Is Collateralised Borrowing an Amplification Mechanism?

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Abstract

The effect of permitting collateralised borrowing in an otherwise standard business cycle model is examined. We find that powerful income effects cause consumption to be far more volatile than in the standard model, and cause far higher demand for leisure following a positive productivity shock than is usual. These effects are shown to inhibit capital accumulation and are capable of dampening the response of output to technology shocks. There are implications for existing models with credit market imperfections that abstract from labour supply behaviour.

1 Introduction

1.1 Motivation

Macroeconomics has recently looked to credit market imperfections as a possible solution to the 'small shocks, large cycles' puzzle that arises in the context of dynamic stochastic general equilibrium (DSGE) models of the business cycle. The problem is that equilibrium models cannot reproduce business cycle fluctuations of the magnitude seen in the data, given shocks of the magnitude seen in the data. We might add 'asymmetric' to the puzzle, since negative shocks appear to have larger effects than positive shocks, and the almost (log) linear nature of the standard DSGE model does not permit substantial asymmetries. But this problem generally receives less attention. The proposed solution to the puzzle is that small shocks are propagated via some amplification mechanism to produce large cycles. The search for such a mechanism (or mechanisms) has dominated research in the

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DSGE field, and several influential papers now support the idea that the credit market is capable of filling the role.

Under perfect markets, all investment with internal rate of return at least as great as the cost of funds is undertaken, and is Pareto optimal. Microeconomic models predict that deviations from the Pareto optimum will occur when credit markets are characterised by asymmetric information, when contracts are incomplete, when there is the potential for 'hold ups' and so on. One particularly fruitful line of research has been to assume the outcome of investment projects is costly for lenders to observe, which under plausible assumptions leads lenders to offer limited liability debt contracts. Credit market equilibrium is then characterised by underinvestment. Bernanke, Gertler, and Gilchrist (1999) provide a sophisticated analysis of the consequences of this type of friction for the dynamic behaviour of the economy. In their paper, firms that borrow to finance a risky investment must pay an interest premium that is inversely proportional to their net worth. This leads to a 'financial accelerator' that operates through the cost of funds: in booms, asset values are high and borrowing is cheap; in slumps, asset values are low and borrowing is costly. This causes investment and output to be more volatile than under perfect markets.

1.2 Overview

This paper exploits a particular form of credit market friction to investigate the degree of amplification caused by agents leveraging themselves against their asset holdings. Borrowing capacity will be limited by the value of their collateral; equivalently, there will be a lower bound on their net worth. A constraint of this form was motivated by Kiyotaki and Moore (1997). It was based on the idea that the borrower can hold up his creditors by threatening to withdraw his labour, causing low output, after funds have been sunk. In the Kiyotaki and Moore economy, borrowers' ability to make these threats means that creditors require them to post collateral, up to the value of the loan. However, a drawback of their model is the raft of non-standard assumptions they employ.

Interest in developing the collateral approach stems partly from its plausibility as a description of real world credit markets. Lending against collateral, especially real estate collateral, is amongst the most frequently observed credit arrangements. It also provides an alternative to the more complex 'dynamic' agency costs of Bernanke, Gertler, and Gilchrist. Here, we will allow agents to collateralise their borrowing against land. The key feature of land for the purposes of the hold up argument is that it cannot be removed; we could equivalently think about two types of capital good, one that is 'bolted down', and the other that is 'transportable'.

This paper retains the spirit of the Kiyotaki and Moore model, but makes two simplifications and one generalisation. The first simplification is that the economy we study deals only with one group of identical agents; the second, that the constraint depending on collateral values is imposed without rigorous micro-foundations. The advantage of this direct approach is that the flavour of the financial accelerator is retained within a tractable set of models. The generalisation is that we employ the canonical stochastic growth model with smooth preferences and production function. Formally, the model is a small open economy with credit constraints, this approach being due to Kocherlakota (2000). We extend the analytical results of his paper firstly by adding technology shocks and capital depreciation. We show analytically in Appendix A that the economy with borrowing against collateral retains the property of saddle path stability; to put it another way, rising asset prices cannot lead to self-sustaining increases in collateralised borrowing. A more complex model which also adds labour supply to the Kocherlakota model, and for which analytical results are not available, is analysed in §3. Simulation results in §4 reveal some substantial differences from the basic DSGE model, namely a very large consumption response to technology shocks coupled with a muted response of capital and countercyclical hours. The economic interpretation of the results are discussed. Finally, by calculating the volatility of output under various parameterisations, we assess the strength of the borrowing channel as an amplification mechanism.

2 The command optimum

In this section we will solve the dynamic optimisation problem faced by a hypothetical planner, which in this case yields allocations identical to those from decentralised markets.

