Monetary policy rules
to preclude booms and busts

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Abstract

This paper examines the issue of the design of monetary policy rules within the canonical New Keynesian model of a small open economy, with the closed economy nested as a special case. Unlike the existing literature, we argue that in order to ensure the implementation of the optimal equilibrium, the monetary policy rule chosen should not only rule out all convergent equilibria other than the optimal one, but also preclude all divergent equilibria which may develop otherwise as the private agents would then expect the central bank to eventually act as a "stabilizer of last resort". We characterize analytically the set of such adequate monetary policy rules, in a flexible exchange rate regime (depending on whether a commitment technology is available or not) and in a fixed exchange rate regime. We show that these rules are necessarily forward-looking in a well-defined manner, so as to insulate the current inflation rate from the private agents' sunspot-prone expectations about the future situation. This result is robust to natural extensions to the canonical New Keynesian framework.

Keywords: canonical New Keynesian model, optimal monetary policy, multiple equilibria, time inconsistency, flexible exchange rate regime, fixed exchange rate regime.

JEL classification system: E31, E52, E58, E61, F33.

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1 Introduction

As stressed by McCallum (1999a), who relates the evolution of monetary policy theory and practice since the early 70's, New Keynesian economics has recently come out as the most celebrated framework for monetary policy analysis. Within this framework, much attention has been paid in particular to the issue of how to design a monetary policy rule so as to avoid (undesirable) multiple equilibria. This issue is arguably of practical importance: according to Clarida, Galí and Gertler (2000) for instance, the American macroeconomic volatility during the pre-Volcker era may be explained by the fact that the monetary policy rule followed by the central bank was compatible with multiple equilibria and hence made way to endogenous fluctuations, born from self-fulfilling expectations.

Our paper aims at giving a new insight into the design of optimal monetary policy rules. In our opinion, the definition of multiple equilibria adopted by the existing literature is too restrictive, as only convergent equilibria are considered. We argue that adequate monetary policy rules should rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria. This correction seems all the more welcome than most post-war American recessions, according to a widespread point of view[1], are due to a monetary policy tightening putting an end to a period of increasing inflation rate. We look for monetary policy rules to avoid the development of such divergent equilibria, in other words for monetary policy rules to preclude booms and busts.

To make our point, we resort to what we call the canonical New Keynesian model, that is to say the New Keynesian model reduced to its simplest form, which has received much attention in the past few years. Its closed economy version is composed of an IS equation, a Phillips curve[2] and a central bank's loss function. Its small open economy version has a very similar structure, as it is composed of the same (in reduced form terms) IS equation, Phillips curve and loss function, to which are added the uncovered interest rate parity and the law of one price relationships.

This intertemporal general equilibrium model manages to combine a highly tractable reduced form with rather sound microfoundations, as the IS equation and the Phillips curve are derived from the optimal behaviour of the representative household and firm respectively, and the central bank's loss function from the representative household's utility function. At the end of the paper, we shortly point to the fact that natural extensions to this canonical framework, making the model more realistic but resting on more or less arbitrary assumptions, would not alter our results qualitatively speaking.

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[1] Paul Samuelson, quoted by *The Economist* ("What a peculiar cycle", March 8, 2001), expresses this point of view in the following way: American recessions are stamped "made in Washington by the Federal Reserve". Alternatively, as Rudiger Dornbusch puts it, again quoted by *The Economist* ("Of shocks and horrors", September 28, 2002), "none of the postwar expansions died of old age, there were all murdered by the Fed".

[2] The New Keynesian model differs from its New Classical counterpart in particular in that its Phillips curve involves the present anticipation of the future inflation rate, due to a price-setting specification *à la* Calvo (1983), and not the past anticipation of the present inflation rate (Lucas' supply curve).

We follow a two-step approach. First, we fully derive the model's analytical results, which describe the optimal macroeconomic adjustment process to demand and cost-push shocks, for a small open economy (with the closed economy nested as a special case) in four alternative configurations: a flexible exchange rate regime without commitment (FL1), a flexible exchange rate regime with commitment (FL2), a(n *ex post*) fixed exchange rate regime with commitment (FI1) and an irrevocably (*ex ante*) fixed exchange rate regime with commitment (FI2). In so doing, we fill a gap in the literature, as these analytical results are absent from most of the existing studies, which are satisfied with calibrating and simulating the model, and incomplete in the few other studies.

Second, we characterize the set of monetary policy rules ensuring the implementation of this optimal adjustment process, in each of the relevant cases considered (FL1, FL2 and FI1), while existing studies attempt to do it only in the FL2 case. Unlike the existing literature, we look for stabilizing feedback rules which rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria. The vast majority of existing studies focus on the dynamics of macroeconomic variables conditionally on their convergence, disregarding the possibility of their divergence, and thus of all the (usually simple) rules examined in the literature, none ensures in our opinion the implementation of the unique desired equilibrium.

We argue that ruling out divergent equilibria is a highly desirable feature for a monetary policy rule, even though our log-linear approximation of the model enables us to consider only small macroeconomic fluctuations around the steady state. Suppose indeed that the central bank adopts a monetary policy rule which does not preclude the development of divergent equilibria. Then divergent paths may start in the neighbourhood of the steady state, so that we can at least appreciate their initial development before losing track of them. Moreover, the central bank will eventually act as a "stabilizer of last resort", that is to say sooner or later abandon its monetary policy rule in order to bring such a divergent path back to the neighbourhood of the stationary state.

This monetary policy reaction will take place even in the presence of a commitment technology, by which the central bank has committed itself to sticking to its rule, provided that we (in our opinion relevantly) deal with an endogenous commitment technology (coming from reputation effects for instance) rather than an exogenous one, so that the central bank still weighs the *pro* and *contra* before deciding whether to stick to its rule. In the end, what we call a divergent path may therefore actually remain constantly in the neighbourhood of the steady state, thus violating neither the transversality condition nor (in the case of a small open economy) the long-run Purchasing Power Parity (PPP) condition. Of course, such paths prove undesirable as they amount to booms and busts which but bring additional noise into the system.

It turns out that we are able to design monetary policy rules which preclude the development of divergent equilibria. Of course, this additional requirement substantially reduce the set of adequate monetary policy rules. We show in particular that these rules are necessarily forward-looking in a well-defined manner, so as to insulate the current inflation rate from the private agents' sunspot-prone

expectations about the future situation.

The remaining of the article is organized as follows: section 2 presents both the closed economy and the small open economy versions of the canonical New Keynesian model. Section 3 determines analytically the optimal equilibrium, in each of the cases considered (FL1, FL2, FI1 and FI2). Section 4 shows how a monetary policy rule can be chosen which ensures the implementation of this optimal equilibrium. Section 5 characterizes the set of such adequate monetary policy rules. Section 6 concludes, and section 7 provides a technical appendix.

2 Presentation of the model

This section presents the canonical New Keynesian model of a small open economy, with the closed economy nested as a special case.

The canonical New Keynesian model of a closed economy has been used notably by Bernanke and Woodford (1997), Clarida, Galí and Gertler (1999, 2000), Woodford (2003). Other works, listed by Woodford (2003, chap. 7), adopt a very similar, if not identical framework. McCallum (1999a) assesses and discusses the recent popularity of this model.

The canonical New Keynesian model of a small economy has been laid out by Clarida, Galí and Gertler (2001), as well as Galí and Monacelli (2002)[3] from whom we borrow our presentation. (A few other works use slightly different versions of this model.)

2.1 Main assumptions

We focus here on the main assumptions of the model, essentially in order to introduce the parameters featuring in the closed form, and refer the reader to Galí and Monacelli (2002) for a more detailed presentation.

The representative household in the small open economy maximizes the following utility at date t:

$$U_t \equiv E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[\frac{C_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{N_{t+k}^{1+\varphi} - 1}{1+\varphi} \right] \right\},$$

where N_{t+k} represents hours of labour and C_{t+k} a composite consumption index at date $t+k$, while E_t stands for the expectation operator conditionally on the information available at date t. We assume $0 < \beta < 1$, $\sigma > 0$ and $\varphi > 0$.

Note that money does not enter the utility function and will be disregarded thereafter. Woodford (2003, chap. 2) gives three alternative justifications for this Wicksellian specification. First, we may deal with a genuinely cashless economy, with the implication that money (the unit of account) must earn the same rate of return as other riskless assets. Second, there may be some

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[3] According to McCallum and Nelson (2000, p. 11), "the GM [Galí and Monacelli (2002)] model has a strong claim to be viewed as a canonical NOEM [New Open Economy Macroeconomics] model, owing to its elegance and tractability".

monetary frictions, so that money does actually enter the utility function, but if preferences are additively separable between consumption and real balances, then money is residually determined by an LM equation and plays no role in what follows, except for its direct contribution to the utility level which we assume is negligible. Third, even if it enters the utility function in a non-separable way, money will not matter in the case of what Woodford (2003, chap. 2, p. 60) calls a "cashless limiting economy".

The composite consumption index is defined by:

$$C_{t+k} \equiv \left[\alpha^{\frac{1}{\mu}} C_{H,t+k}^{\frac{\mu-1}{\mu}} + (1-\alpha)^{\frac{1}{\mu}} C_{F,t+k}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}},$$

where $C_{H,t+k}$ and $C_{F,t+k}$ are CES indices of domestic and foreign goods consumption:

$$C_{H,t+k} \equiv \left[\int_0^1 C_{H,t+k}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{F,t+k} \equiv \left[\int_0^1 C_{F,t+k}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Parameter μ measures the elasticity of substitution between domestic and foreign goods, whereas parameter ε measures the elasticity of substitution between the varieties of the differentiated good produced in a given country. We assume $\mu > 0$ and $\varepsilon > 1$.

The utility maximization is subject to a sequence of intertemporal budget constraints of the form

$$\int_0^1 \left[P_{H,t+k}(i) C_{H,t+k}(i) + P_{F,t+k}(i) C_{F,t+k}(i) \right] di +$$
$$E_t \left\{ Q_{t+k,t+k+1} D_{t+k+1} \right\} \leq D_{t+k} + W_{t+k} N_{t+k} + T_{t+k}$$

for $k = 0, 1, 2...$, where $P_{H,t+k}(i)$ and $P_{F,t+k}(i)$ denote the prices of domestic and foreign good i respectively, W_{t+k} the nominal wage and T_{t+k} lump-sum transfers or taxes at date $t + k$, while D_{t+k+1} the nominal payoff at date $t + k + 1$ of the portfolio held at the end of period $t + k$ (which includes shares in firms). All the previous variables are expressed in units of domestic currency. $Q_{t+k,t+k+1}$ represents the stochastic discount factor for nominal payoffs. We assume that households have access to a complete set of contingent claims, traded internationally.

Each firm produces a variety i of the differentiated good with a linear technology described by the following production function:

$$Y_t(i) \equiv A_t N_t(i),$$

with $\ln A_t = \overline{a} + \varepsilon_t^a$, where $\overline{a} \neq 0$ and where ε_t^a is an exogenous technology shock with zero mean. We thus disregard investment dynamics: private spending has no effect upon the economy's productive capacity, as we deal with

non-durable consumption expenditure. Woodford (2003, chap. 4) finds that relaxing this assumption leads to (to some extent) qualitatively similar results.

We assume the existence of an employment subsidy, whose role is to offset the monopolistic distortions at the steady state. Firms set prices in a staggered fashion, *à la* Calvo (1983): each firm can modify its price at date t only with probability $(1 - \theta)$ strictly comprised between 0 and 1. (This time-dependent price-setting rule may seem less realistic than state-dependent ones, but proves more convenient to handle analytically.) The model thus incorporates a temporary nominal rigidity which will result in a short-run trade-off for the central bank between inflation and output gap deviations from their targets. Of course, each firm sets its price, when allowed to change it, so as to maximize the discounted value of its profits.

We also assume that there is no local currency pricing, that is to say that the price of each variety of the differentiated good is denominated in the producer's currency, not in the consumer's. This assumption ensures that the variations in the nominal exchange rate impact on aggregate demand by modifying the price of the goods produced in one country and consumed in the other country. Besides, even though we do not rule out pricing to market, that is to say even though each producer can make its price depend on whether its good is sold on the domestic market or on the foreign market, each producer ends up choosing the same price on both markets, as she faces the same elasticity of substitution here and there. As a consequence, the law of one price holds.

Contrary to prices, wages are assumed to be perfectly flexible. This assumption enables us to analyze inflation and output gap dynamics without any reference to the labour market. Woodford (2003, chap. 3), who relaxes this assumption, finds that wage *vs.* price stickiness (more precisely staggered wage-setting *vs.* straggered price-setting) matters essentially for the loss function.

The foreign economy is modelized in the same way as the domestic one. The corresponding parameters are signalled by an asterisk. As the foreign economy is large compared to the domestic one, α^* is close to zero and domestic fluctuations have therefore no impact on the foreign economy. To keep things simple, we assume that the foreign economy remains constantly at its steady state, experiencing no fluctuations.

2.2 Closed form

The closed form of the model, log-linearized around its steady state, is essentially composed of an IS equation, derived from the representative household's utility maximization; of a Phillips curve, derived from the producers' price-setting decisions; and of a loss function, which derives from the representative household's utility function and which the central bank seeks to minimize. We refer the reader to Galí and Monacelli (2002) for a detailed derivation of this closed form.

Let us note R_t the gross return of a riskless one-period bond denominated in domestic currency, Y_t the aggregate output index, $P_{H,t}$ the producer price

index (PPI), $P_{F,t}$ the price index for imported goods and P_t the consumer price index (CPI):

$$R_t \equiv \frac{1}{E_t\{Q_{t,t+1}\}}, \qquad Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \qquad P_{F,t} \equiv \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}},$$

$$P_t \equiv \left[\alpha P_{H,t}^{1-\mu} + (1-\alpha) P_{F,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}.$$

Let us also note $\widetilde{Y}_t$ the level of production obtained at date t when prices are perfectly flexible ($\theta = 0$); $y_t \equiv \frac{Y_t - \widetilde{Y}_t}{\widetilde{Y}_t}$ the rate of deviation of Y_t from this level, more concisely called the output gap; and $r_t \equiv \frac{R_t - \overline{R}}{\overline{R}}$ the rate of deviation of R_t from its non-zero stationary value $\overline{R} = \frac{1}{\beta}$. Assuming that y_t and r_t are close to zero, we can approximate $\ln(1 + y_t)$ by y_t and $\ln(1 + r_t)$ by r_t. Besides, if sufficiently close to zero, the CPI inflation rate between dates t and $t+1$ can be written Δp_{t+1}, where $p_t \equiv \ln P_t$ and where Δ is the first difference operator, as the first-order approximation $\Delta p_{t+1} = \frac{P_{t+1} - P_t}{P_t}$ then holds. Similarly, $\Delta p_{H,t+1}$ (where $p_{H,t} \equiv \ln P_{H,t}$) represents the PPI inflation rate.

