Investments, Financial Structure and Insiders' Control of the Cash-Flow: An Intertemporal Discrete-Time Framework and a Qualitative Analysis

Marco Mazzoli*

March 30, 2003

Abstract. This paper deals with the problem of simultaneity between the …rm's investments and …nancial structure in a context of dynamic optimization where the process of information spreading that associates the pro…tability of the …rm to its share price only takes part gradually, due to market imperfections and diverging incentives incentive between shareholders and managers. In particular, the latter are assumed to hold the control of the …rm and decide upon the allocation of its cash-‡ow. This establishes a link between cash-‡ow and rate of discount of future pro…ts and generates a "…nancial channel of transmission" of the real shocks.

Keywords: investment, intertemporal …rm choice, capital structure, …nancing policy

JEL Classi…cation: D92, G32

* Department of Economic and Social Sciences, Catholic University, Via Emilia Parmense 84, Piacenza, 29100, Italy, e-mail: mazzoli62@interfree.it. I am very grateful to Marzio Galeotti for his extremely helpful comments and criticism. Previous versions of this paper have bene…ted, at di¤erent stages, from discussions and comments by Vela Velupillai, Steve Fazzari, Annalisa Cristini, Sergio Pastorello, Daniele Ritelli, Geo¤ Harcourt, Pino Marotta, Mimmo Marino and the participants into various seminars at the Universities of Bergamo, Urbino and Nottingham Trent. Obviously I alone am responsible for any mistakes or shortcomings.

1 Introduction

The behaviour of the stock markets at the end of the 1990's has been the object of a long and lively debate. High volatility and abnormal capital gains, specially in hi-tech sectors, have successfully attracted for a few years an increasing number of …nancial investors. The assumption of market e¢ciency has been questioned by many empirical studies, following, in particular, the literature on "excess volatility" (Shiller, 1989), while other empirical contributions, concerned with the interaction between managers and shareholders, provide evidence on the fact that …rms tend to accumulate cash windfalls without distributing them to shareholders (Blanchard et al., 1994) showing that the . Along these lines, the data reported in Blanchard (1993) showed that the dividend-price ratio had fallen constantly and substantially since the late 1970's, independently on the interest rate on bonds; Shiller's (2000) best-seller, named after the famous statement by Alan Greenspan, after showing data on the complete absence of correlation between the increasing trend of the U.S. stock market at the end of the 1990's and the pro…ts of the …rms in the same period, provides a number of explanation based on "non-economic" factors a¤ecting the "irrational" investment choices of millions of individuals. Miller et al. (2002) provide a sophisticated explanation of the incredible value reached by the U.S. stock at the end of 1998 (over 13 billion dollars, i.e. twice the U.S. GDP and half of the world GDP) based on moral hazard induced into the …nancial investors by "jumps" in the expected attitude of the U.S. Central Banker.

Leaving aside the well-known controversies on how to interpret the behaviour of the highly volatile stock market indices of the last decade, two issues are taken into account in this paper. First, how to provide a rational explanation to the question of why do …rm tend to accumulate cash windfalls. In particular, is there a connection between the allocation of cash windfalls, stock price, dividend policy and real investment decisions? Second, how

could we modify our standard investment model in order to account for the empirical evidence of a large part of developed economies (for instance in Continental Europe) are still characterized by "bank-oriented" …nancial systems (see, in this regard, Deutsche Bundesbank, 1999, Schmidt, 1999, Allen and Gale, 2000) where hostile takeovers are rather rare, the control of large …rms is often sold through private negotiations among controlling groups and, as a consequence, the market for shares is not necessarily associate to the market for …rms' control?

Of course, any view in this regard depends on what are one's opinion about market e¢ciency, in particular, whether or not share prices re‡ect the net present value of dividends, whether or not dividend payments adjust completely to changes in the ‡ow of pro…ts. Any hypothesis on the relation between stock prices and pro…ts should (at least implicitly) rely on some assumptions concerning the di¤usion of information about the pro…ts and the pro…tability of the …rm.

Kurz's (1994a, 1994b) "rational belief" theory provides an alternative notion of rationality vis à vis the conventional rational expectations approach. His theory is based on the consideration that individuals cannot possibly have structural knowledge of the data generating processes because a relevant part of uncertainty about the future is determined by the joint actions of other individuals: in this sense Kurz criticizes the conventional way of formalizing uncertainty by means of exogenous random shocks, since most of the uncertainty is actually endogenously determined by agents' future behaviour. Kurz (1994a) contains a theorem showing that when agents do not have structural knowledge and one uses only statistical regularity as a foundation for a rational theory of belief, the objective rationality criteria can only provide some asymptotic restrictions on the possibly "true" model consistent with observable data. This means that there is a whole set of sign…cantly di¤erent theories completely consistent with the data, even if one put no restrictions whatsoever on the agents' ability to collect and process

data. The selection by each agent of a particular theory among all those consistent with the data must necessarily be based on subjective criteria (in Kurz's terminology, individual theories about the environment). In Kurz's views, one could identify a society by the "distribution of beliefs" and such distributions can be as important as the distributions of preferences for the explanation of economic performance.

Beltratti and Kurz (1996) show that the rational belief theory can explain (both theoretically and empirically) the so-called equity premium puzzle, by taking into account the endogenous propagation of expectations, in a mathematical formalization of Keynes' beauty context.