The representative agent is assumed to have an instantaneous CRRA utility function

$$u(c_t) = \frac{(c_t l_t^{\theta})^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

in consumption and leisure; we assume each agent is endowed with one unit of time per period. This form of the utility function ensures that steady state hours are invariant to the level of productivity[1]. Capital in the economy obeys the law of motion

$$k_t = (1-\delta)k_{t-1} + i_t \tag{2}$$

where δ is the rate of depreciation and i is gross investment. The production technology is Cobb-Douglas in capital, land (denoted x) and hours

$$y_t = z_t k_{t-1}^{\alpha_1} x_{t-1}^{\alpha_2} h_t^{\alpha_3} \tag{3}$$

where z is a stochastic technology shock, and there are constant returns to scale. The possibility of borrowing from the foreign sector, and of effecting land sales or purchases[2], modifies the standard budget constraint so that time t resources are given by

$$y_t + (1-\delta)k_{t-1} + b_t + q_t x_{t-1} \tag{4}$$

where b is borrowing and q is the price of land. Expenditures include debt repayments at the world rate of interest R,

$$(1+R)b_{t-1} + q_t x_t + c_t + k_t.$$

The modification to this fairly standard setup is to introduce a collateral constraint such that agents may borrow only up to the value of their land holdings,

$$q_t x_t - b_t \geq 0. \tag{5}$$

We will follow Kocherlakota in initially assuming no utility from leisure ($\theta = 0$) or labour in production ($\alpha_3 = 0$).

[1]King and Rebelo (1999, p. 945) show this is true for the more general form $u(c,l) = [cv(l)]^{1-\sigma}/(1-\sigma)$, under certain regularity conditions for v. These conditions are satisfied for our parameterisation.

[2]We assume land can only be held by domestic residents.

The Lagrangian for this problem is

$$\begin{aligned} L &= \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} + \\ & \sum_{t=0}^{\infty} \beta^t \pi_t \{ z_t k_{t-1}^{\alpha_1} x_{t-1}^{\alpha_2} + (1-\delta) k_{t-1} + b_t - (1+R) b_{t-1} + q_t (x_{t-1} - x_t) - c_t - k_t \} + \\ & \sum_{t=0}^{\infty} \beta^t \lambda_t \{ q_t x_t - b_t \} \end{aligned}$$

The first order conditions for this problem include

$$u'(c_t) = \mathrm{E}_t \beta u'(c_{t+1})(1 + rr_{t+1}) \tag{6}$$

where rr is the net rate of return on capital between t and $t+1$, and the marginal utility of consumption is equal to the Lagrange multiplier π. Equation [6] has the usual interpretation that the gain from a marginal addition to the capital stock is equal to the gain from a marginal addition to consumption at an optimum. Since the extra utility derived from more capital accrues tomorrow, we discount it so that both quantities are measured in terms of today's utility.

Secondly, the dual variable λ (≥ 0) measures the shadow price of collateral. It is straightforward to see that the credit constraint must bind ($\lambda > 0$) in the steady state, and in every period, so long as $rr > R$. This has the straightforward interpretation that agents want to add to their stock of debt only when returns in the domestic economy make it profitable to do so. To see this, substitute [6] into the first order condition for borrowing to find

$$\begin{aligned} \lambda_t &= \mathrm{E}_t[\pi_t - \beta(1+R)\pi_{t+1}] \\ &= \mathrm{E}_t \pi_{t+1}(1 + rr_{t+1}) - \mathrm{E}_t \pi_{t+1}(1+R) \\ &= \mathrm{E}_t \pi_{t+1}(rr_{t+1} - R) \geq 0 \end{aligned} \tag{7}$$

In the steady state, this condition is satisfied by assuming that agents' discount rates exceed the world interest rate, since the steady state rate of return on capital coincides with its reciprocal. Agents are therefore impatient enough to want to bring future resources into the present, and thereby make the issue of credit constraints relevant. Giving different groups of agents different discount rates is a standard simplifying device in this literature.

We also have from the first order condition for land that

$$u'(c_t) = \mathrm{E}_t \beta u'(c_{t+1}) \left[\frac{\alpha_2 y_{t+1} + q_{t+1}}{q_t} \right] + \lambda_t \tag{8}$$

where the term in square brackets is equal to the return on land (exploiting a normalisation in the aggregate quantity of land to unity, and the form of the production function). Say we are faced with allocating a marginal unit of resources either to current consumption or to purchasing land; the benefit of consumption now is the current marginal utility; the benefit of land has two components. Firstly, tomorrow's extra output plus the capital gain (or loss) enjoyed from holding land; secondly, the benefit today of having more collateral and therefore more borrowing, to finance current consumption or investment.

Equation [8] can be combined with the first order conditions for borrowing to find

$$0 = \mathrm{E}_t \beta u'(c_{t+1}) \left[\frac{\alpha_2 y_{t+1} + q_{t+1} - (1+R) q_t}{q_t} \right];$$

land must then be priced according to the following arbitrage relation

$$1 = (1+R)^{-1} \mathrm{E}_t \left[\frac{q_{t+1} + \alpha_2 y_{t+1}}{q_t} \right] + \zeta_{t+1}, \tag{9}$$

where ζ is a risk premium[3]. Recalling that the expression $1 + R$ is the (given) return on lending; then as the quantity of borrowing and the value of land holdings move one for one, returns to lender and borrower are equalised, up to a (second order) risk premium. As we have a representative agent economy, price will necessarily adjust such that land is in zero net supply. The expression [9] can be rearranged in terms of q_t, and iterated forward to give the standard discounted dividend formula.

