Business Survey Forecasts and Measurement of Output Trends in Five European Economies*

by

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Abstract

A framework for modelling actual and expected output growth is described when direct measures of expectations are available, and alternative measures of trend output are suggested. The relative merits of the alternatives are discussed and an illustative stochastic growth model is described where the trends are interpreted as potential output measures. Direct measures of expectations are derived from survey data in five European economies over 1968-1998. No evidence is found to reject rationality in any of the derived series, and the bivariate models of actual and expected outputs, and the associated trends, are shown to outperform those based on actual data alone.

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1 Introduction

The decomposition of output movements into a trend growth component and a cyclical component has been a central issue in macroeconomics and measures of 'trend', 'normal' or 'potential' output, of 'underlying economic activity', and of 'output gaps' are regularly produced by academics and policy-makers. These measures are obtained using a wide variety of econometric methods and are at the heart of decision making in many different contexts, including the timing and conduct of macroeconomic policy.

In this paper, we provide two alternative measures of trend output in the manufacturing sectors of five European countries over the period between the late 1960's and the late 1990's; the countries are France, Germany, Italy, the Netherlands, and the United Kingdom. The methods employed to obtain the measures make use of forecast-based decompositions of output into permanent and transitory components following the method of Beveridge and Nelson (1981) [BN]. The novelty of the measures proposed in the paper is that they make use of actual output data and direct measures of expected output levels as provided in Business Surveys. In each country, the two series constitute separate sources of information on current and future output levels. The actual and expected output series can be modelled in the context of a Vector Autoregressive (VAR) model subject to innovations which reflect the arrival of news about current and (expected) future output levels. Alternative forecast-based measures of trend output can be derived from the VAR models estimated for each country depending on assumptions on how the news is used.

The analysis relies on the availability of quantitative measures of expected output levels. These are derived from the qualitative information on output expectations provided by Business Surveys conducted in the five countries and published by the Directorate General for Economic and Financial Affairs of the Commission of the European Communities.[1] The derivation of the expected output series is based on the procedure described in Lee (1994) in which measurement errors are taken into account using survey responses on future expectations and on outcomes which have been realised in the past. Having obtained direct observations on expected output, the role played by expectations in the dynamic

[1]Details are provided in the Data Appendix.

evolution of output can be considered without recourse to any (possibly ad hoc) assumptions on the underlying behavioural model of output determination and without use of a (possibly contentious) structural econometric model. It is also possible to investigate empirically the nature of expectations formation, including its rationality, and the paper also reports results on this for our five European countries.

Of course, there are a wide variety of alternative statistical characterisations of output series (taken in isolation or in conjunction with other series) and various alternative methods have been employed in the literature to separate output into trend and cycles.[2] One advantage of the forecast-based decomposition method is that it is often possible to establish a link, through reference to an explicit economic model, between the series derived using these statistical techniques and meaningful economic magnitudes. Hence, for example, Evans (1989a,b) and Attfield and Silverstone (1998) have employed a BN decomposition in a bivariate model of output and unemployment to obtain measures of potential output as defined with reference to Okun's (1962) gap relationship. Or in King et al.'s (1991) influential paper, a similar concept was identified through a BN decomposition where the output trend was given by an accumulation of stochastic productivity shocks. In a similar vein, in this paper, we describe a simple stochastic growth model to illustrate how the proposed trend measures can be interpreted in terms of potential output. Given the use of this concept in Taylor (1993) rules, now widely employed in monetary policy formation, this suggests that the proposed measures could be of considerable practical use.[3]

The plan of the remainder of the paper is as follows. In Section 2, we present the modelling framework and define the alternative measures of trend output which we believe to be of interest. In Section 3, we present the simple illustrative stochastic growth model in which the proposed trend measures are interpreted as potential output measures. Section

[2]Alternative econometric methods employed to separate output into trend and cycles are discussed in Harvey (1985), Watson (1986), Evans (1989), Stock and Watson (1989), Evans and Reichlin (1994), and Kuttner (1994), for example.

[3]Indeed, the measurement of potential output has become increasingly important as theoretical work has focused on the welfare implications of different policy rules (see, for example, Svensson (1997), or Woodford (1999)).

4 provides an overview of the data for the five countries, concentrating on the derivation of the quantitative series on expected outputs and a description of their properties, including tests for rationality in expectation formation. Section 5 presents the estimated VAR models of actual and expected outputs in the five countries and discusses the trend output series obtained.[4] Section 6 concludes.

2 Measuring trend output using a VAR model of expected and actual outputs

2.1 The modelling framework

For each country, we shall model the process simultaneously determining (the logarithm of) actual output, denoted y_t at time t, and (the logarithm of) measured expected output, where (the logarithm of) the expectation of output at time t, formed by agents on the basis of information available to them at time $t-1$, is denoted y_t^*. We assume that actual output is first-difference stationary, and that expectational errors are stationary; the first of these assumptions is supported by considerable empirical evidence, and the latter assumption is consistent with a wide variety of hypotheses on the expectations formation process, including the Rational Expectations hypothesis (REH).[5] Under these assumptions, actual and expected output growth have the following fundamental Wold representation:

$$\begin{bmatrix} y_t - y_{t-1} \\ y_{t+1}^* - y_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \mathbf{A}(\mathbf{L}) \begin{bmatrix} \varepsilon_t \\ \xi_t \end{bmatrix}. \tag{2.1}$$

Here, α_1 is mean output growth, α_2 is mean expected output growth, $\mathbf{A}(\mathbf{L})=\sum_{j=0}^{\infty} \mathbf{A}_j(L)$, where the $\{\mathbf{A}_j\}$ are 2×2 matrices of parameters, assumed to be absolutely summable, and L is the lag-operator. Also, ε_t and ξ_t are mean zero, stationary innovations, with non-singular covariance matrix $\Psi = (\psi_{jk})$, $j, k = 1, 2$. Both actual output growth at time t and the growth in output expected to occur in time $t+1$, based on information at time t, are determined at time t; the actual and expected mean growth rate are provided by the

[4] The derived series are available at http://www.le.ac.uk/economics/kcl2/.

[5] Expected growth in output at time $t+1$, $y_{t+1}^* - y_t$, is also stationary, therefore, since it can be decomposed into actual output growth $(y_{t+1} - y_t)$ and expectational error $(y_{t+1}^* - y_{t+1})$.

deterministic component $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)'$, where $\alpha_1 = \alpha_2$ if there is no bias in expectations, and the random innovations at time t are represented by the vector $\mathbf{v}_t = (\varepsilon_t, \xi_t)'$.

Note that the error term ε_t is naturally interpreted as "news on output growth in time t becoming available at time t", while ξ_t is "news on output growth expected in time $t+1$ becoming available at time t". Both types of news are important in the simultaneous determination of actual and expected output growth; interdependencies in their joint determination are accommodated directly in (2.1) through the lag filter $\mathbf{A}(\mathbf{L})$ and indirectly through the covariance matrix Ψ. The model therefore incorporates the direct effects of news on actual and expected output growth, and the influences of feedbacks which exist in the determination of expected future output growth and actual output growth.

The general model in (2.1) can be expressed in a variety of different ways. For example, assume that $\mathbf{A}^{-1}(\mathbf{L})$ can be approximated by the p-order lag polynomial $\mathbf{A}^{-1}(\mathbf{L}) = \mathbf{B}_0 + \mathbf{B}_1\mathbf{L} + .. + \mathbf{B}_{p-1}\mathbf{L}^{p-1}$, where $\mathbf{B}_0 = \mathbf{I}_2$ without loss of generality. In this case, (2.1) can be rewritten to obtain the AR representation

$$\begin{bmatrix} y_t - y_{t-1} \\ y^*_{t+1} - y_t \end{bmatrix} = \mathbf{A}^{-1}(\mathbf{1})\boldsymbol{\alpha} - \mathbf{B}_1 \begin{bmatrix} y_{t-1} - y_{t-2} \\ y^*_t - y_{t-1} \end{bmatrix} - ... - \mathbf{B}_{p-1} \begin{bmatrix} y_{t-p+1} - y_{t-p} \\ y^*_{t-p+2} - y_{t-p+1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \xi_t \end{bmatrix} \tag{2.2}$$

and hence

$$\begin{bmatrix} y_t \\ y^*_{t+1} \end{bmatrix} = \mathbf{a} + \Phi_1 \begin{bmatrix} y_{t-1} \\ y^*_t \end{bmatrix} + \Phi_2 \begin{bmatrix} y_{t-2} \\ y^*_{t-1} \end{bmatrix} + ... + \Phi_p \begin{bmatrix} y_{t-p} \\ y^*_{t-p+1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}, \tag{2.3}$$

where $\mathbf{a} = \mathbf{M}_0^{-1}\mathbf{A}^{-1}(\mathbf{1})\boldsymbol{\alpha}$, $\Phi_j = \mathbf{M}_0^{-1}\mathbf{M}_j$, $j = 1, ..., p$, and

$$\mathbf{M}_0 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \ \mathbf{M}_p = \mathbf{B}_{p-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \text{ and } \mathbf{M}_j = \mathbf{B}_{j-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \mathbf{B}_j \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix},$$

for $j = 1, ..., p-1$. The error terms $\mathbf{u}_t = (\varepsilon_t, \eta_t)'$ are defined by

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} = \mathbf{M}_0^{-1} \begin{bmatrix} \varepsilon_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} \varepsilon_t \\ \varepsilon_t + \xi_t \end{bmatrix},$$

and the covariance matrix of the $\mathbf{u}_t$ is denoted $\Omega = (\sigma_{jk})$, $j, k = 1, 2$, where $\sigma_{11} = \psi_{11}$, $\sigma_{21} = \psi_{11} + \psi_{12}$, and $\sigma_{22} = \psi_{11} + 2\psi_{12} + \psi_{22}$. Note that ε_t has the interpretation of "news on output level in time t becoming available at time t", which is equivalent to news on output growth given that y_{t-1} is known. On the other hand, η_t is interpreted as "news on the level of output expected in time $t+1$ becoming available at time t" which causes expectations of output in time $t+1$ to be revised. This type of news encompasses the news on output levels at time t and the news on growth expected to be experienced over the coming period ($\eta_t = \varepsilon_t + \xi_t$). In this sense, the news conveyed by η_t dominates that conveyed by ε_t.

