Monetary Policy Amplification Effects through a Bank Capital Channel

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Abstract

This paper improves the analysis of the role of financial frictions in the transmission of monetary policy, by bringing together the borrowers' balance sheet channel with an additional channel working through bank capital, considering capital adequacy regulations and households' preferences for liquidity.

Detailing a dynamic new Keynesian general equilibrium model, in which households require a (countercyclical) liquidity premium to hold bank capital, we find that the introduction of bank capital amplifies monetary shocks to the macroeconomy through a liquidity premium effect on the external finance premium. This effect, together with the financial accelerator, generates quantitatively large amplification effects.

Keywords: Bank capital channel; Bank capital requirements; Financial accelerator; Liquidity premium; Monetary transmission mechanism

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1 Introduction

The precise mechanisms through which monetary policy affects real activity are not completely clarified. Some recent literature explores the possible explanation of monetary policy effects through financial imperfections. Our work fits in this line of research by centering its attention on how microeconomic structures, such as the bank funding structure and the relationship between the bank and the borrower, interact with macroeconomic business conditions. In particular, we contribute to clarify the role of bank capital in lending conditions and, consequently, in the transmission of monetary policy.

The existence of empirical evidence that the bank funding structure, or, more specifically, the bank capital, affects its supply of loans and, consequently, real activity, has motivated our modelization of the banking relationships in the context of a dynamic general equilibrium model.

First, one strand of this empirical literature indicates that lending growth after a monetary policy shock depends on the level of bank capital: using U.S. data from 1980 to 1995, Kishan and Opiela (2000) predict that poorly capitalized banks experience more significant declines in their lending, following monetary contractions. Their results are in line with Van den Heuvel (2002b), which, also using U.S. data, from 1969 until 1995, shows that the real effects of monetary policy are stronger when banks have low capital relative to the existing bank capital requirements: when a U.S. state's banking sector starts out with a low capital-asset ratio, subsequent output growth in that state is more sensitive to changes in the Federal funds rate or other indicators of monetary policy (namely, the Bernanke-Mihov indicator). Also in this line of research, Hubbard *et al.* (2002) find that, even after controlling for information costs and borrower risk, the cost of borrowing from low-capital banks is higher than the cost of borrowing from well-capitalized banks; that is, capital positions of individual banks affect the rate at which their clients borrow.

Second, as mentioned by Van den Heuvel (2002a), the importance of bank capital on lending has increased since the implementation of the 1988 international Basel Accord, which, by imposing risk-based capital requirements, limits the ability of banks with a shortage of capital to supply loans. There is a considerable number of papers that test the hypothesis of a "credit crunch" - a significant leftward shift in the supply curve for bank loans - that may have occurred in the U.S. during the early 90s, simultaneously with the implementation of the new banking system regulation established by the Basel Accord. The idea behind those studies is that, given the common perception that bank capital is more costly than alternative funding sources (such as deposits), regulatory capital requirements can have real effects: in order to satisfy those requirements, banks may choose to reduce loans and, in such an event, otherwise worthy borrowers cannot obtain them. The allocation of credit away from loans can, in turn, potentially cause a significant reduction in macroeconomic activity, given that many borrowers cannot easily obtain substitute sources of funding in public markets. On this credit crunch literature, see, for instance, Bernanke and Lown (1991), Peek and Rosengren (1995, 2000), and Sharpe (1995) for a review.[1]

Some studies in this literature are, however, quite skeptical that the credit crunch played a major role in worsening the 1990 recession in the U.S. (e.g., Bernanke and Lown, 1991): they suggest that demand factors, including the weakened state of borrowers' balance sheet, were instead the major cause of the lending slowdown. In contrast, Peek and Rosengren (2000), for instance, focusing on the strong downward pressure on capital positions of Japanese banks with branches in the U.S., identified an independent loan supply disruption and established that this shock had substantial effects on real economic activity in the U.S..

This controversy illustrates one of the major difficulties of this type of analysis: it is hard to distinguish between movements in loan demand and movements in loan supply, especially because, as mentioned by Van den Heuvel (2002b), there is no interest rate summarizing the effective cost of financing, since this cost depends not only on the contractual interest rate, but also on collateral requirements, the extent of rationing, and other contractual features. Therefore, although there is some evidence that bank capital affects bank lending and, consequently, real activity, these studies are not completely successful in distinguishing loan demand shifts from loan supply shifts, leaving the question of the relative importance of different effects unanswered.

Our model contributes to evaluate the relative importance of these loan supply and demand effects, by bringing together the borrowers' balance sheet channel developed by Bernanke *et al.* (1999), with an additional channel working through bank capital, which also amplifies the real effects of exogenous nominal and real shocks. That is, taking the Bernanke *et al.*'s dynamic general equilibrium model as a starting point, we add banks that face financial frictions when raising funds - due to the imposition of regulatory capital requirements and risk sensitive deposit insurance rates.

Theoretically, our model has been decisively motivated by Bernanke *et al.* (1999)'s suggestion, in their concluding remarks, to introduce the specific role of banks in cyclical fluctuations. Although excluding some bank activities, for simplicity, our model explicitly assumes the role of banks in financing two entities in financial deficit (the public sector and nonfinancial firms) using the funds of households (the entity in financial surplus). Based on the Basel Accord of 1988, we assume that, beside the risk sensitive deposit insurance rate, banks are constrained by a risk-based capital ratio requirement according to which the ratio of bank capital to nonfinancial loans cannot fall below a regulatory minimum exogenously set. Banks are, thus, limited

[1] See also Jackson *et al.* (1999) for a broader survey of studies that analyze the effects of the Basel Accord of 1988.

in their lending to nonfinancial firms by the amount of bank capital that households are willing to hold. Taking into account that bank capital is more expensive to raise than deposits, due to households' preferences for liquidity, and that this difference tends to widen (narrow) during a recession (expansion), we explore the additional channel through which the effects of exogenous shocks on real activity are amplified - the bank capital channel. We borrow this denomination from another approach, by Van den Heuvel (2002a), on the role of bank capital in the transmission of monetary policy.

Related theoretical literature

The theoretical literature distinguishes three channels of propagation of the effects of monetary policy, through mechanisms related to financial imperfections: *(i)* the bank lending channel, based on reserve requirements by monetary authorities; *(ii)* the borrowers' balance sheet channel, focusing on borrowers' financial position and, more recently, *(iii)* the bank capital channel.

Our model abstracts, for simplicity, from the bank lending channel (as Bernanke *et al.*, 1999) to focus on the other two - it combines loan demand shifts arising from the borrowers' balance sheet channel with loan supply shifts related to the bank capital channel. Since the borrowers' balance sheet channel has been more extensively studied,[2] we focus this brief review on the bank-capital-channel theoretical models.

These models can be divided into two groups: one that focus on bank *market* capital requirements and another based on bank *regulatory* capital requirements. Chen (2001) and Meh and Moran (2004) belong to the first group and are built upon Holmstrom and Tirole (1997) formulation that features two sources of moral hazard (between banks and borrowers and between banks and depositors). In particular, according to Meh and Moran's model, a contractionary monetary policy raises the opportunity cost of the external funds that banks use to finance investment projects and leads the market to require banks and firms to finance a larger share of investment projects with their own net worth. Since banks and firms' net worth is largely predetermined, bank lending must decrease to satisfy those market requirements, thereby leading to a decrease in investment. This mechanism implies a decrease in banks and firms' earnings and, consequently, a decrease in banks and firms' net worth in the future. Therefore, there is a propagation of the shock over time after the initial impulse to the interest rate has dissipated.

Among the studies in the second group, *i.e.*, assuming *regulatory* bank capital requirements, are Blum and Hellwig (1995), Repullo and Suarez (2000), Bolton and Freixas (2001), Chami and Cosimano (2001), Van den Heuvel (2002a, 2005) and Berka and Zimmermann (2004).

According to Van den Heuvel (2002a)'s model, an increase in funds rate (due to a contrac-

[2] See Bernanke (1993) and Bernanke and Gertler (1995) for a review.

tionary monetary policy) and, consequently, an increase in bank's cost of funding, leads to a decrease in bank's profits, given the maturity mismatch on the bank's balance sheet. This, in turn, weakens the bank's future capital position, increasing the likelihood that its lending will be constrained by an inadequate level of capital. Therefore, new lending overreacts to the monetary policy shock, when compared to a situation of unconstrained banks. As mentioned above, Van den Heuvel refers to this channel by which monetary policy influences the supply of bank loans through its impact on bank capital as the bank capital channel. The strength of this channel depends on the capital adequacy of the banking sector and the distribution of capital across banks. In particular, lending by banks with low capital is delayed and then amplified in reaction to interest rate shocks, relative to well capitalized banks.

Bolton and Freixas (2001), Chami and Cosimano (2001) and Berka and Zimmermann (2004) also assume regulatory capital requirements, but, in contrast with Van den Heuvel (2002a), consider the possibility of bank capital issuance.

Capital issuance may involve costs, as in Bolton and Freixas (2001), which consider a cost of outside capital for banks by assuming information dilution costs in issuing bank capital: outside equity investors, having less information about the profitability of bank loans, will tend to misprice the capital issues of the most profitable banks. In this context, the presence of regulatory (and binding) capital requirements may magnify the effects of a contractionary monetary policy, since this policy may trigger a decrease (or prevent an increase) in bank capital: a contractionary monetary policy may render bank loans insufficiently lucrative when information dilution costs in issuing bank capital are taken into account.

According to Berka and Zimmermann (2004), when an initial negative shock hits the economy, bank capital becomes more risky and households channel their savings away from capital and into deposits. The banks are then squeezed by the (binding regulatory) capital requirements and have to decrease their loan supply and invest more into government bonds. Without capital requirements, banks could supply more loans, in principle, by charging even higher loan rates, and entrepreneurs would still be ready to pay these rates.

In a recent paper, Van den Heuvel (2005) quantifies the welfare costs of bank capital requirements by embedding the role of liquidity creation by banks in a general equilibrium model, with no aggregate uncertainty. The households' preferences for liquidity play here a major role: equilibrium asset returns reveal the strength of these preferences and allow the quantification of the ("*neither trivial nor gigantic*", according to the author) welfare costs of bank capital requirements. Regulators, thus, face a trade-off between keeping the effective capital requirement ratio as low as possible while keeping the probability of bank failure acceptably low.

Our model relates to this literature by accounting for the interactions between bank capital and macroeconomic conditions. We assume that banks issue capital to satisfy regulatory capital requirements (as Bolton and Freixas, 2001, Chami and Cosimano, 2001 and Berka and Zimmermann, 2004) and that households' preferences for liquidity matter for banks' funding structure. By doing this, our model yields a liquidity premium effect on the external finance premium, a mechanism through which bank capital affects the transmission of monetary policy to the real economy.[3] This additional mechanism is able to account for a substantial amplification of the immediate effects of a monetary policy shock.

The paper is organized as follows. After this introduction, section 2 develops and calibrates a new Keynesian dynamic general equilibrium model, with particular attention to the banking relationships with entrepreneurs and households. Section 3 simulates a monetary policy shock under several variants of the model, in order to analyze the role of bank capital in the monetary policy transmission mechanism and the relative importance of demand and supply-of-loans effects. These simulations are subject to some sensitivity and robustness checks, namely the inclusion of habit formation and different investment adjustment costs, technology and government shocks, and differences in deposit insurance parameters. Section 4 offers some preliminary conclusions on this ongoing research.

2 The Model

In order to analyze the role of bank capital in the transmission mechanism of monetary policy, we develop now a dynamic general equilibrium model, assuming five types of agents in the economy:

- Households, who work, consume and invest their savings in bank deposits and bank capital;
- Entrepreneurs, who need external (bank) finance to buy capital, which is used in combination with hired labor to produce (wholesale) output;
- Banks, which, using the funds of households, finance and monitor (*ex post*) the entrepreneurs;
- Retailers, added in order to incorporate inertia in price setting;
- Government, which conducts both monetary and fiscal policy and regulates banks.

[3] Our acceptation of the bank capital channel, thus, differs from Van den Heuvel (2002a)'s, in which households and liquidity preferences are absent.

2.1 Entrepreneurs

The analysis of entrepreneurs' behavior follows very closely the model of Bernanke, Gertler and Gilchrist (1999), BGG hereafter.

In each period each entrepreneur buys capital to his firm in order to, in combination with labor, produce output in the next period. More specifically, at time t, entrepreneur j purchases homogeneous capital[4] for use at $t+1$, K_{t+1}^{j}. The return to capital is sensitive to both aggregate and idiosyncratic risk. The *ex post* gross return on capital for firm j is $\omega_{t+1}^{j} R_{t+1}^{K}$, where ω_{t+1}^{j} is an idiosyncratic disturbance to firms j's return and R_{t+1}^{K} is the *ex post* aggregate return to capital. The random variable ω^{j} is independently and identically distributed (i.i.d.) across time and across firms, with a continuous and once-differentiable cumulative distribution function (c.d.f.), $F(\omega)$, over a non-negative support, and $E\left(\omega^{j}\right)=1$.

At the end of period t, entrepreneur j has available net worth N_{t+1}^{j} which he entirely uses to finance the acquisition of K_{t+1}^{j}. To finance the difference between his expenditures on capital goods and his net worth, he must borrow an amount $L_{t+1}^{j}=Q_{t} K_{t+1}^{j}-N_{t+1}^{j}$, where Q_{t} is the price paid per unit of capital at time t. Each entrepreneur then borrows from a financial intermediary (bank) which imposes a required return on lending between t and $t+1$, R_{t+1}^{F}. This relationship embodies an asymmetric information problem between each entrepreneur and the bank: only the entrepreneur observes costlessly the return of his project. The financial contract, established between these two agents, is then designed to minimize the expected agency costs. That is, as in BGG, we assume a costly state verification (CSV) problem, in which the bank must pay a fixed monitoring cost in order to observe an individual borrower's realized return.[5] This monitoring cost is assumed to equal a proportion μ of the realized gross payoff of the firm's capital:

$$\mu \omega_{t+1}^{j} R_{t+1}^{K} Q_{t} K_{t+1}^{j},$$

where $0<\mu<1$. The idiosyncratic disturbance ω_{t+1}^{j} is unknown to both the entrepreneur and the bank prior to the investment decision.[6] After the investment decision is made, the bank can only observe ω_{t+1}^{j} by paying the monitoring cost.

Given $Q_{t} K_{t+1}^{j}$, L_{t+1}^{j} and R_{t+1}^{K}, the optimal contract is characterized by a gross non-default loan rate, Z_{t+1}^{j}, and a cutoff value $\bar{\omega}_{t+1}^{j}$, such that, if $\omega_{t+1}^{j} \geqslant \bar{\omega}_{t+1}^{j}$, the borrower pays the lender the amount $\bar{\omega}_{t+1}^{j} R_{t+1}^{K} Q_{t} K_{t+1}^{j}$ and keeps the remaining $\left(\omega_{t+1}^{j}-\bar{\omega}_{t+1}^{j}\right) R_{t+1}^{K} Q_{t} K_{t+1}^{j}$. That is, $\bar{\omega}_{t+1}^{j}$ is defined by

[4] It is assumed that each entrepreneur purchases his entire capital stock each period.

[5] It is also assumed that households cannot observe the outcome of entrepreneurs' projects at any cost.