[3] $\zeta = (1+R)^{-1} \mathrm{Cov}_t[M_{t+1}, (q_{t+1} + \alpha_2 y_{t+1})/q_t] / \mathrm{E}_t M_{t+1}$, where M is the stochastic (or 'utility') discount factor $\beta \pi_{t+1}/\pi_t$.

3 Quantitative dynamics

This section analyses the quantitative properties[4] of a stochastic calibrated version of the model studied in Appendix A. There are several potential gains from this approach: we can analyse a more general environment; we can make comparisons with other models; and we can make comparisons between the theoretical moments predicted by our model and moments observed in the data. As we will see, the current framework proves less successful on the last point, although it does shed some light on the first two. The model is a variation on the standard DSGE model described by King and Rebelo (1999), and this will form a natural point of comparison. However, the assumptions we make in this paper are geared towards uncovering the ability of the collateralised borrowing mechanism to act as an amplification channel, rather than fitting data.

3.1 Formulation and calibration

The first order conditions describing the behavioural relations of the economy are demanding to solve even numerically. The standard method is to examine linearised relationships in the neighbourhood of the steady state[5]. Here we briefly review the steady state properties of the model. The steady state values of the variables are calculated by assuming that any random variables are at their unconditional means. Then steady state land prices are simply the discounted future value of land

$$q_* = \frac{1}{R}\alpha_2 y_* \tag{10}$$

[4]Throughout, one must pay particular attention to avoiding parameterisations that violate the conditions under which the model solutions were obtained. I am thinking of constraint qualification and the conditions for the credit constraint to bind, in particular. The validity of the linear approximation is assumed, partly on the basis of the results contained in the Appendix A.

[5]This is a completely symmetric procedure to that employed in Appendix A since a first order Taylor expansion of a function $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ around $\mathbf{x}_0$ is given by

$$f(\mathbf{x}) - f(\mathbf{x}_0) = \mathbf{J}(\mathbf{x} - \mathbf{x}_0) + O(\mathbf{x}^2),$$

and in fact we will see below that we can recover numerically identical roots of the Jacobian $\mathbf{J}$ as found there analytically.

and, from the budget constraint, consumers take a fraction of output less the cost of depreciation

$$c_* = (1 - \alpha_2)y_* - \delta k_*. \tag{11}$$

The steady state capital stock is given by

$$k_* = \left(\frac{\alpha_1}{rr_* + \delta} \right)^{1/(1-\alpha_1)} \tag{12}$$

where the denominator is the gross rate of return on capital, $rr_* = \alpha_1 y_*/k_* - \delta$, and where I have normalised the (irrelevant) mean productivity term z_* in the numerator to unity. As α_1 is increased towards unity, the steady state capital stock explodes. This means that the steady state gross investment to output level must increase also, to replace all the worn out machinery. Although consumption is increasing also, it is increasing less fast than investment, so its share in output falls. Finally, suppose utility is gained from leisure, $b > 0$, and labour is an input in production, $\alpha_3 > 0$. For an approximate fit to the data, set the steady state proportion of time allocated to work h_* to be one third. Now work backwards to determine the parameter in the utility function

$$\theta = \frac{\alpha_3 y_* (1 - h_*)}{c_* h_*}. \tag{13}$$

I will follow the usual practice of allowing the technology shock to be autocorrelated, with an iid shock ϵ

$$\ln z_t = (1 - \psi) \ln z_* + \psi z_{t-1} + \epsilon_t,$$

so the percentage deviations of the process from its mean (denoted by the circumflex) are well approximated by

$$\hat{z}_t = \psi \hat{z}_{t-1} + \epsilon_t \tag{14}$$

so long as the support of ϵ is 'small'.

Our baseline parameterisation is given in Table 1, and some key macroeconomic ratios in Table 2. Parameter values were chosen to be close to the standard DSGE setup given in King and Rebelo (1999). The intertemporal elasticity of substitution is unity, giving log-separable preferences in consumption and leisure; the intratemporal elasticity of substitution between consumption and leisure θ is set as in [13]. On the production side,

Table 1: Baseline parameter values

α_1	α_2	α_3	β	σ	θ	R	δ	ψ
0.325	0.075	0.60	0.975	1.0	1.525	0.02	0.019	0.979

Table 2: Steady state values at baseline parameter values

c_*/y_*	i_*/y_*	h_*	$R\ q_*/y_*$	rr_*	ε_w^h
0.78	0.14	0.33	0.075	0.026	2.0

the labour share is set close to the standard value, but we deviate from usual practice by splitting the remaining payments between two types of capital good, the 'bolted down' and the 'transportable' mentioned in §1.2. Turning to the macroeconomic ratios, the shares of consumption and investment in GDP are close to the values seen in data, and the net rate of return on capital rr is equal to the rate of time preference implied by the consumer's discount factor, at just over 2.5% (note this exceeds the world interest rate). The labour supply elasticity is 2, and note in particular that this is independent of the value of the intertemporal substitution elasticity (see [15] below). Finally, the term Rq/y indicates the share of output going to debt service at 7.5%; total debt stands at 350% of GDP. These rather high numbers are chosen to better determine the impact of leverage on dynamics. It is to this that we now turn.