In this paper, we take the "critical" view on stock market e¢ciency expressed in the above-mentioned contributions and assume that the behaviour of the …rm's share prices may substantially diverge, at least in the short run, from that implied by the net present value of future dividends. This a¤ects the dividend policy and the …nancial decisions of the …rm in a context where the …rm chooses its investments and …nancial structure simultanously and the managers hold the control of the …rm and decide upon the allocation of the internally generated cash-‡ow.

The relevance of the …rm's …nancial structure for investments has been the object of a great deal of contributions in macroeconomics and monetary economics (after the seminal contribution by Fazzari et al., 1988), industrial economics (for instance, within the literature on the "deep pocket argument", by Telser, 1966, Benoît, 1984, and Poitervin, 1989a and within the literature on the "limited liability e¤ect", by Brander and Lewis, 1985 and Poitervin 1989b) and, obviously, …nancial economics. However, modelling simultaneous decisions of investment and …nancial structure in an intertemporal context is still controversial. In the conventional "principal-agent" problem between managers and shareholders it is common to assume that the managers enjoy some discretional power in allocating the internally generated cash-‡ow; if we further admit that in an imperfectly competitive framework a …rm might

have incentive not to reveal the amount of pro…ts associated to with a certain level of physical capital, we might turn out to have some di¢culty in de…ning the concept of expected pro…ts generated by the newly installed capital[1].

In a world of …nancial market imperfections, where, due to information asymmetries, the internally generated cash-‡ow constitutes a cheaper source of …nance than borrowing and issuing new shares, the behaviour of the share price and capital gains may a¤ect the dividend policy of the management, which, again, a¤ects the …rm's …nancial structure by determining the rate of pro…ts retention, which, in its turn, determines the volume of investments …nanced by internal …nance. All that may have relevant implications for the standard intertemporal investment decision. If the …rm's …nancial structure is a¤ected by pro…ts retention and if the cost of …nancial capital is a¤ected by the …rm's …nancial structure, then for the intertemporal …rm's investment decision, to the extent that the (…rm speci…c) discount factor is a¤ected by the cost of …nancial capital, a causal link is established between the …rm's pro…ts, …nancial structure and discount rate for the future pro…ts. Timing in the coordination process between …nancial and investment decisions is essential for the de…nition of ‡ow variables. For this reason we introduce here a discrete-time optimal control model with a recursive structure, with …nancial markets imperfections and diverging incentives between the management and the external shareholders.

The next section contains the description of the model, section 3 contains a solution approach and the last section contains a few concluding remarks.

[1] Several examples on how di¤erent individuals might attribute di¤erent value and profitability to capital goods are provided by the literature on contracts and …rm's …nancial structure à la Grossman, Hart and Moore. In this regard, see, for instance, Hart (1995).

2 The model

In this model …nancial and real investment decisions take place simultaneously. The goods market is assumed to be imperfectly competitive, although perfect competition can be a particular case. On the basis of the assumptions summarized in the previous section, the management is assumed to be able to decide how to allocate the …rm's cash ‡ow, once the creditors are repaid and the shareholders have been remunerated consistently with a yield wichi depends on the average market yield on shares. The average market yield on shares will, of course, in‡uence the remuneration that the owners expect from their …nancial investments, but, given that the managers are assumed to fully control the …rm and the allocation of its cash-‡ow, the actual amount of dividends paid out to the shareholders is the result of an implicit negotiation between management and external shareholders: it can be a¤ected by a number of factors, in general related to the existing relation between management and external shareholders. In particular, the management may or may not have incentive to reveal information on the …rm's pro…tability. If so, the stock price might not react (at least in the short run) to changes in the …rm's pro…tability. On the other hand, if we allow for the possibility (at least in the short run) of speculative bubbles, and if we admit that in the short run the share price might overshoot with respect to its theoretical level impied by the net present value of the future pro…ts, we must include this fact in the rational …nancial choice of the management.

We therefore assume, more generically, that the management pay out an amount of dividends consistent with a share remuneration assumed to be a function of the (exogenously given) market yield on shares, which, in the short run, as we said, is to be a¤ected by a number of factors not necessarily correlated to the actual pro…ts of the …rm and, therefore, is not under the direct control of the managers. For instance, if the …rm's shares experience a positive bubble characterized by persistent abormal capital gains, the man-

agement probably needs to pay less dividends in order to keep the external shareholders happy and remunerate them at a satisfactory yield. Having assumed that the …nancial and real investment decisions take place simultaneously, the point is: in which way (given imperfect …nancial markets and diverging incentives between managers and external shareholders) does the share price a¤ect dividend policy, pro…t retention, risk premium on external …nance and hence real investments?

The model is formalized with the optimal control approach, in order to explicitly refer to the standard investment model and emphasize how di¤erent can be the results by simply introducing some common assumptions of …nancial market imperfections, diverging incentives between managers and shareholders, when the share market is not necessarily associated to the market for control. In order to take into account the relevance of timing in real and …nancial decisions (which cannot really be fully captured in detali within an optimal control model in continuous time) we introduce here a recursive structure in the intertemporal problem of the …rm's investments.

