

# Inside the Black Box: Permanent vs Transitory Components and Economic Fundamentals

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## Abstract

Any non-stationary series can be decomposed into permanent (or "trend") and transitory (or "cycle") components. Typically some atheoretic pre-filtering procedure is applied to extract the permanent component. This paper argues that analysis of the fundamental underlying stationary economic processes should instead be central to this process. This argument is not, in itself new, since the links between multivariate Beveridge-Nelson trends and cointegration have been known for some time. But, despite the early work of King *et al.* (1991), there have been relatively few applications of the approach, possibly due to the perceived deficiencies of Beveridge-Nelson trends as derived from univariate representations, and a lack of transparency. We present an alternative derivation, whereby transitory components can be derived explicitly as a weighting of observable stationary processes, that have clear economic interpretations. We illustrate with two examples: from Garratt *et al's* (2003) model of the UK economy; and from Robertson and Wright's (2002) model of the US stock market.

# 1 Introduction

Macroeconomic analysis is largely (with the exception of endogenous growth theory) formulated in terms of stationary processes, yet most economic magnitudes are trending. Probably the majority of economists assume that the trend element is not purely deterministic, but also includes stochastic elements. There is as a result widespread use of a range of de-trending procedures, usually of the “black box” variety, where a trend is extracted by some pre-filtering procedure, usually univariate in nature. This paper argues that analysis of economic fundamentals should instead be central to this process.

We argue that the process of detrending should be viewed from a different angle. Rather than deriving trends, and then analysing the properties of whatever stationary processes are left over, we argue that economists should look first for the underlying stationary processes, in terms of identifiable economic fundamentals, ideally with a clear basis in theory. Once you pin down the stationary processes that link a set of variables (most of which are likely to be non-stationary) and, crucially, have identified their predictive power for changes in the underlying variables, deviations from trend<sup>1</sup> are simply projections from current values of the underlying stationary processes. By implication, the trends themselves effectively drop out as whatever is left over. The nature of the trends will thus depend on the nature of the fundamental stationary processes.

This argument is not, in itself new, since, for example, the links between multivariate Beveridge-Nelson (henceforth B-N) trends and cointegration have been known for some time. It is widely known that cointegration implies common stochastic trends: in a set of  $n$  variables with  $r$  stationary (usually, but not necessarily, cointegrating) relations, there will be  $n - r$  common trends (Stock and Watson 1988a,b). Multivariate equivalents of B-N (1981) trends for each of the  $n$  variables can then be derived as weightings of these underlying  $n - r$  trend elements (Newbold and Arino, 1998). However, despite the early work of King *et al* (1991) there have been relatively few applications of this approach. This may possibly be due to the perceived deficiencies of B-N trends as derived from univariate representations, that are frequently viewed as “too volatile” (see, for example Massmann and Mitchell 2002; Favero, 2001). It may also reflect a lack of transparency of the process by which the trends are derived.

We focus on the B-N decomposition, because the trend elements that

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<sup>1</sup>These are frequently referred to as estimates of the “cycle” (most notably when the variable in question is some measure of output). This is however increasingly a misnomer, since by no means all detrending procedures result in series that are in any sense necessarily cyclical.

result, as infinite horizon forecasts, must by definition be limiting forecasts of *any* permanent component that can be derived by alternative techniques. Any alternative trend, or permanent component, can thus always be expressed as the sum of the B-N trend plus a stationary component.

While B-N trends are typically derived from the moving average representation, we show that the equivalent derivation from the vector autoregressive representation has the distinct advantage that deviations from trend can be related directly to the underlying observable stationary processes that drive the system. In principle these may be derived on purely statistical grounds; however there is a well-known difficulty in clearly identifying the true value of  $r$ , the number of stationary relations (which are typically cointegrating relations, but may in principle include some series that are univariate stationary), in a set of  $n$  variables. We show that, when  $r$  is chosen atheoretically, the resulting deviations from trend can be highly sensitive to the chosen value of  $r$ , implying considerable uncertainty about whether a series is even above or below trend, let alone by how much.

It would be very satisfactory if theory entirely eliminated this form of uncertainty; but that would realistically be too much to hope for. Even when we have theory to assist us, we cannot avoid a degree of uncertainty about whether the data reject the theory. Thus even when we use theory to derive cointegrating relations we still suffer both from an element of rank uncertainty, *and* from uncertainty, for a given rank, as to which cointegrating relations to include in the system. Our empirical examples do suggest a limited amount of evidence that, when cointegrating relations are chosen on the basis of theory, the resulting detrended series inhabit a somewhat narrower space. But the much more crucial factor in our analysis is that, even when significant differences remain, our framework allows us to relate these differences directly to the inclusion or exclusion of certain economic relationships from the system.

This could be argued to make perfect sense. If we assert, for example, that output, or the value of the stock market, is “above trend” or “above its equilibrium” by some amount, we must always at least implicitly be positing some underlying disequilibrium, or set of disequilibria that will, in unconditional expectation, be expected to disappear. Crucially, we are also assuming that a fall in output or the stock market will be an important part of that adjustment process, and that this fall is predictable. In our framework, we can directly identify the link between deviations from trend and the underlying economic disequilibria. But, since there is almost always doubt about the statistical credentials of any process that is assumed to mean-revert, we must inevitably end up with different answers about whether output or the stock market is above or below “trend”, depending on which underlying

mean-reverting processes we believe in.

Equally, if we start from the fundamental stationary processes (that may in principle reflect frictions such as adjustment costs) that have predictive power, we cannot have clear priors on whether the resulting trends should be "smooth". The notion that Beveridge-Nelson trends are of necessity volatile is easily shown to be a fallacy; but equally, it is an unavoidable consequence of this approach that the nature of the trends must be determined by the nature of the underlying stationary processes.

We illustrate our analysis with two empirical examples, with a particular focus on two series, real UK GDP and the real value of the US stock market, that have the common property that they appear, as univariate processes, to be very close to being random walks. A univariate B-N decomposition would accordingly imply that each series was very close to, or equal to its trend, all the time. Once we derive trends in a multivariate context, however, and in particular, once we allow for cointegration, we show that estimated deviations from trend become distinctly more significant, and more persistent. We show that both the magnitude and direction of deviations can be very sensitive to our assumptions about the underlying stationary processes; but, crucially, the source of these differences is clear, because they can be related directly to economic fundamentals. Thus, we argue, theory can help to illuminate the interior of the black box.

Section 2 describes our approach to detrending, first outlining a general case then describing our detrending in the context of the VECM form used in the next sections. Section 3 then outlines a purely atheoretical multivariate approach to the derivation of trends in the context of our two examples enabling a direct contrast to be made with the results in section 4, which adopt an approach which uses the fundamental economic relationships outlined in the work of Garratt *et al.* (2003a) and Robertson and Wright (2003). Section 5 describes the results and section 6 concludes.

