

# Effects of the Exchange-Rate Regime on Trade: The Role of Price Setting

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## Abstract

In a stochastic new open-economy macroeconomics (NOEM) model which parallels alternative invoicing conventions, namely consumer's currency pricing (CCP) vs. producer's currency pricing (PCP), we revisit the question whether the exchange-rate regime matters for intra-industry trade. We show analytically that under full symmetry, only money shocks and separable but otherwise very general utility, it is irrelevant in affecting expected trade-to-output ratios. A peg-float comparison is nevertheless meaningful under PCP, although not CCP, in terms of volatility of national trade shares: by shutting down the pass-through and expenditure-switching channel, a peg then stabilizes equilibrium trade-to-GDP at its expected level.

**JEL Classification:** F10, F33, F41.

**Keywords:** alternative price setting, international intra-industry trade, exchange-rate regimes, stochastic NOEM models.

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# 1 Introduction

The present paper belongs to the rapidly growing new open-economy macroeconomics (NOEM) literature.<sup>1</sup> Our objective is to revisit, within this sticky-price optimizing approach and explicitly accounting for monetary uncertainty in general equilibrium, the classic subject of exchange rate and trade determination. In particular, we here reconsider in a fully-symmetric NOEM context and under *alternative price-setting conventions* the question whether the exchange-rate regime matters for international *intra*-industry trade. Comparing *consumer's* currency pricing (CCP) vs. *producer's* currency pricing (PCP), we are able to answer in what sense this is the case. In a self-contained theoretical analysis that explicitly parallels a CCP to a PCP model version, we derive from first (micro-)principles important (macro-)outcomes. Some of them are really novel while the positive and normative implications of other have been debated for long, but largely within ad-hoc frameworks in the Mundell-Fleming-Dornbusch tradition.

More precisely, this paper builds on the stochastic representative agent set-up under CCP proposed in Bacchetta and van Wincoop (1998, 2000 a). As noted by these authors, their "benchmark monetary model" of international intra-industry trade – together with the similar ones developed in Obstfeld and Rogoff (1998, 2000) under PCP, we would add – is intended as a starting point in modern research on monetary policy in open economies. Its main contribution, which we pursue here as well, is to recast traditional comparisons of exchange-rate arrangements in a *general equilibrium* setting that explicitly considers the role of *macroeconomic uncertainty*. We thus explore further Bacchetta and van Wincoop's (2000 a) single-period benchmark, focusing our attention on trade prices and flows, to essentially compare its equilibrium outcomes under assumptions of polar invoicing practices in cross-border transactions, namely CCP vs. PCP. Theoretical analysis of these extremes allows us to draw some clear-cut, mostly qualitative conclusions on the effects of the exchange-rate regime – modelled simply as *float* vs. *peg* – on relative prices and key trade measures as well as on the underlying consumption and labor/leisure choices.

Our principal import is to demonstrate that price-setting assumptions, fundamental in any open-economy model with nominal rigidity, affect in a crucial way optimal consumption allocations under *monetary* uncertainty and, consequently, any microfounded international trade analysis. In a preview of our results we can state that irrespective of the invoicing assumed, the exchange-rate regime does not matter for the *expected level* of trade-to-output ratios by country, which is always 1 given symmetry and frictionless trading, as well as for the ex-ante and ex-post trade balance, always 0. Yet under PCP, but not CCP, it matters for the *volatility* of national trade shares. A peg would thus stabilize, under PCP, the equilibrium trade share in each country across states of nature at its expected level. This latter level coincides with the one under CCP, which is the same ex-ante as ex-post. We identify the difference in the effects of the

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<sup>1</sup>As defined and exhaustively classified in the recent survey by Lane (2001). A narrower and more technical summary of the basic NOEM methodology is also provided in Sarno (2001).

exchange-rate regime on equilibrium trade flows as originating in the particular currency denomination of transactions, hence, the implied *exchange-rate pass-through*, and, ultimately, *expenditure switching*. In our symmetric framework, this major channel of international spillover of monetary shocks is *absent* under CCP and float. As to the PCP model version, a peg effectively *shuts it down*, by equalizing at the neutral unitary level the relative price of foreign goods in terms of domestic analogues agents in both countries face ex-post.

We would not survey here the voluminous literature, classic as well as modern, on the subject we are interested in. We briefly discuss instead only those lines of relevant research that have strongly influenced our motivation for the paper as well as our modelling strategy. In doing so, we also highlight in the next two subsections *two essential features* of our set-up which would have important implications in any open-economy model with price rigidity.

## 1.1 Monetary Uncertainty in General Equilibrium

*Monetary uncertainty* generating *exchange-rate risk* is inherent in issues related to international trade, welfare and macroeconomic policy analysis in which risk-averse agents are involved. To be properly studied, such issues have therefore to be cast in *general equilibrium* frameworks that are *explicitly stochastic*.<sup>2</sup> That is why we have purposefully chosen to follow a recent approach in NOEM theoretical modelling, introduced by Obstfeld and Rogoff (1998, 2000) and Bacchetta and van Wincoop (1998, 2000 a). It extends the deterministic "redux" exchange rate model of Obstfeld and Rogoff (1995, 1996: Chapter 10) and its variations in Corsetti and Pesenti (1997, 2001 a, b, 2002). To our knowledge, the "redux" model was the first microfounded open-economy general-equilibrium framework with rigid prices and monopolistic competition designed to explain exchange-rate *dynamics*. Traditional research on exchange rates and trade was either general-equilibrium but flexible-price,<sup>3</sup> or sticky-price but ad-hoc.<sup>4</sup> If the impact of uncertainty on exchange rates and, hence, trade and consumption flows was at all considered in it, analysis was restricted to partial-equilibrium models, as duly pointed out in Bacchetta and van Wincoop (2000 a).<sup>5</sup>

To allow for *analytical* solutions, the explicitly stochastic NOEM literature has been technically implemented under simplifying assumptions. *Log-normal* processes for shocks, and, consequently, for the endogenous variables, as well as rather *restrictive* specifications of utility are usually imposed in such models, e.g. in Obstfeld and Rogoff (2000). Often, it is also assumed that the Law of One Price (LOP) and, hence, *Purchasing Power Parity* (PPP) hold<sup>6</sup> so that the

<sup>2</sup>Earlier models usually considered impulse responses to just a *single (one-time) shock* in otherwise a completely deterministic setting. Accordingly, although sometimes named "stochastic", they are essentially not.

<sup>3</sup>E.g. Helpman and Razin (1979, 1982, 1984), Helpman (1981) and Lucas (1982).

<sup>4</sup>Here one could enumerate papers in the Mundell-Fleming-Dornbusch tradition of the 1960s and 1970s.

<sup>5</sup>See the references cited in their footnote 7, p. 1096. Good surveys can be found in Côté (1994) and in Glick and Wihlborg (1997).

<sup>6</sup>Obstfeld and Rogoff (1995, 1996: Chapter 10, 1998, 2000) and Devereux (2000), among

real exchange rate (RER) is constant. To benefit from the insights provided by an analytical solution, we likewise limit our set-up in this initial study to a single period with only monetary uncertainty, as in Bacchetta and van Wincoop's (2000 a) benchmark. Yet in a pursuit of greater generality of our conclusions, we do not restrict attention to neither a CCP nor a PCP-LOP-PPP model version only. Furthermore, we need not to specialize, for our purposes here, to a log-normal distribution of disturbances or to a particular class of utility. With respect to the stochastic processes, it proves sufficient to invoke no more restrictions than a jointly *symmetric* distribution for the national money stock growth rates. As to the utility function, we essentially assume that it is well-behaved and separable. These features make our analysis less restrictive than related earlier work, with a few exceptions we know about such as Bacchetta and van Wincoop (1998, 2000 a). The latter two authors do not, however, explicitly consider *PCP* and its *pass-through* and *expenditure switching* implications as well as the effects of the exchange-rate regime on the trade *balance* and *international relative prices*.

## 1.2 Alternative Price Setting in Open Economies

Another important development in NOEM research has been to incorporate considerations of the earlier international trade literature, such as Helpman and Razin (1984) to mention an outstanding example, regarding alternative price setting. Contributions in this particular direction have been due to Betts and Devereux (1996, 2000), Bacchetta and van Wincoop (1998, 2000 a, b, 2001), Devereux and Engel (1998, 1999, 2000), Devereux (2000) and Engel (2000). Extending the original Obstfeld-Rogoff – Corsetti-Pesenti framework of non-segmented markets, these authors introduced international *market segmentation* in the goods market and what they usually call *pricing-to-market* (PTM)<sup>7</sup> behavior of monopolistically competitive firms, engaging at the same time in microfounded welfare comparisons of *exchange-rate regimes*. PTM is often alternatively denoted *local* currency pricing (LCP),<sup>8</sup> but to avoid ambiguity we would rather use a terminology that is hopefully more precise in our context: *producer's* currency pricing (PCP) and *consumer's* currency pricing (CCP).<sup>9</sup>

We already noted in what our analysis differs from, or rather extends and complements, the one in Bacchetta and van Wincoop (1998, 2000 a). As to the remaining NOEM literature cited in the preceding paragraph, our study is justified at least in the following three aspects. First, we examine the effects of the exchange-rate regime on *trade* prices and quantities (no matter that our equilibrium allocations, including imports and exports, have also been the result of underlying optimal consumption/leisure choices), whereas attention in

others.

<sup>7</sup> A term coined by Krugman (1987).

<sup>8</sup> A coinage due to Devereux (1997) to refer to the *special* case of PTM where prices are always set in the currency of the *destination* market.

<sup>9</sup> Since we do not explicitly distinguish an intermediary import/export sector in the two-country economy we study, as Tille (2000) has first done within NOEM, CCP and PCP are equivalent here to, respectively, *importer's* (buyer's) and *exporter's* (seller's) currency pricing.

all quoted papers is focused on welfare issues. Second, and as a consequence of not undertaking welfare analysis here, we are able to allow for a *more general* utility specification, while the other authors use quite restrictive utility subclasses, perhaps narrowing the scope of validity of their findings. Third, under *uncertainty* and in *cash-in-advance* (CiA) sticky-price frameworks – as emphasized in the insightful methodological books by Magill and Quinzii (1996) and Walsh (1998), among others – the assumed *timing of decisions* and *price-setting behavior* are crucial to model outcomes. Recognizing these facts and, more importantly, studying their *interaction* in a symmetric *open-economy* context that makes an explicit parallel between CCP and PCP invoicing is another novel feature of our approach.

The paper is further down organized as follows. Section 2 outlines the stochastic NOEM model of exchange rate and trade determination we employ and highlights the differences in its initial assumptions under CCP vs. PCP. The third section studies, under *float* and full symmetry, the role alternative price setting plays in agents' optimization and in deriving key equilibrium relationships. Section 4 then focuses on the effects of the exchange-rate regime on international relative prices and trade flows, by discussing if and how a *peg* would change the float allocations of the preceding section, and section 5 concludes. The optimization problems and the equilibrium model outcomes are systematized in more detail in *Appendix A* whereas *Appendix B* contains the proofs of propositions.

## 2 A Stochastic NOEM Model of Intra-Industry Trade

The present section serves to introduce the model we study. We first describe the *basic* set-up that underlies both our model versions. The essential differences between the *CCP vs. PCP* cases, originating in the relevant currency denomination assumptions and reflected in our invoicing-specific notation, are highlighted next.

### 2.1 Basic Set-Up

The artificial economy we analyze is made up of two countries, *H(ome)* and *F(oreign)*, assumed of *equal* size. A continuum of differentiated *brands* belonging to the same good *type* is available for consumption. These *highly substitutable* brands are indexed by  $i$  if made in  $H$  and by  $i^*$  if made in  $F$ . Each such brand is produced and sold by a *single* monopolistically competitive firm, also indexed by  $i$  in  $H$  and  $i^*$  in  $F$ . Firms in Home are *uniformly* distributed on the *unit* interval  $[0, 1]$ . Likewise, firms in Foreign produce on  $(1, 2]$ .

To obtain (short-run) money non-neutrality, we assume sticky prices motivated by menu costs.<sup>10</sup> Moreover, monopolistic competition enables each firm

<sup>10</sup>As first suggested by Mankiw (1985). To recall a classic result in Lucas (1982), with

to optimally choose the price(s) at which it sells its product. Prices are *set in advance*, i.e. in our *ex-ante* state 0 (*before* uncertainty has been resolved), and remain *valid for just one period*, i.e. for the *ex-post* state  $s \in S$  we consider (*after* uncertainty has been resolved).<sup>11</sup> Preannounced prices result, in turn, in demand-determined output, on an individual-firm as well as on an aggregate level.<sup>12</sup> In such a (New-)Keynesian situation, technology shocks do not influence production possibilities and output quantities sold.<sup>13</sup>

**Governments and Shocks** In each country, there is a government whose only (passive) role is to proportionally transfer cash denominated in national currency to all domestic households in a *random* way.<sup>14</sup> We interpret such a money supply behavior, equivalent in our context to a *flexible* exchange-rate system, as *exogenous* "monetary policy" and model it in terms of stochastic money stock *growth rates*. Moreover, we restrict it to be *jointly symmetric*, in the sense we explain next.<sup>15</sup>

For  $\forall s \in S$ ,  $\mu_s$  and  $\mu_s^*$  are, respectively, *H*-money stock and *F*-money stock *net* rates of growth, having the same means and variances. For the sake of symmetry, *ex-ante* (state 0) national money holdings of the representative households in Home and Foreign are assumed *identical* in terms of units of each country's currency:<sup>16</sup>  $M_0 = M_0^*$ . The *ex-post* (state  $s$ ) cash balances, i.e. the domestic-currency budgets with which Home and Foreign households dispose for transactions purposes in any realized state of nature  $s \in S$ , are then respectively given by  $M_s \equiv M_0 + \mu_s M_0 = (1 + \mu_s) M_0$  and  $M_s^* \equiv M_0^* + \mu_s^* M_0^* = (1 + \mu_s^*) M_0^*$ . Any state of nature in the model is thus uniquely characterized by the *ex-post* money stocks available to buy consumption goods:  $s \equiv (M_s, M_s^*)$ . Taking into consideration the identical *initial* money stocks,  $M_0 = M_0^*$ , each state can ultimately be identified by the *joint* realization – in the beginning of the single un-

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perfectly flexible prices the exchange-rate regime does not matter, even under uncertainty, for optimal real allocations. As to the locus of rigidity, some authors prefer to model sticky (nominal or real) wages, following Taylor (1979) and the earlier Keynesian tradition, while others give preference to sticky prices, following Rotemberg (1982) and Calvo (1983), and as Kimball (1995) has notably insisted. In essence, the two approaches are not so different and – within NOEM – often imply each other, as Hau (2000) and Obstfeld and Rogoff (2000) have recently argued.

<sup>11</sup>Since our focus is not on inflation dynamics (in general) or inflation persistence (in particular), the static stochastic framework we borrow from Bacchetta and van Wincoop (2000 a) and the related NOEM research seems not too constraining.

<sup>12</sup>For this to be realistic, we note that our subsequent analysis applies only to money growth shocks of a sufficiently *small* magnitude.

<sup>13</sup>That is why we abstract here from modelling also a productivity shock. Even if explicitly accounted for, it will not change much in the present single-period setting.

<sup>14</sup>One could argue that monetary authorities are ultimately unable to perfectly control the money *supply* or precisely estimate the *demand* for money in order to always equilibrate them.

<sup>15</sup>The symmetry of shocks we impose at this point is conceptually close to the *explicitly* assumed in Kraay and Ventura (2002) as well as to the one *implicit* in Bacchetta and van Wincoop (2000 a). Our assumption is quite general since it allows for different *classes* of symmetric distributions, as will become clearer from the proof of Proposition 7 and the simulation described in section 4.4.

<sup>16</sup>At an initial *equilibrium* exchange rate  $S_0 = 1$ , as will be discussed later.

certain period – of the stochastic (net) rates of money growth in both modelled economies,  $\mu_s$  and  $\mu_s^*$ :  $s \equiv (\mu_s, \mu_s^*)$ . An  $s_H$ -indexing of variables from now on will summarize all states of the world in which Home *relative* monetary expansion is observed ( $\mu_{s_H} > \mu_{s_H}^*, \forall s_H \in S_H \subset S$ ). By analogy, an  $s_F$ -indexing will denote all *respective mirror-states* in which Foreign *relative* monetary expansion *of the same magnitude* has occurred ( $\mu_{s_F} < \mu_{s_F}^*, \forall s_F \in S_F \subset S$ ). Finally, an  $s_e$ -indexing will apply to all remaining states of nature in which *relative* monetary equilibrium has materialized (coinciding growth rates  $\mu_{s_e} = \mu_{s_e}^*, \forall s_e \in S_e \subset S$ ), no matter at what *common* (absolute) magnitude.

The only difference between float vs. peg in terms of the (conditional) joint distribution (up to second moments, inclusive) of national money *growth* shocks ( $\mu_s, \mu_s^*$ ) and, hence, of the resulting ex-post money *stocks* ( $M_s, M_s^*$ ) thus arises from their *covariance* terms. It is imposed by the definition itself of a *fixed* vs. *flexible* exchange-rate regime: under (pure) *float*, the (conditional) *correlation* of national money stocks is 0; under (credible) *peg*, this (conditional) *correlation* is 1. In modelling a *peg* regime and in interpreting it as the equivalent of (a type of) *endogenous* monetary policy, we assume by necessity that at least one of the central banks is able to immediately (or rather simultaneously) imitate, i.e. reproduce exactly, the money supply behavior of the other, if not that the two can "draw" in cooperation the *common* rate of money stock growth required to credibly fix the exchange rate (across states of nature). In essence, our fixed exchange-rate version is thus isomorphic to a model where a monetary union or a single currency area is hit by just one, common money shock.

After all transactions are made, the Home government imposes a tax of  $M_s$  on its representative resident, and the Foreign government does the same in the amount of  $M_s^*$ . This assumption is standard in *finite*-horizon models with a cash-in-advance (CiA) constraint<sup>17</sup> like ours. It is needed to ensure that sellers of goods are willing to accept money in the last period (here, in the ex-post state).

**Timing of Events** In the single period we analyze, decisions are made in *two stages*, ex-ante and ex-post. These stages are defined – and distinguished as the *ex-ante* state 0 and the *ex-post* state  $s$  – by the moment of the *resolution of monetary uncertainty*.

**Ex-Ante Behavior** Only *firms* optimize ex-ante, solving a *stochastic* optimization problem. Before knowing the particular state of the world that will materialize but having common views on the joint distribution of the symmetric monetary shocks, they *preannounce prices*. Due to (prohibitive) menu costs, they cannot change ex-post these optimally prefixed prices.

**Ex-Post Behavior** After observing the state of the world, firms *employ labor* to produce goods. Output, hence, labor input and, ultimately, leisure hours are simply determined in any realized state of nature by the *optimal*

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<sup>17</sup>To be soon formally incorporated into the framework we describe.

*consumption demand* for the respective differentiated product each one of the firms produces.

Households, contrary to firms, optimize only *ex-post*. After receiving their random cash, they allocate total money balances across the differentiated goods which make up the real consumption composite. Because of demand-determined output and labor input, households are thus not free to adjust their labor/leisure trade-off once a given state of nature has materialized.

**Households** In each country,  $H$  and  $F$ , there is a continuum of *identical* households. The population in each of these economies is assumed constant and is normalized to 1. The representative household (in  $H$  as well as in  $F$ ) likes diversity and *consumes all brands* on the interval  $[0, 2]$ . It also *supplies labor*, earning the equilibrium wage, and *owns an equal proportion of domestic firms*, receiving their profits (in the form of dividends).

The representative household in Home<sup>18</sup> maximizes its *ex-post* (state  $s$ ) utility:

$$\underset{c_s, l_s}{Max} \quad u(c_s, l_s), \quad \forall s \in S. \quad (1)$$

Our utility function is assumed to be *well-behaved*, i.e. to exist, be continuous, twice differentiable and concave.  $l_s$  is (hours of) leisure (demand) and  $c_s$  is a *constant* elasticity of substitution (CES) *real* consumption (demand) index we define later. The components of the otherwise general utility are, in addition, supposed *separable*.

In this representative agent economy, the aggregate constraints on (per-) household behavior coincide with those of the identical households. They are standard in NOEM but, for completeness, we briefly present them below.

**Time Endowment Constraint** The endowment of hours to the representative household (in Home) is normalized to 1 in each state,

$$l_s + n_s \equiv 1, \quad \forall s \in S, \quad (2)$$

so that  $n_s \equiv 1 - l_s$  is (Home) household's (hours of) labor (supply).

**Cash-in-Advance (CiA) Constraint** Households need to carry *cash* before going to the goods market.<sup>19</sup> Moreover, we restrict them to hold and receive from their monetary authority only *domestic* currency. Thus (for Home)

<sup>18</sup>The notation in which the model is further on set out generally refers to Home, but for Foreign symmetric relationships hold and can usually be verified in the relevant appendices unless otherwise stated (on this particular point, see *Appendix A.1*).