The capital is installed at time $t - 1$, and is …nanced with the …nancial sources raised by the …rm at time t-1 with a contract establishing also the debt remuneration at time t: The investment decisions, the production process (generating the pro…ts $¼_t$) as well as the payment of the interests on borrowed capital and the dividends on the own capital take place at time t . $©^{¤}_{t-1}$ is the weighted average of the cost of own capital and borrower capital, established at time $t - 1$ and paid at time t.[2]

We assume that the time horizon of the decision makers (the management) corresponds to their expected residual time m in power at the company. [3]

[2] For the discrete time extension and applicability of Pontryagin maximum principle with …nite time horizon, see Seierstad and Sydsæter (1987, pp. 207-210 and pp. 370-377) and Tu (1991).

[3] This assumption is actually as arbitrary as assuming that the time horizon is in…nite.

The problem of the …rm may be represented in the following way.

$$V_t = \sum_{t=1}^{\infty} \left(\pounds \; \imath \; k_{t\,i\,1} j \sigma; \overline{!\,l} \; \cent \; i \; I_t \; \pi \; \cent \; \frac{1}{1 + \copyright^{\pi}_{t\,i\,1}} \; \cent_{t\,i\,1} \right) \qquad (1)$$

where

$\imath \; k_{t\,i\,1} j \sigma; \overline{!\,l} \; \cent$ is de…ned as the (strictly concave) maximum value function, conditional on the parameter σ (describing the market structure and the competitive environment) and on the labour costs $\overline{!\,l}$. In what follows we assume σ and $\overline{!\,l}$ to be given and will omit them in the rest of the paper.[4]

$k_{t\,i\,1}$ is the capital installed at time $t \; i \; 1$, I_t is the amount of investments (decided at time t that will contribute to determine the stock of capital at time t + 1)

The maximand 1 is subject to the following constraints 2, 3, 4:

$$I_t = k_t \; i \; (1 \; i \; \pm) k_{t\,i\,1} \qquad (2)$$

$$\pi_t (k_{t\,i\,1}) \; i \; \copyright^{\pi}_{t\,i\,1} k_{t\,i\,1} + \cent B_t + \cent E_t = I_t \qquad (3)$$

with $0 < \pm < 1$ and where

B_t represents the borrowed …nance (and, of course, $\cent B_t = B_t \; i \; B_{t\,i\,1}$)

E_t represents the existing stock of the …rm's shares, valued at their issue price

$\pm$ is the rate of capital depreciation.

The non-distributed pro…ts can be interpreted - loosely speaking - as …nancial reserves accumulated inside the …rm and are de…ned as follows

[4] To justify this sort of "ceteri paribus" assumption we can think of a labour market characterized by a simpli…ed "e¢ciency wages" mechanism, where wages and employment are …xed in the short run and are mainly a¤ected by macroeconomic factors

$$¼_t (k_{t-1}) - ©^{¤}_{t-1} k_{t-1} = W_t:$$

We de…ne the newly borrowed …nance as $¢B_t = B_t - B_{t-1} = X_t$

3 is a ‡ow-of-funds condition saying that the new investments I_t can be …nanced either by issuing new shares, or by borrowing money or with the residual cash ‡ow which is left after remunerating the …nancial capital (employed to …nance the physical capital k_t).

The latter is de…ned as the weighted average of borrowing and non-distributed pro…ts (i.e. pro…ts in excess of the dividends that the …rms pays out to remunerate the shareholders at the exogenously determined yield r^s_t). For the purpose of our model we rule out the case of new shares issue, i.e. we consider the case where the time path and (exogenously given) variation of the share price and dividends is such that $¢E_0 = 0$. [5]

Therefore, we rewrite 3 as follows:

[5] Introducing the possibility for the …rm to issue new shares would not have modi…ed the qualitative meaning of our analysis. For instance, if we de…ne $r^{¤}_{s;t}$ as the yield on the …rm's share at time t; $p_{s;t}$ the share price, $¢p_{s;t}$ its variation with respect to time $t-1$; N_t the number of existing shares, we could interpret the situation where the …rm issues new shares as the case where, given $\overline{D} = r^{¤}_{s;t} ¢p_{s;t} ¢ N_t - ¢p_{s;t} ¢ N_t$, we have $¢p_{s;t} ¢ N_t > r^{¤}_{s;t} ¢p_{s;t} ¢ N_t$, i.e. a case of "negative dividends (like in Greenwald and Stiglitz, 1988, 1990) generated by an extremely favorable valuation of the …rm by the market, which makes extremely convenient for the management to raise risk capital.

$$\frac{1}{4}_t (k_{t-1}) - \Phi^{\alpha}_{t-1} k_{t-1} + X_t = I_t \qquad 3'$$

Furthermore, Φ^{α}_t is de…ned in the following way:

$$\Phi^{\alpha}_t = \min_{\frac{1}{t}} \left[(1 - \frac{1}{t}) i_t + \frac{1}{t} r^f_t + \acute{A}(\frac{1}{t}) \right] \qquad (4)$$

where r^f_t is the risk-free interest rate, $\acute{A}(\frac{1}{t})$ is the risk premium on the interest rate on the …rm's borrowing, which is assumed to be a monotonically increasing function of the gearing ratio $\frac{1}{t} = B_t/k_t$.

Φ^{α}_t represents here the minimum value function of the …rm's …nancial cost minimization problem.