## 2 Beveridge Nelson Trends as Conditional Cointegrating Equilibrium Values

### 2.1 A General Definition

The most general definition of B-N trends is as limiting forecasts, absent deterministic growth as the forecast horizon goes to infinity. Thus, for any individual scalar variable,  $x_t$  define its B-N trend,  $\hat{x}_t$ , by

$$\hat{x}_t = \lim_{h \rightarrow \infty} E_t x_{t+h} - gh$$

where  $g$ , the element of deterministic growth, is typically a constant, but may in principle be a deterministic function of  $h$ . If  $\Delta x_t$  can be given a stationary moving average representation of the form

$$\Delta x_t = g + C(L)\varepsilon_t$$

(the deterministic growth element is, it should be noted, typically ignored) then the B-N trend can be expressed as

$$\Delta \hat{x}_t = g + C(1)\varepsilon_t$$

and is thus by definition a random walk with drift. This representation generalises easily to vector processes (Newbold and Arino, 1998).

The random walk feature of B-N trends is sometimes represented as a disadvantage, but is a necessary consequence of their forward-looking nature. Thus, suppose we take any arbitrary partitioning of  $x_t$  into a permanent and transitory component, of the form

$$x_t = x_t^P + x_t^T$$

then, since the transitory component must always, satisfy

$$\lim_{h \rightarrow \infty} E_t x_{t+h}^T = 0$$

then it must trivially follow that

$$\lim_{h \rightarrow \infty} E_t x_{t+h}^P = \hat{x}_t$$

and hence

$$x_t^P = \hat{x}_t + v(L)\varepsilon_t$$

where  $v(L)$  is some stationary lag polynomial. Thus any possible permanent component can always be partitioned into the random walk B-N trend and some other stationary component.

## 2.2 Beveridge-Nelson Trends in a Cointegrating VAR

When a vector of time series can be given a vector autoregressive representation B-N trends can be derived in a form that is readily interpretable in terms of the underlying stationary processes.<sup>2</sup> Assume a cointegrating VAR in  $n$  variables, of rank  $r$ :

$$\Delta \mathbf{x}_t = \Psi + \Phi \Delta \mathbf{x}_{t-1} + \alpha \beta' \mathbf{x}_{t-1} + \varepsilon_t \tag{1}$$

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<sup>2</sup>Note that univariate reduced forms will typically be higher order ARIMA processes. We would rationalise MA error terms in univariate representations as typically capturing missing cointegrating relations.

or equivalently

$$(\Delta \mathbf{x}_t - \mathbf{g}) = \Phi(\Delta \mathbf{x}_{t-1} - \mathbf{g}) + \alpha(\beta' \mathbf{x}_{t-1} - \kappa) + \varepsilon_t \quad (2)$$

where  $\mathbf{x}$  and  $\Psi$  are  $n \times 1$  vectors,  $\Phi$  is an  $n \times n$  matrix,  $\alpha$  is an  $n \times r$  matrix,  $\beta'$  is an  $r \times n$  matrix and  $\varepsilon$  is a  $n \times 1$  vector of error terms. The  $n \times 1$  vector  $\mathbf{g}$  and the  $r \times 1$  vector  $\kappa$  are the trend growth rates in the variables and the steady state values of the stationary relationships respectively and represent the deterministic components of the system. Note that these vectors of constants can (as shown below) be derived directly from the intercepts in the estimated VAR, and hence the data require no pre-filtering (cf Newbold and Arino, 1998; Rotemberg and Woodford 1996).

Higher order VARs can be dealt with by creating new variables for lagged differences, that do not, however, enter the cointegrating relations ( $n$  increases, but  $r$  does not). Note also that  $\beta'$  may in principle include columns in which there is only a single non-zero element, thus nesting systems in which one or more series is independently stationary (as, for example, in Blanchard and Quah (1989) -type bivariate representations of output growth and unemployment).

The nonstationary system as specified in (1) and (2) can be given equivalent stationary representations. For (1), this is given by,

$$\mathbf{y}_t = \begin{bmatrix} \Psi \\ \mathbf{0} \end{bmatrix} + \mathbf{C}\mathbf{y}_{t-1} + \mathbf{v}_t$$

where

$$\mathbf{y}_t = \begin{pmatrix} \Delta \mathbf{x}_t \\ \beta' \mathbf{x}_{t-1} \end{pmatrix}, \quad \mathbf{v}_t = \begin{pmatrix} \varepsilon_t \\ \mathbf{0} \end{pmatrix} \quad (3)$$

$$\mathbf{C} = \begin{bmatrix} \Phi + \alpha\beta' & \alpha \\ \beta' & \mathbf{I}_r \end{bmatrix} \quad (4)$$

where  $\mathbf{C}$  ( $(n+r) \times (n+r)$ ) is of full rank. $\beta$ .

This can be expressed as a zero mean system

$$\tilde{\mathbf{y}}_t = \mathbf{C}\tilde{\mathbf{y}}_{t-1} + \mathbf{v}_t \quad (5)$$

where

$$\begin{aligned} \tilde{\mathbf{y}}_t &= \begin{pmatrix} \Delta \tilde{\mathbf{x}}_t \\ \beta' \tilde{\mathbf{x}}_{t-1} - \kappa \end{pmatrix} \\ \Delta \tilde{\mathbf{x}}_t &= \Delta \mathbf{x}_t - \mathbf{g}, \\ \tilde{\mathbf{x}}_t &= \mathbf{x}_t - \mathbf{g}\mathbf{t} - \gamma, \end{aligned}$$

where  $\gamma$  is any vector in the space defined by  $\beta'\gamma = \kappa$ . and

$$\begin{pmatrix} \mathbf{g} \\ \boldsymbol{\kappa} \end{pmatrix} = \begin{bmatrix} \boldsymbol{\Psi} \\ \mathbf{0} \end{bmatrix} [\mathbf{I} - \mathbf{C}]^{-1}$$

The system in (5) can be rewritten entirely in deterministically detrended form, given  $\beta'\mathbf{g} = \mathbf{0}$ . The forecast values for the (deterministically) detrended system in  $\tilde{\mathbf{x}}_t$ ,  $h$  periods ahead of period  $t$  are given by

$$E_t \tilde{\mathbf{x}}_{t+h} = \tilde{\mathbf{x}}_t + \mathbf{J} \sum_{i=1}^h \mathbf{C}^i \tilde{\mathbf{y}}_t$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix}$$

In the system with drift, as  $h$  goes to infinity, conditional forecasts from period  $t$  go to infinity, but in the driftless system they will go to finite values:

$$\begin{aligned} E_t \tilde{\mathbf{x}}_{t+\infty} &= \tilde{\mathbf{x}}_t + \mathbf{J} \sum_{i=1}^{\infty} \mathbf{C}^i \tilde{\mathbf{y}}_t \\ &= \tilde{\mathbf{x}}_t + \mathbf{F} \tilde{\mathbf{y}}_t \\ \text{where } \mathbf{F} &\equiv \mathbf{J} \mathbf{C} [\mathbf{I}_{n+r} - \mathbf{C}]^{-1} \equiv \begin{bmatrix} \mathbf{F}_{\Delta} & \mathbf{F}_{ECM} \end{bmatrix} \end{aligned}$$