<sup>19</sup>The alternative would be to introduce money and, hence, the nominal exchange rate whose determination and regimes we wish to analyze, via a *money-in-the-utility* (MiU) function, also common in monetary general-equilibrium models. Our modelling choice here is anyway not crucial, since Feenstra (1986) has demonstrated the equivalence of these two approaches.



$$\underbrace{c_s P_s}_{H \text{ national expenditure (in } H \text{ currency)}} \leq \underbrace{M_s}_{\text{available cash in } H \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \quad (3)$$

**National Money Market Equilibrium** Since CiA constraints are *binding*<sup>20</sup> and there is no investment and government spending in the model, the nominal value of national output sold (for consumption) is equal to the total stock of money in each of the countries. For Home:

$$Y_s = M_s, \quad \forall s \in S. \quad (4)$$

**National Income Identity** With a nominal *wage rate* of  $W_s$  and total *hours of work* amounting to  $1 - l_s$ , the nominal *labor income* of the (Home) representative household is given by  $W_s(1 - l_s)$ . Nominal *dividends* from firm profits earned by this household are denoted by  $\Pi_s$ . *In equilibrium*, all income from the activity of firms is distributed to domestic households who are their ultimate owners, as will be assumed (but this happens only at the end of the one-period framework we consider):<sup>21</sup>

$$\underbrace{\underbrace{W_s(1 - l_s)}_{\text{labor income}} + \underbrace{\Pi_s}_{\text{ownership income}}}_{H \text{ national (factor) income (in } H \text{ currency)}} \equiv \underbrace{Y_s}_{H \text{ national output (in } H \text{ currency)}}, \quad \forall s \in S. \quad (5)$$

**First-Order Conditions** The following "*compact*" FONC can be derived in a familiar way from the above-described constrained optimization problem for the  $H$  representative household:

$$W_s = \frac{u_{l,s}}{u_{c,s}} P_s, \quad \forall s \in S. \quad (6)$$

$u_{l,s}$  and  $u_{c,s}$  in (6) are the marginal utilities of leisure and consumption, respectively, in the realized state  $s$ . The *real* wage rate is thus equal, in equilibrium, to the ratio of these marginal utilities.

<sup>20</sup>For at least two reasons in our present set-up: (i) this is implied by the *concavity* of utility we assumed; (ii) it is also the optimal strategy for the representative household when no future (i.e. no dynamics) is allowed for, as in the *one-period* stochastic framework analyzed here. The binding CiA implies, in turn, a *unitary* velocity of (quantity theory) money demand (4), which is, of course, another limitation but one that is common to similar CiA settings.

<sup>21</sup>Factor income is thus not used further on, to buy consumption goods and to lend or borrow, with no future modelled. In related research, we intend to consider a *sequential markets* dynamic set-up that parallels the one studied here but also allows for *saving* decisions.

**Firms** Unlike the NOEM alternative of "yeoman-farmers", firms exist in themselves in our model and effect production. A usual restriction in similar settings we impose at this stage too is that firms are owned by *domestic* households only. In the present study we also abstract from an international stock market, as well as of risk-sharing issues in general. As noted, product differentiation makes firms *monopolistically competitive*. In line with the *intra*-industry trade literature, we focus here on the case where differentiated brands belong to the same *type* of a *homogeneous* good produced in both countries with identical technology *common* to all firms.<sup>22</sup> Just one factor, labor, available in fixed quantities in both economies, is used as input. For Home:

$$y_s = n_s = 1 - l_s. \quad (7)$$

Such a production function does seem simplistic, but is actually sufficient for the purposes of our sticky-price single-period analysis here. The reason is that, given the (New-)Keynesian set-up we described, it is household demand and not productivity that ultimately determines output.

## 2.2 CCP vs. PCP Version

**Currency Denomination of Transactions** As already mentioned, the *combination* of timing and nominal rigidity assumptions plays an important role in similar stochastic CiA models. In our case, it affects in a crucial way the nature of optimization under CCP vs. PCP. More precisely, the exchange rate does not matter for households decision problem under CCP but becomes a key consideration under PCP. The reason is in the particular currency denomination implied by our alternative invoicing assumptions.

Under *CCP*, households pay for imports as well as for home-produced goods *directly* in their national currency. The equilibrium exchange rate, although observed (calculated implicitly) at the moment of the realization of the national money growth shocks, does not play a role in consumer optimization. It only matters for firms, as their profits from exports denominated in foreign currency are converted back into domestic currency. In short, the ex-post exchange rate does not affect households optimizing behavior and, hence, trade allocations.

Under *PCP*, households use part of their domestic-currency money balances in the realized state of nature to buy, at the equilibrium exchange rate, the foreign currency needed for imports. In short, the ex-post exchange rate now influences the optimizing behavior of households and, hence, trade allocations.

The two model versions we compare further down, under CCP vs. PCP, have imposed a specific notation we clarify next.

**Notation** All our *quantity* variables are denoted by *lowercase* Latin letters. These quantities can be indexed by up to two subscripts and up to two superscripts. A first *subscript* *H* or *F* indicates the *origin* of the respective variable at

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<sup>22</sup>In Mihailov (2003 a) we allow for national good types that *differ* in the sense of being less substitutable than brands, and refer to this case as a simple form of *inter*-industry trade.

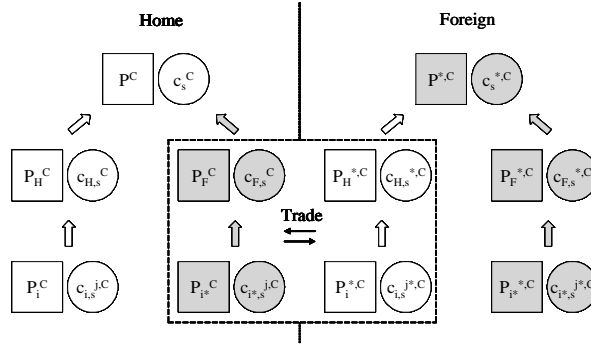


Figure 1: Notation on Price and Quantity Aggregation under CCP

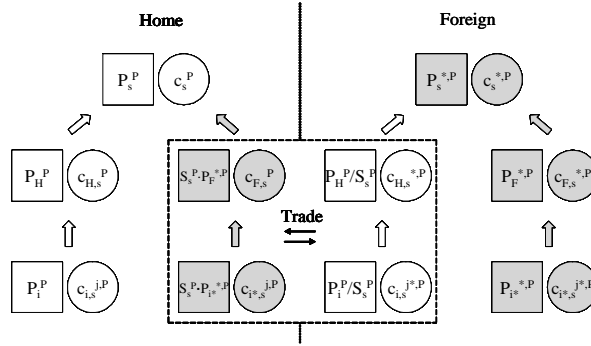


Figure 2: Notation on Price and Quantity Aggregation under PCP

the national-economy level, i.e. the country where a particular good  $i$  or  $i^*$  (first *subscripts* again but at the individual-firm level) has been *produced*. Following the tradition, we use an *asterisk* ( $*$ ) as a first *superscript* to denote that a particular quantity variable has been *consumed* in Foreign. The second *subscript*, 0 for *ex-ante* quantities and  $s$  for *ex-post* quantities, indexes the state of nature whereas the second *superscript*,  $C$  (for  $CCP$ ) or  $P$  (for  $PCP$ ), indicates the assumed price setting. The same notational rules apply to the (*money*) *prices* or *nominal* variables that correspond to all respective quantities in our model, the only difference being that these are denoted by *uppercase* Latin letters. Greek letters, in turn, designate model *parameters* and *shocks*.

For a schematic representation of prices, quantities and their (definitional) interrelations as well as of the general structure of our  $CCP$  vs.  $PCP$  model versions, compare the respective elements and blocks in figures 1 vs. 2. Additional explanatory comments follow suit.

### 3 The Role of Price Setting

The notation we have introduced thus far enables us to draw, in the present section, an explicit parallel between the essential *differences* in the optimizing behavior and the resulting consumption demand and monopolistic pricing functions across alternative price setting. On that basis, a formal definition of equilibrium, in the context of our two model versions, is provided. Under *float* and *symmetry*, we then derive CCP vs. PCP results for the exchange-rate level, international relative prices, cross-country consumption/leisure allocations and, most importantly, some key measures of trade flows. The underlying algebra is systematized in more detail in *appendices A* and *B*.

#### 3.1 Optimization and Equilibrium

**Consumption Demands and Price Levels** In each state of nature  $s \in S$  that has materialized, the representative household in Home (and, analogously, in Foreign) minimizes the cost of buying a unit of real consumption defined by a Dixit-Stiglitz (1977) aggregator:

$$c_s \equiv \left[ \left(\frac{1}{2}\right)^{\frac{1}{\varphi}} (c_{H,s})^{\frac{\varphi-1}{\varphi}} + \left(\frac{1}{2}\right)^{\frac{1}{\varphi}} (c_{F,s})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \quad \forall s \in S. \quad (8)$$

Similarly to Bacchetta and van Wincoop (2000 a) and most NOEM set-ups, we assume that  $\varphi = \varphi^* > 1$ .  $\varphi$  in the above formula turns out to be not only the elasticity of substitution in demand between any two *Home* brands (see  $(3_H^H C)$  and  $(3_H^H P)$  in *Appendix A.1*) but also the *cross-country* substitutability across *brands* of the *homogenous* product *type* modelled here (see (9) and (10) further down).<sup>23</sup> Such assumptions result in the standard expression  $\frac{\varphi}{\varphi-1} > 1$  being the constant markup over price a monopolist would charge (see (16) and (18) under CCP and (17) under PCP further down).

As it becomes clear from the more detailed presentation of the derivation we provide in *Appendix A.1*, the consumption aggregator (8) is only at first sight identical across our alternative price-setting assumptions. The reason is that its components,  $c_{H,s}$  and  $c_{F,s}$ , although seemingly the same, are in fact optimally defined by different expressions under CCP vs. PCP. That is what imposed the more complicated notation we employ in this paper, e.g.  $c_{H,s}^C$  and  $c_{H,s}^P$  vs.  $c_{H,s}^C$  and  $c_{H,s}^P$ , as well as the need to discuss in parallel the key CCP vs. PCP model version differences. They originate in some initial, pricing- and quantity-specific definitions like those above. But as the optimization proceeds on and is nationally aggregated, these differences also feed into the resulting analytical outcomes.

Standard derivations à la Dixit-Stiglitz (1977) under CCP vs. PCP result in Home optimal demands for *H*- (equation (9) below) and *F*-produced (10) brands and the respective price indices at the domestic absorption (11), import demand (12) and consumer (13) levels as follows:

<sup>23</sup>This is, certainly, a restriction on *aggregate* (or national output) substitutability, which we revisit in Mihailov (2003 a).

$$c_{H,s}^C = \frac{1}{2} \left( \frac{P_H^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C} \quad \text{vs.} \quad c_{H,s}^P = \frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}; \quad (9)$$

$$c_{F,s}^C = \frac{1}{2} \left( \frac{P_F^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C} \quad \text{vs.} \quad c_{F,s}^P = \frac{1}{2} \left( \frac{\overbrace{S_s^P P_{F,i^*}^{*,P}}^{\equiv P_{F,s}^P}}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}; \quad (10)$$

with

$$P_H^C \equiv \left[ \int_0^1 (P_i^C)^{1-\varphi} di \right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad P_H^P \equiv \left[ \int_0^1 (P_i^P)^{1-\varphi} di \right]^{\frac{1}{1-\varphi}}; \quad (11)$$

$$P_F^C \equiv \left[ \int_1^2 (P_{i^*}^C)^{1-\varphi} di^* \right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad \underbrace{S_s^P P_{F,i^*}^{*,P}}_{\equiv P_{F,s}^P} \equiv \left[ \int_1^2 \underbrace{(S_s^P P_{i^*}^{*,P})^{1-\varphi}}_{\equiv P_{i^*,s}^P} di^* \right]^{\frac{1}{1-\varphi}}; \quad (12)$$

$$P^C \equiv \left[ \frac{1}{2} (P_H^C)^{1-\varphi} + \frac{1}{2} (P_F^C)^{1-\varphi} \right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad (13)$$

$$P_s^P \equiv \left[ \frac{1}{2} (P_H^P)^{1-\varphi} + \frac{1}{2} \underbrace{(S_s^P P_{F,i^*}^{*,P})^{1-\varphi}}_{\equiv P_{F,s}^P} \right]^{\frac{1}{1-\varphi}}.$$

Clearly, under PCP the exchange-rate pass-through to import prices is *unitary*, while under CCP it is *prefixed* (cf. the CCP vs. PCP expression in (12)). For the same reason, the CPI is constant under CCP,  $P^C$ , but state-dependent under PCP,  $P_s^P$  (cf. equations (13)). This causes demands for even domestically-produced brands, at first sight identical, to be actually different across our alternative price-setting assumptions (cf. the CCP vs. PCP expression in (9)). To be able to go further into this type of parallel analysis, we have to also consider optimal pricing by monopolistically competitive firms.

**Output Prices** Similarly to the consumption aggregator (8), the expected market value of real profits<sup>24</sup> which a Home firm  $i \in [0, 1]$  maximizes is seemingly the same, but is nevertheless differently defined under CCP vs. PCP:

<sup>24</sup>Note that the relevant weights for the states of nature in the formulas we introduce are related to the marginal utility of consumption of the representative shareholder,  $u_{c,s}$ .

$$\underset{P_i^C, P_i^{*,C}}{\text{Max}} E_0 \left[ \frac{u_{c,s}}{P^C} \underbrace{\left( P_i^C c_{i,s}^C + S_s^C P_i^{*,C} c_{i,s}^{*,C} - W_s^C c_{i,s}^C - W_s^C c_{i,s}^{*,C} \right)}_{\equiv \Pi_{i,s}^C} \right], s \in S \quad (14)$$

$$\text{vs. } \underset{P_i^P}{\text{Max}} E_0 \left[ \frac{u_{c,s}}{P^P} \underbrace{\left( P_i^P c_{i,s}^P + P_i^P c_{i,s}^{*,P} - W_s^P c_{i,s}^P - W_s^P c_{i,s}^{*,P} \right)}_{\equiv \Pi_{i,s}^P} \right], s \in S. \quad (15)$$

Under CCP this firm  $i$  – which also turns out to be (see the optimization problem in *Appendix A.2*) the Home *representative* firm – presets *two* prices, one in national currency and the other in foreign currency. Under PCP just *one* price, in national currency, is prefixed. Using the respective first order conditions, CCP vs. PCP optimal prices of the Home representative firm (relevant for consumer households in the domestic and foreign market) are thus:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} \frac{E_0 [u_{c,s} W_s^C M_s]}{E_0 [u_{c,s} M_s]} \text{ vs.} \quad (16)$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0 [u_{c,s} W_s^P M_s] + E_0 [u_{c,s} W_s^P M_s^*]}{E_0 [u_{c,s} M_s] + E_0 [u_{c,s} M_s^*]}; \quad (17)$$

$$P_i^{*,C} = P_H^{*,C} = \frac{\varphi}{\varphi - 1} \frac{E_0 [u_{c,s} W_s^C M_s^*]}{E_0 [u_{c,s} S_s^C M_s^*]} \text{ vs.} \quad (18)$$

$$\underbrace{P_{H,s}^{*,P} \equiv \frac{P_H^P}{S_s^P}}_{\text{LOP}} \Rightarrow \underbrace{P_s^{*,P} = \frac{P_s^P}{S_s^P}}_{\text{PPP}}. \quad (19)$$

As evident from (19), the price at which Home representative firm's product sells in Foreign under PCP,  $P_{H,s}^{*,P}$ , depends on the exchange-rate level that has materialized ex-post,  $S_s^P$ . In fact, it is LOP applied to the homogeneous good *type* (differentiated across monopolistically produced *brands*) in the present intra-industry trade context that underlies the above PCP Foreign import price definition. Moreover as we noted earlier, the price which is preset in the currency of the seller (Home in the case we comment here) under PCP,  $P_H^P$ , becomes state-dependent when converted – via the observed exchange rate,  $S_s^P$  – in the currency of the buyer,  $P_{H,s}^{*,P}$ . This is the major *channel* along which we distinguish and interpret the differences between our model versions under CCP vs. PCP.

**Definition of Equilibrium** We now formally define an equilibrium concept that corresponds to the described sequential optimization.

**Definition 1** *In the context of the model versions we presented, an equilibrium is a set of quantities and prices, such that:*

1. [**Ex-Ante Conditions**] before the resolution of monetary uncertainty but given commonly held views about the joint symmetric distribution of money growth shocks  $(\mu_s, \mu_s^*)$ ;
  - (a) [**Firms Stochastic Optimization**] given their technology constraint and the expected quantities demanded in the goods market,  $\{E_0 [c_{H,s}^C], E_0 [c_{H,s}^{*,C}], E_0 [c_{F,s}^{*,C}], E_0 [c_{F,s}^C]\}$  under CCP or  $\{E_0 [c_{H,s}^P], E_0 [c_{H,s}^{*,P}], E_0 [c_{F,s}^{*,P}], E_0 [c_{F,s}^P]\}$  under PCP, the prices,  $\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\}$  under CCP or  $\{P_H^P, P_F^{*,P}\}$  under PCP, that are optimally preset ex-ante (i.e. in state 0) and bindingly posted to consumer households for transactions ex-post (in state  $s$  for  $\forall s \in S$ ) solve the profit maximization problem of the representative producer firm in Home as well as in Foreign;
2. [**Ex-Post Conditions**] following the resolution of monetary uncertainty and in any state of nature  $s \in S$  that has materialized;
  - (a) [**Households Labor-Leisure Trade-Off**] given its constraints and the posted prices,  $\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\}$  under CCP or  $\{P_H^P, P_F^{*,P}\}$  under PCP, the representative consumer household in Home as well as in Foreign spends up all available cash on its total real consumption  $\{c_s, c_s^*\}$ ; hours of work (employment)  $\{1 - l_s, 1 - l_s^*\}$  are supplied by households until firms demand labor to equilibrate ex-post consumption demand for their differentiated products at the resulting equilibrium real wage rates  $\left\{ \frac{W_s}{P_s}, \frac{W_s^*}{P_s^*} \right\}$ ;
  - (b) [**Households Consumer Basket Allocation**] given the posted prices,  $\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\}$  under CCP or  $\{P_H^P, P_F^{*,P}\}$  under PCP, the consumption quantities  $\{c_{H,s}^C, c_{H,s}^{*,C}, c_{F,s}^{*,C}, c_{F,s}^C\}$  under CCP or  $\{c_{H,s}^P, c_{H,s}^{*,P}, c_{F,s}^{*,P}, c_{F,s}^P\}$  under PCP solve the cost minimization problem à la Dixit-Stiglitz (1977) of the representative consumer household in Home as well as in Foreign;
  - (c) [**Goods Market Clearing**] all quantities under CCP or PCP satisfy the feasibility conditions for each differentiated brand so that all product-brand markets – and, hence, the international product-type market as a whole – clear;
  - (d) [**Forex Market Clearing**] the international forex market clears as well.

### 3.2 Equilibrium Nominal Exchange Rate

The simple symmetric structure of the model we analyze allows an explicit derivation of the equilibrium nominal exchange rate (NER).<sup>25</sup> It solves the international *forex market clearing* condition<sup>26</sup> which states that excess supply of each of the two currencies (expressed in the same monetary unit) is zero in any state of nature  $s \in S$  that has materialized.<sup>27</sup>

$$\underbrace{P_F^C c_{F,s}^C}_{F \text{ export revenues} \Leftrightarrow HC \text{ supply}} - S_s^C \cdot \underbrace{P_H^{*,C} c_{H,s}^{*,C}}_{H \text{ export revenues} \Leftrightarrow HC \text{ demand}} = 0 \quad (20)$$

$$\text{vs.} \quad S_s^P \cdot \underbrace{P_F^{*,P} c_{F,s}^P}_{H \text{ import demand} \Leftrightarrow HC \text{ supply}} - \underbrace{P_H^P c_{H,s}^{*,P}}_{F \text{ import demand} \Leftrightarrow HC \text{ demand}} = 0. \quad (21)$$

Substituting for optimal demands above as well as for the ideal  $H$  and  $F$  CPI definitions further on in the algebraic manipulation derives the following *general* expressions for the equilibrium NER under CCP vs. PCP:

$$S_s^C = \frac{1 + \left(\frac{P_H^C}{P_F^C}\right)^{1-\varphi} M_s}{1 + \left(\frac{P_H^{*,C}}{P_F^{*,C}}\right)^{1-\varphi^*} M_s^*} \quad \text{vs.} \quad S_s^P = \frac{1 + \left(\frac{P_{F,s}^P}{P_H^P}\right)^{1-\varphi} M_s}{1 + \left(\frac{P_{H,s}^{*,P}}{P_F^{*,P}}\right)^{1-\varphi^*} M_s^*}. \quad (22)$$

**Equilibrium NER under Full Symmetry** Under *full symmetry*, i.e. with  $P_H^C = P_F^{*,C}$ ,  $P_F^C = P_H^{*,C}$ ,  $P^C = P^{*,C}$  and  $\varphi = \varphi^*$  under CCP vs.  $P_H^P = P_F^{*,P}$ ,  $P_{F,s}^P \equiv S_s^P P_F^{*,P}$ ,  $P_{H,s}^{*,P} \equiv \frac{P_H^P}{S_s^P}$ ,  $P_s^P = S_s^P P_s^{*,P}$  and (again)  $\varphi = \varphi^*$  under PCP, the above expressions simplify to

$$S_s^C = \frac{M_s}{M_s^*} \quad \text{vs.} \quad S_s^P = \left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}}. \quad (23)$$

The equilibrium exchange rate (23) under CCP vs. PCP only differs in including or not the key model parameter,  $\varphi = \varphi^* > 1$ . This result implies that, in equilibrium, the NER should be *less volatile* under PCP than under CCP,<sup>28</sup> just because of substitutions via imports/exports induced by the pass-through effect under PCP. In both cases, however, the equilibrium exchange rate

<sup>25</sup>The nominal exchange rate is defined in the usual way as the *Home*-currency price of *Foreign* money.