[6] That is, $Q_{t} K_{t+1}^{j}$ and L_{t+1}^{j} are chosen prior to the realization of the idiosyncratic shock.

$$\overline{\omega}^j_{t+1} R^K_{t+1} Q_t K^j_{t+1} = Z^j_{t+1} L^j_{t+1}. \tag{1}$$

If $\omega^j_{t+1} < \overline{\omega}^j_{t+1}$, the borrower receives nothing, while the bank monitors the borrower and receives $(1-\mu)\omega^j_{t+1} R^K_{t+1} Q_t K^j_{t+1}$.

In equilibrium, the contract guarantees the lender an expected gross return on the loan equal to the required return R^F_{t+1} (taken as given in the contracting problem). That is,

$$\begin{aligned}\left[1 - F(\overline{\omega}^j_{t+1})\right] Z^j_{t+1} L^j_{t+1} + (1-\mu) \int_0^{\overline{\omega}^j_{t+1}} \omega^j_{t+1} R^K_{t+1} Q_t K^j_{t+1} f(\omega) d\omega = \\ = R^F_{t+1}(Q_t K^j_{t+1} - N^j_{t+1}),\end{aligned} \tag{2}$$

where $f(\omega)$ is the probability density function (p.d.f.) of ω.

Combining equation (1) with equation (2) yields the following expression:

$$\begin{aligned}\left\{\left[1 - F(\overline{\omega}^j_{t+1})\right] \overline{\omega}^j_{t+1} + (1-\mu) \int_0^{\overline{\omega}^j_{t+1}} \omega^j_{t+1} f(\omega) d\omega \right\} R^K_{t+1} Q_t K^j_{t+1} = \\ = R^F_{t+1}(Q_t K^j_{t+1} - N^j_{t+1}).\end{aligned} \tag{3}$$

Note that an increase in $\overline{\omega}^j_{t+1}$, the idiosyncratic disturbance cutoff value, has two distinct effects on the bank's expected return on the loan: on the one hand, it increases the expected return since it it raises the non-default payoff; on the other hand, it decreases the expected return since it raises the borrower's default probability. As shown in Appendix A, the bank's expected return reaches a maximum at an unique interior value of $\overline{\omega}^j_{t+1}$, $\overline{\omega}^{j^*}_{t+1}$, and equilibrium is characterized by $\overline{\omega}^j_{t+1}$ always below $\overline{\omega}^{j^*}_{t+1}$. Therefore, the hypothesis of an equilibrium with credit rationing is ruled out and the bank's expected return is always increasing in $\overline{\omega}^j_{t+1}$.

With aggregate uncertainty present, $\overline{\omega}^j_{t+1}$ depends on the *ex post* realization of R^K_{t+1}: conditional on the *ex post* realization of R^K_{t+1}, the borrower offers a state-contingent non-default payment that guarantees the lender a return equal in expected value to the required return R^F_{t+1}. Thus, condition (3) implies a set of restrictions, one for each realization of R^K_{t+1}.

The optimal contracting problem, which determines the division of the expected gross payoff to the firm's capital, $E_t\left(R^K_{t+1}\right) Q_t K^j_{t+1}$, between the borrower j and the bank,[7] is defined by the maximization of borrower's payoff, with respect to K^j_{t+1} and $\overline{\omega}^j_{t+1}$, subject to the set of state-contingent constraints implied by (3).

Let l_{t+1} represent $\frac{E_t\left(R^K_{t+1}\right)}{R^F_{t+1}}$, the expected discounted return to capital. Given $l_{t+1} > 1$, the

[7] Where E_t denotes the expectation operator conditional on the information available at time t.

first order conditions of the contracting problem yield the following relationship between $\frac{Q_t K_{t+1}^j}{N_{t+1}^j}$ and the expected discounted return to capital (see appendix A and BGG for details):

$$\frac{Q_t K_{t+1}^j}{N_{t+1}^j} = \varphi\left(\frac{E_t\left(R_{t+1}^K\right)}{R_{t+1}^F}\right),$$

where $\varphi'(.) > 0$ and $\varphi(1) = 1$. Therefore, each borrower's capital expenditures are proportional to his net worth, with a proportionality factor that is increasing in the expected discounted return to capital. As mentioned by BGG, everything else equal, a rise in the expected discounted return to capital reduces the expected default probability. As a consequence, each entrepreneur can borrow more and expand the size of his firm. Since the expected default costs also increase as the ratio of borrowing to net worth increases, the entrepreneur cannot expand the size of his firm indefinitely.

Aggregating the preceding equation over firms we obtain[8]

$$\frac{Q_t K_{t+1}}{N_{t+1}} = \varphi\left(\frac{E_t\left(R_{t+1}^K\right)}{R_{t+1}^F}\right), \tag{4}$$

where K_{t+1} denotes the aggregate amount of capital purchased by (all) entrepreneurs at time t and N_{t+1} the aggregate net worth of those agents.

Equivalently, equation (4) can be expressed as

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{t+1}^F} = l\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right), \tag{5}$$

where $l(.)$ is increasing in $\frac{Q_t K_{t+1}}{N_{t+1}}$ for $N_{t+1} < Q_t K_{t+1}$. Thus, in equilibrium, the expected discounted return to capital, $\frac{E_t\left(R_{t+1}^K\right)}{R_{t+1}^F}$, depends negatively on the share of the firms' capital expenditures that is financed by the entrepreneurs' net worth. As mentioned by Walentin (2003), $l\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right) R_{t+1}^F$ should be interpreted as the return on capital required by banks, in order to grant loans to the firms. Therefore, in an environment where entrepreneurs must borrow, under imperfect information, to buy capital, the expected discounted return to capital, $\frac{E_t\left(R_{t+1}^K\right)}{R_{t+1}^F}$, may be interpreted as an opportunity cost of being an entrepreneur, or, as in BGG's acceptation, as the *external finance premium* faced by entrepreneurs.

Entrepreneurial Net Worth The net worth of entrepreneurs combines profits accumulated from previous capital investment with income from supplying labor. As a technical matter, it is necessary to start entrepreneurs off with some net worth in order to allow them to begin op-

[8]As mentioned by BGG, the assumption of constant returns to scale generates a proportional relationship between net worth and the capital demand at the firm level, with a factor of proportionality independent of firm's specific factors. This facilitates aggregation.

erations: as in BGG, we assume that, in addition to operating firms, entrepreneurs supplement their income by working in the labor market. It is also assumed that the fraction of agents who are entrepreneurs remains constant.

Let V_t be the entrepreneurs' total equity (*i.e.*, wealth accumulated by entrepreneurs from operating firms). Then, normalizing the total entrepreneurial labor to one,

$$N_{t+1} = \gamma V_t + W_t^e, \tag{6}$$

where W_t^e is the entrepreneurial wage and γ is the probability that a entrepreneur survives to the next period.[9]

Note that V_t can be expressed, in equilibrium, as

$$V_t = R_t^K Q_{t-1} K_t - R_t^F (Q_{t-1} K_t - N_t) - \mu\Theta(\overline{\omega}_t) R_t^K Q_{t-1} K_t, \tag{7}$$

where $\mu\Theta(\overline{\omega}_t) R_t^K Q_{t-1} K_t$ are the aggregate default (monitoring) costs (see Appendix A for the definition of these costs). Thus, combining equations (6) and (7), it is straightforward to conclude that N_{t+1} reflects the equity stake that entrepreneurs have in their firms, which accordingly depends on firms' earnings net of interest payments to lenders.

Entrepreneurs who "die" in t are not allowed to buy capital and simply consume their residual equity $(1-\gamma)V_t$. That is,

$$C_t^e = (1-\gamma)V_t, \tag{8}$$

where C_t^e represents the total consumption of entrepreneurs who leave the market.

2.2 Banks

Financial intermediation, consisting of collecting funds from households and granting loans to entrepreneurs, is assured by banks. In this respect we depart from BGG by properly defining the financial intermediaries as banks and, consequently, specifying their behavior.

Banks are not subject to reserve requirements (for simplicity), but are legally subject to a risk-based regulamentary capital requirement imposed by the government. Following a simplified version of what was established in the Basel Accord of 1988, they must hold an amount of equity that covers at least 8% of loans.[10] As in several models reviewed in the introduction (*e.g.*, Bolton and Freixas, 2001), we assume that capital requirements are always binding. Beside loans, banks' assets can also comprise government bonds which, according to the Basel

[9] To avoid the possibility that entrepreneurs accumulate enough net worth to be fully self financed, it is assumed that those agents have finite horizons. The fraction of agents who are entrepreneurs is held constant by the birth of a new entrepreneur for each dying one.

[10] We assume that banks issue equity, but firms do not (as in Bolton and Freixas, 2001, for instance).

Accord, have a weight zero in the risk-based capital requirement since they bear no risk.

Another specificity of banks is the technology needed to monitor entrepreneurs. Since they do not have access to this technology, households delegate monitoring to banks. In this context, the costly state verification framework, just analyzed above, considers that the borrower has all the bargaining power in the relationship with the lender - the contract specifies the maximization of borrower's payoff subject to the constraint that the expected return to the bank (lender) covers its opportunity cost of funds. As mentioned by Gale and Hellwig (1985), p. 651, "*if the expected utility of the entrepreneur is not maximized subject to this constraint, some other investor* [bank] *can offer a contract which is more attractive to the entrepreneur and still make a profit at the going rate of interest.*" In such an environment, we assume a competitive banking system (as in Berka and Zimmermann, 2004, and Yuan and Zimmermann, 2004, for instance) with unrestricted entry, where each bank earns zero profits, in equilibrium, and is allowed to issue equity at any time, on terms that also depend on households' willingness to buy capital in addition to deposits.[11]

In this context, we will now analyze the behavior of a representative bank which maximizes its expected profits, acting as a price (interest rate) taker in a competitive market. Its choice variables are loans, (riskless) government bonds, (insured) deposits and capital. Beside the capital requirements, we will also assume that the bank must buy deposit insurance. More specifically, the bank is subject to a risk-sensitive insurance rate which depends negatively on the level of bank capital.[12]

Finally, in line with the contract established between the representative bank and each entrepreneur, we assume that all bank's assets and liabilities have the same (one period) maturity.

Following Berka and Zimmermann (2004)'s specification of deposit insurance cost, the bank's objective is then given by:

$$\max_{L_{t+1},B_{t+1},D_{t+1},S_{t+1}} \left(R^F_{t+1} L_{t+1} + R_{t+1} B_{t+1} - R^D_{t+1} D_{t+1} - E_t\left(R^S_{t+1}\right) S_{t+1} - \delta_e \frac{D_{t+1}}{S_{t+1}} D_{t+1} \right)$$

$$\text{s.t. } L_{t+1} + B_{t+1} = D_{t+1} + S_{t+1} \text{ (balance sheet constraint)} \tag{9}$$

$$\frac{S_{t+1}}{L_{t+1}} = \alpha_e \text{ (binding capital requirements)} \tag{10}$$

[11] This is in contrast with Van den Heuvel (2002a)'s approach, according to which banks capital consist in retained earnings and banks are not able to issue equity.

[12] There is, however, a strong simplifying assumption, which we maintain in order to guarantee the comparability of our results with BGG's. In our model banks hold a diversified portfolio of loans, guaranteeing the idiosyncratic risk diversification. Deposits do not bear any risk, since the aggregate risk that could be associated to these assets is borne by the entrepreneurs. Bank capital is risky, as we will assume later, but effectively this risk is borne by the representative household which is assumed to own stocks on the bank. Therefore, and in line with Bolton and Freixas (2001), where a sufficiently high capital requirements guarantees that banks do not fail, the representative bank in our model does not go bankrupt.

with $1 > \alpha_e > 0$ and $1 > \delta_e > 0$, and where

- L_{t+1} are the loans granted to all firms from t to $t+1$;
- B_{t+1} are the government bonds held by the bank from t to $t+1$;
- D_{t+1} are the households' deposits;
- S_{t+1} is the bank's capital;[13]
- R^F_{t+1} is the required gross real return on loans between t and $t+1$;
- R_{t+1} is the gross real return on government bonds (B_{t+1});
- R^D_{t+1} is the gross real return on deposits (D_{t+1});
- $E_t\left(R^S_{t+1}\right)$ is the expected real return on bank capital (S_{t+1});
- $\delta_e \frac{D_{t+1}}{S_{t+1}}$ is the risk-based deposit insurance rate;
- α_e is the imposed level of capital requirements.

Note that R^F_{t+1} differs from the (non-default) lending rate (Z_{t+1}): as has been derived above, the difference between the two is due to the possibility of entrepreneur's default and to the existence of monitoring cost, which are taken into account in R^F_{t+1}. The rate of return on bank capital, R^S_{t+1}, is conditional on the realization of date $t+1$ state of nature whereas all the other rates of return are not (R^F_{t+1}, R_{t+1} and R^D_{t+1} are known in t).

The first order conditions of the interior solution (that is, the solution characterized by positive values of B_{t+1}, D_{t+1}, L_{t+1} and S_{t+1})[14] of this problem yield

$$R_{t+1} = R^D_{t+1} + 2\delta_e \left(\frac{D_{t+1}}{S_{t+1}}\right), \tag{11}$$

$$R^F_{t+1} = (1-\alpha_e)R_{t+1} + \alpha_e E_t\left(R^S_{t+1}\right) - \alpha_e \delta_e \left(\frac{D_{t+1}}{S_{t+1}}\right)^2, \tag{12}$$

which satisfy the bank's zero profit condition.

Due to the introduction of (binding) capital requirements, the required return on lending, R^F_{t+1}, becomes dependent on a weighted average of the deposit return and the equity's expected return, whereas in BGG, R^F_{t+1} is equal to the riskless rate, R^D_{t+1}.

[13] L_{t+1}, B_{t+1}, D_{t+1} and S_{t+1} are all defined in real terms.

[14] Which rules out the possibility of obtaining an optimal bank's portfolio with only one asset, and thus brings the model closer to reality.

2.3 Households

The economy is composed of a continuum of infinitely lived identical risk averse households of length unity. Each household works, consumes, and invests his savings in assets which include deposits, that pay a real riskless rate of return between t and $t+1$ of R^D_{t+1}, and (risky) shares of ownership of banks in the economy, that pay R^S_{t+1}.

For simplicity, we assume a (representative) household's instantaneous utility function separable in consumption, liquidity (deposits) and leisure:

$$U_t = \frac{{C_t}^{1-\sigma}}{1-\sigma} + \alpha_0 \frac{{D_{t+1}}^{1-\beta_0}}{1-\beta_0} + \alpha_1 \frac{(1-H^h_t)^{1-\beta_1}}{1-\beta_1},$$

where C_t denotes household real consumption, D_{t+1} the deposits (in real terms) held by the household from t to $t+1$ and H^h_t the household hours worked (as a fraction of total time endowment).

The real level of deposits is included in the instantaneous utility function to indicate the existence of liquidity services from wealth held in the form of that asset. That is, despite of yielding a gross return of R^D, deposits also serve transaction needs,[15] since currency, in contrast with BGG, is absent from our model: we assume that deposits can be used in an almost money like fashion to simplify a variety of transactions. In short, we are assuming that deposits have an advantage in terms of liquidity when compared to bank capital, as in Poterba and Rotemberg (1987),[16] and, more recently, Van den Heuvel (2005).

The representative household chooses consumption, leisure and portfolio to maximize the expected lifetime utility (appropriately discounted) subject to an intertemporal budget constraint that reflects intertemporal allocation possibilities.