The matrices  $\mathbf{F}$  and  $\mathbf{J}$  have dimension  $(n \times (n+r))$ , and the matrices  $\mathbf{F}_{\Delta}$  and  $\mathbf{F}_{ECM}$  dimension  $(n \times n)$  and  $(n \times r)$  respectively. This can be given the “infinite horizon error correction” representation

$$\begin{aligned} E_t \tilde{\mathbf{x}}_{t+\infty} - \mathbf{x}_t &= \boldsymbol{\Phi}_{\infty} \Delta \tilde{\mathbf{x}}_t + \boldsymbol{\alpha}_{\infty} (\beta' \tilde{\mathbf{x}}_t - \boldsymbol{\kappa}) \\ &\text{or equivalently, using the underlying series} \\ \lim_{h \rightarrow \infty} E_t (\mathbf{x}_{t+h} - \mathbf{g}h - \mathbf{x}_t) &= \boldsymbol{\Phi}_{\infty} \Delta (\mathbf{x}_t - \mathbf{g}) + \boldsymbol{\alpha}_{\infty} (\beta' \mathbf{x}_t - \boldsymbol{\kappa}) \quad (6) \\ \text{where } \boldsymbol{\Phi}_{\infty} &= \mathbf{F}_{\Delta} - \mathbf{F}_{ECM} \beta' \\ \boldsymbol{\alpha}_{\infty} &= \mathbf{F}_{ECM} \end{aligned} \quad (7)$$

where  $\boldsymbol{\Phi}_{\infty}$  and  $\boldsymbol{\alpha}_{\infty}$  are of the same dimensions as  $\boldsymbol{\Phi}$  and  $\boldsymbol{\alpha}$ , and can be given a similar interpretation. In the long run any disequilibrium in the cointegrating relations in period  $t$  must, in expectation, be eliminated fully by adjustment of the variables in  $\tilde{\mathbf{x}}$ , with the elements of  $\boldsymbol{\alpha}_{\infty}$  determining the proportion of the adjustment taken up by any given variable.

### 3 Atheoretic Multivariate De-Trending

The above analysis assumes implicitly that the true structure of the model is known. However, uncertainty regarding the correct multivariate empirical representation of the data is extremely high. As a precursor to any estimation we first have to choose what we regard as an appropriate set of variables (presumably motivated by the question being asked and in relation to a theory) and if we assume a VECM form, we would wish to allow for the possibility of incorporating long-run relationships into our model, in which case we would need to know about the orders of integration of the variables being considered. Finally we also need to define the order of the VAR being used. These three preliminary stages in any estimation alone generate considerable uncertainty and any results and implications of the subsequent analysis are always conditional on these choices.

Even when abstracting from these choices there still remains areas of uncertainty concerning the validity with which short and long run restrictions might be imposed on the data. In particular we are still left with considerable uncertainty both about the rank of  $\beta$  (assuming a VECM) and for any given rank, the form of the cointegrating relationships and the true values of their coefficients. The extent of the uncertainty concerning the model is viewed as being a clear weakness in the multivariate approaches to business cycle facts, the results of which are seen as being model specific (see Kozicki, 1999). There is considerable debate about the appropriate method of determining the rank  $r$ . Alternative approaches can yield quite different values, especially in relatively large models, as a our first example shows.

Given this uncertainty our initial aim is to focus on what difference adopting a range of assumptions regarding rank of  $\beta$  makes to the properties of the deviations derived using the approach outlined in section 2. The nature of the exercise is deliberately atheoretical in the sense that it does not impose any restrictions on the matrix  $\beta$  except those required for exact identification. It might be the case that you have no prior regarding the number, if any, of long run relationships that exist in the data and that the uncertainty or low power of tests used to determine the correct rank order is such as to be uninformative. All this is done in the context of two examples, which define a set of variables motivated by a particular exercise, where statistical tests suggest the variables are I(1) and the lag length has been determined. In this sense we focus only on rank uncertainty.

In our first example, taken from Garratt *et al.* (2003a), we consider the following set of variables:

$$\mathbf{x}_t = (p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*, t)'$$
 (8)



where  $p_t^o$  which is the logarithm of oil prices,  $e_t$  is the logarithm of the nominal exchange rate (defined as the domestic price of a unit of the foreign currency, so that a depreciation of the home currency increases  $e_t$ ),  $r_t^*$  is the foreign short term nominal interest rate,  $r_t$  is the domestic short term nominal interest rate variable,  $p_t$  is the logarithm of domestic prices,  $y_t$  is the logarithm of real per capita domestic output,  $p_t^*$  is the logarithm of foreign prices,  $y_t^*$  is the logarithm of real per capita foreign output,  $h_t$  is the logarithm of the real per capita money stock and  $t$  is a deterministic time trend.

Proceeding assuming all the variables are I(1) and using a VAR(2) model with unrestricted intercepts and restricted trend coefficients (see Garratt *et al.* 2003a for details), and treating the oil price variable,  $p_t^o$ , as weakly exogenous for the long-run parameters (or ‘long-run forcing’), Garratt *et al.* (2003a) compute Johansen’s ‘trace’ and ‘maximal eigenvalue’ statistics to test for the appropriate rank of the system. They find that maximal eigenvalue statistic indicates the presence of just two cointegrating relationships or  $r = 2$ , at the 95% significance level whereas the trace statistics reject the null hypotheses that  $r = 0, 1, 2, 3$  and  $4$  at the 5 per cent level of significance but cannot reject the null of hypothesis that  $r = 5$ . In addition if one were to use various model selection criteria then the Akaike Information Criterion (AIC) suggests  $r = 8$ , Schwartz Bayesian Criterion (SBC)  $r = 1$  and the Hannan Quinn Criterion (HQC)  $r = 6$ . Clearly there is a reasonable degree of uncertainty regarding the correct rank.<sup>3</sup>

Given that at this stage we are adopting an atheoretical approach we do not use any theory to further inform our choice but instead adopt a practical view which is to try a range of different rank restrictions and for our purpose of identifying output deviations cycle see what difference it makes to their resulting properties. Hence we analyse a range of rank restrictions,  $r = 0$  through to  $7$ , where the  $\beta$  matrix is exactly identified and in all cases the VAR is of order  $2$ . Note the  $r = 0$  case is the VAR where no long run relationship exists and as such provides us with a useful benchmark with which to compare the effect of imposing long run relationships.

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<sup>3</sup>However we should note that as shown by Cheung and Lai (1993), the maximum eigenvalue test is generally less robust to the presence of skewness and excess kurtosis in the errors than the trace test. Given that there is evidence of non-normality in the residuals of the VAR model used to compute the test statistics (see Garratt *et al.* 2003a), it could be argued that it is more appropriate to base our rank or cointegration tests on the trace statistics. Similarly the power of the model selection criteria is known to be relatively low. Therefore one might argue that the weight of the evidence lies with the trace statistic conclusion of  $r = 5$  but where there is sufficient uncertainty surrounding the exact value of  $r$ .