Manipulation of (2.3) also provides the VECM representation

$$\begin{bmatrix} \Delta y_t \\ \Delta y_{t+1}^* \end{bmatrix} = \mathbf{a} + \Pi \begin{bmatrix} y_{t-1} \\ y_t^* \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \begin{bmatrix} \Delta y_{t-j} \\ \Delta y_{t-j+1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}, \tag{2.4}$$

where $\Delta = (1-L)$ is the difference operator, $\Phi_1 = \mathbf{I}_2 + \Pi + \Gamma_1$, $\Phi_i = \Gamma_i - \Gamma_{i-1}$, $i = 2, 3, .., p-1$, and $\Phi_p = -\Gamma_{p-1}$. Given the form of the Φ_i described in (2.3), it is easily shown that Π takes the form

$$\Pi = \begin{bmatrix} -k_1 & k_1 \\ -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} -k_1 \\ -k_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix},$$

where k_1 and k_2 are scalars dependent on the elements of the $\mathbf{B}_j$, $j = 0, 1, .., p-1$. Hence, the model at (2.1) can be written in a VECM form where $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$ and $\boldsymbol{\alpha}' = [-k_1, -k_2]$ contains the parameters determining the speed of adjustment to equilibrium and $\boldsymbol{\beta}' = [1, -1]$ is the cointegrating vector. The form of the cointegrating vector captures the fact that actual and expected output cannot diverge indefinitely and is incorporated through the inclusion of the error correction term $\boldsymbol{\beta}' \left[y_{t-1}, y_t^*\right]' = y_{t-1} - y_t^*$. This property holds because expectational errors are taken to be stationary in this model, so that actual and expected output levels are cointegrated by assumption.

A final alternative for describing the model is the MA representation obtained through recursive substitution of (2.3):

$$\begin{bmatrix} \Delta y_t \\ \Delta y_{t+1}^* \end{bmatrix} = \mathbf{b} + \mathbf{C}(L) \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}, \tag{2.5}$$

where $\mathbf{b} = \mathbf{C}(1)\mathbf{a}$, $\mathbf{C}(L) = \sum_{j=0}^{\infty} \mathbf{C}_j(L)$, $\mathbf{C}_0 = \mathbf{I}_2$, $\mathbf{C}_1 = \Phi_1 - \mathbf{I}_2$ and $\mathbf{C}_i = \sum_{j=1}^{p} \mathbf{C}_{i-j}\Phi_j$, $i > 1$, $\mathbf{C}_i = 0$, $i < 0$. As is well known, following Engle and Granger (1987), the presence of a cointegrating relationship between the y_t and y_t^* imposes restrictions on the parameters of $\mathbf{C}(L)$; namely, $\boldsymbol{\beta}'\mathbf{C}(1)$=0. Further, given that $\boldsymbol{\beta}' = [1, -1]$, this ensures that $\mathbf{C}(1)$ takes the form

$$\mathbf{C}(1) = \begin{bmatrix} k_3 & k_4 \\ k_3 & k_4 \end{bmatrix} \tag{2.6}$$

for scalars k_3 and k_4.

Although the error terms ε_t and η_t have a natural interpretation in terms of news becoming available at time t, the MA representation given in (2.5) is not unique. Given the dominance of the news incorporated in η_t, we might be interested in identifying the entire effect of this shock, taking into account the interdependencies which are known to exist between the two types of news arriving at time t. If we assume that ε_t and η_t are joint normally distributed, with covariance matrix $\Omega = (\sigma_{jk})$, $j, k = 1, 2$, then we can write $\varepsilon_t = \rho\eta_t + \upsilon_t$ where $\rho = \frac{\sigma_{21}}{\sigma_{22}}$ and υ_t is orthogonal to η_t. An alternative MA representation which is of interest is then given by

$$\begin{aligned} \begin{bmatrix} \Delta y_t \\ \Delta y_{t+1}^* \end{bmatrix} &= \mathbf{b} + \mathbf{C}(L) \begin{bmatrix} 1 & \rho \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon_t \\ \eta_t \end{bmatrix} \\ &= \mathbf{b} + \widetilde{\mathbf{C}}(L) \begin{bmatrix} \upsilon_t \\ \eta_t \end{bmatrix}, \end{aligned} \tag{2.7}$$

where $\widetilde{\mathbf{C}}(L) = \mathbf{C}(L)\mathbf{P}$ and $\mathbf{P} = \begin{bmatrix} 1 & \rho \\ 0 & 1 \end{bmatrix}$ and the covariance matrix of $\widetilde{\mathbf{u}}_t = [\upsilon_t, \eta_t]'$ is diagonal.

The model at (2.1), and the equivalent forms in (2.2), (2.3), (2.4), (2.5) and (2.7), is quite general and has no implications for the expectations formation process. However, the assumption that expectations are formed rationally can be accommodated in the model through the imposition of restrictions. If expectations are formed rationally, the expression for y_t^* given in (the second row of) the lagged version of (2.3) is equal to the mathematical expectation of the expression for y_t given in (the first row of) (2.3).

Equating coefficients on the corresponding terms provides the REH restrictions:

$$\text{first row of } \Phi_1 = \begin{pmatrix} 0 & 1 \end{pmatrix}; \qquad \text{first row of } \Phi_j = \begin{pmatrix} 0 & 0 \end{pmatrix}, \qquad j = 2, ..., p, \tag{2.8}$$

or, equivalently, imposing these restrictions in (2.3),[6]

$$y_t = y_t^* + \varepsilon_t. \tag{2.9}$$

Hence, the deviation of actual output at time t from the level expected in the previous period is equal to the news on the output level becoming available at that time. This news is, by definition, orthogonal to information available at time $t-1$.

2.2 Measuring trend output

Having discussed the various alternative forms of the model of actual and expected outputs that are available, two alternative measures of trend output, based around (a multivariate version of) the BN decomposition procedure follow relatively naturally. The BN decomposition is applicable to models of (vectors of) variables which need to be differenced in order to achieve stationarity and presents the variable(s) as the sum of a stochastic trend, captured by a random walk with drift, and a stationary component. There is considerable evidence to support the view that output is difference stationary so that this decomposition is applicable here. The trend here is the expectation of the limiting value of the forecast of y_t conditional on time t information, or the "long forecast"; i.e. $\lim_{s\to\infty} E[y_{t+s} \mid I_t]$, where $I_t = \{\varepsilon_t, \eta_t, \varepsilon_{t-1}, \eta_{t-1}, ...\}$ is the information set at time t. The trend considers the effect of a (system-wide) shock to the two variables in the model at the infinite horizon; effectively, it abstracts from the cyclical effects of the shocks by concentrating on the infinite horizon only. Defining $\mathbf{C}_0^* = \mathbf{C}_0 - \mathbf{C}(1)$ and $\mathbf{C}_j^* = \mathbf{C}_j + \mathbf{C}_{j-1}^*$, $j > 0$, we can write $\mathbf{C}(\mathbf{L}) = \sum_{j=0}^{\infty} \mathbf{C}_j L^j = \mathbf{C}(1) + (1-L)\mathbf{C}^*(L)$. The model given in (2.5)

[6] Equivalently, in the error correction form of (2.4), the first row of $\Pi = \begin{pmatrix} -1 & 1 \end{pmatrix}$, so that $k_1 = 1$, and $\Gamma_j = \begin{pmatrix} 0 & 0 \end{pmatrix}$, $j = 1, .., p-1$. A similar approach to modelling rationality in expectations is explored in Engsted (1991).

can then be written

$$\begin{bmatrix} y_t \\ y^*_{t+1} \end{bmatrix} = \boldsymbol{\mu}_t + \boldsymbol{\tau}_t, \tag{2.10}$$

where $\boldsymbol{\mu}_t$ and $\boldsymbol{\tau}_t$ are, respectively, the stochastic trend and cyclical components obtained through the BN decomposition, defined by

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \mathbf{b} + \mathbf{C}(1)\mathbf{u}_t \quad \text{and} \quad \boldsymbol{\tau}_t = \sum_{i=0}^{\infty} \mathbf{C}^*_i \mathbf{u}_{t-i}.$$

Empirically, having obtained estimates of the parameters of $\mathbf{C}(L)$ and measures of the $\mathbf{u}_t$, the 'long run trend in output' is defined by

$$\begin{aligned} \overline{y}^P_t &= \begin{bmatrix} 0 & 1 \end{bmatrix} \boldsymbol{\mu}_t \\ &= \begin{bmatrix} 0 & 1 \end{bmatrix} \left(\begin{bmatrix} y_t \\ y^*_{t+1} \end{bmatrix} - \sum_{i=0}^{\infty} \mathbf{C}^*_i \mathbf{u}_{t-i} \right) \end{aligned} \tag{2.11}$$

where, as explained below, the 'P' superscript denotes 'potential' output. In (2.11), we have chosen to look at the long forecast of y^*_{t+1}, as opposed to that of y_t. However, given the cointegrating relation that exists between the variables, there is a single, common stochastic trend which evolves over time depending on the value of $\mathbf{C}(1)\mathbf{u}_t$; i.e., from (2.6),

$$\mathbf{C}(1)\mathbf{u}_t = \begin{bmatrix} k_3 & k_4 \\ k_3 & k_4 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} k_3\varepsilon_t + k_4\eta_t \\ k_3\varepsilon_t + k_4\eta_t \end{bmatrix}.$$

Hence, it is clear that the long forecast of y^*_{t+1} and y_t are equivalent in this case.

