Unemployment, Job Flows and Hours in a New Keynesian Model.*

RICHARD W. P. HOLT
Economics
University of Edinburgh.

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Abstract

New Keynesian models attempt to account for economic fluctuations under nominal rigidities without modelling unemployment. They struggle to generate observed output and inflation persistence. To address these issues, recent research embeds labour search with matching frictions in a New Keynesian framework. Models with labour market search, matching and endogenous job destruction, feature unemployment, but generate an upward sloping Beveridge curve and overly volatile gross job flows. By introducing a second margin, hours, in the adjustment of labour input I obtain a negative unemployment-vacancy correlation and plausible gross job flow volatilities without affecting the desirable persistence properties of the model. I show that these results are affected by real wage rigidity, endogenous job destruction and capital adjustment costs.

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1 Introduction.

Unemployment is a fundamental concern for individuals and policy makers. Yet while unemployment and labour market dynamics ought to be a key element in any account of economic fluctuations, they are notable by their absence in the bulk of the recent New Keynesian (NK) literature. NK models tend instead to assume a Walrasian labour market with frictionless variation of hours worked (the intensive margin), Gali (2003) and Woodford (2003). So, while NK models conform to the strictures of dynamic general equilibrium modelling, they omit the very reason for a 'Keynesian' approach. Another ongoing challenge for the NK literature is the identification of mechanisms which can amplify and propagate impulses so as to match the observed behaviour of macroeconomic aggregates. This is a fundamental precursor to meaningful and coherent policy analysis. Early NK models have difficulty accounting for the persistent nature of output and inflation, Chari, Kehoe and McGrattan (2000). To address these issues, recent research embeds labour search with matching frictions in a New Keynesian framework, Krause and Lubik (2006), Trigari (2005), Walsh (2005). In one sense this is simply a sub-branch of a more general literature that attempts to combine real various real rigidities with the nominal rigidities that characterise the NK approach to address persistence issues. Yet while changes to amplification and propagation mechanisms have been the primary focus of the New Keynesian models with search (NKS), the scope to address the cyclical behaviour of labour market variables has not been fully explored.

The equilibrium labour market search framework, with matching frictions, expounded in Pissarides (2000), provides a natural framework for thinking about the cyclical properties of unemployment and other labour market variables. Mortensen and Pissarides (1994), extend the baseline labour market equilibrium model to endogenise job destruction in order to account for the evidence on the cyclical properties of gross job flows described by Davis, Haltiwanger and Schuh (1996). Using JOLTS data for the period from 2001, Shimer (2005) finds that the cyclical variation of unemployment arises primarily from outflows (job creation) rather than inflows (job destruction and quits). Shimer (2005) argues both that under reasonable calibration the baseline equilibrium labour market search model with matching frictions can produce only 10% of the cyclical variability of unemployment and vacancies and also that shocks to job destruction produce a positive correlation between unemployment and vacancies - a positively sloped Beveridge curve!

The results of Mortensen and Pissarides (1994) and Shimer (2005) are obtained in equilibrium labour market models. From a macroeconomic perspective, it is important to know whether the results are robust in a fully specified dynamic general equilibrium context. In particular this allows for the impact of consumption smoothing and interest rate variation that are absent in the stand alone equilibrium labour market setting. Den Haan, Ramey and Watson (1999) develop a real DGE a model with endogenous job destruction. Trigari (2005), Walsh (2005) and Krause and Lubik (2006) extend to monetary environments with nominal rigidities. The last of these, with only labour as a factor of production and only an extensive margin for variation of labour input, finds evidence to confirm Shimer's conjectures on the volatility of unemployment and vacancies and the sign of the Beveridge curve, while simultaneously demonstrating the irrelevance of real wage rigidity for inflation persistence in an NKS framework. The other papers focus predominantly on persistence issues and on gross job flows rather than the issues raised by Shimer (2005).

The conclusion from both the stand-alone equilibrium labour market search and the dynamic general equilibrium literatures is that one should focus on a labour market search envrionment thath features rigid wages, exogenous job destruction and omits variation in hours. However, Davis, Faberman and Haltiwanger (2006) show that job destruction tends to be dominant in severe recessions - such as those that characterise the 1970s-1990s - prior to the data used in Shimer's work. This casts doubt on the validity of omitting job destruction. Omitting hours from equilibrium labour market search models may be attractive on grounds of parsimony, but hours account for half of the variation in labour input at business cycle frequencies. However, inclusion of the intensive margin may tend to dampen attenuate fluctuation on the extensive margin. This would appear to worsen the unemployment vacancy volatility puzzle. But I show below that in an NKS framework the variability of vacancies and unemployment drops only to 50% of that in the data, rather than to 10% as found by Shimer. I also show that by dampening movements in job destruction, the introduction of hours makes both the correlations between unemployment and vacancies and between job creation and job destruction negative. In a further contrast to the Krause and Lubik (2006), I show that provided wages don't exhibit rigidity and hours are sufficiently elastic hours and employment exhibit a positive and realistic correlation. Trigari (2005) allows variation in hours but focusses on issues relating to inflation persistence. Below I extend her model and use it to address the cyclical behaviour of labour market variables.

The model is outlined in Section 2. Calibration and solution method are discussed in Section 3. Section 4 presents impulse responses to monetary and productivity shocks, stochastic simulation and sensitivity analyses to illustrate the mechanisms at work in the model and assess the contribution of various features in accounting for US business cycles facts. Section 5 concludes.

2 Model

I compare the behaviour of an NK model with search and matching frictions (hereafter *NKS*) with a standard NK model featuring Walrasian labour markets and frictionless adjustment on the intensive labour margin (hereafter *NK*). My *NK* model is a simplified version of CEE (2005).[1]

There are 4 types of agent in the *NKS* economy: intermediate good producers, final goods producers, households and a government. The key differences between the *NKS* and *NK* economies arise in intermediate good production and in the labour market. In *NKS* Production of the intermediate good occurs in matches: single firm - single worker pairs. Labour input can be varied on both extensive and intensive margins. Frictions in the formation of new matches are captured by an aggregate matching function - where the probability of a firm filling a vacancy, and the probability of an unemployed worker finding a job depend on the relative numbers of these two types. The number of vacancies is determined by a free entry condition - which drives the expected value of opening a new vacancy to zero. The flow into unemployment arises through destruction of existing matches. Matches are subject to idiosyncratic productivity disturbances. Both parties in a match with a low productivity realisation may agree to terminate the employment relationship. Upon termination of their relationship, the firm and worker enter the respective matching pools. The job creation and destruction processes are the mechanisms underlying changes

[1] In particular, our model omits three features that CEE consider: 1) staggered nominal wage rigidity (a Calvo-style adjustment rule for wage setting in a monopolistically competitive labour market). 2) A cost channel monetary transmission mechanism. 3) Variable capital utilisation.

in unemployment.

Otherwise the *NKS* economy is as an *NK* economy. Households derive utility from leisure, final goods consumption and holding (real) money balances. They may supply hours of work to intermediate good producers. They can save by accumulating capital which they rent to intermediate good producers and they purchase a basket of final goods from final good producers. Final good producers are monopolistically competitive. They each costlessly produce a differentiated final good by using only the homogeneous intermediate good and set the price of their product intermittently according to a Calvo price adjustment rule. The intermediate good is produced by combining labour and capital. Intermediate good producers are price takers in product and factor markets. The government issues money, collects seigniorage revenues and rebates these to households. It undertakes no other function.

With this basic structure in mind I now fill in some detail by discussing in turn the specification of goods and labour markets, the decision problem of households, my assumptions about the actions of the government and finally the equilibrium characterisation of the economy.

2.1 Goods and Labour Markets

2.1.1 The Intermediate sector

Production Production of intermediate goods takes place in the wholesale sector through matched firm-worker pairs - or, for notational ease, *matches.* Each match consists of one worker and one firm, who together engage in production until the employment relationship is terminated. By assumption, both firms and workers are restricted to a single employment relationship at any given time. Matches are subject to aggregate and idiosyncratic productivity shocks Z_t and X_t respectively, each with unit mean. Following DHRW and others, assume that idiosyncratic productivity disturbances are serially uncorrelated. Date t production occurs after realisation of the date t shocks. A match facing a high realisation of X at date t, greater than some threshold value $\bar{X}_t$ may decide to terminate the matching relationship - see below. At date t an ongoing match facing idiosyncratic shock X_t can combine capital, $\breve{K}(X_t)$ and hours of labour input, $H(X_t)$, to produce

$$Y^w(X_t) = AZ_t\breve{K}(X_t)^{\alpha} H(X_t)^{1-\alpha} + \mathcal{F} - X_t$$

units of intermediate good.[2] The parameter α represents the elasticity of match output with respect to capital input and A and $\mathcal{F}$ are positive constants. Matches are price takers and sell their homogeneous intermediate output at (nominal) price P_t^w. The formal separation of the job-destruction and price-setting decision problems is maintained for tractability. It is consistent with the view that prices are not set at the level of an individual match.

Suppose that the match specific capital stock, $\breve{K}(X_t)$ is chosen to maximise current profits of the match:

$$\max_{\breve{K}} \left\{ \frac{AZ_t\breve{K}(X_t)^{\alpha} H(X_t)^{1-\alpha} + \mathcal{F} - X_t}{\mu_t} - \frac{R_t^K \breve{K}(X_t)}{P_t} - \frac{W(X_t)H(X_t)}{P_t} \right\}.$$

where $\mu_t = \frac{P_t}{P_t^w}$ is the markup of the index of final goods prices over the price of the intermediate good, R_t^K is the (nominal) rental rate on capital, $W(X_t)$ is the match specific (nominal) wage - determined as an outcome of the bargaining process between workers and firms. The first order condition for this problem is

$$\breve{K}(X_t) = \left[\frac{\alpha A Z_t}{\mu_t R_t^K / P_t} \right]^{\frac{1}{1-\alpha}} H(X_t) \tag{1}$$

The optimal choice of capital-hours ratio $\frac{\breve{K}(X_t)}{H(X_t)}$ depends only on aggregate conditions, and is decreasing in the markup and the real return on capital and increasing in aggregate productivity. Using (1) current profits are

$$\Pi(X_t) \equiv (1-\alpha) \left[\frac{AZ_t}{\mu_t} \right]^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{R_t^K / P_t} \right]^{\frac{\alpha}{1-\alpha}} H(X_t) + \frac{\mathcal{F} - X_t}{\mu_t} - \frac{W(X_t)H(X_t)}{P_t} \tag{2}$$

Value Functions Let $h^t = \{h_0, ..., h_t\}$, denote the history of events up to date t, where h_t is the event realisation at date t. The date 0 probability of observing h^t is given by d_t. The initial state h^0 is given so that $d(h^0) = 1$. Henceforth, in order to simplify the notation, define the operator $E_t[\cdot] \equiv \sum_{h_{t+1}} d(h^{t+1}|h^t)$ as the mathematical expectation over all possible states of nature conditional on h^t.

2 An additive match specific shock avoids wide variation of hours across matches, Cooley and Quadrini (1999).

In (3) the date t value, V_t^U, expressed in final goods, of unemployment comprises the consumption value of utility from search, plus the discounted present value, V_{t+1}^U of ongoing unemployment next period, plus the discounted present value of the difference between the value of employment, $V^W(X)$, and that of unemployment in the event that the worker matches this period (with probability κ_t^U) and the match survives to produce next period (with probability $(1-\rho^x) F(\bar{X}_{t+1})$):

$$V_t^U = \frac{(1-e)^{1-\varphi}}{1-\varphi}\frac{1}{\lambda_t} + \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left[V_{t+1}^U + \kappa_t^U (1-\rho^x)\int^{\bar{X}_{t+1}}\left[V^W(X) - V_{t+1}^U\right] dF(X)\right]\right]. \quad (3)$$

Matching and production occur simultaneously, so that a match which is formed in period t cannot produce until period $t+1$, after aggregate and idiosyncratic shocks have been realised. As a result a new match survives with probability $(1-\rho^x) F(\bar{X}_{t+1})$.

