‘Active’ and ‘passive’ monetary policy in a non-Ricardian world*

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Abstract

This paper analyses optimal monetary policy under the fiscal theory of the price level. We use a robustly microfounded New Keynesian model with a substantial degree of persistence. Optimal policy should take into account strategic behaviour by the private sector. We discuss how restricted monetary policy is when the fiscal policy is passive. Our findings reinforce those of Leeper (1991) and Leith and Wren-Lewis (2000). They find that in a the world with non-Ricardian consumers and passive fiscal policy, monetary policy is severely restricted. We find similar constraints apply when policymakers behave optimally subject to a time consistency constraint. We discuss robustness of these results to the specification of the policymaker’s loss function.

Key Words: Monetary Policy, Time Consistency, Feedback Rules, Fiscal Theory of the Price Level

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1 Introduction

This paper studies the interactions between fiscal and monetary policy when fiscal policy is not especially concerned with the level of domestic debt. Governments may find themselves in a situation of too few policy instruments and are therefore forced to devote fiscal policy to other—potentially more politically sensitive—tasks. Such enforced fiscal 'inactivity' will then affect both the behaviour of the private sector, which might become concerned with debt sustainability, and the scope of monetary policy actions, which can become severely restricted. The most extreme case of such interactions can be usefully studied in the framework of the fiscal theory of the price level.

The fiscal theory of the price level has been proposed as the relevant solution for several policy regimes. Leeper (1991) identified two non-conflicting regimes, in one of which (with 'active' fiscal policy and 'passive' monetary policy, to be defined below) the inflation rate is affected by fiscal shocks. Evans and Honkapohja (2002) used the same model to show that these regimes are learnable and so logically can exist. The distinctive feature of these regimes is that monetary policy does not provide a nominal anchor. The short-term interest rate acts as the monetary policy instrument so there is no conflict between fiscal and monetary policies with respect to the price level—the price level is fiscally determined.

The strong effect of fiscal policy on inflation naturally raises the question of what should constitute optimal monetary policy in such a world. Leeper (1991), who began the analytical literature on the subject, has shown that a standard monetary policy rule can have disastrous consequences, namely a substantial rise of the interest rate in response to an inflation shock might destabilise the economy causing spiralling debt accumulation and future inflation (this would be an outcome of two 'active' policies). For a similar setup with Blanchard-Yaari consumers Leith and Wren-Lewis (2000) demonstrate that a monetary policy with strong feedback on inflation is equally undesirable, as such a monetary policy actually performs worse in stabilising inflation than less aggressive policy. In this paper we aim to investigate what an *optimal* monetary policy should look like in a world where price and wage-setters' decisions are sensitive to the prospects of the government's budget stance and this effect is expected to be substantial.

If we classify our setup using Leeper's definitions, we would consider fiscal policy to be 'active', because although it pays no attention to the state of government debt it is devoted to appropriate other tasks. Monetary policy can then be classified as 'passive', both because the monetary authority is a follower and because it is they who have to pay attention to the level of debt. However the correspondence in definitions and results is not exact, especially for monetary policy which has a different information set in our framework. Indeed, in Leeper's economy monetary policy is represented by the class of feedback rules which only have inflation in the information set. Output was kept constant, and the state of debt was ignored. We argue that debt should be in the information set of the monetary authority so that optimal policy can take it into account.

Leeper has shown that if fiscal policy is 'active' then monetary policy must be 'passive' in order to ensure a unique bounded solution. Namely, in his model (with constant output) the monetary policy feedback rule should have a very small feedback coefficient on inflation. Leeper's model only has one channel of transmission of monetary policy shocks—the indirect fiscal channel. A change in the interest rate affects domestic debt so it is inflation that has to react to bring debt back onto the equilibrium path. With an intertemporal IS curve we have an additional (and more conventional) lending channel. A strong positive reaction of the interest rate to inflation shocks can substantially depress output with a consequent disinflationary effect. This might conflict with the desire to have some inflation to reduce the excessive debt, itself

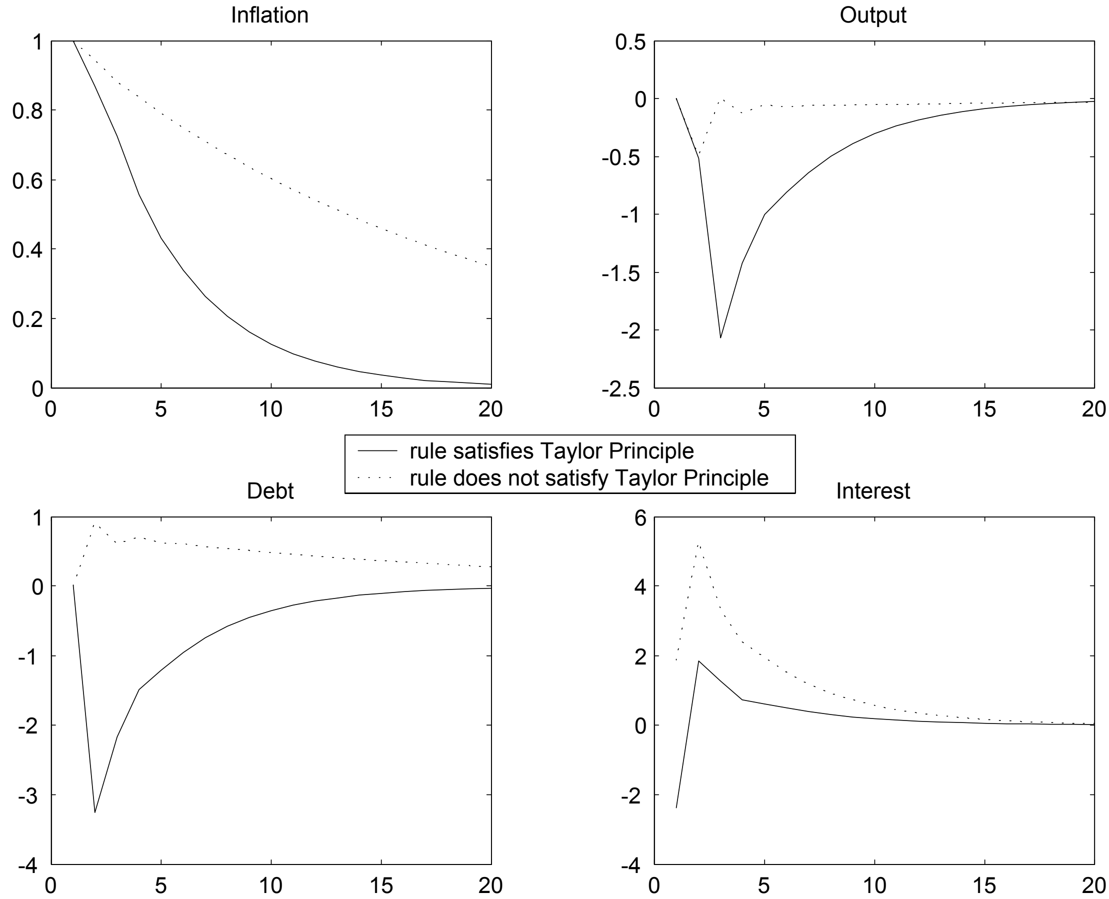

Figure 1: Response to an inflation shock

generated by the high interest rate. Our approach leaves more space for a standard role for monetary policy, so we wish to investigate what would be optimal to do in such a world and how constrained monetary policy will be.

Anticipating some of the results and giving further motivation to the main question, we consider two interest rate policy rules. Both rules feed back on the level of the real domestic debt, inflation and output, $i = \theta_\pi \pi + \theta_x x + \theta_b b$. We consider Rule One, where the feedback coefficients on inflation and output satisfy the Taylor principle, and Rule Two, where the coefficients have unconventional magnitudes and signs.[1] Rule One is 'active' in Leeper's sense so it is expected to destabilise the economy if the fiscal policy is also 'active'. Rule Two is 'passive' in Leeper's sense. We will demonstrate in this paper that the optimal policy may look like Rule Two so the coefficients are explained in the consequent analysis. We can check that both rules generate a unique stationary equilibrium. Figure 1 presents the response of the economy to an inflation shock. Rule One gives notably worse inflation performance but stabilises output reasonably well.

It is not surprising that Leeper's conclusions about the coexistence of passive and active policies fail—we have a richer model so we should expect to find different areas of the model for stability and determinacy. The possibility that debt can now be stabilised by the monetary authority may make the area of stability wider, but the effect of having optimising consumers

[1] Rule One has $(\theta_\pi, \theta_x, \theta_b) = (1.9, 0.25, -1.2)$, and Rule Two has $(\theta\ _{pi}, \theta_x, \theta_b) = (-2.4, -0.25, -1.2)$.

may bring indeterminacy into the model. In this paper we investigate such monetary-fiscal interactions in details.

We also take the opportunity to look at a persistent economy, where we allow for substantial sluggishness in both price-setting and consumption behaviour. Since the fiscal theory of the price level can only work in forward-looking models, a degree of backward-looking behaviour might substantially mitigate all the effects. We aim to investigate this dependence.

The set up of our analysis is as follows. We assume that the instrument of the monetary policy is a short-term interest rate, so that the monetary instrument by itself provides the economy with no nominal anchor. We study optimal policy *represented* by rules. In linear-quadratic models proportional control *is* an outcome of optimisation subject to the time-consistency constraint. Our Non-Ricardian world is described by a model which links inflation, output and debt, so the policymaker's information set includes the level of domestic debt too. We have chosen to study optimal rules simply because a feedback rule is described by vector of its coefficients, and we have a good idea of how a reasonable rule should look—a criterion like Taylor principle might be a sensible way of comparing results. It is more difficult to describe fully optimal time-inconsistent plans on a similar intuitive level, because such a plan contains a substantial element of integral control part. Additionally, it has been shown in the literature (see, e.g. Steinsson (2002)) that similar macroeconomic models suggests little quantitative difference between the outcomes of time-consistent and fully optimal plans, a finding we confirm.