Figure 1 plots the output deviations computed for all eight exact identified cases. Table 1 reports the correlation coefficients, standard deviations and some sign tests for the resulting output deviations, all for the period 1965q1-1999q4 (140 observations). As is evident from the inspection of Figure 1 the output deviations for the range of models examined show a very wide variation (we denote the exactly identified models of ranks 0 through to 7 as  $Ex0, Ex1, \dots, Ex7$ ). On examining the eight variants, it is clear that output deviations computed from the rank 7 model,  $Ex7$ , has very low correlation with any other trend deviation, the highest being 0.46 (with  $Ex4$ ) but where the other correlations are near zero. Note it is also negative for 73% of the period. The output deviation for the most likely rank of 5,  $Ex5$ , are near identical to those generated by the rank 6 model,  $Ex6$ , with a correlation coefficient of 0.99, near identical standard deviations and both are negative for approximately 44% of the period. If the true rank were 5 then imposing a rank of 6 would, in this example, make little difference to the resulting deviation for output. The deviations for output from the zero rank and rank 1 models,  $Ex0$  and  $Ex1$ , also show high comovement with a correlation coefficient of 0.97 but where their correlations with  $Ex5$  and  $Ex6$  are low at approximately 0.45 or less.<sup>4</sup> Hence as is clear from Figure 1 there are large differences in the implied deviations for output. The output deviations for the models of rank 2, 3 and 4,  $Ex2, Ex3$  and  $Ex4$  form another sub-group with correlation coefficients of 0.9 and above. Clearly the rank matters in terms of the properties of the output deviations where, for example, the differences in the output deviation computed from a model of say rank 2 compared to rank 5 (the two ranks highlighted by the the trace and Maximum eigenvalue test statistics) are large. It is worth emphasising that this feature continues to be true when analysing trend deviations for the other variables in the model and for the cases of rank 8 and 9, where the deviations become even more volatile.<sup>5</sup>

It is clear that an atheoretic approach provides very little guidance on a trend and trend deviation (cycle) decomposition as the properties of the deviations vary a lot according the rank we impose which, given our uncertainty on the rank, makes it a difficult method to use.

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<sup>4</sup>The output deviation computed from the rank zero case is in fact very close to that derived using a univariate Beveridge-Nelson decomposition.

<sup>5</sup>It is worth noting that the standard deviation of the output deviations increases as the rank of the model increases. A possible interpretation of this might be that as rank rises, we allow in equilibrium relationships possibly a long way from current values, but with increasingly slower adjustment speeds (Evans and Reichlin 1994) .

## 4 Inside the Black Box: Multivariate De-Trending Using Long Run Theoretical Restrictions

Given that the atheoretic approach does not resolve our uncertainty regarding the form of the model, as an alternative to an atheoretical approach we can posit certain relationships based on theory. In principle these can be long-run or short-run relationships (UIP or random returns being an example of the latter); but in this paper we focus on long run.

### 4.1 Deviations from trend in models with cointegrating relations based on theory

In our first example we adopted an atheoretical approach using the data set of Garratt *et al.* (2003a). However, the main emphasis of their work is to provide a long-run theoretical structure to a VECM model of the UK economy. They described five long-run relationships which were argued to be important to a small open economy like the UK, a detailed account of which, along with an outline of a framework for long run macro-modelling, is given in Garratt *et al.*(2003a). In brief, we note here that the five long-run relationships are: (i) Purchasing Power Parity (PPP), which assumes that, due to international trade in goods, domestic and foreign prices measured in a common currency equilibrate in the long-run (ii) Interest Rate Parity (IRP) which assumes that, under conditions of free capital flows, arbitrage between domestic and foreign bond holdings will, equilibrate domestic and foreign interest rates in the long-run (iii) an “output gap” (OG) relationship implied by a stochastic version of the Solow growth model with a common technological progress variable in production at home and abroad (iv) a real money balance (RMB) relationship, based on the condition that the economy must remain financially solvent in the long run; and (v) the Fisher Interest Parity (FIP) relationship which assumes that, due to inter-temporal exchange of domestic goods and bonds, the nominal rate of interest should in the long-run equate to the real rate of return plus the (expected) rate of inflation.

In contrast to the atheoretical exercise the attempt to relate the long run to explicit theory implies the presence of over identifying restrictions Estimation of the model subject to all the (exact- and over-identifying) restrictions enables a test of the validity of the over-identifying restrictions, and hence the underlying long-run economic theory, to be carried out. Such an empirical exercise is conducted by Garratt *et al.* (2003a) using quarterly UK data over the period 1965q1-1999q4. Their results showed that: (i) a VAR(2) model can adequately capture the dynamic properties of the data; (ii) there are

five cointegrating relationships amongst the nine macroeconomic variables; and that (iii) the over-identifying restrictions suggested by economic theory, cannot be rejected<sup>6</sup>.

The estimated model of Garratt *et al.* (2003a) with rank 5 and the five long-run relationships described above provides us with our benchmark theoretical case. As in the atheoretic case, the degree of uncertainty remains large with respect to the appropriate rank and therefore we would wish to try alternative rank orders. However, given that we have chosen to use a long-run theory to identify a model we also have to consider alternative forms of the  $\beta$  matrix and ask the question what difference does it make to the resulting output deviations. The long-run theory in Garratt *et al.* (2003a) suggests, among others, five cointegrating relationships, so in the first instance it we are able to use this prior to help reduce the number of alternative long run structures considered by ruling out models of rank 6 and above.<sup>7</sup> However, it also expands the number of models by allowing for alternative combinations of models at rank 4, 3, 2 and 1. In this example we confine ourselves to consider all the combinations of the five estimated long run relationships (where no further estimation is done)<sup>8</sup> which requires us to consider five theoretical models with four cointegrating relationships or rank four (each long-run relationship is dropped one at a time), ten models with three cointegrating relationships, ten models with two cointegrating relationships, five models with one long run relationship, which when combined with the benchmark case gives us a total of 31 models.

In what follows we will give a description of the properties of the deviations in output implied by these range of models. However given that theory requires us to impose a set of over identifying restrictions we are in a position to test the restrictions and therefore ascertain how plausible the proposed model might be. We wish here to consider the effects of imposing a long-run theoretical structure on the output cycle, but also to continue considering the effects of model uncertainty. However, before we consider all the alternatives above (see Table 2 for some comparisons) we are in a posi-

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<sup>6</sup>Likelihood ratio tests (reported in Table 2) which use asymptotic critical values reject the theory. But when bootstrapping, due to small samples and large numbers of variables the long-run theory restrictions cannot be rejected.

<sup>7</sup>The theory of Garratt *et al.* (2003a) does allow for the possibility of more than five cointegrating vectors and as such there is uncertainty concerning the prior. However the form of the five long-run relations selected are considered to be the least controversial of the range of possibilities and we therefore use these a means of restricting the choice available to us.