At every time t the …rm is optimizing its …nancial structure by choosing the optimal gearing ratio $\frac{1}{t} = B_t/k_t$ that minimizes the cost of …nancial capital, de…ned as the weighted average between the borrowed and the internally generated …nance. Φ^{α}_t obviously represents the rate at which the …rm can raise external …nance for its investments and transfers resources from time t to time $t + 1$.

The optimized …nancial structure determines the rate of discount appearing in the intertemporal problem, which is conditional on the ‡ow of non-distributed pro…ts of the previous period. In this way the "…rm-speci…c" rate of discount is recursively determined as a function of the lagged stock of physical capital and lagged cost of …nancial capital[6].

[6] The idea of simultaneous optimization of the …rm's …nancial structure (determining the rate of discount of future pro…ts) was …rst introduced by Bernstein and Nadiri (1986) within a continuous time optimal control model, and in a di¤erent context from the one analyzed here, since they assumed no agency problem between the shareholders and the management and no distinction between the controling group and the external shareholders which implies that the management is maximizing the expected wealth of the generical shareholders.

In 4, i_t represents the cost of the internally generated own capital, de…ned as follows:

$$i_t = \frac{\overline{D}}{E_0 + R_t} \tag{5}$$

where

$$E_0 = p_{s;0} ¢ N_t$$

and

$$\overline{D} = r^{¤}_{s;t} ¢ p_{s;t} ¢ N_t \ ¡ \ ¢p_{s;t} ¢ N_t \tag{6}$$

where

$r^{¤}_{s;t}$ is the yield on the …rm's shares at time t,

$p_{s;t}$ is the share price, $¢p_{s;t}$ its variation with respect to time $t ¡ 1$,

N_t is the number of existing shares.

In other words, given the share price, the short run capital gain and the (exogenous) yield $r^{¤}_{s;t}$ that the management allow for the shareholders, we determine the amount of dividends paid out. In this regard, we could have two possible situations: the …rst (and extreme) one is the standard neoclassical investment model; the second one corresponds to the situation where the management strictly pays out an amount of dividends consistent with the market yield on shares and the share price might not always re‡ect (in the short run) the net present value of the future pro…ts.

In order to have the standard neoclassical investment model with e¢cient …nancial markets:

a) share prices adjust perfectly and instantaneously to the value implied by the pro…ts;

b) cash ‡ow (net of adjustment costs of investments) are entirely exhausted into interests and dividends payments (i.e., no agency problem and

no incentive for the managers to keep the cash ‡ow - as far as possible given the yield on shares - inside the …rm).

In all the other cases, the stock price may diverge, in the short run, from the value implied by the net present value of the future pro…ts. This is what we are assuming in the rest of the paper.

If stock prices were to be a¤ected by the endogenous propagation of expectations (like, for instance, in Kurz's "Rational Beliefs" theory) , then the share price would be subject to a number of shocks and show a path apparently uncorrelated (or only very weakly correlated) in the short run to the actual pro…ts.

In order to explain the "irrational exuberance" of some years ago, many mainstream authors (like, for instance, Miller, Weller and Zhang, 2002) had to invoke some sort of long lasting bubble in order to justify the puzzle of the Nasdaq index in 1996-2001: in this context again stock prices would be - in the "short run" - exogenous with respect to the "real" pro…ts, although the "short run" would be in this case, as short as a decade.

As we said, we take Kurz's skeptical view on …nancial markets e¢ciency and admit that stock prices may diverge (although under very particular conditions, such as those witnessed by the Nasdaq index in the last decade) from the value implied by the …rms pro…ts. In particular, it is possible that the share price in the short run is potentially a¤ected by persistent bubbles or even a¤ected by the incentives of the managers to keep the cash-‡ow as much as possible inside the …rm. The managers might not have incentives to fully reveal all the information on the …rm's pro…tability and they tend not to exhaust pro…ts into dividends and interest payments.

Under these assumptions, we can allow ourselves to think of the share price as exogenously determined in the short run. Given the share price and its short run capital gain, the management of the …rm choose a yield $r^{\alpha}_{s;t}$ which determine the amount of dividends they want to pay to the shareholders. $r^{\alpha}_{s;t}$ could be interpreted as a …nancial market constraint on the behaviour of

the management: it represents the remuneration that keeps the shareholders happy. It could be interpreted as the result of an implicit negotiation between the shareholders and the management, with asymmetric information.

Note that for the shareholder the yield on shares is given by $r^{¤}_{s;t} = \frac{\overline{D}}{p_{s;t}¢N_t} + \frac{¢p_{s;t}}{p_{s;t}}$;while for the management the cost of capital is a¤ected by the (exogenous) book value $p_{0;t}¢N_t$ of the shares. Hoever, for a given (and exogenous) value of $\frac{¢p_{s;t}}{p_{s;t}}$, it is easy to verify that if it were subject to shocks, these shocks would have an impact on the dividend policy and, as a consequence, on the …rm's …nancial structure and investment decisions. Note that, due to the assumptions made here about the insiders' control of the cash-‡ow, once the shares have been issued, their market value is relevant to the managers only to the extent that it contributes to the determination of their dividend policy. For this reason, the notation $¢E_t$ and E_t is di¤erent from the notation employed to indicate the value of the newly issued shares.