<sup>26</sup>See also *Appendix A.3*.

<sup>27</sup>Taking the currency of  $H$  as the common unit of account.

<sup>28</sup>A point first made by Betts and Devereux (1996). It is also evident that, for a given *symmetric* distribution of money growth shocks, NER volatility will thus be lower under PCP by a magnitude depending directly on the particular value of *consumption demand substitutability*,  $\varphi$ , or, alternatively, the degree of *monopolistic competition*,  $\frac{\varphi}{\varphi-1}$ .



is a function of *fundamentals*, namely the money stocks in Home and Foreign. The *more general* formula (22) does *not* impose full symmetry in order to apply simplifying substitutions relying on *PPP*,  $P_s^P = S_s^P P_s^{*,P}$  under PCP or even stronger equations such as, in our CCP case,  $P^C = P^{*,C}$ . The benefit from looking at (22) is that this formula makes evident another principal difference between the price-setting assumptions we study here. In general, the equilibrium exchange rate in a sticky-price model of intra-industry trade will depend not only on relative money stocks but also on relative price levels resulting from aggregation of the optimally prefixed prices of domestic and foreign (highly substitutable) brands. This is true for both the cases of CCP and PCP, but the difference is, again, that in our PCP version import prices are state-dependent, and hence sensitive to (or affected by) the ex-post exchange rate, whereas this is not so under CCP. Another parameter that will also, in principle, determine the equilibrium exchange rate in this type of NOEM set-ups could be a nationally-specific elasticity of substitution in consumption,  $\varphi \neq \varphi^*$  (or, equivalently, a nationally-specific degree of product market monopolization,  $\frac{\varphi}{\varphi-1} \neq \frac{\varphi^*}{\varphi^*-1}$ ).

**Optimal Firm Prices under Full Symmetry** Using (6) and its equivalent for Foreign as well as (23) under CCP and PCP to substitute for the endogenous variables  $W_s$ ,  $W_s^*$  and  $S_s$  in (16) through (19), the optimal firm prices derived earlier can now be fully determined. The final model solutions for prices in terms of exogenous variables and parameters only are thus:

$$\begin{aligned}
P_i^C &= P_H^C = \frac{\varphi}{\varphi-1} P^C \frac{E_0 [u_{l,s} M_s]}{E_0 [u_{c,s} M_s]} \text{ vs.} \\
P_i^P &= P_H^P = \frac{\varphi}{\varphi-1} \frac{E_0 [u_{l,s} P_s^P M_s] + E_0 [u_{l,s} P_s^P M_s^*]}{E_0 [u_{c,s} M_s] + E_0 [u_{c,s} M_s^*]}, \\
P_i^{*,C} &= P_H^{*,C} = \frac{\varphi}{\varphi-1} P^{*,C} \frac{E_0 [u_{l,s} M_s^*]}{E_0 [u_{c,s} M_s^*]} \text{ vs.} \\
P_{H,s}^{*,P} &\equiv \underbrace{\frac{P_H^P}{\left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}}}}_{\text{LOP}} \Rightarrow P_s^{*,P} = \underbrace{\frac{P_s^P}{\left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}}}}_{\text{PPP}}.
\end{aligned}$$

As in Bacchetta and van Wincoop (2000 a), it is easily seen that under *CCP* the prices set by the Home representative firm domestically,  $P_H^C$ , and abroad,  $P_H^{*,C}$ , will be the same only if  $E_0 [u_{l,s} M_s] = E_0 [u_{l,s} M_s^*]$ . This will always be true under peg, since then  $M_s^*$  can be substituted by  $M_s$  everywhere in the formulas up to here, but not generally under float. Bacchetta and van Wincoop (2000 a) formally prove, in their Lemma 1 and related Proposition 1, that  $E_0 [u_{l,s} M_s] = E_0 [u_{l,s} M_s^*]$  and, hence,  $P_H^C = P_H^{*,C}$  is true *only* when utility is *separable* in consumption and leisure. To be able to continue now with our focus in this initial study on the fully-symmetric case, in section 2 we purposefully

assumed this less general case which is nevertheless widespread when it comes to modelling preferences. With separable utility under CCP and float, the prices optimally *preset* domestically and abroad will therefore be the *same*, due to symmetry, so that  $P_H^C = P_H^{*,C} = P_F^C = P_F^{*,C}$ .

It is also clear from the respective formula above for Home and the corresponding one for Foreign<sup>29</sup> that under *PCP* and float, when just one price is optimally prefixed in each country, in the domestic currency, the two pre-announced prices will have the same level,  $P_H^P = P_F^{*,P}$ , given symmetry and separability again. Yet the respective ex-post PCP prices in the foreign currency,  $P_{H,s}^{*,P}$  and  $P_{F,s}^P$ , will in general not be equal to those preset domestically. Observe, however, that under PCP and *peg* the domestic-currency prices of home and foreign substitutes faced by consumers in a given country will be the same for any  $s \in S$ , so that  $P_H^P = P_H^{*,P} = P_F^P = P_F^{*,P}$  (ex-post as well as ex-ante).

A final set of key equations in the model provides, under full symmetry, straightforward expressions for some traditional characteristics of international trade. In addition to the *trade share in output by country* considered in Bacchetta and van Wincoop (2000 a) under CCP, in our present extension we also discuss *three other aspects*, missing in their study and central to understanding the CCP vs. PCP outcomes of our analysis. These aspects concern *international relative prices*, the *trade balance* and the *share of world trade in world output*.

### 3.3 Equilibrium Relative Prices

**Relative Price of Foreign to Domestic Goods** We saw that under CCP with jointly symmetric money shocks and separable preferences, all prices are optimally prefixed in the currency of the buyer at the *same* level:  $P_H^C = P_H^{*,C} = P_F^C = P_F^{*,C}$ . As a consequence, the relative price of foreign-produced goods in terms of domestically-produced ones in both countries is predetermined at 1:<sup>30</sup>

$$p_H^C \equiv \frac{P_F^C}{P_H^C} = 1 = \frac{P_H^{*,C}}{P_F^{*,C}} \equiv p_F^{*,C} \text{ for } \forall s \in S. \quad (24)$$

Under PCP, the prices which firms preannounce in their *domestic* currency have likewise the *same* level across countries,  $P_H^P = P_F^{*,P}$ . However, the corresponding *foreign-currency* prices obtained via LOP,  $P_{H,s}^{*,P}$  and  $P_{F,s}^P$ , can remain equal to the domestic-currency ones only if some low-probability state of *relative* monetary equilibrium,  $s_e \in S_e \subset S$ , occurs. In general, the resulting relative prices of foreign-produced goods in terms of domestically-produced ones under PCP are *reciprocal* across countries and reflect directly the ex-post nominal exchange rate:

<sup>29</sup>See *Appendix A.2*.

<sup>30</sup>In such a way, any effects of the ex-post values of these key international relative prices on consumer behavior are precluded under CCP.

$$p_{H,s}^P \equiv \frac{\overbrace{S_s^P P_F^{*,P}}^{\equiv P_{F,s}^P}}{P_H^P} = S_s^P = \left( \frac{\overbrace{P_H^P}^{\equiv P_{H,s}^{*,P}}}{\underbrace{S_s^P}_{P_F^{*,P}}} \right)^{-1} \equiv \left( p_{F,s}^{*,P} \right)^{-1} \neq 1 \text{ unless } s_e. \quad (25)$$

**Terms of Trade** In our symmetric set-up, the terms of trade (ToT) are *inversely* defined – across countries for the same invoicing convention as well as across price setting assumptions for each of the countries with respect to the nominal exchange rate. Our *CCP* model version thus implies a *negative* relationship between the NER and the ToT: a nominal depreciation *improves* the terms of trade. Just the opposite effect is, however, predicted by our *PCP* model version: the relationship between the NER and the ToT is *positive*, so that a nominal depreciation *weakens* the terms of trade and induces, in turn, *expenditure switching*, an international spillover channel largely debated in the Mundell-Fleming-Dornbusch tradition:

$$(ToT)_{H,s}^C \equiv \frac{\overbrace{P_F^C}^{\equiv P_H^{Im,C}}}{\underbrace{S_s^C P_H^{*,C}}_{P_H^{Ex,C}}} = \frac{1}{S_s^C} = \left( \frac{\overbrace{P_H^{*,C}}^{\equiv P_F^{Im,C}}}{\underbrace{P_F^C}_{S_s^C}} \right)^{-1} \equiv \left[ (ToT)_{F,s}^{*,C} \right]^{-1} \neq 1 \text{ unless } s_e \text{ vs.} \quad (26)$$

$$(ToT)_{H,s}^P \equiv \frac{\overbrace{P_{F,s}^P}^{\equiv P_{H,s}^{Im,P}}}{\underbrace{P_H^P}_{\equiv P_{H,s}^{Ex,P}}} = \frac{S_s^P P_F^{*,P}}{P_H^P} = S_s^P = \left( \frac{\overbrace{P_H^P}^{\equiv P_{F,s}^{Im,P}}}{\underbrace{S_s^P}_{P_F^{*,P}}} \right)^{-1} \equiv \left[ (ToT)_{F,s}^{*,P} \right]^{-1} \neq 1 \text{ unless } s_e. \quad (27)$$

This latter result, which we have explicitly derived from microfoundations, is in line with findings in other recent NOEM papers, in particular with the Obstfeld-Rogoff (2000) correlation approach of checking for pricing-to-market in macrodata.<sup>31</sup>

**Real Exchange Rate** In compliance with the PPP literature, our symmetric *PCP* model results in a microfounded real exchange rate (RER) that is *constant* (across states of nature) at 1 in equilibrium:

<sup>31</sup>Our theoretical point here is the subject of related empirical work in Mihailov (2003 b).

$$(RER)_H^P \equiv \frac{S_s^P P_s^{*,P}}{P_s^P} = \frac{P_s^P}{P_s^P} = 1 = \frac{P_s^{*,P}}{P_s^{*,P}} = \frac{\frac{P_s^P}{S_s^P}}{\frac{P_s^{*,P}}{S_s^P}} \equiv (RER)_{F,s}^{*,P} \text{ for } \forall s \in S. \quad (28)$$

On the other hand, our *CCP* version leads to a parallel equilibrium outcome of a RER that *moves one-to-one* with the NER (across states of nature), as consistent with the higher RER volatility implied by PTM-based models:

$$(RER)_{H,s}^C \equiv \frac{S_s^C P_s^{*,C}}{P_s^C} = S_s^C = \left( \frac{\frac{P_s^C}{S_s^C}}{P_s^{*,C}} \right)^{-1} \equiv \left[ (RER)_{F,s}^{*,C} \right]^{-1} \neq 1 \text{ unless } s_e. \quad (29)$$

### 3.4 Equilibrium Consumption and Leisure across Countries

To better understand the implications of the *microfounded* general-equilibrium framework we study for CCP vs. PCP trade flows, we now have to first consider its outcomes across price setting in terms of the ingredients of the utility function, namely consumption and leisure. Our essential points are summarized in the *propositions* we state in their logical order throughout the present subsection. Proofs, based largely on earlier definitions and derivations, are provided in *Appendix B* whereas interpretations follow further down in the main text.

**Proposition 1** (*Relative Consumption*) *Relative real consumption is ultimately determined by the relative money stock, no matter the particular price setting assumed.*

To put it differently, Proposition 1 establishes that it is national money shocks and, consequently, relative money stocks (or relative wealth in our simple NOEM framework) that really matter – via demand and trade – for ex-post real consumption differences across the ex-ante symmetric countries, irrespective of the invoicing convention. Note, however, that under CCP but not PCP the relative money stock is also the equilibrium nominal exchange rate and that under PCP but not CCP the elasticity of consumption demand,  $\varphi > 1$ , mitigates<sup>32</sup> the effect of relative monetary disequilibria. More importantly, there is another, principal difference between our price-setting assumptions which results from the fact that CCP prevents substitution across borders, and hence expenditure switching, while under PCP such substitution is optimal, as we show next.

**Proposition 2** (*Consumption Bias*) *Under CCP the optimal split-up of real consumption between demand for domestic and foreign goods is always 50 : 50 whereas under PCP it is ultimately determined by the relative money stock.*

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<sup>32</sup>Compared to the CCP case.

Proposition 2 is of major importance for understanding our equilibrium trade share outcomes across price-setting assumptions to be discussed in more detail later on. It implies that in our *CCP* model version a monetary expansion – coordinated under peg or unilateral under float – does *not* induce any *bias* in goods consumption. In the *PCP* case, by contrast, a monetary expansion in one of the countries results – by depreciating (appreciating, for the other country) the equilibrium exchange rate, making imports more expensive (cheaper) and inducing substitution away from (into) them – in a *bias in both countries* favoring consumption of the goods produced in the *expansionary* country.

**Proposition 3** (*Balanced Trade*) *Given full symmetry, the trade balance in each of the two economies is always zero, no matter the particular price setting assumed.*

The ultimate reason for the result in Proposition 3 is the *full symmetry* imposed in our set-up. Under CCP the prices relevant for consumer optimization and, hence, the corresponding cross-country quantities consumed *do not differ*. Under PCP they *exactly compensate* each other. In both model versions then the national currency *value* (quantity multiplied by price) of imports and of exports remains necessarily the same for each of the two countries, irrespective of the particular state of nature  $s \in S$  that has materialized. Consequently, the trade balance is always constant at zero, no matter the invoicing we model.

**Proposition 4** (*Relative Leisure*) *Under CCP equilibrium output, employment and leisure (but not consumption) are always equal across countries whereas under PCP output, employment and leisure (as well as consumption) are ultimately determined by the relative money stock and are thus not generally equal across nations.*

The basic intuition behind Proposition 4 is that under CCP the two countries always produce the *same* real quantities of *output*, no matter the particular state of nature that has occurred. Because of the identical technologies, the two countries furthermore employ the *same* amount of *labor*, i.e. employment is the same as well. Therefore, the hours of *leisure* the representative household in Home and in Foreign enjoys – residually, due to the demand-determined output and, hence, labor input – under CCP are always the *same* too. By contrast, under PCP the two countries do *not* produce the *same* real quantities of *output*, unless some state of nature of relative monetary equilibrium  $s_e$  has materialized. Due to the identical technologies again, the two countries do *not* employ the *same* amount of *labor*. Consequently, the hours of *leisure* the representative household in Home and in Foreign enjoys under PCP are generally *not* the *same* either.

To provide certain parallels between the present set-up and the preceding related literature, we finally consider the traditional example of the impact of a one-time money supply shock. Since the model here is explicitly stochastic, we shall rather be talking about *relative* monetary expansion or *relative* money stock disequilibrium, situations summarized by the convenient  $s_H \in S_H \subset S$

and  $s_F \in S_F \subset S$  notation we introduced earlier. In order not to violate the credibility of our sticky-price environment, we more precisely analyze ex-post allocations in response to money stock growth shocks of a *small* magnitude occurring after the initial symmetric equilibrium.

**Proposition 5** (*Impact of Monetary Expansion*) *In our CCP model version, any relative monetary disequilibrium under float increases the ex-post utility of the residents of the expansionary country relative to the ex-post utility of the residents of the contractionary country. Interestingly, PCP inverts this conclusion.*

The logic underlying Proposition 5 is that in our *CCP* model version households in both economies enjoy equal amount of leisure in any state that has materialized, but at the same time those in the expansionary economy consume more *relative to* their neighbors in the contractionary economy. So overall, ex-post utility is higher in the expansionary country, a result reminiscent of (but not identical to) "beggar-thy-neighbor" policies debated in the Mundell-Fleming-Dornbusch tradition. Under *PCP*, by contrast, the gain of the Home representative household in consumption relative to the Foreign one is lower than its simultaneous relative loss in leisure (when consumption and leisure are *equally* valued, as we assume for our purposes here). Under *PCP*, therefore, Home residents are worse-off than Foreign ones in *cross-country* ex-post utility terms following a Home *relative* monetary expansion, a finding similar (but not equivalent) to classic and more recent "beggar-thyself" reasoning.

### 3.5 Equilibrium Trade Flows

**Trade Shares by Country** Under *CCP* vs. *PCP*, Home<sup>33</sup> equilibrium (ex-post) *foreign trade / GDP* ratio in each state of nature  $s \in S$  is defined by

$$(ft)_{H,s}^C \equiv \frac{(Ex)_{H,s}^C + (Im)_{H,s}^C}{(DA)_{H,s}^C + (Ex)_{H,s}^C} = \frac{S_s^C \cdot P_H^{*,C} \cdot c_{H,s}^{*,C} + P_F^C \cdot c_{F,s}^C}{P_H^C \cdot c_{H,s}^C + S_s^C \cdot P_H^{*,C} \cdot c_{H,s}^{*,C}} \text{ vs.} \quad (30)$$

$$(ft)_{H,s}^P \equiv \frac{(Ex)_{H,s}^P + (Im)_{H,s}^P}{(DA)_{H,s}^P + (Ex)_{H,s}^P} = \frac{P_H^P \cdot c_{H,s}^{*,P} + \overbrace{S_s^P \cdot P_F^{*,P} \cdot c_{F,s}^P}^{\equiv P_{F,s}^P}}{P_H^P \cdot c_{H,s}^P + P_H^P \cdot c_{H,s}^{*,P}}, \quad (31)$$

where  $(Ex)_{H,s}^C$  denotes Home exports,  $(Im)_{H,s}^C$  Home imports and  $(DA)_{H,s}^C$  Home domestic absorption, all these three Home-currency *values* (prices multiplied by quantities) under *CCP* and in any state  $s \in S$  that has materialized.  $(Ex)_{H,s}^P$ ,  $(Im)_{H,s}^P$  and  $(DA)_{H,s}^P$  are, of course, the respective *PCP* values.

<sup>33</sup>For Foreign, the corresponding expressions are symmetric, as can be verified in *Appendix A.4*.

Substitutions for optimal demands and use of the Home ideal CPI definition derive – under *full symmetry* and *separable preferences* – the CCP vs. PCP *trade share curve* for Home:

$$(ft)_H^C = \frac{2}{\left(\frac{P_H^C}{P_H^{*,C}}\right)^{1-\varphi} + 1} = \frac{2}{\left(\frac{E_0[u_{i,s}M_s]}{E_0[u_{i,s}M_s^*]}\right)^{1-\varphi} + 1} = \text{const} = 1 \text{ vs.} \quad (32)$$

$$(ft)_{H,s}^P = \frac{2}{\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\varphi-1} + 1} = \frac{2}{(S_s^P)^{\varphi-1} + 1} = \frac{2}{\left(\frac{M_s}{M_s^*}\right)^{\frac{\varphi-1}{\varphi}} + 1} \neq 1 \text{ unless } s_e. \quad (33)$$

The corresponding *trade share curve* for *Foreign* is symmetrically given by

$$(ft)_F^C = \frac{2}{\left(\frac{P_F^C}{P_H^C}\right)^{1-\varphi} + 1} = \frac{2}{\left(\frac{E_0[u_{i,s}^*M_s^*]}{E_0[u_{i,s}M_s]}\right)^{1-\varphi} + 1} = \text{const} = 1 = (ft)_H^C \text{ vs.} \quad (34)$$

$$(ft)_{F,s}^P = \frac{2}{\left(\frac{P_s^{*,P}}{P_s^P}\right)^{\varphi-1} + 1} = \frac{2}{\left(\frac{1}{S_s^P}\right)^{\varphi-1} + 1} = \frac{2}{\left(\frac{M_s^*}{M_s}\right)^{\frac{\varphi-1}{\varphi}} + 1} \neq 1 \neq (ft)_{H,s}^P \text{ unless } s_e. \quad (35)$$

These two pairs of equations compare directly the impact of our alternative price-setting assumptions on trade, measured relative to output.<sup>34</sup> Under CCP, (32) and (34) show that the *equilibrium* trade share is constant at 1 in each country and in any state of nature that has materialized. Under PCP, by contrast, this is not generally the case: as clear from (33) and (35), national trade-to-output ratios now both become state-dependent, i.e. volatile, unless some low-probability state  $s_e$  of relative monetary equilibrium occurs.