We show that the optimal time-consistent monetary policy can satisfy the Taylor principle for the very backward-looking private sector, otherwise it requires small, sometimes even negative, feedback on both inflation and output so it is completely devoted to the stabilisation of the domestic debt. We show that the 'active' fiscal policy is costly, and the cost is higher than the one obtained under some (non-optimal) Taylor-type-rules that are aggressive enough to ensure a unique solution.

The paper is organised as follows. The next section formulates the model and briefly discusses the solution method. We then discuss our definition of active and passive monetary policies and present results. Section 6 concludes.

2 The Model

We consider the following timing of events. At time $t-0$ the nominal debt stock B_t is observed by all the participants. At time t the government sets the interest rate, taking into account the observed state of the economy, $(\pi_{t-1}, x_{t-1}, b_t)$ where $b_t = B_t/P_{t-1}$ and then, at time $t+0$, the private sector solves its optimisation problem choosing (π_t, x_t).Therefore, we consider a game between a policymaker and the private sector, in which the policymaker (monetary authorities) acts a Stackelberg leader, and sets interest rate as a reaction to the state of the economy in which inflation, output and domestic debt are the only observable variables. The fiscal authorities behave in a non-strategic way so they are not considered as a player. In this setup the policymaker's optimal time-consistent behaviour is characterised by feedback rules.

We refer reader to the work of Rotemberg and Woodford (1997) and Steinsson (2002) when deriving the key equations of the model. We repeat in the Appendix the most important discussions of the key properties of the model. The next section includes main assumptions and key results only.

2.1 Behaviour of the private sector

Our economy is inhabited by a large number of individuals. Each representative individual is a yeoman-farmer, who specialises in the production of one differentiated good, denoted by z, and spends $h(z)$ of effort on its production. He also consumes a consumption basket C, and ξ are shocks. Preferences are assumed to be:

$$\max_{\{C_s,h_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}[u(C_s,\xi_s) - v(h_s(z),\xi_s)] \tag{1}$$

An individual chooses optimal consumption and work effort to maximise the criterion (1) subject to the demand system and the intertemporal budget constraint.

We assume that the household *must* satisfy intertemporal budget constraint, which can be solved forward to yield:

$$\sum_{s=t}^{\infty} \mathcal{E}_t(R_{t,s}P_sC_s) = B_t + \sum_{s=t}^{\infty} \mathcal{E}_t(R_{t,s}[w_s(z)h_s(z) - T_s])$$

where $\mathcal{E}_t(R_{t,s}) = \prod_{k=t}^{s-1} \frac{1}{1+i_k}$, and i_t is short-term interest rate.

Therefore, the first order conditions with respect to consumption after linearisation, leads to the familiar Euler equation (intertemporal IS curve): where $x_s = \widehat{Y}_s - \widehat{Y}_s^n$, Y denotes output (we have aggregated consumption over the closed economy) and 'hat' denotes distance from equilibrium values.

$$x_t = x_{t+1} - \sigma(\widehat{i}_t - \pi_{t+1}) + \varepsilon_{11t}$$

We generalise the model by introducing proportion $(1-\xi)$ of rule-of-thumb consumers, who choose their consumption (output) as $x_t = x_{t-1} + \varepsilon_{12t}$ so the aggregate output evolves as explained by the intertemporal IS curve:

$$x_t = \xi x_{t+1} + (1-\xi)x_{t-1} - \xi\sigma(\widehat{i}_t - \pi_{t+1}) + \varepsilon_{1t} \tag{2}$$

In order to describe price setting decisions we, following Steinsson (2002), split individuals into three groups according to their pricing behaviour. A proportion of agents is able to reset their price every period: with probability $1-\gamma$ they re-contract new price. For the rest of the household sector the price will rise at the steady state rate of domestic inflation with probability γ. Those who recontract a new price (with probability $1-\gamma$), are split into backward-looking individuals and forward-looking individuals, in proportion ω. Backward-looking individuals use rule-of-thumb in their price setting decisions, see Appendix A and Steinsson (2002) for detailed derivation. Finally, the optimal price-setting equation, which can be derived from the second first-order-condition, can be written as:

$$\pi_t = \chi^f \beta\pi_{t+1} + \chi^b \pi_{t-1} + \kappa_1 x_t + \kappa_2 x_{t-1} + \varepsilon_{2t} \tag{3}$$

and all coefficients are given in Appendix A.

The Phillips curve (3) has familiar lag structure where both current and past output have an effect on inflation. Its final specification was discussed in Steinsson (2002) and we briefly repeat it in Appendix A. In the case when all consumers are forward-looking, $\omega = 0$, this Phillips curve collapses to the standard forward-looking Phillips curve, see Rotemberg and Woodford (1997).

If all consumers use the rule-of-thumb in price-setting decisions, $\omega = 1$, it can be brought into the form of 'accelerationist' Phillips curve.

The system (2) and (3) is formally equivalent to the optimising behaviour of a representative agent who maximises (1) subject to an aggregate 'law of motion' of the economy (the demand system, the intertemporal budget constraint and pricing decisions) when policymaker's behaviour is taken to be an exogenous process, independent of the individual's actions.

A significant feature of this system is that both of the state variables, π and x, for both countries are non-predetermined, or jump, variables.

2.2 Behaviour of the Fiscal Authorities

The government does not have to satisfy intertemporal budgets constraint (Δ_t is real primary deficit),

$$\frac{B_t}{P_t} = -\sum_{s=t}^{\infty} \beta^{s-t} \frac{u_C(C_s, \xi_s)}{u_C(C_t, \xi_t)} \Delta_s, \tag{4}$$

in the following sense. If the budget constraint (4) is disturbed, the it is not taxes/spendings must be amended, but the market-clearing mechanism moves the price level P_t to restore equality. In other words, this constraint does not hold *for all* price paths. In this sense it is non-Ricardian. The evolution of the nominal debt stock can be written as:

$$B_{t+1} = (1 + i_t)(B_t + P_t \Delta_t) \tag{5}$$

If the fiscal authorities behave in Ricardian way, i.e. by instantaneously changing surplus in reaction to shocks, then (4) has no effect on the price level and (5) is not included in the description of the economic behavior.

Linearisation of equation (5) yields (at the point of equilibrium $i_\infty = \beta^{-1} - 1$):

$$b_{t+1} = \widehat{i}_t + \frac{1}{\beta}(b_t - \pi_t + \widehat{\Delta}_t) \tag{6}$$

where $\widehat{\Delta}_t = (\widetilde{\Delta}_t - \widetilde{\Delta}_\infty)/\widetilde{\Delta}_\infty$, $\widetilde{\Delta} = \Delta_\infty / B_\infty$ and $b_t = \ln(B_t / P_{t-1})$

We assume predetermined path for the real primary deficit as an autoregressive process:

$$\widehat{\Delta}_t = \nu \widehat{\Delta}_t + \zeta_3, 0 < \nu < 1$$

so we can derive the optimal monetary reaction function when the fiscal policy is 'active' (we fix notation later in Section 3). As a benchmark, we consider Ricardian behavior of the fiscal authorities, where the budget constraint (4) is an identity. In what follows we adopt this strategy and $\widehat{i}_t$ becomes the only policy variable.

2.3 Central Bank's actions

The Central Bank's control variable is the short-term interest rate. We assume it seeks to maximise the aggregate utility function:

$$\max_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s^i, \xi_s) - \int_0^1 v(h_s(z), \xi_s) dz \right].$$

It has been shown in Steinsson (2002) that the government's objective function can be reduced to the following quadratic form:

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}[\pi_s^2 + \lambda_1 x_s^2 + \lambda_2(\Delta\pi_s)^2 + \lambda_3 x_{s-1}^2 + \lambda_4 x_{s-1}\Delta\pi_s] \tag{7}$$

where all coefficients are given in Appendix B.

The main difference of this loss function with the one, traditionally assumed in the literature, $\pi_s^2 + 0.5x_s^2$, is that we have non-zero weights on terms with change in inflation. As Steinsson (2002) discusses, with high proportion of backward-looking population (with $\omega \to 1$) large weight is put on stabilisation of change in inflation. Anticipating the results of calibration, we also have low weight on output stabilisation, see Table 1, but this can be a consequence of having no involuntary unemployment in this model. We discuss later in the text how one can bring the coefficient on output up to more reasonable value.

2.4 Optimal feedback rules

Finally, we have three equations (2), (3) and (6) explaining the evolution of the system. The budget equation (6) is only present in non-Ricardian world. The evolution of this system can be written in a matrix form as:

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U_t + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \varepsilon_{t+1} \tag{8}$$

where $Y_t = (\pi_{t-1}, x_{t-1}, b_t)'$ is predetermined state, $X_t = (\pi_t, x_t)'$ is non-predetermined state, and $U_t = (\widehat{i_t})$ is control variable (all the matrices and their partitioning are given in the Appendix).

The instrument of the central bank is the short-term interest rate and it feeds back on the set of latest information, $(\pi_{t-1}, x_{t-1}, b_t)$. Essentially, the central bank is chosing a rule, $U_t = -FY_t$, which feeds back on the set of predetermined variables, such that the objective function (7) is minimised. The problem can be solved numerically, see Söderlind (1999) for the easy-to-use solution procedure[2].

Finally, the system under control can be written as:

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} - B_1F & A_{12} \\ A_{21} - B_2F & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \varepsilon_{t+1} \tag{9}$$

and we refer to matrix $\begin{bmatrix} A_{11} - B_1F & A_{12} \\ A_{21} - B_2F & A_{22} \end{bmatrix}$ as to the transition matrix.

3 Active and Passive Policies

A strength of response of a monetary policy reaction rule can be conveniently described in terms of whether it satisfies Taylor principle. To fix the notation we start with discussion of the Ricardian setup, i.e. when the fiscal policy reacts instantaneously to any deviations of the debt from the target path. We denote the vector of feedback coefficients as $F = (\theta_\pi, \theta_x, \theta_b)$.