The household's problem is then given by

$$\max_{C_t, H^h_t, D_{t+1}, S_{t+1}} E_t \sum_{k=0}^{\infty} \beta^k \left[\frac{(C_{t+k})^{1-\sigma}}{1-\sigma} + \alpha_0 \frac{(D_{t+k+1})^{1-\beta_0}}{1-\beta_0} + \alpha_1 \frac{(1-H^h_{t+k})^{1-\beta_1}}{1-\beta_1} \right] \quad (13)$$

$$\text{s.t. } C_t = W^h_t H^h_t - T_t + \Pi_t + R^D_t D_t - D_{t+1} + R^S_t S_t - S_{t+1}, \quad (14)$$

with $0 < \beta < 1$. β is the subjective discount factor, W^h_t is the real wage, T_t represents lump sum taxes, Π_t dividends received from ownership of imperfect competitive retail firms, and S_t the real bank capital held by the household from $t-1$ to t.

The maximization of the objective function (13) subject to the budget constraint (14) yields

[15] In line with Bolton and Freixas (2001) and Chami and Cosimano (2001).

[16] Poterba and Rotemberg (1987) permit the utility funcion to capture the liquidity services of money, certain time deposits and government securities. The alternative to holding assets which yield liquidity services (deposits, in our model) is to hold assets with uncertain returns (bank capital, in our model).

the following first order conditions:

$$(C_t)^{-\sigma} = \beta R^D_{t+1} E_t \left[(C_{t+1})^{-\sigma} \right] + \alpha_0 D_{t+1}{}^{-\beta_0}, \tag{15}$$

which takes into account that the gross real rate of return on deposits, R^D_{t+1}, is certain at time t (is known ahead of time);

$$(C_t)^{-\sigma} = \beta \left\{ E_t \left(R^S_{t+1} \right) E_t \left[(C_{t+1})^{-\sigma} \right] + cov_t \left(R^S_{t+1}, (C_{t+1})^{-\sigma} \right) \right\}; \tag{16}$$

and the labor supply

$$\alpha_1 (1 - H^h_t)^{-\beta_1} = (C_t)^{-\sigma} W^h_t. \tag{17}$$

In this representation, the expected excess return on the risky asset (bank capital) is linked both to covariance of aggregate consumption and equity's return (risk premium) and to deposits liquidity (liquidity premium, as will become clear later on).

2.4 Bank Capital

Before proceeding we need to specify the return on bank capital. Following a simplified version of what was established in the Basel Accord of 1988, we assume that the (representative) bank must hold an amount of capital which covers 8% of its loans. Loans are thus financed by bank capital and deposits, and households can, in turn, invest their savings in those two financial assets. A spread between the expected real return on bank capital and the real return on deposits is then justified by the liquidity services provided by deposits and by the riskless return on this asset, i.e., $E_t \left(R^S_{t+1} \right) - R^D_{t+1} > 0$.

In addition, based on an arbitrage argument, we assume that the real returns of bank capital and physical capital are equal:

$$R^S_{t+1} = R^K_{t+1}.$$

The arbitrage argument holds since both returns are subject to the same aggregate risk and neither bank capital nor physical capital provide liquidity services to the households.

2.5 General Equilibrium

Now, following the modeling strategy of BGG, we embed the solution of the partial equilibrium contracting problem within a dynamic new Keynesian general equilibrium model, also taking into account the results obtained in the household and the bank optimization problems.

As mentioned above, in each period t each entrepreneur j acquires physical capital, K^j_{t+1}, which is used in combination with hired labor to produce output in period $t+1$. Following BGG, we specify each entrepreneur's investment decisions, under adjustment costs, assuming that each entrepreneur j purchases the capital goods from some other competitive firms, producers of capital. More specifically, each entrepreneur sells his entire stock of capital at the end of each period t to the capital producing firms at price $\overline{Q}_t$. These firms also purchase raw output as a materials input, I_t (total investment expenditures), and combine it with the aggregate capital stock in the economy (K_t) to produce new capital goods via the production function $\Xi\left(\frac{I_t}{K_t}\right)K_t$, where $\Xi(.)$ is an increasing and concave function, with $\Xi(0)=0$. The function $\Xi(.)$ is concave in investment to capture the difficulty of quickly changing the level of capital installed in the firms (and is thus called the adjustment cost function). The new capital goods, jointly with the capital used to produce them, are then sold to each entrepreneur j at the price Q_t. The "rental rate" $\left(\overline{Q}_t - Q_t\right)$ is determined by the investment sector zero-profit condition.[17]

In this context, the *aggregate capital stock* follows an intertemporal accumulation equation with adjustment costs:

$$K_{t+1} = \Xi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t, \tag{18}$$

where δ denotes the depreciation rate. The introduction of adjustment costs permits variation in the price of a unit of capital in terms of the numeraire good, Q_t, which is given by[18]

$$Q_t = \frac{1}{\Xi'\left(\frac{I_t}{K_t}\right)}. \tag{19}$$

The price of capital is, thus, an increasing function of the quantity invested.[19]

Aggregate Production Function The physical capital acquired at period t by each entrepreneur is then combined with labor to produce output in period $t+1$, by means of a constant returns to scale technology. This allows us to write the production function as an aggregate relationship:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} \tag{20}$$

[17]In steady state $\overline{Q} = Q = 1$, and around the steady state the difference between $\overline{Q}_t$ and Q_t is second order and thus ignored.

[18]Equation (19) is derived from the first order condition for investment for one of the capital producer firms mentioned above.

[19]This is the external adjustment costs approach to investment, which from a macroeconomic perspective is equivalent to the internal adjustment costs approach, as detailed in Sala-i-Martin (2000), for instance.

with $0 < \alpha < 1$ and where Y_t represents the aggregate output of wholesale goods, H_t the labor input and A_t an exogenous technology term.

The final output may then be either transformed into a single type of consumption good, invested, consumed by the government (G_t) or used in monitoring costs:

$$Y_t = C_t + C_t^e + I_t + G_t + \mu\Theta(\overline{\omega}_t)R_t^K Q_{t-1}K_t. \tag{21}$$

After producing output, entrepreneurs sell it to retailers at a relative price of $\frac{1}{X_t}$.[20] Therefore, the expected gross return to holding a unit of capital from t to $t+1$ can be written as:

$$E_t\left(R_{t+1}^K\right) = E_t\left[\frac{\frac{1}{X_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)}{Q_t}\right], \tag{22}$$

where $\frac{1}{X_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}$ represents the rent paid to a unit of capital in $t+1$.

In turn, and as already mentioned, the supply of investment capital is described by the return on physical capital the (representative) bank requires in order to grant loans to the firms (see equation (5) on page 9).

Concerning the labor input, it is assumed that

$$H_t = \left(H_t^h\right)^{\Omega}\left(H_t^e\right)^{1-\Omega},$$

with $1 < \Omega < 0$, and where H_t^h represents the households labor, and H_t^e the entrepreneurial labor.

Therefore, we can rewrite (20) as

$$Y_t = A_t K_t^{\alpha}\left[\left(H_t^h\right)^{\Omega}\left(H_t^e\right)^{1-\Omega}\right]^{1-\alpha}. \tag{23}$$

Equating marginal product with the wage, for each case, we obtain:

$$W_{t+1}^h = (1-\alpha)\Omega\frac{1}{X_{t+1}}\frac{Y_{t+1}}{H_{t+1}^h} \tag{24}$$

and

$$W_{t+1}^e = (1-\alpha)(1-\Omega)\frac{1}{X_{t+1}}\frac{Y_{t+1}}{H_{t+1}^e}, \tag{25}$$

where W_{t+1}^h represents the real wage for households labor and W_{t+1}^e the real wage for entrepreneurial labor. As mentioned, and following BGG, we assume that entrepreneurs supply one unit of labor inelastically to the general labor market: $H_t^e = 1, \forall t$.

[20] X_t is the gross markup of retail goods over wholesale goods.

Now, taking into account equations (7), (20) and (25), we can rewrite (6) - see p. 9 - as

$$N_{t+1} = \gamma \left[R_t^K Q_{t-1} K_t - R_t^F (Q_{t-1} K_t - N_t) - \mu \Theta(\overline{\omega}_t) R_t^K Q_{t-1} K_t \right] + \\ +(1-\alpha)(1-\Omega) \frac{1}{X_t} A_t K_t^{\alpha} (H_t^h)^{\Omega(1-\alpha)}. \quad (26)$$

The Retail Sector and Price Setting To increase the empirical relevance of the model concerning price inertia, we introduce sticky prices in it using standard devices used in new Keynesian research. Namely, we incorporate monopolistic competition and costs of adjusting nominal prices by distinguishing between entrepreneurs and retailers (since assuming that entrepreneurs are imperfect competitors complicates aggregation): entrepreneurs produce wholesale goods in competitive markets, and then sell their output to retailers who are monopolistic competitors. Retailers do nothing other than buy goods from entrepreneurs, differentiate them (costlessly), and then resell them to households. They are included only in order to introduce price inertia in a tractable manner: following Calvo (1983), it is assumed that the retailer is free to change its price in a given period only with probability $1-\theta$ (with $0 < \theta < 1$). The profits from retail activity are rebated lump-sum to households (Π_t in equation (14)).[21]

Government[22] Government finances its expenditures, G_t, by lump-sum taxes, T_t, and by issuing securities (government bonds, B_{t+1}):

$$G_t = B_{t+1} - B_t R_t + T_t + J_t,$$

where J_t represents other costs and revenues and includes the deposit insurance premium paid by the banks to the regulatory authority.

In particular, the government adjusts the mix of financing between bonds issuance and lump-sum taxes to support an interest rate monetary policy rule, to be defined below. To implement its choice of the nominal interest rate, the government adjusts the supply of government bonds to satisfy the bank's demand for this asset.

2.6 The Linearized Model and Calibration

According to the model just described, and in the absence of exogenous shocks, the economy converges to a steady state growth path, along which all variables are constant over time (including prices, which implies a zero inflation rate in steady state).

[21] Detailed derivation not presented here, since it is standard in New Keynesian framework.

[22] We assume that the government also comprises the central bank, thus conducting both monetary and fiscal policies, as well as regulating banks (conflicting differences between policies are internalized within the agent government since we do not aim at exploring those differences).

To linearize the preceding equations, we use a first order Taylor series expansion around the steady state. Let the lower case letters denote the percentage deviation of each variable from its steady state level: $x_t = \ln\left(\frac{X_t}{X}\right)$, where X is the value of X_t in nonstochastic steady state.

Aggregate Demand

Starting by log-linearizing the Euler equations, equation (15), on page 13, becomes (assuming that $\sigma = \beta_0$):

$$-\sigma c_t = -\sigma\beta R^D E_t\left(c_{t+1}\right) + \beta R^D r^D_{t+1} - \alpha_0\sigma\left(\frac{C}{D}\right)^{\sigma} d_{t+1}. \tag{27}$$

Concerning equation (16), on page 13, we take a first-order Taylor approximation around the steady state ignoring the second order terms (or assuming that they are constant over time: $cov_t(.) = cov(.), \forall t$) and obtain:[23]

$$-\sigma c_t = -\sigma\beta R^K E_t\left(c_{t+1}\right) + \beta R^K E_t\left(r^K_{t+1}\right). \tag{28}$$

Note that we assume that $R^S_{t+1} = R^K_{t+1}$, as argued above.

Concerning the entrepreneurs' consumption (equation 8, on page 10), we will follow BGG and assume in simulations that

$$c^e_t = n_{t+1}. \tag{29}$$

In turn, the aggregate resource constraint (21), on page 15, becomes

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{C^e}{Y}c^e_t + \frac{G}{Y}g_t + \xi^y_t, \tag{30}$$

where

$$\xi^y_t = \frac{\mu\Theta(\overline{\omega})KR^K}{Y}\left[\ln\left(\frac{\mu\Theta(\overline{\omega}_t)Q_{t-1}K_tR^K_t}{\mu\Theta(\overline{\omega})KR^K}\right)\right].$$

This term (ξ^y_t) is ignored in the simulations. Note that $\frac{\mu\Theta(\overline{\omega})KR^K}{Y}$, the share of expected monitoring costs in output, is quite small (even smaller than $\frac{C^e}{Y}$).

In what concerns the relationship between the external finance premium and the ratio of capital expenditures to net worth, equation (5), p. 9, can be written in log-linear form as

$$E_t(r^K_{t+1}) - r^F_{t+1} = v(k_{t+1} + q_t - n_{t+1}), \tag{31}$$

where v is the steady state elasticity of $\frac{E_t(R^K_{t+1})}{R^F_{t+1}}$ with respect to $\frac{Q_tK_{t+1}}{N_{t+1}}$, *i.e.*, the steady state

[23] Thus, the difference between $E_t\left(r^K_{t+1}\right)$ and r^D_{t+1} rests solely on liquidity purposes.

elasticity of the external finance premium with respect to the ratio of entrepreneurs' capital expenditures to net worth:

$$v = \frac{l'(\overline{\omega}_{SS})}{k'(\overline{\omega}_{SS})}\frac{k(\overline{\omega}_{SS})}{l(\overline{\omega}_{SS})}.$$

We follow Gertler *et al.* (2003) to compute v (see Appendix B).

Log-linearization of (19), p. 15, implies that

$$q_t = \varphi\left(i_t - k_t\right), \tag{32}$$

where φ is the elasticity of the price of capital with respect to $\frac{I}{K}$:

$$\varphi = -\frac{\Xi''\left(\frac{I}{K}\right)}{\Xi'\left(\frac{I}{K}\right)}\frac{I}{K}.$$

Log-linearization of (22), p. 15, in turn, renders

$$r_t^K = (1-\varepsilon)(y_t - k_t - x_t) + \varepsilon q_t - q_{t-1} \tag{33}$$

with

$$\varepsilon = \frac{1-\delta}{(1-\delta) + \alpha\frac{Y}{XK}}.$$

Aggregate Supply

In a log-linearized version of the model, equation (23), p. 16, becomes

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t^h, \tag{34}$$

and the labor market clearing condition, taking into account both equations (17), p. 13, and (24), p. 16, is given by

$$\left(1+\frac{1}{\eta}\right)h_t^h = y_t - x_t - \sigma c_t \tag{35}$$

where $\eta = \frac{\partial H^h}{\partial W^h}\frac{W^h}{H^h} = \frac{1}{\beta_1}\frac{1-H^h}{H^h}$.

Finally the Phillips curve (or the price adjustment equation) is given by

$$\pi_t = \beta E_t \pi_{t+1} - \kappa x_t, \tag{36}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation from $t-1$ to t, $p_t = \ln\left(\frac{P_t}{P}\right)$, and P is the price index.

This equation is derived from the optimal (staggered) price setting by the retail sector.

