to start a new industry, and it generates a sunk cost ($\Omega_t = w_t.\varpi$).

Taking the quantities produced by other producers in industry j ($k \neq i \in J$) as given, the 'objective' demand faced by firm i is represented by

$$p_{jt} = \left(n_t^{1-\lambda}.\frac{y_{it} + \sum_{k \neq i \in J} y_{kt}}{D_t} \right)^{-\frac{1}{\sigma}} .P_t \quad , \tag{14}$$

where $D_t = C_t+G_t$ represents aggregate demand. Since n_t is large, firms are small at the economy level so that macroeconomic variables (D_t, P_t, n_t, a_t) are seen as exogenous by the firm.

Considering there is no accumulation variable (e.g. capital stock) or lag structure, the maximisation problem is equivalent to static repeated game. If we ignore the multiple-equilibria problem arising with the Folk theorem, the solution, under an intra-industrial symmetric equilibrium, is given by the simple Cournot-Bertrand-Nash price-setting rule

$$p_{jt}.\left(1-\mu_{jt}\right)=\frac{w_t}{A_t} \ , \tag{15}$$

where μ_{jt} = $1/(\sigma.m_{jt})$ represents the Lerner index in industry j. Notice the left-hand side of equation (15) gives us the marginal revenue and the right-hand side is the marginal cost for a representative firm in industry j. Here, profits can be expressed as a function of total revenues and total fixed costs in the period

$$\Pi_{it}=\mu_{jt}.\left(p_{jt}y_{jt}\right)-w_t.\left(\Phi+H_{jt}.\varpi\right) \ , \tag{16}$$

With the entry structure described in section 2, mark-up levels may differ between new (μ^{N} = $1/\sigma$) and old [μ_t^{O} = $1/(\sigma.m_t^{\mathrm{O}})$] industries.

3.4. Macroeconomic variables

Let us define aggregate output as total value added, measure in terms of aggregate good units

$$Y_t=\sum_{j=1}^{n_t}\frac{p_{jt}}{P_t}.Y_{jt} \ , \tag{17}$$

where $Y_{jt}=\sum_{i\in J}y_{it}$ stands for total output in industry j, and in equilibrium, we have Y_t = D_t . Notice this structure is equivalent to an economy where households and government only consume a final good, produced in a competitive sector using Y_{jt} as the only (intermediate) inputs, and with a production technology identical to (4). In that case, λ would measure increasing returns to specialisation instead of love for variety.[4]

We use the aggregate good as the *numéraire* in this economy, so that $P_t = 1$, for all t. Non-wage income is given by the sum of profits of all firms $\Pi_t = \Sigma_i \Pi_{it}$.

Finally, if the labour market is in equilibrium, we have $L_t = \Sigma_i L_{it} \geq 0$, and w_t automatically adjusts to reach it.

4. Equilibrium and dynamics

4.1. The initial steady state

First, we suppose the economy is at its steady-state equilibrium in period $t = 0$ and the number of industries is given by $\overline{n}_0$, an arbitrarily large number. If there are no shocks, there is no reason for new industries to be created. Therefore, all industries are of the 'mature' type, and firms within each industry are identical. Also, in the symmetric equilibrium, and given (5) firms post a price of $p_{j0} = \overline{n}_0^{\lambda/(\sigma-1)} \geq 1$.

In the absence of barriers to entry in each industry (and considering a stationary number of varieties), profits are pressed down to zero and equation (16) determines the equilibrium number of firms per industry (m_0). Using equations (7), (9), (15), and (17), we may reduce the system to a pair of equations in C_0 and m_0:

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[4] See Devereux et al. (1996) to see the consequences of changing n_t in this alternative environment with $m_t = 1$ ($\varpi = 0$).

$$C_0=\left[A_0.\overline{n}_0^{\frac{\lambda}{\sigma-1}}.\left(1-\frac{1}{\sigma.m_0}\right)\right]^{\frac{1+\chi}{\chi}}.\left(b.C_0^{\ \theta}\right)^{-\frac{1}{\chi}}-G_0\ , \tag{18}$$

$$C_0=\left[\frac{A_0}{b}.\overline{n}_0^{\frac{\lambda}{\sigma-1}}.\left(1-\frac{1}{\sigma.m_0}\right)\right]^{\frac{1}{\theta}}.\left(\sigma.m_0^{\ 2}.\overline{n}_0.\Phi\right)^{-\frac{\chi}{\theta}}\ . \tag{19}$$

In equilibrium, both equations produce the same value for consumption. Therefore, we can generate an equilibrium function expressed in terms of the mark-up level:

$$F(\mu)\equiv h(\mu)+q(\mu)=0\ , \tag{20}$$

where

$$h(\mu)=\mathrm{K}_0.\mu^c.(1-\mu)^d\geq 0 \qquad , \qquad q(\mu)=G.\mu^2+\mathrm{K}_2.\mu-\mathrm{K}_2$$

$$\mathrm{K}_0=\left(\frac{A_0}{b}\right)^{\frac{1}{\theta}}.\left(\frac{\sigma}{\Phi}\right)^{\frac{\chi}{\theta}}.\overline{n}_0^{\frac{\lambda-\chi.(\sigma-1)}{\theta.(\sigma-1)}}>0 \qquad , \qquad \mathrm{K}_1=\left(A_0.\overline{n}_0^{\frac{\lambda}{\sigma-1}}\right).b^{-\frac{1}{\chi}}>0\ .$$

$$\mathrm{K}_2=\mathrm{K}_0^{-\frac{\theta}{\chi}}.\mathrm{K}_1>0 \qquad , \qquad c=2.\left(1+\frac{\chi}{\theta}\right)>2 \qquad , \qquad d=\frac{1}{\theta}>0$$

We can also notice that $C_0 = h(\mu)/\mu^2$ and $Y_0 = \mathrm{K}_2.(1-\mu)/\mu^2$.

Proposition 1: For a given number of products an equilibrium solution exists and it is unique.