Let $V^W(X_t)$ denote the date t value, expressed in terms of consumption goods, to a worker of employment in an ongoing match with idiosyncratic shock X_t.

$$\begin{aligned} V^W(X_t) &= \frac{W(X_t) H(X_t)}{P_t} + \kappa_H \frac{(1-H(X_t))^{1-\varphi}}{1-\varphi}\frac{1}{\lambda_t} \\ &+\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left[V_{t+1}^U + (1-\rho^x)\int^{\bar{X}_{t+1}}\left[V^W(X) - V_{t+1}^U\right] dF(X)\right]\right]. \end{aligned} \quad (4)$$

Then in equation (4), the worker supplies $H(X_t)$ hours of labour to the firm for real hourly wage $\frac{W(X_t)}{P_t}$. Both wage and hours are outcomes of a bargaining process. Hours worked generates income, but hours spent in the workplace reduce utility. These constitute the first two terms in (4). The remainder of the date t value to an employed worker from the ongoing match is the discounted present value, $\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^U\right]$, of unemployment (where Λ_t is the marginal utility of consumption in date t) plus the difference between the value of employment, $V^W(X)$, and that of unemployment in the event that the match continues to produce next period (where I sum across values of X which do not lead to termination prior to date $t+1$ production).

The date t value, $V^J(X_t)$, of a firm, with current match specific shock X_t, that forms part of an ongoing match, consists of current profits plus the appropriately discounted value to the firm of the sum of a date $t+1$ vacancy, V_{t+1}^V, in the event that the match terminates prior to production in period $t+1$ (where termination occurs with probability $\rho_{t+1} = \rho^x + (1-\rho^x)(1 - F(\bar{X}_{t+1})))$

and the expected value in the event that the match continues to produce in $t+1$;

$$V^J(X_t) = \Pi(X_t) + \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left[\rho_{t+1}V^V_{t+1} + (1-\rho^x)\int^{\bar{X}_{t+1}} V^J(X)\,dF(X)\right]\right].$$

I assume that it costs κ per period to post a vacancy. Then the value in date t of a firm that has an unfilled vacancy, V^V_t, reflects the cost of posting that vacancy plus the present value of firm, V^V_{t+1}, in the event that the firm fails to fill the vacancy or else the event that the vacancy is filled but the match is terminated prior to production in period $t+1$ (this occurs for a sufficiently adverse realisation of the idiosyncratic shock), and the value $V^J(X)$ in the event that it fills the vacancy and the period $t+1$ match-specifc shock takes a value X, that does not lead to termination

$$V^V_t = -\kappa + \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left[\left(1-\kappa^V_t(1-\rho^x)F\left(\bar{X}_{t+1}\right)\right)V^V_{t+1} + \kappa^V_t(1-\rho^x)\int^{\bar{X}_{t+1}} V^J(X)\,dF(X)\right]\right].$$

The free entry condition on vacancies drives the value of a vacancy to zero, $V^V_t = 0$, $\forall t$. So the Bellman equations for $V^J(X_t)$, and V^V_t become

$$V^J(X_t) = \Pi(X_t) + (1-\rho^x)\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\int^{\bar{X}_{t+1}} V^J(X)\,dF(X)\right] \tag{5}$$

$$\kappa = \kappa^V_t(1-\rho^x)\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\int^{\bar{X}_{t+1}} V^J(X)\,dF(X)\right]. \tag{6}$$

Using (5), we can re-write (6) as a Bellman equation for κ^V_t:

$$\frac{\kappa}{\kappa^V_t} = \beta(1-\rho^x)E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\int^{\bar{X}_{t+1}}\left[\Pi(X) + \frac{\kappa}{\kappa^V_{t+1}}\right]dF(X)\right]. \tag{7}$$

Bargaining: Hours and Wages Assume that for each match engaged in production, the firm and worker bargain over hours worked and the hourly wage. The division of the match surplus,

$$S(X_t) = V^W(X_t) - V^U_t + V^J(X_t) - V^V_t = V^W(X_t) - V^U_t + V^J(X_t), \tag{8}$$

is determined on a period by period basis through Nash bargaining:

$$\max_{W(X_t),H(X_t)} \left[V^W(X_t) - V^U_t\right]^\eta \left[V^J(X_t) - V^V_{t+1}\right]^{1-\eta}.$$

The first order conditions for hours and wages respectively are

$$\eta V^J(X_t)\left[\frac{W(X_t)}{P_t} - \kappa_H\frac{(1-H(X_t))^{-\varphi}}{\Lambda_t}\right] = -\left\{\begin{array}{c}(1-\eta)\left(V^W(X_t) - V^U_t\right)\cdot \\ \left[(1-\alpha)\left[\frac{AZ_t}{\mu_t}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R^K_t/P_t}\right]^{\frac{\alpha}{1-\alpha}} - \frac{W(X_t)}{P_t}\right]\end{array}\right\}, \tag{9}$$

$$\eta V^{J}\left(X_{t}\right)=(1-\eta)\left(V^{W}\left(X_{t}\right)-V_{t}^{U}\right). \tag{10}$$

Optimal hours worked are thus

$$\kappa^{H}\frac{\left(1-H\left(X_{t}\right)\right)^{-\varphi}}{\Lambda_{t}}=\kappa^{H}\frac{\left(1-H_{t}\right)^{-\varphi}}{\Lambda_{t}}=(1-\alpha)\left[\frac{AZ_{t}}{\mu_{t}}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R_{t}^{K}/P_{t}}\right]^{\frac{\alpha}{1-\alpha}}\qquad\forall X_{t}\leq\bar{X}_{t}. \tag{11}$$

Hours worked by workers in ongoing matches are decreasing in the rental rate on capital, and the markup, but increasing in aggregate productivity. Note that hours are independent of the match specific shock: $H\left(X_{t}\right)=H_{t}$. From (1) capital is also independent of the match specific shock: $\breve{K}\left(X_{t}\right)=\breve{K}_{t}$.

Using equations (3) and (4)

$$V^{W}\left(X_{t}\right)-V_{t}^{U}=\begin{array}{c}\frac{W(X_{t})H_{t}}{P_{t}}+\kappa_{H}\frac{(1-H_{t})^{1-\varphi}}{1-\varphi}\frac{1}{\lambda_{t}}-\frac{(1-e)^{1-\varphi}}{1-\varphi}\frac{1}{\lambda_{t}}\\+\left(1-\kappa_{t}^{U}\right)\beta\left(1-\rho^{x}\right)E_{t}\left[\frac{\Lambda_{t+1}}{\Lambda_{t}}\int^{\bar{X}_{t+1}}\left[V^{W}\left(X\right)-V_{t+1}^{U}\right]dF\left(X\right)\right].\end{array}$$

Using (10) and (6) it follows that

$$V^{W}\left(X_{t}\right)-V_{t}^{U}=\frac{W\left(X_{t}\right)H_{t}}{P_{t}}+\kappa_{H}\frac{\left(1-H_{t}\right)^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_{t}}-\frac{\left(1-e\right)^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_{t}}+\frac{\eta}{1-\eta}\left(1-\kappa_{t}^{U}\right)\frac{\kappa}{\kappa_{t}^{V}}.$$

Combining (5) and (6)

$$V^{J}\left(X_{t}\right)=(1-\alpha)\left[\frac{AZ_{t}}{\mu_{t}}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R_{t}^{K}/P_{t}}\right]^{\frac{\alpha}{1-\alpha}}H_{t}+\frac{\mathcal{F}-X_{t}}{\mu_{t}}-\frac{W\left(X_{t}\right)H_{t}}{P_{t}}+\frac{\kappa}{\kappa_{t}^{V}}.$$

So the optimal wage for a match with idiosyncratic shock X_t becomes

$$\frac{W\left(X_{t}\right)H_{t}}{P_{t}}=\begin{array}{c}\eta\left[(1-\alpha)\left[\frac{AZ_{t}}{\mu_{t}}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R_{t}^{K}/P_{t}}\right]^{\frac{\alpha}{1-\alpha}}H_{t}+\frac{\mathcal{F}-X_{t}}{\mu_{t}}+\kappa\frac{\kappa_{t}^{U}}{\kappa_{t}^{V}}\right]\\+(1-\eta)\left[\frac{(1-e)^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_{t}}-\kappa_{H}\frac{(1-H_{t})^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_{t}}\right].\end{array}$$

Define aggregate labour income as $\frac{W_{t}H_{t}}{P_{t}}=H_{t}\int^{\bar{X}_{t}}\frac{W(X_{t})}{P_{t}}dF\left(X\right)$. Then

$$\frac{W_{t}H_{t}}{P_{t}}=\left\{\begin{array}{c}\eta\left[(1-\alpha)\left[\frac{AZ_{t}}{\mu_{t}}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R_{t}^{K}/P_{t}}\right]^{\frac{\alpha}{1-\alpha}}H_{t}+\frac{1}{\mu_{t}}\left[\mathcal{F}-\frac{\int^{\bar{X}_{t}}X\,dF(X)}{F\left(\bar{X}_{t}\right)}\right]+\kappa\frac{\kappa_{t}^{U}}{\kappa_{t}^{V}}\right]\\+(1-\eta)\left[\frac{(1-e)^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_{t}}-\kappa_{H}\frac{(1-H_{t})^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_{t}}\right]\end{array}\right\}F\left(\bar{X}_{t}\right) \tag{12}$$

Separation For values of the match specific shock above a certain threshold level, $\bar{X}_t$, separation occurs. The condition $S\left(\bar{X}_{t}\right)=0$, pins down this threshold value of the match specific shock. Combining (8) and (10), $V^J(X_t) = (1-\eta) S(X_t)$. So $\bar{X}_t$ is determined by the condition $V^J(\bar{X}_t) = 0$:

$$(1-\alpha)\left[\frac{AZ_t}{\mu_t}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R_t^K/P_t}\right]^{\frac{\alpha}{1-\alpha}} H_t + \frac{\mathcal{F}-\bar{X}_t}{\mu_t} - \frac{W(\bar{X}_t)H_t}{P_t} + \frac{\kappa}{\kappa_t^V} = 0.$$

Substituting for the match specific wage, the threshold value $\bar{X}_t$ is determined by

$$(1-\eta)\left[\begin{array}{c}(1-\alpha)\left[\frac{AZ_t}{\mu_t}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{R_t^K/P_t}\right]^{\frac{\alpha}{1-\alpha}} H_t + \frac{\mathcal{F}-\bar{X}_t}{\mu_t} \\ -\left[\frac{(1-e)^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_t} - \kappa_H\frac{(1-H_t)^{1-\varphi}}{1-\varphi}\frac{1}{\Lambda_t}\right]\end{array}\right] - \eta\kappa\frac{\kappa_t^U}{\kappa_t^V} + \frac{\kappa}{\kappa_t^V} = 0. \tag{13}$$

2.1.2 Labour Market Flows

The match specific production, bargaining and separation decisions described above depend on the probability that unemployed workers find jobs and the probability that vacancies are filled. In this section I discuss these probabilities and the associated labour market flows.