We have already noted that the news content of η_t dominates that of ε_t in the sense that the former contains information on output levels at time $t+1$, and therefore subsumes information on output at time t. In expressing their opinion on output levels in $t+1$, respondents are explicitly taking into account movements in ε_t and, in particular, any knowledge that they have on the 'unsustainable' component of ε_t (which influences their view on output growth in $t+1$). A second measure of trend output which might be of interest, therefore, focuses on the infinite horizon effect of shocks but abstracts from the effects of shocks which survey respondents consider to be unsustainable. To motivate the measure, we note first from (2.7) that

$$\mathbf{C}(1)\mathbf{u}_t = \widetilde{\mathbf{C}}(1)\widetilde{\mathbf{u}}_t = \begin{bmatrix} k_3 & k_4 \\ k_3 & k_4 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} k_3\upsilon_t + (k_4 + \rho k_3)\eta_t \\ k_3\upsilon_t + (k_4 + \rho k_3)\eta_t \end{bmatrix},$$

so that the long run trend in output underlying $\overline{y}_t^P$ in (2.11) can be expressed equivalently in terms of the elements of $\mathbf{u}_t$ or $\widetilde{\mathbf{u}}_t$. The innovations υ_t have been constructed to be orthogonal to the η_t and are associated with the unsustainable part of news on y_t which respondents discount in forming their expectations on output levels in time $t+1$. Of course, contemporaneous movements in output are not entirely unsustainable, and that part of news on y_t which is associated with a sustained effect (and correlated with η_t therefore) is acknowledged to have an effect on y_t and y_{t+1}^* through the $\rho\eta_t$ term. The complete effect of the innovations η_t on the long run forecast of actual and expected output levels are captured in the composite term $(k_4 + \rho k_3)\eta_t$. The proposed second measure allows for the feedbacks between actual and expected outputs over the (infinite) forecast horizon, but allocates the dynamic effects of the unsustainable innovations υ_t to the cyclical component. Hence, we have

$$\Delta\overline{y}_t^S = \Delta\overline{y}_t^P - k_3\upsilon_t \tag{2.12}$$

where $\overline{y}_t^S$ is considered the 'sustainable' growth trend. This measure corresponds to the unique decomposition of y_{t+1}^* into orthogonal permanent and transitory components discussed in Quah (1992), where 'orthogonality' here means that the innovation in the trend component is uncorrelated with the cycle at all leads and lags.[7]. Such a decomposition was employed in Blanchard and Quah (1989), and has been widely used since that paper. However, the orthogonality restrictions used in these decompositions are typically motivated by behavioural economic models which may or may not be considered realistic and so these restrictions are often contentious. In contrast, the discussion above indicates

[7]Clearly, neither Δy_{t+1}^* nor Δy_t are Granger causally prior to the other; under REH, for example, it is apparent from (2.8) that Δy_t^* helps in the forecast of Δy_t, and it is unlikely that lagged values of Δy_t provide no explanatory power in forecasting Δy_{t+1}^* beyond that provided by lags of Δy_{t+1}^* itself. Theorem 4.1 of Quah (1992) establishes that in these circumstances, there exists an orthogonal decomposition of either of the integrated series and that this decomposition is unique.

that the orthogonality restriction used in this paper has a relatively firm basis. Here, the transitory component is associated with that part of news on y_t arriving at time t which is revealed to be discounted by survey respondents as having an unsustainable effect on output when forming their expectations of next period's output.

Discussion in the literature of the choice between alternative decompositions has focused on the size of the trend and cycle. For example, Quah (1992) noted that there are an infinite number of decompositions available and that, in general, a decomposition can be chosen such that the trend is arbitrarily smooth (i.e. the variance of increments in the permanent component can be infinitely close to zero). If attention is restricted to MA representations, however, then there is a minimum bound for this variance and this minimum falls towards zero as the order of the MA process increases. In this sense, the BN decomposition (which defines the permanent component as a random walk) will *maximise* the variance of the permanent component. Evans and Reichlin (1994) establish that a multivariate version of the BN decomposition generates a smaller trend-cycle variance ratio than that obtained applying the BN decomposition to a univariate model, and that this ratio becomes smaller as the information set used to forecast output is expanded.[8] However, it is worth noting that additional variables are unlikely to have substantial explanatory power over and above that provided by the direct measure of output expectations (and indeed they will have no additional explanatory power under the REH) so that the derived trends will be smooth within the class of trends obtained from an MA representation. In this sense, the apparent arbitrariness of the smoothness of the derived trend (according to the size of the VAR) is avoided in the empirical application of this paper.

While the relative smoothness of a trend output series is clearly of interest, the choice of the measure of trend output should depend on the use to which it will be put and the measure should be judged according to its relevance to its purpose rather than on its size or statistical properties. From this perspective, it is useful to relate the derived

[8] This result matches that of Quah (1992) since the extra information provided by the multivariate VAR effectively provides for a more complicated dynamic specification and this is equivalent to extending the order of the MA representation in a univariate model.

series to an economically meaningful concept. To this end, the following section describes a stylised stochastic growth model with which the trend measures can be related to a potential output concept.

3 Potential output in a stochastic growth model

To illustrate the usefulness of the derived trend series, consider the following stylised model of output determination:

$$y_t = y_t^n + z_{1t} + \beta z_{1,t-1}, \tag{3.13}$$

$$y_t^n = \lambda y_{t-1}^n + (1-\lambda) y_t^p, \tag{3.14}$$

$$y_t^p = g + y_{t-1}^p + z_{2t}. \tag{3.15}$$

Here, in (3.13), actual output, y_t, deviates from the (unobserved) natural level of output, y_t^n, in the presence of nominal shocks, z_{1t}. In (3.14), the natural level of output adjusts slowly over time to the (unobservable) steady-state or potential level of output, y_t^p, while the steady-state level itself evolves over time according to a random walk with drift, g, driven by real shocks, z_{2t}, in (3.15). Equations (3.14) and (3.15) involve real magnitudes only and can be interpreted with reference to the Solow growth model: in this case, y_t^p is the output level associated with full employment and with capital stock at its steady-state level, while y_t^n represents full-employment output obtained with a capital stock that might differ from its steady state level. The dynamics of the real economy are provided by the partial adjustment process in (3.14) and by the unit root process described (3.15). The dynamic time path of actual output levels is influenced by the processes influencing the real economy and through the influence of nominal shocks, which cause actual output levels to differ from y_t^n for up to two periods in (3.13).[9] The nominal shocks, z_{1t}, have a transitory effect on output while the z_{2t} are permanent innovations. Note, however, that these shocks are not necessarily orthogonal, so that we might observe that, in the long

[9]The model of (3.13)-(3.15) represents a simplified version of the stochastic Solow growth model described in more detail in Lee et al. (1997).

run, adverse nominal shocks are associated with output levels which are systematically lower than they would have been had the shock not occurred.[10]

Casual inspection of the system in (3.13)-(3.15) suggests that the time path followed by actual output may have complicated dynamics, but that its long run properties will be dominated by the unit root process of (3.15). This is established formally in the algebra of the Appendix. In particular, solving (3.13)-(3.15) to eliminate the unobservable terms y_t^n and y_t^p and, assuming rational expectations, we obtain a MA representation for $\left(\Delta y_t, \Delta_t y_{t+1}^*\right)$of the form given in (2.5) where

$$\mathbf{u}_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} 1 & 1-\lambda \\ \beta & (1-\lambda)(1+\lambda) \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix}. \tag{3.16}$$

and the $\mathbf{C}(L)$ lag polynomials are functions of the underlying model parameters. Note that news arriving on output levels to be experienced in time t and $t+1$, ε_t and η_t, comes in the form of linear functions of nominal and real shocks, neither of which are observed directly. It is readily shown that

$$\mathbf{C}(1)\mathbf{u}_t = \begin{bmatrix} \frac{-\beta}{(1+\lambda-\beta)(1-\lambda)} & \frac{1}{(1+\lambda-\beta)(1-\lambda)} \\ \frac{-\beta}{(1+\lambda-\beta)(1-\lambda)} & \frac{1}{(1+\lambda-\beta)(1-\lambda)} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} z_{2t} \\ z_{2t} \end{bmatrix} \tag{3.17}$$

and

$$\overline{y}_t^P = \overline{y}_{t-1}^P + g + z_{2t},$$

so that the trend defined in (2.11) is indeed driven by the permanent shocks z_{2t} only and corresponds precisely with the measure of potential output defined in (3.15).

The alternative trend measure defined in (2.12), $\overline{y}_t^S$, also has a clear economic meaning in the context of the model of (3.13)-(3.15). With $[\varepsilon_t,\ \eta_t]'$ defined in (3.16), the orthogonalisation discussed in (2.7) provides an expression for v_t which is a complicated function of model parameters.[11] But with the value of k_3 defined in (3.17), we obtain a relatively

[10]Hence, the model can accommodate possible hysteresis effects.

[11]In the simple case where $\beta = 0$ and $cov(z_{1t}, z_{2t}) = 0$, $v_t = \varepsilon_t - \frac{1}{(1+\lambda)}\eta_t = z_{1t}$ and the orthogonalisation splits out the effect of the nominal and real shocks. In this case, the interpretation of v_t as the 'unsustainable' component of the growth in output at time t is straightforward since the effects of the nominal shocks die away within one period and the stated expectation of output in $t+1$ shows the impact of the real shock directly.

simple expression for $\Delta\overline{y}_t^S$ from (2.12); namely,

$$\begin{aligned}\Delta\overline{y}_t^S &= g + \frac{cov(z_{2t}, \eta_t)}{var(\eta_t)}\eta_t \\ &= g + E\left[z_{2t} \mid \eta_t\right].\end{aligned}$$

Hence, the proposed alternative trend measure also tracks the level of potential output, y_t^P, but evolves over time driven by agents' best guess of the permanent component of today's news, as revealed in the survey of agents' expectations of output, rather than the shock itself.

The precise details of the relationship between the trend measures and the economic concepts will change if a different economic model is used for the purpose of interpretation. But the above discussion shows that, while the proposed trend measures can be motivated purely on the statistical grounds discussed in Section 2, they also have a clearly defined economic meaning in the context of a relatively simple macroeconomic model and one that is likely to carry over to a number of more sophisticated economic models.

4 Analysing qualitative survey data in five European countries

In this section, we first discuss the general method by which directly observed measures of expectations of variables are obtained from survey data. Then, in Section 4.2, we apply the methods to Survey data for our five European countries and describe the properties of the expectations series that are derived, showing that the data provides support for the view that expectations are formed rationally in all five countries.

4.1 Deriving series on output expectations from Surveys

The measurement of expectations based on surveys is complicated by the fact that surveys typically provide only qualitative data on expected events which have to be converted to a quantitative series. For example, in the Surveys that we employ here, information is provided on the proportion of respondents in the Survey who report that they expect the volume of their output to "rise", "stay the same", or "fall" over a given future period. The Survey also provides the equivalent information on what respondents report actually happened to output volumes over a given period in the past. Various conversion procedures have been proposed in the literature for converting the qualitative data to quantitative series,[12] but all procedures suffer from the problem that series derived from the qualitative data provide imperfect measures of the true series, and that the form of the conversion error contained in the derived series is unknown.