PROOF: First, notice $F(0) = -\mathrm{K}_2 < 0$. Furthermore, $F'(\mu) = h'(\mu)+q'(\mu)$ and $q'(\mu) = 2.G.\mu+\mathrm{K}_2 > 0$. Thus, we know that

$$h'(\mu)=C_0.\frac{c-(c+d).\mu}{1-\mu}\ .$$

Considering $\mu = c/(c+d) \in (0, 1)$ is the only solution for $h'(\mu) = 0$,[5] function $h(.)$ is increasing for $\mu \in [0, c/(c+d)]$. Furthermore, $F[c/(c+d)] = h[c/(c+d)]+G.[c/(c+d)]^2+K_2.d/(c+d) > 0$. Therefore, there is one equilibrium for the model in the interval $\mu \in (0, c/(c+d)]$.

Since $q[c/(c+d)] > 0$, $F(\mu)$ is always positive for $\mu \in [c/(c+d), 1)$. Therefore no equilibrium can exist in this interval, despite the fact $h'(.) < 0$ here. Considering $F(1) = G$, $\mu = 1$ is a solution for (20) when there is no public consumption, but this is not an equilibrium.

Thus, $0 < \mu_0^* < \min\{c/(c+d), 1/\sigma\}$, such that $F(\mu_0^*) = 0$, is the unique equilibrium solution for the system, given n_0. ■

Second, if we assume the number of firms per industry is fixed and given by $\overline{m}_0$ - i.e., the mark-up is fixed and given by $\overline{\mu}_0 = 1/(\sigma.\overline{m}_0)$ -, the number of industries is determined by (18) and (19). We can also combine both equations and obtain an equation similar to (20)

$$\Lambda(n) \equiv \Sigma_0.n^f - \Sigma_2.n^g + G = 0 \ ,$$

where

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[5] We also have $\mu = 0$ (perfect competition) as a solution in the interval. However, this cannot be an equilibrium since $\Phi > 0$ implies increasing returns to scale at the firm level. Likewise, have $\mu = 1$ is also a solution, but it implies an infinite price or a zero marginal cost.

$$\Sigma_0 = \left[\frac{A_0}{b}.(1-\bar{\mu}_0).\left(\frac{\sigma.\bar{\mu}_0^{\;2}}{\Phi}\right)^{\chi}\right]^{\frac{1}{\theta}} > 0 \quad , \qquad \Sigma_1 = \left[A_0.(1-\bar{\mu}_0)\right]^{\frac{1+\chi}{\chi}}.b^{-\frac{1}{\chi}} > 0$$

$$\Sigma_2 = \Sigma_1.\Sigma_0^{-\frac{\theta}{\chi}} > 0 \quad , \qquad f = \frac{\lambda-\chi.(\sigma-1)}{\theta.(\sigma-1)} \quad , \qquad g = 1+\frac{\lambda}{\sigma-1} > 1 .$$

Notice both f and g are increasing in the level of love for variety. Furthermore, f is negative for $\lambda < \chi.(\sigma\text{-}1)$ and positive if λ is larger than this value.[6]

Proposition 2: For a given number of firms per industry an equilibrium solution exists and it is unique for all values of λ *when* $\theta \geq 1$, *and for* $\lambda < (\chi+\theta).(\sigma\text{-}1)/(1\text{-}\theta)$ *when* $\theta < 1$.

PROOF: First, if love for variety is not very high - i.e. if $\lambda < \chi.(\sigma\text{-}1)$ -, $f < 0$. Therefore, $\Lambda(0)$ is infinite for that range. Furthermore, $\Lambda'(n) = f.\Sigma_0.n^{f-1}\text{-}g.\Sigma_2.n^{g-1}$ is always negative in the interval considered, which means only one equilibrium can exist here. Considering $\lim_{n\to\infty}\Lambda(n) = -\infty$, an equilibrium solution, $n_0^{\;*}>0$, exists.

If $\lambda = \chi.(\sigma\text{-}1)$, i.e. $f = 0$, $\Lambda(0) = \Sigma_0+G > 0$, and considering the equilibrium function tends to minus infinity when n is infinite, we also have a unique equilibrium solution.

For a large taste for variety, we may have $0 < f \leq 1$. In this case, $\Lambda(0) = G \geq 0$. Furthermore, notice that $\Lambda'(0) > 0$.[7] We also have a unique solution for $\Lambda'(n) = 0$,

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[6] Considering most empirical studies point to a small elasticity of intertemporal substitution in labour supply, we have $\chi > 1$. Moreover, for plausible values of σ (e.g., for $\sigma > 2$), the level of love for variety implied in Dixit and Stiglitz (1977) would not be high enough to generate $f > 0$. Notice $f > 0$ would mean the individual profit function is increasing in n.

given by $n_\Lambda = [g.\Sigma_2/(f.\Sigma_0)]^{1/(f-g)} > 0$ and $\Lambda(n)$ tends to $-\infty$ when n tends to infinity. Thus, a unique equilibrium exists: $n_0^* > 0$ such that $\Lambda(n_0^*) = 0$.

Considering now $1 < f < g$, we have $\Lambda'(0) = 0$ (in fact, this is valid for all $f > 1$). Additionally, notice that $\Lambda(n_\Lambda) = (g-f).\Sigma_0.n_\Lambda^f/g+G$, and this value is larger than G in this interval. Therefore, $\Lambda(n_\Lambda)$ is still the maximum value for $\Lambda(n)$ and this function still tends $-\infty$ when n tends to infinity (for finite values of Σ_2). Thus, a unique equilibrium exists in the interval: $n_0^* \in (n_\Lambda, +\infty)$.

For an even larger love for variety we may have $f = g$, i.e. $\lambda = (\chi+\theta).(\sigma-1)/(1-\theta)$. Notice this can only happen for a large elasticity of marginal utility of consumption, i.e. for $\theta < 1$. Here, a unique equilibrium exists and it is given by $n_0^* = [G/(\Sigma_2-\Sigma_0)]^{1/g}$, provided that $\Sigma_2 > \Sigma_0$.