Define the number of matches at the beginning of period t as $N_t \in [0,1]$. I allow some job destruction in the form of quits which are taken as exogenous and independent of the match-specific productivity. I capture this by allowing a fraction, ρ^x, of matches to separate prior to the realisation of period t (productivity) shocks. Subsequently, idiosyncratic productivity disturbances are realised, and a match may choose to break up if the value of the match surplus is negative. Endogenous separation thus occurs with probability $\rho^n(\bar{X}_t) = 1 - \int^{\bar{X}_t} dF(X)$, where $dF(\cdot)$ is the probability density function over X. The overall separation rate in period t is

$$\rho_t = \rho^x + (1-\rho^x)\left(1 - F(\bar{X}_t)\right). \tag{14}$$

Next consider the matching frictions. I model this rigidity using an aggregate matching function. Matching occurs at the same time as production. I assume, following Pissarides (2000), DHRW, that there is a continuum of potential firms, with infinite mass, and a continuum of workers of unit mass. Unmatched firms choose whether or not to post a vacancy given that it costs κ per period to post a vacancy. Free entry of firms determines the size of the vacancy pool. Define the mass of firms posting vacancies in period t as V_t. Let the mass of searchers, unmatched workers, be U_t. All unmatched workers may enter the matching market in period t - even if their match dissolved at the start of period t, so

$$U_t = 1 - (1 - \rho_t) N_t. \tag{15}$$

New matches in date t begin production in date $t+1$, while unmatched workers remain in the worker matching pool. The flow of successful matches created in period t is given by the constant returns matching function

$$\mathcal{M}_t = \mathfrak{M} (e \cdot U_t)^{\gamma} V_t^{1-\gamma}. \tag{16}$$

where $\gamma, e \in (0,1)$ and $\mathfrak{M} > 0$. The parameter e represents the efficiency with which unemployed workers engage in search. Thus the number of employment relationships at the start of period $t+1$ is

$$N_{t+1} = (1 - \rho_t) N_t + \mathcal{M}_t. \tag{17}$$

Denote the probability that a vacancy is filled in date t as

$$\kappa_t^V = \frac{\mathcal{M}_t}{V_t}, \tag{18}$$

and the probability that an unemployed worker enters employment in period t as

$$\kappa_t^U = \frac{\mathcal{M}_t}{U_t}. \tag{19}$$

Gross job destruction is the employment relationships that separate less exogenous separations that rematch within period

$$JD_t = \frac{\rho_t N_t - \kappa_t^V \rho^x N_t}{N_t} = \rho_t - \kappa_t^V \rho^x. \tag{20}$$

Gross job creation is the flow of new matches (as a fraction of existing employment) less matches due to firms filling vacancies that resulted from exogenous separations

$$JC_t = \frac{\mathcal{M}_t - \kappa_t^V \rho^x N_t}{N_t} = \frac{\mathcal{M}_t}{N_t} - \kappa_t^V \rho^x. \tag{21}$$

2.1.3 Final Goods Sector

Assume that there is a continuum of final goods producers, with unit mass. Final good firm z acquires the wholesale good at price P_t^w and costlessly transforms it into the divisible final good

z which is then sold directly to households at price $p_t(z)$. Define $P_t = \left(\int_0^1 p_t(z)^{1-\varepsilon} dz\right)^{\frac{1}{1-\varepsilon}}$ as the utility based price index associated with both the investment and consumption composites. The market for final goods is characterised by monopolistic competition - ε represents the elasticity of substitution across varieties of final good. The aggregate demand for the final good z in period t is then

$$y_t(z) = c_t(z) + i_t(z),$$

where $c_t(z)$ represents consumption demand for final good z output in period t and $i_t(z)$ represents gross investment demand for final good z output for capital accumulation. The optimal choice of consumption and investment expenditures on final good z are then

$$c_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} C_t, \qquad i_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} I_t,$$

where aggregate consumption, $C_t = \left(\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{1-\varepsilon}}$, aggregate investment, $I_t = \left(\int_0^1 i_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{1-\varepsilon}}$ and aggregate final good output $Y_t = \left(\int_0^1 y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{1-\varepsilon}}$ are composite indices of final goods.

Suppose that final goods prices exhibit nominal rigidities which follow a Calvo style adjustment scheme. Assume that with probability $(1-\omega)$ a final good producer can set the price of its output in period t. Let this probability be independent of when the firm last adjusted price. Then the average price for final goods producers who do not adjust their price is simply P_{t-1}. Suppose that the average price set by firms who do adjust price is $\bar{p}_t$.

Since pure forward looking price adjustment schemes seem not to account adequately for observed inflation dynamics, I work with a hybrid scheme (following Gali and Gertler (1999)). Assume that a fraction $(1-\tau)$ of the final goods producers are forward looking and set prices optimally (to maximise expected discounted profits given the probability of future adjustment).[3] Define the price set by forward looking producer z at date t as $p_t(z)$. Since all forward looking firms setting price at date t face the same expected future demand and cost conditions they choose the same price, so $p_t(z) = p_t^*$, where

$$p_t^* = \frac{\varepsilon}{1-\varepsilon} \frac{E_t \sum_{s=0}^{\infty} \omega^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{p_t^*}{P_{t+s}}\right)^{1-\varepsilon} Y_{t+s} P_{t+s}^w}{E_t \sum_{s=0}^{\infty} \omega^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{p_t^*}{P_{t+s}}\right)^{1-\varepsilon} Y_{t+s}} \tag{22}$$

3 This structure is used by Trigari (2003), Walsh (2005) uses CEE's indexation approach.

The remaining fraction τ of firms setting price in period t are assumed to set prices equal to the average of the last period reset prices, corrected for inflation:

$$p_t^b = \bar{p}_{t-1}\pi_{t-1}. \tag{23}$$

The average price set in period t is $p_t^{1-\varepsilon} = (1-\tau)(p_t^*)^{1-\varepsilon} + \tau\left(p_{t-1}^b\right)^{1-\varepsilon}$, and the aggregate retail price index evolves according to

$$P_t^{1-\varepsilon} = (1-\omega)(\bar{p}_t)^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon}. \tag{24}$$

2.2 Households

Assume that the economy contains a continuum of households of unit mass. Households own all retail and wholesale firms. They can save by accumulating capital, which they rent to wholesale firms, by holding (nominal 1 period discount) bonds, or non-interest bearing money balances. To avoid the distributional issues that arise because some workers are unmatched, assume that complete asset markets allow workers to insure themselves against (cross-section) variation in the marginal utility of consumption. Under this simplifying assumption, household behaviour can be analysed in terms of a representative consumer.[4] Assume that the representative consumer derives utility from consumption, leisure, and services provided by holding real money balances; that the instantaneous utility function is not time-separable in consumption, and that the household chooses consumption and money balances to maximise expected utility over her lifetime

$$E_\tau\left[\sum_{t=0}\beta^t\left[\frac{\left(C_t^h-\kappa_C C_{t-1}^h\right)^{1-\phi}}{1-\phi}+\frac{\kappa_{\frac{M}{P}}}{1-\xi}\left(\frac{M_t^h}{P_t}\right)^{1-\xi}+U_t\frac{(1-e)^{1-\psi}}{1-\psi}-(1-U_t)\frac{\kappa_H}{1-\psi}(1-H_t)^{1-\psi}\right]\right].$$

where β the discount factor, κ_C, $\kappa_{\frac{M}{P}}$, κ_H, ϕ, ξ, and ψ are all positive constants, and variables superscripted h are elements of the household decision problem. When employed, hours of work are determined through bargaining (with wholesale firms) rather than being unilaterally determined by the individual household.

The representative consumer maximises expected lifetime utility subject to the following se-

[4] This sort of assumption is a common simplification in the literature on business cycle fluctuations under labour market search designed to facilitate tractability, see e.g. Andolfatto (1996), Merz (1995).

quence of constraints

$$\begin{aligned} P_t C_t^h + P_t I_t^h + E_t \left\{ v_{t,t+1} D_{t+1}^h \right\} + M_t^h &= R_t^K K_t^h + \mathcal{I}_t^h + D_t^h + M_{t-1}^h + P_t T_t^h, \\ K_{t+1}^h &= (1-\delta) K_t^h + I_t^h + \frac{\chi}{2} \frac{\left(I_t^h - \delta K_t^h \right)^2}{K_t^h}, \qquad t \geq 0. \end{aligned} \tag{25}$$

Where D_{t+1}^h represents nominal payoff in period $t+1$ of the portfolio held at the end of period t, $v_{t,t+1}$, is the stochastic discount factor for one period ahead nominal payoffs relevant to the representative household. M_t^h represents holdings of nominal money balances at the end of period t, $P_t T_t^h$ represents a lump-sum nominal transfer from households due to rebated seigniorage revenues. C_t^h and I_t^h are household consumption expenditures and gross fixed capital formation expenditures respectively, K_{t+1}^h represents capital stock carried held at the end of period t. The positive constants δ and χ capture (geometric) capital depreciation and the magnitude of (quadratic) adjustment costs (on net investment).[5] R_t^K is the rental return on capital. $\mathcal{I}_t^h$ is the household's nominal income (labour income, plus firms' profits net of expenditures on vacancies).[6]

The solution to the representative consumer's problem is characterised by first-order conditions for bond holdings, B_t^h, consumption C_t^h, money balances M_t^h, investment I_t^h and capital stock K_{t+1}^h. Define the gross return on a riskless asset paying off one unit of currency in date $t+1$ as $R_t^n = \frac{1}{E_t[v_{t,t+1}]}$, where $E_t[v_{t,t+1}]$ is the price of that asset, and the date t shadow value of capital at date $t+1$ as Q_t. The resulting first order conditions can be written as:

$$1 = \beta R_t^n E_t \left[\frac{P_t}{P_{t+1}} \frac{\Lambda_{t+1}}{\Lambda_t} \right]. \tag{26}$$

$$\Lambda_t = \left(C_t^h - \kappa_C C_{t-1}^h \right)^{-\phi} + \beta E_t \left[\left(C_{t+1}^h - \kappa_C C_t^h \right)^{-\phi} \right]. \tag{27}$$

$$\kappa_{\frac{M}{P}} \left(\frac{M_t^h}{P_t} \right)^{-\xi} - \Lambda_t = \beta E_t \left[\Lambda_{t+1} \frac{P_{t+1}}{P_t} \right] \tag{28}$$

$$\Lambda_t = Q_t \left[1 + \chi \frac{I_t^h - \delta K_t^h}{K_t^h} \right] \tag{29}$$

5 Costs of capital adjustment are associated with aggregate net investment, while capital can be costlessly re-allocated across intermediate producers.

6 Christiano et al (2005) incoporate habit persistence both in consumption and in investment. I do not consider the latter because I am unaware of any microeconometric evidence which supports adjustment costs of this form.

$$Q_t = \beta E_t \left[\Lambda_{t+1} \frac{R^K_{t+1}}{P_{t+1}} + Q_{t+1} \left[1 - \delta - \frac{\chi}{2} \left[\left(\frac{I^h_{t+1}}{K^h_{t+1}} \right)^2 - \delta^2 \right] \right] \right] \tag{30}$$

2.3 Monetary and Fiscal Policy and Exogenous Driving Processes

I set government spending to zero and assume that the government maintains a balanced budget by rebating seigniorage revenues to households in the form of lump-sum transfers. The government budget constraint is thus $P_t T_t = M_t - M_{t-1}$,where M_t is the aggregate money stock. Monetary policy is specifed by

$$M_t = M_{t-1} e^{\upsilon_t} \tag{31}$$

where υ_t evolve according to the AR(1) process

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_{\upsilon,t}. \tag{32}$$

The logarithm of aggregate productivity also follows an AR(1) process:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_{Z,t} \tag{33}$$

where $\varepsilon_{\nu,t}$ and $\varepsilon_{Z,t}$ are independent mean zero processes.

2.4 Equilibrium

Equilibrium in the rental market for capital requires that $K_t = (1-\rho^x) N_t \int_0^{\bar{X}_t} \breve{K}_t dF(X) = (1-\rho^x) F(\bar{X}_t) N_t \breve{K}_t$. Using (1) gives

$$K_t = (1-\rho^x) F(\bar{X}_t) N_t \left[\frac{\alpha A Z_t}{\mu_t R^K_t / P_t} \right]^{\frac{1}{1-\alpha}} H_t. \tag{34}$$

Under the representative consumer framework, household choices (superscript h) are common across households. There is a unit mass of households, so in equilibrium $M^h_t = M_t$ etc, in (25) to (30).