<sup>8</sup>To examine the alternatives in the true sense would require us to re-estimate the parameters and restrictions for each specification. Our aim here is to assess the sensitivity of the results to dropping combinations of the long-run relations.

tion to test the overidentifying restrictions implied by our set of alternative models and where clear rejections are apparent rule we ought then to be in a position not to consider them as plausible alternatives. For example, when using model selection criteria there might be a subset of models which it is difficult to choose between in which case model uncertainty needs serious consideration, however there also might be a large subset of models that one can with some reasonable degree of confidence rule out on statistical criteria. Hence in Table 2, we report the likelihood ratio test of the overidentifying restrictions for each of our 31 models, where for each rank the overidentified restrictions are tested against the exactly identified model of the same rank. Using the asymptotic critical values all the alternatives are clearly rejected.

It would appear then that only the benchmark case has some validity in that it is not rejected by the data. However, despite these rejections we think it is still instructive to examine model uncertainty so as to get an understanding of the elements of the benchmark structure which contribute most the properties of the cycle in output growth. By examining the empirical structure using our long-run theory we avoid the data mining critiques of the atheoretical approach. Further support for the form of the structure used here is also provided in Garratt *et al.* (2003b) who find that the benchmark model performs well as compared to an exactly identified rank 5 model in an out of sample forecasting exercise. It may also be the case that measurement errors may obscure the true relationships and hence the diagnostics may, for example, reject the PPP restrictions of  $1 - 1 - 1$  in favour of  $1 - \alpha - \beta$ .

Figure 2 plots the output deviations derived from the benchmark overidentified model which imposes the long run theory (*Ov5*) alongside the atheoretical exactly identified case where the impose the rank of 5 (*Ex5*). That is we might imagine that there is reasonable evidence regarding the rank but wish to know what difference imposing the long-run theory makes. From the plot we observe a limited degree of co-movement between the two cycles, where the correlation coefficient is relatively low, at 0.36, and the standard deviation are for *Ov5* and *Ex5* are 2% and 3% respectively. The size of the deviations also differ significantly and it is clear that imposing the long run restrictions has implications for output deviations over and above just imposing the most likely rank restriction. The plot is consistent with the asymptotic rejection of the long-run theoretical restrictions, which would more than likely not be rejected if indeed the two deviations were similar.

A consideration, not addressed so far, is whether the deviations themselves make sense in terms of actual economic events which occurred during the period. Interpreting what is a sensible or plausible deviation is of course very subjective and is made especially difficult in the UK as, there is no equivalent to the NBER dating committee on peaks and troughs which are

often used as a benchmark points, for the US. However, other researches have focussed on the UK, as well as European business cycles and so it is useful to use them as, at the very least, a point of comparison. For example, Birchenhall, Osborn and Sensier (2000) suggest the following dates for peaks and troughs in the cycle or deviation of UK GDP growth; peaks: 1973q3, 1979q2 and 1990q2 and troughs: 1975q3, 1981q1 and 1992q2. In the case of the two deviations plotted in Figure 2, the theoretical deviation (*Ov5*) peaks are in 1973q3, 1979q4 and 1990q1 and troughs are in 1972q1, 1977q3, 1982q4 and 1993q2 whereas the atheoretical deviation (*Ex5*) peaks are in 1974q4, 1979q4, 1989q2 and troughs are in 1971q3, 1977q3, 1986q4 and 1994q1. The suggestion here is that the deviations derived from this approach approximately correspond to previously identified peaks and troughs. This does not necessarily validate the derived deviations nor does it suggest a preference for the theoretical deviation, although the timing is clearly an issue of interest. The possible advantage of the theoretical deviation in this instance is that the proposed decomposition which the long run framework of the model allows may aid the identification of the deviation in relation to known economic events or additional information thought to be relevant at the time of specific output deviations.

Model uncertainty remains an issue despite the results rejecting the range of alternatives models. Hence in Figure 3 we plot the benchmark models output deviation alongside the output deviations generated from the five models with four long run or cointegrating relationships, where each of the models five long run terms are dropped one at a time. This allows for model uncertainty with respect to the  $\beta$  matrix to be examined plus it allows us highlight which of the long run relationships are important as regards the properties of the output deviation. Figure 3 suggests that the degree of uncertainty concerning the output deviation is as great when considering alternative  $\beta$  matrices as it is when considering different ranks. However, it proves distinctly easier to place some understanding of this uncertainty. For example, the deviation computed when excluding the FIP relationship is near identical to that of the benchmark model (the correlation coefficient is one) and as such it does not contribute to the uncertainty regarding the output deviation. This is also the case when excluding the IRP relationship where the resulting deviation has a correlation of 0.98 with the benchmark deviation and to a lesser extent is true when dropping the PPP relationship which produces a deviation which has a correlation coefficient of 0.80 with the benchmark deviation. However, excluding the OG relationship appears to make a significant difference to the output deviation which becomes more volatile and has a correlation coefficient of just 0.32 (see Table 2) with the benchmark theory based output deviation.

Before we proceed to examine the source of the uncertainty for the output deviation and to motivate why the breaking down of the output deviation movements is of interest we first need to make clear the contribution the short run dynamics to the output deviation movements. Recall that equation (6) breaks down the movements in the deviation into both short and long run components and therefore we need to be clear on the relative contributions of each of the sources. Figure 4 performs this task and plots the breakdown of the contribution of the long-run and short run components to the output deviation. The overall implication of Figure 4 is that contribution of the short run dynamics to the output deviation is relatively small and whilst their contribution is non-trivial at certain points it is not persistent and it does appear to be the case that the cyclical elements in output is driven largely by the cointegrating relationships.

## 5 A long-run decomposition

An alternative representation given in Figure 5, shows the output deviations or cycles alongside the contributions of the five cointegrating relations where on the evidence presented in Figure 4 we exclude the contribution of the short run dynamics. The breakdown of the long-run contributions serve to emphasise a number of points. As described above, and emphasised in Figure 3 and by the correlations reported in Table 2, it is clear that there is no contribution from the real interest rate (FIP) relationship on the output deviation. Thus real interest rate movements whose effects would be channelled through investment, the capital stock and output are not significant in explaining short run movements. Small but nonetheless prominent roles are apparent for deviations from UIP and PPP, which according to the correlation coefficients in Table 2, were suggested to have relatively small effects. For example, in the periods 1965-1972, 1979-1983 and 1993-1997 deviations from PPP are of some influence on the output deviation. It would appear that the long-run relation in real money balances (RMB) appears to have very large effects; these are however to a significant extent negatively related with the contribution of the “convergence” long-run relation, in  $y - y^*$ . This negative correlation is however readily explicable by the fact that both relations include output, but with opposite signs. An alternative representation is given in Figure 6, where the contributions of the two long run relations involving output are combined.<sup>9</sup> In this representation, the combined impact of the two output long-runs is quite strongly related to the implied output

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<sup>9</sup>Note that the ECMs themselves are *not* combined (which would clearly eliminate any impact of output at all).

deviation itself. There remains however, roles for deviations from PPP (i.e movements in the real exchange rate) and for the difference between domestic and foreign interest rates.