Finally, we may have a very large taste for variety such that $f > g$, i.e. $\lambda > (\chi+\theta).(\sigma-1)/(1-\theta)$ (again, only possible with $\theta < 1$). In this case, $\Lambda(n_\Lambda)$ is a minimum of $\Lambda(.)$ and we have $\lim_{n\to\infty}\Lambda(n) = +\infty$. If $G > (f-g).\Sigma_0.n_\Lambda^f/g > 0$, an equilibrium does not exist in this interval. For $G = (f-g).\Sigma_0.n_\Lambda^f/g$, a unique equilibrium exists and it is given by $n_0^* = n_\Lambda$. However, if $G < (f-g).\Sigma_0.n_\Lambda^f/g$, $\Lambda(n_\Lambda)$ is negative and there is a pair of solutions to $\Lambda(n_\Lambda) = 0$: one to the left of n_Λ and another one on its right.

■

[7] The value of the derivative is zero for $f > 1$ and it is $\Sigma_0 > 0$ for $f = 1$.

ASSUMPTION 2: *We suppose* $\lambda \in [0, \min\{1, \chi.(\sigma-1)\})$*, i.e. increasing returns are sufficiently low to generate a single equilibrium with an individual profit function decreasing in n (f < 0).*

Of course both n and μ should be endogenously determined in the model. However, as noticed in Costa (2004), the same zero-profit condition cannot be used to obtain both values. Notice the equilibrium function in (20) can be written as

$$X(\mu, n) = 0, \tag{21}$$

where $F(\mu) = X(\mu, \bar{n}_0)$ and $\Lambda(n) = X(\bar{\mu}_0, n)$. Therefore, for $\mu < c/(c+d)$ this function is increasing in μ, and for $\lambda < (\chi+\theta).(\sigma-1)/(1-\theta)$ when $\theta < 1$, it is decreasing in n in the range where the equilibrium occurs. Thus, Figure 1 represents the multiplicity of equilibria that corresponds to $X(\mu,n) = 0$ as an increasing schedule in the (n,μ) space, or alternatively a decreasing schedule in the (n,m) space.

However, the equilibrium cannot be sustained unless there is no incentive for new industries to be created, i.e. if the additional cost of creating a new industry is sufficiently high

$$\begin{gathered}\frac{1}{\sigma}.\frac{Y_0^*}{n_0^{*1-\lambda}}.\left[\frac{A_0.\left(1-\mu^{\mathrm{N}}\right)}{w_0^*}\right]^{\sigma-1} - w_0^*.(\Phi+\varpi) \leq \mu_0^{*2}.\sigma.\frac{Y_0^*}{n_0^{*1-\lambda}}.\left[\frac{A_0.\left(1-\mu_0^*\right)}{w_0^*}\right]^{\sigma-1} - w_0^*.\Phi \Leftrightarrow \\ \Leftrightarrow \phi \equiv \frac{\varpi}{\Phi} \geq \Upsilon\left(\mu_0^*\right) - 1 \equiv \frac{1}{\sigma^2.\mu_0^{*2}}.\left(\frac{1-\mu^{\mathrm{N}}}{1-\mu_0^*}\right)^{\sigma-1} - 1 \geq 0\end{gathered}. \tag{22}$$

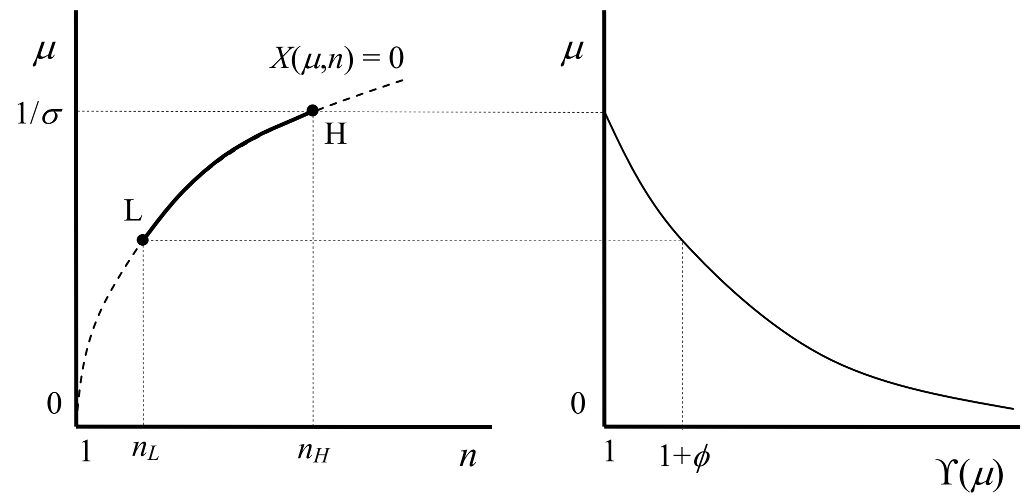

FIGURE 1 – BOUNDARIES TO MARK-UPS AND NUMBER OF INDUSTRIES

Since $\Upsilon(\mu)$ is decreasing with the mark-up level, the relative size of the sunk cost determines a lower bound for the number of industries (n_L). If $\phi = 0$ (i.e. if $\varpi = 0$), it is not possible for a mature industry with $\mu_0^* < \mu^N = 1/\sigma$ to subsist, as creating a new industry is always preferable to keeping this mark-up level. In this case $\Upsilon(1/\sigma) = 1$ and the number of industries has to be given by $n^* = n_L = n_H$. This is the traditional monopolistically competitive solution.

If $\phi > 0$ there is a range of values for $n^* \in [n_L,n_H]$ that are compatible with more than one firm per industry, and this solution is sustainable, as the sunk cost is sufficiently high to preventing new industries from blossoming.

For sake of simplicity, we assume profits of creating a new industry are exactly zero in the initial steady state, i.e. the mark-up level is simply given by $\mu_0^* = \Upsilon^{-1}(1+\phi)$.

In this case, the equilibrium number of industries corresponds to point L (n_L) in Figure 1, i.e. it is the solution to $X[\Upsilon^{-1}(1+\phi),n_0^*] = 0$.