Now aggregate labour income, $\mathcal{I}_t$ comprises labour income, plus profits of final goods producers, plus profits of intermediate goods producers net of vacancy posting costs $\mathcal{I}_t = (1-\rho^x) N_t W_t H_t + P_t \Pi^F_t + P_t \Pi^w_t$. Here, nominal final goods profits are $P_t \Pi^F_t = \int p_t(z) y_t(z) dz - P^w_t \int y_t(z) dz = P_t Y_t - P^w_t Y^w_t$, and

$$Y^w_t = (1-\rho^x) N_t \int_0^{\bar{X}_t} \left[A Z_t \breve{K}^\alpha_t H^{1-\alpha}_t + \mathcal{F} - X \right] dF(X) - \kappa \mu_t V_t \tag{35}$$

denotes aggregate intermediate output net of vacancy posting costs.[7] Nominal intermediate good producers' profit can be written as the sum of output net of vacancy costs, less aggregate wage payments and capital rental payments: $P_t\Pi_t^w = P_t^w Y_t^w - (1-\rho^x) N_t W_t H_t - R_t^K (1-\rho^x) N_t \int_0^{\bar{X}_t} \breve{K}_t dF(X)$. Using these insights and cancelling terms gives

$$\mathcal{I}_t = P_t Y_t - R_t^K K_t$$

In equilibrium, the household budget constraint reduces to the aggregate (final) goods market equilibrium condition

$$Y_t = C_t + I_t \tag{36}$$

Thus the system of equations governing equilibrium in the economy consists of the numbered equations (2), (7) and (11) - (36).

3 Calibration & Model Solution Method

I log-linearise the model about its (zero-inflation, zero growth) steady state and use impulse response analysis and dynamic simulations to tease out the dynamic structure of the economy. Model solution requires choice of several parameters governing steady state values of labour and goods market variables; nominal rigidity, and household preferences. I also specify the processes governing idiosyncratic productivity and money supply growth. These parameters are chosen to match properties of the US economy. The parameter values are summarised in Table 1, Appendix A contains discussion of the rationale for and origins of these choices.

Table (1) about here.

4 Results

I examine the impulse responses to productivity and monetary shocks in order to shed light on the mechanisms at work in the baseline **NKS** model. Next, I evaluate the quantitative performance of the **NKS** model against US **Data** and a number of model variants using stochastic simulation. Finally, I examine the robustness of the results to parameter variation.

7 Note the relationship between Y_t and Y_t^w. $Y_t^w = \int_0^1 y_t(z)\,dz$. Using the demand function for final good z: $y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} Y_t$, we have $Y_t^w = \int_0^1 \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} Y_t dz = \left(\frac{P_t}{\tilde{P}_t}\right)^{\varepsilon} Y_t$, where $\tilde{P}_t = \int_0^1 p_t(z)^{-\varepsilon}\, dz$, is an auxilliary price index.

4.1 Qualitative Response to Monetary and Productivity Shocks

The impulse responses of the **NKS** model under the baseline calibration to monetary and productivity shocks are illustrated in Figures (1) an (2) respectively.

Figure (1) here.

Figure (1) illustrates the *NKS* response to a 1% monetary growth innovation. Output, consumption and investment increase in response to a monetary innovation. Habit persistence leads to a hump-shaped response in consumption, whereas the output and investment responses are front-loaded. Long-run neutrality of money guarantees that steady state capital stock is unaffected by the monetary shock, and, despite ongoing depreciation, the sharp initial rise in investment is followed, after 6 quarters, by a period of disinvestment. With nominal rigidities in price setting, a monetary growth shock generates inflation. Under Calvo-style price-setting this occurs when there is a decline in the markup (a rise in marginal costs) associated with greater production. Although this markup effect is frontloaded, the backward-looking element in price-setting leads to a hump-shaped inflation response. Associated with the front-loaded rise in marginal cost, there is a front-loaded rise in the real wage, which is associated with an immediate rise in hours worked. These variables gradually return towards steady state. The monetary shock also triggers an initial increase in employment above steady state, which is achieved through a sharp rise in job creation and a sharp decline in job destruction. Improved economic conditions following the monetary expansion lead to a spike in vacancies - consistent with a spike in job creation. The decline in unemployment in the aftermath of the shock makes it relatively difficult for firms to fill vacancies so both vacancies and job creation rebound after their initial increases. Vacancies return to just above steady state, but job creation falls below steady state - a consequence of the degree of labour market tightness. This subsequent lack of new job creation, combines with a near steady-state job-destruction rate to eliminate the initial rise in employment (after about 6 quarters) and leads to above steady-state unemployment for the remainder of the transient response.

Figure (2) here.

Figure (2) shows the *NKS* response to a 1% productivity shock. Output, consumption and investment display a persistent hump-shaped response, with investment displaying the largest percentage deviation. With nominal rigidities in price setting, the productivity shock generates disinflation. This is associated with a rise in the markup (a decline in marginal costs). The markup effect is front-loaded, but the backward-looking element in price-setting generates a hump-shaped inflation response. Since money is neutral in the long-run, cumulative inflation is zero. There is a front-loaded decline in the real wage. This is associated with an immediate decline in hours worked, which subsequently returns to steady state. Since hours worked returns to steady state and capital is higher than in steady state, real wage rises above steady state for a period. The productivity shock also triggers an initial decline in employment below steady state - this is more than reversed after 4 quarters - as new matches continue to be formed to take advantage of temporarily high productivity and capital stock. A period of above steady state employment ensues, as existing matches take advantage of the favourable economic conditions (high levels of capital stock) and continue production at temporarily high aggregate productivity levels. In the immediate aftermath of the productivity shock the inauspicious employment prospects lead to a decline in vacancies - consistent with a decline in job creation. However, the initial rise in unemployment makes it relatively easy for firms to fill vacancies and both vacancies and job creation rebound after their initial declines to several percentage points above steady state. This ongoing job creation, combined with a rate of job-destruction that subsequently remains near the steady state level eliminates the initial unemployment (after about 4 quarters) allowing below steady state unemployment for the remainder of the transient response.

4.2 Quantitative Assessment

4.2.1 Relative Variabilities

Table (2) shows the relative variability of key variables. Each column corresponds to a particular model. In this section we focus on columns (2) - (6). Column (2) records US Data (**Data**). Column (3) (**NKS**) reports results for the baseline calibration. Column (4) (**NK**) is a standard New Keynesian model with a Walrasian labour market. This model omits parameters ρ, N, κ^V, ρ^x, σ_X, γ and η , relating to search, bargaining and separation, but is otherwise calibrated identically to **NKS**.[8]. Column (5), **Fix** H, presents results for the case where I eliminate variation of hours. Column (6) **Fix** W presents results for the case where wage rigidity is imposed. The remaining column - **Fix** K, - is discussed in Section **4.4**.

Table (3) here.

In conducting the simulations of **NKS**, the standard deviation of productivity shocks in the baseline model outlined in Section (2) is set to 1.1% in order to match the variability of (detrended) quarterly GDP in US data. This value of σ_Z is used in all model variants, in order to be able to compare the relative strength of the amplitude and propagation mechanisms in each variant.[9] The first row of the table records the volatility of output in the model variants relative to the standard deviation of the deviation of HP-detrended logged US GDP, which is 1.61% over the sample period (1972:2 - 1993:4).[10] For a given column in Table (2), the entry in the row labelled **output** indicates the variability of output in column **X** ($\neq$**Data**) relative to the variability of output in the US **Data**. The other entries in column **X** correspond to the variability relative that of output generated by model **X.** This permits comparison both across models and within models across variables.

Comparison with Walrasian Model Consider **NKS**, **NK** and **Data** columns of Table (3). **NKS** outperforms **NK** along several dimensions. First, given σ_Z, it generates greater output variability than **NK** while capturing the relative variability of consumption and investment. Second **NKS** does a better job of capturing the variability of inflation than **NK** - where the reliance on variation in hours requires more extreme real wage variation, and therefore a more immediate rise in marginal costs and a greater front loading of inflation. Third, by decomposing labour input variation into changes along extensive and intensive margins, **NKS** reduces the variability of hours compared with **NK**. For **NKS**, employment variability is close to that in the data. Yet hours are still more variable in **NKS** than in the data, as are real wages. Unemployment and vacancies exhibit less variability in **NKS** than in the data. Finally, the variability of job flows is of similar magnitude to that in the data, although job destruction is insufficiently volatile. Overall, this evi-

8 Steady state hours are equal in **NK** and **NKS**.

9 This seems preferable to the alternative approach of treating σ_Z as a free paramter and recalibrating so that in each model variant the variability of output matches that in US **Data**

10 The length of this sample period is dictated by the availability of the job flows series.

dence supports the prevalent view that inclusion of search, matching frictions and job destruction into an NK framework improves performance over the basic NK framework.

Comparison with Fixed Hours Worked Variant Next, consider the role of variable hours by contrasting the performance in **NKS** with that in **Fix** H. Absence of an intensive margin for labour input variation is the approach adopted by DHRW, Walsh (2003, 2005), Krause and Lubik (2006).[11] The key effect in this model variant is that with variation in hours suppressed, **Fix** H relies to a greater extent than **NKS** on variation in employment. This increases the variability of unemployment and vacancies (by a factor of two and four times the **NKS** values respectively) to levels close to that in US **Data**. Notice that the increase in unemployment variability arises even without the imposition of wage rigidity suggested by Shimer (2005) and confirmed by Krause and Lubik (2006). Suppression of hours variation also affects both the absolute and relative properties of gross flows. The latter effect is not consistent with the data. Job creation is as volatile as in **NKS** whereas job destruction is almost three times greater in **Fix** H than in **NKS** and twice that in **Data**. The real wage is less volatile in **Fix** H than in **NKS**, though more volatile than in **Data**. Hours variation appears to affect wages (and inflation) variability, but wages are affected reflect labour market tightness rather than mere variation in hours. Finally, variation in hours does not alter the variability of output in **Fix** H relative to **NKS** and the relative variability of consumption and investment is unaffected.

Comparison with Wage Rigidity Variant The standard deviation of wage in **NKS** and **Fix** H is too large. Shimer argues that wage rigidity with search and matching can better account for variability of unemployment and vacancies. Krause and Lubik (2006) show that Shimer's insight applies in a New Keynesian framework with search and matching frictions only when endogenous job destruction, is absent. Column (6) **Fix** W confirms these insights also hold when job destruction is endogenous provided variation of hours and capital accumulation are permitted. Consistent with the rise in variability of vacancies and unemployment, the volatility of gross flows rises compared with **NKS**. However, with variation in wages (rather than hours) suppressed job

[11] The **Fix** H model is obtyained by letting $\psi = 100$. An alternative, that is closer to the approach used elsewhere, DHRW, Walsh (2003, 2005), Krause and Lubik (2005), is simply to suppress hours variation altogether. The latter approach generates greater variability in real wages (six times that in the data) - see Holt (2006).

creation rises relative to job destruction, with the former being three times that in **NKS** and **Data**. As a consequence, the variability of employment rises to almost thrice that in **NKS** and **Data**. The variability of hours increases to almost double that in **NKS** and **Data**, since the dampening effect of wage variability is absent. Since, output is no more variable than in **NKS** or **Data**, a rise in the variability of hours and of employment can be offset by a decline in the variability of capital (and hence investment declines).[12] In the face of this decline in the variability of investment, the relative variability of consumption rises to maintain the volatility of output. Whereas Krause and Lubik (2006) find that real wage rigidity does not impact on inflation, I find that wage rigidity tends to reduce the variability of inflation. In **NKS** the variation in hours associated with economic fluctuations is positively correlated with variation in wages (this correlation is less than perfect due to the impact of labour market tightness under matching frictions). By contrast in **Fix** W, wage igidity allows variation in hours without variation in wages, and hence marginal costs and inflation exhibit less variability. Whereas in Krause and Lubik's work, that eliminates variation in hours, suppression of real wage rigidity does not impact on marginal costs and inflation.