An alternative way to bring out this feature is shown in Table 2 which shows that the output deviation series from the benchmark model is very similar to the series derived from a number of lower rank models: one of rank three, excluding FIP and UIP, the other of just rank two, where the two relations included are just the two “output long-runs”. However, in sharp contrast, when either of these two key relations is excluded, the correlation falls much more markedly - especially so when the output gap is excluded [correlation falls to 0.3]. Thus output deviations in the model are essentially driven by these two relations; and when one or other is excluded, given the distinct differences between two underlying long-run relations, the implied output deviations series is in consequence significantly different. Note also that the inference regarding the output deviation or cycle is more robust to the exclusion of the RMB long-run (the correlation of the resulting deviation with the benchmark falling to 0.8) than it is to exclusion of output gap.

## 6 Conclusion

We have illustrated a method of computing and interpreting multivariate B-N trends which explicitly allows for the use of stationary fundamental economic relationships. It does not resolve the issue of uncertainty with respect to the wide range of deviations which can be derived depending on the model structure, most clearly illustrated in our atheoretical examples. It will always be difficult to avoid uncertainty regarding the definition of a deviation given that we exist in a world of uncertain data and models. However our framework at least enables you to understand the nature of the uncertainty, and relate it to economic fundamentals as illustrated in our two examples.

Note also that although our emphasis has been on long-run or cointegrating relations, it is in fact just as applicable to *any* stationary process that has predictive power for the variable of interest. In what we’ve looked at short run dynamics or delta terms have very little impact; but they might in principle for, e.g. inflation, where growth rates may be much more persistent (though ambiguity about whether inflation has a constant steady state). In principle other mean-reverting processes might also have predictive power without explicitly being cointegrating relations, e.g., capacity utilisation; unemployment (cf Blanchard and Quah, 1989); mean-reverting financial ratios. Possible areas for further work includes; investigating the impact of imposing



short-run as well as long-run restrictions. e.g., unpredictable returns; strict UIP; granger causality restrictions; analysing the impact on inference when deviations are of very low rank (e.g., re-examine Rotemberg and Woodford's 1996 covariance structure), deriving directly from the VAR, rather than estimating using generated deviations. and investigating the link with Blanchard and Quah (1989).

Table 1 : Statistics for the Output deviations Derived from the Exactly Identified Models

Model	Correlation Coefficients								Std Dev	Sign Tests
	Ex0	Ex1	Ex2	Ex3	Ex4	Ex5	Ex6	Ex7		
<b>Ex0</b>	1.00								0.00566	82 (58.6%)
<b>Ex1</b>	0.97	1.00							0.00676	77 (55%)
<b>Ex2</b>	0.57	0.55	1.00						0.01897	81 (57.9%)
<b>Ex3</b>	0.60	0.61	0.90	1.00					0.02076	84 (60%)
<b>Ex4</b>	0.57	0.56	0.91	0.98	1.00				0.02097	85 (60.7%)
<b>Ex5</b>	0.43	0.45	0.69	0.71	0.74	1.00			0.03024	61 (43.6%)
<b>Ex6</b>	0.41	0.43	0.67	0.70	0.73	0.99	1.00		0.02906	62 (44.3%)
<b>Ex7</b>	0.13	0.10	0.32	0.45	0.46	-0.03	-0.03	1.00	0.03201	102 (72.9%)

Notes: The sign test states the number of times during the period 1965q1-1999q4 (140 observations) a negative value of the trend deviation or deviation is recorded, the term in the brackets records this same number as a percentage of the total outcomes

Table 2 : Alternative deviations and Likelihood Ratio Tests of Overidentified Restrictions

Model	Correlation	Std Dev	LR Test	p-value
Benchmark	1.00	0.02078	71.49 (23)	0.00
Ov4: ex PPP	0.88	0.02055	81.32 (22)	0.00
Ov4: ex IRP	0.98	0.02037	82.72 (22)	0.00
Ov4: ex OG	0.32	0.02998	71.55 (22)	0.00
Ov4: ex RMB	0.80	0.01238	95.35 (24)	0.00
Ov4: ex FIP	1.00	0.02079	114.32 (22)	0.00
Ov3: ex PPP, IRP	0.91	0.01965	70.13 (19)	0.00
Ov3: ex PPP, OG	0.42	0.02719	72.29 (19)	0.00
Ov3: ex PPP, RMB	0.79	0.01209	89.89 (21)	0.00
Ov3: ex PPP, FIP	0.88	0.02061	116.17 (19)	0.00
Ov3: ex IRP, OG	0.38	0.02661	70.69 (19)	0.00
Ov3: ex IRP, RMB	0.59	0.00819	98.17 (21)	0.00
Ov3: ex IRP, FIP	0.98	0.02041	114.72 (19)	0.00
Ov3: ex OG, RMB	0.35	0.01138	86.02 (21)	0.00
Ov3: ex OG, FIP	0.32	0.03103	107.58 (19)	0.00
Ov3: ex RMB, FIP	0.80	0.01236	131.22 (21)	0.00

Table 2:Continued

Model	Correlation	Std Dev	LR Test	p-value
Benchmark	1.00	0.02078	71.49 (23)	0.00
Ov2: ex PPP, IRP, OG	0.51	0.02378	48.14 (14)	0.00
Ov2: ex PPP, IRP, RMB	0.58	0.00817	77.16 (16)	0.00
Ov2: ex PPP, IRP, FIP	0.91	0.01981	93.59 (14)	0.00
Ov2: ex PPP, OG, RMB	0.63	0.00989	71.69 (16)	0.00
Ov2: ex PPP, OG, FIP	0.48	0.02637	98.66 (14)	0.00
Ov2: ex PPP, RMB, FIP	0.79	0.012	115.43 (16)	0.00
Ov2: ex IRP, OG, RMB	-0.17	0.01015	80.08 (16)	0.00
Ov2: ex IRP, OG, FIP	0.35	0.029	93.08 (14)	0.00
Ov2: ex IRP, RMB, FIP	0.60	0.00844	111.91 (16)	0.00
Ov2: ex OG, RMB, FIP	0.34	0.012	113.65 (16)	0.00
Ov1: inc PPP	-0.17	0.01022	80.51 (9)	0.00
Ov1: inc IRP	0.67	0.01	83.96 (9)	0.00
Ov1: inc OG	0.56	0.0082	77.62 (9)	0.00
Ov1: inc RMB	0.55	0.024	56.59 (7)	0.00
Ov1: inc FIP	0.26	0.00563	43.54 (9)	0.00