[2] The solution is typically obtained by using Oudiz and Sachs (1985) iterative procedure derived from the dynamic programming principle. The solution is easy to obtain, but it does not reflect the strategic behaviour of players. Alternatively it can be obtained as an outcome of an iterative process, explicitely modelling the infinite dynamic game, see Blake (2002). This also implies that every feedback rule which is obtained as a 'discretion' outcome should be checked as delivering determinacy to an economy.

In the Ricardian world, an optimal policy reaction function is written as a feedback rule: $\widehat{i}_t^R = \theta_\pi^R \pi_{t-1} + \theta_x^R x_{t-1}$. A condition on its coefficients which ensures determinacy of the economy under control (Taylor principle) states that[3]:

$$\theta_\pi + \theta_x \frac{1 - \chi^f \beta - \chi^b}{\kappa_1 + \kappa_2} > 1 \tag{10}$$

We discuss the derivation of this formula for the persistent economy in Appendix E. We now extend this notion on the non-Ricardian world, and say that a monetary policy satisfies Taylor principle if the weighted feedback coefficient on inflation and output satisfy inequality (10), regardless of any feedback coefficients on other variables. The Taylor Principle in the Non-Ricardian world is not a condition for the unique solution but a statement of the fact that monetary policy responds positively to inflation and output; so if a weighted feedback coefficient on inflation and output is large then we have monetary policy that seems to be doing a conventional task. If the policy does not satisfy Taylor principle we simply call it non-satisfying Taylor principle. However, a policy which does not satisfy Taylor principle in the Ricardian world necessarily implies indeterminacy of the economy.

We focus on two polar cases of fiscal policy's strength of response. The fiscal policy is either does not control the real domestic debt at all (e.g., real deficit is an autoregressive process), or corrects for shocks to the budget constraint promptly. In the latter case the budget constraint is always in equilibrium, and the bounded behaviour of the debt is ensured, so prices do not react to the level of domestic debt.

The important consequence of the adopted approach to study optimal feedback rules, is that it makes little sense to discuss the strength of response of the policy for the predominantly forward-looking private sector. For the entirely forward-looking and unconstrained private sector, inflation and output are non-predetermined variables, so the policymaker does not react on them. It is persistence in the economy that requires the monetary policy to react to accumulated shocks, i.e. the persisting variables which become predetermined variables (lagged inflation, output and debt in our model). Obviously, when ω approaches zero and consumption is unconstrained ($\xi = 1$), we have θ_π, θ_x converge to zero too, so the optimal policy is classified as non-satisfying Taylor principle for both Ricardian and Non-Ricardian frameworks.

4 Calibration

To calibrate inflation persistence we use parameter $\gamma = 0.85$ (15% of population are able to renegotiate contracts each period), and a range of $\omega \in (0, 1)$. Fuhrer and Moore (1995) notice that $\chi^f = \chi^b = 0.5$ matches the pattern of the US data much better than pure forward- or backward-looking models. Fuhrer (1997) on US data and Blake and Westaway (1996) on UK data claim that χ^f should be close to 0.2. Gali and Gertler (1999) get $\chi^f = 0.8$ but with the different measure of demand pressure. Table 1 presents the choice of parameters and what they imply for χ^f and χ^b. It is apparent that we can reach $\chi^b_{\max} = 0.54$ for the microfounded model with $\gamma = 0.85$.

There is a substantial number of estimates for parameter σ. Estimates for the US range from 0.16 (McCallum and Nelson (1999)) to 6.0 (Rotemberg and Woodford (1997)). We consider

[3] In fact, this is the condition of that the second biggest eigenvalue is outside the unit circle. In our model we have two jump variables, inflation and output, so this is the condition of when economy obtains two unstable roots and becomes determinate. It is possibly that we can pick up coefficients of the model such that we will have more than two unstable eigenvalues. The model will be unstable, but it will not be indeterminate according to our definition of determinacy.

$\sigma = 0.5$.We follow Steinsson (2002) when calibrating most of the parameters: $\psi = 2$, $\epsilon = 5$, $\beta = 0.99$.

This model assumes that the economy is on the labour supply curve and there is no involuntary unemployment. This model gives unrealistically small coefficients on demand $\kappa_1 + \kappa_2$ in the Phillips curve, see Table 1. To comply with more realistic coefficients, we slightly change the model (see Appendix C) and calibrate ν (parameter on output pressure in wage-setting equation) such that $\kappa_1 + \kappa_2$ would be between 0.2 and 0.5. This, in turn, gives us more realistic coefficient on output deviations in the loss function, $\lambda_1 + \lambda_2$, see 1. We refer to the model with these changes as the model with imperfect labour markets.

We also consider both theoretical loss function and more traditional objective function, which puts fixed weight on output volatility and ignores inflation inertia, see Table 1.

5 Results

5.1 Optimal monetary policy

We illustrate our findings with Figures 2-3 which plot feedback parameters θ_π, θ_x and θ_b for the optimal feedback rule $\widehat{i} = \theta_\pi \pi + \theta_x x + \theta_b b$. The horizontal axis measures the fraction of the backward-looking private sector, ω. $\omega = 0$ corresponds to all forward-looking individuals in the economy. Each panel plots two curves, one for the Ricardian (R) world and another for the non-Ricardian (NR) world. The line is always solid if the policy is active. In Ricardian world, a passive policy leaves economy indeterminate so it is of no policy relevance and is not plotted. In Non-Ricardian world a passive policy is a dotted line, but the economy is still determinate under the passive policy.

We also complement the analysis with two Figures 4-5, which plot response of the economy to an inflation shock[4]. These figures also include additional lines of response of the economy under the optimal commitment plan[5] (C); we included them to illustrate optimality of feedback rules (F) but we discuss it later in the text.

As discussed above, we study the optimal monetary policy with respect to the two alternative welfare criteria. There are two main differences between them. The theoretical central bank's objective function emphasises the society's preferences for sluggish inflation adjustment when the private sector is very backward-looking. The traditional objective function accounts for stronger needs of stable output than suggested by the monopolistic competition model, and ignores inflation inertia terms.

We start with the model with traditional loss function in a Ricardian world which is much studied in the literature. It has the property that for the *very backward-looking* consumers the Phillips curve is close to its accelerationist form so inflation is very persistent. To stabilise inflation in such a world, the monetary policy should be very aggressive in its reaction to output and inflation shocks. For the *sufficiently forward looking* consumers, the private sector's expectations can ajdust quickly to bring inflation down, the active monetary policy is not needed. Additionally, as we discussed earlier in Section 3, we simly cannot investigate the behaviour of the *very forward-looking private* sector in the current framework – in a model without persistence all feedback coefficients converge to zero. To summarise, the traditional model of a Ricardian

[4]These figures also include responses of the economy under passive rules in Ricardian regime. Passive rules in Ricardian regime are consistent with the equilibrium, but they leave economy indeterminate. The response is plotted simply for comparison, no policy implications should be made out of it.

[5]See Söderlind (1999) for easy-to-use solution procedure.

world requires active monetary policy would accompany the fiscal policy which instantaneously reacts to shocks. This is what we plot in Figure 2, left panel.

The Non-Ricardian world is very different. For the very backward-looking consumers, a strong reaction of the interest rate to higher inflation would leave inflation high and stable, and this generates welfare losses. At the same time, when the private sector is sufficiently forward-looking, an optimal monetary policy can satisfy Taylor principle. In this case an active monetary policy causes sharp contraction of output, but it is brought back immediately because inflation expectations stay high so the real interest falls quickly, see Figure 2, left panel. The gain from faster output stabilisation outweighs losses from slower inflation adjustment, compare two panels of Figure 4. The effect of demand in the Phillips should not be too large, otherwise the expected inflation is not high enough to bring output back quickly through the lending channel. Therefore, we can only find it active in the specification of the model leading to small coefficients κ_1 and κ_2, as our calibration suggests (Table 1).

In the theoretical model with inflation inertia terms in the loss function, the requirement of slow inflation adjustment has a strong implications for the activity of an optimal monetary policy. The calibration shows that the term with inflation inertia in the central bank's loss function is always substantial in size and it completely dominates all other terms in the government's loss function if consumers are very backward-looking. Therefore, in the Ricardian world the optimal monetary policy can be passive. Otherwise an active policy can bring inflation down too quickly and this would cause welfare losses. We observe, indeed, that the feedback coefficients diminish when ω is close to one. Again, inflation expectations of the sufficiently forward-looking private sector can adjust quickly to bring inflation down, so the active policy may not be required either. However, there is an area in between with *sufficiently backward-looking* private sector where expectations are not that strong, but the loss function already requires fast convergence of *level* inflation back to the equilibrium. Here an active monetary policy might increase welfare.

The stabilisation in the Non-Ricardian world for the very backward-looking private sector, however, may be consistent with active responses. Since it is optimal for the society that inflation would fall slow, following an inflation shock, interest rate may rise in response to the shock. A rise in interest rate will increase domestic debt, and inflation will remain high simply because it will be also reacting to the level of domestic debt. A rise in interest rate also depress output, so if the demand pressure has some effect on the inflation process, then there will be a counterveiling effect. Which effect dominates we can only say using a numerical excersize. The numerical experiment presented in Figure 2, right panel shows that for our calibration the optimal feedback coefficients increase for large ω indeed, although he optimal policy remains passive. If consumers are not that backward-looking, the optimal policy does not satisfy Taylor principle – inflation should fall reasonably fast, and it cannot be achieved with aggressive policy rules. The optimal feedback on inflation and output parameters can even go into the negative area.

It is apparent from this analysis that our conclusions are much dependent on the size of demand effect in the Phillips curve. We are using a microfounded model which produces small coefficients κ_1 and κ_2, so there is a disconnect between conventional lending channel of monetary policy and the fiscal channel which works through the debt channel. This model is close to the one of Leeper (1991) and the results presented in Figure 2 can be interpreted as the results on optimal monetary policy in in the Leeper's world.

We can investigate the optimal monetary policy with strong conventional lending channel if we impose realistic coefficients κ_1 and κ_2. For example, we may amend the model and try to account for the imperfect competition at the labour markets. This will give us more realistic coefficients on demand pressure and coefficient on output in the central bank's objective function, and we still keep inflation inertia terms. We present some explanations of this amendment in Appendix C and the result of calibration is included in Table 1.