To see the intuition behind, assume a Home *relative* monetary expansion and compare the numerator and denominator in (30) under float. Under CCP, *no substitution* occurs between domestic and foreign brands of the same product type we model here, due to the preset *buyer's* currency prices and the resulting foreign/domestic relative price *equality* across countries in (24). That is why the additional (or excessive, with respect to Foreign) Home cash in the observed state of nature  $s_H \in S_H \subset S$  splits up evenly (50 : 50) into a domestic demand increase and an import demand increase:<sup>35</sup>  $(DA)_{H,s_H}^C \uparrow = (Im)_{H,s_H}^C \uparrow$ . Thus, the denominator in (30) changes by the same amount as the numerator, and the trade/output ratio remains constant (across states).

<sup>34</sup>The first equality in the formulas expresses the trade/GDP ratio as a function of *price levels*. The last equality is, in turn, the reduced-form version which expresses trade relative to output as a function of the *exogenous variables* only.

<sup>35</sup>As formally shown in Proposition 2.

Under PCP, by contrast, prices are prefixed in the currency of the *seller*. Therefore, the observed nominal exchange rate affects import prices, and hence consumer price levels, thus partly "flexibilizing" our otherwise fix-price model. The ex-post NER feeds on into the foreign/domestic relative price *reciprocity* across countries highlighted in (25). This key relative price is now state-dependent and, in turn, influences itself optimal consumer decisions on cross-border *substitution*<sup>36</sup> in demand. Home import demand falls as more expensive imports resulting from the depreciated exchange rate (relative to its ex-ante equilibrium of 1) are substituted away and into domestic analogues so that domestic demand rises, as well as Home exports, for the same (or rather symmetric) reason applied to Foreign importing households:<sup>37</sup>  $(Im)_{H,s_H}^P \downarrow = (Ex)_{H,s_H}^P \uparrow = (DA)_{H,s_H}^P \uparrow$ . Thus, the denominator in (31) goes up whereas the numerator stays flat, as rising exports and falling imports *compensate exactly* each other *in value*.<sup>38</sup> The equilibrium trade share in Home is consequently less than its CCP value of 1, and the trade share in Foreign is more than 1, following a Home *relative* monetary expansion.

To illustrate the interpretation suggested above, we present in Figure 3 the *PCP trade share curves* for Home, equation (33), and for Foreign, equation (35), according to a baseline computation we have performed setting  $\varphi = 11$ . This latter value of the elasticity of substitution in consumption demand is consistent with a markup  $\frac{\varphi}{\varphi-1}$  of 10%, a largely consensual estimate in empirical studies. For completeness, we have also studied the cases of a very elastic demand,  $\varphi = 101$ , which corresponds to a tiny markup of only 1% as in Figure 4 and of almost inelastic demand,  $\varphi = 2$ , corresponding to a huge markup of 100% as in Figure 5. The graphs show the trade share in output  $(ft)_s^P$  (on the vertical axis) under PCP, float and full symmetry as a function of the equilibrium nominal exchange rate  $S_s^P$  or, ultimately, the underlying relative money stock  $\frac{M_s}{M_s^*}$  (on the horizontal axis).

A comparison among the reported three cases shows that the *degree* of substitutability  $\varphi > 1$  across the individualized brands that nations exchange within the same type of good under PCP intra-industry trade – or, alternatively, the *degree* of imperfect competition identified by the monopolistic markup  $\frac{\varphi}{\varphi-1} > 1$  charged over price – matters a lot in related analyses. In particular, PCP trade share curves are much flatter and more curved in the vicinity of 1 under *low* substitutability and *highly monopolized* world market structure relative to the "normal" situation ( $\varphi = 11$ ). By contrast, these same curves are almost vertical and straight in the *near* vicinity of 1 with *high* substitutability and competition *close to perfect*.

## World Trade Share

<sup>36</sup> Whose *degree* depends on the particular value of the key *elasticity* parameter  $\varphi = \varphi^* > 1$ .

<sup>37</sup> According to Proposition 2, again.

<sup>38</sup> Recall the PCP reasoning in the proof of Proposition 3.



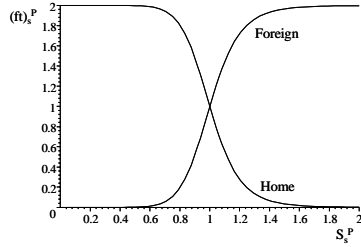


Figure 3: PCP Trade Share Curves under "Usual" Monopolistic Competition (for a markup of 10%, i.e.  $\varphi = 11$ )

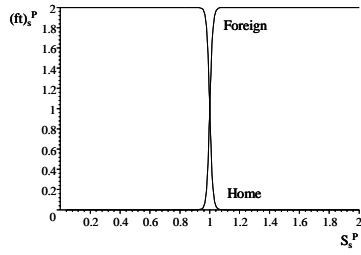


Figure 4: PCP Trade Share Curves under Near-Perfect Competition (for a markup of 1%, i.e.  $\varphi = 101$ )

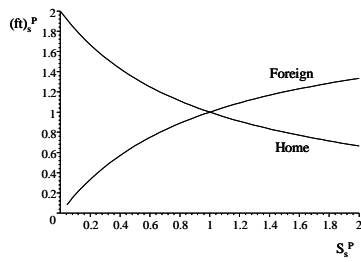


Figure 5: PCP Trade Share Curves under High Monopolistic Competition (for a markup of 100%, i.e.  $\varphi = 2$ )

**Proposition 6** (*World Trade Share*) *For the world economy as a whole, the trade-to-output ratio is constant at 1 in any state of nature  $s \in S$ , due to symmetry and no matter the particular price setting assumed.*

Under *CCP*, *Home* nominal trade is always equal to *Home* nominal output so that the *Home* trade share in output is constant at 1, irrespective of the state of nature that has materialized. The same is true for *Foreign*, and as a consequence the (equally-weighted) world trade-to-GDP ratio is also 1 for  $\forall s \in S$ .<sup>39</sup>

Under *PCP*, by contrast, *Home* and *Foreign* trade shares in output are stochastic and *not* equal to each other and to 1 unless relative monetary equilibrium occurs ( $s_e \subset S$ ). However, as can also be verified by looking at figures 3, 4, and 5 the *Home* and *Foreign* trade share curves are *complementary* in the sense that at each point they sum to 2, so that world trade equals world output for  $\forall s \in S$ .

**Trade Balance** Derivations identical to those for the trade share in output above but with a *minus* sign in the numerator of formulas (30) and (31) always (i.e. in any  $s \in S$ ) derive a *zero* trade balance, no matter the specific invoicing assumed. The logic behind this result was highlighted in Proposition 3.

## 4 Effects of the Exchange-Rate Regime

Making further use of the equilibrium solutions under float we characterized thus far, the present section summarizes the implications of a peg, and therefore of the alternative exchange-rate regimes we study here, for international *intra*-industry trade prices and flows. Our regime comparisons discussed below are made along two dimensions, namely with respect to *ex-post* (*equilibrium*) and *ex-ante* (*expected*) trade measures. The reason is that when evaluating float vs. peg under (monetary) uncertainty it is the expected levels of the relevant variables, i.e. integrated over the entire distribution of shocks, that can be meaningfully compared, the *ex-post* ones being stochastic, i.e. state-specific. We saw, however, that our equilibrium model outcomes concerning, in particular, the share of nominal trade in nominal output by country were not necessarily stochastic, and whether they were or not depended on the currency of invoicing assumed. Moreover, the *equilibrium* solutions are a necessary first *step* in deriving the *expected* ones. That is why we also retain in what follows the *ex-post* dimension of our analysis and then simulate a possible *ex-post trade stabilization* role for a *fixed* exchange-rate regime under *PCP* (but not *CCP*).

### 4.1 Comparative Synthesis of Equilibrium Results

Table 1 captures in a synthetic form the effects we evoked in our propositions up to now. It compares the *equilibrium* model outcomes under a *flexible* exchange-

<sup>39</sup>This latter equality does not, however, also mean that real consumption is equal in the two countries, which will be true only under equal money growth rates in a given state of nature  $s_e$  (recall Proposition 1).

rate regime across the alternative price-setting conventions studied.

	CCP	PCP
<i>NER</i>	$S_s^c = \frac{M_s}{M_s^*} \neq 1$ unless $s_e$	$S_s^P = \left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}} \neq 1$ unless $s_e$
<i>relative prices</i>		
foreign/home	$p_H^C = p_F^{*,C} = 1$	$\overbrace{p_{H,s}^P = \left(p_{F,s}^{*,P}\right)^{-1} = S_s^P = 1}^{\neq 1 \text{ unless } s_e}$
ToT	$(ToT)_{H,s}^C = \frac{1}{(ToT)_{F,s}^{*,C}} = \frac{1}{S_s^C} =$	$\overbrace{= (ToT)_{H,s}^P = \frac{1}{(ToT)_{F,s}^{*,P}}}^{\neq 1 \text{ unless } s_e}$
RER	$= \frac{1}{(RER)_{H,s}^C} = (RER)_{F,s}^{*,C} \neq 1$ unless $s_e$	$(RER)_H^P = (RER)_F^{*,P} = 1$
<i>consumption</i>		
relative	$c_s^C \neq c_s^{*,C}$ unless $s_e$	$c_s^P \neq c_s^{*,P}$ unless $s_e$
split-up	$\frac{c_{H,s}^C}{c_{F,s}^C} = \frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = 1, \forall s$	$1 \neq \frac{c_{H,s}^P}{c_{F,s}^P} \neq \frac{c_{F,s}^{*,P}}{c_{H,s}^{*,P}} \neq 1$ unless $s_e$
aggregates	$c_{H,s}^C = c_{F,s}^C \neq c_{H,s}^{*,C} = c_{F,s}^{*,C}, \forall s$	$c_{H,s}^P \neq c_{F,s}^P \neq c_{H,s}^{*,P} \neq c_{F,s}^{*,P}$ unless $s_e$
<i>labor/leisure</i>		
employment	$n_s^C = n_s^{*,C}, \forall s$	$n_s^P \neq n_s^{*,P}$ unless $s_e$
leisure	$l_s^C = l_s^{*,C}, \forall s$	$l_s^P \neq l_s^{*,P}$ unless $s_e$
<i>trade-to-output</i>		
by country	$(ft)_H^C = (ft)_F^{*,C} = 1$	$\overbrace{(ft)_{H,s}^P}^{\neq 1} \neq \overbrace{(ft)_{F,s}^{*,P}}^{\neq 1}$ unless $s_e$
world	$\frac{1}{2}(ft)_H^C + \frac{1}{2}(ft)_F^{*,C} = 1$	$\frac{1}{2}(ft)_{H,s}^P + \frac{1}{2}(ft)_{F,s}^{*,P} = 1, \forall s$
<i>trade balance</i>	$(TB)_H^C = (TB)_F^{*,C} = 0$	$(TB)_H^P = (TB)_F^{*,P} = 0$

Table 1: Equilibrium Results under Float

Similarly, Table 2 provides a compact account of our CCP vs. PCP *equilibrium* findings under a *fixed* exchange-rate regime, i.e. with  $M_s \equiv M_s^*$  for  $\forall s \in S$ . It helps clarify in an explicit manner the parallels and divergencies with regard to the corresponding float results in Table 1.

On the basis of these two comparative tables, we next discuss the impact of alternative exchange rate-regimes on trade prices and flows, given CCP or PCP.

## 4.2 Relative Prices under Peg

As far as the key international prices are concerned, a peg makes a difference with respect to a float in that it ensures all three relative prices we considered

	CCP	PCP
<i>NER</i>	$S_s^C = \frac{M_s}{M_s} = 1, \forall s$	$S_s^P = \left(\frac{M_s}{M_s}\right)^{\frac{1}{\varphi}} = 1, \forall s$
<i>relative prices</i>		
foreign/home	<i>same as under float</i>	$p_H^P = p_F^{*,P} = S^P =$
ToT	$(ToT)_H^C = (ToT)_F^C = S^C =$	$= (ToT)_H^P = (ToT)_F^P = 1$
RER	$= (RER)_H^C = (RER)_F^C = 1$	<i>same as under float</i>
<i>consumption</i>		
relative	$c_s^C = c_s^{*,C}, \forall s$	$c_s^P = c_s^{*,P}, \forall s$
split-up	<i>same as under float</i>	$\frac{c_{H,s}^P}{c_{F,s}^P} = \frac{c_{H,s}^{*,P}}{c_{F,s}^{*,P}} = 1, \forall s$
aggregates	$c_{H,s}^C = c_{F,s}^C = c_{H,s}^{*,C} = c_{F,s}^{*,C}, \forall s$	$c_{H,s}^P = c_{F,s}^P = c_{H,s}^{*,P} = c_{F,s}^{*,P}, \forall s$
<i>labor/leisure</i>		
employment	<i>same as under float</i>	$n_s^P = n_s^{*,P}, \forall s$
leisure	<i>same as under float</i>	$l_s^P = l_s^{*,P}, \forall s$
<i>trade-to-output</i>		
by country	<i>same as under float</i>	$(ft)_H^P = (ft)_F^P = 1$
world	<i>same as under float</i>	<i>same as under float</i>
<i>trade balance</i>	<i>same as under float</i>	<i>same as under float</i>

Table 2: Equilibrium Results under Peg

– the foreign/domestic output price, the ToT and the RER – to be equal to 1, i.e. to the *fixed* NER (cf. tables 1 and 2) not only ex-ante (in expectation) but also ex-post (in equilibrium) in any realized state. Consequently, Home as well as Foreign agents perceive these prices in the *same neutral* way which does not induce substitutions in consumption via pass-through and expenditure switching. Under float and *CCP* (see Table 1), this is not generally the case for the ToT and the RER, no matter that the relative price of foreign-produced goods in terms of domestic goods is always predetermined at 1 (so that the expenditure-switching channel is inoperative). Under float and *PCP* (see again Table 1), it is not generally the case for this latter relative price (so that now NER pass-through induces optimal expenditure switching) and for the ToT, no matter that the equilibrium RER is always 1, due to PPP.

### 4.3 Expected Trade Flows

**Proposition 7** (*Expected Trade Share*) *Under full symmetry and separable preferences, the expected trade-to-output ratio in each of the countries is always 1, no matter the particular price setting and exchange-rate regime modelled.*

Under CCP, equations (32) and (34) we derived earlier showed that the value of trade is equal to the value of output, irrespective of the particular state of nature that has materialized. To put it differently, both trade and output do vary in *value* across states, but under CCP when there is no consumption bias this

variation is in the same direction and proportion so that their *ratio* always remains constant, at 1 under full symmetry and separable preferences. Therefore, *expected* trade-to-output is also 1 under CCP, given the above assumptions:

$$E_0 \left[ (ft)_H^C \right] = E_0 \left[ (ft)_F^C \right] = E_0 [1] = 1, \quad s \in S. \quad (36)$$

Taking expectations from the equilibrium trade share formulas, (33) and (35), under PCP with float and *full symmetry* is shown in the *Appendix B* to derive the same result:

$$E_0 \left[ (ft)_{F,s}^P \right] = 1 = E_0 \left[ (ft)_{H,s}^P \right], \quad s \in S. \quad (37)$$

We thus conclude that *expected* trade-to-output is 1 under PCP too.

To sum-up, our alternative assumptions on invoicing and monetary arrangements are neutral to *expected* trade shares, the *relevant* measure to compare them under uncertainty, as in our framework. Moreover, these same key assumptions are also neutral to the trade balance – expected *as well as* equilibrium.<sup>40</sup>

However, there is one essential way, valid only under PCP, in which the exchange-rate regime does matter for trade in our set-up. It is that a peg eliminates – by preventing any exchange-rate pass-through on relative prices and, hence, by shutting down the expenditure switching channel – the volatility of trade in terms of output across states of nature. Comparing the trade share formulas (33) and (35) makes it easy to see that a peg under PCP *restores* in any  $s \in S$  the ex-post equality, typical under CCP with float, between nominal trade and nominal output in each of the countries. This interesting parallel is highlighted next.

**Corollary 1** (*Trade-Output Equalization under PCP with Peg*) *A fixed exchange-rate regime, by maintaining relative money stock equilibrium in any state of nature, guarantees under PCP – via the optimal consumption split-up channel – equilibrium trade to be equal to output in both countries modelled.*

**Proof.** Follows directly from the proofs of propositions 2 and 5. ■

Note that trade-output equalization obtains always under CCP even with float,<sup>41</sup> so a peg is in that case not needed to bring about such a result.

#### 4.4 How Much Trade Stabilization under PCP with Peg?

In Corollary 1 we showed that under PCP a peg can stabilize the ex-post trade share in each of the economies at its CCP level of 1 (expected as well as actual in any state of the world). But what is the likely *degree* of such trade stabilization?

To answer this aspect of our analysis, we finally simulated our PCP model version under two *classes* of jointly symmetric money growth shock processes: (i) a *standard normal* (discretized) distribution,  $\mathcal{N}(0, 1)$ , and (ii) a *uniform*

<sup>40</sup>Under full symmetry and separable preferences, again.

<sup>41</sup>Given full symmetry and separable preferences.

(discretized) distribution with 101 equally-spaced values centered around zero,  $\mathcal{U}[-5.0, -4.9, \dots, -0.1, 0, 0.1, \dots, 4.9, 5.0]$ . In the second case we have thus implicitly assumed a volatility of the forcing variables in the model, the two monetary disturbances, which is 2.93 times *higher* than in the first one in terms of (relative) standard deviation. In both cases the simulated magnitudes of the shocks are directly interpretable as *percentage* growth rates of the money stock under (pure) float, i.e. given the assumption that the shocks in Home and in Foreign are jointly drawn from identical but independent distributions.

100 random draws were generated from the described two classes of bivariate symmetric distributions for each money shock. We then computed, for the obtained 100 states of nature, the equilibrium NER and trade shares under CCP and PCP as well as their descriptive statistics, given  $\varphi = 11$ . 10 such simulation exercises were performed in total, from which we finally computed the respective *mean* descriptive statistics, as they are reported in Table 3.<sup>42</sup>

Reported values are means of 10 simulations with 100 states ( $M_s, M_s^*$ ) generated in each								
	Equilibrium NER				Equilibrium Trade Shares			
	$\mathcal{N}(0, 1)$ shocks		$\mathcal{U}[-5, 5]$ shocks		$\mathcal{N}(0, 1)$ shocks		$\mathcal{U}[-5, 5]$ shocks	
	CCP	PCP	CCP	PCP	Home	Foreign	Home	Foreign
Mean	1.0002	1.0000	1.0004	1.0000	0.9993	1.0007	1.0024	0.9976
Median	1.0005	1.0000	0.9994	1.0000	0.9974	1.0026	1.0032	0.9968
Maximum	1.0391	1.0035	1.0954	1.0083	1.1880	1.2061	1.4596	1.4596
Minimum	0.9657	0.9968	0.9129	0.9918	0.7939	0.8120	0.5404	0.5404
Std. Dev.	0.0145	0.0013	0.0419	0.0038	0.0787	0.0787	0.2196	0.2196
Observations	100	100	100	100	100	100	100	100

Table 3: Volatility of the Equilibrium NER and PCP Trade Shares

As clear from the table – and as implied by our earlier equilibrium NER formulas – the exchange rate is more volatile under CCP than PCP, no matter the particular shock process (once it is jointly symmetric across identical nations). *How much* more volatile corresponds roughly to the value of the elasticity of (cross-country) demand parameter,  $\varphi > 1$ . Since  $\varphi = 11$  in our simulation, the standard deviation of the CCP NER is 11 times higher than that of the PCP NER, for the standard normal as well as for the uniform distribution. As we noted earlier, the degree of consumption substitutability thus mitigates the impact of exogenous monetary policies under PCP (relative to CCP). Nominal exchange-rate volatility is roughly 3 times higher for the uniform distribution, since the forcing shocks in it were chosen to be 2.93 times more volatile than those in the standard normal distribution, as already explained. The variability of the NER in the model is thus ”inherited” – although to a different degree under CCP vs. PCP<sup>43</sup> – from the variability of the underlying symmetric money

<sup>42</sup>The GAUSS program for the simulation and a more detailed presentation of the results are available upon request.

<sup>43</sup>In the PCP case the former volatility is much lower than the latter in terms of standard deviation, 0.13% vs. 1% under the normal shocks and 0.38% vs. 2.93% under the uniform

growth disturbances.