Lee (1994) describes a procedure to obtain a quantitative expectations series from the Survey responses which takes into account the presence of conversion error by using the forward-looking responses and the backward-looking responses obtained in the Survey in a particular way. Briefly, the procedure focuses first on the backward-looking survey responses and derives a measure of 'realised' output growth over the previous period by applying any one of the available conversion procedures to the qualitative data. Conversion error is measured by the gap between this derived 'realised' output growth measure and the output growth which was actually observed. Any systematic patterns in the conversion error are identified through a regression model in which the conversion error at time t is regressed on a vector of specified variables dated at time $t-1$ and before, denoted $\mathbf{h}_{t-1}$. Next, the conversion procedure that was applied to the backward-looking survey responses is applied to the forward-looking survey responses to produce a quantitative series on expected output; this is denoted y_t^e and differs from the true expectations series, y_t^*, if conversion error is present. The procedure of Lee (1994) assumes that the conversion error contained in the measure y_t^e is of the same form as that contained in the backward-looking series and, on this assumption, the derived expectations series can be 'purged' of conversion error using the regression results. The discrepancy between this purged measure of expected growth and observed growth can be interpreted as pure 'expectational' error and the expectation formation process can be examined directly by analysing these expectational errors.[13]

[12]Pesaran (1987) and McAleer and Smith (1995) provide discussions of various alternative conversion procedures and their relative merits.

[13]For example, rationality requires these expectational errors to be orthogonal to known information.

4.2 Expected output series for five European countries

The empirical work of the paper investigates the survey responses given by samples of firms in the manufacturing sectors of five European countries. The countries are France, Germany, Italy, the Netherlands and the UK and these were selected on the basis of data availability. The survey questions in every country refer to the respondent firms' own past and future, seasonally-adjusted output levels,[14] although the time horizon specified in the survey questions differ across countries. Hence, for Germany, Italy and the Netherlands, the backward-looking part of the question refers to output trends over the past month, while the question considers the last three months for France and the last four months for the UK. For the UK, the forward-looking question refers to the next four months; for the other countries, the specified time horizon is the next three months. All the surveys are conducted monthly, but the empirical work is conducted using quarterly data to match the time horizon over which survey respondents are typically asked to form their expectations.[15] The sample period mainly runs from the late 1960's to the late 1990's, although these also differ across countries: data for Germany and Italy are available over 1968q1-1998q1; France covers 1969q1-1998q1; the Netherlands covers 1972q1-1998q1; and the UK data period is 1975q3-1998q2.

The method chosen for converting the qualitative survey responses into quantitative series is the widely-used 'Probability Method'; the application of this method to the backward-looking and forward-looking survey responses provided the 'realised' output growth series and the (unpurged) expected output growth series, $y_t^e - y_{t-1}$, respectively.[16]

[14]For example, for the UK, the responses relate to the question "Excluding seasonal variation, what has been the trend over the past four months, and what are the expected trends over the next four months, with regard to the volume of output ?". For Italy, there is ambiguity in the survey questionnaire over whether the forward-looking responses are seasonally adjusted or not (although there is no ambiguity in the backward-looking part of the survey).

[15]Hence, for the forward-looking expectations series, the analysis considers only the survey reponses published in January, April, July and October of each year.

[16]The Probability Method, described in detail in Pesaran (1987), requires an assumption to be made on the form of the subjective probability distribution of firms' future output change and the construction of a scaling parameter. Here, the distribution is assumed to be normal and the scaling parameter is the ratio of the sum of the absolute changes in actual output to the sum of the absolute values of the unscaled

Where the backward-looking survey responses relate to a one month period, a monthly realised series was derived, using all of the monthly surveys, and monthly conversion errors were obtained by subtracting the realised series from actual monthly data. A quarterly conversion error series was then obtained by averaging the monthly errors over successive three month intervals. The vector of specified variables (dated at quarterly intervals), $\mathbf{h}_{t-1}$, which is assumed to be known to agents at time t and which is used in the regression explaining the backward-looking conversion error includes: a lagged dependent variable; up to four lags of manufacturing output growth; two lags of the interest rate; and two lags of the exchange rate of each respective country.[17] A specification search was undertaken to obtain a well-specified model of the conversion error for each country,[18] and these were then used to construct expected output growth series, $y_t^* - y_{t-1}$ which are purged of conversion error employing the method described in Section 4.1 above.

Table 1 presents summary statistics of the properties of the actual and expected output growth series derived from the Survey data. The first two columns of the Table present Augmented Dickey-Fuller (ADF) statistics calculated to investigate the order of integration of the actual output data.[19] The unit root hypothesis cannot be rejected when applied to the (log) output data (y_t), but is comprehensively rejected when applied to the output growth data (Δy_t). These results confirm that Manufacturing Sector output can be considered an I(1) process, as assumed in the analysis of Section 2. The third column provides the mean (quarterly) growth rates of Manufacturing Sector output in the five countries during their respective sample periods and shows the wide variety of rates experienced across the countries over the last two decades.

There follows two sets of statistics in Table 1 relating to the (unpurged) derived expectations series, $y_t^e - y_{t-1}$, and the purged series, $y_t^* - y_{t-1}$. In these, we find first that

expected output series derived from the survey.

[17]The interest rate used is the discount rate, and the exchange rate is the average exchange rate of the country currency to the US Dollar over the quarter.

[18]Hence, we ensured that the 'backward-looking' regression model exhibited no serial correlation, parsimony, stability in the parameters, and satisfied optimal information criteria.

[19]The orders of augmentation were selected on the basis of the Akaike and Schwarz-Bayesian information criteria. No more than two lags were required for any of the countries.

contemporaneous correlations between actual output growth and the unpurged expected output growth series are positive in all countries, but small in most cases, averaging 0.28. In comparison, contemporaneous correlations between the actual and the 'purged' expected output growth series are positive and larger for each of the countries, averaging 0.44. Second, the reported ADF statistics indicate that a hypothesised unit root in the expectational errors can be rejected for both expectation series in all of the countries. Given that the actual output growth series have been shown to be I(0), this result implies that the actual and expected output series are both I(1) and cointegrated with cointegrating vector $(1\ -1)$. Third, the skewness statistic provides no evidence of asymmetries in the responsiveness of expectation formation to increases and decreases in output in either of the expectation series for any country. Fourth, the 'SC' statistics show that there is evidence of (first-order) autocorrelation present in the unexpected output growth series based on y_t^e in the UK, but there is no such evidence in the 'purged' expectational errors in any country. Finally, the 'H' statistics show that the expectational errors are strongly related to actual output growth in both series, with large errors made at times when output growth, in absolute terms, is relatively large.[20]

Finally in Table 1, statistics d1-d3 are presented to test the orthogonality of the various types of error to information which is known to agents in the industry when expectations are formed, $\mathbf{h}_{t-1}$. In each case, the statistics are to be compared with the χ^2 distribution with six degrees of freedom.[21] The statistics denoted 'd1' test the orthogonality of the expectational errors based on y_t^e. These effectively test the rationality of expectation formation under the assumption that expectational conversion errors are orthogonal to known information. This hypothesis is strongly rejected in all five countries. The statistic 'd2' provides the corresponding test of the hypothesis that the backward-looking conversion error is orthogonal to known information. These also provide strong evidence with which to reject the hypothesised orthogonality in all but one economy (the Netherlands).

[20]This observation is consistent with the conservatism in expectation formation described in Lee (1994) and Lee and Shields (2000)'s analysis of price, cost and output expectations in the industries within UK manufacturing.

[21]The reader is referred to Lee (1994) and Lee and Shields (2000) for further details of the test statistics.

This indicates that an adequate treatment of the conversion errors is required before a test of rationality can be carried out, and certainly suggests that the 'd1' statistics should be interpreted with caution. Finally, the statistics denoted 'd3' test the orthogonality of the expectational errors based on the 'purged' expectations series y_t^*. These provide a test of the rationality of expectations formation under the assumption that the expectational conversion error is of the same form as the realisation conversion error. In this case there is no evidence with which to reject the hypothesised orthogonality in any country. Given that the assumptions underlying this final test of rationality are relatively weak, these results provide support for the view that expectations on manufacturing output growth are formed rationally in our five countries.

5 Trend output measures in five European countries

In this section, we describe the time series analysis of the actual and expected output series for our five countries and consider the associated measures of trend output. For the most part, we shall consider the trend measures $\overline{y}_t^S$ and $\overline{y}_t^P$ obtained from the estimated bivariate models of actual and expected output in each country. But as a point of reference, we shall also consider a trend measure $\overline{y}_t$ obtained by applying the BN decomposition to a univariate model of actual output growth.

We begin by estimating univariate models for actual and expected output growth in each country and testing for the presence of feedbacks between the actual and expected series. Specifically, in the column of Table 2 headed '$F1$', we report the test of the joint significance of four lagged values of expected output growth when added to a $AR(4)$ model of actual output growth. In each case, the variable addition test shows that the expected output growth series make a statistically significant contribution to the regression over and above that provided by the lagged actual output series. This is not surprising, given the results of the rationality test, which suggested that $(y_t^* - y_{t-1})$ would have considerable explanatory power for $(y_t - y_{t-1})$. In the column of Table 2 headed '$F2$', we next report the test of the joint significance of four lagged values of actual output growth when added to a AR(4) model of expected output growth. These show that there are also statistically significant feedbacks from actual to expected output series (in addition to those provided by lagged expected output growth) at least in the cases of Germany and Italy.[22] Taken together, these results confirm that there are important interactions between actual and expected output growth series and that a bivariate model of actual and expected output growth will outperform a univariate model of actual output growth in terms of statistical fit. This also suggests that the trend measures $\overline{y}_t^S$ and $\overline{y}_t^P$ are preferred to $\overline{y}_t$ on statistical grounds.

Table 3 provides the parameter estimates for the bivariate VAR models given in (2.4) which are used to derive the measures of trend output $\overline{y}_t^P$ and $\overline{y}_t^S$ for each of the five countries.[23] The reported equations represent the outcome of a specification search which starts from an unrestricted model including two lags of Δy_t and Δy_{t+1}^* and the error correction term $(y_t^* - y_{t-1})$ and excludes variables whose (absolute) t-ratios are less than unity.[24] These bivariate models allow for various interactions between actual and expected outputs which could not be captured within a univariate model (and which provide a substantially more complicated dynamic specification than could be provided by any univariate model). In particular, the models incorporate the effects of the cointegrating relationships between y_t and y_t^*; this effect could not be included in a univariate model of actual (or expected) output growth and its omission represents a misspecification in the univariate model.[25] Further, the bivariate model, and its associated trend measures, can take into account any possible contemporaneous correlations that exist between innovations in actual and expected future outputs. As it turns out, the estimated value of this correlation, given by ρ in Table 3, averages 0.72 across the five countries, showing that this is an empirically

[22]For Italy, the regressions reported in Tables 2 and 3 also include simple quarterly dummy variables to take into account the possibility that there are seasonal effects in the expectations series for this country.