The law of one price implies the following first-order approximation:

$$\Delta p_t = \alpha \Delta p_{H,t} + (1-\alpha) \Delta e_t, \tag{1}$$

where e_t denotes the log of the nominal exchange rate at date t (value of one foreign currency unit expressed in domestic currency). Under the assumption of complete international financial markets, the dynamics of the nominal exchange rate is described by the uncovered interest rate parity relationship, which holds up to a first-order approximation too. The nominal interest rate being constantly equal to its stationary value in the foreign country, this UIP relationship is written:

$$E_t\{\Delta e_{t+1}\} = r_t. \tag{2}$$

The Euler equation and the goods market clearing condition, together with equations (1) and (2), lead to the following equation:

$$y_t = E_t\{y_{t+1}\} - \eta(r_t - E_t\{\Delta p_{H,t+1}\}) + \varepsilon_t^{is}, \tag{3}$$

where $\eta \equiv \frac{1+(1-\alpha)(1+\alpha)(\mu\sigma-1)}{\sigma}$ and where ε_t^{is} represents an exogenous shock with mean zero occurring at date t. Equation (3) corresponds to the standard IS equation of the New Keynesian model. The shock ε_t^{is}, which has been added in an *ad hoc* fashion, can be interpreted as a temporary demand shock, corresponding for instance to an unexpected exogenous public spending. Alternatively, it

could derive from an adequately specified preference shock ξ_t entering the utility function (so that the factor β^k is replaced by $\beta^k \xi_k$), as shown by Ireland (2002). For simplicity, we assume it is not autocorrelated.

Equation (3) directly derives from the Euler equation in the closed economy case, where $c_t = y_t$ and $\Delta p_{H,t} = \Delta p_t$ at each date t. Its interpretation is then straightforward: the present output gap is expressed as an increasing function of the expected future output gap and a decreasing function of the *ex ante* real interest rate, due to income and substitution effects. Two points are worth noting in the small open economy case. First, the deflator in the expression of the real interest rate is the PPI inflation rate, not the CPI inflation rate; but $r_t - E_t\{\Delta p_{H,t+1}\}$ and $r_t - E_t\{\Delta p_{t+1}\}$ are proportional to each other, due to the law of one price (1) and the uncovered interest rate parity (2) relationships. Second, the equation involves $E_t\{\Delta y_{t+1}\}$, rather than $E_t\{\Delta c_{t+1}\}$ as in the Euler equation; but $E_t\{\Delta c_{t+1}^*\}$ is proportional to the variation in the terms of trade $E_t\{\Delta e_{t+1} - \Delta p_{H,t+1}\}$ as a first-order approximation, due to our CES consumption index assumption, and $E_t\{\Delta e_{t+1} - \Delta p_{H,t+1}\}$ is itself proportional to $r_t - E_t\{\Delta p_{H,t+1}\}$, due to the UIP relationship.

The optimization programme of the representative household does not only lead to the IS equation (*via* the Euler equation). Indeed, as in all frameworks with infinitely-lived utility-maximizing agents, there is also a transversality condition attached to this programme. In what follows, this transversality condition will be satisfied even along what we call "divergent paths", as made clear by subsection 4.3, because these paths are actually bounded, as the central bank eventually reacts to them so as to bring them back to the neighbourhood of the stationary state. As a consequence, we proceed as if the private agents, knowing that the behaviour of the central bank will ensure that the transversality condition is satisfied, did optimize without taking this condition into account.

The price-setting decisions of firms lead to the following equation:

$$\Delta p_{H,t} = \beta E_t\{\Delta p_{H,t+1}\} + \gamma y_t + \varepsilon_t^{pc}, \tag{4}$$

where $\gamma \equiv \frac{(1-\theta)(1-\beta\theta)\left(\frac{1}{\eta}+\varphi\right)}{\theta}$ and $\varepsilon_t^{pc} \equiv -\frac{(1-\theta)(1-\beta\theta)(1+\varphi)}{\theta}\varepsilon_t^a$. Equation (4) corresponds to the standard Phillips curve of the New Keynesian model. It is forward-looking because firms know that the price they choose today will remain effective for a (random) number of periods. Like the demand shock ε_t^{is}, the cost-push shock ε_t^{pc} is assumed not to be autocorrelated: in the same way as Woodford (2003, chap. 7), we will thus focus on monetary policy inertia which does not stem from any lagged variables in the structural equations, nor from any serial correlation in the exogenous disturbances.

As no lagged (hence pre-determined) variable enters equation (4) at first sight, the inflation rate appears as a jump variable. As a consequence, the New Keynesian Phillips curve has been criticized for failing to provide enough inflation inertia: one had to appeal - so was it argued - to adaptative expectations to reconcile this equation with the data. However, lagged variables can enter the equation through the output gap term, if the monetary policy rule is backward-looking, so that this criticism need not hold. (As next section makes

clear, the first-best monetary policy does actually involve nominal interest rate and inflation rate inertia.) Moreover, the empirical investigations of Galí and Gertler (1999), Sbordone (2002) indicate that forward-looking behaviour matters more than backward-looking behaviour in the price-setting process. (What these authors question, however, is the empirical relevance of the theoretical link between real marginal costs and the output gap, so that they estimate equation (4) with real marginal costs instead of the output gap.)

Clarida, Galí and Gertler (2001), Galí and Monacelli (2002) show that the quadratic approximation of the representative household's utility function, taken in the neighbourhood of the stationary equilibrium where a system of lump-sum transfers or taxes exactly offsets the monopolistic distortions, leads to the following social loss function in the special case $\mu = \sigma = 1$:

$$L_t^S = E_t \left\{ \sum_{k=0}^{+\infty} \delta_S^k \left[(\Delta p_{H,t+k})^2 + \lambda_S (y_{t+k})^2 \right] \right\},$$

where $(\delta_S, \lambda_S) \equiv \left(\beta, \frac{(1-\theta)(1-\beta\theta)(1+\varphi)}{\varepsilon\theta} \right)$, and Woodford (2002; 2003, chap. 6) derives the equivalent social loss function in the closed economy case, corresponding to $\alpha = 1$. We assume the existence of such an optimal subsidy scheme so as to focus on the welfare losses associated with price stickiness and imperfect stabilization of shocks, because monetary policy is not aimed at addressing first-order distortions. Under this optimal subsidy scheme, first-order effects disappear, only second-order effects remain (in this second order approximation). There is no apparent "terms of trade gap" term in L_t^S because this gap turns out to be proportionate to the output gap (in what can be interpreted as a goods market clearing condition) and can therefore be included in the output gap term.

The presence of a PPI inflation term in L_t^S comes from the fact that variability in the general level of prices p_H creates discrepancies between relative prices, due to the absence of synchronization in the adjustment of the prices of different goods, and these relative price distortions lead in turn to an inefficient sectoral allocation of labour, even when the aggregate level of output is correct, *i.e.* even when the output gap is nil. These distortions matter all the more than the elasticity of substitution between goods is large and than the frequency of price adjustment is low, hence λ_S depends negatively on ε and θ. Besides, λ_S depends positively on φ, as the welfare costs of fluctuations in the output gap increase with the elasticity of the utility function with respect to labour.

Of course, L_t^S arises as the natural choice for the central bank's loss function in the case $(\mu, \sigma) = (1, 1)$. Now in order to handle other cases as well, we assume more generally that the central bank chooses the nominal interest rate r_t so as to minimize the following quadratic loss function[4]:

$$L_t = E_t \left\{ \sum_{k=0}^{+\infty} \delta^k \left[(\Delta p_{H,t+k})^2 + \lambda (y_{t+k})^2 \right] \right\}, \tag{5}$$

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[4]This loss function, though admittedly *ad hoc*, is widely used in the literature.

where (δ, λ) is a pair of positive parameters, possibly different from (δ_S, λ_S), even if the specific case $(\mu, \sigma) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ will naturally be examined at regular intervals in the following. The monetary authorities seek therefore anyway to maintain the PPI inflation rate and the output gap as close as possible from their respective values at the stationary state.

Finally, as shown by Galí and Monacelli (2002), the initial conditions can be chosen for the sake of convenience and without loss of generality so that the condition that PPP should hold (or equivalently here that trade should be balanced) in the long run can be written: $(p_{H,t+k} - e_{t+k}) \to 0$ as $k \to +\infty$. As a consequence, we get, if the infinite sum in the right-hand side does converge:

$$\Delta e_t = \frac{p_{t-1} - e_{t-1}}{\alpha} + \Delta p_{H,t} + \sum_{k=1}^{+\infty} \left(E_t \left\{ \Delta p_{H,t+k} \right\} - E_t \left\{ \Delta e_{t+k} \right\} \right). \qquad (6)$$

With a flexible exchange rate regime, the closed form of our small open economy model is made of equations (1), (2), (3), (4), (5) and (6). (With a fixed exchange rate regime, (5) should be replaced by the condition $\Delta e_{t+k} = 0$ for $k \geq 0$.) Note that the structure of the system is block-recursive: y, Δp_H and r are derived from equations (3), (4) and (5) only, with Δp and Δe being residually determined with the help of equations (1), (2) and (6). As for the closed form of the closed economy model, it is made of equations (3), (4) and (5), with $\alpha = 1$. In both the closed economy and the small open economy versions of the canonical New Keynesian model, y, Δp_H and r are therefore derived from the same (qualitatively speaking) IS equation, Phillips curve and central bank's loss function[5].

The stationary state of the small open economy, obtained in the absence of shocks ε_t^{is} and ε_t^{pc}, is characterized by $y_t = \Delta p_t = \Delta p_{H,t} = \Delta e_t = r_t = 0$ at each date t[6]. Note that the model provides no inflationary bias à la Barro and Gordon (1983a, 1983b), since the output gap and inflation objectives of the central bank coincide with the stationary values of these variables; still, the first-best monetary policy will be temporally inconsistent, as will be seen below.

Of course, this stylized model is too simple to be realistic. In particular, the absence of inertial terms in the structural equations can be criticized. As stressed by Woodford (1999; 2003, chap. 7, p. 12) however, what matters is that "it incorporates forward-looking private sector behavior in three respects, each of which is surely of considerable importance in reality".

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[5] Clarida, Galí and Gertler (2001) were the first to show this isomorphism between the reduced forms of the closed economy and the small open economy versions of the canonical New Keynesian model.

[6] The stationary value of the gross nominal interest rate R_t is $\frac{1}{\beta}$, and the net nominal interest rate $i_t = R_t - 1$ thus fluctuates around $\frac{1-\beta}{\beta}$. For small enough fluctuations, it does not reach therefore its lower bound 0.

3 Analytical resolution of the model

This section determines analytically and comments on the optimal equilibrium of the model, depending on whether the exchange rate is flexible or fixed, (when flexible) on whether a commitment technology is available or not[7], and (when fixed) on whether this commitment applies to a monetary policy rule or to the fixity of the exchange rate. The results obtained are summarized in **table 1**.

We thus consider four alternative configurations for our small open economy: a flexible exchange rate regime without commitment (FL1), a flexible exchange rate regime with commitment (FL2), a(n *ex post*) fixed exchange rate regime with commitment (FI1) and an irrevocably (*ex ante*) fixed exchange rate regime with commitment (FI2). When $\alpha = 1$, the FL1 and FL2 cases respectively correspond to that of a closed economy without commitment (CE1) and a closed economy with commitment (CE2).

To our best knowledge, most of the analytical results displayed in this section are new, in the sense that they have not been obtained by the existing literature. The only impulse-response functions already known are those of Δp_H, y and r in the FL1 case. All the others, namely the impulse-response functions of Δe and Δp in the FL1 case, those of Δp_H, y, r, Δe and Δp in the FL2, FI1 and FI2 cases, have been incompletely characterized by some existing studies, but fully derived by none[8], as shown in **table 2** which makes a (to our knowledge exhaustive) inventory of existing studies based on the canonical New Keynesian model.

It is of no consequence in this section to proceed as if the commitment technology were exogenous, but subsection 4.3 will make clear that this commitment technology should more relevantly be considered as endogenous in the FL2 and FI1 cases. (And actually our results on the design of optimal monetary policy rules will rest on this endogeneity.)

Before solving the minimization problem faced by the central bank, we need to specify the model timing. We suppose that the private agents form their (rational) expectations and the monetary authorities choose the nominal interest rate after the realization and the observation of shocks ε_t^{is} and ε_t^{pc}. There is therefore no informational asymmetry between the private agents on the one hand and the monetary authorities on the other hand.

3.1 Flexible exchange rate regime without commitment (FL1)

This first subsection examines the case (labelled FL1) of a small open economy with a flexible exchange rate regime and without commitment, which corresponds to the case (labelled CE1) of a closed economy without commitment when $\alpha = 1$. By "without commitment", we mean that only time-consistent

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[7]McCallum (1999b) discusses this distinction and reviews the corresponding literature.

[8]Galí and Monacelli (2002) do actually derive analytically the optimal equilibrium in the FL2, FI1 and FI2 cases, but not as a function of the exogenous shocks only, *i.e.* not in the form of impulse-response functions.

monetary policies are credible for the private agents. When no commitment technology is available, the private agents expect the central bank to re-optimize at each period, that is to say to choose r_{t+n} (for each $n \geq 0$) only after the realization of shocks ε_{t+n}^{is} and ε_{t+n}^{pc}. As a consequence, their expectations about the future situation in our purely forward-looking framework do not depend on the present monetary policy decision, and the central bank takes therefore these expectations as given when choosing r_t.

The resulting outcome, usually named discretionary equilibrium, or time-consistent plan, or non-reputational solution, is easily determined. Because the central bank takes expectations as given when choosing r_t at date t, the first-order condition of the minimisation of L_t (which corresponds to the derivative of L_t with respect to r_t being zero) is written: $\lambda y_t + \gamma \Delta p_{H,t} = 0$. Facing the same optimization programme in the future, the central bank will behave in a similar way and the private agents expect therefore: $\lambda E_t \{y_{t+n}\} + \gamma E_t \{\Delta p_{H,t+n}\} = 0$ for $n \geq 1$. Using (3) and (4), under the assumption that the inflation rate expectations at all horizons are bounded ($\exists A, \quad \forall n \geq 1, \quad |E_t \{\Delta p_{H,t+n}\}| \leq A$), and we will go back to this assumption in section 4, we then obtain the following impulse-response functions:

$$\Delta p_{H,t} = \frac{\lambda}{\gamma^2 + \lambda} \varepsilon_t^{pc} \quad \text{and} \quad \Delta p_{H,t+n} = 0 \text{ for } n \geq 1,$$

$$y_t = \frac{-\gamma}{\gamma^2 + \lambda} \varepsilon_t^{pc} \quad \text{and} \quad y_{t+n} = 0 \text{ for } n \geq 1,$$

$$r_t = \frac{1}{\eta} \varepsilon_t^{is} + \frac{\gamma}{(\gamma^2 + \lambda) \eta} \varepsilon_t^{pc} \quad \text{and} \quad r_{t+n} = 0 \text{ for } n \geq 1.$$

Note that we choose in this section to express all the results in the form of impulse-response functions. These impulse-response functions characterize the effect of shocks ε_t^{is} and ε_t^{pc} (at the exclusion of any other shock) on the paths followed by the different variables. In other words, they isolate the effect of the present shocks on the dynamics of the economy. This restriction takes place without any loss of generality, as past, present and future shocks are orthogonal to each other.