The above assumptions generate not only a recursive structure in the problem but also a certain persistence of the past pro…ts in‡uence on the discount rate. The extent of this persistence is implicitly limited by the rate of capital depreciation ±:

Since the internally generated …nance is pre-determined (by the non-distributed pro…ts at time "t ¡ 1"), by choosing the value X_t of the newly borrowed …nance, the …rm also determine the maximum amount of feasible new investments at time "t" and the gearing ratio at time "t", which will be incorporated in the new debt contracts that the …rm issues in order to …nance part of its investments.

Let us now analyse the minimum value function $©^{¤}_t$: Assuming that the second order conditions be satis…ed, the …rst order conditions are the following:

$$d©^{¤}_t/d¹ = r^f_t + Á(¹_t) + ¹_t Á'(¹_t) ¡ i_t = 0$$

The above equation (stating that in equilibrium the marginal cost of

borrowing equals the marginal cost of the internally generated …nance) can be simpli…ed by assuming that »(θ_t) = Á(θ_t) + θ_tÁ$'$(θ_t) can be rearranged into a monotonically increasing and invertible function of θ_t. One can easily verify that this would be always true if Á(θ_t) is convex[7] in θ_t , as we are actually assuming in this model.

In this case we get

$$\theta_t = »^{-1}(i_t - r_t^f) \tag{7}$$

This means, in other words, that the gearing ratio is an increasing function of the di¤erence between the cost of own capital i_t and the interest rate on risk-free assets r_t^f, since, for a given r_t^f, the higher is the cost of own capital, the higher is the incentive for the …rm to borrow by increasing the gearing ratio. At each time the managers, by choosing the level of debt, simultaneously a¤ect the investments (i.e. the control variable), the …nancial structure and the cost of …nance

By looking at the constraints 2 and 3', one immediately sees that they both are dynamic equations putting into relation two ‡ow variables (I_t and X_t) with the state variable k at two di¤erent moments in time.(t - 1 and t).

In particular, while I_t relates the state variables k_{t-1} and k_t for a given rate of discount $©_t^¤$; X_t does the same job and in addition determines (together with k_t) the optimal rate of discount. In other words, di¤erently from the

[7] This would be true also if Á(θ_t) were concave but with a second derivative su¢ciently small in absolute value, i.e. if its curvature is "relatively ‡at". However the assumption of convexity for Á(θ_t) is rather general, since it could capture the situation where highly indebted …rms would have to pay an extremely high risk premium on borrowed capital. Furthermore, if the analytical form of Á(θ_t) were such that it tended asymptotically to in…nite when θ_t approaches 1, one could reproduce the case of credit rationing by introducing appropriate analytical form and parameters for the function Á(θ_t):

conventional neoclassical intertemporal investment models, it is not I_t but X_t that acts as a control variable in this context.

Since we know from 3' that $¼_t(k_{t-1}) - I_t = ©^{¤}_{t-1}k_{t-1} - X_t$, we may express 1 in terms of the control variable X_t and the state variable k_{t-1}, while by putting together the two constraints 3' and 2 we can eliminate I_t and express the intertemporal constraints too in terms of X_t. Therefore the …rm's problem can be rede…ned as follows:

$$V_t = (©^{¤}_{t-1} ¢ k_{t-1} - X_t) + \sum_{t=1}^{\infty} \frac{(©^{¤}_t ¢ k_t - X_{t+1})^{½}}{(1 + ©^{¤}_t)^t}^{¾} \qquad (8)$$

s.t.

$$k_t = (1 - ±)k_{t-1} + ¼_t(k_{t-1}) - ©^{¤}_{t-1}k_{t-1} + X_t$$

if one allowed for shocks in the pro…t function $¼_t$, for instance, by letting σ be subject to shocks, these shocks would be transferred to the rate of discount of the future pro…ts from the next period on. In addition, as we can see again from 5, 7 and 4, the …rm's discount rate is a¤ected by the share price and in its variations. In other words, a …nancial shock modifying the optimal dividend policy of the …rm's managers would also modify the cost of own capital, the optimal gearing ratio, and, as a consequence, the discount rate. Of course, the speci…c nature of these causal links would depend on the nature of the connections between ¼ and p, i..e. how e¢cient is the …nancial market and how fast does the information process go.

3 A slightly unconventional result

We are now enabled to write down the discrete Hamiltonian as follows:

$$H_t = (\Omega^{\alpha}_{t-1} \cdot k_{t-1} - X_t) + \sum_{i=t}^{\infty} \frac{(\Omega^{\alpha}_i \cdot k_i - X_{i+1})}{(1+\Omega^{\alpha}_i)^i} +$$
$$+ \varsigma_t(X_t - k_t + (1-\delta)k_{t-1} + \varphi_t(k_{t-1}) - \Omega^{\alpha}_{t-1} \cdot k_{t-1}$$

where $\Omega^{\alpha}_i = \Omega^{\alpha}_i(\tau_i(i - r^f_t))$ and

$$i_t = \frac{r^{\alpha}_{s;t} \cdot p_{s;t} \cdot N_t - \cdot p_{s;t} \cdot N_t}{E_0 + \sum_{i=0}^{t}(\varphi_i(k_{i-1}) - \Omega^{\alpha}_{i-1}k_{i-1})}$$

The de…nition for i_t allows us to clarify the link between pro…ts, information spreading, share price and dividend policy. For instance, if the managers do not have incentives to reveal information on the pro…tability of the …rm, the share price might not react (at least in the short run) to increases in the pro…ts. Therefore the numerator of i_t would not change and the denominator would increase. This means that an increase in $\varphi_i(k_{i-1})$ would be associated to a reduction in the cost of the own capital and, hence, on the average cost of capital.