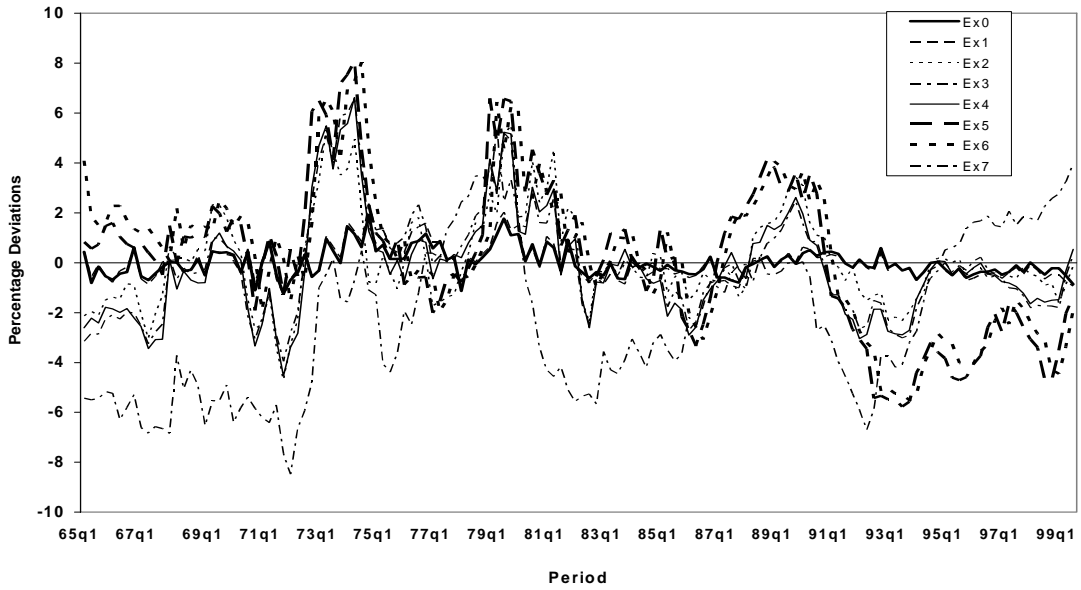
Notes: The term "Ov" refers to overidentified, the number given is the number of long run relations in the overidentified model and the term "ex" refers to the long-run relations excluded from the model which are then listed. The final set lists the long run relations included and therefore uses the term "inc" The correlation coefficient is the correlation of the benchmark model output deviation with the output deviation computed using the alternative model, Std Dev is the output deviation standard deviation and LR Test is the likelihood ratio test of the overidentified restrictions where the number being tested is given in brackets

Table 3 : Statistics for the Output deviations Derived from the Over Identified Models Excluding each Long Run Relation.

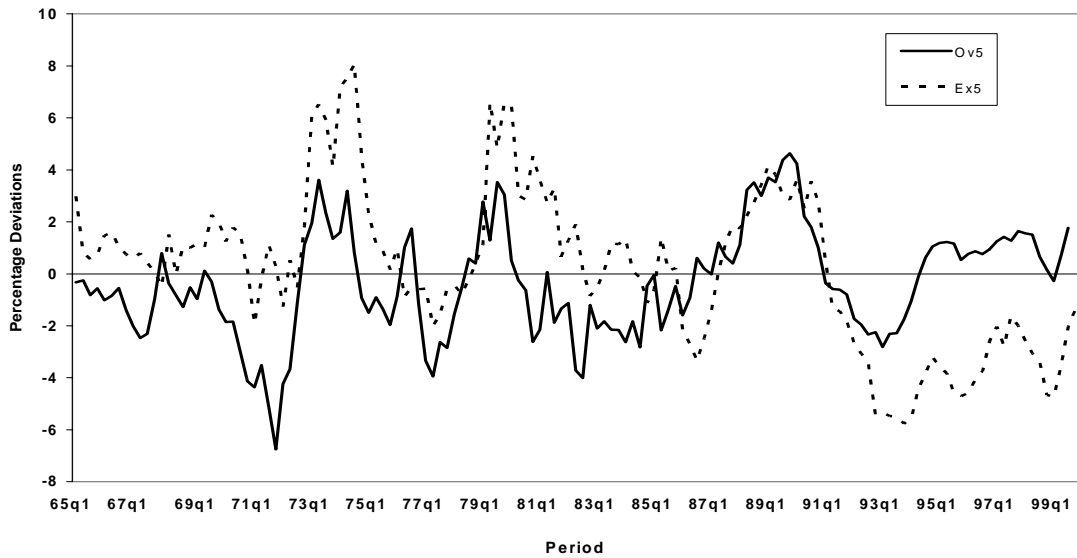
Model							Std Dev
<b>Ov5</b>	<b>Ov5</b>	<b>Ex PPP</b>	<b>Ex IRP</b>	<b>Ex OG</b>	<b>Ex RMB</b>	<b>Ex FIP</b>	
<b>Ov5</b>	1.00						0.0208
<b>Ex PPP</b>	0.88	1.00					0.0205
<b>Ex IRP</b>	0.98	0.85	1.00				0.0204
<b>Ex OG</b>	0.32	0.52	0.26	1.00			0.0300
<b>Ex RMB</b>	0.80	0.66	0.88	-0.08	1.00		0.0124
<b>Ex FIP</b>	0.99	0.88	0.98	0.32	0.80	1.00	0.0208

Notes: Ov5 denotes our benchmark model with five cointegrating vectors and the terms PPP, IRP, OG, RMB and FIP denote the cointegrating relationship excluded from the model with four cointegrating vectors, Ov4.. The sign test states the number of times during the period 1965q1-1999q4 (140 observations) a negative value of the trend deviation or deviation is recorded, the term in the brackets records this same number as a percentage of the total outcomes

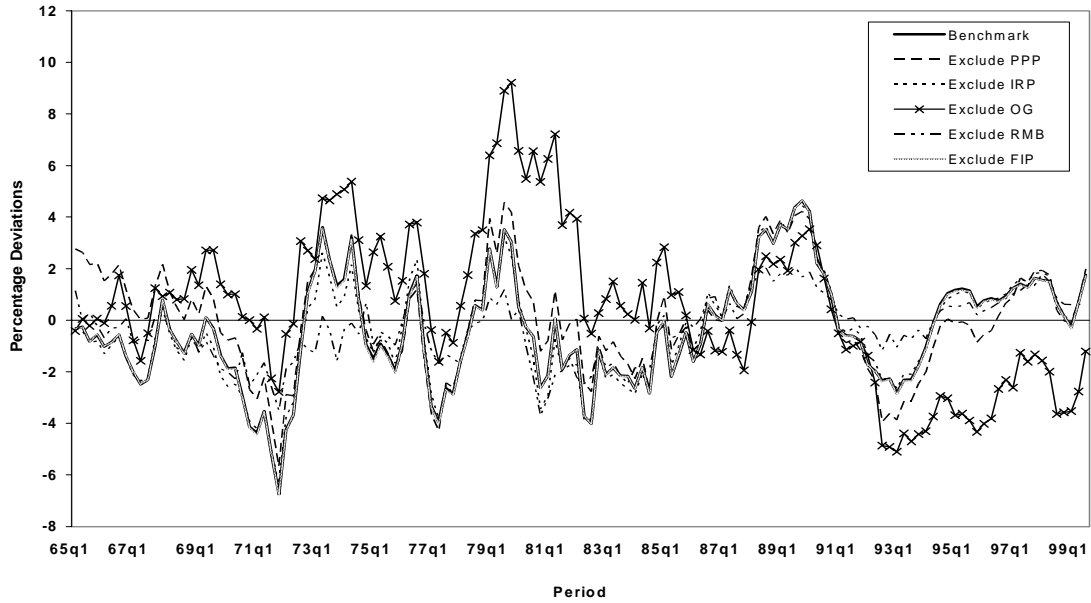
**Figure 1: Output Deviations Derived From a Range of Exactly Identified Models**