These impulse-response functions for Δp_H, y and r characterize completely the optimal equilibrium in the CE1 case, and incompletely the optimal equilibrium in the FL1 case. (In the latter case, they will be completed by the impulse-response functions of Δe and Δp.) They are discussed in details by Clarida, Galí and Gertler (1999). In brief, they indicate that demand shocks ε^{is} are entirely countered by monetary policy and have therefore no impact on the output gap and the inflation rate. (In other words, output gap stabilization and inflation stabilization are then mutually compatible.) On the contrary, cost-push shocks ε^{pc} are not entirely countered, and the central bank faces a trade-off between a higher inflation rate and a lower output gap following such a shock. In both cases (ε^{is} or ε^{pc}), the effect of the shock is one-shot, that is to say that the variations in Δp_H, y and r display no inertia.

Besides, equation (6) holds as the infinite sum in its right-hand side does converge. Acknowledging that the past term $\frac{p_{t-1}-e_{t-1}}{\alpha}$ cannot depend on present shocks and using the non covered interest rate parity equation, we then obtain the following impulse-response functions:

$$\Delta e_t = -\frac{1}{\eta}\varepsilon_t^{is} + \frac{\eta\lambda - \gamma}{(\gamma^2 + \lambda)\eta}\varepsilon_t^{pc},$$

$$\Delta e_{t+1} = \frac{1}{\eta}\varepsilon_t^{is} + \frac{\gamma}{(\gamma^2 + \lambda)\eta}\varepsilon_t^{pc} \quad \text{and} \quad \Delta e_{t+n} = 0 \text{ for } n \geq 2,$$

$$\Delta p_t = \frac{-(1-\alpha)}{\eta}\varepsilon_t^{is} + \frac{(\eta\lambda - \gamma) + \gamma\alpha}{(\gamma^2 + \lambda)\eta}\varepsilon_t^{pc},$$

$$\Delta p_{t+1} = \frac{1-\alpha}{\eta}\varepsilon_t^{is} + \frac{(1-\alpha)\gamma}{(\gamma^2 + \lambda)\eta}\varepsilon_t^{pc} \quad \text{and} \quad \Delta p_{t+n} = 0 \text{ for } n \geq 2.$$

These results indicate that the effect of the shocks ε_t^{is} and ε_t^{pc} on Δp and Δe is spread on dates t and $t+1$. It is therefore more prolonged than the effect of the same shocks on y, Δp_H and r, due to the non covered interest rate parity equation.

Following a positive ε_t^{is} shock, the nominal exchange rate appreciates at date t, then depreciates at date $t+1$ to go back to its initial value. This depreciation at date $t+1$ is the consequence (*via* the non covered interest rate parity equation) of the increase in the nominal interest rate at date t. The producers price level being left unchanged by the shock ε_t^{is}, PPP holds in the long run if and only if the final value of the nominal exchange rate equals its initial value: the nominal exchange rate must therefore appreciate at date t to offset its depreciation at date $t+1$. The evolution of the consumers price level follows then accurately that of the nominal exchange rate with the multiplicative factor $(1-\alpha)$, since the producer price level remains unchanged.

Following a positive ε_t^{pc} shock, the nominal exchange rate depreciates in a two-period time: $e_{+\infty} - e_{t-1} = e_{t+1} - e_{t-1} = \frac{\lambda}{\gamma^2 + \lambda}\varepsilon_t^{pc}$, in order to compensate the effect of a higher producer price level on the long run real exchange rate. This overall depreciation is unevenly spread on each of the two periods: at date $t+1$, we do have a depreciation, which results from the increase in the nominal interest rate at date t, *via* the non covered interest rate parity equation; but at date t, we can have either a depreciation (if $\eta\lambda > \gamma$), or an appreciation (if $\eta\lambda < \gamma$). Note that in the special case $(\sigma, \mu) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ considered above, the condition $\eta\lambda < \gamma$ is necessarily satisfied, as it is equivalent to $\varepsilon > 1$, so that the nominal exchange rate appreciates at date t.

The nominal exchange rate is the more likely to appreciate at date t, the lower is the elasticity η of the output gap with respect to the nominal interest rate (because then the increase in the nominal interest rate at date t is sizeable, and therefore so is the nominal exchange rate depreciation at date $t+1$), or the lower is the relative weight λ of the central bank output gap objective (because

then the increase in the producer price level is small, and therefore so is the nominal exchange rate depreciation required to satisfy the long run PPP).

As for the evolution of the consumer price index, it is explained by that of the nominal exchange rate and the producer price index: p increases therefore during the two periods considered as a whole ($p_{+\infty} - p_{t-1} = p_{t+1} - p_{t-1} = \frac{\lambda}{\gamma^2 + \lambda}\varepsilon_t^{pc}$), since so does p_H and since e depreciates; p increases at date $t+1$ too, since p_H remains unchanged at this date and since e depreciates; finally, p can either increase or decrease at date t, depending on the sign of $(\eta\lambda - \gamma) + \gamma\alpha$, and decreases only if e appreciates sufficiently to do more than compensate the effect of the increase in p_H on p.

Let L^{FL1} denote the mean $E\{L_t\}$ of the loss function in the FL1 case. Because shock ε^{pc} is serially uncorrelated, we obtain:

$$L^{FL1} = \frac{\lambda}{(1-\delta)(\gamma^2 + \lambda)}V(\varepsilon^{pc}),$$

where $V(\varepsilon^{pc})$ denotes the variance of ε^{pc}.

3.2 Flexible exchange rate regime with commitment (FL2)

This second subsection considers the case (labelled FL2) of a small open economy with a flexible exchange rate regime and with commitment, which corresponds to the case (labelled CE2) of a closed economy with commitment when $\alpha = 1$. By "with commitment", we mean that the central bank can (credibly) commit itself to following a time-inconsistent monetary policy rule.

When no commitment technology is available, the central bank cannot conduct the first-best monetary policy, because this policy does not fulfill the temporal consistency requirement, as will be seen below: the central bank will face the incentive not to act tomorrow according to what it announces today. Announcing that the first-best monetary policy will be conducted is therefore not credible.

The existence of a commitment technology enables the central bank to avoid the trap of discretionary optimization by tying its hands: announcing that the first-best monetary policy will be conducted is then credible, because the central bank will be compelled to meet its obligations. In this case, it does not re-optimize at each period, but only implements the policy decided beforehand.

By first-best monetary policy, we mean the unique impulse-response function for variable r which is compatible (*via* the IS equation) with the first-best equilibrium. And by first-best equilibrium, we mean the unique impulse-response functions for variables Δp_H and y which minimize the loss function L_t subject to the constraint represented by the Phillips curve.

In other words, we specify the variables as time-invariant linear combinations of the complete history of the exogenous disturbances, from date t onwards, up through the current date $t + n$: $\Delta p_{H,t+n} = \sum_{k=0}^{n}\left(u_k\varepsilon_{t+n-k}^{pc} + u_k'\varepsilon_{t+n-k}^{is}\right)$, $y_{t+n} = \sum_{k=0}^{n}\left(v_k\varepsilon_{t+n-k}^{pc} + v_k'\varepsilon_{t+n-k}^{is}\right)$, $r_{t+n} = \sum_{k=0}^{n}\left(w_k\varepsilon_{t+n-k}^{pc} + w_k'\varepsilon_{t+n-k}^{is}\right)$ for $n \geq 0$, and we determine these linear combinations which minimize the loss function subject to the constraints represented by the structural equations.

Note that there are two steps in our approach. The first step, involving (4) and (5), determines the optimal impulse-response functions (that is to say the optimal patterns of responses to disturbances, or equivalently the optimal state-contingent paths) for Δp_H and y. For either variable, the impulse-response function thus defined turns out to be unique. The second step, using (3), residually determines the (here again unique) impulse-response function for r associated with the ones obtained for Δp_H and y.

In so doing, we leave temporarily aside the question of whether the impulse-response function obtained for r is compatible only with the (optimal) impulse-response functions obtained for Δp_H and y, or with other (non-optimal) impulse-response functions for these two variables as well. This question obviously matters, as it amounts to ask whether or not the central bank should express its instrument r *ex ante* in the form of this impulse-response function. This question matters so much actually that we choose to devote the next two sections to answering it. (The answer will be negative.)

Note also that unlike so many studies, we are not optimizing over a low-dimensional parametric family of monetary policy rules (usually Taylor-type rules). We are seeking what Clarida, Galí and Gertler (1999) call the "unconstrained optimal rule", that is to say the optimum within the class of rules which are function of the entire history of shocks. To our knowledge, only Clarida, Galí and Gertler (1999), Woodford (2003, chap. 8), Giannoni and Woodford (2002) adopt this approach. Oddly enough, they do not go the whole way and are satisfied with considering the first-order conditions of the optimization problem. We go further and obtain the impulse-response functions of each variable.

Before turning to the results, let us consider one moment the optimum within the class of rules which specify the nominal interest rate r_t as a linear combination of the current shocks ε_t^{is} and ε_t^{pc} only. As noted by Clarida, Galí and Gertler (1999), Woodford (2003, chap. 7), the consideration of this (arbitrarily) restricted family of rules has a pedagogical virtue, as it shows whether what matters is commitment with inertia or commitment without inertia. The resolution of the optimization problem is then very simple: it amounts to follow the procedure presented above while imposing the restriction $\forall k \geq 1$, $u_k = u_k' = v_k = v_k' = w_k = w_k' = 0$. Because shocks are assumed to be serially uncorrelated, we find that the corresponding optimum coincides with the optimal solution in the absence of any commitment technology. In other words, commitment to a non-inertial behaviour is not welfare-improving (relatively to no commitment) in our framework.

The resolution of the model (in the general case) is given in appendix in subsection 7.1. We obtain the following impulse-response functions:

$$\Delta p_{H,t} = \frac{\delta z}{\beta} \varepsilon_t^{pc} \quad \text{and} \quad \Delta p_{H,t+n} = \frac{-\gamma^2 \delta z^{n+1}}{\beta \lambda \left(1 - \beta z\right)} \varepsilon_t^{pc} \text{ for } n \geq 1,$$

$$y_{t+n} = \frac{-\gamma \delta z^{n+1}}{\beta \lambda} \varepsilon_t^{pc} \text{ for } n \geq 0,$$

$$r_t = \frac{1}{\eta}\varepsilon_t^{is} + \frac{\gamma\delta z\left[\beta z^2 - (1 + \beta + \gamma\eta)z + 1\right]}{\beta\eta\lambda(1 - \beta z)}\varepsilon_t^{pc},$$

$$r_{t+n} = \frac{\gamma\delta\left[\beta z^2 - (1 + \beta + \gamma\eta)z + 1\right]z^{n+1}}{\beta\eta\lambda(1 - \beta z)}\varepsilon_t^{pc} \text{ for } n \geq 1,$$

where z is a constant, expressed in subsection 7.1 as a function of the parameters. These results characterize completely the optimal equilibrium in the CE2 case, and need to be completed by the impulse-response functions of Δe and Δp to characterize completely the optimal equilibrium in the FL2 case. As in the previous subsection, monetary policy insulate the output gap and the inflation rate from the effects of demand shocks ε^{is} (by adopting a "leaning against the wind" attitude), but not from those of cost-push shocks ε^{pc}.

The main difference between these results and those of the previous subsection is that the effect of ε_t^{pc} is more prolonged here. The shock ε^{pc} is one-shot, but the variations in Δp_H, y and r display some inertia. This is because the central bank can now trade off not only between a higher inflation rate and a lower output gap at a given date, but also between the present and the future situations. In other words, the commitment technology enables it to spread the burden of the adjustment to the shock over several periods. Note that commitment (which enables the central bank to credibly choose the entire future state-contingent evolution of the nominal interest rate, once and for all, at date t) does matter here, because the central bank faces no actual incentive to go on reacting to bygone shocks.

This inertial feature of the first-best monetary policy is interpreted by Woodford (2003, chap. 7) in the following way: as implicitly stated by the (iterative) IS equation, the effect of monetary policy goes through the long term interest rate, which is determined by market expectations of future short-term interest rates, so that the central bank must make the private sector expect future short term interest rates maintained at given levels to substantially affect the current output gap and inflation rate. To support this interpretation, Woodford (2003, chap. 7) reports the results of empirical studies providing evidence that the variations in long-term interest rates are contemporaneously affected by those in short-term interest rates.

Let us focus one moment on the impulse-response functions of the different variables to the cost-push shock. Following a positive shock ε_t^{pc}, the price level increases at date t, then decreases and tends exponentionally towards its long run value. The latter, noted $p_{H,+\infty}$, is characterized by $p_{H,+\infty} - p_{H,t-1} = \frac{(\delta - \beta)z}{\beta(1 - z)}\varepsilon_t^{pc}$: the final value of the price level is therefore higher than its initial value if and only if $\delta > \beta$, that is to say if and only if the monetary authorities are more patient than the private agents. In the meantime, the output gap decreases at date t, then increases and tends exponentially towards its stationary value ($y_{+\infty} = 0$).

The central bank reacts to the initial positive shock ε_t^{pc} either by increasing or decreasing the nominal interest rate, depending on z being respectively lower or

higher than the unique real root[9] in-between 0 and 1, noted x, of the polynome $P(q) = \beta q^2 - (1 + \beta + \gamma\eta) q + 1$. This value x corresponds indeed to what could be called a natural harmonic of the system, that is to say a root of the characteristic polynomial of the system's recurrence equation when monetary policy is passive ($r_{t+n} = 0$ for $n \geq 0$).

When $z < x$, the central bank wants the different variables to tend towards their long run values more rapidly than allowed by the economic system left by itself: it increases then the nominal interest rate in order to speed up this convergence process. On the contrary, when $z > x$, the central bank wants to slow down the convergence of the different variables towards their long run values, which makes it decrease the nominal interest rate following a positive cost-push shock.

Note that if $\delta = \delta_S$, the condition $z < x$ is equivalent to $\eta\lambda < \gamma$: we find again here the inequality obtained in the previous subsection. Thus, in the special case $(\sigma, \mu) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ considered above, this condition is necessarily satisfied, as it is equivalent to $\varepsilon > 1$, so that monetary policy is always tightening in reaction to a positive cost-push shock (*i.e.* a negative productivity shock), all the more so than the elasticity of substitution ε between the varieties of the differentiated good is large. (Indeed, a larger ε implies a larger welfare cost of inflation and therefore a larger weight on the inflation objective of the central bank.)

This outcome, under our preferred specification $(\sigma, \mu) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$, proves in accordance with the conventional wisdom, which states that monetary policy, when aimed at stabilizing aggregate output, should react procyclically in the case of productivity shocks and countercyclically in the case of demand shocks, because in so doing it replicates the behaviour of the (real) economy under flexible prices. The mechanisms at work are different here, as the inflation rate enters the loss function in our framework, but the conclusion is the same.

In both cases ($z < x$ or $z > x$), the nominal interest rate, after its reaction at date t, tends exponentially towards its initial value ($r_{+\infty} = 0$). In the intermediate case $z = x$, the nominal interest rate keeps equal to zero ($r_{t+n} = 0$ for $n \geq 0$): the central bank remains passive *ex post*, but active *ex ante*, since it follows a monetary policy rule, as indicated in sections 4 and 5.