[23]In view of the the support provided for the REH in Table 1, the restricted parameters of the first row of (2.4) are provided by the REH restriction in (2.9).

[24]The reported SC tests show that a second-order autoregression is sufficient to capture the series' dynamics adequately, and the reported LM tests show the imposed restrictions do not violate the data in any case.

[25]The error correction term does not show significantly in the regressions of Table 3 for France, Italy, or the UK. But even in these cases, the inclusion of the expression of (2.9) in the bivariate VAR accommodates the cointegrating relationship by construction.

important feature of the data.[26]

The results of Tables 2 and 3 suggest that trend measures derived from the bivariate model for each country, $\overline{y}_t^P$, will be preferred to the trends derived from the univariate (actual output growth) models on statistical grounds, $\overline{y}_t$. Figures 1a-1e plot the two trend measures[27] and demonstrate that, while the two trends move in broadly similar ways relative to the actual output series, the two series differ substantially in terms of their relative volatility around the actual output level. Given that both measures are based on the BN decomposition, a large part of these differences reflect differences in the measures of the *persistence* of shocks to output obtained from the models. In a univariate model, persistence relates to the infinite horizon effect of a shock to output where the shock causes output to rise by 1% on impact.[28] In our bivariate context, the persistence relates to the size of the infinite horizon impact on actual output of a *system-wide* shock to actual and expected output that causes *actual* output to increase by one percent on impact. The persistence measures for the models in Table 3 are denoted $P_{\overline{y}^P}$.[29] Comparison of persistence measures for the univariate and bivariate models show that the measure is larger in the bivariate model than the univariate model for all five countries. It appears that the additional dynamic sophistication of the bivariate model (including the effect of the feedbacks between actual and expected outputs captured by the error correction term) allows for a more prolonged effect of shocks and one in which the accumulation of effects over time is larger. In terms of the measures of output trends, this is reflected by

[26]No attempt has been made to adjust the models for the effect of once-and-for-all events (such as price shocks or national strikes) which result in outliers and which help explain some of the statistically significant diagnostic statistics in Table 3.

[27]The underlying univariate models are obtained following the same specification search procedure as that used in Table 3. Simple AR(2) specifications in actual output growth were adequate for France and Germany, while AR(1) specifications were adequate for Italy, the Netherlands and UK.

[28]If the univariate model is written in its MA form, $\Delta y_t = b + C(L)\epsilon_t$, then persistence is given by $C(1)$. In the univariate models underlying $\overline{y}_t$, the estimated persistence measures are 0.987 (0.140), 1.014 (0.143), 0.836 (0.064), 0.758 (0.056), and 1.749 (0.298) for France, Germany, Italy, the Netherlands and UK respectively (with standard errors in parentheses).

[29]For further details of measures of persistence in the context of a multivariate framework, see Pesaran, Pierse and Lee (1993).

the volatile movements of the trend series derived from the bivariate models around the actual output level.

Figures 2a-2e plot the alternative trend measures $\overline{y}_t^S$ and $\overline{y}_t^P$ for each of the five countries. In the event, these two measures coincide for France, Italy and the UK based on the regression models of Table 3. This is a consequence of the absence of an error correction term in the Δy_{t+1}^* regressions in these countries which ensures that the long run properties of the output series are independent of the ε_t.[30] Hence, k_3, the coefficient on ε_t in $C(1)u_t$ from (2.12), is zero and, given that $\overline{y}_t^S$ differs from $\overline{y}_t^P$ by the magnitude $-k_3\upsilon_t$, the two measures are the same in these countries. In Germany and the Netherlands, where the error correction term shows significantly in the Δy_{t+1}^* regression, the two alternative trends again move in broadly similar ways relative to the actual output series, but the $\overline{y}_t^S$ series shows more volatility than that of $\overline{y}_t^P$.

To summarise the relative smoothness of the three series, Table 4 gives the values for R for each of the trend output measures in the five countries, where this measures the ratio of the sample variance in the change in cycle to the sample variance in the change in trend output, and provides an indication of the smoothness of the different trend measures. According to the discussion in Section 2.2, we expect $\text{var}(\Delta\overline{y}_t)$ to exceed $\text{var}(\Delta\overline{y}_t^P)$ and, in turn, we expect $\text{var}(\Delta\overline{y}_t^P)$ to be greater than $\text{var}(\Delta\overline{y}_t^S)$. This is shown to be the case in Table 4. Here, the calculated R statistics based on $\overline{y}_t^S$ and $\overline{y}_t^P$ are broadly comparable with each other and are both substantially larger than those based on $\overline{y}_t$, illustrating the relative smoothness of the trend measures obtained from the bivariate model.

6 Discussion

The empirical work of the previous sections provides some important empirical insights when considering growth dynamics. Significantly, on the basis of the expected output series derived from Business Surveys, we found no evidence with which to reject rationality

[30] In the absence of an error correction term in the second line of (2.4), Δy_{t+1}^* is determined by η_t and lagged values of Δy_t and Δy_{t+1}^* only. Noting from (2.9) that $\Delta y_t = \Delta y_t^* + \Delta\varepsilon_t$, this means that ε_t influences Δy_{t+1}^* in differences only (and not in levels). In these circumstances, the output series evolve independently of the ε_t in the long run.

in expectation formation in any of the five countries considered. This immediately suggests that the inclusion of direct measures of output expectations will enhance any statistical analysis of actual output data. Of course, the inclusion of additional explanatory variables will always improve the fit of a model explaining output, but the use of direct measures of expected outputs achieves this in the most parsimonious way (and indeed the evidence on rationality implies that the inclusion of further variables would have no further explanatory power in a regression of actual output). As it turns out, our time series analysis of the output data of our five European economies demonstrated that there are indeed important feedbacks between actual and expected output series (both contemporaneous and lagged). The estimated bivariate model for each economy is able to capture sophisticated dynamic responses to innovations which could not be accommodated in any simple univariate model of output growth and we would argue therefore that, on statistical grounds alone, there is a strong case for the use of bivariate models of the sort described in the paper when investigating growth dynamics.

The primary purpose of this paper, however, is to suggest some alternative measures of trend output based on the VAR model of actual and expected output series. The VAR modelling framework that is described provides a framework within which output growth can be analysed without relying on any (possibly contentious) behavioural economic assumptions. Innovations in the model are interpreted in terms of news of different types and the two proposed measures of trend show agents' perceptions of the infinite horizon level of output according to their use of this news. The proposed trends have the same statistical properties of any trend/cycle decompositions obtained using the BN decomposition. But the discussion of Section 2 notes that the use of expectations data in the bivariate VAR provides trend measures which are smooth (but not arbitrarily so). Further, the $\overline{y}_t^S$ measure represents the unique decomposition into orthogonal permanent and transitory components. Hence, the trends have a reasonable motivation on purely statistical grounds. Moreover, as illustrated in Section 3, the trends are readily interpretable as measures of potential output in the context of a simple stochastic growth model. Given the widespread use of this concept in applied macroeconomics, the proposed measures could have considerable practical use.

7 Appendix

Solving (3.13)-(3.15) to eliminate the unobservable terms y_t^n and y_t^p, we obtain the following representation for output growth:

$$\begin{aligned} y_t - y_{t-1} &= (1-\lambda)g + \lambda(y_{t-1} - y_{t-2}) \\ &\quad + z_{1,t} + (1+\lambda-\beta)z_{1,t-1} + (\lambda - (1+\lambda)\beta)z_{1,t-2} + \lambda\beta z_{1,t-3} + (1-\lambda)z_{2t}. \end{aligned} \tag{7.18}$$

Moving (7.18) forward one period and taking expectations under the REH provides the associated expression for expected output growth. Taken together, these provide a VARMA(1,3) model for $(\Delta y_t, \Delta_t y_{t+1}^*)$ as follows:

$$\begin{bmatrix} \Delta y_t \\ \Delta_t y_{t+1}^* \end{bmatrix} = \begin{bmatrix} (1-\lambda)g \\ (1-\lambda)(1+\lambda)g \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ \lambda^2 & 0 \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \Delta_{t-1} y_t^* \end{bmatrix} + \mathbf{u}_t + \mathbf{D}_1\mathbf{u}_{t-1} + \mathbf{D}_2\mathbf{u}_{t-2} + \mathbf{D}_3\mathbf{u}_{t-3}$$

where

$$\mathbf{D}_1 = \begin{bmatrix} -(1+\lambda) & 1 \\ \frac{-(\lambda^2+\beta)(1+\lambda)}{1+\lambda-\beta} & \frac{\lambda^2+\beta}{1+\lambda-\beta} \end{bmatrix}, \quad \mathbf{D}_2 = \begin{bmatrix} \frac{(\lambda-\beta-\lambda\beta)(1+\lambda)}{1+\lambda-\beta} & \frac{-(\lambda-\beta+\lambda\beta)}{1+\lambda-\beta} \\ \frac{\lambda^2(1-\beta)(1+\lambda)}{1+\lambda-\beta} & \frac{-\lambda^2(1-\beta)}{1+\lambda-\beta} \end{bmatrix} \quad \mathbf{D}_3 = \begin{bmatrix} \frac{\lambda\beta(1+\lambda)}{1+\lambda-\beta} & \frac{-\lambda\beta}{1+\lambda-\beta} \\ \frac{\lambda^2\beta(1+\lambda)}{1+\lambda-\beta} & \frac{-\lambda^2\beta}{1+\lambda-\beta} \end{bmatrix},$$

and

$$\mathbf{u}_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} 1 & 1-\lambda \\ \beta & (1-\lambda)(1+\lambda) \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix}. \tag{7.19}$$

Through recursive substitution, we can obtain a MA representation as given in (2.5) where

$$\mathbf{C}_0 = \mathbf{I}_2, \quad \mathbf{C}_1 = \begin{bmatrix} \lambda & 0 \\ \lambda^2 & 0 \end{bmatrix} + \mathbf{D}_1, \quad \mathbf{C}_2 = \begin{bmatrix} \lambda & 0 \\ \lambda^2 & 0 \end{bmatrix} \mathbf{C}_1 + \mathbf{D}_2, \quad \mathbf{C}_3 = \begin{bmatrix} \lambda & 0 \\ \lambda^2 & 0 \end{bmatrix} \mathbf{C}_2 + \mathbf{D}_3,$$

and

$$\mathbf{C}_j = \begin{bmatrix} \lambda & 0 \\ \lambda^2 & 0 \end{bmatrix} \mathbf{C}_{j-1} \text{ for } j \geq 4.$$

Hence, we have

$$\begin{aligned} \mathbf{C}(1)\mathbf{u}_t &= \begin{bmatrix} \frac{-\beta}{(1+\lambda-\beta)(1-\lambda)} & \frac{1}{(1+\lambda-\beta)(1-\lambda)} \\ \frac{-\beta}{(1+\lambda-\beta)(1-\lambda)} & \frac{1}{(1+\lambda-\beta)(1-\lambda)} \end{bmatrix} \begin{bmatrix} 1 & 1-\lambda \\ \beta & (1-\lambda)(1+\lambda) \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} \\ &= \begin{bmatrix} z_{2t} \\ z_{2t} \end{bmatrix}, \end{aligned}$$

as discussed in the text.