Section Summary The results described here suggest that the baseline **NKS** model does a reasonable job of matching a the relative volatility of a number of macroeconomic variables, but generates insufficient variability in unemployment and vacancies (and consequently in gross job flows). This can be addressed to some extent by decreasing the elasticity of labour supply to attenuate variation in hours, or by imposing real wage rigidity while allowing labour input to vary on both intensive and extensive margins. However, both approaches generate *too much* variability in vacancies, unemployment and gross flows, with the former leading to excessive job destruction and the latter leading to excessive job creation.

4.2.2 Labour Market Cross-Correlations

To shed further light on the role of intensive and extensive margins for adjusting labour input and the impact of imposing real wage rigidity I discuss the effect of suppressing variation in hours worked on the slope of the Beveridge curve, and the dynamic interaction of gross flows with each

[12] In Section 4.3 below I show that greater variability in employment and hours is partly offset by a more negative correlation of employment and hours.

other, unemployment and inflation.

The motivation for the analysis in this section is Shimer's (2005) observation that models of labour market search with matching studies which incorporate endogenous job destruction have a tendency to produce a positively-sloped Beveridge curve. Table (6) sheds light on this issue. It contains data on the correlation of unemployment and vacancies and the correlation of gross flows. In terms of the model variants, the key determinant of the slope of the Beveridge curve is the variability of hours. When the intensive margin for adjusting labour input is unavailable the correlation becomes close to zero - and a positive correlation emerges for gross job flows.[13] Imposing real wage rigidity - thereby increasing the variability of unemployment, vacancies and gross flows - generates an unemployment-vacancy correlation that is closer to the data than in **NKS,** yet produces a job flows correlation identical to that in **NKS** . The column **Fix** K will be discussed in Section **4**.

Table (3) here.

Figure (3) displays the cross-correlation structure of job creation and job destruction with each other and with unemployment and inflation for **Data, NKS**, **Fix** H and **Fix** W. Discrepancies between **NKS** responses and the data are related to the lack of persistence of the job creation and job destruction responses revealed in the impulse responses in Figures (1) and (2). In the data, lagged and contemporaneous job creation are negatively correlated with current job destruction, while leads of job creation at 2-3 quarters are positively correlated with job destruction suggesting that job creation rises in the wake of a spike in job destruction. By contrast in **NKS**, lagged job creation is only weakly negatively correlated with current job destruction, while contemporaneous job creation and destruction are more strongly negatively correlated than in the data. Also in contrast to the data, and consistent with the rebound observed in the impulse responses, contemporaneous job destruction is positively correlated with 1, 2 and 3 quarter ahead job creation. This behaviour carries over to the relationship between gross flows and other variables where much more of the movement in the correlograms occurs around the 0 lag point in **NKS** than in **Data**.

[13] If the intensive margin is completely eliminated as in the literature, rather than suppressed by letting $\psi = 100$, then the slope of the Beveridge curve become positive - see supplementary notes.

That aside, **NKS** seems to capture the broad features of the data.

Figure (3) here.

Figure (3) also sheds some light on how the differences in the margins for adjustment across models affect the responce of gross job flows to shocks. The responses for **Fix** H and **Fix** W in Tables (2) and (3) suggest that suppression of hours variation and imposotion real wage rigidity will reduce our ability to capture some behaviour of gross job flows. From a simple accounting viewpoint, a rise in unemployment can be achieved either through a rise in job destruction and a fall in job creation, or by one of these in isolation with no change in the other, or by a *rise* in job creation combined with a larger rise in job destruction. The first of these is what occurs in US **Data**, *and* in **NKS**, whereas the last of these options captures what occurs in **Fix** H without variation in hours. Although from Table (2) it appeared that the primary effect of suppressing variation in hours was to make job destruction more volatile, Figure (3) shows that for cross correlations this impacts to a greater extent on job creation decisions. Suppressing the variation of hours appears to alter the volatility of job destruction without altering its correlation structure with unemployment or inflation (relative to that in **NKS**). By contrast, suppressing variation in hours, by generating such a volatile response of job destruction, generates an increase in job creation prior to, contemporaneously with and following a rise in unemployment: so job creation rises in a recession. This unfortunate feature is also replicated in the correlation of job creation with inflation. This is exacerbated by the fact that the contemporaneous accociation between unemployment and inflation is itself too strong in the model.

Turning to the case of wage rigidity. While Table (2) suggests that the impact of real wage rigidity (**Fix** W) on job creation is greater (more extreme) than on job destruction, Figure (3) suggests that the correlation structure for job creation with unemployment is in fact closer to that in the data than obtained using **NKS.** In particular, with wage variation suppressed a rise in unemployment is heralded by slightly higher than average job creation, and uncorrelated with contemporaneous job creation. However, the price of this improvement in the cross correlation structure of job creation (with unemployment and inflation) is a worsened cross-correlation structure for job destruction. In contrast to **NKS** and US **Data** job destruction in **Fix** W declines in before and during a rise in unemployment, while - other than the contemporaneous value - the signs of the cross correlation of job creation with job destruction are reversed compared with **NKS** and US **Data**. The increased volatility of job creation relation to job destruction in **Fix** W means that job creation plays a greater role in unemployment fluctuations. Suppression of real wage rigidity affects the cross correlation structures with inflation with that of job creation worsening and job destruction improving with respect to **NKS** and US **Data.** In **Fix** W the combined effects of the changes in job flows on unemployment and the suppression of wage rigidity on inflation also feed through to the unemployment-inflation relationship where the magnitude of the correlations decline in relation to the data.

4.3 Role of Labour Supply Elasticity, Wage Rigidity & Job Destruction

The baseline **NKS** model is calibrated to unit-elastic labour labour supply. This figure is of similar magnitude to that in real business cycle models that incorporate labour market search with hours variation while neglecting endogenous job destruction Andolfatto (1995), standard New Keynesian models without labour market search, Woodford (2003) and could be justified by appealing to microeconometric work that considers female, as well as male labour supply, or for male labour supply alone Domeij and Floden (2005). However, it is not uncontroversial. In this section I examine the robustness of the results on absolute and relative variabilities, on the sign and magnitude of the slope of the Beveridge curve and the correlation of gross job flows to variation in the degree of labour supply elasticity.

I consider three variants in Figures (4)-(6): the baseline model **NKS**, the model with wage variation suppressed **Fix** W and a version of **Fix** W in which endogenous job destruction is suppressed. Here I explain how other studies are nested as special cases of the one considered here. In the limit, as φ becomes large, labour supply elasticity drops and each variant approximates a limit case for which hours variation is suppressed ($\varphi \to \infty$). For **NKS** we have already considered the approximation to the limit case **Fix** H in some detail above, and our emphasis will be on how quickly the limiting behaviour emerges as labour supply elasticity falls. The baseline **NKS** model corresponds to that of Trigari (2005) extended to allow for capital accumulation. The limit case **Fix** H approximates the models of Walsh (2005) and Krause and Lubik (2006) extended to allow for capital accumulation (and additionally habit persistence respectively). The suppression of wage variability in isolation was discussed above in **Fix** W. The focus here on robustness to variation in labour supply elasticity allows us consider the limit case in which variation in both wages and hours is suppressed. This limit case of **Fix** W corresponds to that considered by Krause and Lubik (2006) when assessing in a New Keynesian set up Shimer's claims on the role of wage rigidity on the volatility of unemployment and vacancies. However, Krause and Lubik (2006) find that the sign of the slope of the Beveridge curve is positive when endogenous job destruction is permitted - which they interpret as supporting Shimer's critique of models featuring job destruction. Consequently I also consider the impact of variation of labour supply elasticity in a variant of **Fix** W for which job destruction is exogenous - call this **Fix** WX. For **Fix** WX, the limit case where φ becomes large corresponds to the version of Krause and Lubik's model that generates adequate volatility in unemployment and vacancies *and* a negatively sloped Beveridge curve - one might call this limit case **Fix** WHX.

Exogenous Job Destruction & Wage Rigidity It is easiest to understand the contribution of labour supply elasticity, wage variation and endogenous job destruction by adding these effects in separately. So I start with the variant of **Fix** WX in Figure (4). Variation in φ has little impact on the variability of output. When φ is low, around the unit elastic case both hours and employment exhibit several times more variability than in the data. As φ increases, (labour supply becomes less elastic), the variability of hours and employment both decline. This seems a little odd as output variability is almost unchanged and one might imagine that the employment and hours are substitutes. In fact hours and employment are strongly negatively correlated at low φ. This correlation rises but remains negative even when labour supply is relatively inelastic. Since employment variability rises at low φ and the rate of job destruction is exogenous. Variation in the outflow from unemployment (job creation) is required to generate the variation in both employment and unemployment. This is reflected in the high standard deviation of vacancies, unemployment and employment at low φ. High φ generates increased inflation variability. This appears to be because (given constant wages) greater elasticity of hours allows firms to rely on variation in labour input rather than capital stock - since altering the latter is costly. Thus, with wage variation suppressed and the rate of job destruction fixed exogenously - the environment supported by the work of Shimer (2005) and Krause and Lubik (2006) - allowing hours to vary and using elastic labour supply produces labour market behaviour that is inconsistent with the data, but as was discussed above the approach favoured in the literature - suppressing variation in hours, wages and job destruction - fails to capture cyclical variation in the labour market.

Endogenous Job Destruction & Wage Rigidity Next consider how the behaviour of the economy is modified as ψ varies if one allows job destruction to be determined endogenously, while imposing wage rigidity - see Figure (5). The key feature of the response is that, once ψ becomes sufficiently large (once labour supply becomes sufficiently inelastic tending in the limit to **Fix** WH - the case dismissed by Shimer (2005) and Krause and Lubik (2006) in favour of **Fix** WHX) firms rely heavily on the extensive margin rather than the intensive margin to adjust labour input, and use job destruction rather than job creation once the former can be varied. By contrast with the case of exogenous job destruction, variability in hours declines much more rapidly (with ψ) precisely because of the extra flexibility on the extensive margin. In fact this quickly leads to the elimination of hours as an important source of labour input variation and an overreliance on variation in employment. This variation in employment reflects the variability in unemployment brought about by the increase in job destruction. The lack of variation in hours renders as irrelevant the otherwise plausible employment-hours correlation obtained at high values of ψ. At low values of ψ (close to the values used in the unit elastic calibration of **Fix** W) the absence of wage variation leads to high variation in hours, and a negative correlation between hours and employment, even though the correlation of unemployment and vacancies and of job creation and job destruction are reasonable. This suggests an intermediate value of ψ might be a reasonable compromise. Unfortunately, at these intermediate values the job-creation - job destruction and unemployment-vacancies correlations would be implausible.

Endogenous Job Destruction & Flexible Wages The only environment which appears to deliver plausible values for the hours-employment, unemployment-vacancies and job creation-job destruction correlations is the baseline **NKS** parameterisation of the model with endogenous job destruction and flexible wages - see Figure (6). As noted above, this generates plausible variation in job creation and job destruction, but insufficient variation in unemployment and vacancies. As ψ rises from this unit elastic case towards the limit case **Fix** H the pattern of overreliance on job destruction and a rapid, unrealistic elimination of variation in hours occurs. The correlation of job flows and of unemployment and vacancies is also adversely affected at intermediate ψ. However, unlike the fixed wage cases just considered, wage variability appears to attenuate the response of hours at low $\psi$and so hours and employment exhibit plausible correlation. A key difficulty with the unit elastic baseline calibration is the degreee of real wage variation that it implies. This suggests that the introduction of a limited amount of real wage rigidity, might be a useful extension.