State Variables

Log-linearization of (26), on page 16, implies that the entrepreneurs' net worth evolves according to (ignoring the monitoring costs):

$$
\begin{aligned}
n_{t+1} = {} & \gamma R^F n_t + \gamma R^F \left(1 - \frac{K}{N}\right) r_t^F + \\
& + \left(\gamma \frac{K}{N} R^K\right) r_t^K + \gamma \frac{K}{N} \left(R^K - R^F\right) q_{t-1} + \\
& + \gamma \frac{K}{N} \left(R^K - R^F\right) k_t + (1-\alpha)(1-\Omega) \frac{Y}{N} \frac{1}{X} (y_t - x_t). \qquad (37)
\end{aligned}
$$

Concerning the capital stock, the log-linearized version of (18), p. 15, is

$$k_t = \delta i_{t-1} + (1-\delta) k_{t-1}. \qquad (38)$$

Representative Bank

Equations (11), p. 12, and (12), p. 12, derived from the first order conditions of the bank's profit maximization problem, can be written in log-linear form as (taking into account the assumption that $R_{t+1}^S = R_{t+1}^K$):

$$r_{t+1} = \frac{R^D}{R} r_{t+1}^D + \frac{2\delta_e \frac{D}{S}}{R} (d_{t+1} - s_{t+1}) \qquad (39)$$

and

$$r_{t+1}^F = \alpha_e \frac{R^K}{R^F} E_t \left(r_{t+1}^K\right) + (1-\alpha_e) \frac{R}{R^F} r_{t+1} - \frac{2\alpha_e \delta_e \left(\frac{D}{S}\right)^2}{R^F} (d_{t+1} - s_{t+1}). \qquad (40)$$

The capital requirement constraint $S_{t+1} = \alpha_e (Q_t K_{t+1} - N_{t+1})$, turns into:

$$s_{t+1} = \frac{K}{L} \left(k_{t+1} + q_t\right) - \frac{N}{L} n_{t+1}. \qquad (41)$$

Monetary Policy Rule and Shock Processes

The interest rate rule is given by

$$r_{t+1}^n = \rho r_t^n + \varsigma \pi_{t-1} + \varepsilon_t^{r^n} \qquad (42)$$

where $r_{t+1}^n \equiv r_{t+1} + E_t \pi_{t+1}$ is the nominal interest rate from t to $t+1$ (with $\pi_{t+1} \equiv p_{t+1} - p_t$) and $\varepsilon_t^{r^n}$ an i.i.d. disturbance at time t. As in BGG, we standardly assume that the current nominal interest rate responds to the lagged inflation rate and the lagged interest rate.

Concerning the exogenous disturbances to government spending and technology, they follow, as in BGG, stationary autoregressive processes:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \quad (43)$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (44)$$

where ε_t^g and ε_t^a are i.i.d. disturbances.

The complete description of all variables in the model is provided in Appendix C.

Calibration

We calibrate the model assuming that a period is a quarter. To evaluate some of the model's parameters and variables in steady state (SS), we follow BGG, which, focusing on U.S. data, consider (recall that, according to our notation, a variable without the time subscript indicates its steady state value):

Entrepreneurial Consumption/Output in SS	$\frac{C^e}{Y}$	0.01
Government Expenditure/Output in SS	$\frac{G}{Y}$	0.2
Gross Markup of Retail Goods over the Wholesale Goods in SS	X	1.1
Price of Capital in SS	Q	1
Entrepreneurial Labor	H_t^e	1
Elasticity of the price of capital with respect to I/K	φ	0.25
Capital Share	α	0.35
Household Labor Share	$(1-\alpha)\Omega$	0.64
Labor Supply Elasticity	η	3
Depreciation Rate	δ	0.025
Interest Rate Smoothing	ρ	0.9
Coefficient on inflation in the interest rate rule	ς	0.11
Prob. that an entrepreneur survives to the next quarter	γ	0.9728
Probability of a firm does not change its price within a given period	θ	0.75
Serial correlation parameter for technology shock	ρ_a	1
Serial correlation parameter for gov. expend. shock	ρ_g	0.95
Standard Deviation of $\ln(\omega)$	$\sigma_{\ln\omega}$	0.28
Monitoring Costs Parameter	μ	0.12
Preference Parameter	β_1	1

Table 1: Calibration I

Concerning the parameters related to the financial contract, we choose the same values as BGG for the probability that an entrepreneur survives to the next quarter ($\gamma = 0.9728$) and for the monitoring cost parameter ($\mu = 0.12$). For the standard deviation of $\ln(\omega)$, we assume that $\sigma_{\ln\omega} = 0.28$. According to Carlstrom and Fuerst (1997), a standard deviation of ω of around

0.2 is comparable to the corresponding empirical standard deviation reported by Boyd and Smith (1994). These assumptions allowed us to approximate, with good accuracy, the three steady state outcomes pointed out by BGG: a financing premium of 2% per year; $\frac{K}{N} = 2$ (which implies a leverage ratio, $\frac{L}{K}$, of 0.5) and an annualized business failure rate, $F(\overline{\omega}) = 3\%$.[24]

There are, however, some other parameters and variables in steady state which must be defined and which are specific to our model, namely:

Loans/Deposits in SS	$\frac{L}{D}$	0.75
Bank Capital Requirement	α_e	0.08
Deposit Insurance Costs Parameter (risk sensitive dep. ins. rate)	δ_e	0.0000045
Preference Parameter	σ	1
Preference Parameter	β_0	1

Table 2: Calibration II

$\frac{L}{D}$: In steady state, and according to the model, $L = K - N$, where L represents loans without collateral that are granted to entrepreneurs who buy capital to produce the final good. Therefore, real estate and consumer loans should not be included in L, as well as loans secured by collateral. In other words, L should only comprise commercial and industrial (C&I) loans which are not secured by collateral.[25] The Survey of Terms of Business Lending, published by the Federal Reserve,[26] provides some data which allowed us to compute the amount of C&I loans not secured by collateral in percentage of all C&I loans (made by all U.S. commercial banks), in each quarter from 1997:2 to 2004:4:

$$\frac{L^{C\&I}_{WithoutCol}}{L^{C\&I}}.$$

Then, using the (U.S., quarterly) banking data, from 1997:2 to 2004:4, available at the Federal Reserve Bank of St. Louis,[27] on *(a)* the total loans at all commercial banks and *(b)* the deposits at all commercial banks, we computed the ratio $(a)/(b)$, from which we could proceed, assuming

$$\frac{L^{Total}}{D^{Total}} = \frac{(a)}{(b)},$$

and

[24] Data for the U.S. on the financing premium is available at http://research.stlouisfed.org/fred2/, whereas data on the leverage ratio is available in Rajan and Zingales (1995).

[25] As mentined by Bernanke and Lown (1991), p. 215, C&I lending epitomizes what theory would identify as a special function of banks (concerning their ability to evaluate and monitor borrowers).

[26] And available at http://www.federalreserve.gov/releases/e2/.

[27] See http://research.stlouisfed.org/fred2/.

$$\frac{L^{C\&I}}{D^{C\&I}} = \frac{L^{Total}}{D^{Total}} \left(\equiv \frac{(a)}{(b)} \right),$$

where $D^{C\&I}$ denotes the deposits that are used in financing $L^{C\&I}_{WithoutCol}$, *i.e.*, the deposits relevant to our model.

Finally, we assumed that $\frac{L}{D}$ corresponds to the average value of

$$\frac{L^{C\&I}_{WithoutCol}}{D^{C\&I}} = \frac{L^{C\&I}_{WithoutCol}}{L^{C\&I}} \frac{L^{C\&I}}{D^{C\&I}} \simeq 0.75.$$

Concerning the bank capital requirement (α_e), and as mentioned above in 2.2, we assume a simplified version of the rules imposed by the Basel Accord, by setting α_e equal to 0.08.

To calibrate the deposit insurance parameter (δ_e) we use the data from the Federal Deposit Insurance Corporation (FDIC)[28] and assume that, in steady state, the representative bank of our model is an adequately capitalized firm (since we are assuming that the capital requirement is always binding) and belongs to the subgroup A, which consists of financially sound institutions with only a few minor weaknesses, and generally corresponds to the primary federal regulator's composite rating of "1" or "2" - see http://www.fdic.gov/deposit/insurance/risk/rrps_ovr.html. Therefore, the deposit insurance rate corresponds to 3 cents per $100 of deposits in annual terms. In quarterly terms, this means that

$$\delta_e \frac{D}{S} = 0.000075.$$

Since we are assuming that, in steady state, $\frac{L}{D} = 0.75$, and that $\frac{S}{L}$ is always equal to 0.08 (in the benchmark case),

$$\frac{D}{S} = 16.6(6) \Longrightarrow \delta_e = 0.0000045.$$

Note that outside the steady state the risk sensitive deposit insurance rate may be different from 0.000075, since $\frac{D}{S}$ may vary (*e.g.*, the bank, although maintaining the adequate level of capital, may change from subgroup A to subgroup B due to an increase in $\frac{D_{t+1}}{S_{t+1}}$).[29]

Finally, concerning the preference parameters, we assume, for simplicity, that $\sigma = \beta_0$ (by assuming that, we only need to compute the deposit to consumption ratio in steady state $\left(\frac{D}{C}\right)$ to solve the model, instead of defining both variables, C and D, separately) and set σ equal to 1 (log preferences), which is assumed by many business cycle models, including BGG.

The other parameters and variables in steady state are set in the following way:

1. v, $\frac{R^K}{R^F} = l$, $\frac{QK}{N}$, and Z follow from the computation of $\overline{\omega}$ (see Appendix B);

[28] Available at http://www.fdic.gov/deposit/insurance/risk/assesrte.html.

[29] See the Supervisory Subgroup Descriptions by FDIC at http://www.fdic.gov/deposit/insurance/risk/rrps_ovr.html.

2. $\varepsilon = \frac{1-\delta}{(1-\delta)+\alpha\frac{Y}{XK}}$;
   $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

3. The variables and parameters must satisfy the following steady state equations (derived from the model's FOC and optimization constraints):

   Equation (9) $\Longrightarrow B + L = D + S$

   Equation (10) $\Longrightarrow \alpha_e = \frac{S}{L}$

   Equation (11) $\Longrightarrow R = R^D + 2\delta_e \frac{D}{S}$

   Equation (12) $\Longrightarrow R^F = (1-\alpha_e)R + \alpha_e R^K - \alpha_e \delta_e \left(\frac{D}{S}\right)^2$

   Equation (15) $\Longrightarrow 1 = \beta R^D + \alpha_0 \left(\frac{D}{C}\right)^{-\sigma}$

   Equation (16) $\Longrightarrow R^K = \frac{1}{\beta}$

   Equation (21)[30] $\Longrightarrow \frac{C}{Y} + \frac{C^e}{Y} + \frac{I}{Y} + \frac{G}{Y} = 1$

   Equation (22) $\Longrightarrow R^K = \frac{\frac{1}{X}\alpha\frac{Y}{K}+(1-\delta)}{1}$.

R represents the quarterly steady state real gross return on government bonds. Taking into account the first Euler equation (15) evaluated in steady state,

$$1 = \beta R^D + \alpha_0 \left(\frac{D}{C}\right)^{-\sigma}$$

and the relationship between R and R^D,

$$R = R^D + 2\delta_e \frac{D}{S},$$

we set the parameter α_0 to guarantee $R = 1.01$ in steady state (a value which is assumed by many other business cycle models, including BGG, for the riskless real rate of return, since it guarantees an average riskless interest rate of 4% per year), with $\frac{L}{D} = 0.75$.

After log-linearizing the model, we applied the computational procedure used for solving linear rational expectations models developed by McCallum (1999).

3 Results

3.1 Simulating the effects of a monetary policy shock

In order to analyze the role of bank capital in the transmission of monetary policy, we present now some quantitative experiments focusing on the economy response to an unanticipated temporary negative monetary policy shock.

[30] Ignoring monitoring costs.

Concerning the channels through which monetary policy affects real activity, our model, derived above, brings together *(i)* the standard interest rate channel of monetary policy transmission - according to which an unanticipated increase in the nominal interest rate depresses the demand for physical capital, which, in turn, reduces investment and the price of capital; *(ii)* the borrowers' balance sheet channel; and *(iii)* the bank capital channel.

The borrowers' balance sheet channel predicts that the decline in asset prices (physical capital price in our model), due to a contractionary monetary policy, decreases borrowers' net worth, raising the external finance premium and, consequently, forcing down investment. This, in turn, will reduce asset prices and borrowers' net worth, further pushing down investment, and thus giving rise to the financial accelerator effect, which amplifies the impact of the monetary shock on borrowers' spending decisions. Finally, the bank capital channel, in contrast with the borrowers' balance sheet channel, works through the supply of funds side and is related to the introduction of the specific role of banks in cyclical fluctuations, as implied by our model.

The amplification effects

To analyze the bank capital channel, we begin by comparing the effects of a negative innovation in the nominal interest rate (which corresponds to an annual increase of 25 basis points) under three distinct hypotheses:

- Variant 1: Baseline model derived previously, assuming a risk-based capital ratio requirement of 8%;
- Variant 2: Model without capital requirements, *i.e.*, excluding the capital requirement constraint from the baseline model (equation 10);
- Variant 3: Model with no capital requirements nor financial accelerator, which is generated by fixing the external finance premium, in variant 2, at its steady state level.

Figures 1 and 2 illustrate the impulse response functions of several variables under these three variants (variant 1: solid line; variant 2: dashed line; and variant 3: dashed-dotted line), using the calibrated model economy.[31]

The increase in the nominal interest rate triggers an immediate decline in output, investment and consumption below their steady-state values, after which the economy returns gradually to its steady state. As predicted by the Phillips curve in a sticky prices context, inflation also decreases in response to the output decline, and then gradually reverts to its stationary value. Inflation behavior, in turn, influences the nominal interest rate through the monetary policy

[31] The time horizon is always in quarters and the variables are expressed as percentage deviations from their steady state values.

rule - the monetary authority sets the nominal interest rate in response to lagged inflation and lagged nominal interest rate.

Figure 2 depicts the financial sector variables response. As in BGG, the external finance premium evolves countercyclically, increasing in response to the deterioration of entrepreneurs' financial position following the decline in assets prices. In fact, both variant 1 and 2 comprise the financial accelerator effect embedded within equation (31):

$$E_t(r^K_{t+1}) - r^F_{t+1} = v(k_{t+1} + q_t - n_{t+1}).$$

That is, both variants 1 and 2 comprise the effects of monetary policy arising from the loan demand side (due to the informational asymmetry between each entrepreneur and the bank, which gives rise to the financial accelerator effect, the basis of the borrowers' balance sheet channel).[32] However, the effects of the monetary policy shock are stronger in the presence of capital requirements (that is, are stronger in variant 1 when compared to variant 2). This amplification effect can be explained through the analysis of bank and household behavior:

Recall the log-linearized equations (39) and (40) which have been derived from the representative bank's profit maximization problem:[33]

$$r_{t+1} = \frac{R^D}{R} r^D_{t+1} + \frac{2\delta_e \frac{D}{S}}{R}(d_{t+1} - s_{t+1})$$

and

$$r^F_{t+1} = \alpha_e \frac{R^K}{R^F} E_t\left(r^K_{t+1}\right) + (1-\alpha_e)\frac{R}{R^F} r_{t+1} - \frac{2\alpha_e\delta_e\left(\frac{D}{S}\right)^2}{R^F}(d_{t+1} - s_{t+1}).$$

Combining these two equations it is straightforward to derive the following condition:

$$E_t(r^K_{t+1}) - r^F_{t+1} = \left(1 - \alpha_e \frac{R^K}{R^F}\right) E_t(r^K_{t+1}) - (1-\alpha_e)\frac{R^D}{R^F} r^D_{t+1} - \\ - \left[(1-\alpha_e)\frac{2\delta_e \frac{D}{S}}{R^F} - \frac{2\alpha_e\delta_e\left(\frac{D}{S}\right)^2}{R^F}\right](d_{t+1} - s_{t+1}). \quad (45)$$

Taking into account the steady state conditions derived in the previous section, it is also straightforward to conclude that $\left(1 - \alpha_e \frac{R^K}{R^F}\right) \simeq (1-\alpha_e)\frac{R^D}{R^F}$.[34] Therefore, equation (45) may be

[32] In line with the analysis in 2.1, above, the demand effects are based on the prediction that the external finance premium facing a borrower should depend on borrower's financial position. In particular, the greater the borrower's self-financing ratio is, the lower the external finance premium should be. Intuitively, a stronger financial position enables a borrower to reduce his potential conflict of interests with the bank, by self-financing a greater share of his investment project.