The orthogonalisation discussed in (2.7) defines v_t by $v_t = \varepsilon_t - \rho\eta_t$, where $\rho = \frac{cov(\varepsilon_t,\ \eta_t)}{var(\eta_t)}$. With $[\varepsilon_t,\ \eta_t]'$ defined in (7.19), and with the value of k_3 defined in (7.20), we find that the term distinguishing $\overline{y}_t^P$ and $\overline{y}_t^S$ is of the form

$$k_3 v_t = (1-\gamma)z_{1t} + \gamma z_{2t},$$

where $\gamma = \frac{-\beta var(\eta_t)+\beta(1+\lambda)cov(\varepsilon_t,\ \eta_t)}{(1+\lambda-\beta)var(\eta_t)}$. Noting that

$$\begin{aligned} \Delta\overline{y}_t^S &= \Delta\overline{y}_t^P - k_3 v_t \\ &= z_{2t} - k_3 v_t, \end{aligned}$$

we find that

$$\Delta\overline{y}_t^S = \frac{(1-\gamma)}{(1+\lambda)\,(1-\lambda)}\eta_t. \qquad (7.20)$$

But, writing ε_t, in terms of η_t by eliminating z_{1t} using (7.19), we observe that

$$\beta cov(\varepsilon_t, \eta_t) - var(\eta_t) = -\,(1-\lambda)\,(1+\lambda-\beta)\,cov(z_{2t}, \eta_t),$$

so that (7.20) can be written in terms of $cov(z_{2t}, \eta_t)$, rather than $cov(\varepsilon_t, \eta_t)$, as follows:

$$\Delta\overline{y}_t^S = \frac{cov(z_{2t}, \eta_t)}{var(\eta_t)}\eta_t = E\left[z_{2t} \mid \eta_t\right].$$

8 Data Appendix

The expectations data for France, Germany, Italy and the Netherland has been obtained from two consecutive publications of the Directorate General for Economic and Financial Affairs of the Commission of the European Communities; namely, the **Report of the Results of the Business Survey carried out among Heads of Enterprises in the Community**, 1967-1975, and **Results of the Business Survey carried out among Managements in the Community**, 1976-1998. The survey question on production expectations has been published since 1967; the realised output survey data prior to 1980 was provided directly by the Commission of the European Communities. The expectations data for the UK has been taken from successive issues of the CBI's **Survey of Industrial Trends**. This Survey has been carried out since 1958, and published quarterly since 1972. However, the responses to the output volume question have been published since 1975q3; prior to that date, the question was phrased in terms of output *values* as opposed to output *volumes*.

The index of production for the Total Manufacturing industry for each country (except the UK) has been taken from successive issues of two consecutive OECD publications; **Industrial Production, Quarterly Supplement to Main Economic Indicators**, 1967-1978, and **Indicators of Industrial Activity**, 1979-1998. The output data for the UK has been taken from various issues of the CSO's **Monthly Digest of Statistics**. Seasonally-adjusted monthly output indices are used to calculate output growth rates, measured as the percentage change in the output index from its level in an earlier month where the period is chosen so that the time horizon matches that of the question posed in the corresponding Survey. An adjustment has been made to the data point in Germany for May 1984 when industrial disputes in Heavy Manufacturing sector lead to a large and unprecedented fall in the level of output. To adjust for this, we replaced the original observation by an average of the index of production for April and June.

Finally, the discount rates and exchange rates (defined as the average exchange rate of the country currency to the US Dollar) are obtained from DATASTREAM at monthly intervals, with growth rates being calculated as above.

Table 1: Summary Statistics relating to actual and derived expected output growth series

Country (Sample)	ADF		$E(\Delta y_t)$ (%)	$\varepsilon_t = (y_t - y_t^e)$					$\varepsilon_t = (y_t - y_t^*)$					d1	d2	d3
	y_t	Δy_t		r^e	ADF	Skew	SC	H	r^*	ADF	Skew	SC	H			
France (69q1-98q1)	-3.34	-12.05†	0.469	0.31	-5.15†	0.14	1.04	6.59†	0.52	-6.97†	-0.24	-0.27	11.09†	98.82†	193.33†	1.59
Germany (68q1-98q1)	-3.21	-11.93†	0.598	0.25	-5.67†	0.26	0.77	6.61†	0.33	-7.10†	-0.08	-0.33	8.04†	60.04†	91.65†	11.98
Italy[1] (68q1-98q1)	-2.97	-5.58†	0.667	0.18	-5.06†	0.16	-0.08	5.92†	0.40	-6.40†	0.28	-0.16	5.92†	68.28†	109.43†	12.18
Neth. (72q1-98q2)	-2.99	-6.87†	0.604	0.19	-4.48†	0.14	-0.14	5.31†	0.36	-4.42†	0.40	-0.06	4.83†	70.65†	11.85	8.71
UK (75q3-98q2)	-1.85	-6.11†	0.303	0.45	-4.42†	-1.49	2.77†	10.47†	0.59	-5.02†	-1.27	0.59	14.53†	28.26†	116.72†	1.43

Notes: 'ADF y_t and Δy_t', denotes the Augmented Dickey-Fuller statistic testing the null that there is a unit root in y_t and Δy_t, respectively, where an intercept, a time trend, and lagged dependent variables are selected according to the Akaike and Schwarz-Bayesian criteria. '*E(.)*' indicates the sample mean of a variable. 'r^+', where + = *e* or *, indicates the contemporaneous correlation between $(y_t - y_{t-1})$ and $(y_t^+ - y_{t-1})$. The ADF statistic is of tests of a unit root in ε_t^+, defined by $\varepsilon_t^+ = (y_t^+ - y_t^e)$, where the underlying regressions include an intercept and two lags in the dependent variable. 'Skew' is the coefficient of skewness estimated on the ε_t^+, 'SC' are t-values on the estimated values of ρ in the AR1 specification of $\varepsilon_t^+ = \mu + \rho\varepsilon_{t-1} + \xi_t$, and 'H' are t-values on the estimated values of ρ in the regression $\varepsilon_t^{+2} = \mu + \rho(y_t - y_{t-1})^2 + \xi_t$. Statistics denoted 'd1', 'd2' and 'd3' relate to the tests of the orthogonality of expectational errors to information known when expectations are formed as described in the text.

[1]This statistic takes account of any seasonality that may exist in the derived series for Italy - see text for details.

'†' denotes significance at the 5% level.

Table 2: Importance of Actual and Expected Output Growth Series

Country (Sample)	F_1	F_2
France (70q1-98q1)	9.780†	1.178
Germany (69q1-98q1)	5.179†	4.461†
Italy[1] (69q1-98q1)	8.187†	6.224†
Netherlands (73q1-98q1)	4.396†	0.028
UK (76q3-98q2)	7.373†	0.550

Notes: Statistics denoted F_i , *i=1,2,* relate to the following regressions conducted separately for each country:

$$y_t - y_{t-1} = a_0 + \sum_{j=1}^{4} a_j (y_{t-j} - y_{t-j-1}) + \sum_{k=0}^{3} \alpha_k (y^*_{t-k} - y_{t-k-1}) + u_{1t} \qquad (A)$$

$$y^*_{t+1} - y_t = b_0 + \sum_{k=0}^{3} b_k (y^*_{t-k} - y_{t-k-1}) + \sum_{j=1}^{4} \beta_j (y_{t-j} - y_{t-j-1}) + u_{2t} \ . \qquad (B)$$

F_1 is the F-statistic testing the null hypothesis that α_k , *k=0,1,2,3* are jointly equal to zero in regression *(A)*, and F_2 is the F-statistic testing the null hypothesis that β_j , *j=1,2,3,4* are jointly equal to zero in regression *(B)*.

[1] This statistic takes account of any seasonality that may exist in the expected growth series for Italy - see text for details.

'$\dagger$' denotes significance at the 5% level.