4.2. Entry

Since we assumed barriers to entry in an existing industry disappear completely in the period after the industry creation, profits will be pressed to zero in the future. Thus, the zero-profit condition for new industries holds in the short-run. Thus substituting (14) and (15) in (16), we obtain the 'optimal' number of varieties

$$N_t = \left\{ \frac{Y_t.\left[\left(1-\mu^N\right).A_t\right]^{\sigma-1}}{\sigma.\left(\Phi + u_t.\varpi\right).w_t^{\sigma}} \right\}^{\frac{1}{1-\lambda}}, \qquad u_t = \begin{cases} 1 & \Leftarrow N_t > n_{t-1} \\ 0 & \Leftarrow N_t < n_{t-1} \end{cases}. \tag{23}$$

Notice that if $\lambda = 1$ (the Dixit-Stiglitz case), this equation is not able to determine N, as the sharing and the love-for-variety effects exactly cancel each other and profits in new industries do not depend on n_t. For $\lambda > 1$ the preference for variety is so strong that profits depend positively on the number of industries.

Therefore, profits in new industries are smaller when the number of industries increases. Notice this is not a sufficient condition for the existence of an equilibrium with a fixed m, as we may have $(\sigma\text{-}1).(\chi+\theta)/(1\text{-}\theta) < \lambda < 1$. Furthermore, if $\lambda > 1$ (and ϖ is small), we would observe counter-cyclical business creation (of new industries) which is not observed in real economies.

Furthermore, the effect of the sunk cost is asymmetric, since the economic rational for destroying an existing industry cannot be the same for creating a new one. However, as we considered equation (2), the number of industries (n_t) is only given by (23) when $u_t = 1$. Figure 2 represents the equilibrium number of industries in period t given $Q_t \equiv Y_t.[(1-\mu^N).A_t]^{\sigma-1}/(\sigma.\Phi.w_t^{\sigma})$: [8]

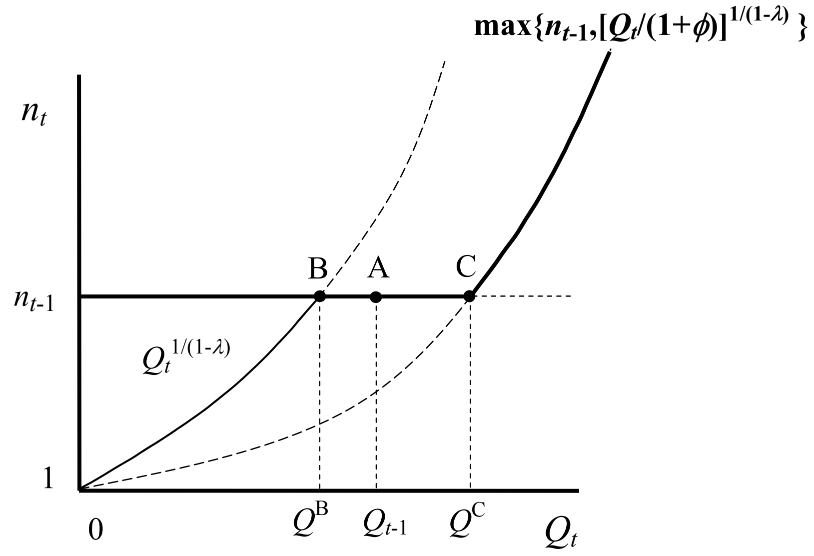

FIGURE 2 – THE EQUILIBRIUM NUMBER OF INDUSTRIES

Assume the values of Q (a business-cycle position indicator) and n at the period t-1 are represented by point A. For $Q^B < Q_t < Q^C$, even ignoring the irreversibility in product innovation, there is no incentive for new industries to be created or destroyed, since the sunk cost introduces a discontinuity in the function representing the 'optimal' number of industries. For $Q_t < Q^B$, the 'optimal' number of firms would be smaller than n_{t-1}, but irreversibility prevents it. For $Q_t > Q^C$, the sunk cost is not enough to detain product creation. Thus, $Q_t \in [0, Q^C)$ can be seen as a band of inertia.

[8] Notice $N_t = [Q_t/(1+\phi)]^{1/(1-\lambda)}$.

4.3. The aggregate production function

Considering entry eliminates pure profits at every moment in time (for two distinct reasons) and since there is no investment, aggregate output has to be equal to labour income. Furthermore, we can use the aggregate output definition in (17) – or its CES reduced-form formulation in alternative – and substitute outputs using (14) and (15). If the product market is in equilibrium ($D_t = Y_t$), we find the wage rate required by labour demand, and using it in the aggregate budget constraint we obtain a reduced-form aggregate production function given by

$$Y_t = A_t.\Gamma_t.L_t\,,^{9} \tag{24}$$

where Γ_t is an efficiency index represented by

$$\Gamma_t \equiv \Gamma\left(n_t, n_{t-1}, \mu_t^O, \lambda\right) \equiv n_t^{-\frac{1-\lambda}{\sigma-1}}.\left[n_{t-1}.\left(1-\mu_t^{\mathrm{O}}\right)^{\sigma-1} + \Delta n_t.\left(1-\mu^{\mathrm{N}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} > 0\ .$$

We can notice its partial derivatives are given by

---

[9] Notice that $w_t = A_t.\Gamma_t$.

$$R_t = \left(\frac{1-\mu^{\mathrm{N}}}{1-\mu_t^{\mathrm{O}}}\right)^{\sigma-1} \in (0,1]$$

$$\frac{\partial \Gamma_t}{\partial n_t} = \frac{\Gamma_t}{n_t.(1-\sigma)}.\left[(\lambda-1)+\frac{n_t.R_t}{n_{t-1}+\Delta n_t.R_t}\right]$$

$$\frac{\partial \Gamma_t}{\partial n_{t-1}} = \frac{\Gamma_t^{\,2-\sigma}.n_t^{\,\lambda-1}.\left(1-\mu_t^{\mathrm{O}}\right)^{\sigma-1}}{\sigma-1}.\left(1-R_t\right) \geq 0\ .^{10}$$

$$\frac{\partial \Gamma_t}{\partial \mu_t^{O}} = -n_{t-1}.n_t^{\,\lambda-1}.\left(\frac{1-\mu_t^{\mathrm{O}}}{\Gamma_t}\right)^{\sigma-2} < 0$$

$$\frac{\partial \Gamma_t}{\partial \lambda} = \frac{\Gamma_t.\ln n_t}{\sigma-1} > 0$$