4.4 Role of Capital and Capital Adjustment Costs

Besides introducing an intensive margin for labour input adjustment, the **NKS** model allows a further margin for adjustment - by altering capital stock. I show that the impact of capital accumulation on the quantitative performance of the model depend on the specification of capital adjustment costs, χ. Table (2) & (3) contain entries for models **Fix** K. These attempt to capture alternative approaches to suppressing the effects of capital accumulation. In **Fix** K I eliminate *variation* in aggregate investment by imposing very high capital adjustment costs (I set $\chi = 1000000$, otherwise I follow the calibration of **NKS**) on positive or negative net investment, but retain capital as a component of production. DHRW adopt this approach to the suppression of capital accumulation.

Relative Variability Consider the relative volatilities illustrated in Table (2). In **Fix** K output is only 63% as volatile as in **NKS.** Suppressing investment fluctuations in this way leads to substantially greater variation of labour input (both hours and employment) than in **NKS**. As a consequence, unemployment, vacancies are more variable than in **NKS**. Surprisingly, given the increase in unemployment and vacancy variability, the relative variability of gross job flows is lower than in **NKS**. To draw forth this increased labour input variation the real wage must be substantially more variable in **Fix** K than in **NKS**. This suggests that marginal cost is likely to be more volatile in **Fix** K. Consistent with this view, inflation is substantially more volatile in **Fix** K than in **NKS** and the **Data**. Consumption (expressed as a percentage deviation from steady state) is more volatile in relation to output precisely because investment is non-zero *and* exhibits no cyclical variation so it is no surprise that **Fix** K performs less well at matching the relative volatility of investment and consumption.

Cross Correlations While suppressing capital accumulation limits the ability of the model to account for the relative volatilities of key business cycle variables. By contrast, Table (2) and Figure (3) offer no firm answer as to the impact of capital accumulation, since capital suppression moves the gross job flow and Beveridge curve correlations in opposite directions relative to the data.

Sensitivity Analysis Figure (7) illustrates the sensitivity to variations in the (log of) magnitude of adjustment costs, χ. The empirically plausible region for χ is in the interval 5-25. In the limit, a low adjustment cost environment, $\chi < 0.1$ tends to the no adjustment cost framework of DHRW and Cooley and Quadrini (1999), while the high adjustment cost environment, $\chi > 100$ tends to behave as in the suppressed capital accumulation case of **Fix** K. In the low adjustment cost environment output is around 6 times more volatile than in **NKS,** whereas under high adjustment costs, where investment essentially remains at its steady state value, output is less variable than in **NKS.**

Figure (7) here.

In a high adjustment cost environment, the variability of thge real wage relative to output is roughly twice that observed in a low adjustment cost environment. The variability of hours worked (wrt output) is relatively insensitive to adjustment costs, whereas employment displays much more variability (wrt output) in a high adjustment cost environment. This behaviour merits further discussion.

In a high adjustment cost environment, the marginal cost of changing aggregate capital stock in response to shocks is comparatively high. This reduces the variability of investment, which of itself (holding wage variability constant) would tend to reduce the variability of hours (due to the complementarity of hours and capital at match level), and, in turn, increase employment variability in a high adjustment cost environment. However wages are not constant, so greater employment variation tends to be associated with higher wage variability and the effect of employment variability on wage variability increases the variability of hours (which in turn reduces the extent of any increase in employment variability). In equilibrium, the variability of hours worked is almost unaffected by the size of investment adjustment costs, while employment variability is three times higher in a high adjustment cost environment and wages are twice as high.

The results give the impression that capital adjustment is relatively unimportant in determining the correlation structure of gross flows. However, Figure (7) demonstrates that flexibility of capital adjustment does matter. In a low capital adjustment cost environment the low variability of employment affects gross job flows in three ways. First, low employment variability in a low adjustment cost environment reduces the sum of job creation variability and job destruction variability (which is loosely related to variability of gross job reallocation). A low adjustment cost environment also increases the importance of job destruction variability, relative to job-creation variability. Thirdly, and interestingly, a low adjustment cost environment affects the sign of the correlation of gross flows and that of unemployment with vacancies In fact, Figure (7) shows that, even with variable hours, the correlation of gross flows and that of unemployment with vacancies are both positive. Only for adjustment costs close to and above the baseline calibration and in the limit does the negative relationship between gross flows and between unemployment and vacancies emerge. Fortunately, this interval incorporates the empirically plausible range of adjustment costs. Although the variability of employment relative to output is low in a low adjustment cost environment, nonetheless, the absolute variability of employment is high. This generates substantial variability in job flows. A change (say a reduction) in employment may arise in several ways. The data suggest that a rise in unemployment is brought about by above average job destruction and below average rates of job creation. However, as a matter of accounting, nothing prevents a rise in unemployment being brought about by a decline in job destruction and an even faster decline in job creation or even by a rise in job creation and an even larger rise in job destruction. In contrast to the data, it seems to be the latter which occurs in a low adjustment cost environment.

Inflation is roughly one tenth as volatile with respect to output in a low adjustment cost environment, but five times more volatile in a high adjustment cost environment. In the absence of adjustment costs on investment, it is possible to adjust output substantially in response to demand and productivity shocks without immediately raising marginal costs by increasing investment, so that inflation is relatively stable. By contrast, in a high adjustment cost environment, capital adjustment becomes prohibitively expensive, and in the absence of capital accumulation the diminishing marginal product of labour tends to drive up the marginal cost of production associated with drawing out a given level hours.

5 Conclusions

In this paper I investigated the role of hours in a New Keynesian model with labour market search, matching frictions endogenous job destruction and capital accumulation. This framework, more general than one used heretofore, can be used to address a number of the issues from the literature on labour market search in a full business cycle context. It augments the basic Labour market equilibrium model (Pissarides) with consumption smoothing, capital accumulation, nominal rigidities, monetary innovations and variable interest rates. In this framework, it appears that even in the absence of wage rigidity - and in contrast to the results of Shimer (2005) and Krause and Lubik (2006), the variability of unemployment and vacancies around one half of that observed in the data. This contrasts favourably with the standard equilibrium labour market search model, which as Shimer demonstrates captures only 10% of the variation in unemployment and vacancies.

Motivated by the data-inconsistency of the current consensus in the literature - that endogenous job destruction and variation in hours should be suppressed to capture the cyclical behaviour of unemployment and vacancies, I use the framework developed in Section 2 to consider the interaction of labour supply elasticity, wage rigidity and endogenous job destruction. I find that under wage rigidity and with exogenous job destruction, a relatively elastic labour supply produces implausible behaviour. While vacancies and unemployment display sufficient variability in this environment, the associated variation in employment is too great, and is offset by a similarly excessive variation in hours. So employment and hours would exhibit strong negative correlation.

By allowing endogenous job destruction while suppressing wage rigidity, and calibrating to highly inelastic labour supply, I am able to reproduce results akin to those of Krause and Lubik (2006) and Shimer (2005) on the slope of the Beveridge curve - but this is achieved by eliminating cyclical variation in hours. It arises from excessive variation in job destruction. By contrast, a unit-elastic labour supply instead creates excessive variation in hours, and a negative correlation between hours and employment. In addition the transition from use of intensive margin to extensive margin is much more abrupt if job destruction is endogenous. When variability in wages and job destruction are permitted together, then the switch from variation on intensive margin (hours) to extensive margin (job destruction) still arises, and inelastic labour supply produces realistic unemployment and vacancy volatility only at the cost of overvolatile job destruction which distorts the slope of the Beveridge curve. However, with near unit-elastic labour supply, as in the baseline calibration of NKS, firms tend to rely to a greater and more realistic extent on hours and employment variation in conjunction. Variation in wages tends to temper hours variation, so that hours and employment exhibit plausible positive correlation. While unemployment and vacancies only display around half of the volatility observed in US data, the correlation of job creation and job destruction and even of unemployment and vacancies are plausible. The main problem with the NKS calibration is that the wage is too volatile. This suggests that in future work it may be worth introducing some element of rigidity into wages. I find that the speed at which capital accumulation can react to shocks plays a key role in determining the slope of the Beveridge curve. Only above capital adjustment costs of around 10 (an empirically plausible value) does the Beveridge curve attain a plausible negative slope. Yet if capital adjustment costs become too large ($\chi > 100$) then capital stock is effectively fixed, and this drives up the variability of employment and wages.

In summary, my analysis confirms and refines the role played by labour market search in economic fluctuations by embedding this feature in a more complete business cycle framework that might be used for policy analysis. I leave that topic for future research.

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6 Appendix A: Calibration & Model Solution Method

6.1 Labour Market Flows

I specify the following labour market parameters ρ, N, κ^V, ρ^x, σ_X, e and γ. I use this information to compute $F\left(\bar{X}\right)$, $\bar{X}$, $\mathcal{F}$, U/N, V/N, JC, JD, $\mathfrak{M}$.

Following DHRW I assume that 10% of employment relationships separate each quarter: $\rho = 0.1$. I also set $N = 0.9$, lower than DHRW (0.94) but higher (and more realistic) than T (0.75) and Andolfatto (0.54). From equation (15) the ratio of searchers to employment U/N is $\frac{U}{N} = \frac{1}{N} - (1-\rho)$. Therefore the ratio of workers searching for jobs to employed workers, U/N, is 0.211. In steady state, the employment evolution equation (17) can be written as, $\rho = \frac{\mathcal{M}}{N}$, while from (18): $\kappa^V = \frac{\mathcal{M}}{V}$. I set $\kappa^V = 0.7$, as in DHRW. Therefore $\frac{V}{N} = \frac{\rho}{\kappa^V} = \frac{1}{7}$.

In steady state the probability of separation, ρ, can be written as $\rho = \rho^x + (1-\rho^x)\left(1 - F\left(\bar{X}\right)\right)$. Following DHRW, I assume that the probability of exogenous separation, $\rho^x = 0.068$. So the probability of endogenous separation is $1 - F\left(\bar{X}\right) = (\rho - \rho^x)/(1-\rho^x) = 0.034$. To compute $\bar{X}$ and $\mathcal{F}$, we assume that X is lognormally distributed (μ_X, σ_X). I normalise $\mu_X = 1$. There is little empirical evidence to guide the choice of σ_X. DHRW set $\sigma_X = 0.1$, W1 and W2 set $\sigma_X = 0.12$ but X enters multiplicatively in their models, and in *Base* this generates too much variability in labour flows data. I set $\sigma_X = 0.08$ to match the variability of labour market flows data. Then, in steady state, the threshold value of the idiosyncratic cost shock is $\bar{X} = F^{-1}(0.034) = 1.157$. The elasticity $\epsilon_{\bar{X}} = 0.716$. I set $\mathcal{F}$ to equal the average cost shock for those matches that produce in steady state, $\mathcal{F} = \frac{E\left[X|X<\bar{X}\right]}{F\left(\bar{X}\right)} = 0.996$, so that the cost shock does not impact on steady state wholesale production.

In steady state, the job creation and job destruction rates are equal, so $JC = JD = \rho - \rho^x\kappa^V = 0.052$, consistent with DHRW.

The steady state condition for the employment evolution equation (17) is

$$\rho = \mathfrak{M}\left(\frac{eU}{N}\right)^{\gamma}\left(\frac{V}{N}\right)^{1-\gamma}.$$

I set the parameter γ in the matching function at 0.6, in the light of Petrongolo and Pissarides survey of empirical results and $e = 0.5H$. Then $\mathfrak{M} = 1.231$.

6.2 Preferences

The preferences of the representative household are characterised by the parameters β, ψ, H, ξ, κ_C, C, ϕ, Λ and κ_H. The final two are computed from the steady state of the system.