As far as national trade-to-output ratios are concerned, our simulation indicates a much higher volatility. In compliance with the theoretical implications of the PCP model version, it is the *same across countries*. Trade share variability under PCP is, of course, a function of the NER variability (given  $\varphi = 11$ ) and, ultimately, of the volatility of the driving monetary shocks. It is of the order of a standard deviation of 7.87% for the jointly symmetric normal disturbances and of 21.96% for the uniform ones. To judge about the magnitude of these fluctuations, observe that the underlying standard deviations of the PCP NER are of only 0.13% and 0.38% for the normal and uniform distribution cases, respectively (cf. Table 3).

All in all, the above-simulated magnitudes of the (short-run) fluctuations of the PCP NER are much too insignificant compared to what is usually observed, for example, in real-world macrodata of a monthly frequency. Those of the CCP NER, 1.45% and 4.19%, respectively, seem rather small as well. By contrast, the simulated (short-run) volatility of national trade shares in output appears exaggerated even under symmetric standard normal shocks (7.87%), to say nothing about the excessive swings driven by the three times larger symmetric uniform shocks (21.96%). More work is, evidently, needed in order to better design and calibrate the simple NOEM set-up we employed in our theoretical analysis and trade variability simulation.

One thing is, nevertheless, clear: given the *equal* preference for Home and Foreign product brands in the model of intra-industry trade we analyzed, there may be some role for a peg in eliminating ex-post bias in consumption under float and, consequently, stabilizing the equilibrium trade-to-output in both countries. With view to the high trade share volatility resulting from shock distributions with a (much) weaker dispersion, there is a (significant) trade stabilization potential of a fixed exchange-rate regime under PCP. But rough calculations within the context of this model have indicated that there is a cost of such stabilization in terms of some loss (not a big one, it is true) of *world* consumption relative to a PCP with float. Since (slightly) reduced real world consumption implies, in this framework, (slightly) increased world leisure, a deeper welfare analysis of the set-up we considered requires an explicit specification of the utility function, which we preferred to keep general for our purposes here, and thus goes beyond the scope of the present study.

## 5 Concluding Comments

The objective of this paper was to analyze the implications of alternative price setting in evaluating the effects of the exchange-rate regime on *intra*-industry trade. The recent NOEM modelling approach underlying much related research has provided a modern toolkit to revisit this classic but still unresolved issue. To study it within an appropriate framework, we essentially extended Bacchetta and van Wincoop's (2000 a) stochastic "benchmark monetary model" based shocks, reflecting the influence of the substitutability parameter  $\varphi > 1$ .

on consumer's currency pricing (CCP) to a producer's currency pricing (PCP) version as well.

Our analysis confirmed in a broader context their conclusion that a peg does not necessarily imply a higher trade share in output relative to a float, for any of the two identical countries or currency blocs modelled as well as for the world economy as a whole. With full symmetry, only monetary shocks and separable but otherwise very general utility, the exchange-rate regime does not matter for the *expected level* of trade-to-output ratios across nations, irrespective of the assumed price setting. This important result was explicitly derived from microfoundations and formally proved within our purposefully kept simple analytical framework. We also pointed out that once nominal rigidity is distinguished across open-economy invoicing practices, a comparison of exchange-rate regimes is nevertheless meaningful under *PCP*, although not *CCP*, in terms of *volatility* of relative prices and, hence, national trade shares. More precisely, the equilibrium trade share *by country* becomes volatile across states of nature under *PCP*, although it is still constant at 1 for the *world* as a whole, just like in the *CCP* model version. Our simulation has shown that this trade-to-output variability under *PCP*, equal for both countries due to symmetry, is much higher than that of the exchange rate and the underlying money growth shocks, for a reasonable parametrization. There is, thus, an effect of a peg under *PCP*, absent under *CCP*, in *stabilizing* across states of nature equilibrium trade-to-output in each of the economies at its expected level of 1.

We identified the difference in the impact of exchange-rate regimes on national trade share variability as originating in the currency denomination of transactions and, hence, the *exchange-rate pass-through* implied by our alternative price-setting assumptions. Consequently, the expenditure-switching channel functions well under (full) *PCP* but not at all under (full) *CCP*. We showed, in particular, that under *both CCP* and *PCP* *relative* real consumption is determined in equilibrium by the *relative* money stock, although in a different way, and that the trade balance is always zero, thus being independent of invoicing practices given symmetry. We also demonstrated that the optimal split-up of real consumption between demand for domestic and foreign goods is 50 : 50 under *CCP* no matter the state of nature, so any kind of monetary expansion – coordinated under peg or unilateral under float – does *not* induce *bias* in consumption. Under *PCP*, this optimal split-up depends instead on the *relative* money stock in the realized state. Thus, a monetary expansion under float in one of the countries results in a *bias in both countries* favoring consumption of the goods produced in the *expansionary* country. Finally, we proved that under *CCP* equilibrium output, employment and, ultimately, leisure (but not consumption) are always the same across countries, whereas under *PCP* they are determined (as well as consumption) by the *relative* money stock and are therefore not equal between nations unless in the case of relative monetary equilibrium.



## A Optimization Problems and Equilibrium Model Outcomes

### A.1 Households Optimization

As noted in the main text, households optimize only ex-post, under certainty, and in two dimensions, or rather stages. We now present, in turn, the essential algebra behind these two stages of their *nested* optimization problem à la Dixit-Stiglitz (1977).

#### Households *Labor-Leisure* Optimization

**Constrained Maximization Problem** Having observed the realization of shocks and subject to the constraints (2), (3), (4) and (5) specified in subsection 2.1, the *representative* consumer-household in  $H$  first chooses its *trade-off between labor and leisure*, maximizing its utility within state  $s$ :

$$\underset{c_s, l_s}{Max} \quad u(c_s, l_s), \quad \forall s \in S.$$

The analogous expression for the representative household in  $F$  is

$$\underset{c_s^*, l_s^*}{Max} \quad u(c_s^*, l_s^*), \quad \forall s \in S.$$

**First-Order Conditions** The following "*compact*" FONC can be derived in a standard way from the above-described constrained optimization problem for the  $H$  representative household:

$$W_s = \frac{u_{l,s}}{u_{c,s}} P_s, \quad \forall s \in S.$$

For  $F$ , the analogous expression is, of course, symmetric:

$$W_s^* = \frac{u_{l,s}^*}{u_{c,s}^*} P_s^*, \quad \forall s \in S.$$

The equilibrium wages in the realized state of nature are thus determined in the competitive labor market we model.

**Households *Consumption Basket* Optimization** The details of households *consumption basket* optimization in each realized state of nature  $s$  differ across our price-setting assumptions, so we present in turn the CCP and the PCP cases.

**CCP Optimization of Home Households** Under CCP, a  $H$  household's  $j \in [0, 1]$  total real consumption demand is defined by a Dixit-Stiglitz (1977) aggregator of the following form:

$$c_s^{j,C} \equiv \left[ \left(\frac{1}{2}\right)^{\frac{1}{\varphi}} \left(c_{H,s}^{j,C}\right)^{\frac{\varphi-1}{\varphi}} + \left(\frac{1}{2}\right)^{\frac{1}{\varphi}} \left(c_{F,s}^{j,C}\right)^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \quad \forall s \in S$$

Standard *representative* household's *cost minimization* ex-post, i.e. under certainty for any realized state of nature  $s$ , then progressively derives the expressions reported in the summary tables below.

**CCP:  $H$  Domestic Absorption ( $c_{i,s}^{j,C} \rightarrow c_{H,s}^C$ ) and PPI ( $P_i^C \rightarrow P_H^C$ ) Aggregation**

CCP SUMMARY TABLE 1H	
$c_{i,s}^{j,C}, P_i^C, j \in [0, 1], i \in [0, 1], \forall s \in S$	
(1 <sup>H</sup> )	$c_{H,s}^{j,C} \equiv \left[ \int_0^1 \left(c_{i,s}^{j,C}\right)^{\frac{\varphi-1}{\varphi}} di \right]^{\frac{\varphi}{\varphi-1}}$ by <i>index</i> definition
(2 <sup>H</sup> )	$P_i^C$ given (preset in $HC$ by a $H$ firm $i$ ) $\Leftrightarrow$ state-independent
(3 <sup>H</sup> $C$ )	$c_{i,s}^{j,C} = \left(\frac{P_i^C}{P^C}\right)^{-\varphi} c_{H,s}^{j,C} \Rightarrow c_{i,s}^C = \left(\frac{P_i^C}{P^C}\right)^{-\varphi} c_{H,s}^C$
(4 <sup>H</sup> )	$P_H^C \equiv \left[ \int_0^1 (P_i^C)^{1-\varphi} di \right]^{\frac{1}{1-\varphi}}$ defined as the price of a <i>unit</i> of $c_{H,s}^C$
(5 <sup>H</sup> $C$ )	$c_{H,s}^{j,C} = \frac{1}{2} \left(\frac{P_H^C}{P^C}\right)^{-\varphi} \frac{M_s^j}{P^C} \Rightarrow c_{H,s}^C = \frac{1}{2} \left(\frac{P_H^C}{P^C}\right)^{-\varphi} \frac{M_s}{P^C}$
(3a <sup>H</sup> $C$ )	$c_{i,s}^{j,C} = \frac{1}{2} \left(\frac{P_i^C}{P^C}\right)^{-\varphi} \frac{M_s^j}{P^C} \Rightarrow c_{i,s}^C = \frac{1}{2} \left(\frac{P_i^C}{P^C}\right)^{-\varphi} \frac{M_s}{P^C}$

**CCP:  $H$  Import Demand ( $c_{i^*,s}^{j,C} \rightarrow c_{F,s}^C$ ) and Import Price Index ( $P_{i^*}^C \rightarrow P_F^C$ ) Aggregation**

CCP SUMMARY TABLE 2H

$c_{i^*,s}^{j,C}, P_{i^*}^C, \quad j \in [0,1], \quad i^* \in [1,2], \quad \forall s \in S$	
(1 $_F^H$ )	$c_{F,s}^{j,C} \equiv \left[ \int_1^2 \left( c_{i^*,s}^{j,C} \right)^{\frac{\varphi-1}{\varphi}} di^* \right]^{\frac{\varphi}{\varphi-1}}$ by <i>index</i> definition
(2 $_F^H C$ )	$P_{i^*}^C$ given (preset in $HC$ by a $F$ firm $i^*$ ) $\Leftrightarrow$ state-independent
(3 $_F^H C$ )	$c_{i^*,s}^{j,C} = \left( \frac{P_{i^*}^C}{P_F^C} \right)^{-\varphi} c_{F,s}^{j,C} \Rightarrow c_{i^*,s}^C = \left( \frac{P_{i^*}^C}{P_F^C} \right)^{-\varphi} c_{F,s}^C$
(4 $_F^H C$ )	$P_F^C \equiv \left[ \int_1^2 \left( P_{i^*}^C \right)^{1-\varphi} di^* \right]^{\frac{1}{1-\varphi}}$ defined as the price of a <i>unit</i> of $c_{F,s}^C$
(5 $_F^H C$ )	$c_{F,s}^{j,C} = \frac{1}{2} \left( \frac{P_F^C}{P^C} \right)^{-\varphi} \frac{M_s^j}{P^C} \Rightarrow c_{F,s}^C = \frac{1}{2} \left( \frac{P_F^C}{P^C} \right)^{-\varphi} \frac{M_s^j}{P^C}$
(3a $_F^H C$ )	$c_{i^*,s}^{j,C} = \frac{1}{2} \left( \frac{P_{i^*}^C}{P^C} \right)^{-\varphi} \frac{M_s^j}{P^C} \Rightarrow c_{i^*,s}^C = \frac{1}{2} \left( \frac{P_{i^*}^C}{P^C} \right)^{-\varphi} \frac{M_s^j}{P^C}$

**CCP:  $H$  CPI ( $P^C$ ) Aggregation**

CCP SUMMARY TABLE 3H

$P^C, \quad i \in [0,1] \cup i^* \in [1,2], \quad \forall s \in S$	
(6 $^H C$ )	$P^C \equiv \left[ \frac{1}{2} \left( P_H^C \right)^{1-\varphi} + \frac{1}{2} \left( P_F^C \right)^{1-\varphi} \right]^{\frac{1}{1-\varphi}}$
(6a $^H C$ )	$P^C \equiv \left\{ \frac{1}{2} \left[ \int_0^1 \left( P_i^C \right)^{1-\varphi} di \right] + \frac{1}{2} \left[ \int_1^2 \left( P_{i^*}^C \right)^{1-\varphi} di^* \right] \right\}^{\frac{1}{1-\varphi}}$

**CCP Optimization of Foreign Households** Under CCP, a  $F$  household's  $j^* \in (1,2]$  total real consumption demand is analogously (or symmetrically) defined by the Dixit-Stiglitz (1977) aggregator:

$$c_s^{j^*,C} \equiv \left[ \left( \frac{1}{2} \right)^{\frac{1}{\varphi^*}} \left( c_{F,s}^{j^*,C} \right)^{\frac{\varphi^*-1}{\varphi^*}} + \left( \frac{1}{2} \right)^{\frac{1}{\varphi^*}} \left( c_{H,s}^{j^*,C} \right)^{\frac{\varphi^*-1}{\varphi^*}} \right]^{\frac{\varphi^*}{\varphi^*-1}}, \quad \forall s \in S$$

Standard *representative* household's *cost minimization* under certainty, i.e. for any realized state of nature  $s$ , then progressively derives the expressions below for Foreign that *parallel* (or, more precisely, are the mirror image of) those for Home.

**CCP:  $F$  Domestic Absorption ( $c_{i^*,s}^{j^*,C} \rightarrow c_{F,s}^{*,C}$ ) and PPI ( $P_{i^*}^{*,C} \rightarrow P_F^{*,C}$ ) Aggregation**

CCP SUMMARY TABLE 1F

$c_{i^*,s}^{j^*,C}, P_{i^*}^{*,C}, j^* \in [1, 2], i^* \in [1, 2], \forall s \in S$	
(1 <sup>F</sup> )	$c_{F,s}^{j^*,C} \equiv \left[ \int_1^2 \left( c_{i^*,s}^{j^*,C} \right)^{\frac{\varphi^*-1}{\varphi^*}} di^* \right]^{\frac{\varphi^*}{\varphi^*-1}}$ by <i>index</i> definition
(2 <sup>F</sup> )	$P_{i^*}^{*,C}$ given (preset in $FC$ by a $F$ firm $i^*$ ) $\Leftrightarrow$ state-independent
(3 <sup>F</sup> $C$ )	$c_{i^*,s}^{j^*,C} = \left( \frac{P_{i^*}^{*,C}}{P_F^{*,C}} \right)^{-\varphi^*} c_{F,s}^{j^*,C} \Rightarrow c_{i^*,s}^{*,C} = \left( \frac{P_{i^*}^{*,C}}{P_F^{*,C}} \right)^{-\varphi^*} c_{F,s}^{*,C}$
(4 <sup>F</sup> )	$P_F^{*,C} \equiv \left[ \int_1^2 \left( P_{i^*}^{*,C} \right)^{1-\varphi^*} di^* \right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a <i>unit</i> of $c_{F,s}^{*,C}$
(5 <sup>F</sup> )	$c_{F,s}^{j^*,C} = \frac{1}{2} \left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{F,s}^{*,C} = \frac{1}{2} \left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^*}{P^{*,C}}$
(3a <sup>F</sup> )	$c_{i^*,s}^{j^*,C} = \frac{1}{2} \left( \frac{P_{i^*}^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{i^*,s}^{*,C} = \frac{1}{2} \left( \frac{P_{i^*}^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^*}{P^{*,C}}$

**CCP:  $F$  Import Demand ( $c_{i,s}^{j^*,C} \rightarrow c_{H,s}^{*,C}$ ) and Import Price Index ( $P_{i^*}^{*,C} \rightarrow P_H^{*,C}$ ) Aggregation**

CCP SUMMARY TABLE 2F

$c_{i,s}^{j^*,C}, P_i^{*,C}, j^* \in [1, 2], i \in [0, 1], \forall s \in S$	
(1 <sup>F</sup> )	$c_{H,s}^{j^*,C} \equiv \left[ \int_0^1 \left( c_{i,s}^{j^*,C} \right)^{\frac{\varphi^*-1}{\varphi^*}} di \right]^{\frac{\varphi^*}{\varphi^*-1}}$ by <i>index</i> definition
(2 <sup>F</sup> $C$ )	$P_i^{*,C}$ given (preset in $FC$ by a $H$ firm $i$ ) $\Leftrightarrow$ state independent
(3 <sup>F</sup> $C$ )	$c_{i,s}^{j^*,C} = \left( \frac{P_i^{*,C}}{P_H^{*,C}} \right)^{-\varphi^*} c_{H,s}^{j^*,C} \Rightarrow c_{i,s}^{*,C} = \left( \frac{P_i^{*,C}}{P_H^{*,C}} \right)^{-\varphi^*} c_{H,s}^{*,C}$
(4 <sup>F</sup> $C$ )	$P_H^{*,C} \equiv \left[ \int_0^1 \left( P_i^{*,C} \right)^{1-\varphi^*} di \right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a <i>unit</i> of $c_{H,s}^{*,C}$
(5 <sup>F</sup> $C$ )	$c_{H,s}^{j^*,C} = \frac{1}{2} \left( \frac{P_H^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{H,s}^{*,C} = \frac{1}{2} \left( \frac{P_H^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^*}{P^{*,C}}$
(3a <sup>F</sup> $C$ )	$c_{i,s}^{j^*,C} = \frac{1}{2} \left( \frac{P_i^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{i,s}^{*,C} = \frac{1}{2} \left( \frac{P_i^{*,C}}{P^{*,C}} \right)^{-\varphi^*} \frac{M_s^*}{P^{*,C}}$

**CCP:  $F$  CPI ( $P^{*,C}$ ) Aggregation**

CCP SUMMARY TABLE 3F	
$P^{*,C}, i^* \in [1, 2] \cup i \in [0, 1], \forall s \in S$	
(6 <sup>F</sup> C)	$P^{*,C} \equiv \left[ \frac{1}{2} \left( P_{F^*}^{*,C} \right)^{1-\varphi^*} + \frac{1}{2} \left( P_{H^*}^{*,C} \right)^{1-\varphi^*} \right]^{\frac{1}{1-\varphi^*}}$
(6a <sup>F</sup> C)	$P^{*,C} \equiv \left\{ \frac{1}{2} \left[ \int_1^2 \left( P_{i^*}^{*,C} \right)^{1-\varphi^*} di^* \right] + \frac{1}{2} \left[ \int_0^1 \left( P_i^{*,C} \right)^{1-\varphi^*} di \right] \right\}^{\frac{1}{1-\varphi^*}}$

**PCP Optimization of *Home* Households** Under PCP, a  $H$  household's  $j \in [0, 1]$  total real consumption demand is again defined by a Dixit-Stiglitz (1977) index of the same form as under CCP but with different resulting domestic and external demands for goods (hence the  $P$  superscript indexing for PCP now in place of the  $C$  superscript indexing for CCP earlier) :

$$c_s^{j,P} \equiv \left[ \left( \frac{1}{2} \right)^{\frac{1}{\varphi}} \left( c_{H,s}^{j,P} \right)^{\frac{\varphi-1}{\varphi}} + \left( \frac{1}{2} \right)^{\frac{1}{\varphi}} \left( c_{F,s}^{j,P} \right)^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \quad \forall s \in S$$

Standard representative household's *cost minimization* ex-post, i.e. for any realized state of nature  $s$ , derives again the expressions in the summary tables below that parallel those reported for the CCP model version.