Note $\partial\Gamma_t/\partial n_t$ is positive if and only if $\lambda > [\Delta n_t+(1-R_t).n_{t-1}]/(n_t+\Delta n_t.R_t)$. If we evaluate the term on the right-hand side at the initial steady-state equilibrium we obtain $\lambda > 1-R_0^*$ as an equivalent condition. Therefore, if there is no love for variety, this efficiency index is decreasing with n_t, but it is non-decreasing with n_{t-1}.

Let us analyse Γ_t with some particular cases:

- In a Walrasian model all the firms face an infinitely elastic demand function, i.e. $\sigma\to\infty$, and there can be no equilibrium with increasing returns to scale, i.e. $\Phi = \varpi = 0$ and $\lambda = 0$. In this case both new- and mature-industries mark-ups are zero and $\Gamma_t = 1$, i.e. $Y_t = A_t.L_t$.

- If monopolistic competition always holds ($\varpi = 0$) and there is no love for variety ($\lambda = 0$), the mark-up is always fixed and equal to μ^{N}. Here, Γ_t = 1-

[10] Note $\Upsilon(\mu_t^{\mathrm{O}}) = R_t/(\sigma.\mu_t^{\mathrm{O}})^2$.

μ^N, i.e. the model works like a constrained Walrasian model. See Dixon and Lawler (1996) and Startz (1989), *inter alia*, for models in this class.

- If the number of industries is fixed, entry leads to more intra-industry competition ($\phi \to \infty$), and if there is no taste for variety, all industries are mature and the mark-up is equal to μ_t^O. In this case, $\Gamma_t = 1-\mu_t^O$ fluctuates in the opposite direction of the endogenous mark-up. See *inter alia* Costa (2004) and Portier (1995).

- If monopolistic competition always holds, but there is love for variety ($\lambda >$ 0), we have $\Gamma_t = n_t^{\lambda/(\sigma-1)}.(1-\mu^N)$, i.e. there are increasing returns due to entry. This is the case in Devereux et al. (1996) and Heijdra (1998), amongst others.

4.4. Stability and dynamics

To study stability in this model we log-linearise the dynamic system about its initial steady-state equilibrium, reduce it to a one-dimensional system in n, and compute the dynamic eigenvalue.

Using (23) we obtain a log-linearised version for the 'optimal' number of industries:

$$\hat{N}_t = \frac{1}{1-\lambda}.\left[\hat{Y}_t + (\sigma - 1).\hat{A}_t - \sigma.\hat{w}_t\right] , \qquad (25)$$

where $\hat{X}_t \equiv dX_t / X_0^*$ represents the proportional deviation of variable X from its initial steady-state value. Notice the number of industries is pro-cyclical (relative to aggregate output) as we assumed that $\lambda < 1$, and it would be counter-cyclical if we considered the possibility of having $\lambda > 1$. This happens because for $\lambda > 1$ the love for variety is so strong that it offsets the sharing effect and profits would depend positively on the number of varieties.

Equation (9) gives rise to

$$\hat{L}_t = \frac{1}{\chi}.\hat{w}_t - \frac{\theta}{\chi}.\hat{C}_t \; . \tag{26}$$

The equilibrium macroeconomic condition in product markets implies that

$$\hat{Y}_t = s^*.\hat{C}_t + (1-s^*).\hat{G}_t \; , \tag{27}$$

where $s^* \equiv C_0^*/Y_0^*$ the private consumption share in aggregate demand in the initial steady state.

Considering firms are free to enter or leave mature industries (provided there is at least one), the mark-up level in this type of industries evolves according to

$$\hat{\mu}_t^{\mathrm{O}} = -\frac{1}{k^*}.\left[\hat{Q}_t - (1-\lambda).\hat{n}_t\right] \; , \tag{28}$$

where $k_t \equiv [2/(1+\sigma) - \mu_t^{\mathrm{O}}].(1+\sigma)/(1-\mu_t^{\mathrm{O}}) > 0$, as $\mu_t^{\mathrm{O}} < 1/\sigma < 2/(1+\sigma)$. Notice steady-state profits in mature industries react positively to changes in the mark-up level, as $\partial\Pi_t^{\mathrm{O}}/\partial\mu_t^{\mathrm{O}} = k_t.(w_t+\Phi)/\mu_t^{\mathrm{O}} > 0$. Furthermore, changes in the business-cycle index (Q) can be expressed as

$$\hat{Q}_t = \hat{Y}_t + (\sigma-1).\hat{A}_t - \sigma.\hat{w}_t \; . \tag{29}$$

Then, equation (24) gives rise to

$$\hat{Y}_t = \hat{w}_t + \hat{L}_t \ , \tag{30}$$

with the following equilibrium real wage given by labour demand

$$\hat{w}_t = \hat{A}_t + \hat{\Gamma}_t \ , \tag{31}$$

and the consequent definition for the efficiency index

$$\begin{aligned} &\hat{\Gamma}_t = -\left(\gamma_1 - \frac{\lambda}{\sigma-1}\right).\hat{n}_t + \gamma_1.\hat{n}_{t-1} - \gamma_2.\hat{\mu}_t^{\mathrm{O}} \\ &\gamma_1 = \frac{1-R_0^*}{\sigma-1} \geq 0 \qquad , \qquad \gamma_2 = \frac{\mu_0^*}{1-\mu_0^*} > 0 \end{aligned} \ . \tag{32}$$