[33] Recall also that, according to the notation used in the preceding section, a variable without the time subscript indicates its steady state value.

[34] More specifically, the difference between these two coefficients relies in the presence of deposit insurance costs (and, consequently, vanishes when we set $\delta_e = 0$). When $\alpha_e = 0.08$ the coefficients associated with

rewritten as

$$E_t(r^K_{t+1}) - r^F_{t+1} \simeq \left(1 - \alpha_e \frac{R^K}{R^F}\right)\left[E_t(r^K_{t+1}) - r^D_{t+1}\right] +$$
$$+ \left[\frac{2\alpha_e \delta_e \left(\frac{D}{S}\right)^2}{R^F} - (1-\alpha_e)\frac{2\delta_e \frac{D}{S}}{R^F}\right](d_{t+1} - s_{t+1}) \,. \tag{46}$$

According to this expression, the external finance premium, $E_t(r^K_{t+1}) - r^F_{t+1}$, depends positively on $E_t(r^K_{t+1}) - r^D_{t+1}$, which we will refer to as the liquidity premium.[35] The external finance premium also depends on the deposit-bank capital ratio, $d_{t+1} - s_{t+1}$, through the deposit insurance costs, but this effect is relatively small.[36] Therefore, we focus our attention now on the relationship between the liquidity premium and the external finance premium.

As illustrated in figure 2, a contractionary monetary policy shock leads to an increase in the level of capital issued by the bank (s_{t+1}) in variant 1. This happens for two reasons: *(i)* the level of commercial and industrial (both uncollateralized) loans also increases - although entrepreneurs invest less [$\swarrow (Q_t K_{t+1})$], the sharp decrease in their net worth ($\swarrow\swarrow N_{t+1}$) leads to an increase of $L_{t+1} (= Q_t K_{t+1} - N_{t+1})$ above its steady state level;[37] and *(ii)* as bank capital requirements are binding in variant 1, the bank may only grant more credit if it issues more capital. To hold more bank capital during the recession, households in turn require an increase in the liquidity premium, $E_t(r^K_{t+1}) - r^D_{t+1}$, since they must reduce the amount of deposits to attenuate the decline in consumption (in line with Gorton and Winton, 2000's model, for instance).[38]

$E_t(r^K_{t+1})$ and r^D_{t+1}, in the right hand side of equation (45), are 0.9194 and 0.9193, respectively.

[35] Note that, since we use a first-order Taylor approximation around the steady state (ignoring the second order terms) to linearize the model (see section 2), $E_t(r^K_{t+1}) - r^D_{t+1}$ is not the equity premium, defined as the extra return required by risk averse households to compensate them for the covariance between equity returns and the stochastic discount factor. Instead, $E_t(r^K_{t+1}) - r^D_{t+1}$ reflects the deposit advantage, over bank capital, in terms of liquidity. That is why it is properly called liquidity premium, instead of equity premium.

[36] For example, in variant 1, the immediate decrease in deposit-bank capital ratio [second term on the right hand side of (46)] accounts for less than 1% of the increase in the external finance premium. This small impact is confirmed in the robustness analysis conducted below, which includes the sensitivity to δ_e.

[37] As mentioned by Gertler and Gilchrist (1993), following a tight monetary policy, banks may not be able to reduce lending immediately because they may force many borrowers prematurely into bankruptcy. Credit demand may actually rise, because of the need to finance inventories. In fact, Gertler and Gilchrist find that consumer and real estate loans fall after a contractionary monetary policy shock but commercial and industrial loans do not (see also, more recently, Den Haan *et al.*, 2004). Considering only the manufacturing firms, loans actually rise after a Funds rate increase. However, this movement, seems to be driven mainly by large firms: bank loans to small firms (which, for informational or incentive reasons, tend to face a proportionately larger premium for external finance) eventually decline in the wake of tight money.

[38] Note that in terms of the bank's balance sheet, the increase in loans and the decrease in deposits are compensated by a reduction in bonds held by the bank.

To clarify this last effect recall the log-linearized Euler equations (27) and (28) derived in section 2,

$$-\sigma c_t = -\sigma\beta R^D E_t(c_{t+1}) + \beta R^D r^D_{t+1} - \alpha_0\sigma\left(\frac{C}{D}\right)^{\sigma} d_{t+1}$$

and

$$-\sigma c_t = -\sigma\beta R^K E_t(c_{t+1}) + \beta R^K E_t\left(r^K_{t+1}\right).$$

Combining these two equations, with the calibrated $\sigma = 1$, yields

$$\beta R^K E_t\left(r^K_{t+1}\right) - \beta R^D r^D_{t+1} = \left(R^K - R^D\right)\beta E_t(c_{t+1}) - \alpha_0\frac{C}{D}d_{t+1} \tag{47}$$

where $\alpha_0\frac{C}{D} > 0$, which confirms that the liquidity premium required by the households depends negatively on deposits (d_{t+1}).

In sum, after the contractionary monetary policy shock in variant 1, the level of loans can only increase above its steady state level if the bank issues more capital (due to the binding capital requirements). Households in turn require an increase in the liquidity premium to hold more bank capital and less deposits - note that, as illustrated in figure 2, the liquidity premium under variant 1 increases with a simultaneous decrease in deposits' level and in the deposit-bank capital ratio. The larger the increase in the liquidity premium the larger will be the increase in the external finance premium (see equation 46):

$$\swarrow d_{t+1} \Longrightarrow \nearrow \left[E_t(r^K_{t+1}) - r^D_{t+1}\right] \Longrightarrow \nearrow \left[E_t(r^K_{t+1}) - r^F_{t+1}\right].$$

We call this relationship between deposits and the external finance premium (through the liquidity premium), the *liquidity premium effect*. This effect is strictly related to the financial accelerator effect. That is, in variant 1 of the model, the external finance premium increases not only because the net worth of firms decreases (due to the decline in asset prices), but also because the liquidity premium required by the households increases (a cost that is passed on to firms):

$$\begin{array}{llcll} \text{(A)} & \text{Liquidity Premium Effect:} & \Delta^- D & \Longrightarrow & \Delta^+ \frac{E_t R^K_{t+1}}{R^D_{t+1}} \Longrightarrow \\ \text{(B)} & \text{Financial Accelerator Effect:} & \Delta^- Q & \Longrightarrow & \Delta^- \frac{N_{t+1}}{Q_t K_{t+1}} \Longrightarrow \end{array} \quad \Delta^+ \frac{E_t R^K_{t+1}}{R^F_{t+1}}.$$

Comparing variants 1 and 2 further clarifies the liquidity premium effect. In contrast with variant 1, the negative monetary shock in variant 2 leads to a decrease in bank capital, since banks are no longer forced to issue equity to finance a given percentage of loans.[39] As illustrated in figure 2, after the negative shock both deposits and bank capital decrease in variant 2 (the increase in loans is compensated by a decrease in bonds held by the bank),[40] and the deposit-bank capital ratio increases (in contrast with variant 1).

Even though $d_{t+1} - s_{t+1}$ increases, in variant 2, the liquidity premium required by the households still rises after the shock, although less than in variant 1. This can again be explained through the analysis of equation (47) above, according to which the liquidity premium required by the households depends negatively on d_{t+1}. In variant 2, households reduce the amount of bank capital held after the shock, and, consequently, reduce the level of deposits to a smaller extent than in variant 1. Therefore, the increase in the liquidity premium is smaller than in variant 1, as predicted by equation (47). This, in turn, implies a smaller increase in the external finance premium through effect (A) above, reducing the effects of the exogenous shock on investment and output (see figure 1).[41]

We may then conclude that the introduction of capital requirements - in a model with bank capital, but where banks were not constrained by capital requirements - amplifies the effects of monetary policy on real activity through the liquidity premium effect. Other experiments conducted by us, but not reported here, assuming different levels of risk-based capital requirements (α_e), show that the same conclusion applies to an exogenous increase in capital requirements imposed by the authorities (increase in α_e).

Finally, variant 3 excludes both the liquidity premium and the financial accelerator effects. As figures 1 and 2 show, there are considerable differences between variants 1 and 2, on the one hand, and variant 3, on the other. The effects of a monetary policy shock are much weaker in variant 3. Concerning, for instance, the immediate effect on real output and inflation, output decreases 1.44% in variant 1 and only 0.52% in variant 3, while inflation decreases 0.52% and 0.18% in variants 1 and 3, respectively.[42]

BGG predict that the financial accelerator amplifies monetary shocks by about 50% (in terms of real output response). According to Quadrini (2001), in his comment to Carlstrom

[39] Although the variant 2 of the model assumes no capital requirements, the bank still issues some capital due to the risk-sensitive deposits insurance rate: recall that the deposit insurance rate depends negatively on the level of bank capital. In variant's 2 steady state, the bank sets an equity-loan ratio of 3.2%, approximately.

[40] In fact, Gertler and Gilchrist (1993) present evidence that, after a negative monetary shock and to offset the potential decline in deposits relative to loans, banks sell securities and issue money market liabilites as large certificates of deposits.

[41] Note that in the variant without capital requirements, equation (45) is replaced by

$$E_t(r^K_{t+1}) - r^F_{t+1} = E_t(r^K_{t+1}) - \frac{R^D}{R^F} r^D_{t+1} - \frac{2\delta_e \frac{D}{S}}{R^F}(d_{t+1} - s_{t+1}),$$

which corresponds to (45) when $\alpha_e = 0$.

[42] This difference in inflation response justifies the contrast in nominal interest rate behavior following the initial shock, shown in figure 1.

and Fuerst (2001), 50% is still relatively small: "*Based on this result, it is hard to claim that financial frictions are the main mechanism through which monetary shock get propagated in the economy. If we eliminate financial market frictions, the impact of monetary shocks will be reduced by only one third.*" (p. 31) Our model responds to this insufficiency. If we eliminate financial market frictions, that is, if we compare variant 3 with variants 1 and 2, the impact of the monetary shock is reduced by much more than one third: 63.63% and 56.84% from variants 1 and 2, respectively, to variant 3.

Decomposing the amplification effects

To confirm and then explain this discrepancy in the magnitude of results, we compare, in figure 3, the effects of the negative innovation in the nominal interest rate under variants 1, 3 and a BGG variant, that is, a variant derived as our baseline model but treating the bank as the financial intermediary in BGG's model, thus excluding bank capital and eliminating deposits from households' utility function.

Variant 1 includes both the financial accelerator and the liquidity premium effects, variant BGG only comprises the financial accelerator effect and variant 3 excludes both effects (the external finance premium does not depart from its steady state value). Or, in other words, variant 1 comprises the effects arising from the loan demand side (due to the informational asymmetry between each entrepreneur and the bank, which gives rise to the financial accelerator effect) and the effects arising from the loan supply side (due to the presence of bank capital in the model, which gives rise to the liquidity premium effect); variant BGG, in turn, only comprises loan demand effects and variant 3 excludes both effects.

As illustrated in figure 3, the real effects of monetary policy are in fact much stronger in variant 1 than in variant BGG: concerning real output, once more, whereas it initially decreases 1.44% in variant 1, it only decreases 0.685% in variant BGG. In other words, whereas the introduction of an informational asymmetry between each entrepreneur and the bank amplifies monetary shocks by about 30% in our model (variant BGG *vs* variant 3), the introduction of that same information asymmetry jointly with the imposition of bank capital minimum levels (through a risk-sensitive deposit insurance rate and capital requirements) amplifies monetary shocks by significantly more than 100% (variant 1 *vs* variant 3).[43]

In variant 1 the external finance premium set by the bank must not only compensate the bank for the costs of mitigating incentive problems due to informational asymmetries (as in variant BGG), but also the return required by the households to hold bank capital. That is, the external finance premium, in variant 1, is not only influenced by the self financing ratio, $\frac{N_{t+1}}{Q_t K_{t+1}}$, but also by the liquidity premium required by the households, $\frac{E_t R^K_{t+1}}{R^D_{t+1}}$. Since

[43] The behavior of the nominal interest rate after the initial period is, once more, justified by the response of inflation, which is much stronger in variant 1 than in variants 3 and BGG.

the liquidity premium required by the households is countercyclical in variant 1 (see figure 2), due to deposits response, the countercyclical movement in the external finance premium is exacerbated (see figure 3: the external finance premium initially increases 0.066% in variant 1 vs 0.036% in variant BGG). This explains why real effects are much stronger in variant 1 than in variant BGG.

In sum, the amplification effects are much stronger in variant 1 (as well as in variant 2) than in variant BGG. The reason is summarized in table 3: in addition to the borrowers' balance sheet channel of monetary policy transmission (also included in variant BGG), variants 1 and 2 comprise the *bank capital channel*, which, through the liquidity premium effect, further amplifies the monetary policy shock effects. In turn, the amplifying effects are somewhat stronger in variant 1 than in 2, since in variant 1 banks must issue more capital to comply with the binding capital requirements.

	Standard interest rate channel	Borrowers' balance sheet channel	Bank capital channel
		(amplifiers of monetary policy effects)	
Variant 1	✓	✓	✓
Variant 2	✓	✓	✓
Variant 3	✓		
Variant BGG	✓	✓	

Table 3: Monetary policy transmission channels

Our result of a much more powerful propagation than in BGG's model, is in line with Kocherlakota (2000)'s argument - credit constraints can help to explain the properties of output fluctuations in the U.S., including the large movements in aggregate output. According to this author, these large movements cannot be explained by large shocks (those "*are hard to find in the data,*" p. 3), but by mechanisms which transform "*small, barely detectable, shocks to some or all parts of the economy into large, persistent, asymmetric movements in aggregate output.*"

As for persistence over time of the effects of the monetary policy shock, using the half-life (from the initial impact) criterion, as in Carlstrom and Fuerst (2001), none of the amplification channels generate higher persistence: the output response, for instance, reaches half life between the second and the third quarters in variants 1, 2, 3 and BGG.

3.2 Some Sensitivity and Robustness Checks

Shocks to technology and to government expenditures We have also analyzed the effects of two alternative shocks in our model: a permanent technology shock and a temporary shock to government expenditures, under variants 1, 2, 3 and BGG. The results, not reported here, show that as in the monetary policy shock, the financial accelerator and the liquidity premium effects amplify the real effects of both shocks.

The effect of the deposit-bank capital ratio on the external finance premium, through the deposit insurance costs We return now to equation (46),

$$E_t(r^K_{t+1}) - r^F_{t+1} \simeq \left(1 - \alpha_e \frac{R^K}{R^F}\right) \left[E_t(r^K_{t+1}) - r^D_{t+1}\right] + \\ - \left[(1-\alpha_e)\frac{2\delta_e \frac{D}{S}}{R^F} - \frac{2\alpha_e \delta_e \left(\frac{D}{S}\right)^2}{R^F}\right] (d_{t+1} - s_{t+1}),$$

and evaluate the effect of $d_{t+1} - s_{t+1}$ on the external finance premium, through the deposit insurance costs. As mentioned this effect is relatively small, when compared to the effect associated with the liquidity premium. Besides, since it works through the deposit insurance costs, if we set $\delta_e = 0$ the effect vanishes.