The presence of relatively strong lending channel has a substantial effect. The left panel of Figure 3 plots the results for the model with traditional loss function. In the Ricardian world the area of active optimal policy widens. To have the same disinflation path as in the base model, we need to deal with smaller output deviations from the target. So the monetary policy should be more active in general and react more agressively to the ouput in particular. This is also consistent with having higher unemployment aversion in the loss function. In the non-Ricardian world, as we discuss above, an active monetary policy is more likely to cause welfare losses and Figure 3, left panel suggests that this is now the case.

The model with theoretical loss function suggests that in the Ricardian world the monetary policy should be passive for the whole range of ω, so it is not plotted. In the Non-Ricardian world, however, the optimal monetary policy is now active for the very backward-looking population – the debt effect stil outweigh the demand effect on inflation, but the requirement of bigger unemployment aversion requires more activity.

Overall, the model with traditional loss function and with reasonable calibration parameters does not suggest any more that in the Non-Ricardian world the monetary policy should be active. The effect is only present for the very backward-looking population where we have requirement of smooth inflation adjustment. Additionally, in all our excersizes the negative feedback coefficient on the level of domestic debt was found quite stable and sufficiently large. Therefore we can conclude that the fiscal (debt) channel is the main channel of transmission of shocks into the inflation process, the optimal monetary policy is passive regardless of the presence of the lending channel, thus it is completely devoted to the stabilisation of domestic debt.

5.2 Welfare evaluation

The cost of 'active' fiscal policy can be assessed by comparing the cost-to-go under Ricardian and non-Ricardian regimes[6]. These costs are plotted in Figures 6-7, top rows. We plot log difference from the costs incurred under the Ricardian, fully optimal time-inconsistent regime (R C). For the theoretical model with traditionall loss function, Figure 6, top left panel, the cost of having active fiscal policy is below 0.1 for the backward-looking population. Here the monetary policy ensures determinacy in the Ricardian world and does not satisfy Taylor Principle under the FTPL. However, the gap widens dramatically with lower ω, i.e. for more forward-looking consumers, when *all* costs are small, so this is not surprising.

The bottom row of each figure evaluates feedback rules in the non-Ricardian world, which were optimal with respect to either theoretical or traditional loss function, with respect to an alternative loss function. It is apparent that if the alternative welfare criterion was used, then the welfare loss can be substantial, sometimes exceeding 100%. The results are more robust to the choice of the welfare creterion in the case with more realistic coefficients, see Figure 7, where there is a very large difference is only or extremely backward-looking consumers. Therefore, it might be crucial whether a policymaker uses an approprate welfare function.

Additionally, it is apparent, (see also Steinsson (2002)) that the difference in welfare due to different loss functions is much bigger than the difference in welfare of two policies, time-consistent feedback rules and time-inconsistent optimal commitment plans.

[6] See, e.g. Currie and Levine (1993), pp. 107-111 on how to compute it. We assumed an autoregressive process for the disturbances.

5.3 Activity of the Fiscal Policy

Suppose that the fiscal authorities operate with a feedback rule, $\widehat{\Delta}_t = -\mu b_t$, $0 \leq \mu \leq 1$ (note that b_t is observable). From the budget constraint $b_{t+1} = \widehat{i}_t + \frac{1}{\beta}(b_t - \pi_t + \widehat{\Delta}_t) = \widehat{i}_t + \frac{1}{\beta}((1-\mu)b_t - \pi_t)$ it follows that as soon as $\frac{1-\mu}{\beta} \geq 1$ inflation might need to react to the level of domestic debt. Indeed, the debt accumulation process is not a stationary process on its own so a conceivable outcome of optimal policy is where both interest rate and inflation react with a unique stationary equilibrium. If $\frac{1-\mu}{\beta} < 1$ then the debt accumulation is a stationary process, and inflation does not have to react to ensure the bounded solution. Again, it might be optimal for the private sector to react to the level of debt in order to maximise welfare, but if the Ricardian world has indeterminacies under the optimal monetary policy, they will remain in the system. If $\frac{1-\mu}{\beta}$ is small enough, i.e. the response of the fiscal policy is strong, the debt will be close to its equilibrium path and the private sector's inflation might not be affected. Our simulations show that with $1 \geq \mu > \mu_R$ we get a Ricardian result: price and wage-setting decisions of the private sector are not affected by the fiscal stance and for the monetary policy it is not optimal to react to the level of domestic debt. We plot μ_R in Figure 8. Feedback coefficient μ_R is on average of 0.2-0.3, and it goes down with more backward-looking private sector. A backward-looking private sector is using past information predominantly, so it is enough to ensure that the debt is on non-explosive path (this can be done with small μ, $\frac{1-\mu}{\beta} < 1$) so the private sector stops reacting on it.

It is apparent that the threshold feedback coefficient on debt in the fiscal policy rule, μ_R, is much smaller in absolute value than the coefficient on debt needed in the optimal interest rate rule (a typical feedback coefficient on debt in a interest rate rule is between 1 and 2). Obviously, there is an interaction between inflation and interest rate in the debt accumulation equation so we cannot expect a symmetrical solution for the deficit and interest rate rules, but, nevertheless one can conclude that even if the monetary policy has to take debt into account, its stabilisation abilities are much worse the one of fiscal policy.

6 Conclusions

In this paper we characterised the optimal monetary policy under the fiscal theory of the price level. We have shown that an 'active' fiscal policy is costly. If the fiscal authorities pay no attention to the level of domestic debt, fiscal disturbances can strongly affect the wage-price setting behaviour of the household sector. In this environment the monetary policy is severely restricted in its ability to stabilise inflation and output fluctuations. As a result, the optimal monetary policy is concerned with stabilisation of domestic debt and only slightly feeds back on inflation and output – the channel of transmission is predominantly fiscal. We have also shown that a moderate feedback on debt for a fiscal authorities will make the model observationally equivalent to the one with Ricardian fiscal policy.

Table 1: Values for the structural parameters

ω	0.01	0.7	0.85	0.99	0.01	0.7	0.85	0.99	0.01	0.7	0.85	0.99
	$\gamma=0.6,\ \sigma=0.5,\ \nu=0$				$\gamma=0.85,\ \sigma=0.5,\ \nu=0$				$\gamma=0.85,\ \sigma=2.0,\ \nu=0$			
χ^f	0.98	0.46	0.42	0.38	0.99	0.55	0.50	0.46	0.99	0.55	0.50	0.46
χ^b	0.02	0.54	0.59	0.62	0.01	0.45	0.50	0.54	0.01	0.45	0.50	0.54
κ_1	0.19	-0.04	-0.06	-0.07	0.02	0.00	-0.01	-0.01	0.01	0.00	0.00	0.00
κ_2	0.01	0.23	0.25	0.27	0.00	0.02	0.02	0.02	0.00	0.02	0.02	0.02
$\kappa_1+\kappa_2$	0.19	0.19	0.19	0.19	0.02	0.02	0.02	0.01	0.01	0.02	0.02	0.02
λ_1	0.04	0.04	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
λ_2	0.02	3.89	9.44	165.0	0.01	2.75	6.67	116.4	0.01	2.75	6.67	116.4
λ_3	0.00	0.15	0.35	6.17	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
λ_4	0.01	1.50	3.65	63.80	0.00	0.11	0.27	4.65	0.00	0.03	0.07	1.30
$\lambda_1+\lambda_3$	0.04	0.18	0.39	6.21	0.00	0.01	0.01	0.05	0.00	0.00	0.00	0.00
	$\gamma=0.6,\ \sigma=0.5,\ \nu=30$				$\gamma=0.85,\ \sigma=0.5,\ \nu=30$				$\gamma=0.85,\ \sigma=0.5,\ \nu=30$			
χ^f	0.98	0.46	0.42	0.38	0.99	0.55	0.50	0.46	0.99	0.55	0.50	0.46
χ^f	0.02	0.54	0.59	0.62	0.01	0.45	0.50	0.54	0.01	0.45	0.50	0.54
κ_1	2.42	-0.46	-0.72	-0.92	0.25	-0.06	-0.09	-0.12	0.24	-0.05	-0.09	-0.11
κ_2	0.05	1.48	1.62	1.72	0.00	0.13	0.14	0.15	0.00	0.13	0.14	0.15
$\kappa_1+\kappa_2$	2.47	1.03	0.89	0.79	0.25	0.07	0.05	0.04	0.24	0.08	0.06	0.04
λ_1	0.04	0.04	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
λ_2	0.02	3.89	9.44	165.0	0.01	2.75	6.67	116.4	0.01	2.75	6.67	116.4
λ_3	0.11	24.5	59.6	1042	0.00	0.19	0.45	7.86	0.00	0.17	0.40	7.01
λ_4	0.08	19.55	47.47	829.4	0.01	1.43	3.46	60.5	0.01	1.35	3.27	57.1
$\lambda_1+\lambda_3$	0.15	24.6	59.7	1042	0.00	0.19	0.45	7.86	0.00	0.17	0.40	7.01
λ	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Note: The model is:

$$
\begin{aligned}
\pi_t &= \chi^f \beta\pi_{t+1} + \chi^b\pi_{t-1} + \kappa_1 x_t + \kappa_2 x_{t-1} + \widetilde{\varepsilon}_{1t} \\
x_t &= \xi x_{t+1} + (1-\xi)x_{t-1} - \xi\sigma(\widehat{i}_t - \pi_{t+1}) + \widetilde{\varepsilon}_{2t} \\
b_t &= \widehat{i}_t + \frac{1}{\beta}(b_{t-1} - \pi_t) + \widetilde{\varepsilon}_{3t} \\
L_t^I &= \pi_t^2 + \lambda_1 x_t^2 + \lambda_2(\Delta\pi_t)^2 + \lambda_3 x_{t-1}^2 + \lambda_4 x_{t-1}\Delta\pi_t \\
L_t^{II} &= \pi_t^2 + \lambda x_t^2
\end{aligned}
$$