Finally, we log-linearise equation (2) and reduce the system to a single dynamic equation, that can be used to analyse its stability:

$$\hat{n}_t = \max\left\{\varepsilon.\hat{n}_{t-1} + \eta_G.\hat{G}_t + \eta_A.\hat{A}_t, \hat{n}_{t-1}\right\} \ , \tag{33}$$

where

$$\begin{aligned} &\varepsilon = -\frac{\left(1-R_0^*\right).a^*}{R_0^*.a^* - \left(\theta - s^*\right).\left(1-\lambda\right)} \quad , \qquad a^* = s^*.\left[\chi.\left(\sigma-1\right)-1\right] + \sigma.\theta \\ &\eta_G = -\frac{\theta.\left(1-s^*\right)}{\gamma_1.a^*}.\varepsilon \qquad , \qquad \eta_A = \frac{\theta - s^*}{\gamma_1.a^*}.\varepsilon \end{aligned} \ .$$

We assume the time series for fiscal or productivity shocks are given by

$$\hat{G}_{t+1} = \alpha_G.\hat{G}_t \quad , \quad \hat{A}_{t+1} = \alpha_A.\hat{A}_t \quad , \quad \alpha_G, \alpha_A \in [0,1] \quad , \quad \forall t \geq 1 \ . \tag{34}$$

If a temporary negative shock hits the economy (a decrease in Q[11]), the number of firms stays at its previous level, i.e. $\hat{n}_t = \hat{n}_{t-1}$.

If a temporary positive shock hits the economy (an increase in Q), the number of firms changes, i.e. $\hat{n}_t = \hat{N}_t$.[12] Here, we can face four cases:

- If $\varepsilon > 1$, the dynamics of the number of industries is unstable and there is nothing that can prevent n of moving towards infinity after a positive shock in Q.[13]

- If $\varepsilon < -1$, the model would be unstable if irreversibility did not prevent the number of new industries from decreasing. Thus, when a positive shock hits the economy we have an increase in the number of industries in period $t = 1$, and it stays at that level afterwards.

- If $-1 \leq \varepsilon < 0$, results are similar to the previous case.

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[11] Notice an increase in productivity (A) may have a negative effect on Q due to its positive impact in real wages.

[12] Assuming $\mu_0^* = \Upsilon^{-1}(1+\phi)$, means the economy is, in $t = 1$, at point A = C in Figure 2 . Thus, there is no inertia band at the right of it.

[13] This source of instability may disappear in a model with capital accumulation or in an open-economy framework, as the transversality condition works by keeping the discounted utility finite.

- Finally, if $0 \leq \varepsilon \leq 1$, the number of industries may increase for a while, depending on the persistency of the shock, but it eventually stops at its new steady-state value.

Thus, the stability features of the model depend solely on ε. Unfortunately, it is a very complicated function of the fundamental parameters and it is not possible to obtain an unambiguous sign or range for values. There are three reasons for this ambiguity:

i) The effect of a wage increase on the 'optimal' number of industries is unknown and its sign is the opposite of the sign of a^*. The direct effect of wages on Q is negative and so is its indirect effect when it increases consumption, decreasing employment. However, its income effect on output is positive.

ii) The effect of the number of industries on wages (via efficiency index) depends on love for variety.

iii) The absolute value of both previous effects combined is also important.

5. Describing the Business Cycle

5.1. The average mark-up

First, if the path of the shocks (temporary or permanent) is known at $t = 1$ (the short run), then we know the path for n_t. Second, using (28) and (33) we can observe that changes in the mark-up level of mature firms are given by

$$\hat{\mu}_t^{\mathrm{O}} = \begin{cases} 0 & \Leftarrow \hat{Q}_t > (1-\lambda).\hat{n}_{t-1} \\ -\dfrac{\hat{Q}_t - (1-\lambda).\hat{n}_{t-1}}{k^*} & \Leftarrow \hat{Q}_t < (1-\lambda).\hat{n}_{t-1} \end{cases} . \tag{35}$$

This equation can be interpreted as follow: if there is a positive shock in period $t = 1$ new products will be created (remember we assumed the profit of a new firm was exactly zero in the initial steady state) up to the number that erodes all the entry incentive. In this case, the mark-up level of mature firms remains unchanged, but the average mark-up in the economy increases in the short-run due to the entry of new monopolists.[14] However, if the shock is negative irreversibility prevents the number of products from decreasing and the mark-up in mature industries increases.

Thus, there is not a clear cyclical pattern for the average mark-up in the economy, since it is pro-cyclical for positive and counter-cyclical for negative shocks. In a real economy permanently hit by several shocks, the inertia band may be active for positive shocks and the time-series for average mark-ups shows a moderately counter-cyclical pattern consistent with the findings in the empirical literature.[15]

5.2. A contractionary increase in productivity?

Let us assume there is an increase in A in period $t = 1$ and government consumption stays at its steady-state level. Using equations (26), (27), (30), and (31), we can notice the immediate impact on aggregate output is given by

[14] We can define the average mark-up as $\mu^{\mathrm{A}} = 1\text{-}n_t^{-\lambda/(\sigma-1)}.\Gamma_t$.

[15] See Martins and Scarpetta (1999), *inter alia*.

$$\hat{Y}_1 = \frac{1+\chi}{\theta+\chi}.\left(\hat{A}_1 + \hat{\Gamma}_1\right) \ .$$

If the efficiency index were constant, output would increase. This is what happens in fixed-mark-up models. However, there is the clear possibility of a decrease in Γ following the productivity shock, when the number of firms goes up, i.e., when $\hat{Q}_1 > 0$.

Proposition 3: Assuming θ > 1 and λ = 0, a productivity increase implies an output reduction if the set of parameter values generates an initial mark-up level in the following region:

$$0 < \mu_0^* < 1 - \Xi(\sigma, a) < \frac{1}{\sigma} < 1$$

$$\text{with } \Xi(\sigma, a) \equiv \frac{\sigma-1}{\sigma}.\left(\frac{\sigma-a}{1-a}\right)^{\frac{1}{\sigma-1}} \ , \ a = \frac{1}{1+\dfrac{\theta - s^*}{s^* + s^*.\chi}} \in (0,1) \ .$$

PROOF: See Appendix.