**PCP:  $H$  Domestic Absorption ( $c_{i,s}^{j,P} \rightarrow c_{H,s}^P$ ) and PPI ( $P_i^P \rightarrow P_H^P$ ) Aggregation**

PCP SUMMARY TABLE 1H	
$c_{i,s}^{j,P}, P_{i,s}^P, j \in [0, 1], i \in [0, 1], \forall s \in S$	
(1 <sup>H</sup> )	$c_{H,s}^{j,P} \equiv \left[ \int_0^1 \left( c_{i,s}^{j,P} \right)^{\frac{\varphi-1}{\varphi}} di \right]^{\frac{\varphi}{\varphi-1}}$ by <i>index</i> definition
(2 <sup>H</sup> )	$P_i^P$ given (preset in $HC$ by a $H$ firm $i$ ) $\Leftrightarrow$ state-independent
(3 <sup>H</sup> P)	$c_{i,s}^{j,P} = \left( \frac{P_i^P}{P_H^P} \right)^{-\varphi} c_{H,s}^{j,P} \Rightarrow c_{i,s}^P = \left( \frac{P_i^P}{P_H^P} \right)^{-\varphi} c_{H,s}^P$
(4 <sup>H</sup> )	$P_H^P \equiv \left[ \int_0^1 \left( P_i^P \right)^{1-\varphi} di \right]^{\frac{1}{1-\varphi}}$ defined as the price of a <i>unit</i> of $c_{H,s}^P$
(5 <sup>H</sup> P)	$c_{H,s}^{j,P} = \frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s^j}{P_s^P} \Rightarrow c_{H,s}^P = \frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}$
(3a <sup>H</sup> P)	$c_{i,s}^{j,P} = \frac{1}{2} \left( \frac{P_i^P}{P_s^P} \right)^{-\varphi} \frac{M_s^j}{P_s^P} \Rightarrow c_{i,s}^P = \frac{1}{2} \left( \frac{P_i^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}$

**PCP:  $H$  Import Demand ( $c_{i^*,s}^{j,P} \rightarrow c_{F,s}^P$ ) and Import Price Index**  
**( $P_{i^*}^{*,P} \xrightarrow{S_s^P} P_{i^*,s}^P \rightarrow P_{F,s}^P$ ) Aggregation**

PCP SUMMARY TABLE 2H

$c_{i^*,s}^{j,P}, P_{i^*}^{*,P} \xrightarrow{S_s^P} P_{i^*,s}^P, j \in [0, 1], i^* \in [1, 2], \forall s \in S$	
(1 $_F^H$ )	$c_{F,s}^{j,P} \equiv \left[ \int_1^2 \left( c_{i^*,s}^{j,P} \right)^{\frac{\varphi-1}{\varphi}} di^* \right]^{\frac{\varphi}{\varphi-1}}$ by index definition
(2 $_F^H P$ )	$P_{i^*}^{*,P}$ given (preset in $FC$ by a $F$ firm $i^*$ ) $\Leftrightarrow$ state independent
(3 $_F^H P$ )	$c_{i^*,s}^{j,P} = \left( \frac{P_{i^*}^{*,P}}{P_F^{*,P}} \right)^{-\varphi} c_{F,s}^{j,P} \Rightarrow c_{i^*,s}^P = \left( \frac{P_{i^*}^{*,P}}{P_F^{*,P}} \right)^{-\varphi} c_{F,s}^P$
(4 $_F^H P$ )	$\underbrace{S_s^P P_F^{*,P}}_{\equiv P_{F,s}^P} \equiv \left[ \int_1^2 \underbrace{\left( S_s^P P_{i^*}^{*,P} \right)^{1-\varphi}}_{\equiv P_{i^*,s}^P} di^* \right]^{\frac{1}{1-\varphi}}$ defined as the price of a unit of $c_{F,s}^P$
(5 $_F^H P$ )	$c_{F,s}^{j,P} = \frac{1}{2} \left( \frac{\underbrace{S_s^P P_F^{*,P}}_{\equiv P_{F,s}^P}}{P_s^P} \right)^{-\varphi} \frac{M_s^j}{P_s^P} \Rightarrow c_{F,s}^P = \frac{1}{2} \left( \frac{\underbrace{S_s^P P_F^{*,P}}_{\equiv P_{F,s}^P}}{P_s^P} \right)^{-\varphi} \frac{M_s^j}{P_s^P}$
(3a $_F^H P$ )	$c_{i^*,s}^{j,P} = \frac{1}{2} \left( \frac{\underbrace{S_s^P P_{i^*}^{*,P}}_{\equiv P_{i^*,s}^P}}{P_s^P} \right)^{-\varphi} \frac{M_s^j}{P_s^P} \Rightarrow c_{i^*,s}^P = \frac{1}{2} \left( \frac{\underbrace{S_s^P P_{i^*}^{*,P}}_{\equiv P_{i^*,s}^P}}{P_s^P} \right)^{-\varphi} \frac{M_s^j}{P_s^P}$

**PCP:  $H$  CPI ( $P_s^P$ ) Aggregation**

PCP SUMMARY TABLE 3H	
$P_s^P, i \in [0, 1] \cup i^* \in [1, 2], \forall s \in S$	
(6 <sup>H</sup> P)	$P_s^P \equiv \left[ \frac{1}{2} (P_H^P)^{1-\varphi} + \frac{1}{2} (P_{F,s}^P)^{1-\varphi} \right]^{\frac{1}{1-\varphi}} \equiv$ $\equiv \left[ \frac{1}{2} (P_H^P)^{1-\varphi} + \frac{1}{2} (S_s^P P_{F^*,P}^P)^{1-\varphi} \right]^{\frac{1}{1-\varphi}}$
(6a <sup>H</sup> P)	$P_s^P \equiv \left\{ \frac{1}{2} \left[ \int_0^1 (P_i^P)^{1-\varphi} di \right] + \frac{1}{2} \left[ \int_1^2 (P_{i^*}^P)^{1-\varphi} di^* \right] \right\}^{\frac{1}{1-\varphi}} \equiv$ $\equiv \left\{ \frac{1}{2} \left[ \int_0^1 (P_i^P)^{1-\varphi} di \right] + \frac{1}{2} \left[ \int_1^2 (S_s^P P_{i^*}^P)^{1-\varphi} di^* \right] \right\}^{\frac{1}{1-\varphi}}$

**PCP Optimization of Foreign Households** Under PCP, a  $F$  household's  $j^* \in (1, 2]$  total real consumption demand is defined, analogously to that of a  $H$  household  $j \in [0, 1]$ , by a Dixit-Stiglitz (1977) index of the same form:

$$c_s^{j^*,P} \equiv \left[ \left( \frac{1}{2} \right)^{\frac{1}{\varphi^*}} (c_{F,s}^{j^*,P})^{\frac{\varphi^*-1}{\varphi^*}} + \left( \frac{1}{2} \right)^{\frac{1}{\varphi^*}} (c_{H,s}^{j^*,P})^{\frac{\varphi^*-1}{\varphi^*}} \right]^{\frac{\varphi^*}{\varphi^*-1}}, \quad \forall s \in S$$

Standard representative household's cost minimization under certainty, i.e. for any realized state of nature  $s$ , then derives the expressions in the summary tables that follow.

**PCP:  $F$  Domestic Absorption ( $c_{i^*,s}^{j^*,P} \rightarrow c_{F,s}^{*,P}$ ) and PPI ( $P_{i^*}^{*,P} \rightarrow P_F^{*,P}$ ) Aggregation**

PCP SUMMARY TABLE 1F	
$c_{i^*,s}^{j^*,P}, P_{i^*}^{*,P}, j^* \in [1, 2], i^* \in [1, 2], \forall s \in S$	
(1 <sup>F</sup> )	$c_{F,s}^{j^*,P} \equiv \left[ \int_1^2 (c_{i^*,s}^{j^*,P})^{\frac{\varphi^*-1}{\varphi^*}} di^* \right]^{\frac{\varphi^*}{\varphi^*-1}} \quad \text{by index definition}$
(2 <sup>F</sup> )	$P_{i^*}^{*,P} \text{ given (preset in } FC \text{ by a } F \text{ firm } i^*) \Leftrightarrow \text{state-independent}$
(3 <sup>F</sup> P)	$c_{i^*,s}^{j^*,P} = \left( \frac{P_{i^*}^{*,P}}{P_F^{*,P}} \right)^{-\varphi^*} c_{F,s}^{j^*,P} \Rightarrow c_{i^*,s}^{*,P} = \left( \frac{P_{i^*}^{*,P}}{P_F^{*,P}} \right)^{-\varphi^*} c_{F,s}^{*,P}$
(4 <sup>F</sup> )	$P_F^{*,P} \equiv \left[ \int_1^2 (P_{i^*}^{*,P})^{1-\varphi^*} di^* \right]^{\frac{1}{1-\varphi^*}} \quad \text{defined as the price of a unit of } c_{F,s}^{*,P}$
(5 <sup>F</sup> P)	$c_{F,s}^{j^*,P} = \frac{1}{2} \left( \frac{P_F^{*,P}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{F,s}^{*,P} = \frac{1}{2} \left( \frac{P_F^{*,P}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^*}{P_s^{*,P}}$
(3a <sup>F</sup> P)	$c_{i^*,s}^{j^*,P} = \frac{1}{2} \left( \frac{P_{i^*}^{*,P}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{i^*,s}^{*,P} = \frac{1}{2} \left( \frac{P_{i^*}^{*,P}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^*}{P_s^{*,P}}$

**PCP:  $F$  Import Demand ( $c_{i,s}^{j^*,P} \rightarrow c_{H,s}^{*,P}$ ) and Import Price Index**  
**( $P_i^P \xrightarrow{S_s^P} P_{i,s}^{*,P} \rightarrow P_{H,s}^{*,P}$ ) Aggregation**

PCP SUMMARY TABLE 2F

$c_{i,s}^{j^*,P}, P_i^P \xrightarrow{S_s^P} P_{i,s}^{*,P}, j^* \in [1,2], i \in [0,1], \forall s \in S$	
(1 <sup>F</sup> <sub>H</sub> )	$c_{H,s}^{j^*,P} \equiv \left[ \int_0^1 \left( c_{i,s}^{j^*,P} \right)^{\frac{\varphi^*-1}{\varphi^*}} di \right]^{\frac{\varphi^*}{\varphi^*-1}}$ by <i>index</i> definition
(2 <sup>F</sup> <sub>H</sub> P)	$P_i^P$ given (preset in <i>HC</i> by a $H$ firm $i$ ) $\Leftrightarrow$ state-independent
(3 <sup>F</sup> <sub>H</sub> P)	$c_{i,s}^{j^*,P} = \left( \frac{P_i^P}{P_H^P} \right)^{-\varphi^*} c_{H,s}^{j^*,P} \Rightarrow c_{i,s}^{*,P} = \left( \frac{P_i^P}{P_H^P} \right)^{-\varphi^*} c_{H,s}^{*,P}$
(4 <sup>F</sup> <sub>H</sub> P)	$\underbrace{\frac{P_H}{S_s^P}}_{\equiv P_{H,s}^{*,P}} \equiv \left[ \int_0^1 \underbrace{\left( \frac{P_i}{S_s^P} \right)^{1-\varphi^*}}_{\equiv P_{i,s}^{*,P}} di \right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a <i>unit</i> of $c_{H,s}^{*,P}$
(5 <sup>F</sup> <sub>H</sub> P)	$c_{H,s}^{j^*,P} = \frac{1}{2} \left( \frac{\underbrace{\frac{P_H^P}{S_s^P}}_{\equiv P_{H,s}^{*,P}}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{H,s}^{*,P} = \frac{1}{2} \left( \frac{\underbrace{\frac{P_H^P}{S_s^P}}_{\equiv P_{H,s}^{*,P}}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^*}{P_s^{*,P}}$
(3a <sup>F</sup> <sub>H</sub> P)	$c_{i,s}^{j^*,P} = \frac{1}{2} \left( \frac{\underbrace{\frac{P_i^P}{S_s^P}}_{\equiv P_{i,s}^{*,P}}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{i,s}^{*,P} = \frac{1}{2} \left( \frac{\underbrace{\frac{P_i^P}{S_s^P}}_{\equiv P_{i,s}^{*,P}}}{P_s^{*,P}} \right)^{-\varphi^*} \frac{M_s^*}{P_s^{*,P}}$

**PCP:  $F$  CPI ( $P_s^{*,P}$ ) Aggregation**

PCP SUMMARY TABLE 3F

$P_s^{*,P}, i^* \in [1,2] \cup i \in [0,1], \forall s \in S$	
(6 <sup>F</sup> P)	$P_s^{*,P} \equiv \left[ \frac{1}{2} \left( P_F^{*,P} \right)^{1-\varphi^*} + \frac{1}{2} \left( P_{H,s}^{*,P} \right)^{1-\varphi^*} \right]^{\frac{1}{1-\varphi^*}} \equiv$ $\equiv \left[ \frac{1}{2} \left( P_F^{*,P} \right)^{1-\varphi^*} + \frac{1}{2} \left( \frac{P_H^P}{S_s^P} \right)^{1-\varphi^*} \right]^{\frac{1}{1-\varphi^*}}$
(6a <sup>F</sup> P)	$P_s^{*,P} \equiv \left\{ \frac{1}{2} \left[ \int_0^1 \left( P_{i^*}^{*,P} \right)^{1-\varphi^*} di^* \right] + \frac{1}{2} \left[ \int_1^2 \left( P_i^{*,P} \right)^{1-\varphi^*} di \right] \right\}^{\frac{1}{1-\varphi^*}}$ $\equiv \left\{ \frac{1}{2} \left[ \int_0^1 \left( P_{i^*}^{*,P} \right)^{1-\varphi^*} di^* \right] + \frac{1}{2} \left[ \int_1^2 \left( \frac{P_i^P}{S_s^P} \right)^{1-\varphi^*} di \right] \right\}^{\frac{1}{1-\varphi^*}}$



## A.2 Firms Optimization

Unlike households, firms optimize only ex-ante. The details of their optimization problem *across* states of nature differ under our alternative price setting, so we present in turn the CCP and PCP cases.

### CCP

***H Producer Firm Optimal Pricing*** Under CCP, a *H* firm  $i \in [0, 1]$  maximizes its expected (real) profit by setting *two* prices, one in national currency and the other in foreign currency:

$$\begin{aligned} & \underset{P_i^C, P_i^{*,C}}{\text{Max}} E_0 \left[ u_{c,s} \frac{\Pi_{i,s}^C}{P^C} \right], \quad s \in S \\ & \quad \quad \quad \updownarrow \\ & \underset{P_i^C, P_i^{*,C}}{\text{Max}} E_0 \left\{ \frac{u_{c,s}}{P^C} \underbrace{\left[ P_i^C c_{i,s}^C + S_s^C P_i^{*,C} c_{i,s}^{*,C} - W_s^C c_{i,s}^C - W_s^C c_{i,s}^{*,C} \right]}_{\equiv \Pi_{i,s}^C} \right\}, \quad s \in S \end{aligned}$$

subject to expected *domestic* and *external* (real) demand for its differentiated product  $i$ .

The two FONCs of *H* representative firm *maximization* problem under uncertainty derive, respectively, the two optimal prices it sets at home and abroad:

CCP SUMMARY TABLE 4H	
$P_i^C, P_i^{*,C}, \quad i \in [0, 1], \quad j \in [0, 1] \cup j^* \in [1, 2]$	
$(7_H^C)$	$P_i^C = P_H^C = \frac{\varphi}{\varphi-1} \frac{E_0[u_{c,s} W_s^C M_s]}{E_0[u_{c,s} M_s]}$
$(8_H^C)$	$P_i^{*,C} = P_H^{*,C} = \frac{\varphi}{\varphi-1} \frac{E_0[u_{c,s} W_s^C M_s^*]}{E_0[u_{c,s} S_s^C M_s^*]}$

***F Producer Firm Optimal Pricing*** Under CCP, a *F* firm  $i^* \in (1, 2]$  analogously maximizes:

$$\begin{aligned} & \underset{P_{i^*}^{*,C}, P_{i^*}^C}{\text{Max}} E_0 \left[ u_{c,s}^* \frac{\Pi_{i^*,s}^{*,C}}{P^{*,C}} \right], \quad s \in S \\ & \quad \quad \quad \updownarrow \end{aligned}$$

$$\underset{P_{i^*}^{*,C}, P_{i^*}^C}{Max} E_0 \left[ \frac{u_{c,s}^*}{P^{*,C}} \underbrace{\left( P_{i^*}^{*,C} c_{i^*,s}^{*,C} + \frac{1}{S_s^C} P_{i^*}^C c_{i^*,s}^C - W_s^{*,C} c_{i^*,s}^{*,C} - W_s^{*,C} c_{i^*,s}^C \right)}_{\equiv \Pi_{i^*,s}^{*,C}} \right]$$

by setting *two* prices and subject to expected *domestic* and *external* (real) demand for its differentiated product  $i^*$ .

$F$  representative firm *maximization* under uncertainty thus symmetrically derives:

CCP SUMMARY TABLE 4F	
	$P_{i^*}^{*,C}, P_{i^*}^C, \quad i^* \in [1, 2], \quad j \in [0, 1] \cup j^* \in [1, 2]$
(7 <sub>F</sub> <sup>F</sup> C)	$P_{i^*}^{*,C} = P_F^{*,C} = \frac{\varphi^*}{\varphi^* - 1} \frac{E_0[u_{c,s}^* W_s^{*,C} M_s^*]}{E_0[u_{c,s}^* M_s^*]}$
(8 <sub>F</sub> <sup>H</sup> C)	$P_{i^*}^C = P_F^C = \frac{\varphi^*}{\varphi^* - 1} \frac{E_0[u_{c,s}^* W_s^{*,C} M_s^*]}{E_0[u_{c,s}^* \frac{1}{S_s^C} M_s^*]}$

## PCP

**$H$  Producer Firm Optimal Pricing** Under PCP, a  $H$  firm  $i \in [0, 1]$  maximizes its expected (real) profit by setting just *one*, national-currency price:

$$\underset{P_i}{Max} E_0 \left[ u_{c,s} \frac{\Pi_{i,s}^P}{P_s^P} \right], \quad s \in S$$

⇕

$$\underset{P_i^P}{Max} E_0 \left[ \frac{u_{c,s}}{P_s^P} \underbrace{\left( P_i^P c_{i,s}^P + P_i^P c_{i,s}^{*,P} - W_s^P c_{i,s}^P - W_s^P c_{i,s}^{*,P} \right)}_{\equiv \Pi_{i,s}^P} \right]$$

subject to expected *domestic* and *external* (real) demand for its differentiated product  $i$ .

The FONC of  $H$  representative firm *maximization* problem under uncertainty derives the optimal price in national currency for the domestic market; *LOP* then transforms this price into its foreign currency equivalent relevant for the foreign market:

PCP SUMMARY TABLE 4H	
$P_i^P, \quad i \in [0, 1], \quad j \in [0, 1] \cup j^* \in [1, 2]$	
$(7_H^H P)$	$P_i^P = P_H^P = \frac{\varphi}{\varphi-1} \frac{E_0[u_{c,s} W_s^P M_s] + E_0[u_{c,s} W_s^P M_s^*]}{E_0[u_{c,s} M_s] + E_0[u_{c,s} M_s^*]}$
$(8_H^F P)$	$\underbrace{P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}}_{\text{LOP}} \Rightarrow \underbrace{P_s^{*,P} = \frac{P_s^P}{S_s^P}}_{\text{PPP}}$

**F Producer Firm Optimal Pricing** Under PCP, a  $F$  firm  $i^* \in (1, 2]$  analogously maximizes:

$$\begin{aligned} & \underset{P_{i^*}^{*,P}}{\text{Max}} E_0 \left[ u_{c,s}^* \frac{\Pi_{i^*,s}^{*,P}}{P_{i^*}^{*,P}} \right], \quad s \in S \\ & \quad \quad \quad \updownarrow \\ & \underset{P_{i^*}^{*,P}}{\text{Max}} E_0 \left[ \frac{u_{c,s}^*}{P_s^{*,P}} \underbrace{\left( P_{i^*}^{*,P} c_{i^*,s}^{*,P} + P_{i^*}^{*,P} c_{i^*,s}^P - W_s^{*,P} c_{i^*,s}^{*,P} - W_s^{*,P} c_{i^*,s}^P \right)}_{\equiv \Pi_{i^*,s}^{*,P}} \right] \end{aligned}$$

by setting just *one*, national-currency price and subject to expected *domestic* and *external* (real) demand for its differentiated product  $i^*$ .

The FONC of  $F$  representative firm *maximization* problem under uncertainty derives the optimal price in national currency for the domestic market; *LOP* then transforms this price into its foreign currency equivalent relevant for the foreign market:

PCP SUMMARY TABLE 4F	
$P_{i^*}^{*,P}, \quad i^* \in [1, 2], \quad j \in [0, 1] \cup j^* \in [0, 1]$	
$(7_F^F P)$	$P_{i^*}^{*,P} = P_F^{*,P} = \frac{\varphi^*}{\varphi^*-1} \frac{E_0[u_{c,s}^* W_s^{*,P} M_s^*] + E_0[u_{c,s}^* W_s^{*,P} M_s]}{E_0[u_{c,s}^* M_s^*] + E_0[u_{c,s}^* M_s]}$
$(8_F^H P)$	$\underbrace{P_{F,s}^P = S_s^P P_{i^*}^{*,P}}_{\text{LOP}} \Rightarrow \underbrace{P_s^P = S_s^P P_{i^*}^{*,P}}_{\text{PPP}}$

### A.3 Equilibrium Nominal Exchange Rate

**CCP** Under CCP, the equilibrium (ex-post) NER solves the foreign exchange market clearing condition (20). Substituting for optimal  $c_{F,s}^C$  and  $c_{H,s}^{*,C}$  in (20) as well as for the ideal  $H$  and  $F$  CPI definitions further on in the algebraic manipulation derives (for the *full-symmetry* case expression, refer to (9CFS) in the table below):<sup>44</sup>

CCP SUMMARY TABLE 5	
$S_s^C$	
(9C)	$S_s^C = \frac{1 + \left(\frac{P_H^C}{P_F^C}\right)^{1-\varphi}}{1 + \left(\frac{P_H^{*,C}}{P_F^{*,C}}\right)^{1-\varphi^*}} \frac{M_s}{M_s^*}$
(CFS)	$P_H^C = P_F^{*,C}, \quad P_F^C = P_H^{*,C}, \quad \varphi = \varphi^*$
(9CFS)	$\Rightarrow S_s^C = \frac{M_s}{M_s^*} \neq 1 \text{ unless } s_e$

**PCP** Under PCP, the equilibrium (ex-post) NER solves, analogously to the CCP case, the foreign exchange market clearing condition (21). Substituting for optimal  $c_{F,s}^P$  and  $c_{H,s}^{*,P}$  in (21) as well as for the ideal  $H$  and  $F$  CPI definitions further in the algebraic manipulation derives (for the *full-symmetry* case expression, refer to (9PFS) in the table below):<sup>45</sup>

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<sup>44</sup>Alternatively, we could have derived the same CCP exchange rate equation by writing the *money market equilibrium* condition for one of the countries and then substituting out optimal demands and using ideal CPI definitions to solve for  $S_s^C$ , as in Bacchetta and van Wincoop (2000 a, b), e.g. taking *Home*:

$$M_s \equiv Y_s^C \equiv \underbrace{P_H^C c_{H,s}^C}_{(DA)_{H,s}^C} + S_s^C \underbrace{P_H^{*,C} c_{H,s}^{*,C}}_{(Ex)_{H,s}^C}$$

This second approach is possible because of the *fully symmetric two-country* set-up considered.