On the other hand, if an increase in the …rm's pro…tability determine an increasing and persistent capital gain, the numerator of i_t would be small again: in other words, the own capital would become relatively cheap (as long as $\cdot p_{s;t}$ increases) since, due to the capital gains, the management needs to pay less dividends to the external shareholders in order to keep them happy.

Given the assumptions we made on the cost of the own capital and dividends determination, any shock to the exogenous share price would be transferred to the dividends and hence to i_t and the optimal …nancial structure τ, which determines (through 7) the rate of discount of future pro…ts. In other words, by substituting , 5, 6 and 7 into 4, Ω^{α}_icould be de…ned as the following generical function:

$$\Omega^{\alpha}_t = \Omega^{\alpha}_{t-1}\tau_t(r^f_t; r^{\alpha}_{s;t}; \varphi_t;)jp_{s;t}; \cdot p_{s;t}$$

Assuming now that the regularity conditions for H_t are satis…ed, an easy and straightforward application of TU (1991, pp. 261-264) de…nition of the "Discrete Maximum Principle" yields the following results:

$$\frac{@H_t}{@X_t} = 0 =) \quad ,_t = 1 \tag{9}$$

$$\frac{@H_t}{@k_{t\,i\,1}} = ,_t$$

which imply

$$\frac{@¼_t}{@k_{t\,i\,1}} \; i \; \pm \; = \; \left[\frac{@©^¤_t}{@W_t}\left(\frac{@¼_t}{@k_{t\,i\,1}} \; i \; ©^¤_{t\,i\,1}\right) \frac{©^¤_t k_t \; i \; X_{t+1}}{(1+©^¤_t)^2} \; i \right. \tag{10}$$

$$\left. i \; \frac{1}{1+©^¤_t}\frac{@©^¤_t}{@W_t}\left(\frac{@¼_t}{@k_{t\,i\,1}} \; i \; ©^¤_{t\,i\,1}\right) ¢ k_t + \right.$$

$$\left. + \sum_{i=t+1} \frac{1}{(1+©^¤_i)^i}\frac{@©^¤_i}{@W_i}\left(\frac{@¼_i}{@k_{i\,i\,1}} \; i \; ©^¤_{i\,i\,1}\right) ¢ \frac{©^¤_i k_i \; i \; X_{i+1}}{1+©^¤_i} \; i \; k_i \right]$$

The left-hand side of 10 is, of course, the marginal pro…tability of capital, net of the rate of depreciation of k. The right-hand side of 10 is composed of three addends, one for each row. The …rst one can be thought of as the e¤ect of how the modi…cations in the discount rate generated by a change in the state variable a¤ect the way the future values of the net …nancial ‡ows ($©^¤_t k_t \; i \; X_t$) are discounted.

The second addend (second row) describes how again the same modi…-cations in the discount rate modify the ‡ow of dividends and interest rates that have to be paid on the future capital k_t (which, given the balance sheet constraint of the …rm, is equal to the …nancial capital $B_t + R_t$) .

The third addend (third row) jointly represents the two above-mentioned e¤ects for the remaining future periods.

The intuitive interpretation of all that goes as follows: …rst, any shock to the pro…t function on the left-hand side of the above equation (i.e. any shock a¤ecting the functional link between pro…ts and capital, such as technology shocks, but also shocks in the market structure or in the degree of competition among …rms) is propagated to the cost of …nancial capital, and, therefore, to the rate of discount of future pro…ts, the …rst addend on the right-hand side of the above equation. This happens because in imperfect …nancial markets, the cost at which the management is able to raise funds is bound to be a¤ected by the risk premium and by the cash-‡ow. The conventional neoclassical investment models, where the discount rate is …xed and exogenous, miss the potential causal link between cash-‡ow, risk premium cost of …nance and rate of discount of future pro…ts. In this sense they might not be the more appropriate tool to analyse investment decisions with …nancial market imperfections where the managers control the …rm's cash-‡ow and have incentive not to disclose all the information on cash-‡ow.

In addition, the converse is also true: any (exogenous in this framework) shock to the discount rate (caused, for instance, by a speculative bubble increasing the share price) a¤ects the cost of external …nance (since the manager need to pay less dividends to the shareholders in order to keep them happy) and hence the rate of discount, by increasing the right-hand side of the above equation. All this brings about a movement along the optimal point in the …rm's pro…t function, i.e. a modi…cation in the marginal pro…tability of capital, in the left-hand side of the above equation.