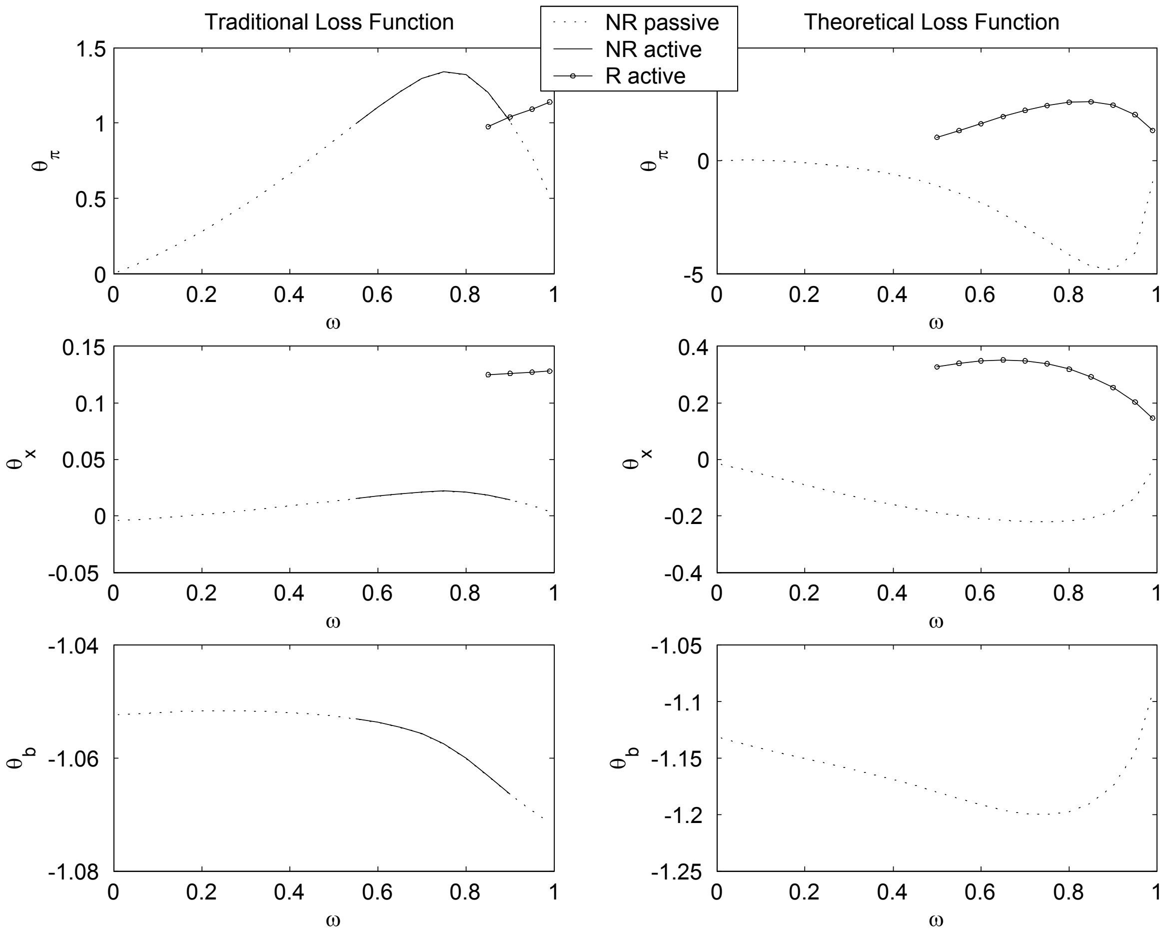

Figure 2: Model with perfect labour markets. Feedback parameters for $\sigma = 0.5$, $\xi = 0.95$, $\kappa_1 + \kappa_2 = 0.015$, $\lambda_1 + \lambda_3 = 0.01$

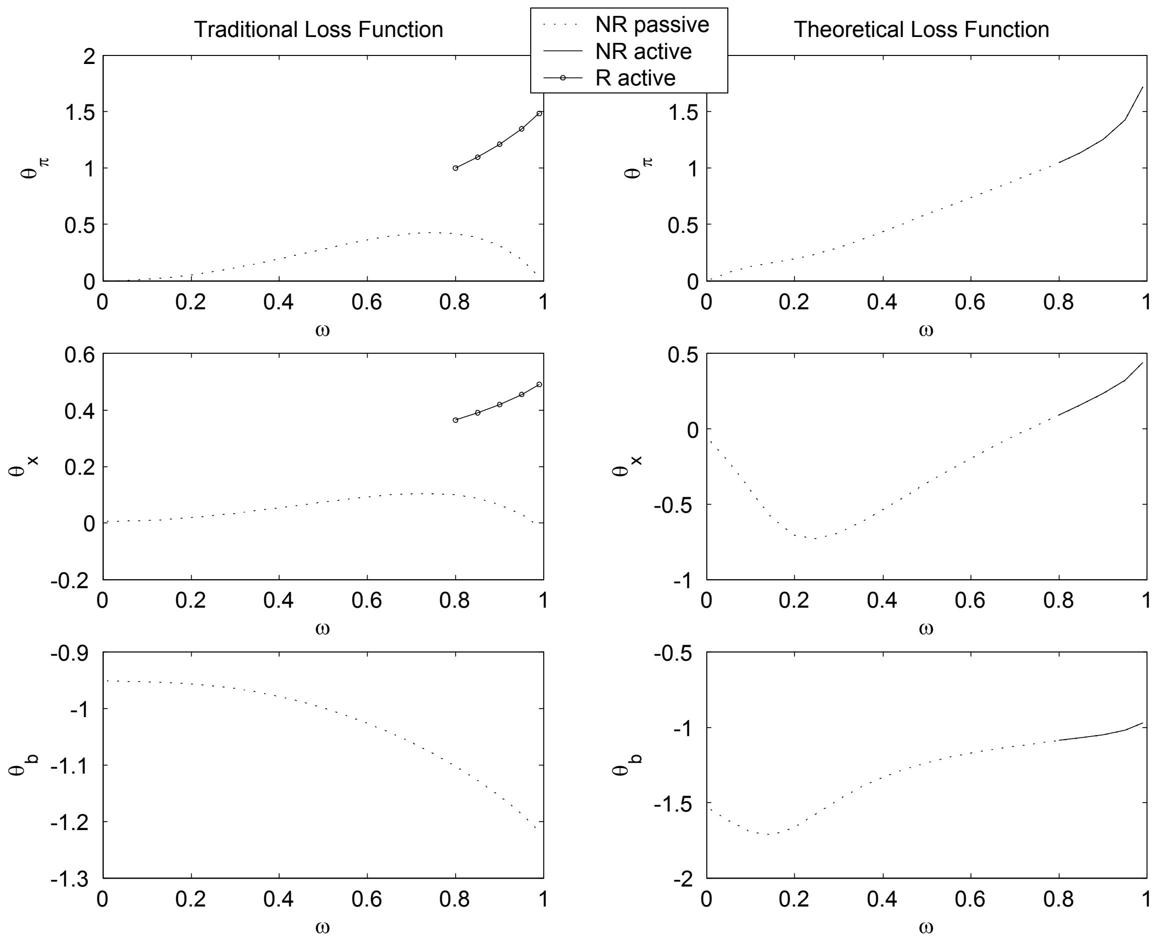

Figure 3: Model with imperfect labour markets. Feedback parameters for $\sigma = 0.5$, $\xi = 0.95$, $\kappa_1 + \kappa_2 = 0.05$, $\lambda_1 + \lambda_3 = 0.45$

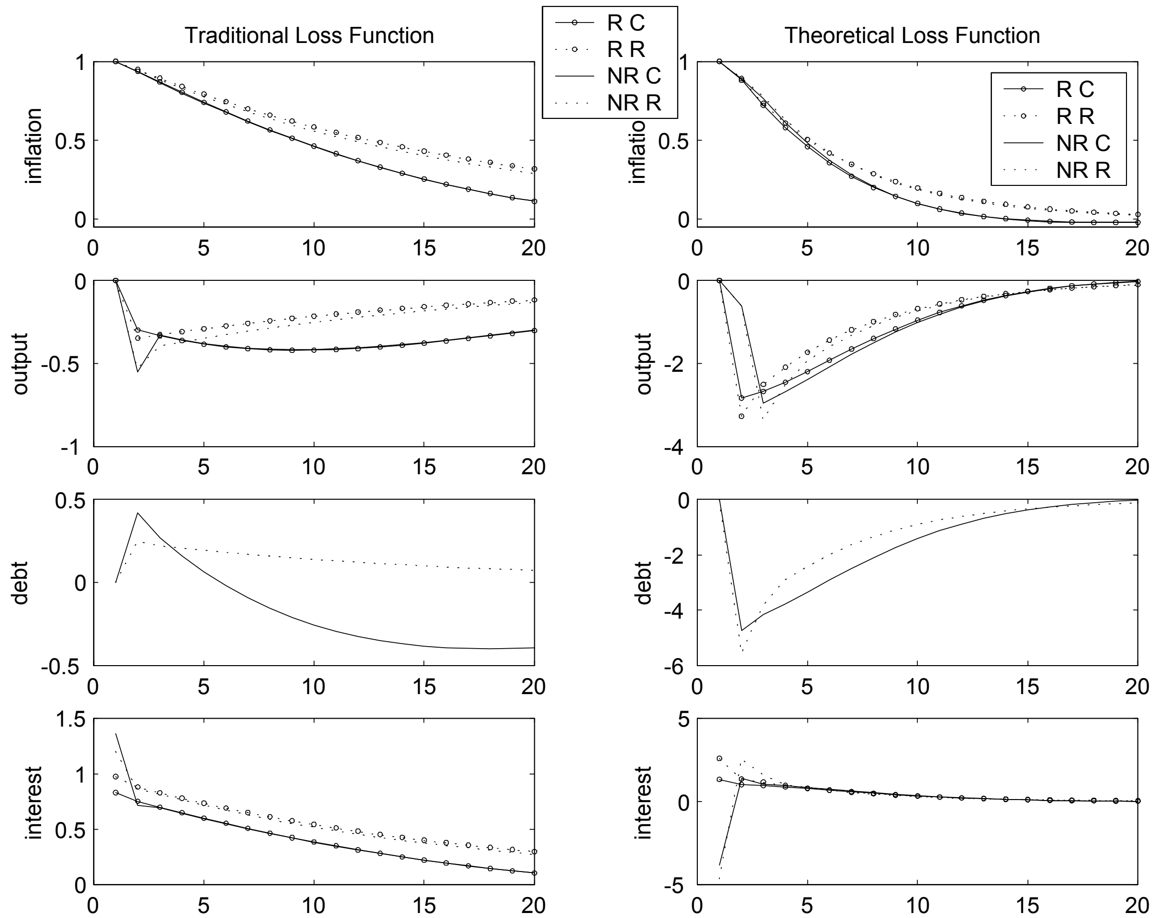

Figure 4: Model with perfect labour markets. Feedback parameters for $\sigma = 0.5$, $\xi = 0.95$, $\kappa_1 + \kappa_2 = 0.015$, $\lambda_1 + \lambda_3 = 0.01$