<sup>45</sup>Again, we could have derived the same PCP exchange rate equation by writing the *money market equilibrium* condition for one of the countries and then substituting out optimal demands and using ideal CPI definitions to solve for  $S_s^P$ , by analogy with Bacchetta and van Wincoop (2000 b), e.g. taking *Home*:

$$M_s \equiv Y_s^P \equiv \underbrace{P_H^P c_{H,s}^P}_{(DA)_{H,s}^P} + \underbrace{P_H^{*,P} c_{H,s}^{*,P}}_{(Ex)_{H,s}^P}$$

PCP SUMMARY TABLE 5

$$\begin{aligned}
 & S_s^P \\
 (9P) \quad & S_s^P = \frac{1 + \left( \frac{P_{F,s}^P}{P_H^P} \right)^{1-\varphi} M_s}{1 + \left( \frac{P_{H,s}^{*,P}}{P_F^{*,P}} \right)^{1-\varphi^*} M_s^*} = \\
 (PFS) \quad & P_H^P = P_F^{*,P}, \quad P_{F,s}^P = S_s^P P_F^{*,P}, \quad P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}, \quad \varphi = \varphi^* \\
 & = \frac{1 + \left( \frac{S_s^P P_F^{*,P}}{P_H^P} \right)^{1-\varphi} M_s}{1 + \left( \frac{P_H^P}{S_s^P P_F^{*,P}} \right)^{1-\varphi} M_s^*} = \frac{1 + (S_s^P)^{1-\varphi} M_s}{1 + \left( \frac{1}{S_s^P} \right)^{1-\varphi} M_s^*} = \frac{1 + (S_s^P)^{1-\varphi} M_s}{\frac{(S_s^P)^{1-\varphi+1}}{(S_s^P)^{1-\varphi}} M_s^*} = (S_s^P)^{1-\varphi} \frac{M_s}{M_s^*} \\
 & \Leftrightarrow (S_s^P)^\varphi = \frac{M_s}{M_s^*} \\
 (9PFS) \quad & \Rightarrow S_s^P = \left( \frac{M_s}{M_s^*} \right)^{\frac{1}{\varphi}} \neq 1 \text{ unless } s_e
 \end{aligned}$$

#### A.4 Equilibrium Trade Shares

**CCP** Under CCP and  $H$ - $F$  symmetry, the equilibrium trade-to-GDP ratio in each  $s$  is defined for *Home* as:

$$(ft)_{H,s}^C \equiv \frac{(FT)_{H,s}^C}{Y_{H,s}^C} \equiv \frac{(Ex)_{H,s}^C + (Im)_{H,s}^C}{(DA)_{H,s}^C + (Ex)_{H,s}^C} = \frac{S_s^C \cdot P_H^{*,C} \cdot c_{H,s}^{*,C} + P_F^C \cdot c_{F,s}^C}{P_H^C \cdot c_{H,s}^C + S_s^C \cdot P_H^{*,C} \cdot c_{H,s}^{*,C}}$$

and for *Foreign* as:

$$(ft)_{F,s}^C \equiv \frac{(FT)_{F,s}^{*,C}}{Y_{F,s}^{*,C}} \equiv \frac{(Ex)_{F,s}^{*,C} + (Im)_{F,s}^{*,C}}{(DA)_{F,s}^{*,C} + (Ex)_{F,s}^{*,C}} = \frac{\frac{P_F^C}{S_s^C} \cdot c_{F,s}^C + P_H^{*,C} \cdot c_{H,s}^{*,C}}{P_F^{*,C} \cdot c_{F,s}^{*,C} + \frac{P_F^C}{S_s^C} \cdot c_{F,s}^C}$$

Using optimal domestic and external demands for  $H$  and  $F$  output and the ideal  $H$  and  $F$  CPI definitions, substitutions under *full symmetry* derive:

CCP SUMMARY TABLE 6H

$$\begin{aligned}
 & \overline{\overline{(ft)_{H,s}^C FS = (ft)_H^C FS = (ft)_F^C FS = (ft)_W^C FS = 1}} \\
 & \left( \begin{array}{l} 10HCFS: H \text{ relative} \\ \text{nontraded output price} \end{array} \right) \quad (ft)_{H,s}^C = (ft)_H^C = \frac{2}{\underbrace{\left(\frac{P_H^C}{P_H^{*,C}}\right)^{1-\varphi}}_{=1} + 1} = \\
 & \left( \begin{array}{l} 10HCFS: H \text{ expected} \\ \text{relative cash under peg} \end{array} \right) = \frac{2}{\underbrace{\left\{\frac{E_0[u_{l,s} M_s]}{E_0[u_{l,s} M_s^*]}\right\}}_{=1} + 1} = 1, \forall \varphi \in (1, \infty) \\
 & \left( \begin{array}{l} 10HCFS: H \text{ expected} \\ \text{relative cash under float} \end{array} \right) = \frac{2}{\underbrace{\left\{\frac{E_0[u_{l,s} M_s]}{E_0[u_{l,s} M_s^*]}\right\}}_{=1} + 1} = 1, \forall \varphi \in (1, \infty)
 \end{aligned}$$

and:

CCP SUMMARY TABLE 6F

$$\begin{aligned}
 & \overline{\overline{(ft)_{F,s}^C FS = (ft)_F^C FS = (ft)_H^C FS = (ft)_W^C FS = 1}} \\
 & \left( \begin{array}{l} 10FCFS: F \text{ relative} \\ \text{nontraded output price} \end{array} \right) \quad (ft)_{F,s}^C = (ft)_F^C = \frac{2}{\underbrace{\left(\frac{P_F^{*,C}}{P_H^C}\right)^{1-\varphi}}_{=1} + 1} = \\
 & \left( \begin{array}{l} 10FCFS: F \text{ expected} \\ \text{relative cash under peg} \end{array} \right) = \frac{2}{\underbrace{\left\{\frac{E_0[u_{l,s}^* M_s^*]}{E_0[u_{l,s}^* M_s^*]}\right\}}_{=1} + 1} = 1, \forall \varphi \in (1, \infty) \\
 & \left( \begin{array}{l} 10FCFS: F \text{ expected} \\ \text{relative cash under float} \end{array} \right) = \frac{2}{\underbrace{\left\{\frac{E_0[u_{l,s}^* M_s^*]}{E_0[u_{l,s}^* M_s^*]}\right\}}_{=1} + 1} = 1, \forall \varphi \in (1, \infty)
 \end{aligned}$$

**PCP** Under PCP and  $H$ - $F$  symmetry, the ex-post trade-to-GDP ratio for *Home* is defined as:

$$(ft)_{H,s}^P \equiv \frac{(FT)_{H,s}^P}{Y_{H,s}^P} = \frac{(Ex)_{H,s}^P + (Im)_{H,s}^P}{(DA)_{H,s}^P + (Ex)_{H,s}^P} = \frac{P_H^P \cdot c_{H,s}^{*,P} + \overbrace{S_s^P \cdot P_F^{*,P}}^{\equiv P_{F,s}^P} \cdot c_{F,s}^P}{P_H^P \cdot c_{H,s}^P + P_H^P \cdot c_{H,s}^{*,P}}$$

and for *Foreign* as:

$$(ft)_{F,s}^P \equiv \frac{(FT)_{F,s}^{*,P}}{Y_{F,s}^{*,P}} = \frac{(Ex)_{F,s}^{*,P} + (Im)_{F,s}^{*,P}}{(DA)_{F,s}^{*,P} + (Ex)_{F,s}^{*,P}} = \frac{P_F^{*,P} \cdot c_{F,s}^P + \overbrace{\frac{P_H^P}{S_s^P}}^{\equiv P_{H,s}^{*,P}} \cdot c_{H,s}^{*,P}}{P_F^{*,P} \cdot c_{F,s}^{*,P} + P_F^{*,P} \cdot c_{F,s}^P}$$

Using optimal domestic and external demands for  $H$  and  $F$  output and the ideal  $H$  and  $F$  CPI definitions, substitutions under *full symmetry* derive:

PCP SUMMARY TABLE 6H	
$(ft)_{H,s}^P FS \neq \begin{cases} 1 \\ const \end{cases}$	(unless $s_e$ ) but $(ft)_{W,s}^P FS = (ft)_W^P FS = 1, \forall s \in S$
(10PFS: NER)	$(ft)_{H,s}^P = \frac{2}{(S_s^P)^{\varphi-1} + 1} =$
(10PFS: relative CPIs)	$= \frac{2}{\left[\frac{P_s^P}{P_s^{*P}}\right]^{\varphi-1} + 1} =$
$\left( \begin{array}{l} 10PFS: actual \\ relative cash under peg \end{array} \right)$	$= \frac{2}{\left(\frac{M_s}{M_s^*}\right)^{\frac{\varphi-1}{\varphi}} + 1} = \frac{2}{(1)^{\frac{\varphi-1}{\varphi}} + 1} = 1, \forall \varphi \in (1, \infty)$
$\left( \begin{array}{l} 10PFS: actual \\ relative cash under float \end{array} \right)$	$(ft)_{H,s}^P = \frac{2}{\left(\frac{M_s}{M_s^*}\right)^{\frac{\varphi-1}{\varphi}} + 1} \neq \begin{cases} 1 \\ const \end{cases}$ (unless $s_e$ )

and:

PCP SUMMARY TABLE 6F	
$(ft)_{F,s}^P FS \neq \begin{cases} 1 \\ const \end{cases}$	(unless $s_e$ ) but $(ft)_{W,s}^P FS = (ft)_W^P FS = 1, \forall s \in S$
(10PFS: NER)	$(ft)_{F,s}^P = \frac{2}{\left(\frac{1}{S_s^P}\right)^{\varphi-1} + 1} = \frac{2}{(S_s^P)^{1-\varphi} + 1} =$
(10PFS: relative CPIs)	$= \frac{2}{\left[\frac{P_s^{*P}}{P_s^P}\right]^{\varphi-1} + 1} = \frac{2}{\left[\frac{P_s^P}{P_s^{*P}}\right]^{1-\varphi} + 1} =$
$\left( \begin{array}{l} 10PFS: actual \\ relative cash under peg \end{array} \right)$	$= \frac{2}{\left(\frac{M_s^*}{M_s}\right)^{\frac{\varphi-1}{\varphi}} + 1} = \frac{2}{(1)^{\frac{\varphi-1}{\varphi}} + 1} = 1, \forall \varphi \in (1, \infty)$
$\left( \begin{array}{l} 10PFS: actual \\ relative cash under float \end{array} \right)$	$(ft)_{F,s}^P = \frac{2}{\left(\frac{M_s^*}{M_s}\right)^{\frac{\varphi-1}{\varphi}} + 1} \neq \begin{cases} 1 \\ const \end{cases}$ (unless $s_e$ )

## B Proofs of Propositions

### B.1 Proof of Proposition 1 (Relative Consumption)

**Proof.**

- Under *CCP* and *full symmetry* with *separable* preferences (recall that in this case  $P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$  and thus  $P^C = P^{*,C}$ ), *relative* real consumption can be expressed as follows:

$$\begin{aligned}
\frac{c_s^C}{c_s^{*,C}} &\equiv \frac{c_{H,s}^C + c_{F,s}^C}{c_{F,s}^{*,C} + c_{H,s}^{*,C}} = \frac{\frac{1}{2} \left( \frac{P_H^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C} + \frac{1}{2} \left( \frac{P_F^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C}}{\frac{1}{2} \left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}} + \frac{1}{2} \left( \frac{P_H^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}}} = \\
&= \frac{\left( \frac{P_H^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C}}{\left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}}} = \frac{M_s}{M_s^*} = S_s^C \neq 1 \text{ unless } s_e \subset S;
\end{aligned}$$

- Under *PCP* and *full symmetry* with *separable* preferences (so that  $P_H^P = P_F^{*,P}$ ,  $P_{F,s}^P = S_s^P P_F^{*,P}$ ,  $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}$  and thus  $P_s^P = S_s^P \Leftrightarrow P_s^{*,P} = \frac{P_s^P}{S_s^P}$ ), analogous reasoning derives *relative* real consumption to be:

$$\begin{aligned}
\frac{c_s^P}{c_s^{*,P}} &\equiv \frac{c_{H,s}^P + c_{F,s}^P}{c_{F,s}^{*,P} + c_{H,s}^{*,P}} = \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P} + \frac{1}{2} \left( \frac{P_F^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}}{\frac{1}{2} \left( \frac{P_F^{*,P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}} + \frac{1}{2} \left( \frac{P_H^{*,P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}}} = \\
&= \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P} + \frac{1}{2} \left( \frac{S_s^P P_F^{*,P}}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}}{\frac{1}{2} \left( \frac{P_F^{*,P}}{S_s^P} \right)^{-\varphi} \frac{M_s^*}{P_s^P} + \frac{1}{2} \left( \frac{P_H^{*,P}}{S_s^P} \right)^{-\varphi} \frac{M_s^*}{P_s^P}} = \\
&= \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P} \left[ 1 + (S_s^P)^{-\varphi} \right]}{\frac{1}{2} \left( \frac{P_F^{*,P}}{P_s^P} \right)^{-\varphi} \frac{M_s^*}{P_s^P} \left[ (S_s^P)^{-\varphi} S_s^P + S_s^P \right]} = \\
&= \frac{M_s}{M_s^*} \frac{\left[ 1 + (S_s^P)^{-\varphi} \right]}{\left[ (S_s^P)^{-\varphi} + 1 \right] S_s^P} = (S_s^P)^\varphi \frac{1}{S_s^P} = \\
&= (S_s^P)^{\varphi-1} = \left( \frac{M_s}{M_s^*} \right)^{\frac{\varphi-1}{\varphi}} \neq 1 \text{ unless } s_e \subset S.
\end{aligned}$$

This completes our proof. ■

## B.2 Proof of Proposition 2 (Consumption Bias)

**Proof.**

- Under *CCP* and *full symmetry* with *separable* preferences ( $P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$  and  $P^C = P^{*,C}$ ), the optimal *split-up* of real consumption between demand of domestic and foreign goods can be expressed as follows:



$$\text{for Home: } \frac{c_{H,s}^C}{c_{F,s}^C} = \frac{\frac{1}{2} \left( \frac{P_H^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C}}{\frac{1}{2} \left( \frac{P_F^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C}} = 1 \Leftrightarrow c_{H,s}^C = c_{F,s}^C \text{ for } \forall s \in S,$$

$$\text{for Foreign: } \frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = \frac{\frac{1}{2} \left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}}}{\frac{1}{2} \left( \frac{P_H^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}}} = 1 \Leftrightarrow c_{F,s}^{*,C} = c_{H,s}^{*,C} \text{ for } \forall s \in S.$$

- Under *PCP* and *full* symmetry with *separable* preferences ( $P_H^P = P_F^{*,P}$ ,  $P_{F,s}^P = S_s^P P_F^{*,P}$ ,  $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}$  and  $P_s^P = S_s^P P_s^{*,P} \Leftrightarrow P_s^{*,P} = \frac{P_s^P}{S_s^P}$ ), analogous reasoning derives the optimal *split-up* of real consumption to be:

$$\text{for Home: } \frac{c_{H,s}^P}{c_{F,s}^P} = \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}}{\frac{1}{2} \left( \frac{S_s^P P_F^{*,P}}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}} = (S_s^P)^\varphi = \frac{M_s}{M_s^*} \neq 1 \text{ unless } s_e \subset S,$$

$$\text{for Foreign: } \frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = \frac{\frac{1}{2} \left( \frac{P_F^{*,P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}}}{\frac{1}{2} \left( \frac{\frac{P_H^P}{S_s^P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}}} = (S_s^P)^{-\varphi} = \frac{M_s^*}{M_s} \neq 1 \text{ unless } s_e \subset S.$$

This completes our proof. ■

### B.3 Proof of Proposition 3 (Balanced Trade)

**Proof.**

- Given *full* symmetry with *separable* preferences, the prices for domestic and foreign output a Home or Foreign consumer faces in its *own* national currency under *CCP* are *identical*,  $P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$ , as commented. Proposition 2 then states that the optimal real consumption *split-up* is 50 : 50, for  $\forall s \in S$  in Home as well as in Foreign.<sup>46</sup> In other words, *half* of the total consumption demand of each of the representative households is directed to domestically produced brands and *half* to their foreign analogues, no matter the state of nature that has materialized (i.e. for any level of nominal cash available for consumption ex-post in national currency). Therefore the national-currency *value* (price multiplied

<sup>46</sup>Note, however, that  $c_{H,s}^C = c_{F,s}^C \neq c_{F,s}^{*,C} = c_{H,s}^{*,C}$  unless  $s_e$ , according to this same Proposition 2.

by quantity) of exports equals the national-currency *value* of imports for both countries, so that the equilibrium trade balance is always zero.<sup>47</sup>

- Given *full* symmetry with *separable* preferences again, the prices optimally preset in the currency of the *seller* under *PCP* are prefixed as *identical*,  $P_H^P = P_F^{*,P}$ , as we discussed. It is easy to see then that the state-dependent domestic-currency price of Home imports,  $P_{F,s}^P = S_s^P P_F^{*,P}$ , multiplied by the imported quantity  $c_{F,s}^P$  gives exactly the same value as the preset domestic-currency price of Home exports,  $P_H^P$ , multiplied by the exported quantity  $c_{H,s}^*$ :

$$\begin{aligned} \frac{P_{F,s}^P c_{F,s}^P}{P_H^P c_{H,s}^*} &= \frac{S_s^P P_F^{*,P}}{P_H^P} \frac{\frac{1}{2} \left( \frac{S_s^P P_F^{*,P}}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}}{\frac{1}{2} \left( \frac{P_H^P}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}}} = S_s^P \frac{\left( \frac{S_s^P P_F^{*,P}}{S_s^P P_s^{*,P}} \right)^{-\varphi} \frac{M_s}{S_s^P P_s^{*,P}}}{\left( \frac{P_H^P}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}}} = \\ &= S_s^P \frac{\left( \frac{P_F^{*,P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s}{M_s^* S_s^P}}{\left( \frac{P_H^P}{P_s^{*,P}} \right)^{-\varphi} \left( \frac{1}{S_s^P} \right)^{-\varphi}} = S_s^P \left[ \frac{M_s}{M_s^*} \left( \frac{1}{S_s^P} \right)^{1+\varphi} \right] = \\ &= S_s^P \left[ (S_s^P)^\varphi (S_s^P)^{-1-\varphi} \right] = S_s^P (S_s^P)^{-1} = 1 \text{ for } \forall s \in S \end{aligned}$$

Ex-post prices and quantities thus *exactly compensate* each other under *PCP*, due to the full symmetry imposed, so that the national-currency *value* (quantity multiplied by price) of imports and of exports remains necessarily the same for each of the two countries no matter the particular state of nature  $s \in S$  that has materialized, just like under *CCP*.

This completes our proof. ■

## B.4 Proof of Proposition 4 (Relative Leisure)

**Proof.**

- Under *CCP* and *full* symmetry with *separable* preferences ( $P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$  and  $P^C = P^{*,C}$ ), *relative* real output can be expressed as:

$$\begin{aligned} \frac{y_s^C}{y_s^{*,C}} &\equiv \frac{c_{H,s}^C + c_{H,s}^{*,C}}{c_{F,s}^{*,C} + c_{F,s}^C} = \frac{\frac{1}{2} \left( \frac{P_H^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C} + \frac{1}{2} \left( \frac{P_H^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}}}{\frac{1}{2} \left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{M_s^*}{P^{*,C}} + \frac{1}{2} \left( \frac{P_F^C}{P^C} \right)^{-\varphi} \frac{M_s}{P^C}} = \\ &= \frac{\frac{1}{2} \left( \frac{P_H^C}{P^C} \right)^{-\varphi} \frac{1}{P^C} (M_s + M_s^*)}{\frac{1}{2} \left( \frac{P_F^{*,C}}{P^{*,C}} \right)^{-\varphi} \frac{1}{P^{*,C}} (M_s^* + M_s)} = 1 \Leftrightarrow y_s^C = y_s^{*,C} \text{ for } \forall s \in S. \end{aligned}$$

<sup>47</sup> Yet this latter conclusion does not mean that *total* real consumptions are equal in Home and Foreign, which will also be true only in some  $s_e$  as we saw in Proposition 1.