Equation 10 can be written more compactly in the following way:

$$\frac{@¼_t}{@k_{t\,i\,1}} \;i\; ± = \left(\frac{@¼_i}{@k_{i\,i\,1}} \;i\; ©^¤_{i\,i\,1}\right) \sum_{i=t}^{½} \frac{1}{(1+©^¤_i)^i} \frac{@©^¤_i}{@W_i} ¢ \left(\frac{©^¤_i k_i \;i\; X_i}{1+©^¤_i} \;i\; k_i\right)$$

Equation 10 can be rearranged as follows:

$$\frac{\partial \pi_t}{\partial k_{t-1}} - \rho = \left(\frac{\partial \Omega_t^*}{\partial W_t}\right)\left(\frac{\partial \pi_t}{\partial k_{t-1}} - \Omega_{t-1}^*\right)\left(\frac{1}{(1+\Omega_t^*)}\right)\left(\pi_t - I_t - (1+\Omega_t^*)k_t\right) + \quad (11)$$
$$+ \sum_{i=t+1}\left[\frac{1}{(1+\Omega_i^*)^i}\frac{\partial \Omega_i^*}{\partial W_i}\left(\frac{\partial \pi_i}{\partial k_{i-1}} - \Omega_{i-1}^*\right)\left(\frac{\pi_i - I_i - (1+\Omega_i^*)k_i}{1+\Omega_i^*}\right)\right]$$

The expression $I_t + (1+\Omega_t^*)k_t$ might be interpreted as the total capital absorption (i.e. capital stock plus investments) plus capital remuneration at time t. Since the marginal pro…tability of capital associates a change in pro…ts to a change in the stock of capital, the …rst line of 11 contains the di¤erence between pro…ts and capital absorption and remuneration $\pi_t - I_t - (1+\Omega_t^*)k_t$

The term $\frac{\partial \pi_t}{\partial k_{t-1}} - \Omega_{t-1}^* \frac{1}{(1+\Omega_t^*)}$ is the present value of the spread between the $\frac{\partial \pi_t}{\partial k_{t-1}}$ and Ω_{t-1}^* at time t+1. The term $\frac{\pi_t - I_t - (1+\Omega_t^*)k_t}{(1+\Omega_t^*)}$ is the present value of the di¤erence between pro…ts at time t+1 and capital absorption and remuneration at time t+1

11 allows us to interpret and decompose the marginal pro…tability of capital (net of the depreciation rate) which appears on the left-hand side of 11

The term $\frac{\partial \Omega_t^*}{\partial W_t}$ is the impact of the …rm's wealth on the risk premium and hence on the capital cost. Therefore the marginal pro…tabiliy of capital (net of depreciation) may be decomposed into $\frac{\partial \Omega_t^*}{\partial W_t}\left(\frac{\partial \pi_t}{\partial k_{t-1}} - \Omega_{t-1}^*\right)\frac{1}{(1+\Omega_t^*)}$ and $\frac{1}{(1+\Omega_t^*)}\left(\pi_t - I_t - (1+\Omega_t^*)k_t\right)$ as well as their future net present discounted values. Loosely speaking, 11 could be interpreted as a link between the marginal pro…tability of the capital and the …nancial value of the …rm. In other words

MARGINAL PROFITABILITY OF CAPITAL (NET OF THE RATE OF DEPRECIATION)

=

FUTURE DISCOUNTED VALUE OF THE FOLLOWING:

IMPACT OF CHANGES IN THE FIRM'S FINANCIAL RESERVES ON CAPITAL COST

times

CHANGES IN THE FIRM'S RESERVES FINANCIAL , i.e. DIFFERENCE BETWEEN CASH FLOW AND CAPITAL ABSORPTION AND REMUNERATION

times

SPREAD BETWEEN MAGINAL PROFITABILITY OF CAPITAL AND AVERAGE COST OF EXTERNAL CAPITAL

This means that the part of marginal pro…tability of capital which is not paid out by the management as remuneration for the shares and debt, has an impact on the …rm's …nancial reserves, and hence on the discount rate of future pro…ts and on the value of the …rm.

The approach and results presented here slightly diverge from the conventional neoclassical optimal control investment model because the assumptions made on the control of the cash-‡ow by the managers, the fact that market for shares is not necessarily associated with the market for the …rm's control, and, …nally, …nancial market imperfections (and imperfect adjustment of the share price to the value implied by the discounted future pro…ts) introduce a causal link between the ‡ow of pro…ts, the …rm's …nancial structure and the rate of discount of the future ‡ows of pro…t. This can be interpreted as an "inside the …rm" channel of transmission of …nacnial shocks to the real investments. This link between the real and the …nancial side of the economy and its underlying idea is broadly consistent with the "excess sensitivity" empirical literature à la Fazzari, Hubbard and Petersen (1988).

This framework could also help to explain some recent empirical results claiming that including appropriate measures for stock market yields and capital gains would make internal cash ‡ow statistically non signi…cant in investments regressions based on …rms panel data (for instance, Gomes, 2001). In fact, to the extent that both current pro…ts and stock prices simultaneously contribute to determine the (endogenous) rate of discount of future pro…ts, they could turn out to be statistically co-determined and simultaneously correlated with the investments through the …rm speci…c rate of discount of future pro…ts. If the …rms enjoys a long period of high pro…ts and its stock price overshoots (like in the excess volatility case à la Shiller) with respect to the value implied by the pro…ts, so that the …rms experiences a persistent long period of increasing capital gains (like in the "irrational exuberance" case) the results gets even stronger. In other words, an increasingly overvalued share price makes the internally generated …nance cheaper because it allows the managers to pay out less dividends (and keep the shareholders satis…ed, since they are remunerated by the capital gain). This could contribute to explain why some recent empirical analyses (like Gomes, 2001) …nd out that introducing in an investment regression appropriate measures for the stock market prices seem to reduce the statistical signi…cance of the internally generated csh-‡ow