Therefore, to evaluate the effect of $d_{t+1} - s_{t+1}$ on the external finance premium, through the deposit insurance costs, we compare the impact of an increase in capital requirements (from $\alpha_e = 4\%$ to $\alpha_e = 8\%$), in the monetary policy transmission mechanism, under $\delta_e = 0.0000045$ (variant 1) and $\delta_e = 0$.

As mentioned, the increase in capital requirements leads, through the liquidity premium effect, to an amplification of the real effects of the monetary shock. According to our results, this amplification effect is not significantly affected by the deposit insurance cost parameter, δ_e:

1. If $\delta_e = 0.0000045$, real output initially decreases 1.31% under $\alpha_e = 4\%$ *vs* 1.44% under $\alpha_e = 8\%$. That is, under $\delta_e = 0.0000045$ the increase in capital requirements amplifies the initial impact of monetary policy on real output by about 10%;

2. If $\delta_e = 0$, real output initially decreases 1.29% under $\alpha_e = 4\%$ *vs* 1.37% under $\alpha_e = 8\%$. That is, under $\delta_e = 0$ the increase in capital requirements amplifies the initial impact of monetary policy on real output by about 6%.

We perform a similar exercise assuming a flat deposit insurance rate, instead of a risk sensitive deposit insurance rate: $\left[\delta_e \frac{D_{t+1}}{S_{t+1}}\right]$ was replaced by cD_{t+1} $(= 0.000075 D_{t+1})$ in the bank's profit maximization problem. The results obtained are similar to those reported in 2.: assuming a flat deposit insurance rate, the increase in capital requirements, from $\alpha_e = 4\%$ to $\alpha_e = 8\%$, amplifies the initial impact of monetary policy shock on real output by about 6%.[44]

[44]Note that if we assume a flat deposit insurance rate, the first order conditions derived from the bank's profit

Therefore, we confirm that the effect of the deposit-bank capital ratio on the external finance premium, through the deposit insurance costs, has a minor role in the amplification of the monetary policy shock real effects.

Investment adjustment costs in the production of capital Since the financial accelerator effect in BBG's model is based on the unanticipated change of asset prices, we have also considered the hypothesis of removing the adjustment costs in the production of capital (which leads to a constant price of capital). The differences between the impulse response functions of variants 1 and 2, on the one hand, and variant 3, on the other hand, remain notorious, now due mainly to the liquidity premium effect: in variant 3 the initial impact of the monetary policy shock on real output is reduced by 43.26% (vs 63.63% in the benchmark case) when compared to variant 1.

Introducing habit formation and alternative investment adjustment costs In general, evidence shows that exogenous shocks lead to a delayed and hump-shaped response of real output. For instance, Christiano *et al.* (2005)'s estimates of how major macroeconomic variables respond to a monetary policy shock in the US economy, show that after an expansionary monetary policy shock, output, consumption, and investment respond in a hump-shaped fashion, peaking after about one and a half years and returning to pre-shock levels after about three years. Inflation also responds in a hump shaped fashion, peaking after about two years. Our baseline model, presented in the previous section, doesn't match these hump shaped responses: output, consumption, investment and inflation peak immediately after the shock and then revert to their steady state levels. To address this issue, we now consider the two following modifications to the baseline model:

(i) Introduction of (internal) habit formation in consumption: the household's momentary utility increases with the current period consumption and decreases with the previous period consumption. That is, we replace the level of consumption, in the utility function, with its growth rate. As mentioned by Christiano *et al.*, with habit formation households relate the change in the growth rate of consumption to the interest rate. With a low

maximization problem are different from those derived above. More specifically, the log-linearized equations derived from those first order conditions become

$$
\begin{aligned}
r_{t+1} &= \frac{R^D}{R} r_{t+1}^D \\
r_{t+1}^F &= \alpha_e \frac{R^K}{R^F} E_t\left(r_{t+1}^K\right) + (1-\alpha_e)\frac{R}{R^F} r_{t+1},
\end{aligned}
$$

and, in steady state, $R = R^D + c$ and $R^F = (1-\alpha_e)R + \alpha_e R^K$. Therefore, the effect of $(d_{t+1} - s_{t+1})$ on the external finance premium through the deposit insurance rate, present in equation (46), disappears.

interest rate, households choose a consumption profile characterized by a declining growth rate of consumption: a low real interest rate is associated with high current consumption growth relative to the future, a pattern found in the data (whereas, a model with standard, time-separable preferences cannot be made consistent with this pattern).[45]

(ii) Modification of investment adjustment costs, assuming that those costs depend directly on changes in investment, as in Christiano *et al.* (2005).

Recall that our baseline model assumes that the aggregate (physical) capital stock follows an intertemporal accumulation equation with adjustment costs:

$$K_{t+1} = \Xi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t,$$

where I_t denotes the investment expenditures, δ the depreciation rate, and $\Xi\,(.)$ an increasing and concave function with $\Xi\,(0) = 0$. The introduction of adjustment costs permitted variation in the price of a unit of capital in terms of the numeraire good, Q_t, which is given by

$$Q_t = \frac{1}{\Xi'\left(\frac{I_t}{K_t}\right)}.$$

Now we assume instead that the aggregate investment expenditures I_t yield a gross output of new capital goods $F\,(I_t, I_{t-1})$. This function summarizes the technology that transforms current and past investment into capital for use in the following period:

$$K_{t+1} = F\,(I_t, I_{t-1}) + (1-\delta)K_t.$$

The functional of $F\,(I_t, I_{t-1})$ adopted by Christiano *et al.*, which penalizes the *change* in I_t, is motivated by their empirical finding that investment exhibits a hump-shaped response to a monetary policy shock. Using numerical experiments, Christiano *et al.* found that in practice the specifications that penalize the *level* of investment instead are not able to generate hump shaped response of investment. Therefore, following those authors, we assume that

$$F\,(I_t, I_{t-1}) = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

[45] In particular, we assume the following instantaneous utility function:

$$U_t = \frac{(C_t - bC_{t-1})^{1-\sigma}}{1-\sigma} + \alpha_0 \frac{D_{t+1}{}^{1-\beta_0}}{1-\beta_0} + \alpha_1 \frac{(1-H_t^h)^{1-\beta_1}}{1-\beta_1},$$

where $b = 0.7$ (based on Fuhrer, 2000, Boldrin *et al.*, 2001 and Christiano *et al.*, 2005's estimates).

where $S(1) = S'(1) = 0$ and $S''(1) > 0$. To calibrate the model, we assume that $S''(1) = 2.5$ (in the presence of consumption habit formation), as Christiano *et al.*.

In our model, the introduction of habit formation alone generates a hump-shaped consumption profile, as expected, but output and investment maintain their "jump variable" behavior. However, combining habit formation in consumption with Christiano *et al.*'s investment adjustment costs, generates hump-shaped responses of investment, consumption and, consequently, output.[46]

Figure 4 compares the effects of the negative innovation in the nominal interest rate assumed before, under the same three distinct hypotheses (variants 1, 2 and 3), but introducing habit formation together with Christiano *et al.*'s investment adjustment costs. The responses of real output, investment and consumption are weaker and more persistent than in the benchmark case: the real output response, for instance, reaches half life between the seventh and the eighth quarters after the initial shock (under the three variants). But our conclusions concerning the amplification effects, under these three variants, remain valid: for instance, taking the peak of the impulse response function of real output for the three variants (third quarter's values), the impact of monetary shock is amplified by 127.85% (*vs* 131.69% in the benchmark case) when the borrowers' balance sheet and the bank capital channels are introduced (variant 2 vs variant 3) and the impact is further amplified by 17.6% (*vs* 18.66% in the benchmark case) with the introduction of capital requirements (variant 1 vs variant 2).

4 Concluding Remarks

Focusing on how microeconomic structures - namely the banks funding structure and the relationship between the banks, entrepreneurs and households - interact with macroeconomic business conditions, we found that, in our model, the bank capital channel amplifies the real effects of a monetary policy shock, through the liquidity premium effect. This effect is strictly related to the financial accelerator effect associated with the borrowers' balance sheet channel: when the liquidity premium and the financial accelerator effects are both present, the external finance premium responds, not only to borrowers' financial position (as in Bernanke *et al.*, 1999), but also to the liquidity premium required by households to hold bank capital. This exacerbates the (coutercyclical) response of the external finance premium to a monetary policy shock, since the liquidity premium also moves countercyclically and influences positively the external finance premium.

The liquidity premium effect rests on the fact that bank capital (mandatory due to risk-based capital requirements) is more expensive to raise than deposits, due to households' preferences for

[46] Although not reported here, employment also decreases in a hump-shaped manner.

liquidity, and that this difference tends to widen (narrow) during a recession (expansion): after a contractionary monetary policy shock, for instance, households tend to decrease the amount of deposits held to attenuate the decline in consumption; since we assume that deposits provide liquidity services, the households, thus, require an increase in liquidity premium, that is, an increase in the difference between the expected return on bank capital (which is also owned by households, but which does not render any liquidity services) and the return on deposits. This cost is then passed on to firms by the bank through an increase of the external finance premium.

Concerning the magnitude of the amplification effects, our results indicate that if we bring together the bank capital with the borrowers' balance sheet channel, financial frictions do seem to be a very important mechanism through which monetary shocks get propagated in the economy. Actually, if, in addition to the informational asymmetry between each entrepreneur and his bank, we introduce in the model other financial frictions related to the imposition of bank capital minimum levels, the role of financial frictions in the mechanism through which monetary shocks are propagated in the economy becomes much more powerful than in Bernanke *et al.*'s model, in line with some arguments in related literature.

Economic policy conclusions should be drawn carefully, however, since the model simplifies and abstracts from many important features of the economy. For example, in our analysis we were not concerned with questions such as whether bank regulation is itself optimal and what type of regulation is more appropriate. We ignore risk and incentives that support capital adequacy regulation (as the social cost of bank failure) and, therefore, our analysis does not support any normative conclusions regarding bank-capital regulation.

Since we have also shown that the magnitude of the amplification effects is sensitive to the level of risk-based capital requirements, future steps of this research will explore in more detail the consequences of different bank capital requirements, namely those proposed under Basel II.

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Appendices

Appendix A: The optimal financial contract (following Bernanke, Gertler and Gilchrist, 1999 - BGG)

According to the model, and as mentioned in 2.1, the return to capital is sensitive to both aggregate and idiosyncratic risk. The *ex post* gross return on capital for firm j is $\omega_{t+1}^{j} R_{t+1}^{K}$, where ω_{t+1}^{j} is an idiosyncratic disturbance to firms j's return and R_{t+1}^{K} is the *ex post* aggregate return to capital. The random variable ω^{j} is i.i.d. across time and across firms, with a continuous and once-differentiable cumulative distribution function (c.d.f.), $F(\omega)$, over a non-negative support, and $E\left(\omega^{j}\right) = 1$.

More specifically, it is assumed that ω follows a log-normal distribution:

$$\ln(\omega) \sim N\left(-\frac{1}{2}\sigma_{\ln\omega}^{2}, \sigma_{\ln\omega}^{2}\right).$$

Note that, under this distribution, the hazard rate $h(\omega)$ satisfies,

$$\frac{\partial\left(\omega h(\omega)\right)}{\partial\omega} > 0, \tag{48}$$

where $h(\omega) = \frac{f(\omega)}{1-F(\omega)}$, and $f(\omega)$ is the probability density function (p.d.f.) of ω.

To describe the optimal contractual arrangement, we follow BGG and analyze the case where the aggregate return on capital, R_{t+1}^{K}, is known in advance. In this context, the only uncertainty about the project's return is idiosyncratic to the firm.

Taking into account the definition of the cutoff value $\overline{\omega}_{t+1}^{j}$ (see equation (1) in section 2), let $\Gamma\left(\overline{\omega}_{t+1}\right)$ be the expected gross share of profit going to the lender,[47]

$$\Gamma\left(\overline{\omega}_{t+1}\right) \equiv \int_{0}^{\overline{\omega}_{t+1}} \omega_{t+1}^{j} f(\omega) d\omega + \overline{\omega}_{t+1} \int_{\overline{\omega}_{t+1}}^{\infty} f(\omega) d\omega, \tag{49}$$

and $\mu\Theta(\overline{\omega}_{t+1})$ the expected monitoring costs,

$$\mu\Theta(\overline{\omega}_{t+1}) \equiv \mu \int_{0}^{\overline{\omega}_{t+1}} \omega_{t+1}^{j} f(\omega) d\omega. \tag{50}$$

Therefore, $\Gamma\left(\overline{\omega}_{t+1}\right) - \mu\Theta(\overline{\omega}_{t+1})$ represents the net share of profits going to the lender and $1 - \Gamma\left(\overline{\omega}_{t+1}\right)$ the share going to the entrepreneur (where, by definition, $\Gamma\left(\overline{\omega}_{t+1}\right)$ satisfies $0 < \Gamma\left(\overline{\omega}_{t+1}\right) < 1$).

In this context, the optimal contracting problem, which determines the division of the

[47] To simplify the notation we now drop the j superscript of $\overline{\omega}_{t+1}^{j}$, since, as will be shown later in this appendix, $\overline{\omega}_{t+1}^{j} = \overline{\omega}_{t+1}, \forall j$.

expected gross payoff to the firm's capital $R^K_{t+1}Q_tK^j_{t+1}$ between the borrower and the lender, may be written in the following way:

$$\begin{aligned}&\max_{K^j_{t+1},\overline{\omega}_{t+1}} (1-\Gamma(\overline{\omega}_{t+1}))R^K_{t+1}Q_tK^j_{t+1}\\ &s.t.\\ &\left[\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})\right]R^K_{t+1}Q_tK^j_{t+1}=R^F_{t+1}(Q_tK^j_{t+1}-N^j_{t+1}).\end{aligned} \tag{51}$$

Defining l_{t+1} as $\frac{R^K_{t+1}}{R^F_{t+1}}$ and using $k^j_{t+1}=\frac{Q_tK^j_{t+1}}{N^j_{t+1}}$ as the choice variable we may redefine (given constant returns to scale) the optimal contracting problem as

$$\begin{aligned}&\max_{k^j_{t+1},\overline{\omega}_{t+1}} (1-\Gamma(\overline{\omega}_{t+1}))l_{t+1}k^j_{t+1}\\ &s.t.\\ &\left[\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})\right]l_{t+1}k^j_{t+1}=(k^j_{t+1}-1).\end{aligned} \tag{52}$$

The first order conditions (FOC) of this problem are then given by

$$\overline{\omega}_{t+1}:\Gamma'(\overline{\omega}_{t+1})-\lambda\left[\Gamma'(\overline{\omega}_{t+1})-\mu\Theta'(\overline{\omega}_{t+1})\right]=0; \tag{53}$$

$$k^j_{t+1}:\left\{(1-\Gamma(\overline{\omega}_{t+1}))+\lambda\left[\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})\right]\right\}l_{t+1}-\lambda=0; \tag{54}$$

$$\lambda:\left[\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})\right]l_{t+1}k^j_{t+1}-(k^j_{t+1}-1)=0, \tag{55}$$

where λ is the Lagrange multiplier of the constraint that the bank earns its required rate of return in expectation.