Under the assumption that 1 period represents 1 quarter I set the discount factor $\beta = 0.989$, in line with the rest of the literature. I assume that a worker has a unit time endowment and in steady state spends a third of her time working: $H = 1/3$. For the preferences specified, the elasticity of labour supply, ϵ_H, is $\frac{1}{\psi}\left[\frac{1-H}{H}\right]$. Estimates of this elasticity vary with gender and other variables, but is typically less than unity. I take the value $\psi = 2$ as a baseline, which gives $\epsilon_H = 1$. The elasticity, ξ, of demand for real money balances with respect to the marginal utility of consumption is set at 1. In line with CEE, I set the curvature parameter for the instantaneous utility function, $\phi = 1$, and $\kappa_C = 0.5$. I also normalise the steady state value of the aggregate consumption index to $C = 1$, then from the steady state version of (27), $\Lambda = \frac{1-\beta\kappa_C}{((1-\kappa_C)C)^{\phi}} = 1.011$. Computation of κ_H will be discussed below.

6.3 Capital Accumulation

The rate of depreciation, δ, is set at the satndard value of 0.025. In the baseline parameterisation adjustment costs are set to $\chi = 12$. In steady state, from the capital accumulation equation (25), we that $I/K = \delta$. From the steady state of equation (29) $Q = \Lambda$. In steady state, the capital euler equation (30) determines the real rental rate on capital as $R^K/P = \beta^{-1} - (1-\delta) \simeq 0.036$.

6.4 Price Rigidity & Price Setting

Calibration of nominal rigidities and price setting by retailers involves specification of ω, τ and ε.

The extent of nominal rigidity in the goods market is determined by ω, which captures the fraction of final goods firms in any period that do not adjust their price and τ which refers to the fraction of final goods firms which set prices in a backward-looking manner. Empirical evidence from studies using aggregate data suggests that prices last for 9-12 months on average corresponding to $\omega \in [2/3, 3/4]$. I take $\omega = 3/4$ as a baseline value. Following Gali and Gertler's estimates I set $\tau = 0.5$.

From steady state of forward-looking price setters behaviour, the steady state mark up as $\mu = \frac{\varepsilon}{(\varepsilon-1)}$. I assume that, ε, the elasticity of demand equals 6, so $\mu = 1.2$.

6.5 Production, Bargaining and Equilibrium

I set ρ_Z, ρ_υ, α and η. Then using (7), (12), (13), (25) (34), (35) and (36) I compute κ, κ_H, A, K, $W/P\ Y$ and C/Y.

Following Cooley and Quadrini (1999), the money supply growth process is assumed to follow an AR(1) process with the autoregressive parameter $\rho_\upsilon = 0.5$, with mean zero normally distributed innovations with standard deviation $\sigma_\upsilon = 0.006$, while aggregate productivity also follows an AR(1) process, with $\rho_Z = 0.95$. I normalise the steady state value of Z to unity. The standard deviation of productivity is chosen in order to match the standard deviation of US quarterly HP-detrended GDP data. I set the elasticity of output with respect to capital, $\alpha = 1/3$, and worker bargaining power (and share of the match surplus) $\eta = 0.6$.

$$K = (1-\rho^x)\, F\left(\bar{X}\right) N \left[\frac{\alpha A}{\mu R^K/P}\right]^{\frac{1}{1-\alpha}} H. \tag{37}$$

Combining (25), (35) and (36) with the definition of the auxilliary price index, $\tilde{P}$, and the capital market equilibrium condition we have the steady state condition

$$Y = C + \delta K = \left(\frac{\tilde{P}}{P}\right)^{\varepsilon} \left[A\left[(1-\rho^x)\, F\left(\bar{X}\right) NH\right]^{1-\alpha} K^{\alpha} - \kappa\mu V\right] \tag{38}$$

This can be combined with (37) to give

$$A^{\frac{1}{1-\alpha}} = \frac{C + \kappa\mu V\left(\frac{\tilde{P}}{P}\right)^{\varepsilon}}{\left(\frac{\alpha}{\mu R^K/P}\right)^{\frac{1}{1-\alpha}} \left[\left(\frac{\tilde{P}}{P}\right)^{\varepsilon} \frac{\mu R^K}{\alpha P} - \delta\right] (1-\rho^x)\, F\left(\bar{X}\right) NH}. \tag{39}$$

The steady state versions of (12) and (7) combined with (2) are

$$\frac{\kappa}{\kappa^V} = \beta\,(1-\rho^x)\left[F\left(\bar{X}\right)\left[\frac{\kappa}{\kappa^V} + (1-\alpha)\left(\frac{A}{\mu}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{R^K/P}\right)^{\frac{\alpha}{1-\alpha}} H\right] - \frac{WH}{P}\right] \tag{40}$$

$$\frac{WH}{P} = \left[\eta\left[(1-\alpha)\left(\frac{A}{\mu}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{R^K/P}\right)^{\frac{\alpha}{1-\alpha}} H + \kappa\frac{V}{U}\right] - \frac{(1-\eta)}{\Lambda}\left[\frac{(1-e)^{1-\psi}}{1-\psi} - \kappa_H\frac{(1-H)^{1-\psi}}{1-\psi}\right]\right] \tag{41}$$

Combining (40) and (41) to eliminate $\frac{WH}{P}$

$$\kappa\left[\begin{array}{c}\frac{1}{\kappa^V}-\frac{\beta(1-\rho^x)F(\bar{X})}{\kappa^V}\\ +\eta\beta\left(1-\rho^x\right)F\left(\bar{X}\right)\end{array}\right]=\left(1-\eta\right)\beta\left(1-\rho^x\right)F\left(\bar{X}\right)\left[\begin{array}{c}\left(1-\alpha\right)\left(\frac{A}{\mu}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{R^K/P}\right)^{\frac{\alpha}{1-\alpha}}H\\ -\frac{1}{\Lambda}\left[\frac{(1-e)^{1-\psi}}{1-\psi}-\kappa_H\frac{(1-H)^{1-\psi}}{1-\psi}\right]\end{array}\right] \quad (42)$$

Combining (39) and the steady state version of (13) to eliminate A gives

$$\frac{1-\eta}{\Lambda}\left[\frac{(1-e)^{1-\psi}}{1-\psi}-\kappa_H\frac{(1-H)^{1-\psi}}{1-\psi}\right]=\left[\begin{array}{c}\eta\kappa\frac{V}{U}-\frac{\kappa}{\kappa^V}-\left(1-\eta\right)\frac{\mathcal{F}-\bar{X}}{\mu}\\ -\left(1-\eta\right)\left(1-\alpha\right)\frac{C+\kappa\mu V\left(\frac{\tilde{P}}{P}\right)^{\varepsilon}}{\left[\left(\frac{\tilde{P}}{P}\right)^{\varepsilon}\frac{\mu R^K}{\alpha P}-\delta\right]\left(1-\rho^x\right)F\left(\bar{X}\right)N}\end{array}\right] \quad (43)$$

Next, substituting (39) and (43) in (42), to elinate A and κ_H and gives the following expression for κ in terms of known parameters:

$$\kappa=-\beta\left(1-\rho^x\right)F\left(\bar{X}\right)\left(1-\eta\right)\left(\frac{\mathcal{F}-\bar{X}}{\mu}\right)\kappa^V=0.04 \quad (44)$$

Combining this value for κ with (43) determines $\kappa_H = 1.370$. Then using (41) $W/P = 2.559$. Finally, substituting for κ in (39) $A = 1.610$, from (37) $K = 8.721$ and from (38) $Y = 1.218$. It follows that $C/Y = 0.821$.

6.6 Model Solution Method

The log-linearsied approximation to the system of equations, (2), (7) and (11) - (36), is stacked in the form

$$\mathcal{A}E_t\left[\mathcal{Y}_{t+1}\right]=\mathcal{B}\cdot\mathcal{Y}_t+\mathcal{C}\cdot\mathcal{Z}_t$$

Where $\mathcal{Z}_t$ is a vector of exogenous state variables ($\hat{z}_t$ and $\hat{v}_t$) and $\mathcal{Y}_t$ is a vector of endogenous jump ($\hat{y}_t$, $\hat{c}_t$, $\hat{\imath}_t$, $\hat{h}_t$, $\hat{u}_t$, $\hat{v}_t$, $\hat{j}c_t$, $\hat{j}d_t$, $\hat{p}_t$, $\hat{w}_t$, $\hat{\mu}_t$, $\hat{\pi}_t$, $\hat{r}^n_t$, $\hat{r}^K_t$, $\hat{m}_t$, $\hat{q}_t$, $\hat{\kappa}^V_t$, $\hat{x}_t$, $\hat{\lambda}_t$) and state ($\hat{k}_t$, $\hat{n}_t$, $\hat{c}_{t-1}$, $\hat{\pi}_{t-1}$, $\hat{m}_{t-1}$, $\hat{p}_{t-1}$) variables, and $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are conformable matrices of coefficients.[14] The system is solved with MATLAB, version 7.0.1, using McCallum's (1998) undetermined coefficients approach to solving linear RE models based on Klein's (1997) generalised Schur decomposition method.

[14]The full system, $\mathcal{Y}_t$, includes a definition of the inflation rate in terms of the price index ($\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$)) updating equations for $\hat{c}_{t-1}$, $\hat{\pi}_{t-1}$, $\hat{m}_{t-1}$ and $\hat{p}_{t-1}$, and additional auxilliary variables including labour market tightness $\hat{\theta}_t = \hat{v}_t - \hat{u}_t$, real wages, output per worker.

Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
ρ	0.1	γ	0.6	ω	0.75
N	0.9	β	0.989	τ	0.5
κ^V	0.143	ϕ	1	ε	6
ρ^x	0.068	κ_C	0.5	α	0.3
σ_X	0.08	C	1	η	0.6
e	0.165	ξ	1	ρ_Z	0.95
H	0.333	χ	12	ρ_v	0.49
ψ	2	δ	0.025	σ_v	0.006

Table 1: Calibration: Assigned Parameters

Standard Deviation w.r.t. GDP	**Data**	**NKS**	**NK**	**Fix H**	**Fix W**	**Fix K**	**No K**	**Freq**
Output	1.00	1.00	0.94	1.00	0.98	0.60	0.71	1.00
Investment	2.81	2.84	2.87	2.86	2.42	-	-	2.60
Consumption	0.81	0.65	0.66	0.64	0.73	1.22	1.00	0.69
Hours per worker	0.57	0.79	1.35	-	1.13	0.98	0.53	0.58
Employment	0.68	0.73	-	0.97	2.02	1.10	0.77	0.36
Unemployment	7.50	3.89	-	7.37	7.37	5.44	3.89	2.20
Vacancies	8.36	2.53	-	10.48	10.65	4.74	3.09	1.66
Job Creation	5.29	5.85	-	6.21	15.28	4.72	3.04	3.67
Job Destruction	9.32	6.01	-	21.66	11.97	4.06	3.05	4.42
Wages	0.41	1.63	3.22	1.34	-	2.24	1.79	1.34
Inflation	0.22	0.31	0.41	0.27	0.24	0.53	0.39	0.51

Table 2: Business Cycle Statistics

Correlation	**Data**	**NKS**	**Fix H**	**Fix W**	**Fix K**	**No K**	**Freq.**
Job Creation - Job Destruction	-0.41	-0.58	0.31	-0.57	-0.67	-0.63	-0.14
Unemployment - Vacancies	-0.94	-0.40	-0.11	-0.59	-0.44	-0.51	-0.06