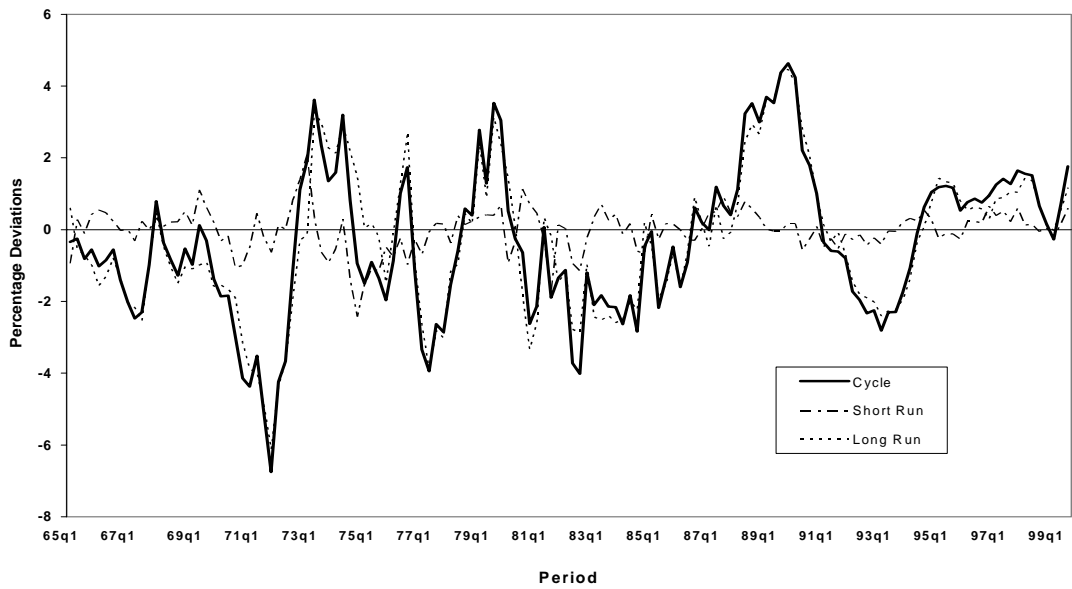
**Figure 2: Output Deviations with and without Long-run Theory (where  $r = 5$ )**



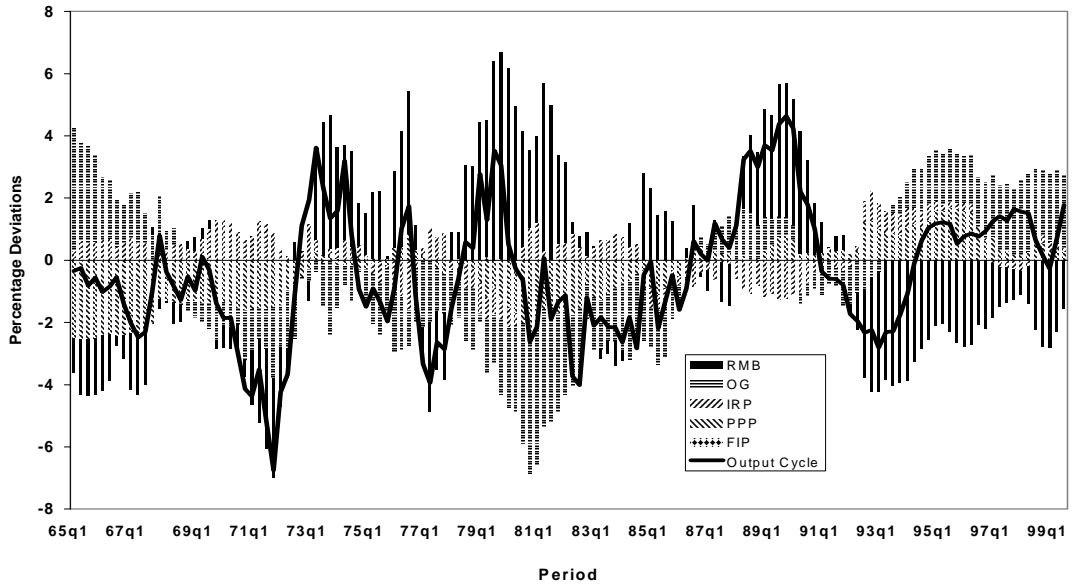
**Figure 3: Impact on Output Deviations of Excluding the Long Run Relationships One at a time**



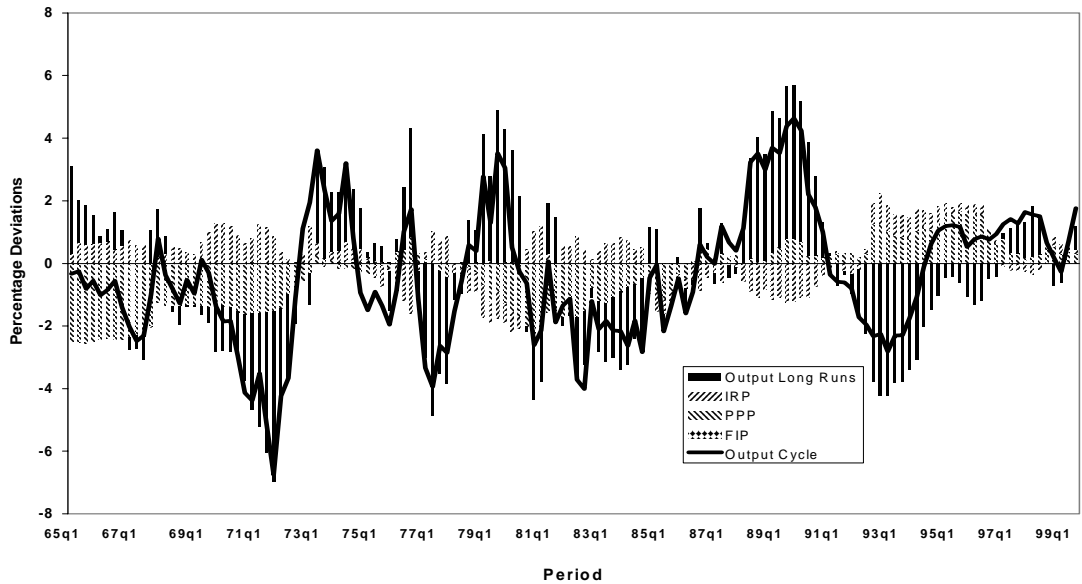
**Figure 4: Decomposition of Benchmark Output Deviations into Long and Short Run Deviations**



**Figure 5: Decomposition of the Benchmark Models Output Deviations into each of the Long-run Relationships Contributions**



**Figure 6: Decomposition of the Benchmark Models Output Deviations into Combined Output and other Long-run Contributions**



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