Now, taking into account the assumptions above, namely $0<\mu<1$, $E(\omega)\equiv\int_0^\infty \omega f(\omega)d\omega=1$, and taking into account equations (49) and (50), it is straightforward to show that, for $\overline{\omega}\in(0,\infty)$, $\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})>0$, or, equivalently, that

$$(1-\mu)\int_0^{\overline{\omega}_{t+1}}\omega^j_{t+1}f(\omega)d\omega+\overline{\omega}_{t+1}\int_{\overline{\omega}_{t+1}}^\infty f(\omega)d\omega>0. \tag{56}$$

Taking limits,

$$\lim_{\overline{\omega}_{t+1}\to 0}\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})=0 \tag{57}$$

$$\lim_{\overline{\omega}_{t+1}\to \infty}\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})=1-\mu. \tag{58}$$

In what follows, we will assume, as in BGG, that $R^K_{t+1}(1-\mu)<R^F_{t+1}$, which implies that

$l_{t+1} < \frac{1}{1-\mu}$.[48]

Differentiating $\Gamma(\overline{\omega}_{t+1}) - \mu\Theta(\overline{\omega}_{t+1})$ yields,

$$\Gamma'(\overline{\omega}_{t+1}) - \mu\Theta'(\overline{\omega}_{t+1}) = [1 - F(\overline{\omega}_{t+1})]\,[1 - \mu\overline{\omega}_{t+1}h(\overline{\omega}_{t+1})]\,, \tag{59}$$

where $F'(\overline{\omega}_{t+1}) > 0$, and, since ω follows a log-normal distribution, $\overline{\omega}_{t+1}h(\overline{\omega}_{t+1})$ is also increasing in $\overline{\omega}_{t+1}$ (see expression (48) on page 41).

Therefore, and also taking into account equations (56), (57) and (58), $\Gamma(\overline{\omega}_{t+1}) - \mu\Theta(\overline{\omega}_{t+1})$ reaches a global maximum at $\overline{\omega}^*_{t+1}$: $\Gamma(\overline{\omega}_{t+1}) - \mu\Theta(\overline{\omega}_{t+1})$ is increasing on $\left(0, \overline{\omega}^*_{t+1}\right)$ and decreasing on $\left(\overline{\omega}^*_{t+1}, \infty\right)$. That is, the bank's expected return reaches a maximum at an unique interior value of $\overline{\omega}_{t+1}$, $\overline{\omega}^*_{t+1}$.

Taking the first order conditions above into account, and assuming that $0 < \overline{\omega}_{t+1} < \overline{\omega}^*_{t+1}$ (interior solution) and that $l_{t+1} > 1$, we derive a one-to-one mapping between the optimal cutoff $\overline{\omega}_{t+1}$ and l_{t+1}, such that,

$$\overline{\omega}_{t+1} = \overline{\omega}\left(l_{t+1}\right),$$

with $\overline{\omega}'\left(l_{t+1}\right) > 0$.

Besides, under the same assumptions, the first order conditions yield the following relationship between entrepreneur's j ratio of capital expenditures to net worth and the cutoff value:

$$k^j_{t+1} = \psi(\overline{\omega}_{t+1}),$$

where $\psi'(\overline{\omega}_{t+1}) > 0$.

Combining these two expressions, we may express the capital-wealth ratio, k^j_{t+1}, as an increasing function of the discounted return to capital:

$$k^j_{t+1} = \vartheta(l_{t+1}),$$

where $\vartheta'(l_{t+1}) > 0$.

Therefore, the first order conditions of the contracting problem give rise to a monotonically increasing relationship between each firm's ratio of capital expenditures to net worth, $\frac{Q_t K^j_{t+1}}{N^j_{t+1}}$, and the discounted return to capital, $\frac{R^K_{t+1}}{R^F_{t+1}}$:

$$\frac{Q_t K^j_{t+1}}{N^j_{t+1}} = \vartheta\left(\frac{R^K_{t+1}}{R^F_{t+1}}\right),$$

with $\vartheta'(.) > 0$ and $\vartheta(1) = 1$, given $R^K_{t+1}/R^F_{t+1} > 1$.

[48] If not, the firm could obtain unbounded profits under monitoring that occurs with probability one.

A detailed solution of this problem is available upon request, as well as the proof that equilibrium is always characterized by $\overline{\omega}_{t+1}$ below $\overline{\omega}^{*}_{t+1}$: $0 < \overline{\omega}_{t+1} < \overline{\omega}^{*}_{t+1}$.[49] See also BGG.

In this context, and as mentioned above, it is also possible to show that $\overline{\omega}^{j}_{t+1}$ is the same for all producers:[50] The intuition is that, facing the same discounted return, producers choose the same leverage ratio, which leads to the same cutoff value; firms with more net worth do not pay a lower interest rate, but have a larger scope of operations.

According to the financial contract signed by the bank and by each entrepreneur, the bank receives an expected gross return on the loan equal to the required return R^{F}_{t+1} (see equation 3 in section 2):

$$\left\{\left[1 - F(\overline{\omega}^{j}_{t+1})\right]\overline{\omega}^{j}_{t+1} + (1-\mu)\int_{0}^{\overline{\omega}^{j}_{t+1}} \omega^{j}_{t+1} f(\omega)d\omega\right\} R^{K}_{t+1} Q_t K^{j}_{t+1} =$$
$$= R^{F}_{t+1}(Q_t K^{j}_{t+1} - N^{j}_{t+1}).$$

This equation, jointly with $k^{j}_{t+1} = \vartheta(l_{t+1})$, implies that

$$\left\{\left[1 - F(\overline{\omega}^{j}_{t+1})\right]\overline{\omega}^{j}_{t+1} + (1-\mu)\int_{0}^{\overline{\omega}^{j}_{t+1}} \omega^{j}_{t+1} f(\omega)d\omega\right\} R^{K}_{t+1}\vartheta\left(\frac{R^{K}_{t+1}}{R^{F}_{t+1}}\right) =$$
$$= R^{F}_{t+1}\left[\vartheta\left(\frac{R^{K}_{t+1}}{R^{F}_{t+1}}\right) - 1\right]. \tag{60}$$

The right hand side of equation (60) does not depend on j (is equal for all entrepreneurs). The left hand side, in turn, can be rewritten as

$$\left[\Gamma\left(\overline{\omega}^{j}_{t+1}\right) - \mu\Theta(\overline{\omega}^{j}_{t+1})\right] R^{K}_{t+1}\vartheta\left(\frac{R^{K}_{t+1}}{R^{F}_{t+1}}\right).$$

Since we are constraining ourselves to the analysis of the interior solution, $0 < \overline{\omega}_{t+1} < \overline{\omega}^{*}_{t+1}$, $\left[\Gamma\left(\overline{\omega}^{j}_{t+1}\right) - \mu\Theta(\overline{\omega}^{j}_{t+1})\right]$, and, consequently, the left hand side of (60) is increasing in $\overline{\omega}^{j}_{t+1}$. Therefore, there exists only one $\overline{\omega}^{j}_{t+1}$ that satisfies equation (60): $\overline{\omega}^{j}_{t+1} = \overline{\omega}_{t+1}, \forall j$.[51]

Following a similar procedure, it is straightforward to show that the non-default lending rate is also the same for all producers: $Z^{j}_{t+1} = Z_{t+1}, \forall j$.

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[49] Therefore, the hypothesis of an equilibrium with credit rationing is ruled out and the bank's expected return is always increasing in $\overline{\omega}_{t+1}$. Note that a borrower would be rationed if $\overline{\omega}_{t+1}$ would not generate the required expected bank's return.

[50] The reasoning here follows an unfinished paper by Refet Gürkaynak who asked not to be cited.

[51] This also applies to the case with aggregate risk. The only difference is that in equation (60), $\vartheta\left(\frac{R^{K}_{t+1}}{R^{F}_{t+1}}\right)$ is replaced by $\varphi\left[\frac{E_t\left(R^{K}_{t+1}\right)}{R^{F}_{t+1}}\right]$.

Contract terms when there is aggregate risk (See also BGG, appendix A.3)

The problem is now given by the maximization of $E_t\left[(1-\Gamma(\overline{\omega}_{t+1}))R^K_{t+1}Q_tK^j_{t+1}\right]$ with respect to K^j_{t+1} and $\overline{\omega}_{t+1}$, subject to a set of state-contingent constraints implied by:

$$\left[\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})\right]R^K_{t+1}Q_tK^j_{t+1}=R^F_{t+1}(Q_tK^j_{t+1}-N^j_{t+1}),$$

where E_t denotes the expectation operator conditional on the information available at time t.

Let l_{t+1} represent now $\frac{E_t\left(R^K_{t+1}\right)}{R^F_{t+1}}$, the expected discounted return to capital. Given $l_{t+1}>1$, the first order conditions yield now the following relationship between $\frac{Q_tK^j_{t+1}}{N^j_{t+1}}$ and l_{t+1}:

$$\frac{Q_tK^j_{t+1}}{N^j_{t+1}}=\varphi\left(\frac{E_t\left(R^K_{t+1}\right)}{R^F_{t+1}}\right),$$

where $\varphi'(.)>0$ and $\varphi(1)=1$.

Appendix B: Computation of parameter v

The log-linearization of equation (5) in section 2,

$$\frac{E_t\left(R^K_{t+1}\right)}{R^F_{t+1}}=l\left(\frac{Q_tK_{t+1}}{N_{t+1}}\right)$$

yields $E_t(r^K_{t+1})-r^F_{t+1}=v(k_{t+1}+q_t-n_{t+1})$, where v is the steady state elasticity of l with respect to k:

$$v=\frac{l'(\overline{\omega}_{SS})}{k'(\overline{\omega}_{SS})}\frac{k(\overline{\omega}_{SS})}{l(\overline{\omega}_{SS})}. \tag{61}$$

Computation of v (following Gertler *et al.*, 2003):

The first order condition (53) - see appendix A - yields the following relationship between λ and $\overline{\omega}_{t+1}$:

$$\lambda(\overline{\omega}_{t+1})=\frac{\Gamma'(\overline{\omega}_{t+1})}{\Gamma'\left(\overline{\omega}_{t+1}\right)-\mu\Theta'(\overline{\omega}_{t+1})}. \tag{62}$$

Defining

$$\rho(\overline{\omega}_{t+1})\equiv\frac{\lambda(\overline{\omega}_{t+1})}{\left[1-\Gamma\left(\overline{\omega}_{t+1}\right)\right]+\lambda(\overline{\omega}_{t+1})\left[\Gamma\left(\overline{\omega}_{t+1}\right)-\mu\Theta(\overline{\omega}_{t+1})\right]}, \tag{63}$$

first order condition (54) implies, in turn, that

$$\rho(\overline{\omega}_{t+1})=l_{t+1}, \tag{64}$$

where l_{t+1} represents the wedge between the rate of return on capital and the return on loans demanded by the bank.

Now, let $\Psi(\overline{\omega}_{t+1}) \equiv [1-\Gamma(\overline{\omega}_{t+1})]+\lambda(\overline{\omega}_{t+1})[\Gamma(\overline{\omega}_{t+1})-\mu\Theta(\overline{\omega}_{t+1})]$. Therefore, equations (63) and (64), can be combined into

$$l_{t+1}(\overline{\omega}_{t+1}) = \frac{\lambda(\overline{\omega}_{t+1})}{\Psi(\overline{\omega}_{t+1})} \tag{65}$$

with

$$l'_{t+1}(\overline{\omega}_{t+1}) = \frac{\lambda'(\overline{\omega}_{t+1})\Psi(\overline{\omega}_{t+1}) - \lambda(\overline{\omega}_{t+1})\Psi'(\overline{\omega}_{t+1})}{[\Psi(\overline{\omega}_{t+1})]^2}, \tag{66}$$

and where

$$\Psi'(\overline{\omega}_{t+1}) = -\Gamma'(\overline{\omega}_{t+1}) + \lambda'(\overline{\omega}_{t+1})[\Gamma(\overline{\omega}_{t+1}) - \mu\Theta(\overline{\omega}_{t+1})] + \lambda(\overline{\omega}_{t+1})[\Gamma'(\overline{\omega}_{t+1}) - \mu\Theta'(\overline{\omega}_{t+1})].$$

Concerning $k_{t+1}\left(\equiv \frac{Q_t K_{t+1}}{N_{t+1}}\right)$ we know that (taking into account the first order condition (55), in appendix A, and equations (63) and (64))

$$\begin{aligned} k_{t+1} &= \psi(\overline{\omega}_{t+1}) \equiv 1 + \frac{\lambda(\overline{\omega}_{t+1})[\Gamma(\overline{\omega}_{t+1}) - \mu\Theta(\overline{\omega}_{t+1})]}{1-\Gamma(\overline{\omega}_{t+1})} = \\ &= \frac{1-\Gamma(\overline{\omega}_{t+1}) + \lambda(\overline{\omega}_{t+1})[\Gamma(\overline{\omega}_{t+1}) - \mu\Theta(\overline{\omega}_{t+1})]}{1-\Gamma(\overline{\omega}_{t+1})} = \\ &= \frac{\Psi(\overline{\omega}_{t+1})}{1-\Gamma(\overline{\omega}_{t+1})}. \end{aligned} \tag{67}$$

Thus,

$$k'_{t+1}(\overline{\omega}_{t+1}) = \frac{\Psi'(\overline{\omega}_{t+1})[1-\Gamma(\overline{\omega}_{t+1})] + \Psi(\overline{\omega}_{t+1})\Gamma'(\overline{\omega}_{t+1})}{[1-\Gamma(\overline{\omega}_{t+1})]^2}. \tag{68}$$

Finally, after evaluating equations (65), (66), (67) and (68) in the steady state, we may then compute v through equation (61). $\overline{\omega}_{SS}$ is, in turn, computed by solving equation,

$$l(\overline{\omega}) - (1-\delta)\frac{1}{R^F} = \frac{\alpha}{(1-\alpha)(1-\Omega)}\left[\frac{1}{R^F}\frac{1}{k(\overline{\omega})} - \gamma l(\overline{\omega})(1-\Gamma(\overline{\omega}))\right],$$

derived from equations (6), (7), (22), (25), and the constraint of the contract optimization problem (51), all evaluated in the steady state.

A detailed procedure to compute v is available upon request.