Table 3: Bivariate Model; Dependent Variable: $y^*_{t+1} - y^*_t$

	France (70q1-98q1)	Germany (69q1-98q1)	Italy[1] (69q1-98q1)	Netherlands (73q1-98q1)	UK (76q3-98q2)
constant	0.004 (0.003)	0.037 (0.003)	0.018 (0.004)	0.007 (0.003)	0.001 (0.003)
$y_{t-1} - y_{t-2}$	0.382 (0.108)	0.015 (0.095)	0.212 (0.094)	-	0.317 (0.118)
$y^*_t - y^*_{t-1}$	-0.365 (0.108)	-	-0.006 (0.086)	-0.180 (0.105)	-
$y_{t-2} - y_{t-3}$	0.112 (0.097)	0.347 (0.010)	-	-	-
$y^*_{t-1} - y^*_{t-2}$	-	-	-0.198 (0.081)	-	-
$y^*_t - y_{t-1}$	-	-0.208 (0.128)	-	-0.255 (0.137)	-
n	111	115	115	99	86
R^2	0.132	0.118	0.374	0.096	0.081
S.E	0.026	0.027	0.033	0.030	0.028
RSS	0.069	0.077	0.114	0.085	0.066
LLF	246.83	251.91	229.43	203.77	181.24
SC	0.108	0.477	0.775	0.099	0.356
FF	0.060	0.022	2.118	2.625	0.385
N	5.693	4.048	3.590	45.787	33.356
H	1.352	1.880	0.988	5.553	1.493
LM	1.432	0.512	2.145	0.608	5.192
ρ	0.699	0.778	0.766	0.655	0.685
$P_{\bar{y}^P}$	1.266 (0.207)	1.285 (0.222)	0.864 (0.104)	0.771 (0.056)	1.951 (0.336)
[k_3, k_4]	[0.000, 1.149] (0.000, 0.188)	[0.246, 1.181] (0.123, 0.260)	[0.000, 1.006] (0.000, 0.121)	[0.178, 0.697] (0.084, 0.068)	[0.000, 1.464] (0.000, 0.252)

Notes: Regression results provide estimates of the parameters (and errors) of the second row of the bivariate VAR model of (2.4). The (entirely) restricted parameters of the first row are provided by the REH restriction described in (2.9). The reported regressions are the outcome of the specification search described in the text. The sample size is denoted by n; R^2 is the Goodness of Fit statistic; S.E. denotes the standard error of the regression; RSS is the Residual Sums of Squares; LLF represents the maximum value of the log-likelihood function; SC gives an LM test of residual serial correlation; FF is a functional form statistic based on the Ramsey RESET test; N denotes a normality test based on a test of skewness and kurtosis of residuals; H is a test statistic for heteroskedasticity based on a regression of squared residuals on squared fitted values; and LM is a (chi-squared) statistic jointly testing the exclusion restrictions in the table.

ρ is the coefficient in expression (2.8) and is defined by the regression $\varepsilon_t = \rho\eta_t + \upsilon_t$. [$k_3$, k_4] refer to the parameters of **C**(1) of expression (2.6). $P_{\bar{y}^P}$ denotes the size of the long-run impact on actual output of a system-wide shock that causes actual output to rise by one percent.

[1] The regression results for Italy take account of any seasonality that may exist in the expected growth series for Italy - see text for details.

Standard errors are in parentheses.

Table 4: Ratio of Variances for Alternative Measures of Trend Output

	R=var(Δcycle)/var(Δtrend)		
	$\bar{y}_t$ (1)	$\bar{y}_t^P$ (2)	$\bar{y}_t^S$ (3)
France	0.156	0.821	0.821
Germany	0.168	0.620	0.658
Italy	0.307	0.948	0.948
Netherlands	0.538	0.931	1.031
UK	0.512	0.631	0.631

Notes: The statistics relate to the ratio of the variance in the change in cycle to the variance in the change in trend. The trend is measured by one of $\bar{y}_t$, $\bar{y}_t^P$ and $\bar{y}_t^S$ as defined in the text, and the cycle is the deviation of the trend from y_t.

Figure 1a: Forecast-based Measures of Trend Output for France: 1971q1-97q4

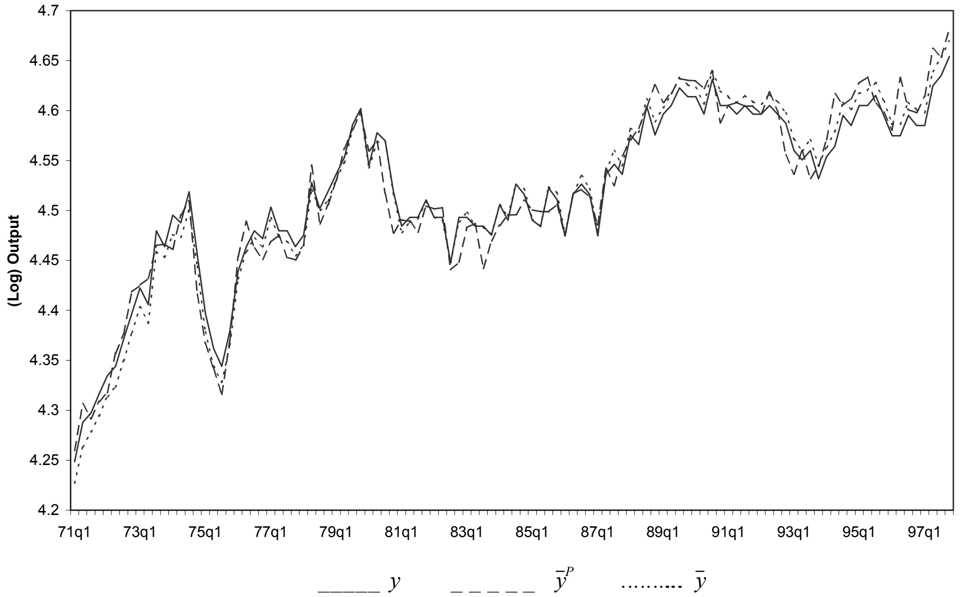


Figure 1b: Forecast-based Measure of Trend Output for Germany: 1970q1-97q4

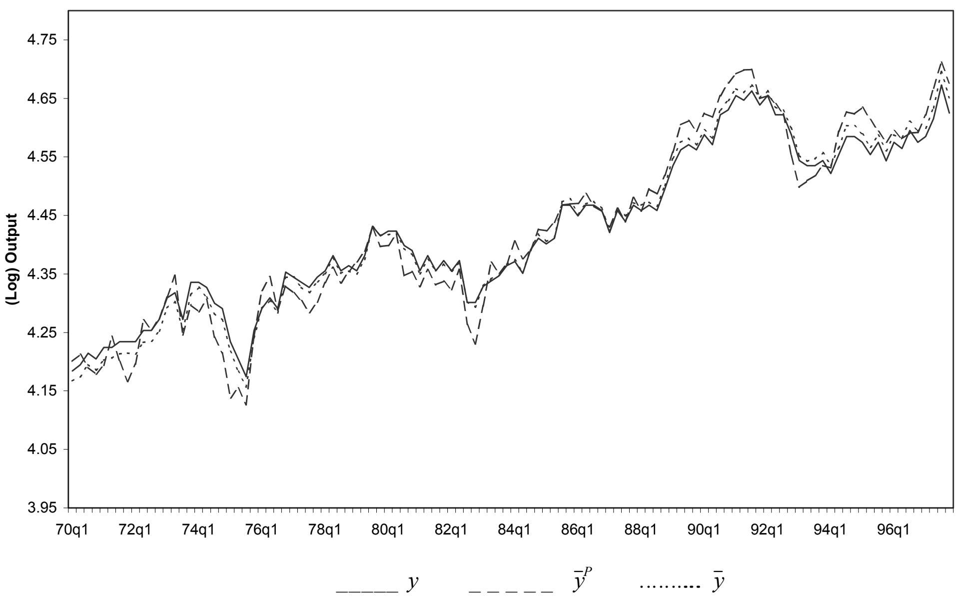


Figure 1c: Forecast-based Measures of Trend Output for Italy: 1970q1-97q4

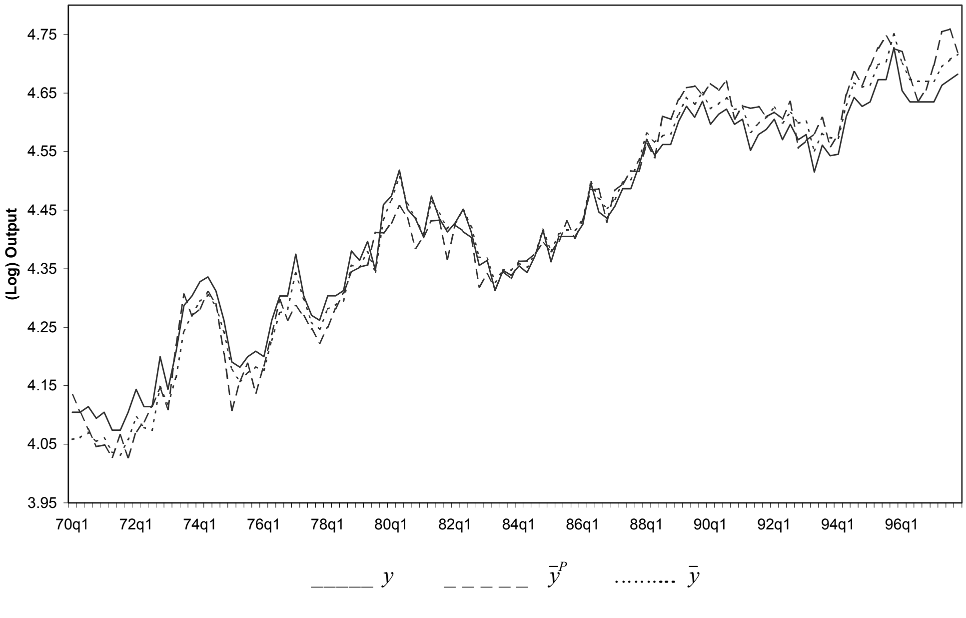
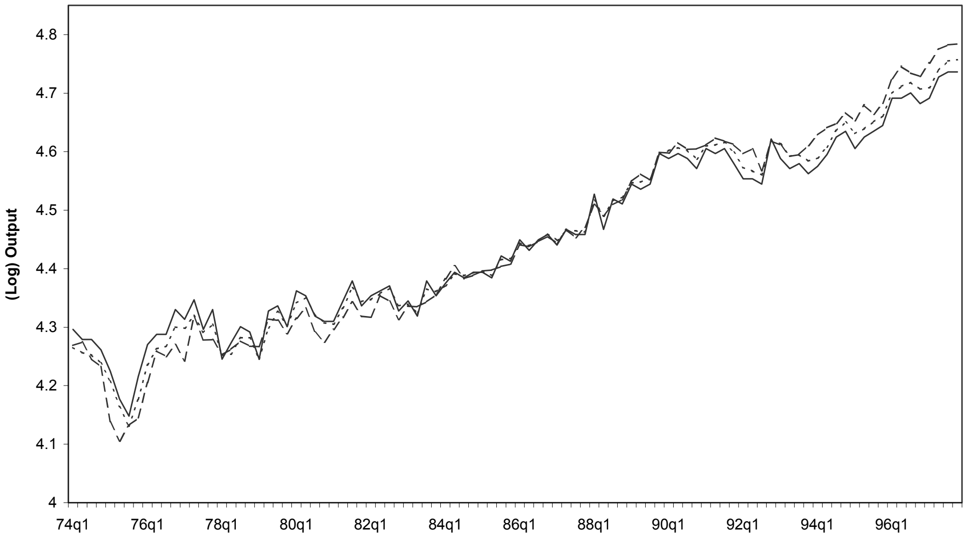
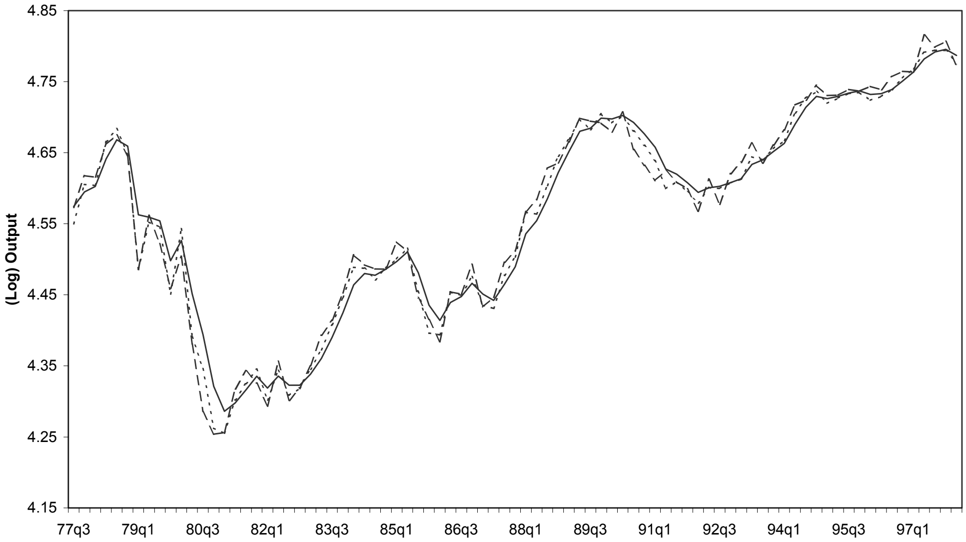