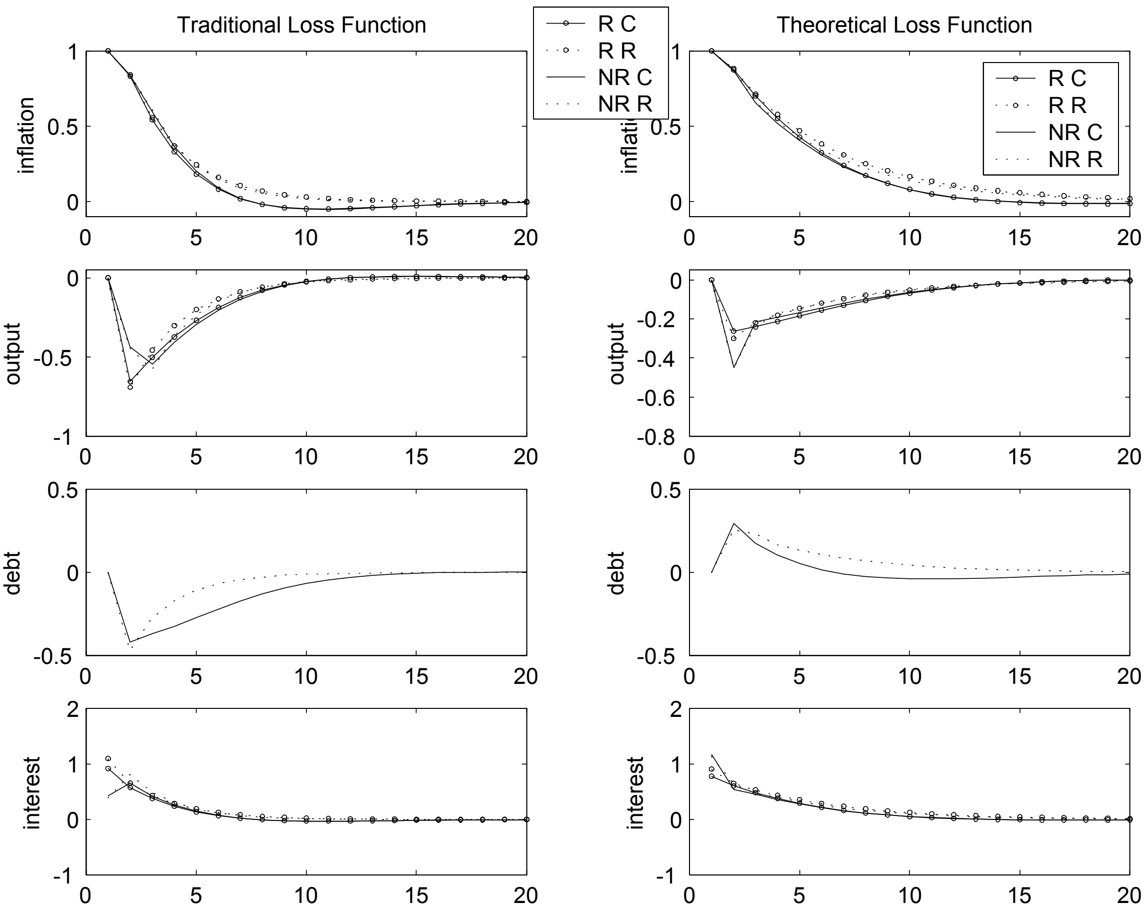

Figure 5: Model with imperfect labour markets. Feedback parameters for $\sigma = 0.5$, $\xi = 0.95$, $\kappa_1 + \kappa_2 = 0.05$, $\lambda_1 + \lambda_3 = 0.45$

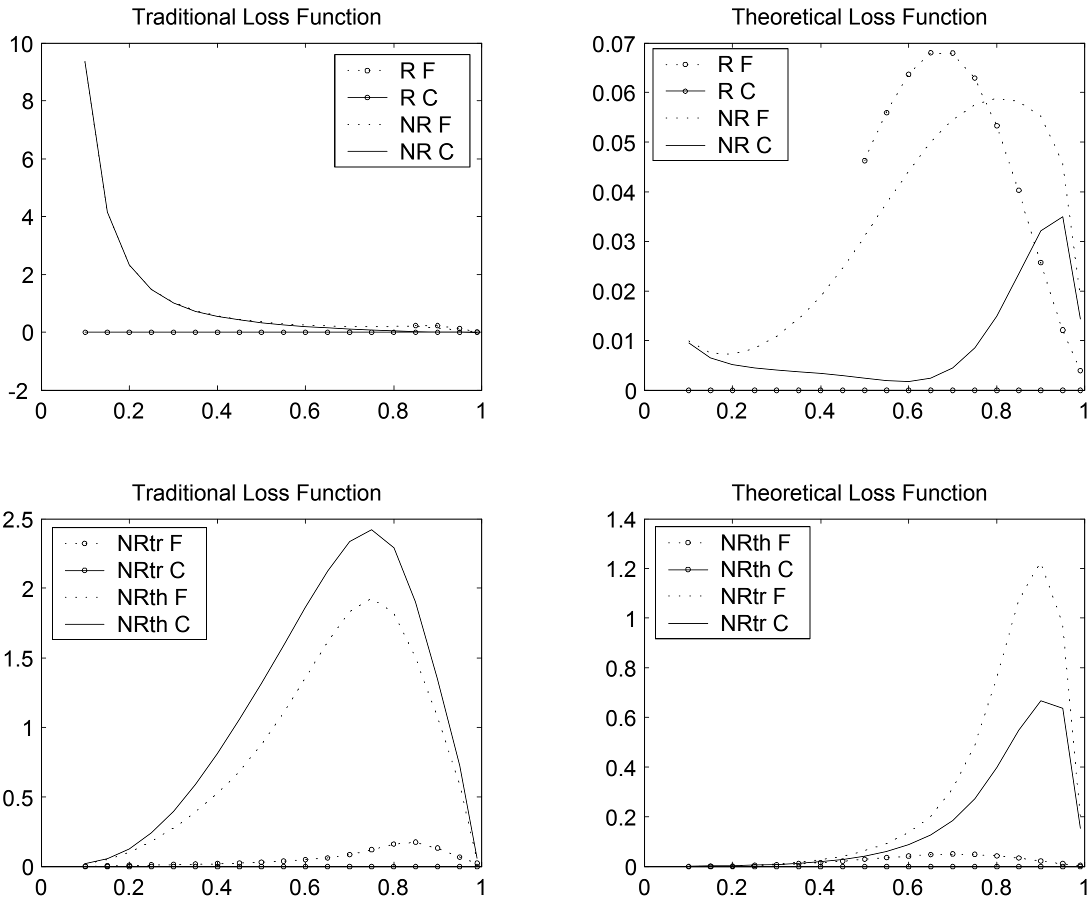

Figure 6: Model with perfect labour markets

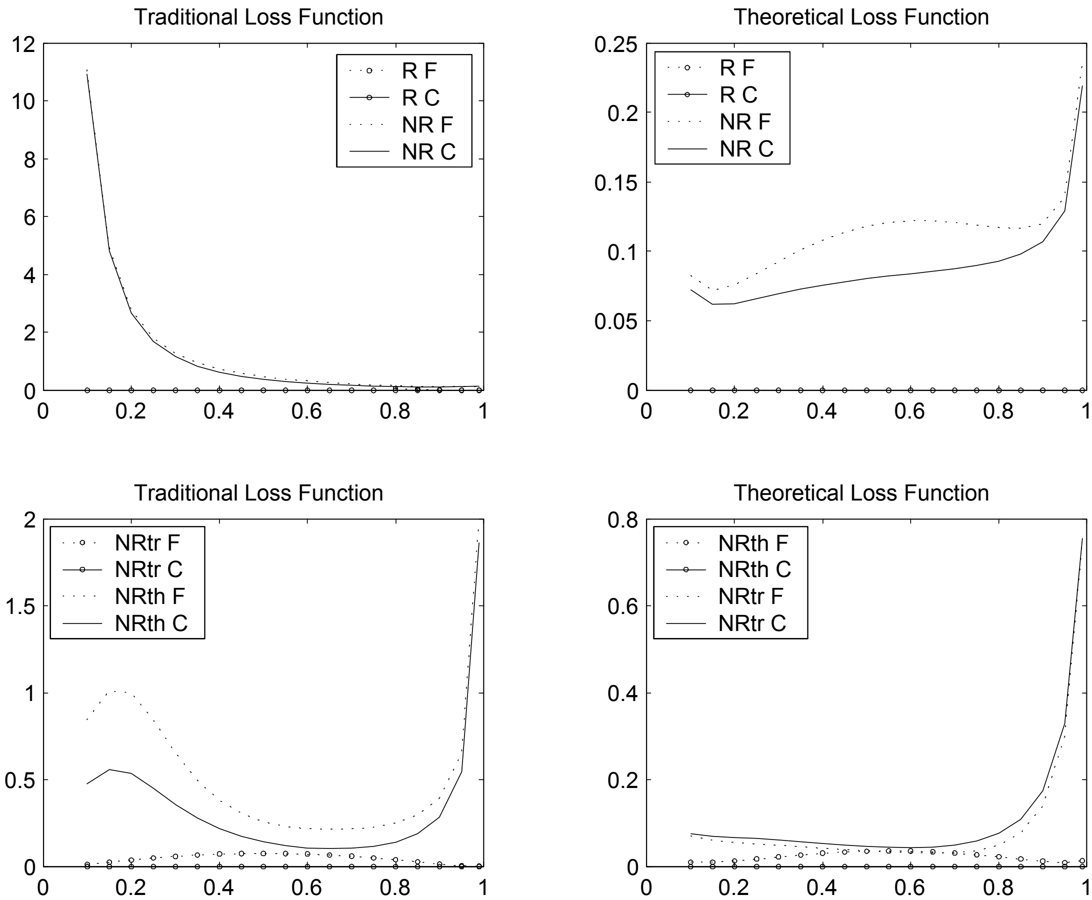

Figure 7: Model with imperfect labour markets.