Appendix C: List of variables

Variables	Definition
K	Aggregate capital stock
K^j	Capital stock of entrepreneur j
N	Aggregate entrepreneurial net worth
N^j	Entrepreneur's j net worth
V	Aggregate entrepreneurial equity
Q	Price of capital
H^h	Household hours worked
H^e	Entrepreneurial labor
H	Total labor input
W^h	Household real wage
W^e	Entrepreneurial real wage
R^K	Ex post aggregate real return to capital
ω^j	Idiosyncratic disturbance to firm's j return
$\overline{\omega}$	Cutoff value of ω^j
R^F	Bank's required real return on lending
Z	Non-default loan rate
L	Total bank loans
L^j	Loan granted to entrepreneur j
D	Bank deposits
S	Bank capital
B	Government bonds held by the bank
R	Gross real return on gov bonds
R^D	Gross real return on deposits
R^S	Gross real return on bank capital ($= R^K$)
C	Household consumption
C^e	Entrepreneurial consumption
T	Lump sum taxes
Π	Dividends (of retail firms)
Y	Aggregate output
I	Investment expenditures
A	Technology
P	Price index
X	Gross markup of retail goods over the wholesale goods
G	Government expenditures
π	Inflation rate

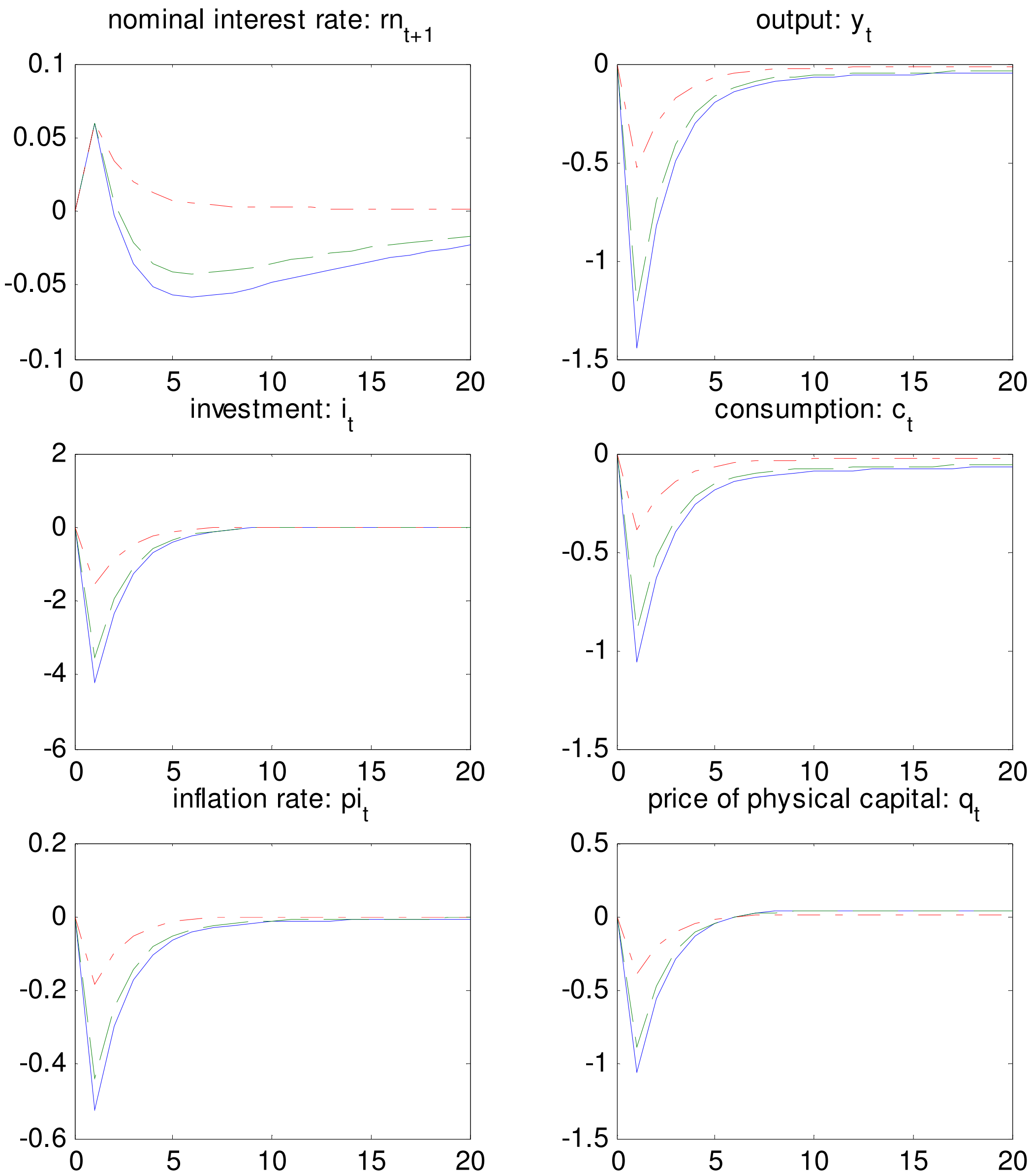

Figure 1: Response of economic activity to a negative monetary policy shock: variant 1 (solid line) - with capital requirements; variant 2 (dashed line) - without capital requirements; variant 3 (dashed-dotted line) - with no capital requirements nor financial accelerator.

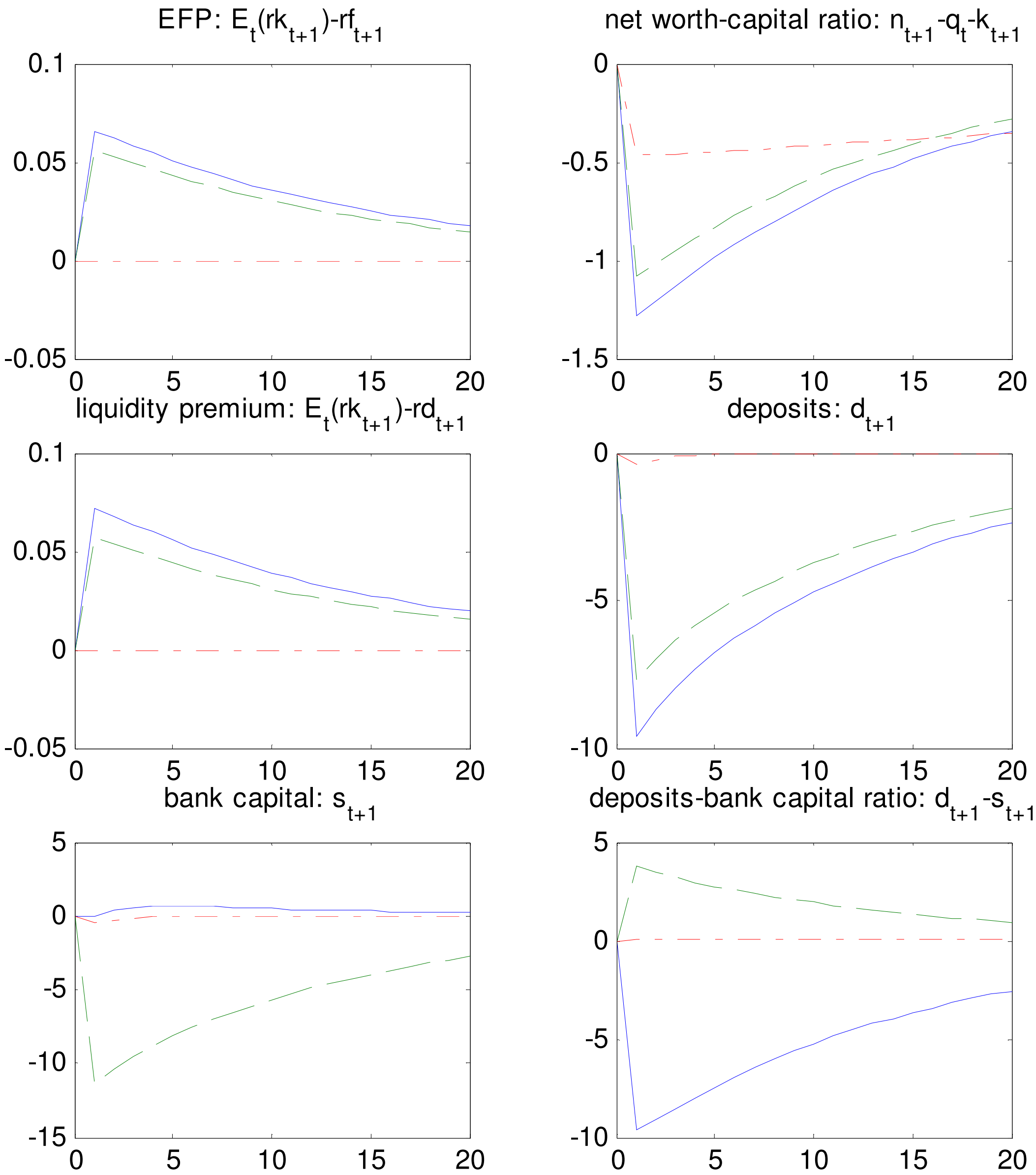

Figure 2: Response of financial variables to a negative monetary policy shock: variant 1 (solid line) - with capital requirements; variant 2 (dashed line) - without capital requirements; variant 3 (dashed-dotted line) - with no capital requirements nor financial accelerator.

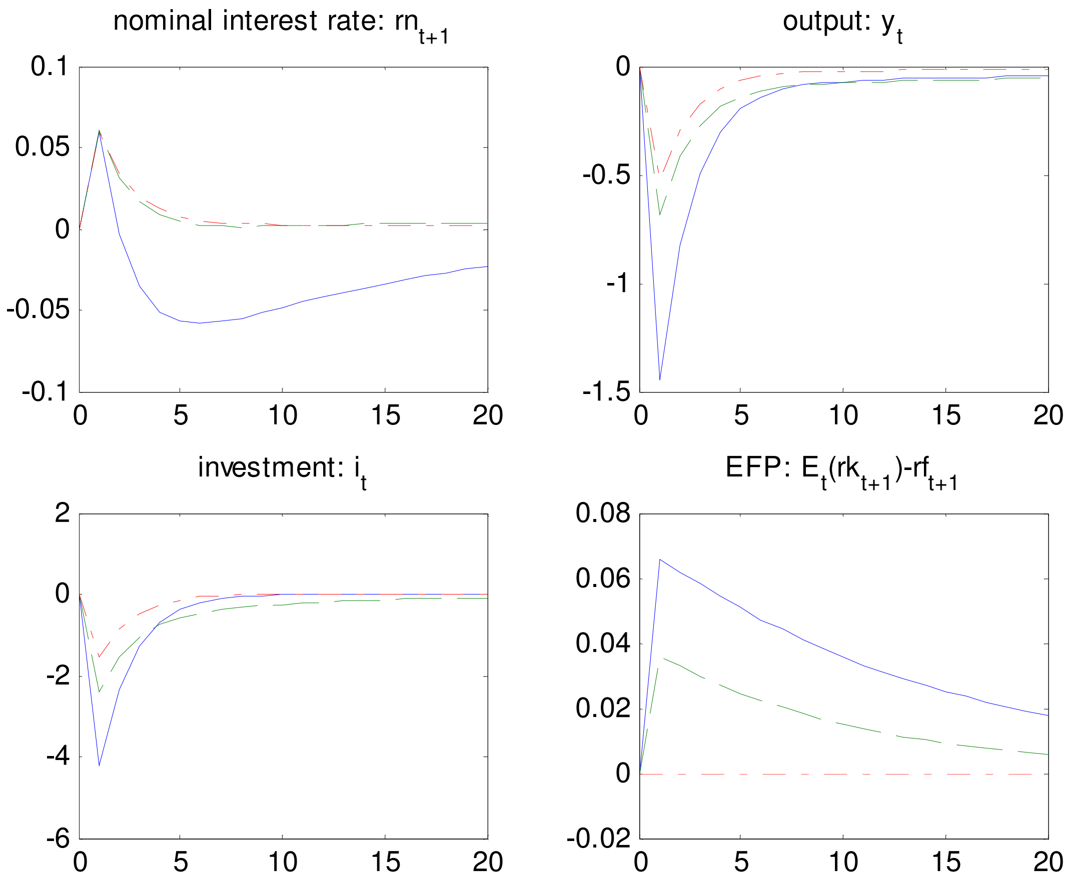

Figure 3: Response of economic activity to a negative monetary policy shock: variant 1 (solid line) - with capital requirements; variant BGG (dashed line) - based on BGG's model; variant 3 (dashed-dotted line) - with no capital requirements nor financial accelerator.

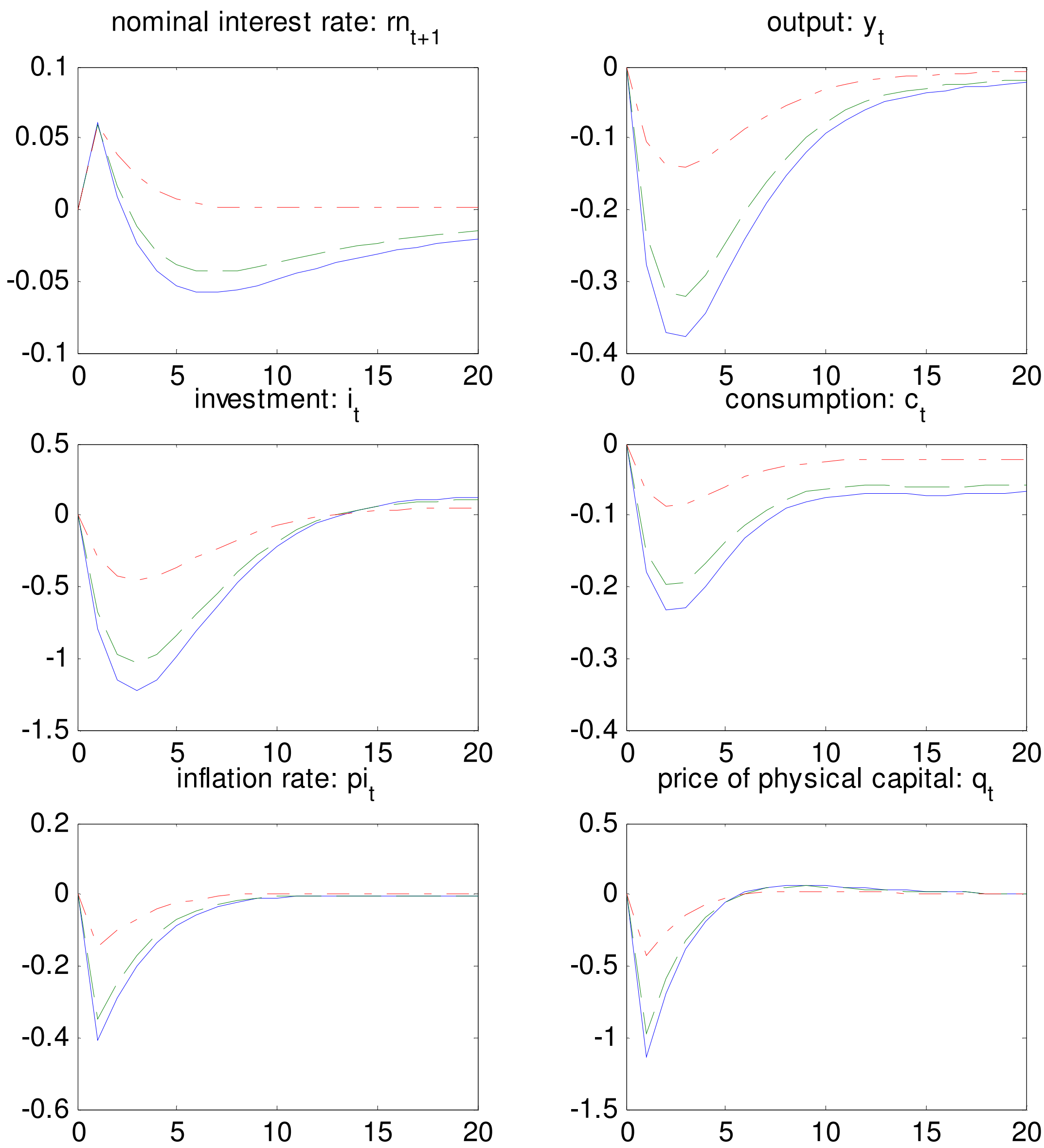

Figure 4: Response of economic activity to a negative monetary policy shock under habit formation and alternative investment adjustment costs: variant 1 (solid line) - with capital requirements; variant 2 (dashed line) - without capital requirements; variant 3 (dashed-dotted line) - with no capital requirements nor financial accelerator.