4 Concluding remarks

The simple qualitative framework considered here describes the simultaneous decisions of …rm's investments and …nancial structure, in a context of discrete-time dynamic optimization with imperfect …nancial markets where management hold the control of the …rm, decide upon the allocation of the …rm's cash-‡ow, and the stock price (due to imperfect information and incentive of the manager not to fully reveal their private information on the

…rm's pro…tability) may deviate, in the short run, from the value implied by the discounted future dividends. The simultaneous optimization of the …rm investment and …nancial structure determines a link between the cash-‡ow and the rate of discount of future pro…ts in the intertemporal optimization problem. All this carries two implications: …rst, any shock in the pro…ts or in the …rm's pro…tability has an e¤ect on the …nancial structure of the …rm and hence on the rate of discount of the future pro…ts; second, an exogenous shock to the stock market a¤ects the dividend policy, the pro…t retention, hence the cost of external …nance and the real investments.

Bibliography

Abel, A., B., Eberly,J.,C.,(1994) 'A Uni…ed Model of Investment Under Uncertainty', American Economic Review, 84, pp. 1369-1384

Allen, F., Gale, D., (2000), "Comparing Financial Systems", Cambridge, MA, M.I.T. Press.

Beltratti, A., Kurz, M., (1996), The Equity Premium is no Puzzle, Banca dItalia, Temi di discussione del Servizio Studi, No. 282, October 1996

Benoît, J.-P. (1984) "Financially constrained Entry in a Game with Incompete Information", Rand Journal of Economics, 15, pp. 490-9.

Bernstein, J., I., Nadiri, M., I., (1986), "Financing and Investment in Plant and Equipment and Research and Development", in Peston , M., H., Quandt, R., E., (eds.) Prices, Competition and Equilibrium, Oxford: Philip Allan.

Blanchard, O., J., (1993), "Movements on the Equity Premium", Brooking Papers on Economic Activity, 2, pp. 75-118.

Blanchard, O., J., Lopez-de-Silanez, F., Shleifer, A., (1994), "What Do Firms Do With Cash Windfalls", Journal of FInancial Economics, 36, pp. 337-60.

Brander, J., A., and Lewis, T. R., (1985), "Oligopoly and the Financial Structure - The limited Liability E¤ect", American Economic Review, 75, pp.956-970.

Deutsche Bundesbank (1999), "Corporate Finance in Germany and France: a Comparative Analysis", Monthly Report, October, pp. 29-45.

Dixit, A., K., Pindyck, R., S., (1994), "Investment Under Uncertainty", Princeton: Princeton University Press.

Fazzari, S., M., Hubbard, G., R., Petersen, B., C., (1988), "Financing Constraints and Corporate Investments", Brookings papers on economic activity, 1, pp.141-206.

Gomes, J., F., (2001), "Financing Investments", American Economic Review, pp. 1286-1310.

Greenwald, B., C., Stiglitz, J., E., (1988), "Imperfect Information, Financial Constraints and Business Fluctuations", in Kohn, M. and Tsiang, S., (eds), Financial Constraints, Expectations and Macroeconomics, Oxford, U.K., Oxford University Press.

Greenwald, B., C., Stiglitz, J., E., (1990), "Macroeconomic Models with Equity and Credit Rationing", in Hubbard, R., G., (ed), Information, Capital Markets and Investments, Chicago, University of Chicago Press.

Hart, O., (1995), Firms, Contracts and Financial Structures, Oxford, U.K., Oxford University Press

Kurz, M., (1994a), 'On the Structure and Diversity of Rational Beliefs', Economic Theory, 4, pp. 877-900.

Kurz, M., (1994b), 'On Rational Beliefs Equilibria', Economic Theory, 4, pp. 859-876.

Mazzoli, M., (1998), Credit, Investments and the Macroeconomy, Cambridge, U.K., Cambridge University Press.

Miller, M., Weller, P., Zhang, L., (2002), "Moral Hazard and the US Stock Market: Analysing the Greenspan Put", Economic Journal, Forthcoming.

Poitervin, M., (1989a), "Financial Signalling and the 'Deep Pocket Argument", Rand Journal of Eocnomics, 20, pp.26-40.

Poitervin, M., (1989b), "Collusion and the Banking Structure of a Duopoly", Canadian Journal of Economics, 22 (2), pp.263-277.

Seierstad, A., Sydsæter, K., (1987) "Optimal Control Theory with Economic Applications", Amsterdam, New York, North Holland.

Schmidt, R., H., (1999), Di¤erences between Financial Systems in European Countries: Consequences for the EMU", mimieo, presented at the Bundesbank conference on The Monetary Transmission Process.

Shiller, R., (1989) Market Volatility, Cambridge, MA. M.I.T. Press.

Shiller, R., (2000), Irrational Exuberance, Princeton, N.J.Princeton University Press.

Telser, L. G., (1966), "Cutthroat Competition and the Long Purse", Journal of Law and Economics, 20, pp. 26-40

Tu , P. N. V., (1991), Introductory Optimization Dynamics, Berlin, Heidelberg, Springer-Verlag.