3.2 The recursive equilibrium law of motion

The linearised first order conditions are shown in [17]-[21] below. Since the variables are approximately percentage deviations from the steady state, the coefficients may be interpreted as elasticities, in the neighbourhood of the steady state. To illustrate the procedure, the first order condition for labour supply is

$$-\theta c_t^{1-\sigma}(1-h_t)^{\theta(1-\sigma)-1} + \pi_t w_t = 0$$

where w is the competitive wage, equal to the value of the marginal product of labour $\alpha_3 y/h$, and π is the shadow resource price discussed in §2. Substituting for this price (and after multiplying by c^σ and dividing by $(1-h)^{\theta(1-\sigma)}$), the elasticity of labour supply can be found to be

$$\varepsilon_w^h = \frac{\theta c_t}{w_t - \theta c_t}. \tag{15}$$

Now substitute for the wage also, and use the Cobb-Douglas assumption to write the marginal product so that

$$\theta c_t \frac{1}{1-h_t} = \alpha_3 \frac{y_t}{h_t}$$
$$\text{or} \quad -\alpha_3 h_* y_* \mathrm{e}^{\hat{h}_t} + \alpha_3 y_* \mathrm{e}^{\hat{y}_t} - \theta c_* h_* \mathrm{e}^{\hat{c}_t + \hat{h}_t} = 0$$

where for example $\hat{h}_t = \log(h_t/h_*)$. The first order Taylor approximation around the steady state (that is, around zero for variables with a circumflex), and the steady state condition delivers the equation [17] in the block below. Since $\hat{w} = \hat{y}_t - \hat{h}_t$, substitution yields an equation in hours, consumption and wages, whereupon

$$\hat{h}_t = \frac{\theta c_*}{w_* - \theta c_*}(\hat{w}_t - \hat{c}_t) \tag{16}$$

which establishes the equality with the labour supply elasticity at the steady state.

$$-\alpha_3 y_* \hat{h}_t + \theta c_* h_* (\hat{y}_t - \hat{c}_t) = 0 \tag{17}$$

$$-\sigma(\mathrm{E}_t \hat{c}_{t+1} - \hat{c}_t) + \theta(1-\sigma) h_* (\mathrm{E}_t \hat{h}_{t+1} - \hat{h}_t) + (1 - \beta[1-\delta])(\mathrm{E}_t \hat{y}_{t+1} - \hat{k}_t) = 0 \tag{18}$$

$$\mathrm{E}_t \hat{q}_{t+1} - (1+R)\hat{q}_t + R\mathrm{E}_t \hat{y}_{t+1} = 0 \tag{19}$$

$$\hat{y}_t - \hat{z}_t - \alpha_1 \hat{k}_{t-1} - (1 - \alpha_1 - \alpha_2)\hat{h}_t = 0 \tag{20}$$

$$y_* \hat{y}_t + (1-\delta) k_* \hat{k}_{t-1} - c_* \hat{c}_t - k_* \hat{k}_t + q_* \hat{q}_t - (1+R) q_* \hat{q}_{t-1} = 0 \tag{21}$$

$$(1 + rr_*) k_* \widehat{rr}_t - \alpha_1 y_* (\hat{y}_t - \hat{k}_{t-1}) = 0 \tag{22}$$

Note that in [19] the risk premium is second order and therefore drops out. There are six equations in six endogenous variables, which can be solved as a block. The relevant solution concept is that of a recursive competitive equilibrium. That is, we seek a law of motion for the variables of the system such that agents maximise their utility and all markets clear. This takes the form of a recursive equation on a minimal set of state variables (in our case, z, k and q). We use the generalised method of undetermined coefficients given in Uhlig (1999) to find this law, from which any desired theoretical moments can be computed.

Table 3: Cross correlations with output (HP Filtered)

Land prices	0.40	0.60	0.82	0.98	0.69	0.45	0.24
Capital	0.44	0.59	0.75	0.81	0.33	0.05	−0.11
Hours	−0.44	−0.58	−0.72	−0.77	−0.27	0.01	0.17
Returns	0.34	0.56	0.80	1.00	0.83	0.63	0.42
Consumption	0.43	0.61	0.81	0.93	0.53	0.27	0.09
Output	0.36	0.58	0.81	1.00	0.81	0.58	0.36
Technology	0.40	0.60	0.82	0.98	0.69	0.44	0.24
$t+j$	−3	−2	−1	0	1	2	3

4 Results

4.1 Is there amplification?

There is a single stable root associated with the state variable of the economy, capital, equal to 0.559. By setting $\alpha_3 = \theta = 0$ we find a root of 0.941, which is equal to the stable root of the Jacobian [26] found in Appendix A. Table 3 shows the cross correlations between the model variables and output at up to three quarters of lead and lag. For example, the number in the top left hand cell of the table gives the correlation of land prices at $t-3$ with output at time t.