Figure 1d: Forecast-based Measures of Trend Output for the Netherlands: 1974q1-97q4



______ y _ _ _ _ _ $\bar{y}^P$ ………. $\bar{y}$

Figure 1e: Forecast-based Measures of Trend Output for the UK: 1977q3-98q1

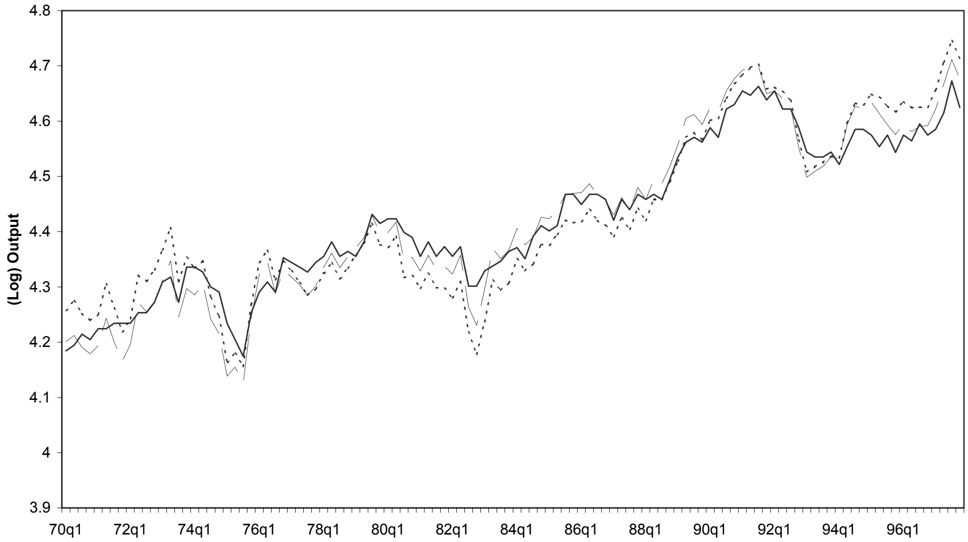


______ y _ _ _ _ _ $\bar{y}^P$ ………. $\bar{y}$

Figure 2a: Alternative Measures of Trend Output for France: 1971q1-97q4

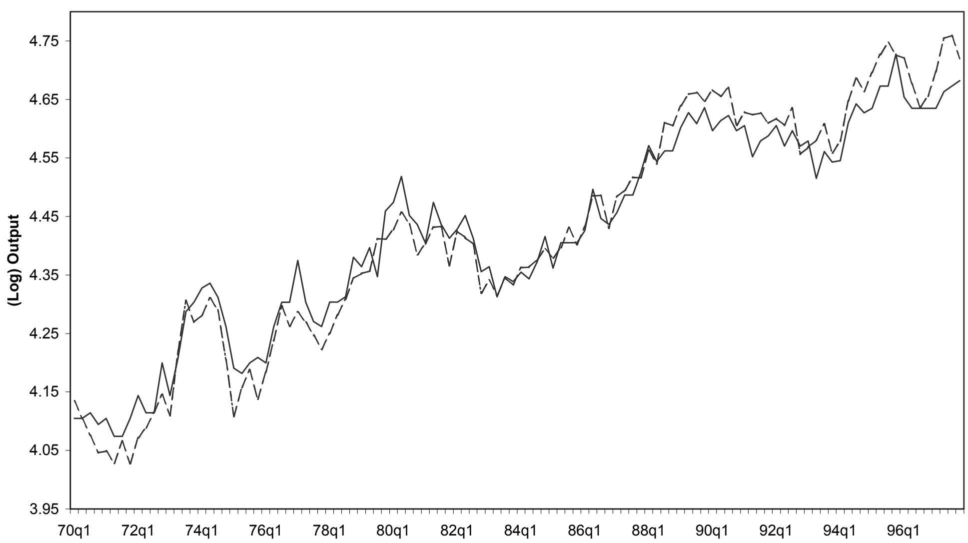
______ y _ _ _ _ _ $\bar{y}^P$ ……... $\bar{y}^S$

Figure 2b: Alternative Measures of Trend Output for Germany: 1970q1-97q4



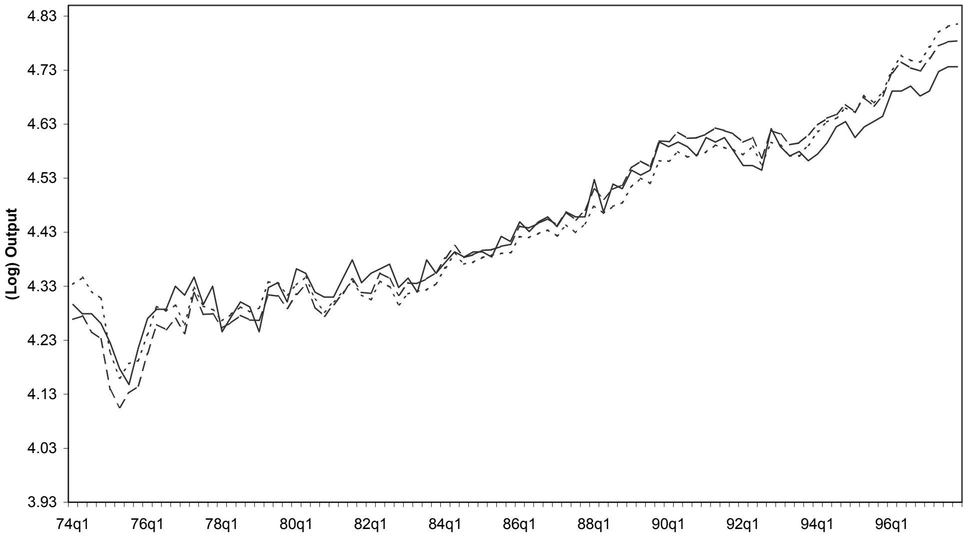
______ y _ _ _ _ _ $\bar{y}^P$ ……... $\bar{y}^S$

Figure 2c: Alternative Measures of Trend Output for Italy: 1970q1-97q4



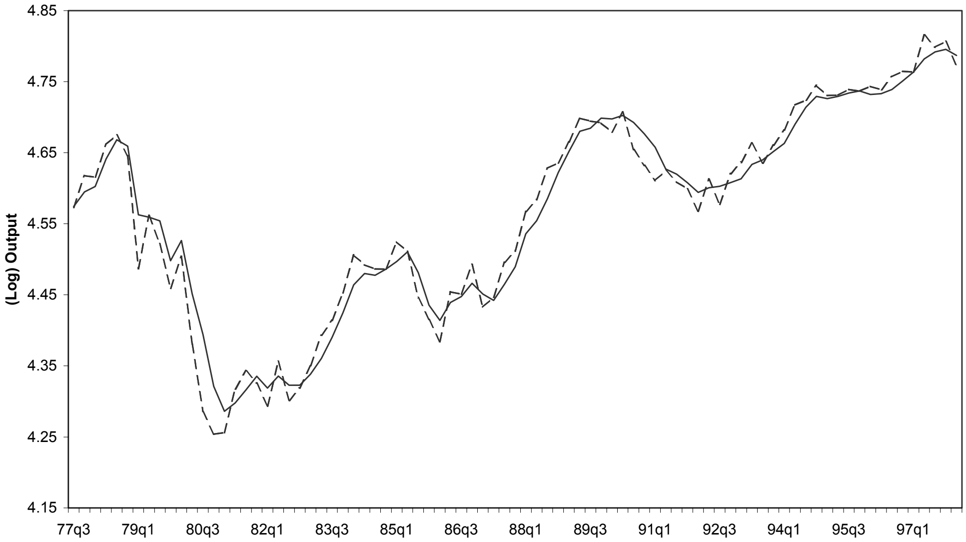
______ y _ _ _ _ _ $\bar{y}^P$ ……... $\bar{y}^S$

Figure 2d: Alternative Measures of Trend Output for the Netherlands: 1974q1-97q4



_____ y _ _ _ _ _ $\bar{y}^P$ ……… $\bar{y}^S$

Figure 2e: Alternative Measures of Trend Output for the UK: 1977q3-98q1



_____ y _ _ _ _ _ $\bar{y}^P$ ……… $\bar{y}^S$

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