The other impulse-response functions are obtained as previously:

$$\Delta e_t = \frac{-1}{\eta}\varepsilon_t^{is} + \frac{\delta(\eta\lambda - \gamma)z}{\beta\eta\lambda}\varepsilon_t^{pc},$$

$$\Delta e_{t+1} = \frac{1}{\eta}\varepsilon_t^{is} + \frac{\gamma\delta\left[\beta z^2 - (1 + \beta + \gamma\eta)z + 1\right]z}{\beta\eta\lambda(1 - \beta z)}\varepsilon_t^{pc},$$

$$\Delta e_{t+n} = \frac{\gamma\delta\left[\beta z^2 - (1 + \beta + \gamma\eta)z + 1\right]z^n}{\beta\eta\lambda(1 - \beta z)}\varepsilon_t^{pc} \text{ for } n \geq 2,$$

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[9]The analytical expression of this root is given in subsection 3.3.

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$$\Delta p_t = \frac{-(1-\alpha)}{\eta}\varepsilon_t^{is} + \frac{\delta\left[(\eta\lambda - \gamma) + \gamma\alpha\right]z}{\beta\eta\lambda}\varepsilon_t^{pc},$$

$$\Delta p_{t+1} = \frac{1-\alpha}{\eta}\varepsilon_t^{is} + \frac{\gamma\delta\left[(1-\alpha)(1-z)(1-\beta z) - \gamma\eta z\right]z}{\beta\eta\lambda(1-\beta z)}\varepsilon_t^{pc},$$

$$\Delta p_{t+n} = \frac{\gamma\delta\left[(1-\alpha)(1-z)(1-\beta z) - \gamma\eta z\right]z^n}{\beta\eta\lambda(1-\beta z)}\varepsilon_t^{pc} \text{ for } n \geq 2.$$

The impulse-response functions of Δe and Δp to the shock ε_t^{is} are identical to those described in the previous subsection, since the central bank reacts in the same way to the shock ε_t^{is} with or without commitment technology. The model therefore predicts in particular that no matter whether a commitment technology is available or not, the nominal exchange rate appreciates at date t and depreciates at date $t+1$ following a positive ε_t^{is} shock, to go back to its initial value.

Following a positive ε_t^{pc} shock, the nominal exchange rate depreciates (if $\delta > \beta$) or appreciates (if $\delta < \beta$) in the long run: $e_{+\infty} - e_{-1} = \frac{(\delta - \beta)z}{\beta(1-z)}\varepsilon_t^{pc}$, in order to offset the increase or decrease in the producer price level. It depreciates from date $t+1$ if and only if $z < x$: we find again naturally the distinction made above. And it depreciates at date t if and only if $\eta\lambda > \gamma$: we find again here, more unexpectedly, the distinction made in the previous subsection.

Thus, in the special case $(\sigma, \mu) = (1,1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ examined above, where both conditions $z < x$ and $\eta\lambda > \gamma$ are equivalent to $\varepsilon > 1$, the nominal exchange rate appreciates instantaneously and depreciates thereafter, following a positive cost-push shock, to go back in the long run to its initial value. Its volatility depends positively on the elasticity of substitution ε between the varieties of the differentiated good, because so does the volatility of the nominal interest rate, as seen above. In the limit case $\varepsilon = 1$, the nominal exchange rate remains fixed, whatever the shocks ε^{pc} affecting the small open economy.

As for the evolution of the consumer price index, it is explained by that of the nominal exchange rate and the producer price index, in the same way as in the previous subsection.

Let L^{FL2} denote the mean $E\{L_t\}$ of the loss function in the FL2 case. Because shock ε^{pc} is serially uncorrelated, we obtain:

$$L^{FL2} = \frac{\delta z}{\beta(1-\delta)}V(\varepsilon^{pc}).$$

In a flexible exchange rate regime, the existence of a commitment technology is of course beneficial:

$$L^{FL1} - L^{FL2} = \frac{\gamma^2\delta z^2}{(1-\delta)(\gamma^2+\lambda)(1-\beta z)}V(\varepsilon^{pc}) > 0.$$

This is because with the help of a commitment technology, the central bank is able to trade off not only between a higher inflation rate and a lower output gap at a given date, but also between the present and future situations.

3.3 Fixed exchange rate regimes with commitment (FI1 and FI2)

This third subsection focuses on the case of fixed exchange rate regimes with commitment. By "with commitment", we mean either commitment to a (time-inconsistent) monetary policy rule ensuring the fixity of the exchange rate, or adoption of an irrevocably fixed exchange rate regime. In the former case, labelled FI1, there are still a national central bank, still a monetary policy rule. In the latter case, labelled FI2, there are no more national central bank, no more monetary policy rule. The fully-fledged dollarization of some small South American economies and the Euro-membership of some small European economies fall into the latter case.

The distinction between FI1 and FI2 obviously matters in terms of monetary policy rules: in the former case, the central bank remains passive *ex post*, but active *ex ante*, since it follows a monetary policy rule, as indicated in sections 4 and 5; in the latter case, the (shadow) central bank is passive *ex ante*. As far as the resolution of the model (*i.e.* the outcome) is concerned however, there is no difference between FI1 and FI2, and we shall speak of the general FI case in the present subsection.

For the nominal exchange rate to remain fixed, we need two (straightforward) conditions to be satisfied: $\Delta e_t = 0$ and $\Delta e_{t+n} = 0$ for $n \geq 1$. The second condition implies, *via* the non covered interest rate parity, that the nominal interest rate should keep constantly equal to its stationary value: $r_{t+n} = 0$ for $n \geq 0$. That $r_{t+n} = 0$ for $n \geq 1$ in particular implies in turn, together with the IS equation and the Phillips curve, that the inflation rates $\Delta p_{H,t+n}$ for $n \geq 1$ (we drop the operator $E_t\{.\}$ to simplify the notations) follow a recurrence equation whose second-order characteristic polynomial has x and x' for roots, where:

$$x = \frac{1 + \beta + \gamma\eta - \sqrt{(1 + \beta + \gamma\eta)^2 - 4\beta}}{2\beta},$$

$$x' = \frac{1 + \beta + \gamma\eta + \sqrt{(1 + \beta + \gamma\eta)^2 - 4\beta}}{2\beta}.$$

We easily check that $0 < x < 1$ and $x' > 1$. The general form of the solution is the following: $\Delta p_{H,t+n} = \phi x^n + \varphi x'^n$ for $n \geq 1$, where ϕ and φ are two real numbers. Now, the conditions $\Delta e_t = 0$ and $\Delta e_{t+n} = 0$ for $n \geq 1$, together with equation (6), imply $\varphi = 0$, if we reasonably assume that $\Delta p_{H,t} \neq +\infty$, in other words if we assume that the inflation rate has a finite value at each date, though allowing this value to become arbitrarily large and to tend towards infinity as time passes.

We thus get: $\Delta p_{H,t+n+1} = x\Delta p_{H,t+n}$ for $n \geq 1$. Then the condition $r_t = 0$, together with the IS equation and the Phillips curve, implies $\Delta p_{H,t} - \frac{1}{x}\Delta p_{H,t+1} = \gamma\varepsilon_t^{is} + \varepsilon_t^{pc}$, and the long-run condition (6) becomes $\Delta p_{H,t} + \frac{1}{1-x}\Delta p_{H,t+1} = 0$. These three equations enable us to get $\Delta p_{H,t+n}$ for $n \geq 0$, from which we

recover y_{t+n} for $n \geq 0$ with the help of the IS equation. We thus obtain the following impulse-response functions[10]:

$$\Delta p_{H,t} = \gamma x \varepsilon_t^{is} + x \varepsilon_t^{pc},$$

$$\Delta p_{H,t+n} = -\gamma (1-x) x^n \varepsilon_t^{is} - (1-x) x^n \varepsilon_t^{pc} \text{ for } n \geq 1,$$

$$y_t = x (1 + \beta - \beta x) \varepsilon_t^{is} - \frac{(1-\beta x)(1-x)}{\gamma} \varepsilon_t^{pc},$$

$$y_{t+n} = - (1-\beta x)(1-x) x^n \varepsilon_t^{is} - \frac{(1-\beta x)(1-x) x^n}{\gamma} \varepsilon_t^{pc} \text{ for } n \geq 1.$$

Note that these impulse-response functions have been obtained without any optimization of the loss function: they actually characterize the only possible equilibrium in a fixed exchange rate regime.

One result contrasts with those of the previous subsections: the output gap and the inflation rate are no longer insulated from the effects of demand shocks ε^{is}. This is because a "leaning against the wind" monetary policy reaction to these shocks would be incompatible with the fixity of the exchange rate. Following a positive shock ε_t^{is}, the price level increases at date t, then decreases and tends exponentially towards its initial value ($p_{H,+\infty} = p_{H,t-1}$), so that PPP holds in the long run. In the meantime, the output gap first increases (at date t), then decreases by more (at date $t+1$), before eventually increasing and tending exponentially towards its stationary value ($y_{+\infty} = 0$).

Following a positive shock ε_t^{pc}, the price level increases at date t, then decreases and tends exponentially towards its initial value ($p_{H,+\infty} = p_{H,t-1}$), so that once again PPP holds in the long run. In the meantime, the output gap decreases at date t, then increases and tends exponentially towards its stationary value ($y_{+\infty} = 0$). The speed of convergence of the variables is measured by parameter x, which corresponds to what we have called a natural harmonic of the system. The higher x, the slower the convergence of these variables towards their long run values.

Note that commitment (whether to a monetary policy rule in FI1 or to the fixity of the exchange rate in FI2) does matter here, because without it the central bank would seek to react to the shocks. Note finally that we also have, of course, $\Delta p_t = \alpha \Delta p_{H,t}$ and $\Delta p_{t+n} = \alpha \Delta p_{H,t+n}$ for $n \geq 1$.

Let us note L^{FI} the mean $E\{L_t\}$ of the loss function. Given that shocks ε^{pc} and ε^{is} are serially uncorrelated and orthogonal to each other, we obtain:

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[10] These results are only partially and incompletely derived and displayed by Galí and Monacelli (2002).

$$L^{FI} = \frac{x^2\left[\left(1+2\beta+\delta+\beta^2\right)-2\left(\beta+\delta+\beta\delta+\beta^2\right)x+\beta\left(\beta+2\delta\right)x^2\right]}{(1-\delta)\left(1-\delta x^2\right)}V\left(\varepsilon^{is}\right)$$
$$+\frac{\gamma^2\left(1+\delta-2\delta x^2\right)x^2+\lambda\left(1-\beta x\right)^2\left(1-x\right)^2}{\gamma^2\left(1-\delta\right)\left(1-\delta x^2\right)}V\left(\varepsilon^{pc}\right),$$

where $V\left(\varepsilon^{is}\right)$ denotes the variance of ε^{is}. The comparison between L^{FI} and L^{FL2} proves easier than that between L^{FI} and L^{FL1}. Indeed, since a non contraint optimization is more performing than a constraint one, we naturally have: $L^{FI} \geq L^{FL2}$. Moreover, we can show that $L^{FI} = L^{FL2} \iff V\left(\varepsilon^{is}\right) = 0$, $\delta = \beta$ and $z = x$. In this case indeed, the optimal monetary policy in the FL2 case is passive (*ex post*) and therefore coincides with the necessary monetary policy reaction in the FI case.

Thus, in the absence of demand shocks and in the special case $(\sigma, \mu) = (1,1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ examined above, where $z = x \iff \varepsilon = 1$, the fixed exchange rate regime is close to the optimal regime if the elasticity of substitution ε between the varieties of differentiated good is close to one. As ε increases (from $\varepsilon = 1$), the welfare cost of inflation increases as well and therefore so does the relative weight of the central bank's inflation objective, so that the optimal monetary policy reaction to a positive cost-push shock in the FL2 case is no longer passivity, but a rise in the nominal interest rate, which is incompatible with the fixity of the exchange rate.

At first sight, the canonical New Keynesian model considered here takes into account none of the advantages usually attributed to the fixed exchange rate regime, because it focuses on its stabilization properties: the flexibility of the nominal exchange rate (FL1 and FL2 vs. FI) enables the central bank to trade off between $\Delta p_{H,t}$ and y_t in very much the same way as the existence of a commitment technology (FL2 vs. FL1) enabled it to trade off between the present and the future situations.

As a consequence, not only does our framework offer a biased point of view on the fixed exchange rate regime, but also it provides no rationale for the adoption of such a regime. Indeed, either no commitment technology is available, and the central bank will not be able to escape the FL1 equilibrium; or a commitment technology is available, and the central bank will prefer to stick to a (time-inconsistent) monetary policy rule implementing the FL2 equilibrium rather than to stick to a (time-inconsistent) monetary policy rule implementing the FI1 equilibrium.

In order to tip the scales towards the fixed exchange rate regime, the consideration of an exogenous shock ε^e affecting the nominal exchange rate under a flexible exchange rate regime, specified as a variable risk-premium added to the non covered interest rate parity equation, would not fit the bill. Indeed, this shock would merely end up as a component of the demand shock, whose effect on the target variables is completely countered by monetary policy in a flexible exchange rate regime.

What we would need instead is an exogenous shock ε^r, added to the monetary policy rule and representing the involuntary and non-systematic deviations of the nominal interest rate from its value prescribed by the monetary policy rule, so as to account for the central bank's shaking hand. The introduction of such a shock would create a non-degenerated trade-off, depending on the variance $V(\varepsilon^r)$, between the FL2 and the FI2 regimes. Alternatively, we could say that one (less easily quantifiable) advantage of the FI2 regime over the FL1, FL2 and FI1 regimes, which will become apparent in section 5, is that the FI2 regime does not make the implementation of the desired equilibrium rest on the perilous application (by the central bank) and the improbable understanding (by the private agents) of a rather complicated monetary policy rule.

4   Adoption of a monetary policy rule

In this section, we first show that the adoption of a monetary policy rule expressing the nominal interest rate as a function only of the exogenous shocks leads to multiple equilibria. We then indicate how the adoption of a monetary policy rule expressing the nominal interest rate as a well-chosen function of the endogenous variables enables the central bank to select the desired equilibrium among these multiple equilibria. Next, we lay emphasis on the importance of ruling out divergent equilibria in particular. Finally, we shortly review the literature on exogenous equilibrium selection and compare it to our endogenous equilibrium selection approach.

4.1   Existence of multiple equilibria

Section 3 shows that a necessary condition for the minimisation of the loss function (in the FL1 and FL2 cases) or for the fixity of the exchange rate (in the FI1 case) is that the nominal interest rate should follow a well-defined state-contingent (*i.e.* expressed as a function of the exogenous shocks) path, which of course depends on the case considered (FL1, FL2 or FI1). This condition is necessary, but not sufficient. Indeed, in the FL1 and FL2 cases, this path $(r_{t+n})_{n\geq 0}$ proves compatible not only with the optimal paths $(\Delta p_{H,t+n})_{n\geq 0}$ and $(y_{t+n})_{n\geq 0}$ obtained in subsections 3.1 and 3.2, but also with an infinity of other paths which, as for them, do not minimize L_t. Similarly, in the FI1 case, the path $r_{t+n} = 0$ for $n \geq 0$ does imply $E_t\{\Delta e_{t+n}\} = 0$ for $n \geq 1$, but not $\Delta e_t = 0$, so that the fixity of the exchange rate is not ensured.