Thus, for a small elasticity of marginal utility of consumption ($\theta > 1$) and ignoring increasing returns ($\lambda = 0$), a productivity increase may decrease the average efficiency level in the economy and the aggregate output, provided the elasticity of substitution between varieties is not too large (it is necessary that $\sigma < 2$).

5.3. The permanent effects of temporary shocks

Now, let us assume the economy is hit by a positive shock (i.e. $\hat{Q}_1 > 0$) in $t = 1$, but its persistency is given by $0 \leq \alpha_i < 1$, with $i = A, G$. This change can be due to either a positive fiscal shock or to a productivity shock. Notice the latter can be either positive or negative since it also affects the real wage that negatively affects Q. Also, let us ignore love for variety for a moment ($\lambda = 0$).

In this case, the number of industries increases in the short run. Due to irreversibility of product innovation, there is an excess of firms in old industries and a shortage in new ones, when the shock fades away. Given the positive relationship between mark-ups and the number of industries, depicted in right-hand panel of Figure 1, the new steady state will exhibit a higher mark-up level. This is a consequence of net business destruction in the long run.

Here, a positive temporary shock has a permanent negative effect on the overall level of efficiency in the economy since it increases the average mark-up once and for all. If we allow for love for variety ($\lambda > 0$), this negative effect may be partially or totally offset by increasing returns arising from growing varieties.

To illustrate this claim, let us use a numerical simulation and compare the outcomes of the model with both a fixed-mark-up and reversible innovation model ($\phi = 0$) and a fixed number of industries model ($\phi \to \infty$). Notice we can choose the appropriate parameter values in order to generate the same initial steady state, namely using $\sigma' = 1/\mu_0^*$ instead of σ in the latter. The parameter values chosen are presented in Table I:

TABLE I

NUMERICAL VALUES FOR THE PARAMETERS IN THE BENCHMARK MODEL

θ	χ	λ	A_0	σ	n_0	B	Φ	G_0
1	1	0	1	2	1000	45/4	1/54000	1/18

The values for θ and χ imply unit elasticities of marginal utility of consumption and labour supply, $\lambda = 0$ eliminates increasing returns in n, n_0 is a large number, $A_0 = 1$ is a normalization, and the rest of the values chosen imply that $\mu_0^* = 1/6$, $L_0^* = 1/3$ and $G_0/Y_0^* = 1/5$. In this case we obtain $\varepsilon = -0.8$, i.e. the model does not converge to the initial steady state after a temporary positive shock in Q. If we simulate a one per cent positive shock in government consumption in $t = 1$ with $\alpha_G = 0.85$, we obtain the following impulse-response pictures:

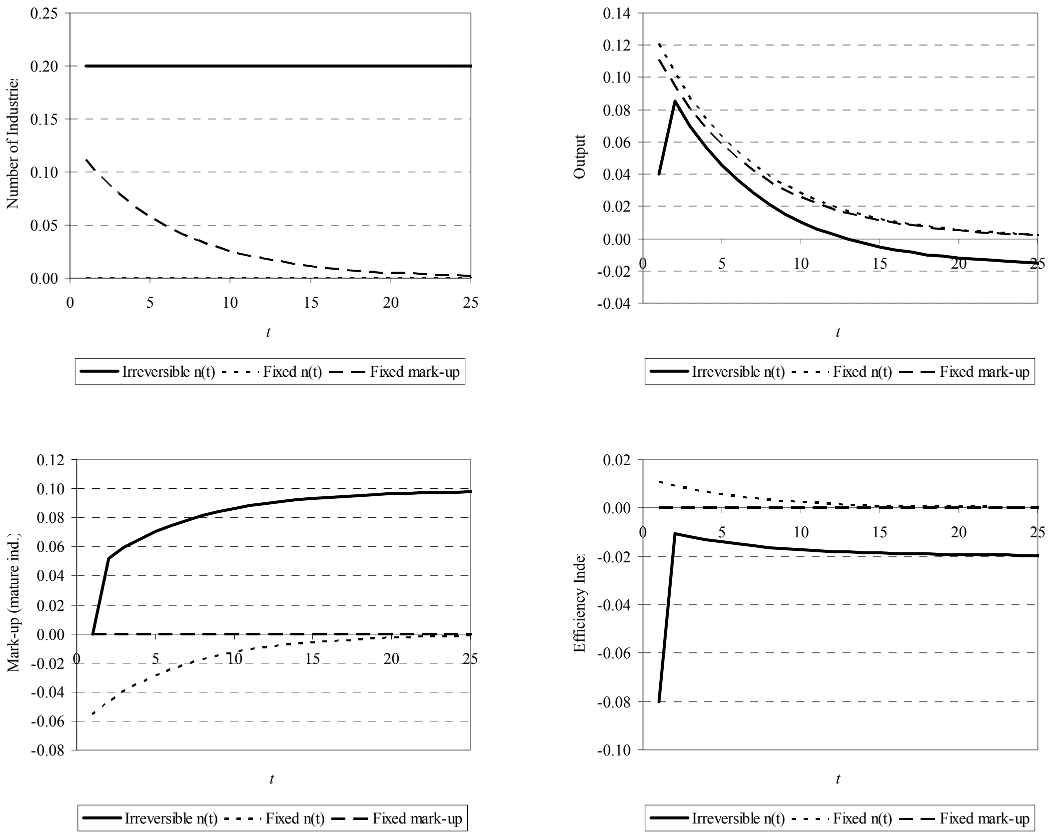

FIGURE 3 – A TEMPORARY FISCAL SHOCK

The negative externality caused by the increase in the number of products when there is no love for variety depresses the efficiency index and thus output permanently. The effect on output is smaller than in the other two models even during the transition, especially due to the big increase in the average mark-up in the short run.