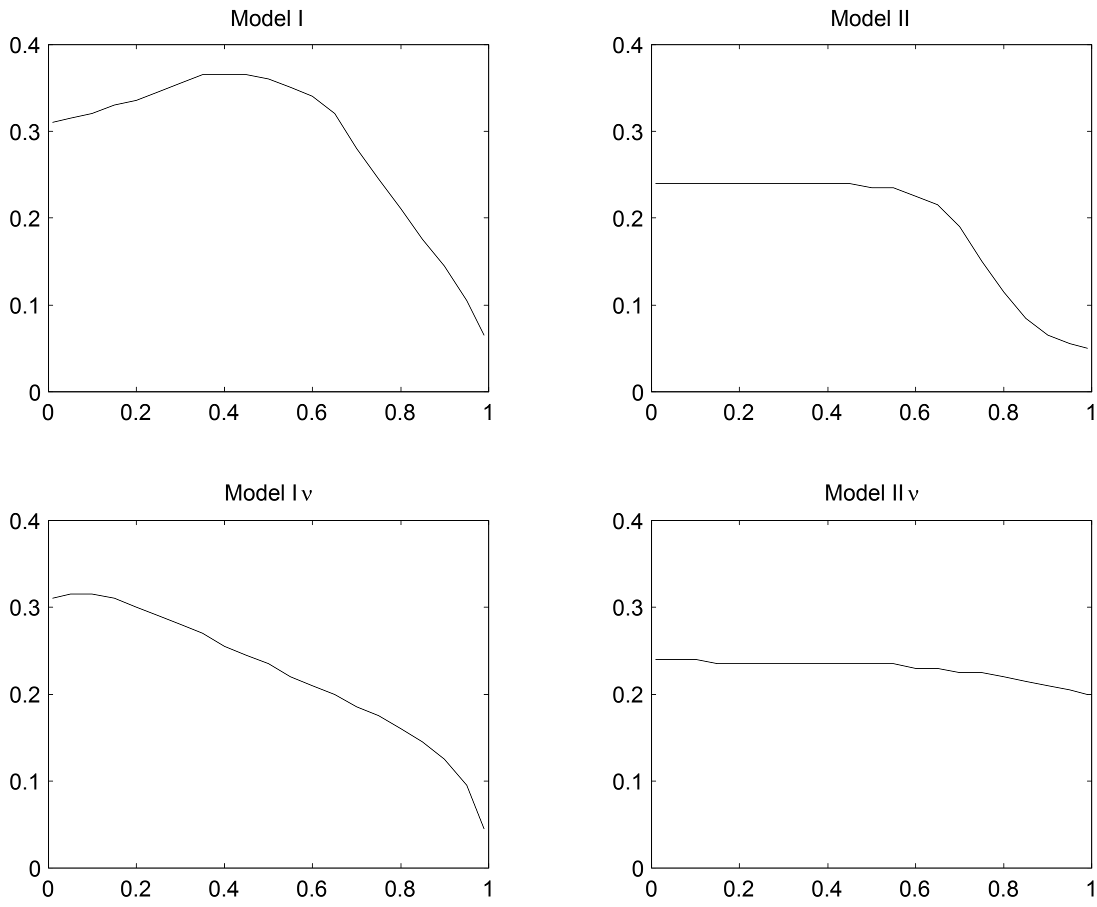

Figure 8: Fiscal feedback parameter μ_R on debt which ensures Ricardian result. Parameters $\gamma = 0.85, \xi = 0.8, \sigma = 0.5$.

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A Aggregate Supply Equation

To introduce inflation persistence into the model we follow Steinsson (2002) and divide individuals into three groups according to their pricing behaviour. A proportion of agents is able to reset their price every period: with probability $1-\gamma$ they re-contract new price P_H^n. For the rest of the household sector the price will rise at the steady state rate of domestic inflation $\overline{\Pi}_H$ with probability γ:

$$P_{H,t} = \overline{\Pi}_H P_{H,t-1} \tag{11}$$

Those who recontract a new price (with probability $1-\gamma$), are split into backward-looking individuals and forward-looking individuals, in proportion ω, such that the aggregate index of prices set by them is

$$P_{H,t}^{\times} = (P_{H,t}^f)^{1-\omega}(P_{H,t}^b)^{\omega}.$$

Backward-looking individuals set their prices $P_{H,t}^b$ according to the rule of thumb:

$$P_{H,t}^b = P_{H,t-1}^{\times}\Pi_{H,t-1}(\frac{Y_{t-1}}{Y_{t-1}^n})^{\delta} \tag{12}$$

where:

$$\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$$

and Y_t^n is the efficient level of output (defined later).

The forward-looking individuals are able to solve the first order conditions and obtain an optimal solution $P_{H,t}^f$. These conditions are discussed in the next section. For the household sector as a whole, the price equation can be written as:

$$P_{H,t} = [\gamma(\overline{\Pi}_H P_{H,t-1})^{1-\epsilon} + (1-\gamma)(1-\omega)(P_{H,t}^f)^{1-\epsilon} + (1-\gamma)\omega(P_{H,t}^b)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}. \tag{13}$$

Steinsson (2002) derives the Phillips curve for our economy of the following form:

$$\pi_t = \chi^f\beta\pi_{t+1} + \chi^b\pi_{t-1} + \kappa_1 x_t + \kappa_2 x_{t-1} \tag{14}$$

where:

$$\chi^f = \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}, \quad \chi^b = \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)},$$
$$\kappa_1 = (\frac{(1-\gamma\beta)(1-\gamma)(1-\omega)}{\sigma(\gamma+\omega(1-\gamma+\gamma\beta))}\frac{(\psi+\sigma)}{(\psi+\epsilon)} - \frac{(1-\gamma)\gamma\beta\omega\delta}{\gamma+\omega(1-\gamma+\gamma\beta)}),$$
$$\kappa_2 = \frac{(1-\gamma)\omega\delta}{\gamma+\omega(1-\gamma+\gamma\beta)}.$$

This equation is valid for $0 \leq \omega < 1$. When $\omega \to 0$ the Phillips curve collapses to the familiar purely forward-looking specification of Woodford (200x):

$$\pi_t = \beta\pi_{t+1} + \kappa_1 x_t \tag{15}$$

with

$$\kappa_1 = \frac{(1-\gamma\beta)(1-\gamma)}{\sigma\gamma}\frac{(\psi+\sigma)}{(\psi+\epsilon)}.$$

For $\omega = 1$ the derivation is incorrect. Taking limit as $\omega \to 1$ we come to the specification:

$$\pi_t = \frac{\gamma\beta}{1+\gamma\beta}\pi_{t+1} + \frac{1}{1+\gamma\beta}\pi_{t-1} - \frac{(1-\gamma)\gamma\beta\delta}{1+\gamma\beta}x_t + \frac{(1-\gamma)\delta}{1+\gamma\beta}x_{t-1}$$

However, it is an illusion that this specification has a non-trivial forward-looking component. The unique bounded solution of this equation is

$$\pi_t = \pi_{t-1} + (1-\gamma)\delta x_{t-1} \tag{16}$$

and it is of the form of the acceleration Phillips curve.

Parameter δ is determined from the following considerations (see Steinsson (2002)). The Phillips curve collapses to the forward-looking specification (15) for $\omega = 0$ and to the 'accelerationist' specification (16) when $\omega = 1$. We require coefficients on output to have the same value, we denote it κ, and that determines $\delta = (1-\gamma\beta)(\psi+\sigma)/(\sigma\gamma(\psi+\epsilon))$. Eliminating δ from the parameters we get:

$$\begin{aligned}
\chi^f &= \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}, \quad \chi^b = \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}, \\
\kappa_1 &= \frac{(1-\gamma)(1-\gamma\beta)(\psi+\sigma)}{(\gamma+\omega(1-\gamma+\gamma\beta))\sigma(\psi+\epsilon)}[1-\omega(1+\beta)], \\
\kappa_2 &= \frac{(1-\gamma)(1-\gamma\beta)(\psi+\sigma)\omega}{(\gamma+\omega(1-\gamma+\gamma\beta))\sigma\gamma(\psi+\epsilon)} \\
\kappa_1+\kappa_2 &= \frac{(1-\gamma)(1-\gamma\beta)(\psi+\sigma)}{(\gamma+\omega(1-\gamma+\gamma\beta))\sigma(\psi+\epsilon)}[1-\omega(1+\beta-\frac{1}{\gamma})] > 0
\end{aligned}$$