The properties of this model appear to be significantly different both from the standard DSGE model reported by King and Rebelo, and from the properties of the data that is the ultimate arbiter of model performance. Consumption is a clear place to start: in the US data[6] from 1950-2000, the contemporaneous correlation with output is 0.78; the standard model produces a correlation of 0.94; our model with borrowing, a figure very close to the standard model of 0.93. However, the relative volatility of consumption, as measured by the standard deviation of consumption divided by the standard deviation of output, in the data the figure is 0.81, for the standard model 0.44, and for our model 1.65 (see Table 4). Turning to hours worked, we see procyclical hours and fairly stable wages in the data, whilst the standard model delivers a rather high positive correlation of both; our model predicts a negative correlation between output and hours, with large changes in wages (in

[6]Logarithms detrended by the HP filter with bandwidth 1600, as is usual in the literature.

Table 4: Standard deviations (%), HP filtered series

	IES* = 1			IES = 1/3	
	No hours**	Borrowing	No borrowing	Borrowing	No borrowing
Land prices	3.00	0.401	0.916	0.419	1.740
Capital	4.75	0.113	0.024	0.232	0.035
Hours	-	0.430	0.071	0.184	0.063
Returns	-	0.034	0.039	0.036	0.039
Consumption	3.59	1.236	0.991	1.005	0.976
Output	1.54	0.748	0.906	0.884	0.918
Technology	1.00	0.935	0.935	0.935	0.935

** Intertemporal Elasticity of Substitution* $(1/\sigma)$
***Figures in the 'no hours' column are relative to the no hours, no borrowing case. Calibration maintains proportionality between capital and land shares.*

Figure 1, the response of the wage is the gap between the lines for output and hours, as discussed in §3.2). On the other hand, hours worked are less volatile in this model than in the standard model with an identical labour supply elasticity.

To ascertain the degree of amplification[7], we compared the standard deviations of the key variables with those from a model with no borrowing permitted (see Appendix B). Table 4 shows that there is much lower volatility in land prices when land is used to collateralise borrowing, but somewhat more volatility in consumption, hours worked and capital. Output is less affected, consistent with the offsetting changes in capital and hours seen in Table 3. The ratio of the standard deviation of output to productivity shocks is close to unity, indicating that in this model, rather than there being an amplification mechanism, shocks are being damped. This compares with a ratio of 1.48 for the standard model, which King and Rebelo describe as 'limited amplification'. This finding appears

[7]The question of amplification was addressed explicitly in the agency cost framework by Bacchetta and Caminal (2000). They consider an overlapping generations model, within which a careful spefication of the type of risks firms face leads to an absence of any external funds premium; rather, firms face quantity rationing as in our model. Amplification effects arise from the redistribution of wealth between credit constrained and unconstrained sectors; since credit constrained firms have less capital, and therefore a higher marginal product, than unconstrained firms, redistributions of capital that favour the constrained lead to larger increases in output than distributionally neutral shocks. Here, we model only the constrained agents, which has the advantage of simplicity. As we are interested in the amplificaton of shocks over and above the usual business cycle mechanisms, we are not impeded by looking solely at the constrained sector.