Consequently:

$$\frac{y_s^C}{y_s^{*,C}} \equiv \frac{n_s^C}{n_s^{*,C}} \equiv \frac{1 - l_s^C}{1 - l_s^{*,C}} = 1 \Rightarrow \frac{l_s^C}{l_s^{*,C}} = 1 \Leftrightarrow l_s^C = l_s^{*,C} \text{ for } \forall s \in S.$$

- Under *PCP* and *full* symmetry with *separable* preferences ( $P_H^P = P_F^{*,P}$ ,  $P_{F,s}^P = S_s^P P_F^{*,P}$ ,  $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}$  and  $P_s^P = S_s^P P_s^{*,P} \Leftrightarrow P_s^{*,P} = \frac{P_s^P}{S_s^P}$ ), analogous reasoning derives *relative* real output to be:

$$\begin{aligned} \frac{y_s^P}{y_s^{*,P}} &\equiv \frac{c_{H,s}^P + c_{H,s}^{*,P}}{c_{F,s}^{*,P} + c_{F,s}^P} = \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P} + \frac{1}{2} \left( \frac{P_{H,s}^{*,P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}}}{\frac{1}{2} \left( \frac{P_{F,s}^{*,P}}{P_s^{*,P}} \right)^{-\varphi} \frac{M_s^*}{P_s^{*,P}} + \frac{1}{2} \left( \frac{P_{F,s}^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}} \\ &= \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P} + \frac{1}{2} \left( \frac{\frac{P_H^P}{S_s^P}}{\frac{P_s^P}{S_s^P}} \right)^{-\varphi} \frac{M_s^*}{\frac{P_s^P}{S_s^P}}}{\frac{1}{2} \left( \frac{P_{F,s}^{*,P}}{\frac{P_s^P}{S_s^P}} \right)^{-\varphi} \frac{M_s^*}{\frac{P_s^P}{S_s^P}} + \frac{1}{2} \left( \frac{S_s^P P_{F,s}^{*,P}}{P_s^P} \right)^{-\varphi} \frac{M_s}{P_s^P}} \\ &= \frac{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{1}{P_s^P} (M_s + S_s^P M_s^*)}{\frac{1}{2} \left( \frac{P_H^P}{P_s^P} \right)^{-\varphi} \frac{1}{P_s^P} \left[ (S_s^P)^{-\varphi} S_s^P M_s^* + (S_s^P)^{-\varphi} M_s \right]} \\ &= \frac{(M_s + S_s^P M_s^*)}{[S_s^P M_s^* + M_s] (S_s^P)^{-\varphi}} = (S_s^P)^\varphi = \\ &= \left[ \left( \frac{M_s}{M_s^*} \right)^{\frac{1}{\varphi}} \right]^\varphi = \frac{M_s}{M_s^*} \neq 1 \text{ unless } s_e \in S. \end{aligned}$$

Hence:

$$\begin{aligned} \frac{y_s^P}{y_s^{*,P}} &\equiv \frac{n_s^P}{n_s^{*,P}} \equiv \frac{1 - l_s^P}{1 - l_s^{*,P}} = \frac{M_s}{M_s^*} = (S_s^P)^{-\varphi} \neq 1 \text{ unless } s_e \in S \Rightarrow \\ &\Rightarrow \frac{l_s^C}{l_s^{*,C}} \neq 1 \Leftrightarrow l_s^C \neq l_s^{*,C} \text{ unless } s_e \in S. \end{aligned}$$

This completes our proof. ■

## B.5 Proof of Proposition 5 (Impact of Monetary Expansion)

**Proof.** To evaluate and compare *ex-post* utility *across countries* following a *relative* monetary expansion under float, we need to take account of the simultaneous effects on *relative* real consumption and *relative* leisure. Assume at this point that a higher positive (or a lower negative) money growth has occurred in Home in a given state  $s_H \in S_H \subset S$ , so that  $\frac{M_{s_H}}{M_{s_H}^*} > 1$ .<sup>48</sup>

- Under *CCP* and *full* symmetry with *separable* preferences, we then obtain directly from Proposition 1 that  $\frac{c_{s_H}^C}{c_{s_H}^{*,C}} > 1 \Leftrightarrow c_{s_H}^C > c_{s_H}^{*,C}$ , so the Home representative household consumes more than the Foreign one in this state of nature. From Proposition 4 we also know that *relative* leisure is independent, under CCP, from money stocks, so  $\frac{l_{s_H}^C}{l_{s_H}^{*,C}} = 1 \Leftrightarrow l_{s_H}^C = l_{s_H}^{*,C}$ . Taking account of both utility index components, namely real consumption and leisure hours, we can conclude that the relative monetary expansion has tilted *relative ex-post* utility in favor of the expansionary country.
- Under *PCP* and *full* symmetry with *separable* preferences, we obtain – again from Proposition 1 – that  $\frac{c_{s_H}^P}{c_{s_H}^{*,P}} > 1 \Leftrightarrow c_{s_H}^P > c_{s_H}^{*,P}$ , so the Home representative household consumes again more than the Foreign one. From Proposition 4 we can see that, under PCP,  $\frac{1-l_{s_H}}{1-l_{s_H}^*} = \frac{M_{s_H}}{M_{s_H}^*} > 1$ , so  $\frac{l_{s_H}}{l_{s_H}^*} < 1$ , with  $0 < l_{s_H} < l_{s_H}^* < 1$ . We now have to know whether the *relative gain in real consumption* in the expansionary economy is higher or lower than the *relative loss in leisure*. This type of calculation depends *qualitatively* on the magnitude of the *relative* monetary disequilibrium,  $\frac{M_{s_H}}{M_{s_H}^*} > 1$ , and *quantitatively* on the degree of substitutability in consumption,  $\varphi > 1$ . For our purposes here, we abstract from unrealistic relative monetary disequilibria<sup>49</sup> and focus on cases that are consistent with our sticky-price set-up. It turns out that under PCP the gain of the Home representative household in consumption relative to the Foreign one is lower than its simultaneous relative loss in leisure, when consumption and leisure are separable and *equally* valued, as we assume for our purposes here.<sup>50</sup> Under PCP, therefore, Home residents are worse-off than Foreign ones in *cross-country* utility terms, following a Home *relative* monetary expansion. Let us take as an illustrative example the (realistic) case where  $\mu_{s_H} = 4\%$  and

<sup>48</sup>Of course, we would arrive at the same conclusions if we start from a symmetric state of the world characterized by a Foreign *relative* monetary expansion  $s_F \in S_F \subset S$ , so that  $\frac{M_{s_F}}{M_{s_F}^*} < 1$ .

<sup>49</sup>Although we have computed such as well, to numerically verify that they do *not* change our conclusions.

<sup>50</sup>And starting from an *initial* symmetric equilibrium with 8 hours of labor and 8 hours of leisure (and 8 hours of sleep), so that  $n_0 = n_0^* = l_0 = l_0^* = \frac{1}{2}$  if our time endowment (less the optimum of sleep) is normalized to 1, as in (2) (and as usual).

$\mu_{s_H}^* = 2\%$  so that  $\frac{M_{s_H}}{M_{s_H}^*} = \frac{104}{102} = 1.0196 > 1$ . Furthermore, consider (as being close to reality) a monopolistic markup of 10%, and thus a corresponding parameter value of  $\varphi = 11$ . Using our model to perform such a calculation,<sup>51</sup> we find that the Home representative household consumes 1.0178 times (+1.78 percentage points) more than the Foreign one but also works 1.0200 times more and so has  $\frac{1}{1.0200} = 0.9804$  times (−1.96 percentage points) less leisure. Note, however, that given our choice of parameter values above (or parameter regions, more generally) considered as realistic given nominal rigidity, the magnitude of *relative* utility effects measured by the reported *difference* in terms of percentage points (+1.78 − 1.96 = −0.18) appears somewhat small to be easily perceptible, and motivating indeed, in the optimizing behavior of the rational agents we model. Another observation to make here is that a *lower* substitutability exacerbates the gap between the relative consumption gain and the relative leisure loss while a *higher* substitutability, by contrast, reduces it.<sup>52</sup> In the limit, when  $\varphi \rightarrow \infty$  and competition is perfect, one could infer from our numerical examples that the gain in relative consumption following a domestic monetary expansion under float and PCP will be *exactly offset* by the loss in relative leisure, in percentage terms, and ex-post *cross-country* utility will remain unchanged. Thus PCP, and hence PPP, with *perfect competition* would act as a risk-sharing device between the two nations we model, a finding that has been pointed out in other NOEM papers as well.

This completes our proof. ■

## B.6 Proof of Proposition 6 (World Trade Share)

**Proof.**

- Under *CCP*, *Home* nominal trade is always equal to *Home* nominal output so that the *Home* trade share in output is constant at 1, irrespective of the state of nature that has materialized. The same is true for *Foreign*, and as a consequence  $(ft)_F^{*,C} = 1 = (ft)_H^C$  so that  $\frac{1}{2}(ft)_H^C + \frac{1}{2}(ft)_F^{*,C} = 1$ , for  $\forall s \in S$ .
- Under *PCP*, by contrast, *both* these trade-to-output ratios are stochastic and generally *not* equal to each other and to 1:  $1 \neq (ft)_{H,s}^P \neq (ft)_{F,s}^{*,P} \neq 1$  unless  $s_e \subset S$ . However due to symmetry, the *Home* and *Foreign* trade shares in GDP are *complementary* in the sense that in any state of nature  $s \in S$  they sum to 2:

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<sup>51</sup>The details are available upon request.

<sup>52</sup>The details of the similar computations we performed with  $\varphi = 2$  and  $\varphi = 101$  (as well as with other values for the relative monetary disequilibrium) are also available upon request.

$$\begin{aligned}
(ft)_{H,s}^P + (ft)_{F,s}^{*,P} &= \frac{2}{(S_s^P)^{1-\varphi} + 1} + \frac{2}{\left(\frac{1}{S_s^P}\right)^{1-\varphi} + 1} = \\
&= \frac{2}{(S_s^P)^{1-\varphi} + \frac{(S_s^P)^{1-\varphi}}{(S_s^P)^{1-\varphi}}} + \frac{2}{\frac{1}{(S_s^P)^{1-\varphi}} + \frac{(S_s^P)^{1-\varphi}}{(S_s^P)^{1-\varphi}}} = \frac{2}{\frac{(S_s^P)^{2(1-\varphi)} + (S_s^P)^{1-\varphi}}{(S_s^P)^{1-\varphi}}} + \frac{2}{\frac{1 + (S_s^P)^{1-\varphi}}{(S_s^P)^{1-\varphi}}} = \\
&= \frac{2(S_s^P)^{1-\varphi}}{(S_s^P)^{1-\varphi} \left[ (S_s^P)^{(1-\varphi)} + 1 \right]} + \frac{2(S_s^P)^{1-\varphi}}{1 + (S_s^P)^{(1-\varphi)}} = \frac{2}{1 + (S_s^P)^{(1-\varphi)}} + \frac{2(S_s^P)^{1-\varphi}}{1 + (S_s^P)^{(1-\varphi)}} = \\
&= \frac{2 + 2(S_s^P)^{1-\varphi}}{1 + (S_s^P)^{(1-\varphi)}} = \frac{2 \left[ 1 + (S_s^P)^{1-\varphi} \right]}{1 + (S_s^P)^{(1-\varphi)}} = 2
\end{aligned}$$

Thus, (equally-weighted) world trade *equals* world output in any state of nature  $s \in S$ :

$$\frac{1}{2} (ft)_{H,s}^P + \frac{1}{2} (ft)_{F,s}^{*,P} = 1, \text{ for } \forall s \in S.$$

This completes our proof. ■

## B.7 Proof of Proposition 7 (Expected Trade Share)

**Proof.** Let us begin by looking closer at the specification of the *joint shock process* which determines the model equilibrium in each realized state of nature  $s \in S$ . As clarified in section 2, we have postulated *symmetric monetary uncertainty* under float as well as under peg.

To visualize this symmetry of money-growth stock disturbances in the simplest way, one could employ the bivariate *uniform* distribution plotted in Figure 6. This latter figure makes evident two essential points, within the context of the *uniform* distribution illustrated. First, monetary uncertainty is not eliminated by a fixed exchange-rate regime: it is only reduced from a plane to a line, i.e. to points where the stochastic growth rate of money is the same in both countries in each state of nature ( $\mu_s \equiv \mu_s^*$  for  $\forall s \in S$ ), by the very definition of a peg. Second, money shock symmetry is with respect to the origin along the peg-line and with respect to the peg-line itself within the whole state-space plane.

The particular symmetric bivariate uniform distribution illustrated in Figure 6 has  $81 = 9 \times 9$  possible states of nature under *float* and just  $9 = \sqrt[2]{81} = (81)^{\frac{1}{2}}$  under *peg*. We shall now prove the present proposition for the general case of any such *symmetric bivariate* distribution, i.e. not only for 81 states and not only for bivariate distribution. It is logical to proceed in our proof from the simplest and most restrictive case to the more general ones. We therefore start by considering the case of  $9 = 3 \times 3$  equally probable states of nature in the

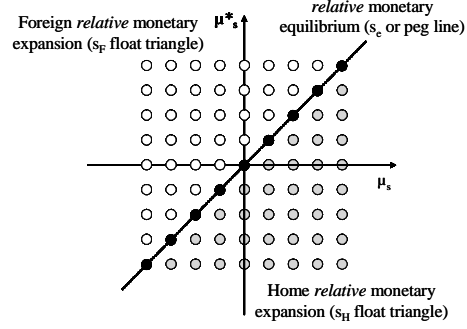


Figure 6: Symmetric Bivariate Uniform Distribution of Money Shocks

illustrated bivariate symmetric uniform distribution, that is, by the nine closest points around the origin in the scheme. Let us introduce the following notation, corresponding to these 9 "core" states in Figure 6:

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

with  $s_{22}$  corresponding to the *center of gravity* of this distribution, i.e. to the  $(0,0)$  point in the scheme.

Using the *Home PCP* trade share formula (33) derived earlier and our result in Proposition 6, we now express analytically the *expected Home* trade-to-output *under PCP and float* for this simplest case of  $9 = 3 \times 3$  states of nature in the symmetric bivariate uniform distribution considered here:

$$\begin{aligned} E_0 \left[ (ft)_{H,s}^P \right] &\approx \frac{1}{9} \sum_s \frac{2}{(S_s^P)^{\varphi-1} + 1} = \\ &= \frac{1}{9} \frac{2}{(S_{11}^P)^{\varphi-1} + 1} + \frac{1}{9} \frac{2}{(S_{12}^P)^{\varphi-1} + 1} + \frac{1}{9} \frac{2}{(S_{21}^P)^{\varphi-1} + 1} + \\ &+ \frac{1}{9} \frac{2}{(S_{31}^P)^{\varphi-1} + 1} + \frac{1}{9} \frac{2}{(S_{22}^P)^{\varphi-1} + 1} + \frac{1}{9} \frac{2}{(S_{13}^P)^{\varphi-1} + 1} + \\ &\frac{1}{9} \frac{2}{(S_{33}^P)^{\varphi-1} + 1} + \frac{1}{9} \frac{2}{(S_{23}^P)^{\varphi-1} + 1} + \frac{1}{9} \frac{2}{(S_{32}^P)^{\varphi-1} + 1} = \\ &= \frac{1}{9} \left[ \frac{2}{(S_{11}^P)^{\varphi-1} + 1} + \frac{2}{(S_{33}^P)^{\varphi-1} + 1} \right] + \\ &+ \frac{1}{9} \left[ \frac{2}{(S_{12}^P)^{\varphi-1} + 1} + \frac{2}{(S_{23}^P)^{\varphi-1} + 1} \right] + \end{aligned}$$

$$\begin{aligned}
& +\frac{1}{9} \left[ \frac{2}{(S_{21}^P)^{\varphi-1} + 1} + \frac{2}{(S_{32}^P)^{\varphi-1} + 1} \right] + \\
& +\frac{1}{9} \left[ \frac{2}{(S_{31}^P)^{\varphi-1} + 1} + \frac{2}{(S_{22}^P)^{\varphi-1} + 1} + \frac{2}{(S_{13}^P)^{\varphi-1} + 1} \right] = \\
& = \frac{1}{9} 2 + \frac{1}{9} 2 + \frac{1}{9} 2 + \frac{1}{9} [1 + 1 + 1] = \\
& = \frac{1}{9} [2 + 2 + 2 + 1 + 1 + 1] = \\
& = \frac{1}{9} 9 = 1 = \text{const}, \quad \forall s \in S.
\end{aligned}$$

We have thus obtained that the *expected Home PCP* trade share in output is the *same* as under peg (with PCP as well as CCP) or under CCP (with float as well as peg).

Analogously, using the *Foreign PCP* trade share formula (35) derived earlier and our result in Proposition 6, we express in turn the *expected Foreign* trade-to-output under *PCP and float*:

$$\begin{aligned}
E_0 \left[ (ft)_{F,s}^P \right] & \approx \frac{1}{9} \sum_s \frac{2}{(S_s^P)^{1-\varphi} + 1} = \\
& = \frac{1}{9} \frac{2}{(S_{11}^P)^{1-\varphi} + 1} + \frac{1}{9} \frac{2}{(S_{12}^P)^{1-\varphi} + 1} + \frac{1}{9} \frac{2}{(S_{21}^P)^{1-\varphi} + 1} + \\
& + \frac{1}{9} \frac{2}{(S_{31}^P)^{1-\varphi} + 1} + \frac{1}{9} \frac{2}{(S_{22}^P)^{1-\varphi} + 1} + \frac{1}{9} \frac{2}{(S_{13}^P)^{1-\varphi} + 1} + \\
& \frac{1}{9} \frac{2}{(S_{33}^P)^{1-\varphi} + 1} + \frac{1}{9} \frac{2}{(S_{23}^P)^{1-\varphi} + 1} + \frac{1}{9} \frac{2}{(S_{32}^P)^{1-\varphi} + 1} = \\
& = \frac{1}{9} \left[ \frac{2}{(S_{11}^P)^{1-\varphi} + 1} + \frac{2}{(S_{33}^P)^{1-\varphi} + 1} \right] + \\
& + \frac{1}{9} \left[ \frac{2}{(S_{12}^P)^{1-\varphi} + 1} + \frac{2}{(S_{23}^P)^{1-\varphi} + 1} \right] + \\
& + \frac{1}{9} \left[ \frac{2}{(S_{21}^P)^{1-\varphi} + 1} + \frac{2}{(S_{32}^P)^{1-\varphi} + 1} \right] + \\
& + \frac{1}{9} \left[ \frac{2}{(S_{31}^P)^{1-\varphi} + 1} + \frac{2}{(S_{22}^P)^{1-\varphi} + 1} + \frac{2}{(S_{13}^P)^{1-\varphi} + 1} \right] =
\end{aligned}$$



$$\begin{aligned}
&= \frac{1}{9}2 + \frac{1}{9}2 + \frac{1}{9}2 + \frac{1}{9}[1 + 1 + 1] = \\
&= \frac{1}{9}[2 + 2 + 2 + 1 + 1 + 1] = \\
&= \frac{1}{9}9 = 1 = \text{const}, \quad \forall s \in S.
\end{aligned}$$

We have thus established that the *expected Foreign PCP* trade share in output is the *same* as under peg (with PCP as well as CCP) or under CCP (with float as well as peg) and is, furthermore, *identical* to that of *Home*.

From the logic of this proof for the simplest case of a jointly symmetric distribution of (money) shocks it follows directly that for any *uniform* distribution with (much) more possible states of the *same* class, i.e. for  $25 = 5 \times 5$  equally probable symmetric states of nature, for  $49 = 7 \times 7$  states, for  $81 = 9 \times 9$  states (as in Figure 6), and so on, that is, generally for any symmetric bivariate uniform distribution centered around 0 of the class  $(2n+1)^2 = (2n+1) \times (2n+1)$  for  $n = 1, 2, \dots, N, \dots$ , where  $(2n+1)^2$  is the relevant state-space, *expected* trade-to-output is *always* constant at 1. This result for bivariate uniforms naturally generalizes to any bivariate distribution centered around 0 once the assumption of *equally probable* states of nature is relaxed, provided that the distribution remains symmetric in the sense that the respective *pairs of symmetric states* remain equally probable.

This completes our proof. ■

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