Table 3: Business Cycle Statistics

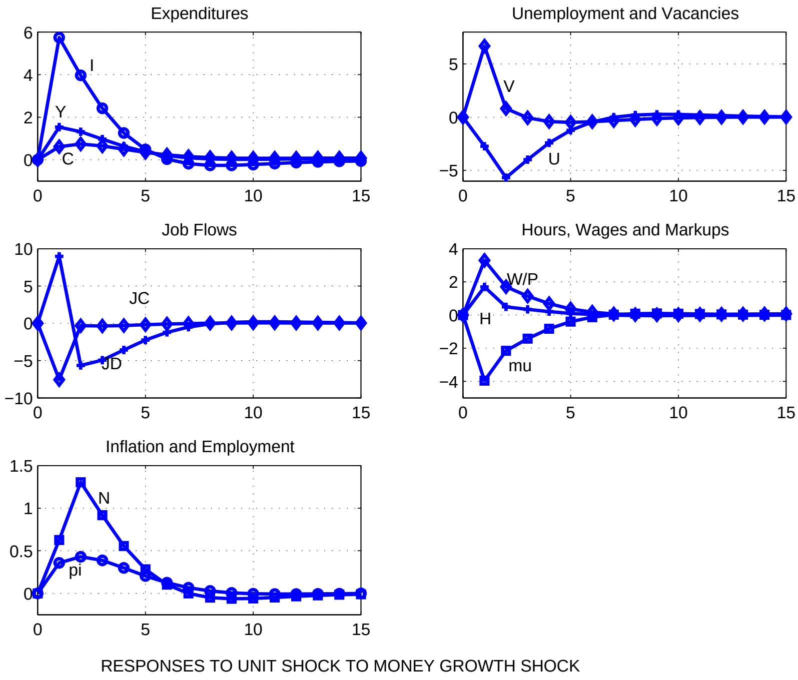

Figure 1:

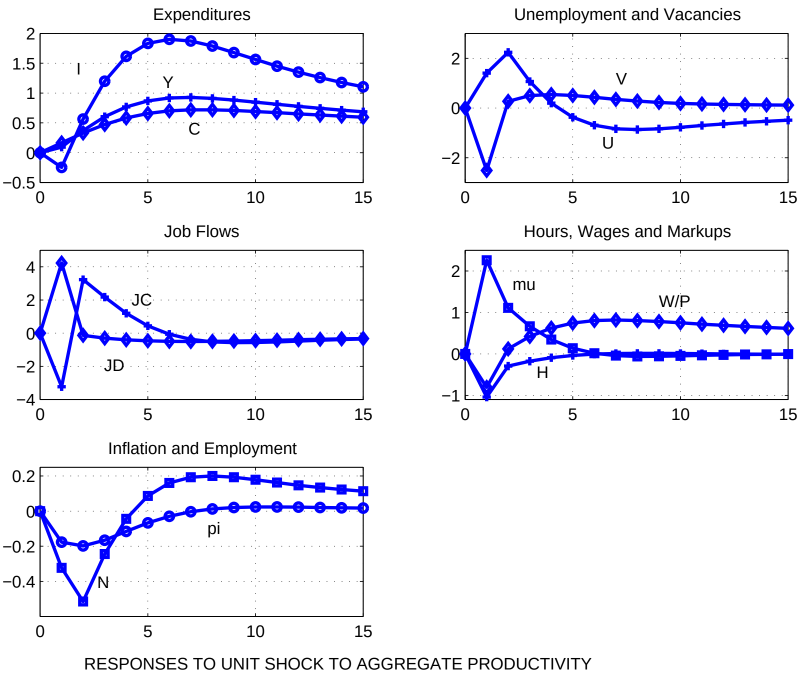

Figure 2:

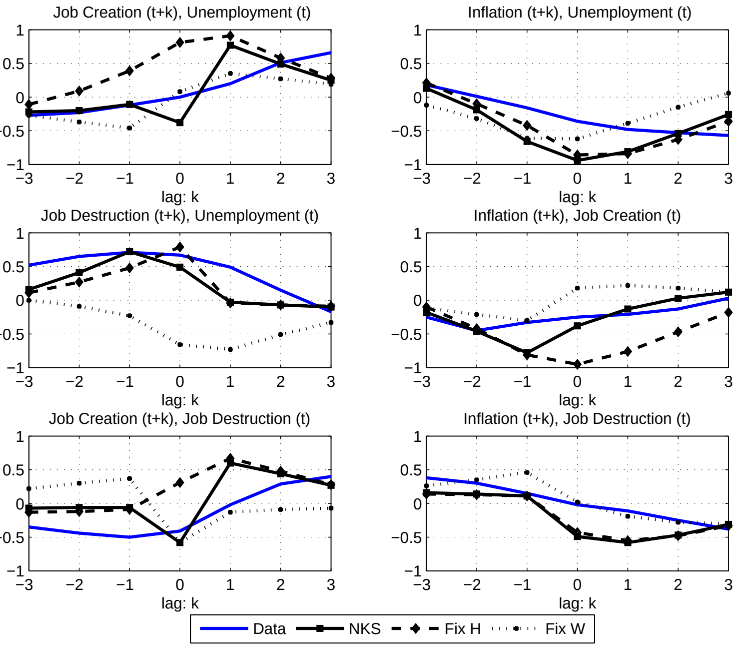

Figure 3: Dynamic Correlation Structure

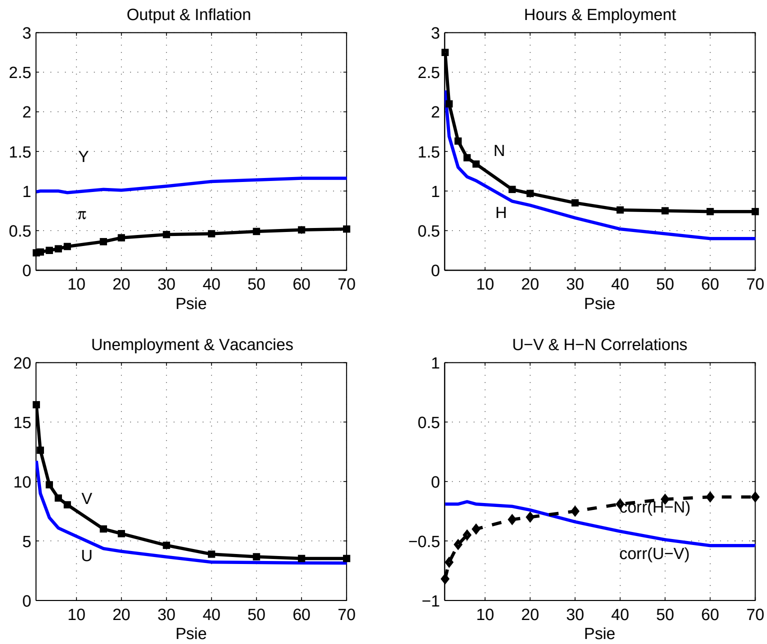

Figure 4: Rigid Wage & Exogenous Job Destruction

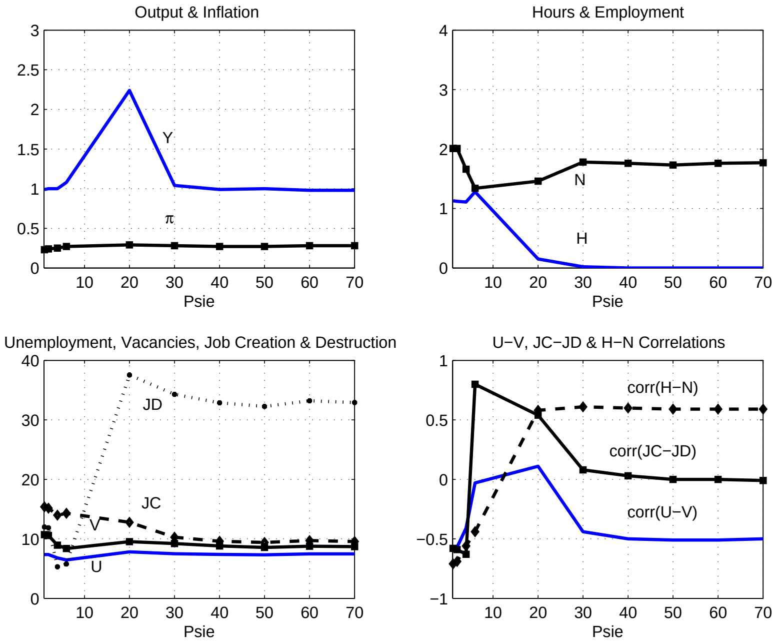

Figure 5: Rigid Wage and Endogenous Job Destruction

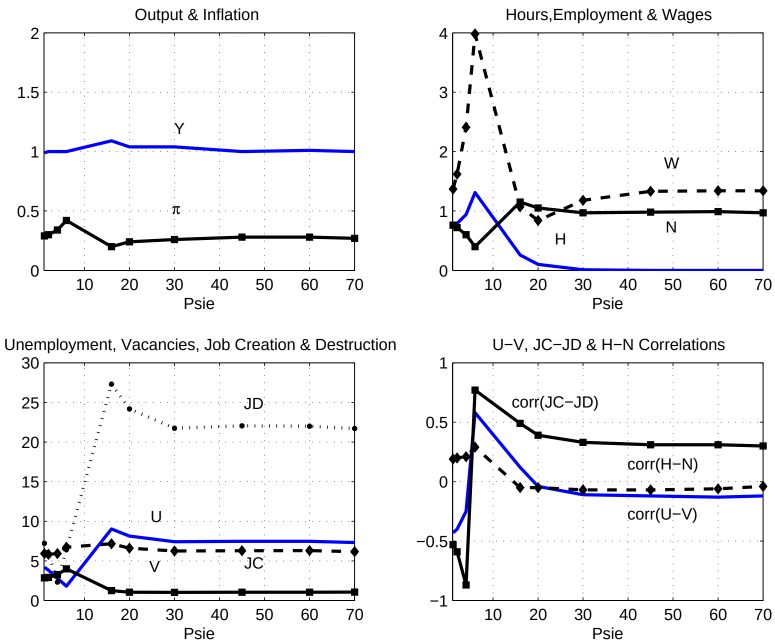

Figure 6: Flexible Wage and Endogenous Job Destruction

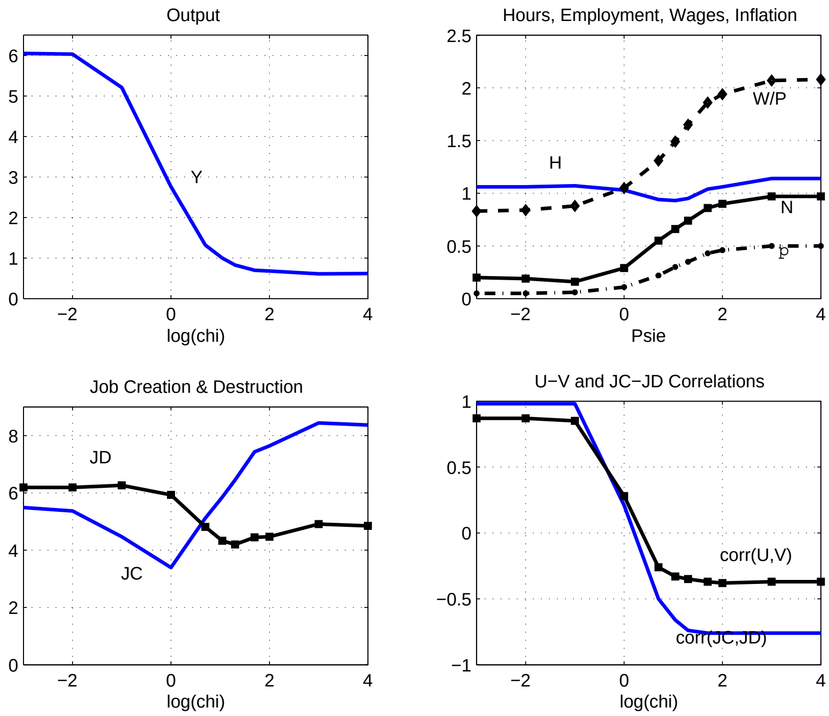

Figure 7: Capital Adjustment Costs