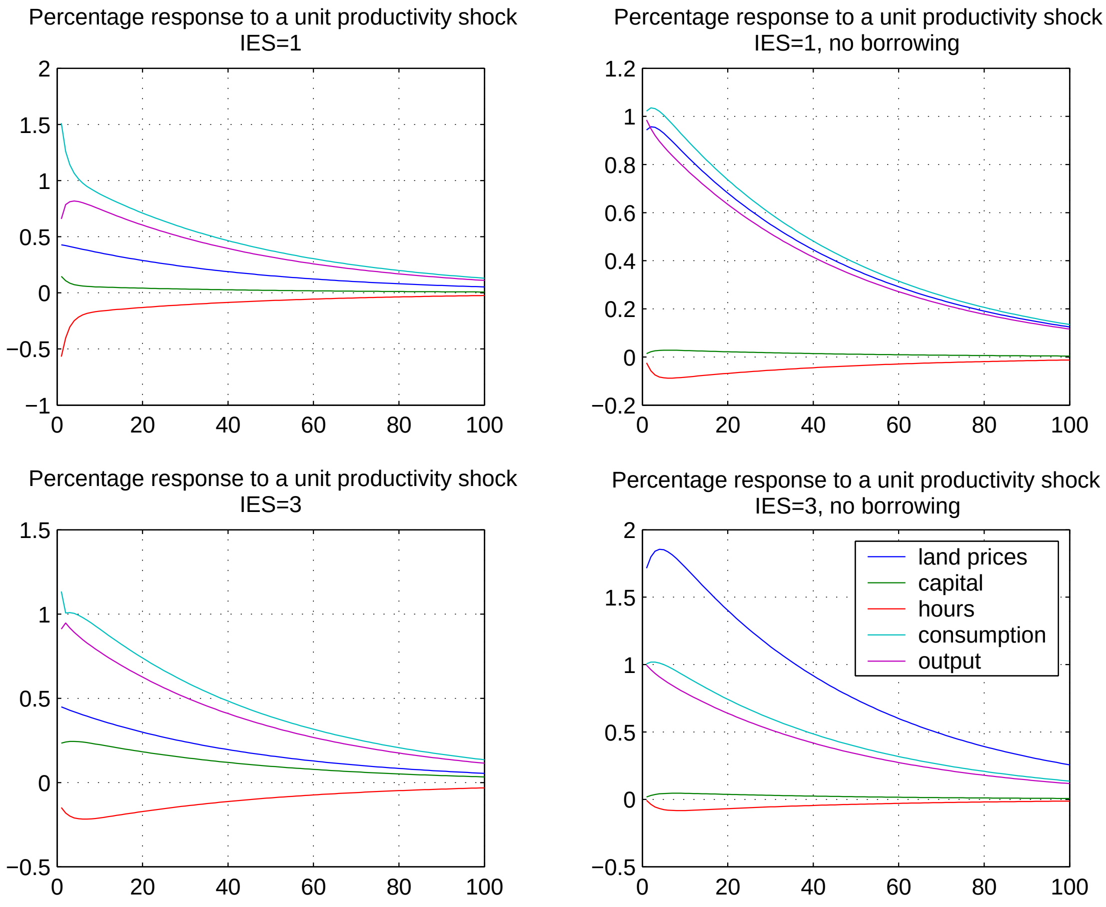
to be a severe drawback of the model, since it exacerbates the 'small shocks, large cycles' puzzle discussed in §1.1. Much more noticeable is the strong amplification when the labour share α_3 is set to zero, with output then being 1.53 times as volatile as technology. Relative standard deviations are given in the first column of Table 4; the standard deviation of output is 1.54 times greater when borrowing than when there is not borrowing. It appears that this is the only case where significant amplification occurs, suggesting that labour market behaviour may be responsible for dampening output fluctuations. A similar negative result is obtained when Kocherlakota introduces a third input to the production process, that is inelastically supplied. He summarises by noting that "it is theoretically possible for small income shocks to lead to arbitrarily large output movements... [however] this possibility is not robust" (p. 7). We now turn to the reasons behind the Kocherlakota findings, and the non-standard behaviour of the leveraged economy.

4.2 Interpreting the results

The response of the model economy to an autocorrelated positive technology shock (the Impulse Response Function) is shown in Figure 1; this function is computed by setting $\epsilon = 1$ for one period, and then recursively calculating values for the endogenous variables. It is noticeable that consumption is much less smooth than in the standard case, and that hours initially rise, but then fall. This information was also contained in Tables 3 and 4. There also appears to be much less capital accumulation than usual. When the intertemporal elasticity of substitution (IES) is smaller, the consumption response is now somewhat smaller, as one would expect, and the response of capital a little larger. However, hours remain negatively correlated with output.

To see why, consider the impact of the shock on the labour market: the marginal product of labour is raised, which raises the competitive wage; the substitution effect of higher wages acts to make leisure relatively less attractive, whilst the income effect makes the agent want to increase both leisure and consumption. There is also an intertemporal substitution effect stemming from the fact that wages are high today relative to the future, that induces extra effort today. However, in this model these effects are augmented by

Figure 1: Impulse response functions



the agent's ability to capitalise his land assets. Thus an increase in asset values leads to a large increase in current income through higher borrowing, and the effect of this extra income is, again, to induce more demand for consumption and leisure.

Recall that it is the real interest rate that determines the strength of the agent's desire to substitute away from current leisure and consumption to exploit the greater returns to current effort that will expand future resources. Here, although the marginal product of capital is boosted by the increase in total factor productivity, the real interest rate remains low, since work effort is both low today and expected to be low in the future. The unresponsiveness of the real interest rate mutes the response of investment to the shock so that little extra capital is accumulated, and the usual output amplification mechanism is closed off.