As an illustration, let us assume that the central bank pledges in a credible way to choose a nominal interest rate following the path obtained in the FL2 case. Equations (3) and (4) then imply that the expected inflation rates $E_t\{\Delta p_{H,t+n}\}_{n\geq 1}$ satisfy a recurrence equation of order three, whose characteristic polynomial has z, x and x' for roots. The general form of the solution is the following: $E_t\{\Delta p_{H,t+n}\} = az^n + bx'^n + cx^n$ for $n \geq 1$, where a, b and c are three real numbers.

We therefore have four unknowns $(a, b, c, \Delta p_{H,t})$, which must be determined by the initial condition(s). (Once the current and expected future inflation rates determined, the current and expected future output gaps are residually obtained through the IS equation.) Now, we only have one initial condition, namely a mix of the IS equation, the Phillips curve and the monetary policy rule taken at date t, involving $E_t \{\Delta p_{H,t+3}\}$, $E_t \{\Delta p_{H,t+2}\}$, $E_t \{\Delta p_{H,t+1}\}$, $\Delta p_{H,t}$, ε_t^{is} and ε_t^{pc}. The results obtained in subsection 3.2 (corresponding in particular to $b = c = 0$) represent of course one possible solution, but there exist an infinity of other solutions, either convergent or divergent, which are usually called "sunspot equilibria" as they do not depend only on the fundamentals.

This multiplicity of equilibria comes from the fact that the present values of the inflation rate and the output gap depend in particular on their expected future values, *via* the IS equation and the Phillips curve. Now, these expected future values cannot be controlled by the central bank: the model says how the private sector's expectations influence the current situation, not the other way round, as Woodford (2003, chap. 2) makes clear[11]. Our framework is thus one in which the current situation depends on expectations about the indefinite future, hence the indeterminacy of the equilibrium.

Woodford (1994), Kerr and King (1996), Bernanke and Woodford (1997), Clarida, Galí and Gertler (2000) were the first to identify this both nominal and real indeterminacy of the equilibrium in the canonical New Keynesian model[12]. Their framework is that of a closed economy with a commitment technology available to the central bank, and the following literature about indeterminacy in the canonical New Keynesian model has stuck to this framework, which corresponds more or less to our CE2 case (*i.e.* our FL2 case with $\alpha = 1$), more or less do we say because most of the existing studies consider arbitrarily chosen (usually Taylor-type) monetary policy rules rather than the ones implementing the optimal equilibrium. We argue that this indeterminacy problem arises not only in the FL2 case, but also in the FL1 and FI1 cases.

4.2 Selection of a unique equilibrium

The remedy advocated by the existing literature to remove (at least partially) this indeterminacy consists in choosing an adequate monetary policy rule expressing the nominal interest rate r_t as a function of past, present or expected

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[11] In Woodford's own terms (2003, chap. 2, p. 78): "Such reasoning involves a serious misunderstanding of the causal logic of [the] difference equation [(4)] [...]. The equation does not indicate how the equilibrium inflation rate in period $t + 1$ is determined by the inflation that happens to have occurred in the previous period. [...] But instead, the equation indicates how the equilibrium inflation rate in period t is determined by *expectations* regarding inflation in the following period. These expectations determine the real interest rate, and hence the incentive for spending [...]".

[12] This problem is also discussed by McCallum (1999c, pp. 24-28), again within the framework of the canonical New Keynesian model, and mentionned by Svensson (1998, p. 7) in a more general framework. McCallum (1999b) discusses the fact that this is indeed no mere nominal indeterminacy. In another context, Sargent and Wallace (1975) were the first to point to the (nominal) indeterminacy of the equilibrium when the monetary policy instrument is the nominal interest rate, rather than the money stock.

future endogenous variables, rather than as a function of the exogenous shocks ε^{is} and ε^{pc} having occurred in the past and occuring in the present (as implicitly done in section 3). Besides, another rationale put forward in the literature for adopting such a monetary policy rule, rather than specifying the nominal interest rate as a function of the complete history of the exogenous disturbances, is that this kind of rule typically requires the knowledge of no more than a few lagged, current and expected future endogenous variables.

In the previous example corresponding to the FL2 case, if r_t is expressed as a function of $\Delta p_{H,t}$ and y_t, or of $E_t\{\Delta p_{H,t+n}\}$ and $E_t\{y_{t+n}\}$ for $n \geq 1$, then the number and the values of the roots of the characteristic polynomial of the recurrence equation followed by the expected inflation rates $E_t\{\Delta p_{H,t+n}\}_{n\geq 1}$ are *a priori* modified, as well as the expression of the initial condition. If r_t is expressed as a function of $\Delta p_{H,t-n}$ and y_{t-n} for $n \geq 1$, it is then not only the number and the values of the roots of the characteristic polynomial, as well as the expression of the initial condition, which are *a priori* affected, but also the number of initial conditions.

Actually, we can independently control the number of roots of the characteristic polynomial and the number of initial conditions. For instance, adding a term $\omega\left(\Delta p_{H,t-1} - \beta\Delta p_{H,t} - \gamma y_{t-1}\right)$, where $\omega \neq 0$, to an otherwise non backward-looking monetary policy rule, provides one more initial condition without affecting the degree of the characteristic polynomial. Indeed, this additional term becomes $\omega\varepsilon_t^{pc}$ in the expression of $E_t\{r_{t+1}\}$ and 0 in the expression of $E_t\{r_{t+n}\}$ for $n \geq 2$, because it corresponds to the deterministic part of the Phillips curve. Adding this term amounts therefore somehow to postpone the starting date of the recurrence equation, without affecting this recurrence equation.

To decrease the number of roots of the characteristic polynomial amounts to decrease the number of unknowns. To increase the number of initial conditions amounts to increase the number of equations. An adequate choice of monetary policy rule can therefore reduce the indeterminacy, and possibly remove it completely.

For instance, we only need one root (equal to z in the FL2 case and to x in the FI1 case) and two initial conditions to ensure the implementation of the results obtained in the FL2 and FI1 cases. Indeed, in each of these two cases, the results can be summarized by the value for $\Delta p_{H,t}$, that for $\Delta p_{H,t+1}$, and the recurrence equation $\Delta p_{H,t+n} = \chi\Delta p_{H,t+n-1}$ for $n \geq 2$, where $\chi = z$ in the FL2 case and $\chi = x$ in the FI1 case. (From the impulse-response function of the inflation rate can then be recovered those of the other variables.) Similarly, no root and one initial condition are enough to ensure the implementation of the results obtained in the FL1 case.

A whole branch of the New Keynesian literature, whose most representative authors are Bernanke and Woodford (1997), Woodford (1999; 2003, chap. 4, 7 and 8), Giannoni and Woodford (2002), aims at characterizing the monetary policy rules ensuring the implementation of the unique optimal equilibrium. As already said, these studies focus on the CE2 case (*i.e.* the FL2 case with $\alpha = 1$), while we flush the indeterminacy problem not only in the FL2 case, but also in the FL1 and FI1 cases. More importantly, the literature has been

concerned only about the possible existence of multiple convergent equilibria[13], which entail endogenous fluctuations, and has disregarded divergent equilibria so far. We do not.

4.3 Ruling out divergent equilibria

All the existing studies[14] concerned about the possible indeterminacy of the equilibrium restrict their attention to bounded paths, and thus are satisfied with obtaining the unicity of the path of each variable conditionally on its boundedness. In other words, they characterize monetary policy rules which rule out all convergent equilibria other than the optimal one, but which do not *a priori* rule out divergent equilibria. In the example of subsection 4.1, with $0 < x < 1 < x'$ and $z < 1$ (subsection 7.1 provides a sufficient condition for the latter inequality to be satisfied), this amounts to let parameter b be free, rather than constrain it to be nil.

The reason usually put forward to justify this restriction is that the linearization of the model is acceptable only for small macroeconomic fluctuations around the steady state, and therefore is not adapted to the study of non-bounded paths; as a consequence, the latter should be ignored. We disagree with this justification for three reasons.

First, the non-bounded paths of the model are explosive paths, of the kind bx'^n to pick up again the example of subsection 4.1. The deviations from the stationary state which they entail can therefore remain of modest size during several periods in a row, before reaching the threshold from which the log-linear approximation of the model can no longer be considered as acceptable. In other words, divergent paths may start in the neighbourhood of the steady state, so that we can at least appreciate their initial development before losing track of them, and hence we can try to find monetary policy rules which preclude their initial development.

Second, suppose that the central bank adopts a monetary policy rule which does not preclude the development of divergent equilibria. If we consider an *ad hoc* exogenous commitment technology (thus forbidding the central bank to abandon its monetary policy rule, whatever the welfare costs caused by the divergent equilibria), then as already said we soon lose sight of these divergent

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[13]Rather an isolated voice, McCallum (1999c, 2000) expresses doubts on the empirical relevance of these multiple equilibria, and thinks that the fundamental (or bubble-free) solution is the most likely to emerge in the economy.

[14]We know of only three exceptions which do not disregard unbounded solutions among all these possible saddle-point equilibria. First, Christiano and Gust (1999) distinguish between determinate, indeterminate and explosive equilibria, but their work hinges on numerical simulations, not analytical results, within the framework of a limited participation model, not the framework of the canonical New Keynesian model. Second, Batini and Haldane (1999) distinguish between explosive and non-explosive (simulated, not analytical) solutions to a New Keynesian model close to our canonical version, but fail to acknowledge the possible existence of multiple (non-explosive) equilibria. Third, in a more general framework than ours, Currie and Levine (1993, chap. 4, section 5) consider "overstable feedback rules" which remove all unstable roots from the system, but these rules do not remove undesired stable roots and hence do not rule out multiple (bounded) equilibria.

paths. In particular, we have no clue about whether these paths eventually violate the transversality condition[15] and (in the case of a small open economy) the long-run PPP condition (6), that is to say about whether these paths actually correspond to solutions of the model.

We lose track of the divergent paths as soon as the variables are sufficiently far away from their stationary values not only because these paths then invalidate our log-linear approximation of the model, but also because they invalidate the model itself, and in particular our price-setting specification *à la* Calvo (1983), our CES modelization of the domestic consumption basket, or our assumption on the currency in which prices are quoted. In the end, we know very little of these divergent paths, not even whether they exist or not, only that they are likely to be welfare-reducing if they exist, so that it seems more prudent to us (as a precautionary measure) to seek to rule them out.

Third, and most importantly, there exists actually no such thing as an exogenous commitment technology, by which the central bank commits itself to sticking to its monetary policy rule. Only in the FI2 case should the commitment technology be considered as exogenous, as the commitment then applies to the fixity of the exchange rate, not to a monetary policy rule. In the FL2 and FI1 cases, it seems more relevant to deal with an endogenous commitment technology, coming from reputation effects for instance[16], so that the central bank weighs the *pro* and *contra* before deciding whether to stick to its monetary policy rule.

If the central bank lets a divergent path develop itself, it will therefore sooner or later abandon its monetary policy rule in order to bring this path back to the neighbourhood of the stationary equilibrium. According to Clarida, Galí and Gertler (1999, p. 1701), this reaction of the monetary authorities, which will not fail to take place, is enough to nip any explosive path in the bud:

> "To avoid global indeterminacy, the central bank may have to commit to deviate from a simple interest rule if the economy were to get sufficiently off track. This threat to deviate can be stabilizing, much the way off the equilibrium path threats induce uniqueness in game theory. Because the threat is sufficient to preclude indeterminate behavior, further, it may never have to be implemented in practice."

We disagree with this analysis. Each private agent has no other individual interest than that of correctly anticipating the future values of the different variables[17]. That the central bank should sooner or later react to a developing

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[15] As Blanchard and Fischer (1989, p. 78) put it: "Of course, the proof that the transversality condition is violated on all but the saddle point path in the linearized system does not establish the fact that the paths of the original system that are not saddle point paths explode (...). A complete proof requires a characterization of the dynamics of the original nonlinear system".

[16] Loisel (2003) represents a first attempt at endogenizing the commitment technology in the canonical New Keynesian model through reputation effects.

[17] Our criticism of Clarida, Galí and Gertler's (1999, p. 1701) argument is to a certain extent reminiscent of McCallum's (1999c) criticism of the minimum-variance criterion used by some

explosive path does not represent a threat for her, but rather a certainty which is all the same to her: she will anticipate the development of this path as well as the end which the central bank will put to it. In other words, a moral hazard problem arises as the private agents know that the central bank will eventually act as a "stabilizer of last resort".

In the end, what we call a divergent path may actually remain constantly in the neighbourhood of the steady state, thus violating neither the transversality condition nor (in the case of a small open economy) the long-run PPP condition (6). Of course, such paths prove undesirable as they amount to booms and busts[18] which but bring additional noise into the system.

Prevention is better than cure: it seems to us preferable to require from the chosen monetary policy rule the property to clear the field of mines, that is to say to defuse any explosive path[19]. This property will enable the central bank to avoid not only indesirable fluctuations in the form of booms and busts, but also violations of the rule which would at best undermine its credibility and at worst, if ceaseless, denature the very idea of a monetary policy rule.

4.4 Endogenous *vs.* exogenous equilibrium selection

Most of the time, economists will not be specifically concerned with equilibria multiplicity and will choose to focus on one particular solution. We call this approach "exogenous equilibrium selection", because the particular solution in question is (more or less arbitrarily) selected by the economists themselves. By contrast, in our "endogenous equilibrium selection" approach, the particular solution in question is selected by one agent of the model, namely the central bank.

The first example of exogenous equilibrium selection goes back to Burmeister, Flood and Garber (1983), who advocate the choice of the solution which reflects the market fundamentals, whose identification has to be made on a model-specific basis. Since then, several alternative exogenous equilibrium selection criteria have been proposed by the literature, as reviewed by McCallum (1999c): Taylor's (1977) minimum-variance criterion, Evans' (1985, 1986) expectation-stability criterion, McCallum's (1983) minimal state variable crite-

economists to select one equilibrium out of several possible equilibria: "the minimum-variance criterion evidently pertains to some social *desideratum*, not anything that could be affected by any single agent's choice. Consequently, the model's agents will have no incentive to select this solution path, so there is no particular reason to believe that it would in fact be empirically relevant."

[18] "Boom and bust" may not seem at first sight the most suited term to qualify the initial development of a divergent path and the eventual monetary policy reaction putting an end to this divergent path. Indeed, as clear from the Phillips curve, a high level of real activity is associated to a decreasing inflation rate in the canonical New Keynesian model, and this does not really correspond to the common definition of a boom. However, as shortly discussed in conclusion, our point still holds in extended versions of the New Keynesian model, which as for them can satisfyingly account for boom periods.