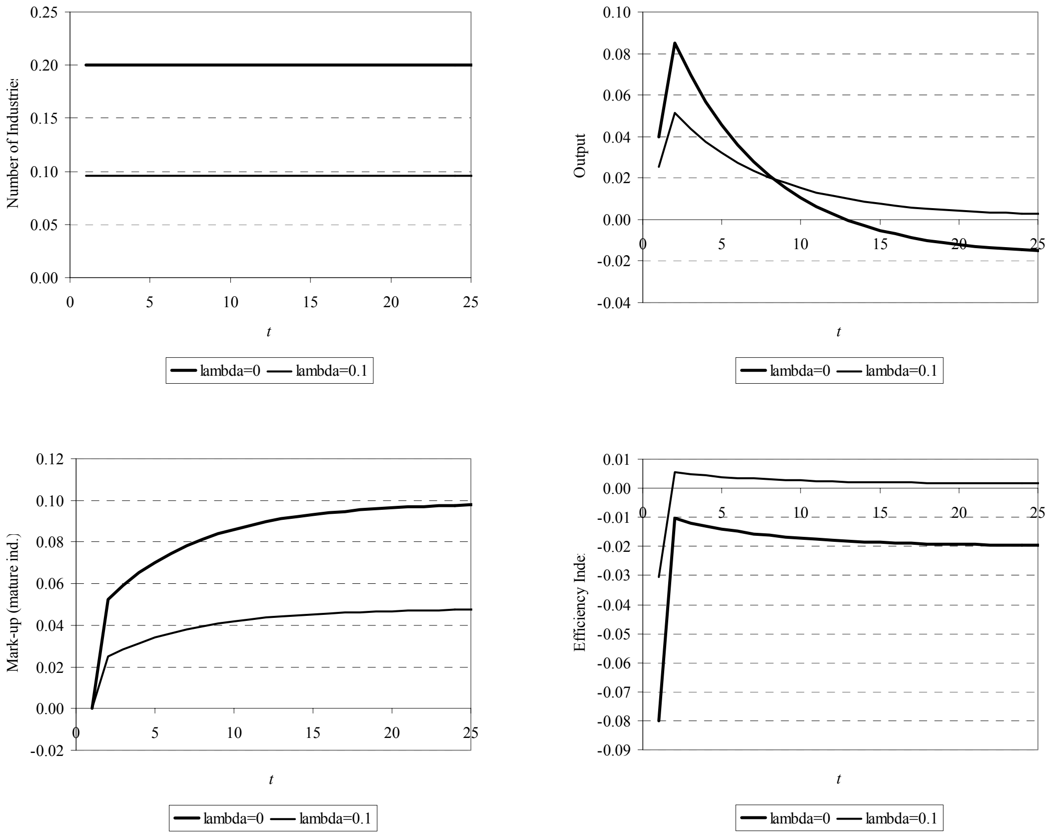

FIGURE 4 – INCREASING RETURNS IN THE NUMBER OF INDUSTRIES

Allowing for a modest level of increasing returns in n ($\lambda = 0.1$) reduces the negative impact of irreversibility, as expected. Figure 4 shows the effects of the same fiscal-policy experiment in four key variables.[16] In this case, the efficiency effect of the positive shock is sufficiently high to produce a higher efficiency index in the long run, despite its under-shooting and the permanent increase in the average mark-up.

[16] Of course the initial steady-states do not produce the same values.

6. Conclusions

In this paper we have developed a dynamic general equilibrium model of imperfect competition where a sunk cost of creating a new product regulates the type of entry that dominates in the economy: new products or more competition in existing industries. We assume the process of product innovation is irreversible, which introduces hysteresis in the business cycle.

The model exhibits some interesting Schumpeterian business-cycle features: i) the economy starts from a business-as-usual steady state where firms compete over existing products and abnormal profits are depressed to zero; ii) when significant profit opportunities are matched by product innovation, considerable market power is gained by the innovator, for a short period of time; and iii) when other firms are able to freely produce the new product, we have a new steady state that corresponds to a new 'prosperity plateau.'

Here, permanent positive technology shocks can have a negative effect on the overall efficiency level of the economy that is absent in most models, as it leads to a permanent increase in the average mark-up, due to irreversible product innovation. Temporary shocks, either in technology or in demand, may also have permanent effects on the efficiency level, especially when the level of increasing returns in the number of products is small.

Appendix

First, if we assume the number of industries increases ($\hat{n}_1 > 0$), we are assuming the business-cycle index increases ($\hat{Q}_1 > 0$). Therefore, considering the ratio between the change in Y and Q is given by

$$\left.\frac{\hat{Y}_1}{\hat{Q}_1}\right|_{\substack{\hat{A}_1 \neq 0 \\ \hat{n}_1 > 0 \\ \lambda = 0}} = -\frac{(1+\chi).s^*.R_0^*}{\theta - s^*} ,$$

and this value is negative for $\theta > 1 > s^*$. Thus output and the business-cycle index go in opposite directions after a productivity shock. Now, Q is affected by productivity in the following way

$$\left.\frac{\hat{Q}_1}{\hat{A}_1}\right|_{\substack{\hat{n}_1 > 0 \\ \lambda = 0}} = -\frac{\theta - s^*}{x} ,$$

where $x = (\theta + s^*.\chi).(1 - \sigma.\gamma_1) + s^*.(1 + \chi).\gamma_1$. Thus, we observe an increase in Q following a productivity increase if $x < 0$. This condition is equivalent to $\sigma < (1 - a)/R_0^* + a$, or to $\mu_0^* < 1 - \Xi(\sigma, a)$. The function $\Xi(.)$ is strictly increasing in a, the function is increasing in σ when its values are close to unity and it approaches unity when σ tends to infinity. Also, it is not difficult to observe that, given $a = a_0$, there is only one solution for $\Xi(\sigma, a_0) = 1$, i.e. the value of σ, and it is $\sigma^*(a_0)$. Thus, this condition imposes a constraint on the range of possible values for σ, and the maximum value that is compatible with $\mu_0^* \in [0, 1)$ is given by $\sigma^*(0) = 2$. When a increases, $\sigma^*(.)$ decreases and so the set of parameter values capable of generating a recession after a productivity increase is reduced, but it is not empty, while a is smaller than unity. ■

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