We must also take into account a second effect that comes specifically from land, rather than from borrowing via land. By [4], the increase in land values raises current resources directly, and this augments the income effect described above. This means that even in the 'no borrowing' economy, there is an additional income effect increasing the demand for leisure that does not exist in the standard model. In fact, this effect is accentuated because land prices react more in the no borrowing case. This is a result of the change in the land pricing relation that occurs when the one-to-one correspondence between land prices and borrowing is broken. Land is no longer discounted at the world interest rate, but rather according to the agent's stochastic discount factor (SDF),

$$1 = \beta \mathrm{E}_t \frac{u'(c_{t+1})}{u'(c_t)} \left[\frac{\alpha_2 y_{t+1} + q_{t+1}}{q_t} \right] \tag{23}$$

(see Appendix B and equation [30] for the derivation). Terms in the SDF are first order in this case; recall that in [9] they were second order. As can be seen from the consumption Euler equation, the SDF falls when the real interest rate rises, and the result of the lower discount factor is as usual that the value of land increases, for any given dividend stream. The simulations show that these additional income effects are strong enough to cause labour supply to fall.

The findings outlined here have implications for other models of credit market imperfections that feature an amplification mechanism that works through entrepreneurial net worth. For example, Bernanke et al. (1999) assume that entrepreneurs supply their labour inelastically to the market; and Bacchetta and Caminal (2000) have two classes of agent, who operate production technologies that do not require labour input. It is likely that suppression of the channel identified in this paper plays a role in their results. However, it is not immediately obvious that entrepreneurs behave in the way that the model considered here predicts; a possible example is the behaviour of entrepreneurs in the technology boom of the late 1990s, many of whom did cash in[8].

4.3 Shortcomings

The symmetrical nature of the model means that the above analysis should apply in reverse when there is a negative income shock. In that case, the agent will ant to increase his hours of work, and dramatically scale back his consumption. However, the assumption that the credit constraint remains binding is perhaps incorrect, as the fall in the RIR may be sufficient to violate condition [7]. It is possible that following a large shock the collateral constraint ceases to be binding. By definition, we have analysed behaviour in a small neighbourhood of the steady state, that is, we rule out the possibility of 'large shocks' moving us into a different regime.

So how good is the approximation likely to be with our parameterisation? One important test is whether the model can erroneously generate negative values for λ. We would like to simulate, but we can only generate observations on $\hat{\lambda}$, which being a log linear approximation implies positive levels by definition. However, a plot of a simulation of the model series suggests that in fact the approximation may be quite poor for the 'no hours' case; the range of variation in all series is less than ± 0.7, except for $\hat{\lambda}$ for which it is more than ± 20. Since the absolute approximation error for this higher figure is 0.018, and the steady state value $\lambda_* = 0.039$, our results could be misleading. The approximation to our model may fail predict that for some large negative shocks, the credit constraint is slack, giving incorrect dynamics. This form of mis-specification is rarely discussed in the literature, partly as it is difficult to diagnose. However, in the model with a labour

[8] Thanks to Tony Atkinson for suggesting this to me.

market, the RIR moves rather little, so the condition for the constraint to be binding is more likely to be satisfied.

A further consequence of this is that we should take the analysis here as indicative of the dynamics of the economy in a credit constrained regime. As it stands, the alternative, non-constrained, regime is inconsistent, in the sense of having more unknown variables than equations. This points to a rather unsatisfactory aspect of the model, namely that it has less to say about the role of credit constraints *per se*, than about the effects of leverage. The credit constraints are a device to make borrowing bounded, but it is the possibility of borrowing that leads to a new type of dynamics.

5 Conclusion

This note has analysed a simple dynamic model of borrowing and credit constraints. Building on Kocherlakota (2000), it extends previous results into the context of the standard business cycle model whilst remaining highly tractable. Simulation results were analysed, and their validity was checked by deriving analytic results in a slightly simplified version of the model.

Some new results on the changes in behaviour induced by access to cheap credit were demonstrated. The first main finding concerned the behaviour of hours worked and consumption. This indicated that there are additional income effects arising from the possibility of borrowing that tend to increase consumption and reduce hours following a positive productivity shock. The second finding, and the central concern of this paper, was that the collateralised borrowing mechanism does not significantly increase output fluctuations except under the special case where labour the market is not included in the model. This finding sounds a cautionary note for other general equilibrium models that incorporate credit market imperfections, as their results may rely partly on the suppression of this channel.

An interesting extension would be to specify a different production technology, that allowed for differential shocks, rather than simply shocks to total factor productivity. For example, if there were shocks that impacted land and capital in different ways, we could see tighter credit constraints at the same time as higher capital productivity. This is left to future research.

A Local dynamics

In this section we will establish the dynamic behaviour of the system under the collateral in advance constraint. The reason for doing this is to gain confidence in the simulation results given in §3. Not all economies have equilibria that lend themselves to study of local behaviour, and many economies have dynamic behaviour that is sensitive to the selected parameterisation. This section shows that the methods we will apply are likely to be valid.