[19] These considerations echo that of Friedman and Schwartz (1963), quoted by Radelet and Sachs (1998, p. 1): "no great strength would be required to hold back the rock that starts a landslide".

rion, and at last but not least, the saddle path or stability or non-explosiveness criterion, notably favoured by Blanchard and Kahn (1980), Whitman (1983), Sargent (1987, pp. 197-199 and pp. 306-307), Blanchard and Fisher (1989, p. 225, p. 260).

Often, these criteria will point to the same solution. Let us shortly define them in turn. First, Taylor's (1977) minimum-variance criterion selects the solution which minimizes the unconditional variance of one endogenous variable of interest. As noticed by McCallum (1999c), this criterion naturally proves problematic when there are several endogenous variables of interest. Second, the expectational stability criterion, first proposed by Evans (1985, 1986) and further developed by Evans (1989), Evans and Honkapohja (1992, 1999), selects rational expectations solutions which are the convergent limits of an iterative revised expectations process[20].

Third, McCallum's (1983) minimal state variable criterion selects the linear function of a minimal set of state variables[21] common to all admissible parameter values, with the additional requirement that whenever an admissible set of parameter values implies zero coefficients in all structural equations for a given variable, the solution-equation coefficient of this variable should also be zero for that set of parameter values. Thus defined, the minimal state variable solution is unique by construction. As shown by McCallum (1999c), it may not coincide with the solution selected by other criteria, and may in particular be divergent.

Fourth, the stability criterion selects the non-explosive solutions. This criterion is the most widely used in the literature. As emphasized in the previous subsection, it is adopted whether explicitly or implicitly by the quasi-totality of the studies based on rational expectations macroeconomic models and concerned with the issue of multiple equilibria. This criterion is also incorporated in particular into most computation algorithms aimed at solving linear systems under rational expectations. According to its supporters, its only drawback lies in that it does not select one unique equilibrium when there are several stable solutions[22]. According to McCallum (1999c) however, the stability criterion suffers from another drawback:

> "the stability criterion [...] is, to a significant extent, self-defeating.
> For the criterion is precisely that the selected solution path must be

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[20] This statement actually corresponds to a sufficient but not necessary condition for an equilibrium to be "expectationally stable". Bullard and Mitra (2000, 2002) as well as Evans and Honkapohja (2003) study this learning process, which specifies how the private agents revise their adaptative expectations iteratively until they converge towards rational expectations, within the framework of the canonical New Keynesian model.

[21] There may actually be several minimal sets of state variables, though one unique minimal state variable solution.

[22] In order to ensure the uniqueness of the equilibrium chosen, Cho and Moreno (2002) propose a criterion which selects what they claim is the "economically relevant solution", actually in our opinion an arbitrary stationary real-valued solution within the class of non-explosive solutions. Note besides that according to Cho and Moreno (2002), Blanchard and Kahn (1980) suggest the choice of the smallest modulus eigenvalues when there are too many eigenvalues of modulus strictly lower than one. We proved however unable to find any reference thereupon in Blanchard and Kahn (1980).

non-explosive - dynamically stable - under the natural presumption that exogenous driving variables (such as shocks and policy instruments) are non-explosive. Yet one important objective of dynamic economic analysis is to determine whether particular hypothetical policy rules - or institutional arrangements - would lead to desirable economic performance, which will usually require stability. Or, to express the point somewhat differently, the purpose of a theoretical analysis will often be to determine the conditions under which a system will be dynamically stable or unstable. But, obviously, the adoption of the stability criterion for selection among solutions would be logically incompatible with use of the models' solution to determine if (or under what conditions) instability would be forthcoming. To the extent, then, that this objective of analysis is important, the stability criterion is inherently unsuitable. One cannot use a model to determine whether property A would be forthcoming, if the model includes a requirement that A must not obtain."

Well, we endorse this criticism of the stability criterion, only to extend it to all other criteria. In our opinion, all these exogenous equilibrium selection approaches are flawed by their *ad hoc* or arbitrary nature, and it is simply wishful thinking to believe that the model's agents will coordinate on the equilibrium pointed to by whichever criterion is considered. Instead, we propose our endogenous equilibrium selection approach, based on the acknowledgement of the existence of one particular agent in the model, namely the central bank, which has both the incentive and the capacity to ensure the uniqueness of the equilibrium.

About the stability criterion in particular, what we are saying here is that the conditions determined by Blanchard and Kahn (1980) for the existence of a unique bounded equilibrium should be considered as of little use in our framework. Indeed, the relevance of these conditions rests on the assumption that the equations of the system remain unchanged - even along divergent paths. Now, we argue that the monetary policy rule would be the first equation to blink along a divergent path, both because it is not structural, contrary to the IS equation and the Phillips curve, by which we mean that it is easier to change, and because the central bank would find it desirable.

In saying so, we are actually questioning the relevance of standard techniques, as the vast majority of the studies solving or estimating rational expectations macroeconomic models do rely on the stability criterion. Our point is therefore quite general, by which we mean that it does not apply only in the specific context of the canonical New Keynesian model or even in the wider context of New Keynesian models in general. We have resorted to the canonical New Keynesian model merely so as to illustrate this point.

5 Characterization of the adequate monetary policy rules

This section characterizes the monetary policy rules ensuring the implementation of the optimal equilibrium determined in section 3, in each of the relevant cases considered (FL1, FL2 and FI1), while the existing literature attempts to do it only in the CE2 case (*i.e.* the FL2 case with $\alpha = 1$). As made clear by section 4, we require (unlike the existing literature) from a monetary policy rule the property to rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria.

Besides, in both the closed economy and the small open economy cases, we restrict our attention, like Woodford (2003, chap. 8), Giannoni and Woodford (2002), to the monetary policy rules which express the nominal interest rate as a function only of the (past and present) exogenous shocks, the (past) nominal interest rates and the (past, present and expected future) target variables. This restriction is merely to keep things as simple and our message as clear as possible: of course, the same reasoning and the same qualitative results would hold, were this assumption to be relaxed. In effect, it amounts to forbid p_H in the closed economy case, p_H, p, e, Δp and Δe in the small open economy case to enter the monetary policy rules considered, as none of these variables is a target variable in our framework (*i.e.* none of them enters the loss function).

Table 2 shows that of all the existing studies based on the canonical New Keynesian model, only Giannoni and Woodford (2002) as well as Woodford (2003, chap. 8) do consider a class of monetary policy rules which includes what we call adequate monetary policy rules. However, because they do not acknowledge the dangerousness of divergent equilibria as we do, they do not require from their optimal monetary policy rules the property to rule them out, so that they end up advocating monetary policy rules which in our opinion should entail macroeconomic instability as they are compatible with multiple equilibria, just like the monetary policy rules considered in all the other existing studies based on the canonical New Keynesian model[23].

5.1 Forward-lookingness

Subsection 7.2 shows that whatever the case considered (FL1, FL2, FI1, as well as in particular CE1 and CE2), the finite linear monetary policy rules ensuring the implementation of the desired equilibrium are necessarily forward-looking[24]

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[23]**Table 2** indicates indeed that none of the economies considered in the existing studies based on the canonical New Keynesian model are exempt from *ex post* macroeconomic instability, which we define as the macroeconomic instability arising from the failure of monetary policy to eliminate multiple equilibria.

[24]Forward-looking monetary policy rules in our framework correspond to "implicit instrument rules" in the terminology of Woodford (2003, chap. 8, p. 8), Giannoni and Woodford (2002, p. 8): "an implicit instrument rule (...) is a formula for setting the policy instrument as a function of other variables, some of which must be *projected* by the central bank in order to implement the rule, with the projections themselves being *conditional upon* (and affected by) the instrument setting". (Authors' emphasis.)

in a well-defined manner, so as to control the effect of the expected future values of the inflation rate and the output gap on their present values.

More precisely, the only way to remove indeterminacy consists in choosing a monetary policy rule whose forward-looking part counters exactly the effect of expected future values of the inflation rate and the output gap on the present value of the inflation rate (effect described by the IS equation in which y_t is expressed as a function of $E_t\{\Delta p_{H,t+1}\}$, $\Delta p_{H,t}$ and ε_t^{pc} with the help of the Phillips curve), that is to say that it opposes this effect so as to cancel it. The present value of the inflation rate is thus completely determined. Each expected future value of the inflation rate is determined in a similar way, using the equation corresponding to the expected application of the monetary policy rule in the future[25]. The present and expected future output gaps are then residually determined by the Phillips curve.

This result can be interpreted in the following way. In the canonical New Keynesian model, current variables depend on expected future variables, so that in order to pin down current variables, monetary policy should first pin down expected future variables. But these expected future variables depend in turn on still further expected future variables, and so on, so that a possible indeterminacy problem arises in this framework. The only way to remove indeterminacy is for monetary policy to be forward-looking so as to disconnect current variables from expected future variables, more precisely to disconnect the current inflation rate from expectations about the future situation. In so doing, the central bank kills two birds with one stone: not only does it insulate the current inflation rate from the sunspot-prone expectations about the future situation, but it does also insulate these expectations from sunspots, as they are similarly disconnected from expectations about the further future situation.

It is well-known that the efficiency of monetary policy in the canonical New Keynesian model mainly depends on the central bank's ability to influence the private agents' expectations[26]. What we argue is that the central bank should actually react to (and in so doing influence) these expectations so as to cancel their effects on the current inflation rate. If the central bank acts differently, the economy is then subject to an infinity of multiple (convergent and/or divergent) equilibria. Of course, such forward-looking rules require from the central bank precise knowledge of the current situation as well as accurate observation of the private agents' expectations (conditional on the monetary policy chosen) about the future situation, not to mention perfect information about the true values of the parameters, which is unlikely to be the case in practice, as argued notably by McCallum (1999b)[27]. But nobody said central banking was easy.

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[25] In other words, $\Delta p_{H,t}$ is determined by the application of the monetary policy rule at date t, while $E_t\{\Delta p_{H,t+k}\}$ for $k \geq 1$ is determined by the expected application of the monetary policy rule at date $t+k$.

[26] In Woodford's (2003) own terms, "markets can to a large extent do the central bank's work for it" (chap.1, p. 20), or, more precisely, "the bond market does the Fed's work for it" (chap. 7, p. 10).

[27] Of all these powers ascribed to the monetary authorities, the ability to observe the private agents' expectations seems to us the less far-fetched for two reasons. First, readily available business and households surveys are more often than not at the disposal of the central bank.

Note that expectations of future variables can be expressed in a backward-looking form in equilibrium (*ex post*), but not out of equilibrium (*ex ante*): it is therefore essential that the monetary policy rule should be explicitly forward-looking. In saying so, we agree with Evans and Honkapohja (2002, 2003), who insist on ruling out indeterminacy by basing monetary policy on observed private expectations[28], but we disagree with Batini and Haldane (1999, p. 161), according to whom "any forward-looking rule can be given a backward-looking representation and respecified in terms of current and previously-dated variables"; similarly with Taylor (1999a, 1999b), who dismisses the very idea of forward-looking monetary policy rules as of little relevance, on the ground that forecasts are based on current and lagged data; again with Levin, Wieland and Williams (2001, p. 3), who argue that "since every forecast can be expressed in terms of current and lagged state variables, a forecast-based rule cannot yield any improvement in macroeconomic stability relative to the fully optimal policy rule (which incorporates all of the relevant state variables)"; and even with Woodford (2003, chap. 8, p. 29), Giannoni and Woodford (2002, p. 29), for whom "if a forecast-based policy rule can be found that is consistent with the desired equilibrium, one can necessarily also obtain a purely backward-looking rule (...) by substituting for the forecast the particular function of predetermined and exogenous variables that represents the rational forecast".

As stressed by Bernanke and Woodford (1997) indeed, what actually matters is not so much the central bank's forecasts as the private sector's expectations, which can be affected by sunspots. Once again, the model says how the private sector's expectations influence the current situation, not the other way round. The rationale for forward-looking monetary policy rules in our framework is not the existence of monetary policy transmission lags, which would require pre-emptive strikes from the central bank, as in Batini and Haldane (1999), Batini and Pearlman (2002): indeed, monetary policy has immediate effects under our specification. It is rather that monetary policy should aim at disconnecting the current situation from the private sector's sunspot-prone expectations about the future situation. In our view, the explicit (and in some cases published) forecasts on which the central banks of Canada, New Zealand and the UK for instance base their monetarty policy should therefore be made conditional on the private sector's expectations.

As shown in **table 2**, the existing literature about forward-looking monetary policy rules within the canonical New Keynesian framework mainly focuses on simple specific families of monetary policy rules, for instance to address the question of the optimal forecast horizon, like Batini and Haldane (1999), Levin, Wieland and Williams (2001), or the question of the equilibrium (in)determinacy, like Clarida, Galí and Gertler (2000), Batini and Pearlman

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Second, much can be derived from the yield curve about the private agents' expectations of future inflation rates and nominal interest rates.

[28]Their point differs from ours however, as they reach this conclusion under adaptative learning by private agents. In particular, they resort to the determinacy conditions of Blanchard and Kahn (1980) like all other studies and in so doing fail to acknowledge the inherent fragility of the monetary policy rule.

(2002). Of course, their results depend on the (arbitrarily chosen) class of rules considered. We adopt the more general approach of Woodford (2003, chap. 8), Giannoni and Woodford (2002), and consider a much broader class of forward-looking monetary policy rules. While these authors require from their monetary policy rules the (robustness) property that they should rule out all convergent equilibria other than the optimal one whatever the statistical properties of the exogenous disturbances, we require from ours the (stability) property that they should rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria. What we then find is that this requirement is enough to entirely pin down (*modulo* the Phillips curve, as made clear by subsection 7.2) the forward-looking part of our monetary policy rules.

5.2 Backward-lookingness

5.2.1 Flexible exchange rate regime without commitment (FL1)

Subsection 7.2 shows that in the FL1 case (which includes the CE1 case), the finite linear monetary policy rules ensuring the implementation of the desired equilibrium can be backward-looking ($N_1 > 0$) or not ($N_1 = 0$), and that the set of these rules of "size" N_1 is a $3N_1 + 2$-dimensional vectorial space.

Let us take an example. The set of adequate finite linear monetary policy rules of size $N_1 = 0$ is a 2-dimensional vectorial space. Among these "minimally history-dependent rules", in the terminology of Woodford (2003, chap. 8), Giannoni and Woodford (2002), there is only one which satisfies to the double constraint $(c_0, d_0) = \left(-\frac{\beta + \gamma\eta}{\beta\eta}, 0\right)$[29]. It is written in the following way:

$$r_t = \frac{1}{\eta} E_t \{y_{t+1}\} - \frac{\beta + \gamma\eta}{\beta\eta} y_t - \frac{\gamma^2}{\beta\lambda} \Delta p_{H,t} + \frac{1}{\eta} \varepsilon_t^{is}.$$

This rule is (by definition) applied at each date. The private agents will find it credible, in spite of the absence of commitment technology, precisely because it implements the optimal solution in the absence of commitment technology. In other words, this rule is temporally consistent: if the private agents expect it to be followed in the future, then the central bank will have no incentive to deviate from it.