B Government's Loss Function

Steinsson (2002) shows that the Central bank's minimisation problem can be reduced down to:

$$\min_{\{i_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}[\pi_s^2 + \lambda_1 x_s^2 + \lambda_2(\Delta\pi_s)^2 + \lambda_3 x_{s-1}^2 + \lambda_4 x_{s-1}\Delta\pi_s]$$

where

$$\begin{aligned}
\phi_1 &= \frac{1}{\psi} + \frac{1}{\sigma}, \quad \phi_2 = (\frac{1}{\psi} + \frac{1}{\epsilon}), \frac{\phi_1}{\phi_2} = \frac{(\sigma+\psi)\epsilon}{(\epsilon+\psi)\sigma} \\
\lambda_1 &= \frac{(1-\gamma\beta)(1-\gamma)}{\gamma\epsilon^2}\frac{\phi_1}{\phi_2}, \lambda_2 = \frac{\omega}{\gamma(1-\omega)}, \lambda_3 = \frac{(1-\gamma)^2\delta^2\omega}{\gamma(1-\omega)}, \lambda_4 = \frac{2\omega\delta(1-\gamma)}{\gamma(1-\omega)}
\end{aligned}$$

This equation is valid for $0 \le \omega < 1$. When $\omega \to 0$ the one-period loss function $L_s = \pi_s^2 + \lambda_1 x_s^2 + \lambda_2(\Delta\pi_s)^2 + \lambda_3 x_{s-1}^2 + \lambda_4 x_{s-1}\Delta\pi_s$ collapses to the familiar $L_s = \pi_s^2 + \lambda_1 x_s^2$ which is reported by Woodford (200x), Chapter 6. As $\omega \to 1$ λ_2, λ_3 and λ_4 become unbounded. However, the relative size of these three terms $\lambda_2+\lambda_3+\lambda_4 = \frac{\omega}{(1-\omega)}\frac{(1+(1-\gamma)\delta)^2}{\gamma}$ remains constant so what really happens is that the size of the first two terms shrinks as the fraction of backward-looking price setters rises. The control of inflation becomes less and less important relative to the control of change in inflation.

C Imperfect competition on Labour Markets

We can modify the model by assuming that the wage-setting curve is above the labour supply curve, namely $w_t(z) = P_t \frac{v_y(y_t(z),\xi_t)}{u_C(C_t,\xi_t)}(1+\varphi)\left(\frac{Y_t}{Y_t^n}\right)^{\nu}$. Here we assume that efficiency wages impose constant mark-up φ on the labour supply curve and term $\left(\frac{Y_t}{Y_t^n}\right)^{\nu}$ explains bargain power when employment rate differs from its equilibrium rate. This leads to the following modifications:

$$\chi^f = \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}, \quad \chi^b = \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)},$$
$$\kappa_1^{\nu} = \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)}{\sigma(\gamma+\omega(1-\gamma+\gamma\beta))}\frac{(\psi+\sigma(1+\nu\psi))}{(\psi+\epsilon)} - \frac{(1-\gamma)\gamma\beta\omega\delta}{\gamma+\omega(1-\gamma+\gamma\beta)},$$
$$\kappa_2^{\nu} = \frac{(1-\gamma)\omega\delta}{\gamma+\omega(1-\gamma+\gamma\beta)}.$$

The requirement to have the same coefficients on output for the forward-looking and accelerationist Phillips curve suggests $\delta = (1-\gamma\beta)(\psi+\sigma(1+\nu\psi))/(\sigma\gamma(\psi+\epsilon))$, so

$$\kappa_1^{\nu} = \frac{(1-\gamma\beta)(1-\gamma)(\psi+\sigma(1+\nu\psi))(1-(1+\beta)\omega)}{(\gamma+\omega(1-\gamma+\gamma\beta))\sigma(\psi+\epsilon)}$$
$$\kappa_1^{\nu}+\kappa_2^{\nu} = \frac{(1-\gamma\beta)(1-\gamma)(\psi+\sigma(1+\nu\psi))(1-\omega(1+\beta-\frac{1}{\gamma}))}{(\gamma+\omega(1-\gamma+\gamma\beta))\sigma(\psi+\epsilon)} > \kappa_1+\kappa_2$$

The derivation of the Central Bank's Loss Function is the same as in Steinsson (2002), so the only effect on coefficients works through different δ that increases λ_3 and λ_4. Therefore, this specification requires more output stability but, simultaneously, the need for inflation inertia is enhanced.

D Canonical form of the model and matrix partitioning

The system in matrix form can be written as:

$$\widehat{\Omega} Z_{t+1} = \widehat{A} Z_t + \widehat{B} U_t + \widehat{E}\widetilde{\varepsilon}_t$$

where $Z_t = (Y_t', X_t')'$ and Y_t is predetermined state and X_t is non-predetermined state. This equation should give a unique solution, given boundary conditions and control U_t. Therefore, $\widehat{\Omega}$ needs to be invertible.

We multiply both sides by $\widehat{\Omega}^{-1}$ to come to the canonical form of representation:

$$Z_{t+1} = AZ_t + BU_t + E\widetilde{\varepsilon}_t$$

The government's loss function (7) can be written in terms of target variables, $G_t = (\pi_t, x_t, \Delta\pi_t, x_{t-1})'$ as

$$\min_{\{\widehat{i}_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} L_s = \min_{\{\widehat{i}_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} G_s' Q G_s$$

and, again, matrix Q is given in Appendix D. The target variable G_t can be linked to the state variable Z_t as $G_t = CZ_t$, so that the criterion can be re-written as:

$$\min_{\{\widehat{i}_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} G_s' Q G_s = \min_{\{\widehat{i}_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_s' C' Q C Z_s = \min_{\{\widehat{i}_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_s' \mathcal{Q} Z_s$$

where $\mathcal{Q} = C'QC$.

D.1 Non-Ricardian world, model with persistence

In this case, the predetermined state is $Y_t = (\pi_{t-1}, x_{t-1}, b_t)'$, and the non-predetermined state is $X_t = (\pi_t, x_t)'$, so matrices are :

$$A_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\beta} \end{bmatrix},\ A_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{1}{\beta} & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -\frac{\chi^b}{\beta\chi^f} & -\frac{\kappa_2}{\beta\chi^f} & 0 \\ \frac{\sigma\chi^b}{\chi^f\beta} & \frac{\sigma\kappa_2}{\chi^f\beta} - \frac{(1-\xi)}{\xi} & 0 \end{bmatrix},\ A_{22} = \begin{bmatrix} \frac{1}{\beta\chi^f} & -\frac{\kappa_1}{\beta\chi^f} \\ -\frac{\sigma}{\chi^f\beta} & \frac{\sigma\kappa_1}{\chi^f\beta} + \frac{1}{\xi} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \sigma \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} -\frac{1}{\chi^f\beta} & 0 & 0 \\ \frac{\sigma}{\chi^f\beta} & -\frac{1}{\xi} & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & \frac{1}{2}\lambda_4 \\ 0 & 0 & \frac{1}{2}\lambda_4 & \lambda_3 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

D.2 Non-Ricardian world, model without persistence

If the economy is closed then there is no persistence in any of the equations, including the budget constraint, and there is only one predetermined variable, $Y_t = (b_{t-1})'$, the non-predetermined state is $X_t = (\pi_t, x_t)'$, so the matrices take the form:

$$A_{11} = \left[\frac{1}{\beta}\right],\ A_{12} = \begin{bmatrix} -\frac{1}{\beta} & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ A_{22} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa_1}{\beta} \\ -\frac{\sigma}{\beta} & 1 + \frac{\sigma\kappa_1}{\beta} \end{bmatrix},$$

$$B_1 = [1], B_2 = \begin{bmatrix} 0 \\ \sigma \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} -\frac{1}{\beta} & 0 & 0 \\ \frac{\sigma}{\beta} & -1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \lambda_1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E Indeterminacy of the price level in the Ricardian setup (Taylor principle)

Consider the Ricardian fiscal policy in a deterministic setup

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \chi^f\beta & 0 \\ 0 & 0 & \xi\sigma & \xi \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ \pi_{t|t+1} \\ x_{t|t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\chi^b & -\kappa_2 & 1 & -\kappa_1 \\ 0 & -(1-\xi) & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \xi\sigma \end{bmatrix} \begin{bmatrix} \widehat{i}_t \end{bmatrix}$$

Bring it to the canonical form:

$$\begin{bmatrix} \pi_t \\ x_t \\ \pi_{t|t+1} \\ x_{t|t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\chi^b}{\beta\chi^f} & -\frac{\kappa_2}{\beta\chi^f} & \frac{1}{\beta\chi^f} & -\frac{\kappa_1}{\beta\chi^f} \\ \frac{\sigma\chi^b}{\chi^f\beta} & \frac{\sigma\kappa_2}{\chi^f\beta} - \frac{(1-\xi)}{\xi} & -\frac{\sigma}{\chi^f\beta} & \frac{\sigma\kappa_1}{\chi^f\beta} + \frac{1}{\xi} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sigma \end{bmatrix} \begin{bmatrix} \widehat{i}_t \end{bmatrix}$$

Suppose the feedback rule is

$$\widehat{i}_t = \theta_\pi \pi_{t-1} + \theta_x x_{t-1}$$

then the system under control is rewritten as

$$\begin{bmatrix} \pi_t \\ x_t \\ \pi_{t|t+1} \\ x_{t|t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\chi^b}{\beta\chi^f} & -\frac{\kappa_2}{\beta\chi^f} & \frac{1}{\beta\chi^f} & -\frac{\kappa_1}{\beta\chi^f} \\ \frac{\sigma\chi^b}{\chi^f\beta} + \sigma\theta_\pi & \frac{\sigma\kappa_2}{\chi^f\beta} - \frac{(1-\xi)}{\xi} + \sigma\theta_x & -\frac{\sigma}{\chi^f\beta} & \frac{\sigma\kappa_1}{\chi^f\beta} + \frac{1}{\xi} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ \pi_t \\ x_t \end{bmatrix}$$

In order the system would have a unique saddle path solution we need two explosive and two non-explosive roots. The necessary condition for the second biggest root to be outside the unit circle is:

$$\theta_\pi > 1 - \theta_x \frac{1 - \chi^f\beta - \chi^b}{\kappa_1 + \kappa_2}$$

We can check that with substitution of the parameters into the system's matrix, it will have exactly one unit root, one root greater than one, and two roots inside the unit circle (we used Maple to get the result and check it for different sets of parameters).