To demonstrate the result, we first exploit the equality between borrowing and the price of land that exists when the credit constraint is binding to eliminate the former from the budget constraint. Further, from [9] the change in land prices can be written in terms of the marginal product of land under the condition that there is 'full collateralisation'. Focusing on the deterministic part of the system, we now have two implicit difference equations in two unknowns, capital and consumption

$$(1-\alpha_2)zk_t^{\alpha_1} + (1-\delta)k_t - c_{t+1} - k_{t+1} = 0 \tag{24}$$

$$\beta c_{t+1}^{-\sigma}\{\alpha_1 z k_t^{\alpha_1 - 1} + (1+\delta)\} - c_t^{-\sigma} = 0 \tag{25}$$

The dynamics can be analysed by examining the Jacobian of the system $\mathbf{y} = G(\mathbf{x})$, where $\mathbf{y}$ is the one period ahead value of $\mathbf{x} = (k, c)'$. This can be calculated by exploiting the fact that if $F(\mathbf{y}, \mathbf{x}) = 0$ implicitly defines such a system in the neighbourhood of the steady state, then the ith column of the Jacobian of G is given by

$$\begin{pmatrix} \frac{\partial y^1}{\partial x^i} \\ \frac{\partial y^2}{\partial x^i} \end{pmatrix} = -(\mathrm{D}F)^{-1} \begin{pmatrix} \frac{\partial F^1}{\partial x^i} \\ \frac{\partial F^2}{\partial x^i} \end{pmatrix}$$

where $\mathrm{D}F$ is the derivative of the F evaluated at $\mathbf{x}_*$.

After some tedious algebra, it can be established that the relevant Jacobian is given by

$$\mathbf{J} = \begin{bmatrix} \frac{1}{\beta} - \alpha_2(1-\beta[1-\delta])(1 + rr_*\frac{\sigma-1}{\sigma}) & -1 \\ -\frac{rr_*}{\sigma}\alpha_2(1-\beta[1-\delta]) & 1 \end{bmatrix} \tag{26}$$

Taking the limit as $\sigma \rightarrow 1$ of the instantaneous utility function [1], logarithmic utility pertains. It can easily be seen that this case corresponds to roots $1/\beta$ and $1 - \alpha_2(1 - \beta[1-\delta])$. Since β is assumed to be less than unity, the first root is (backwards) unstable; the second root must lie in the unit interval, so is stable, making the system a saddle. However, as the land share α_2 approaches zero, the second root approaches unity, at which point the system becomes a source. Land is valued because it contributes to the flow of goods in the economy via the production function[9]. So generally, a lower share of land in production results in more volatile land prices.

Now consider the case where $\sigma \rightarrow \infty$, that is, agents are very unwilling to substitute consumption over time. The roots in this case can be shown to be

$$\mu_1, \mu_2 = \frac{\beta + \Upsilon \pm \sqrt{\{\beta - \Upsilon\}^2}}{2\beta}$$

where $\Upsilon = 1 - \alpha_2[1 - \beta(1-\delta)]$. As agents become very impatient, as well as very unwilling to intertemporally substitute, $\lim_{\beta \rightarrow 0} \mu = \pm\infty$ and the system is again a source. However, as agents become very patient,

$$\lim_{\beta \rightarrow 1} \mu = 1 - \tfrac{1}{2}\left(\delta\alpha_2 \pm \delta\alpha_2\right),$$

and so one root approaches unity, while the other approaches $1 - \delta\alpha_2 < 1$, and the system is a saddle.

Saddle path stability is generic in the representative agent single sector growth model, and has been shown to extend to the case of collateralised borrowing as long as agents display some impatience and some dislike of intertemporal substitution, and as long as land has some role in production (and can therefore be priced)[10].

[9] An alternative modelling strategy is to allow it to enter directly into agents' utility functions.

[10] This result makes it impossible to use Kocherlakota's method for analysing the amplification effects of credit (Kocherlakota, 2000, p. 7).

B The no borrowing case

This appendix solves the representative agent's problem for the economy in which no borrowing is permitted. In this case the maximand remains

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \tag{27}$$

but the resource constraint in period t is now

$$z_t f(k_{t-1}, x_t, h_t) - c_t + (1 - \delta)k_{t-1} - k_t + q_t(x_{t-1} - x_t) \geq 0. \tag{28}$$

This setup is the standard RBC model with a fixed factor of production. The first order conditions include:

$$\beta \mathrm{E}_t u_1(c_{t+1}, 1 - h_{t+1})[z_{t+1} f_1(k_t, x_{t+1}, h_{t+1}) + (1 - \delta)] = u_1(c_t, 1 - h_t) \tag{29}$$

$$\beta \mathrm{E}_t u_1(c_{t+1}, 1 - h_{t+1})[z_{t+1} f_2(k_t, x_{t+1}, h_{t+1}) + q_{t+1}] = u_1(c_t, 1 - h_t) q_t \tag{30}$$

$$u_1(c_t, 1 - h_t) z_t f_3(k_{t-1}, x_t, h_t) = u_2(c_t, 1 - h_t) \tag{31}$$

These are respectively, the consumption Euler equation, the arbitrage pricing relation for land, and the labour supply equation. Aggregate relations exploit identical restrictions on the sale and purchase of land, and the fact of its fixed total supply, as in the case analysed in the text.

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