Let us check that this rule does really implement the desired equilibrium. Suppose this rule is applied at date t. Using the IS equation and the Phillips curve at date t, we then easily obtain $\Delta p_{H,t}$ identical to the result of the subsection 3.1. Suppose moreover that the private agents expect the rule to be applied in the future: using the IS equation and the Phillips curve at these dates, we then get $E_t \{\Delta p_{H,t+n}\} = 0$ for $n \geq 1$, which does correspond to the desired result. The present and expected future output gaps are then determined with the help of the Phillips curve. The present and expected future interest rates

---

[29] We choose this constraint on c_0 throughout the whole subsection because it enables us to express the forward-looking part of the monetary policy rule in a very simple way, more precisely to limit this forward-looking part to the single term $\frac{1}{\eta} E_t \{y_{t+1}\}$.

are eventually obtained with the help of the IS equation. All these results are identical to those obtained in subsection 3.1.

5.2.2 Flexible exchange rate regime with commitment (FL2)

Subsection 7.2 shows that in the FL2 case (which includes the CE2 case), finite linear monetary policy rules ensuring the implementation of the desired equilibrium are necessarily backward-looking ($N_1 \geq 1$), and that the set of these rules of size N_1 is a $3N_1 + 1$-dimensional vectorial space. The (partially) backward-looking nature of these rules offsets the purely forward-looking nature of the IS equation and the Phillips curve, where no lagged variable features. It amounts to introduce at each period predetermined variables which can play a anchoring role and thus provide additional initial conditions.

Let us take an example. The set of adequate finite linear monetary policy rules of size $N_1 = 1$ is a 4-dimensional vectorial space. Among these minimally history-dependent rules, there is only one[30] which satisfies to the quadruple constraint $(b_{-1}, c_0, d_0, d_{-1}) = \left(0, -\frac{\beta + \gamma \eta}{\beta \eta}, 0, 0\right)$. It is written in the following way:

$$r_t = \frac{1}{\eta} E_t \{y_{t+1}\} - \frac{\beta + \gamma \eta}{\beta \eta} y_t + A y_{t-1} - \frac{\gamma^2}{\beta \lambda (1 - \beta z)} \Delta p_{H,t} + B \Delta p_{H,t-1} + \frac{1}{\eta} \varepsilon_t^{is},$$

with

$$A = \frac{\gamma \left[\beta \lambda^2 (1 - \beta z)^2 + \gamma^4 \delta z\right]}{\delta \left[\beta \lambda^2 (1 - \beta z)^3 - \beta \gamma^4 \delta z^2\right]},$$

$$B = \frac{-\gamma^4 z}{\beta \lambda^2 (1 - \beta z)^3 - \beta \gamma^4 \delta z^2}.$$

There also exist infinite linear monetary policy rules implementing the desired equilibrium. Indeed, the unique adequate linear monetary policy rule featuring only the past, present and/or expected future inflation rate (as far as endogenous variables are concerned) is an infinite rule which is written in the following way:

$$
\begin{aligned}
r_t = {} & -\beta E_t \{\Delta p_{H,t+2}\} + (1 + \beta + \gamma \eta) E_t \{\Delta p_{H,t+1}\} \\
& + \frac{\beta - \delta z}{\gamma \delta \eta z} \sum_{i=0}^{+\infty} \left(\frac{\beta}{\delta}\right)^i \Delta p_{H,t-i} + \frac{1}{\eta} \varepsilon_t^{is}.
\end{aligned}
$$

---

[30] Given our requirements, this monetary policy rule happens to be a "direct rule" in the terminology of Woodford (2003, chap. 8), Giannoni and Woodford (2002), that is to say a rule which involves only (lags and leads of) target variables. As should be clear from subsection 7.2 however, there also exist adequate monetary policy rules which are not "direct" in the sense that they involve the lagged nominal interest rate.

5.2.3 Fixed exchange rate regime with commitment (FI1)

Subsection 3.3 has shown that the canonical New Keynesian model as such provides no clear and direct rationale for the adoption of a fixed exchange rate regime of the FI1 type. Let us nonetheless suppose that the small economy embraces such a fixed exchange rate regime. Even though it keeps the nominal interest rate constantly equal to its stationary value, the central bank is not passive: as section 4 makes clear, it has to follow *ex ante* a monetary policy rule in order to ensure the *ex post* fixity of the exchange rate.

Subsection 7.2 shows that in this FI1 case, finite linear monetary policy rules ensuring the implementation of the desired equilibrium are necessarily backward-looking ($N_1 \geq 1$), and that the set of these rules of size N_1 is a $3N_1+1$-dimensional vectorial space. Let us take an example. The set of adequate finite linear monetary policy rules of size $N_1 = 1$ is a 4-dimensional vectorial space. Among these minimally history-dependent rules, there is only one which satisfies to the quadruple constraint $(b_{-1}, c_0, d_0, d_{-1}) = \left(0, -\frac{\beta+\gamma\eta}{\beta\eta}, 0, 0\right)$. It is written in the following way:

$$
\begin{aligned}
r_t &= \frac{1}{\eta} E_t \{y_{t+1}\} - \frac{\beta+\gamma\eta}{\beta\eta} y_t - \frac{1-x}{\beta x} \Delta p_t + \frac{\gamma}{\beta(1-\beta x)} y_{t-1} \\
&\quad + \frac{\beta+\gamma\eta}{\beta\eta} \varepsilon_t^{is} - \frac{\gamma}{\beta(1-\beta x)} \varepsilon_{t-1}^{is}.
\end{aligned}
$$

From these considerations, we may draw the conclusion that an irrevocably (*ex ante*) fixed exchange rate regime (FI2) is preferable to a(n *ex post*) fixed exchange rate regime (FI1) for a small open economy, because though both are immune from multiple equilibria, the former does not make the implementation of the desired equilibrium rest on the perilous application (by the central bank) and the improbable understanding (by the private agents) of a rather complicated monetary policy rule, contrary to the latter. This result proves in accordance with conventional wisdom, which advocates the choice of a "corner solution" for the exchange rate regime.

6 Conclusion

Given how successful the New Keynesian model is nowadays, we found it opportune to examine its canonical version in order to give a new insight into the design of optimal monetary policy rules. Our original contribution is actually twofold, as we first determine analytically the optimal equilibrium and then characterize the monetary policy rules ensuring the implementation of this equilibrium, but we view the latter contribution as much more significant than the former one.

Our first (and minor) contribution thus consists in fully deriving the model's analytical results, which describe the optimal macroeconomic adjustment process to demand and cost-push shocks, for a small open economy (with the closed

economy nested as a special case) in four alternative configurations: a flexible exchange rate regime without commitment (FL1), a flexible exchange rate regime with commitment (FL2), a(n *ex post*) fixed exchange rate regime with commitment (FI1) and an irrevocably (*ex ante*) fixed exchange rate regime with commitment (FI2). Only in a special case (CE1) of the first configuration (FL1) had these results been fully derived in the existing literature.

These results notably indicate that the optimal monetary policy reaction to a cost-push shock, in the FL2 case, can be to raise or to lower the nominal interest rate, depending on the value of the various parameters. Under our preferred specification however, monetary policy should be tightened in response to a positive cost-push shock (*i.e.* a negative productivity shock), in accordance with conventional wisdom. As the elasticity of substitution ε between the varieties of the differentiated good gets closer to one (thus decreasing the welfare cost of inflation and hence the relative weight of the central bank's inflation objective), the optimal monetary policy reaction to a cost-push shock becomes passivity under this specification, so that a fixed exchange rate regime (FI1 or FI2) provides in the limit case $\varepsilon = 1$ the same welfare level as the flexible exchange rate regime (FL2) in the absence of demand shocks.

These results also indicate that all variables (among which the inflation rate, the output gap and the nominal interest rate) are stationary, whatever the (demand or cost-push) shock and the (FL1, FL2, FI1 or FI2) case considered, except the price level and the nominal exchange rate following a cost-push shock in the FL1 case, as well as in the FL2 case when the central bank's degree of patience differs from the society's ($\delta \neq \beta$). This non-stationarity is not obtained by Galí and Monacelli (2002), who disregard the FL1 case and consider the FL2 case only for $\delta = \beta$.

Our second (and major) contribution consists in characterizing the set of monetary policy rules ensuring the implementation of this optimal adjustment process, in each of the relevant cases considered (FL1, FL2 and FI1). By contrast, the existing literature attempts to do it only in the CE2 case (*i.e.* the FL2 case with $\alpha = 1$). Most importantly, unlike the existing literature, we look for stabilizing feedback rules which rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria. We argue indeed that if the monetary policy rule chosen does not preclude divergent equilibria, then these divergent equilibria may develop as the private agents expect anyway the central bank to abandon its monetary policy rule eventually in order to react to them, thus acting as a "stabilizer of last resort".

We show that these adequate rules are necessarily forward-looking so as to insulate the current inflation rate from the private agents' sunspot-prone expectations about the future situation. If the central bank acts differently, the economy is then subject to an infinity of multiple equilibria. Thus, of all the (usually simple) rules considered in the existing literature, for instance those examined by Woodford (2003, chap. 4, 7 and 8), Clarida, Galí and Gertler (1999), none ensures in our opinion the implementation of the unique desired equilib-

rium, as these studies do not acknowledge the well-defined *garde-fou* role[31] that the New Keynesian model assigns to monetary policy.

Using the same canonical New Keynesian model, Clarida, Galí and Gertler (2000) explain the American macroeconomic volatility during the pre-Volcker era by the existence of endogenous fluctuations, born from self-fulfilling expectations. But their definition of multiple equilibria is (from our point of view wrongly) more restrictive than ours, since they restrict their analysis to the sole convergent equilibria. It would therefore be appropriate to resume their empirical analysis widening the set of problematic monetary policy rules to the rules which do not preclude the development of divergent equilibria.

Finally, precisely because we acknowledge the "stabilizer of last resort" role of the central bank, we prove very critical of the widespread use of Blanchard and Kahn's (1980) conditions to solve whatever rational expectations macroeconomic model involves a monetary policy rule in addition to some structural equations. Indeed, the relevance of these conditions rests on the assumption that all equations of the system remain unchanged along divergent paths, but we argue that the monetary policy rule would actually be swiftly abandoned along such paths because the central bank would find it both possible and desirable.

All these conclusions have been reached within the specific context of the canonical New Keynesian model, which has been chosen to illustrate our point in a simple way. But our main results would still hold in a much more general context. Notably, our criticism of the use of Blanchard and Kahn's (1980) conditions naturally bites in the context of most rational expectations macroeconomic models. As the vast majority of the studies solving or estimating these models do rely on these conditions, we are actually questioning the relevance of standard techniques. Our point on what should be an adequate monetary policy rule will similarly still apply in a more general context, while our results on the optimal equilibrium will on the contrary be far from having equally far-reaching consequences. The final part of this conclusion is devoted to an investigation into their robustness to natural extensions of the canonical New Keynesian model.

The very simple framework of the canonical New Keynesian model can be extended in many ways. For instance, many authors specify the shocks as autoregressive processes of order one. This extension would certainly alter the analytical expression of the optimal equilibrium, but would not fundamentally question or invalidate (qualitatively speaking) our conclusions on the optimal implementation of monetary policy. We could also consider other sources of exogenous disturbances, for instance take into account foreign macroeconomic fluctuations in the small open economy model, or introduce a risk-premium shock in the uncovered interest rate parity equation. These extensions would simply add new terms to the analytical expression of the optimal equilibrium, under the natural assumption that all shocks are orthogonal to each other. In our opinion, there should exist monetary policy rules ensuring the implementation

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[31] We choose to use the French word "*garde-fou*", literally "guards madmen", as we find it more evocative than its English counterpart "safeguard".

of this new optimal equilibrium, especially so if we allow the new disturbances to enter the rules considered, and our results on the qualitative properties of adequate monetary policy rules (such as forward-lookingness) should remain robust.

Other natural extensions to the canonical New Keynesian model aim at addressing a criticism often formulated about the purely forward-looking nature of its structural equations. Indeed, it is now widely agreed that some form of costly adjustment or habit formation needs to be introduced into this model in order to match the inertia or the lagged responses which are apparent in the data. In our view, such extensions should dramatically alter the analytical expression of the optimal equilibrium, but would not fundamentally question or invalidate (qualitatively speaking) our conclusions on the optimal implementation of monetary policy, provided that they amount to adding only lagged variables in the Phillips curve and the IS equation, in the same (more or less arbitrary) way for instance as Clarida, Galí and Gertler (1999), Woodford (1999; 2003, chap. 3 and 8), which is the case of most extensions to be encountered in the literature.

Now, some of these extensions introduce additional expected leads of the endogenous variables into the Phillips curve and the IS equation. Such is notably the case of habit formation in consumer preferences. Whenever habit formation simply amounts to introducing additional expected leads of the output gap into the IS equation[32], adequate monetary policy rules will still exist. Indeed, in a similar way as in the canonical version of the New Keynesian model, monetary policy rules can then be found which pin down the inflation rate uniquely, while the output gap is residually determined by the Phillips curve. However, Amato and Laubach (2002) argue that the consideration of habit formation should also make expected leads of the output gap enter the Phillips curve. In this case, the output gap will not be residually determined by the Phillips curve if the monetary policy rule is chosen so as to pin down the inflation rate uniquely. It proves therefore not clear at first sight whether adequate monetary policy rules would then exist. In other words, there may well be a particular and relevant leads structure of the Phillips curve and the IS equation for which no monetary policy rule can ensure the uniqueness of the equilibrium implemented.

7 Appendix

7.1 Analytical resolution of the model (FL2)

One way to proceed is to follow the undetermined coefficients method, noting $\Delta p_{H,t+n} = \sum_{k=0}^{n} \left(u_k \varepsilon_{t+n-k}^{pc} + u_k' \varepsilon_{t+n-k}^{is} \right)$, $y_{t+n} = \sum_{k=0}^{n} \left(v_k \varepsilon_{t+n-k}^{pc} + v_k' \varepsilon_{t+n-k}^{is} \right)$, $r_{t+n} = \sum_{k=0}^{n} \left(w_k \varepsilon_{t+n-k}^{pc} + w_k' \varepsilon_{t+n-k}^{is} \right)$ for $n \geq 0$, expressing the structural equations (3) and (4) as constraints on coefficients u_k, u_k', v_k, v_k', w_k, w_k' for $k \geq 0$, and choosing these coefficients so as to minimize the corresponding Lagrangian.

---

[32]Such is typically the case, as attested by Bouakez, Cardia and Ruge-Murcia (2002), Christiano, Eichenbaum and Evans (2001), Edge (2000), Fuhrer (2000), McCallum and Nelson (1999a).

Note that we choose not to allow for retroaction: the commitment, which is announced at date t and takes place from that date onwards[33], involves no shock having occurred before that date. Allowing for (or rather actually imposing) retroaction would require considering the following linear combinations: $\Delta p_{H,t+n} = \sum_{k=0}^{+\infty} \left(u_k \varepsilon_{t+n-k}^{pc} + u_k' \varepsilon_{t+n-k}^{is} \right)$, $y_{t+n} = \sum_{k=0}^{+\infty} \left(v_k \varepsilon_{t+n-k}^{pc} + v_k' \varepsilon_{t+n-k}^{is} \right)$, $r_{t+n} = \sum_{k=0}^{+\infty} \left(w_k \varepsilon_{t+n-k}^{pc} + w_k' \varepsilon_{t+n-k}^{is} \right)$ for $n \geq 0$[34].

Had we imposed retroaction, the commitment chosen would then have depended on date t (assuming that the shocks having occurred before that date have been observed), because the central bank would take advantage of the fact that expectations formed before date t (when the commitment is both announced and implemented) are given. That the optimal solution should depend on date t is little satisfactory, and we choose therefore, like Clarida, Galí and Gertler (1999), Woodford (2003, chap. 8), Giannoni and Woodford (2002), to adopt a timeless perspective, which in effect amounts to rule out retroaction[35].

Rather than following the undetermined coefficients method to solve the optimization problem, we adopt the approach of Clarida, Galí and Gertler (1999), Woodford (2003), inspired by Currie and Levine (1993). This approach leads directly to the optimal solution in the form of impulse-response functions, *i.e.* to the desired values of $\Delta p_{H,t+n}$, y_{t+n}, r_{t+n} (for $n \geq 0$) as functions of the current shocks ε_t^{pc} and ε_t^{is}. More precisely, this approach consists in choosing the inflation rates and the output gaps so as to minimize the loss function under the constraint imposed by the Phillips curve, the nominal interest rates being residually determined by the IS equation.

Let μ_k be the coefficient corresponding to the constraint represented by the Phillips curve at date $t+k$. We are looking for the values of $\Delta p_{H,t+k}$ and y_{t+k} for $k \geq 0$ which minimize the following Lagrangian: $\sum_{k=0}^{+\infty} \delta^k \left[\left(\Delta p_{H,t+k} \right)^2 + \lambda \left(y_{t+k} \right)^2 \right] - \mu_0 \left(\Delta p_{H,t} - \beta \Delta p_{H,t+1} - \gamma y_t - \varepsilon_t^{pc} \right) - \sum_{k=1}^{+\infty} \mu_k \left(\Delta p_{H,t+k} - \beta \Delta p_{H,t+k+1} - \gamma y_{t+k} \right)$, where we have dropped the operator $E_t \{.\}$ to simplify the notations[36]. The first-order conditions lead to $\mu_k = \sum_{j=0}^{k} \beta^{k-j} \delta^j \Delta p_{H,t+j}$ for $k \geq 0$, then to $y_{t+k} = \frac{-\gamma}{\lambda} \sum_{j=0}^{k} \beta^{k-j} \delta^{j-k} \Delta p_{H,t+j}$ for $k \geq 0$. The IS equation, the Phillips curve and the latter relation then enable us to obtain for $k \geq 0$:

---

[33] In order to simplify notations and without any loss in generality, we choose the same starting date (namely date t) for both the commitment technology considered here in subsection 7.1 and the impulse-response functions presented there in subsection 3.2.

[34] Retroaction does not matter obviously if the economy was at its stationary state until date $t-1$ included (*i.e.* $\varepsilon_{t-k}^{is} = \varepsilon_{t-k}^{pc} = 0$ for $k \geq 1$), or if the economy starts from scratch at date t (with $p_{H,t-1}$ and e_{t-1} being exogenously given). Neither does it matter, more interestingly, if the central bank announced at date $-\infty$ that the rule would be implemented from date t onwards.

[35] If we imposed retroaction while assuming that the shocks having occurred before date t have not been observed, then the optimal solution would not depend on parameter δ, which is also unsatisfactory.

[36] Note that the certainty equivalence property holds here, as in all linear quadratic optimization problems. In other words, the solution will not depend on the variances of the exogenous shocks.

$$1_{k=0}\beta\lambda\varepsilon_t^{pc} = \beta\delta\lambda\Delta p_{H,t+k+2} - \left(\gamma^2\delta + \beta^2\lambda + \delta\lambda\right)\Delta p_{H,t+k+1} + \beta\lambda\Delta p_{H,t+k}, \quad (7)$$

which in particular defines a recurrence equation on the Δp_{t+n} for $n \geq 1$. The corresponding (second-order) characteristic polynomial has two positive real roots, one noted z potentially lower than one, the other noted z' strictly higher than one:

$$z = \frac{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right) - \sqrt{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right)^2 - 4\beta^2\delta\lambda^2}}{2\beta\delta\lambda},$$

$$z' = \frac{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right) + \sqrt{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right)^2 - 4\beta^2\delta\lambda^2}}{2\beta\delta\lambda},$$

where $z < 1$ if and only if $\gamma^2\delta + \beta^2\lambda + \delta\lambda > \beta\delta\lambda + \beta\lambda$. We assume this inequality satisfied in the following. Note that it is indeed satisfied at the point $(\delta, \lambda) = (\delta_S, \lambda_S)$, as well as, by continuity, in the neighbourhood of this point.

We restrict our research to the equilibria for which the inflation rates are bounded: $\exists A, \quad \forall n \geq 0, \quad |\Delta p_{H,t+n}| \leq A$. This restriction (discussed in section 4) imposes that the coefficient associated to z' (in the expression of the general form of the solution to the recurrence equation) should be nil. The coefficient associated to z is then jointly determined with $\Delta p_{H,t}$ by using (4) at date t, (7) for $k = 0$ and the first-order conditions of the Lagrangian minimization. We thus obtain the inflation rates Δp_H, then the output gaps y (with the Phillips curve) and finally the nominal interest rates r (with the IS equation) at all dates.

7.2 Characterization of the adequate monetary policy rules

If, as in Woodford (2003, chap. 8), Giannoni and Woodford (2002), the only endogenous variables allowed to enter the monetary policy rules are the nominal interest rate (r) and the target variables (Δp_H and y in our framework), then the general form of finite linear monetary policy rules is the following, no matter whether we deal with a closed economy or a small open economy:

$$\begin{aligned}
r_t &= \sum_{i=0}^{N_1} a_{-i}\Delta p_{H,t-i} + \sum_{i=0}^{N_1} b_{-i}r_{t-i} + \sum_{i=0}^{N_1} c_{-i}y_{t-i} + \\
&\quad \sum_{i=0}^{N_1} d_{-i}\varepsilon_{t-i}^{pc} + \sum_{i=0}^{N_1} f_{-i}\varepsilon_{t-i}^{is} + \\
&\quad \sum_{i=1}^{N_2} a_i E_t\{\Delta p_{H,t+i}\} + \sum_{i=1}^{N_2} b_i E_t\{r_{t+i}\} + \sum_{i=1}^{N_2} c_i E_t\{y_{t+i}\} \quad (8)
\end{aligned}$$

where $N_1 \geq 0$ and $N_2 \geq 0$. Without any loss of generality, we impose $b_0 = 0$ and $(a_{-N_1}, b_{-N_1}, c_{-N_1}, d_{-N_1}, f_{-N_1}) \neq (0,0,0,0,0)$. The private agents expect the monetary policy rule (8) to be applied in the future: for $k \geq N_1 + 1$, we obtain therefore, with the Phillips curve (4):

$$E_t\{r_{t+k}\} = \sum_{i=-N_1}^{N_2} a_i E_t\{\Delta p_{H,t+k+i}\} + \sum_{i=-N_1}^{N_2} b_i E_t\{r_{t+k+i}\} +$$
$$\frac{1}{\gamma}\sum_{i=-N_1}^{N_2} c_i\left[E_t\{\Delta p_{H,t+k+i}\} - \beta E_t\{\Delta p_{H,t+k+i+1}\}\right] \quad (9)$$

Besides, using the IS equation (3) and the Phillips curve (4), we obtain the condition C_0:

$$\beta E_t\{\Delta p_{H,t+2}\} - (1+\beta+\gamma\eta)\,E_t\{\Delta p_{H,t+1}\} + \Delta p_{H,t} + \gamma\eta r_t - \left(\gamma\varepsilon_t^{is} + \varepsilon_t^{pc}\right) = 0,$$

and the conditions C_k for $k \geq 1$:

$$\beta E_t\{\Delta p_{H,t+k+2}\} - (1+\beta+\gamma\eta)\,E_t\{\Delta p_{H,t+k+1}\} + E_t\{\Delta p_{H,t+k}\} + \gamma\eta E_t\{r_{t+k}\} = 0.$$

These conditions enable us to rewrite equation (9) as a recurrence equation on the expected future inflation rates:

$$\forall k \geq N_1 + 1, \quad \sum_{i=-N}^{N} g_i E_t\{\Delta p_{H,t+k+i}\} = 0,$$

where $N \geq 0$. This recurrence equation holds at least from $k = N_1 + 1$, and potentially before.

Let us note $M \equiv Max\,(i \in \{-N, ..., N\}, \quad g_i \neq 0)$. The monetary policy rule must be chosen such that M exists; indeed, if $\forall i \in \{-N, ..., N\}$, $g_i = 0$, then the expected future inflation rate proves undetermined from a certain date onwards, which is incompatible with the desired results.

This recurrence equation necessitates $N_1 + M + 1$ initial conditions, in order to determine $\Delta p_{H,t}$, $E_t\{\Delta p_{H,t+1}\}$, ..., $E_t\{\Delta p_{H,t+N_1+M}\}$. Now, we have only $N_1 + 1$ initial conditions at our disposal, corresponding to the monetary policy rule taken at dates t, ..., $t + N_1$, rewritten with the help of conditions C_k for $k \geq 0$. We must therefore have $M \leq 0$, that is to say that the monetary policy rule must be forward-looking so as to exactly counter the effect of the expected future values of the inflation rate and the output gap on the present value of the inflation rate.

Note that the forward-looking part of the monetary policy rule is thus uniquely defined *modulo* the Phillips curve, by which we mean that there are an infinity of (distinct though equivalent) expressions for this forward-looking part, which are linked to each other through the Phillips curve. Note also that these expressions depend on the choice of c_0 (once again through the Phillips curve): with $c_0 = -\frac{\beta+\gamma\eta}{\beta\eta}$ for instance, which corresponds to the examples given in subsection 5.2, the forward-looking part of the monetary policy rule can be written $\frac{1}{\eta}E_t\{y_{t+1}\}$, or equivalently written $\frac{1}{\gamma\eta}E_t\{\Delta p_{H,t+1}\} - \frac{\beta}{\gamma\eta}E_t\{\Delta p_{H,t+2}\}$, or still equivalently written as any convex linear combination of these two expressions.

Having characterized its forward-looking part, we now turn to the backward-looking part of the monetary policy rule. We have $5N_1 + 5$ coefficients: a_{-i},

b_{-i}, c_{-i}, d_{-i} and f_{-i} for $i \in \{0, ..., N_1\}$, on which are imposed a certain number of linear constraints. One of these constraints corresponds to the normalization $b_0 = 0$. A number $2(N_1 + 1)$ of other constraints come from the initial conditions.

Indeed, these $N_1 + 1$ initial conditions, which correspond to the application of the monetary policy rule at date t and its expected application at dates $t + k$ for $k \in \{1, ..., N_1\}$, should determine $\Delta p_{H,t}$, $E_t\{\Delta p_{H,t+1}\}$, ..., $E_t\{\Delta p_{H,t+N_1}\}$. In other words, the coefficients a_{-i}, b_{-i}, c_{-i}, d_{-i} and f_{-i} for $i \in \{0, ..., N_1\}$ should ensure that each of these $N_1 + 1$ inflation rates depend on the two shocks ε_t^{is} and ε_t^{pc} in the way described in section 3, which effectively corresponds to $2(N_1 + 1)$ constraints whatever the case considered (be it FL1, FL2 or FI1)[37].

Finally, in the FL2 and FI1 cases, one additional constraint comes from the fact that z or x must be a root of the characteristic polynomial of the recurrence equation on the expected future inflation rates, given the desired results (described in subsections 3.2 and 3.3). This requirement implies moreover that $N_1 \geq 1$, whereas N_1 can be nil in the FL1 case.

Consequently, the set of adequate finite linear monetary policy rules, that is to say rules described by equation (8) and ensuring the implementation of the optimal equilibrium, is a $3N_1 + 2$-dimensional vectorial space[38] (where $N_1 \geq 0$) in the FL1 case, and a $3N_1 + 1$-dimensional vectorial space (where $N_1 \geq 1$) in the FL2 and FI1 cases.

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[37] For instance, in the FL1 and FL2 cases, we must have $f_0 = \frac{-1}{\eta}$ and $b_{-i} = \frac{f_{-i}}{\eta}$ for $i \in \{1, ..., N_1\}$ to get the desired impulse-response function of Δp_H with respect to ε^{is}.

[38] Actually, the vectorial space in question is not the set $\{r\}$ of adequate monetary policy rules *per se*, but the set $\{r - r_0\}$ of adequate monetary policy rules relatively to a given benchmark adequate monetary policy rule r_0.

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Table 1: *ex post* macroeconomic volatility[39].

Exchange rate regime	Equilibrium	Inertial effect of shocks	Conver-gence	Short-run real effects of shock ϵ^{is}	Long-run nominal effects of shock ϵ^{pc}
FL1	optimal	0	immediate	0	+
FL2	optimal	+ or −	in z^t	0	+ or − (0 if $\delta = \beta$)
FI1 and FI2	unique	+ or −	in x^t	+	0

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[39] By *ex post* macroeconomic volatility, we mean the macroeconomic volatility arising in the presence of the optimal monetary policy.

Table 2: *ex post* macroeconomic instability
in the literature based on the canonical New Keynesian model.

Study	Economy [40]	Results [41]	Class of monetary policy rules[42]	*Ex post* macro-economic instability
Batini and Pearlman (2002)	CE	SR	restricted class of BL and/or FL rules, including no optimal rule	✓
Bernanke and Woodford (1997)	CE	IAR	restricted class of BL rules, including no optimal rule	✓
Clarida, Galí and Gertler (1999)	CE	IAR	restricted class of BL and/or FL rules, including no optimal rule	✓
Clarida, Galí and Gertler (2000)	CE	SR	restricted class of BL and/or FL rules, including no optimal rule	✓
Galí and Monacelli (2002)	SOE	IAR and SR	-	-
Giannoni and Woodford (2002), Woodford (2003, chap. 8)	CE	IAR	large class of BL and/or FL rules, including all optimal rules	✓
Kerr and King (1996)	CE	IAR	restricted class of BL rules, including no optimal rule	✓
Levin, Wieland and Williams (2001)	CE	SR	restricted class of BL and/or FL rules, including no optimal rule	✓
Woodford (1999), Woodford (2003, chap. 7)	CE	IAR	large class of BL rules, including no optimal rule	✓
Woodford (2003, chap. 4)	CE	IAR	restricted class of neither BL nor FL rules, including no optimal rule	✓
this study	CE and SOE	CAR	large class of BL and/or FL rules, including all optimal rules	-

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[40] CE: closed economy; SOE: small open economy.

[41] CAR: complete analytical results; IAR: incomplete analytical results (*e.g.* limited to the first-order conditions); SR: simulation results.

[42] BL: backward-looking; FL: forward-looking; the optimal monetary policy rules in question are optimal according to our definition of optimality, that is to say that these rules rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria.