The Problems of Learning Stability and Determinacy in Inflation Targeting Based On Constant Interest Rate Projections*

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Abstract

Monetary policy is sometimes formulated in terms of a target level of inflation, a fixed time horizon and a constant interest rate that is anticipated to achieve the target at the specified horizon under rational expectations These requirements lead to instrument rules for interest rate setting that can be called CIP (constant interest rate projections) rules. We consider the twin questions of determinacy and stability under adaptive learning for CIP interest rate policy using the standard New Keynesian model. It is shown that CIP policy necessarily leads to both indeterminacy of equilibria and instability under learning if the economy is fully forward looking. These unfavourable properties of CIP policy remain in most cases of inertial demand and price behaviour.

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Key words: Inderminacy, instability under learning, inflation targeting, inertia in demand, price inertia

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1 Introduction

A fairly commonly employed method for conducting monetary policy is based on inflation forecast targeting when the forecasts are derived as constant-interest-rate forecasts, see e.g. (Leitemo 2003) and (Svensson 1999). Given a model of the macroeconomy, setting the forecast of inflation based on constant interest rates at a given target level of inflation implies a rule for the interest rate, the actual policy instrument of the central bank. Such an analysis needs to employ a concept of equilibrium and the literature follows the current standard paradigm and assumes that the economy is in a rational expectations equilibrium (REE) and that the central bank is able to use rational expectations (RE) as their forecasting procedure. We will refer to this way of conducting policy as constant interest rate projection (CIP) inflation targeting and corresponding interest rate rules as CIP rules.

CIP inflation targeting has been advocated as an easily understandable and hence practical approach to conducting monetary policy; see eg.(Leitemo 2003) for a good discussion. Arguably, it has even been implemented by some central banks like the Bank of England and Bank of Sweden. The inflation target is set for some given forecast horizon h and policy tries to achieve that target, so that

$$E_t \pi_{t+h} = \bar{\pi}.$$

Here $E_t \pi_{t+h}$ is the forecast of the inflation for period $t+h$ and it is taken to be the rational expectations (RE) forecast as the central bank is, for simplicity, assumed to know the model of the economy.

Since the policy horizon h for deriving policy is usually higher than 1 assumptions have to be made about the level of the interest during the time span from t to $t+h$ before it is possible to derive explicit policy rules. A common assumption here is to assume that the interest rate is held constant during this time interval, so the policy might be referred to as "constant-interest-rate inflation forecast targeting", see e.g. the discussion in (Leitemo 2003).[1]

Since current models for monetary policy are forward-looking, issues of determinacy of equilibria and their stability under (adaptive) learning have

[1] (Svensson 1999) and especially the appendix of the working paper version (see (Svensson 1998)) have a somewhat different formulation of CIP policy. We will discuss it further in Section XX.

been raised in the recent literature. (Bullard and Mitra 2002) have derived constraints on the interest rate instrument rules that achieve stability and determinacy in a standard New Keynesian model of monetary policy. (Evans and Honkapohja 2003b) and (Evans and Honkapohja 2003c) have shown that some standard ways for implementing optimal policy under discretion or commitment can lead to these difficulties. (Evans and Honkapohja 2003a) survey this literature and provide further references.

CIP policies have not thus far been examined for determinacy and stability under learning and our goal in this paper is to fill this gap. We will argue that CIP policies can often lead to unpleasant outcomes, i.e. the resulting RE solution can exhibit both indeterminacy and instability under learning. This problematic outcome necessarily happens in the basic New Keynesian model, which has become a workhorse for the recent research on monetary policy. We then examine the role of inertia for the indeterminacy and instability results. A numerical examination shows that the results for forward-looking models mostly remain unchanged, though for specific parameter configurations determinacy or stability under least squares learning can occur.

2 The Framework

2.1 The Basic Model

The model we employ is the standard New Keynesian model of monopolistic competition and (Calvo 1983) price stickiness. This model has been employed in numerous recent studies, see e.g. (Clarida, Gali, and Gertler 1999) for a survey. The log-linearized model is described by two equations

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t, \tag{1}$$

which is the “IS” curve derived from the Euler equation for consumer optimization, and

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{2}$$

which is the price setting rule for the monopolistically competitive firms.[2] We remark that in a later section we will add inertia terms to (1) and (2).

[2] See e.g. (Woodford 1996) or (Woodford 2000) for further details of the linearization and the original nonlinear model.

The inertia is usually justified by empirical relevance even though the micro foundations of the model are then fairly weak.[3]

Here x_t and π_t denote the output gap and inflation for period t, respectively. i_t is the nominal interest rate, expressed as the deviation from the steady state real interest rate. The determination of i_t will be discussed below. $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ denote the private sector expectations of the output gap and inflation next period. Since our focus is on learning behavior, these expectations need not be rational (E_t without $*$ denotes RE). The parameters φ and λ are positive and β is the discount factor so that $0 < \beta < 1$.

The shocks g_t and u_t are assumed to be observable and follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \tag{3}$$

where

$$F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix},$$

$0 < |\mu| < 1$, $0 < |\rho| < 1$ and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$ are independent white noise. g_t represents shocks to government purchases and or potential output. u_t represents any cost push shocks to marginal costs other than those entering through x_t. For simplicity, we assume throughout the paper that μ and ρ are known (if not, they could be estimated).

For brevity, details of the derivation of equations (1) and (2) are not discussed. The derivation is based on individual Euler equations under (identical) subjective expectations, together with aggregation and definitions of the variables. The Euler equations for the current period give the decisions as functions of the expected state next period. Rules for forecasting the next period's values of the state variables are the other ingredient in the description of individual behavior. Given forecasts, agents are assumed to make decisions according to the Euler equations.[4]

[3] See (Christiano and Evans 2001) and (Galí and Gertler 1999) for possible justifications.

[4] This kind of behavior is boundedly rational but in our view reasonable since agents attempt to meet the margin of optimality between the current and the next period. Other models of bounded rationality are possible. Recently, (Preston 2002b) has proposed a formulation in which long horizons matter in individual behavior. See also (Preston 2002a) and (Honkapohja, Mitra, and Evans 2002) for further discussion.

2.2 The CIP Policy

To derive CIP policy we follow the approach of (Leitemo 2003), which can be consulted for further details. First, we write the model in matrix-vector form

$$\begin{aligned} y_t &= AE_t^* y_{t+1} + Bw_t + Di_t, \\ w_t &= Fw_{t-1} + v_t, \end{aligned} \tag{4}$$

where $y_t = (x_t, \pi_t)'$, $w_t = (g_t, u_t)'$ and $v_t = (\tilde{g}_t, \tilde{u}_t)'$. The coefficient matrices are

$$A = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}, D = \begin{pmatrix} -\varphi \\ -\lambda\varphi \end{pmatrix}. \tag{5}$$

We note that in the derivation of CIP policies it is assumed that expectations are rational.

Next, we introduce the (strict) inflation target

$$E_t \pi_{t+h} = \bar{\pi} = 0, \tag{6}$$

where for simplicity the target is assumed to be zero.[5] We express this constraint as

$$0 = K(E_t w_{t+h}, E_t y_{t+h})', \; K = (0, 0, 0, 1). \tag{7}$$

Rewriting (4) as

$$\begin{pmatrix} w_{t+1} \\ E_t y_{t+1} \end{pmatrix} = \Omega \begin{pmatrix} w_t \\ y_t \end{pmatrix} + \Psi i_t + \begin{pmatrix} v_t \\ 0 \end{pmatrix}, \tag{8}$$

where

$$\Omega = \begin{pmatrix} F & 0 \\ -A^{-1}B & A^{-1} \end{pmatrix}, \Psi = \begin{pmatrix} 0 \\ -A^{-1}D \end{pmatrix},$$

and keeping the interest rates i_{t+s} at constant value i_t we obtain the interest rate rule

$$i_t = G \begin{pmatrix} w_t \\ y_t \end{pmatrix}, \tag{9}$$

[5] This is without loss of generality as the precise values of model constants affect neither determinacy nor stability under learning.

where

$$G = -\left(K \sum_{j=0}^{h-1} \Omega^j \Psi \right)^{-1} K\Omega^h.$$

We will refer to (9) as the CIP-rule I.

The CIP-rule I, equation (9), has the general form

$$i_t = \chi_g g_t + \chi_u u_t + \chi_x x_t + \chi_\pi \pi_t \tag{10}$$

and it is possible to compute (10) explicitly for different values of h. For $h = 2$

$$\chi_g = \varphi^{-1}, \chi_u = -\frac{1+\beta\rho+\lambda\varphi}{\beta\lambda\varphi}, \chi_x = -\frac{1+\beta+\lambda\varphi}{\beta\varphi}, \chi_\pi = \frac{1+\lambda\varphi}{\beta\lambda\varphi}$$

and for $h = 3$ we have

$$\begin{aligned}
\chi_g &= \frac{1+\beta+\beta\mu+\lambda\varphi}{\varphi+2\beta\varphi+\lambda\varphi^2}, \chi_u = -\frac{\beta^2\rho^2+(1+\lambda\varphi)^2+\beta(\rho+\lambda\varphi+\lambda\varphi\rho)}{\beta\lambda\varphi(1+2\beta+\lambda\varphi)}, \\
\chi_x &= -\frac{\beta+\beta^2+2\beta\lambda\varphi+(1+\lambda\varphi)^2}{\beta\varphi(1+2\beta+\lambda\varphi)}, \chi_\pi = \frac{1+(2+\beta)\lambda\varphi+\lambda^2\varphi^2}{\beta\lambda\varphi(1+2\beta+\lambda\varphi)}.
\end{aligned}$$

It is seen that, for $h = 2, 3$, the rule (9) surprisingly has $\chi_x < 0$, i.e. the interest rate should react negatively to the output gap. For higher values of h the expressions become quite cumbersome, but numerical computations indicate that the negative coefficient on the output gap is a robust phenomenon of the CIP-rule I.[6]

This unexpected result can be given an economic interpretation in the case $h = 2$. Shift the New Phillips curve (2) forward and take RE. This yields the positive relation between $E_t\pi_{t+1}$ and $E_t x_{t+1}$. Recalling that the inflation target is assumed to be zero, we have

$$E_t\pi_{t+1} = \lambda E_t x_{t+1},$$

which will pin the expectations terms in (1). By (2) we also have

$$E_t\pi_{t+1} = \beta^{-1}(\pi_t - \lambda x_t),$$

[6] The Mathematica routine is available on request.

which indicates that both $E_t\pi_{t+1}$ and $E_t x_{t+1}$ depend negatively on the current output gap under this policy. Finally, rewriting the IS curve (1) as

$$\begin{aligned}\varphi i_t &= -x_t + \varphi E_t \pi_{t+1} + E_t x_{t+1} \\ &= -(1+\beta^{-1}+\beta^{-1}\lambda\varphi)x_t + (\beta^{-1}\varphi + \beta^{-1}\lambda^{-1})\end{aligned}$$

it is seen that i_t and x_t are negatively related, both directly as part of the IS relationship and indirectly through the negative dependence of $E_t\pi_{t+1}$ and $E_t x_{t+1}$ on the current x_t.

We remark that (9) should be viewed as an instrument rule as, in addition to the observable exogenous shocks, it depends on current endogenous variables. It is an instrument rule in the same sense as the widely discussed rule proposed by (Taylor 1993) is so. (9) can be viewed as a behavioral rule in the same sense as demand and supply functions of private agents are behavioral rules, i.e. the central bank "goes to the market" with that schedule and is assumed to be able to adjust the interest rate within the period. Such rules are sometimes said to be non-operational, see (McCallum 1999) for further discussion.[7]

For this reason a different rule, which depends only on predetermined variables is often suggested instead. It can be derived as follows. Substituting (9) into (8) we have

$$\begin{pmatrix} w_{t+1} \\ E_t y_{t+1} \end{pmatrix} = (\Omega + \Psi G) \begin{pmatrix} w_t \\ y_t \end{pmatrix} + \begin{pmatrix} v_t \\ 0 \end{pmatrix},$$

for which it is possible to derive the MSV solution of the form

$$y_t = H w_t$$

using standard techniques (we omit the precise form of H). Introducing the partition $G = \begin{pmatrix} G_w & G_y \end{pmatrix}$, we can rewrite the interest rule (9) as

$$i_t = (G_w + G_y H) w_t, \qquad (11)$$

which we will call the CIP-rule II.

[7] Alternatively, this kind of rule is referred to as an equilibrium condition; see e.g. (Leitemo 2003).

3 Indeterminacy and Instability

We consider whether CIP interest rate rules, either in the form (9) or (11), yield determinacy and stability under learning. We will assess stability under learning using the concept of E-stability, which is known to be the relevant condition convergence of adaptive learning.[8] The analysis is conducted using the forward-looking model (1) and (2), together with either (9) or (11).

We start with (11). In the basic model it has unpleasant properties on both counts:

Proposition 1 *CIP-rule II, i.e. equation (11), leads to both indeterminacy and instability under adaptive learning.*

Proof. We can directly apply Proposition 2 in (Evans and Honkapohja 2003b), which states that, in this model, any interest rate rule that depends only on the exogenous shocks lead to both indeterminacy and instability under learning. ■

The indeterminacy result means that there other stationary REE to the model under the CIP-rule II besides the MSV solution used above. These equilibria include various sunspot solutions and it is possible to examine whether the non-MSV solutions are stable under learning. The results of Section 3 of (Honkapohja and Mitra 2001) are directly applicable and it can be shown that the non-MSV REE are also E-unstable.[9]

Taken together, the results suggest that, in the forward-looking context, implementation of monetary policy using the CIP-rule II can lead to fluctuations since it is likely to be difficult for the private economy to coordinate on the MSV REE on which the derivation of the rule was based.

The difficulties spelled out by Proposition 1 naturally raise the question whether the instrument rule form of CIP monetary policy, i.e. CIP-rule I given by equation (9) has better determinacy or learnability properties. Unfortunately, this is not the case:

Proposition 2 *CIP-rule I leads to both indeterminacy and E-instability in model (1)-(2) when $h \geq 3$. For $h = 2$ the model has indeterminacy but the unique MSV solution is E-stable.*

[8] (Evans and Honkapohja 2001) provides a treatise on adaptive learning, while (Evans and Honkapohja 1999), (Evans and Honkapohja 1995), (Marimon 1997), (Sargent 1993) and (Sargent 1999) are surveys of the subject.

[9] Under (11) the coefficient matrix of the forecasts is A in (4) and the matrix A can be shown to have an eigenvalue greater than one, whereby we have E-instability.

Proof. To prove the results we first note that it is unnecessary to consider the exogenous shocks for these results. They play no role in indeterminacy and also, in this setting, the E-instability part follows from considering the model without the shocks. We now consider determinacy and E-stability under (10) for different values of h.

We will derive analytic results for $h = 2, 3, 4$ and then illustrate numerically that, using a calibration of the model, the same results remain true for higher values of h.[10] Substituting (10) into (4) and omitting the shocks, we have the system

$$\begin{aligned} My_t &= NE_t^* y_{t+1}, \text{ where} \\ M &= \begin{pmatrix} 1+\varphi\chi_x & \varphi\chi_\pi \\ -\lambda & 1 \end{pmatrix}, N = \begin{pmatrix} 1 & \varphi \\ 0 & \beta \end{pmatrix}. \end{aligned}$$

In the case $h = 2$ matrix M is singular, but we can assess determinacy by computing

$$N^{-1}M = \begin{pmatrix} -\beta^{-1} & (\beta\lambda)^{-1} \\ -\lambda\beta^{-1} & \beta^{-1} \end{pmatrix}.$$

Both eigenvalues of $N^{-1}M$ are zero, so that we have indeterminacy. The analysis of E-stability for $h = 2$ is given in the appendix since the singularity of M raises additional technical and conceptual issues.

In the case $h = 3$ we get

$$M^{-1}N = \begin{pmatrix} \frac{1+2\beta+\lambda\varphi}{\beta} & \frac{-1+(\beta-1)\lambda\varphi}{\beta\lambda} \\ \frac{\lambda(1+2\beta+\lambda\varphi)}{\beta} & \frac{(\beta-1)(1+\beta+\lambda\varphi)}{\beta} \end{pmatrix}.$$

The model is determinate if and only if both eigenvalues of $M^{-1}N$ lie outside the unit circle. Equivalently, it is required that

$$\begin{aligned} 0 &> Abs[Det(M^{-1}N)] - 1 \\ 0 &> Abs[Tr(M^{-1}N)] - 1 - Det(M^{-1}N). \end{aligned}$$

However, it turns out that $Det(M^{-1}N) - 1 = 2\beta + \lambda\varphi > 0$, so that indeterminacy prevails. For E-stability it is required that

$$\begin{aligned} Tr(M^{-1}N - I) &< 0, \\ Det(M^{-1}N - I) &> 0. \end{aligned}$$

[10] The Mathematica routine is available on request.

In this case we have $Tr(M^{-1}N - I) = \beta + \lambda\varphi > 0$.

In the case $h = 4$ we have

$$
\begin{aligned}
Det(M^{-1}N) - 1 &= \frac{3\beta^2 + 3\beta\lambda\varphi + \lambda\varphi(1+\lambda\varphi)}{1 + 2\beta + \lambda\varphi} > 0, \\
Tr(M^{-1}N - I) &= \frac{2\beta^2 + 3\beta\lambda\varphi + \lambda\varphi(1+\lambda\varphi)}{1 + 2\beta + \lambda\varphi} > 0
\end{aligned}
$$

so that both indeterminacy and E-instability prevail.

For higher values of h we have computed the relevant conditions numerically using the calibration $\beta = 0.99$, $\lambda = 0.3$, $\varphi = 1$ due to (Clarida, Gali, and Gertler 2000). We also set $\mu = 0.4$ and $\rho = 0.4$. The results reported in Table 1 for $h = 5, \ldots, 12$ after the proof clearly indicate that the CIP-rule I delivers neither determinacy nor stability under learning. ∎

h	$Det(M^{-1}N) - 1$	$Tr(M^{-1}N - I)$
5	0.986466	0.857115
6	0.856092	0.791627
7	0.791006	0.756621
8	0.756244	0.737238
9	0.737011	0.726297
10	0.726162	0.720055
11	0.719975	0.716473
12	0.716426	0.71441

Table 1: Indeterminacy and E-instability for CIP-rule I

4 Model with Inertia

The model given by (1) and (2) is entirely forward-looking and as a result has difficulty capturing the inertia in output and inflation evident in the data, see (Fuhrer and Moore 1995b), (Fuhrer and Moore 1995a) and (Rudebusch and Svensson 1999). Consequently, we look at an extension of this model considered in (Clarida, Gali, and Gertler 1999), Section 6, with important backward-looking elements. The model now consists of the structural equations

$$
\begin{aligned}
x_t &= -\varphi\left(i_t - \hat{E}_t\pi_{t+1}\right) + \theta\hat{E}_t x_{t+1} + (1-\theta)x_{t-1} + g_t && (12) \\
\pi_t &= \lambda x_t + \beta\gamma\hat{E}_t\pi_{t+1} + (1-\gamma)\pi_{t-1} + u_t && (13)
\end{aligned}
$$

The parameters θ and γ capture the inertia in output and inflation inherent in the model and are assumed to be between 0 and 1. The shocks g_t and u_t continue to follow the process (3). As in Section 2.2, we can write this model in matrix form as

$$\begin{aligned} y_t &= A_1 E_t^* y_{t+1} + L_1 y_{t-1} + B w_t + D i_t, \\ w_t &= F w_{t-1} + v_t, \end{aligned} \tag{14}$$

where $y_t = (x_t, \pi_t)'$, $w_t = (g_t, u_t)'$ and $v_t = (\tilde{g}_t, \tilde{u}_t)'$. The coefficient matrices are

$$A_1 = \begin{pmatrix} \theta & \varphi \\ \lambda\theta & \beta\gamma + \lambda\varphi \end{pmatrix}, L_1 = \begin{pmatrix} 1-\theta & 0 \\ \lambda(1-\theta) & 1-\gamma \end{pmatrix}. \tag{15}$$

with B and D as defined before in (5). Strict inflation targeting is defined as before by equations (6) and (7). This leads to the form corresponding to (8) as

$$\begin{pmatrix} y_{1,t+1} \\ E_t y_{2,t+1} \end{pmatrix} = \Omega_1 \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} + \Psi_1 i_t + \begin{pmatrix} v_t \\ 0 \end{pmatrix}, \tag{16}$$

where $y_{2,t} = (x_t, \pi_t)'$, $y_{1,t} = (g_t, u_t, x_{lt}, \pi_{lt})'$, $v_t = (\tilde{g}_t, \tilde{u}_t)'$, and $x_{lt} \equiv x_{t-1}, \pi_{lt} \equiv \pi_{t-1}$. Also

$$\Omega_1 = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{\theta} & \frac{\varphi}{\theta\gamma\beta} & -\frac{1-\theta}{\theta} & \frac{\varphi(1-\gamma)}{\theta\gamma\beta} & \frac{1+\varphi\lambda\gamma^{-1}\beta^{-1}}{\theta} & -\frac{\varphi}{\theta\gamma\beta} \\ 0 & -\frac{1}{\gamma\beta} & 0 & -\frac{(1-\gamma)}{\gamma\beta} & -\frac{\lambda}{\gamma\beta} & \frac{1}{\gamma\beta} \end{pmatrix}, \Psi_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\varphi}{\theta} \\ 0 \end{pmatrix}.$$

It is possible to compute the interest rule based on constant interest rate projections in the same way as before. The CIP-rule I corresponding to (9) is now

$$i_t = -\left(K \sum_{j=0}^{h-1} \Omega_1^j \Psi_1 \right)^{-1} K \Omega_1^h \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix}, \tag{17}$$

where now $K = (0, 0, 0, 0, 0, 1)$. The CIP-rule I, equation (17), has the general form

$$i_t = ag_t + bu_t + cx_{t-1} + d\pi_{t-1} + ex_t + f\pi_t \tag{18}$$

and it is possible to compute this rule explicitly for different values of h. For instance, with $h = 2$, the rule is

$$\begin{aligned} a &= \varphi^{-1}, b = -\frac{\theta + \beta\gamma\theta\mu + \lambda\varphi}{\beta\gamma\lambda\varphi}, c = \frac{1-\theta}{\varphi}, d = -\frac{(1-\gamma)(\theta + \lambda\varphi)}{\beta\gamma\lambda\varphi}, \\ e &= -\frac{\theta + \beta\gamma + \lambda\varphi}{\beta\gamma\varphi}, f = \frac{\theta\{1 - \beta\gamma(1-\gamma)\} + \lambda\varphi}{\beta\gamma\lambda\varphi}. \end{aligned} \tag{19}$$

Note that as in the non-inertial model, the response of the interest rule to the contemporaneous output gap is negative and that to the contemporaneous inflation term is positive. In addition, the response of the interest rate to the lagged output gap is positive and negative to lagged inflation. Similar qualitative responses follow for the baseline values for other horizons.

The analysis of determinacy and E-stability for CIP-rule I can be conducted using (12), (13), and (18).

4.1 CIP Rule I: Determinacy and E-stability

We now examine determinacy and E-stability for the CIP-rule I for the case when the central bank pursues strict inflation targeting. When the forecasting horizon h equals 2, we are able to obtain analytical results and we analyze this case first. For computation of E-stability, we assume that agents's expectations are based on information at time $t-1$ which we believe is more realistic since contemporaneous data on output and inflation are not usually available for making forecasts; see (McCallum 1999) and the discussion in Section 10.3 of (Evans and Honkapohja 2001).

When $h = 2$, we have indeterminacy for all values of output and inflation inertia. This follows since the matrix for checking determinacy, namely,

$$\begin{pmatrix} -\frac{1}{\beta\gamma} & \frac{1+\beta\gamma(\gamma-1)}{\beta\gamma\lambda} & 0 & \frac{\gamma-1}{\beta\gamma\lambda} \\ -\frac{\lambda}{\beta\gamma} & \frac{1}{\beta\gamma} & 0 & \frac{\gamma-1}{\beta\gamma} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

has all eigenvalues zero. Since there are 2 free and 2 pre-determined variables, the above matrix should have 2 eigenvalues inside and 2 outside the unit circle for determinacy. Since there is indeterminacy, the possibility of multiple stationary MSV solutions arises. However, we now prove that there exists a unique MSV solution when $h = 2$.

Plugging the interest rule, (18), with the coefficients (19), into the system (14), we get the reduced form system

$$
\begin{aligned}
y_t &= A_f E_t^* y_{t+1} + A_l y_{t-1} + A_w w_t, \qquad (20)\\
A_f &= \begin{pmatrix} -(1-\gamma)^{-1} & \frac{1-\beta\gamma(1-\gamma)}{\lambda(1-\gamma)} \\ -\lambda(1-\gamma)^{-1} & (1-\gamma)^{-1} \end{pmatrix}, A_l = \begin{pmatrix} 0 & -\lambda^{-1}(1-\gamma) \\ 0 & 0 \end{pmatrix}\\
A_w &= \begin{pmatrix} 0 & [\lambda(\gamma-1)]^{-1}(1-\gamma+\rho) \\ 0 & -\rho(1-\gamma)^{-1} \end{pmatrix}.
\end{aligned}
$$

Note that the lagged output gap and the g_t shock do not appear in the reduced form system (20); the interest rule has offset both these terms. The MSV solution of (20), consequently, takes the form

$$
\begin{aligned}
x_t &= a_x + b_x \pi_{t-1} + c_x u_t, \qquad (21)\\
\pi_t &= a_\pi + b_\pi \pi_{t-1} + c_\pi u_t. \qquad (22)
\end{aligned}
$$

It is easy to verify that there exists a unique MSV solution of this form and it involves

$$
\begin{aligned}
a_x &= 0, a_\pi = 0, \qquad (23)\\
b_x &= -\lambda^{-1}(1-\gamma), b_\pi = 0.
\end{aligned}
$$

We now check E-stability of this unique MSV solution. Assuming agents have a PLM of the form (21)-(22), they compute their forecasts $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ and these forecasts used in (20) lead to an ALM of the same form. If agents use $t-1$ data to compute their forecasts, the E-stability conditions for such a system are given by Proposition 10.1 in (Evans and Honkapohja 2001). For the constant term, the eigenvalues corresponding to the following characteristic polynomial $p(\tau)$ need to have negative real parts for E-stability.

$$
p(\tau) = \tau^2 + \tau + \frac{\beta\gamma}{\gamma - 1}
$$

However, $p(0) < 0$, $p(\infty) > 0$ which implies that there exists a positive eigenvalue. The discussion is summarized in:

Proposition 3 *CIP-rule I leads to indeterminacy in the model (14) when $h = 2$. Nevertheless, there exists a unique MSV solution which turns out to be E-unstable.*

For $h > 2$, we need to resort to numerics and we let γ and θ take values from 0.1 to 0.9 at intervals of 0.1. Table 2 below reports the results when $h = 3$. In this table, column 3 shows determinacy (D) or indeterminacy (I). Column 4 shows the number of stationary MSV solutions. Obviously, in the determinate case, there is only one stationary solution whereas there may be more than one in the indeterminate region. The final column examines E-stability of the stationary MSV solutions whether in the determinate or indeterminate region.[11]

Table 2. CIP Rule I: E-stability of MSV solution when h=3

γ	θ	Det/Indet	# of stat solns	E-stability
.1	{.1,..,.5}	D	1	No
.1	{.6,..,.9}	I	2	No in both cases
.2	{.1,..,.4}	D	1	No
.2	{.5,..,.9}	I	2	No in both cases
.3	.05,.07,.1	D	1	Yes
.3	{.2,..,.4}	D	1	No
.3	{.5,..,.9}	I	2	No in both cases
.4	.1	D	1	Yes
.4	.2,.3	D	1	No
.4	{.4,..,.9}	I	2	No in both cases
.5	.1,.2	D	1	No
.5	{.3,..,.9}	I	2	No in both cases
.6	{.1,..,.9}	I	2	No in both cases
.7	{.1,..,.3},{.7,..,.9}	I	2	No in both cases
.7	.4,.5,.6	I	3	No in all cases
.8	{.1,..,.5}	I	2	No in both cases
.8	{.6,..,.9}	I	3	No in all cases
.9	.1,.2,.3,.5,.6,.7	I	2	No in both cases
.9	.4,.8,.9	I	3	No in all cases

[11] Agents are assumed to use $t-1$ data in their forecasts as their information set under learning.

The table shows that most values of inflation and output inertia lead to indeterminacy when $h = 3$. Even in cases, when determinacy obtains, the (locally) unique solution is usually E-unstable. In the indeterminate region, all MSV solutions always turn out to be E-unstable. This shows that a constant interest rate policy continues to have undesirable properties in the presence of inertia.

Table 3 below shows that similar qualitative features obtain with CIP rule I when $h = 4$. Thus, we conclude that CIP policy I is not a good policy to adopt even in the presence of empirically realistic features of the data in the model.

Table 3. CIP Rule I: E-stability of MSV solution when h=4

γ	θ	Det/Indet	# of stat solns	E-stability
.1	{.1,..,.5}	D	1	No
.1	{.6,..,.9}	I	2	No in both cases
.2	.1	D	1	Yes
.2	{.2,..,.5}	D	1	No
.2	{.6,..,.9}	I	2	No in both cases
.3	.1	D	1	Yes
.3	.2,.3,.4	D	1	No
.3	{.5,..,.9}	I	2	No in both cases
.4	.1	D	1	Yes
.4	.2,.3	D	1	No
.4	{.4,..,.9}	I	2	No in both cases
.5	.1	D	1	Yes
.5	.2	D	1	No
.5	{.3,..,.9}	I	2	No in both cases
{.6,.7}	{.1,..,.9}	I	2	No in both cases
.8	{.1,..,.6},.9	I	2	No in both cases
.8	.7,.8	I	3	No in all cases
.9	{.1,..,.5},.8	I	2	No in both cases
.9	.6,.7,.9	I	3	No in all cases

4.2 CIP Rule II: Determinacy and E-stability

CIP-rule II is based only on pre-determined variables so that the values of x_t and π_t in CIP-rule I, (18), need to be replaced by their values in the MSV solution. The latter is computed from the model with inertia, namely equations (12) and (13), with i_t given by CIP-rule I, (18). The MSV solution for the model with CIP-rule II will be a function only of the contemporaneous shocks and last period values of inflation and output. Assuming this solution takes the explicit form

$$\begin{aligned} x_t &= a_{xx}x_{t-1} + a_{x\pi}\pi_{t-1} + c_x u_t + d_x g_t, && (24)\\ \pi_t &= a_{\pi x}x_{t-1} + a_{\pi\pi}\pi_{t-1} + c_\pi u_t + d_\pi g_t. && (25) \end{aligned}$$

which when plugged into the interest rule (18) lead to the explicit form of the CIP-rule II:

$$\begin{aligned} i_t &= (c + ea_{xx} + fa_{\pi x})x_{t-1} + (d + ea_{x\pi} + fa_{\pi\pi})\pi_{t-1} + (a + ed_x + fd_\pi)g_t \\ &\quad + (b + ec_x + fd_\pi)u_t. && (26) \end{aligned}$$

Note that the coefficients in (24)-(25) are the ones for the MSV solution computed using CIP-rule I, (18). Determinacy and E-stability for CIP-rule II is then examined with equations (12), (13), and (26), that is, with system (14) and with i_t given by (26). For ease of exposition, we write the latter system explicitly as

$$\begin{aligned} y_t &= A_1 E_t^* y_{t+1} + D_1 y_{t-1} + E w_t, && (27)\\ w_t &= F w_{t-1} + v_t, \end{aligned}$$

where $y_t = (x_t, \pi_t)'$, $w_t = (g_t, u_t)'$ and $v_t = (\tilde{g}_t, \tilde{u}_t)'$. The coefficient matrices in (27) are

$$D_1 = \begin{pmatrix} 1 - \theta - \varphi(c + ea_{xx} + fa_{\pi x}) & -\varphi(d + ea_{x\pi} + fa_{\pi\pi}) \\ \lambda(1-\theta) - \lambda\varphi(c + ea_{xx} + fa_{\pi x}) & 1 - \gamma - \lambda\varphi(d + ea_{x\pi} + fa_{\pi\pi}) \end{pmatrix}.$$

and A_1 defined in (15) (E is not important for our analysis).

We first tackle the case when $h = 2$. In this case, the unique MSV solution with CIP rule I is given by (21)-(22) with the MSV coefficients given in (23). It is easy to verify that this MSV solution remains unaltered even with CIP rule II. However, unlike CIP rule I, when this solution was always E-unstable, the situation is not as straightforward with CIP rule II. As it happens, E-stability depends on the inertial parameters γ and θ. Most values of γ and θ imply E-stability. The possibility of E-instability arises for large enough values of γ. For instance, using the calibrated parameters values in (Woodford 1999), namely $\varphi = (.157)^{-1}, \lambda = .024$, $\beta = .99$ together with $\mu = .35, \rho = .9$, we find that E-stability holds for all $\gamma \leq 0.84$ (independent of θ) and that E-instability holds for all $\gamma \geq 0.87$ (independent of θ).[12]

We now examine the performance of CIP rule II for longer horizons numerically. However, before we do this, a couple of things are worth noting. First, as might be expected, determinacy (or E-stability) computed with CIP-rule I does not necessarily imply determinacy (or E-stability) with CIP-rule II. This will be illustrated below by our numerical results. Second, in cases when we have indeterminacy with the CIP-rule I and there are two (sometimes even three) MSV solutions, we can examine determinacy and E-stability for each such solution with the CIP-rule II. In addition, it is possible that an MSV solution in the indeterminate region with CIP-rule I may nevertheless lead to determinacy with CIP-rule II.[13]

The numerical results below illustrate that the qualitative features of CIP rule II are roughly unchanged from those of CIP rule I in the sense that most parameter values continue to lead to indeterminacy with CIP rule II and all such solutions continue to be E-unstable.[14] So, indeterminacy and E-instability is a robust feature with CIP rule II even in the presence of inertia. As a consequence, for ease of presentation, the tables below only report parameter values for which either determinacy obtains or there is indeterminacy but there exist one (or potentially more) E-stable MSV solution.

We first report in Table 4 the parameter values of output and inflation inertia for which *determinacy* obtains with CIP-rule II. As shown there, E-stability of the determinate solution may or may not obtain.

Table 4. CIP Rule II: E-stability of Determinate MSV solution when h=3

γ	θ	E-stability
.1	.1,.2,.3	Yes
.2	.1,.2	Yes
.2	.3	No

[12]For $\gamma = .85, .86$, E-instability holds for θ large enough. E-stability was computed numerically through the eigenvalues of the relevant matrices using a grid search of 0.01 for both γ and θ in the interval $(0, 1)$.

[13]In some cases of indeterminacy with CIP rule II, we nevertheless have only one MSV solution. Indeterminacy means that there are potentially other solutions though not of the MSV type.

[14]The number of stationary solutions varies from 1 to 5 with CIP rule II.

It is obvious from Table 4 that determinacy prevails for very few values. Table 5 below reports those parameter values for which there exist either 2 (or 3) stationary MSV solutions with CIP rule I *and* E-stability prevails for exactly one of these MSV solutions with CIP rule II. In other words, for these values, one of the MSV solutions is determinate and E-stable with CIP rule II while the remaining (1 or 2) solutions are indeterminate and E-unstable.

Table 5. CIP Rule II: Multiple MSV solutions of which one is E-stable when h=3

γ	θ
.4,.5,.6	.4,.5
.5	.3
.6,.7	.6
.7	.7
.8	.9

Finally, for the parameter values $\{\gamma, \theta\} = \{.6, .3\}, \{.7, .5\}, \{.8, .8, \}$, we have determinacy obtaining with one MSV solution and indeterminacy with the other solutions; however, all these solutions (whether determinate or indeterminate) are E-unstable.

We remind the reader that parameter values not reported in Tables 4 and 5 all lead to indeterminacy with all stationary MSV solutions being E-unstable demonstrating the robustness of such a situation. Appendix 2 shows that these qualitative results remain roughly unchanged even when $h = 4$.

5 Concluding Remarks

The results in this paper suggest that the conduct of inflation targeting by using a fixed target at a fixed horizon and a constant interest rate to meet the target is subject to two fundamental difficulties. First, there may be multiple stationary RE solutions under such a policy. Second, the suggested interest rates rules of this approach lead to instability of these solutions.

We examined the issues of determinacy and stability under learning theoretically in the forward-looking New Keynesian model. On the other hand, extensions of the study to inertia in prices or in demand had to be conducted numerically. In most cases the results remained unchanged, i.e. there both indeterminacy and learning instability problems with this policy.

The analysis can be extended in various ways. First, we made the strong assumption that the central bank knows the structural parameters of the economy. Learning of structural parameters ought to be examined along the lines discussed in (Evans and Honkapohja 2003b) and (Evans and Honkapohja 2003a).

Second, we have limited attention to the formulation of CIP policy suggested by (Leitemo 2003). While Leitemo's approach is very natural, the appendix of (Svensson 1998), which is the unpublished version of (Svensson 1999), suggests a different formulation of what is meant by inflation targeting with a fixed target at fixed horizon. One possible interest rate rule from Svensson's approach leads to a rule, for which the interest rate depends only on exogenous shocks in the basic forward-looking model and a result analogous to Proposition 1 is then applicable. However, other formulations and the model with inertia remains to be examined.

A Appendix 1: E-stability for $h = 2$

Substituting the interest rate rule when $h = 2$ into (4) we can obtain the system

$$\begin{aligned} Qy_t &= AE_t^* y_{t+1} + Pw_t, \qquad (28)\\ Q &= \begin{pmatrix} -\frac{1+\lambda\varphi}{\beta} & \frac{1+\lambda\varphi}{\beta\lambda} \\ -\frac{\lambda(1+\beta+\lambda\varphi)}{\beta} & \frac{1+\beta+\lambda\varphi}{\beta} \end{pmatrix}, P = \begin{pmatrix} 0 & \frac{1+\beta\rho+\lambda\varphi}{\beta\lambda} \\ 0 & \frac{1+\beta+\beta\rho+\lambda\varphi}{\beta} \end{pmatrix} \end{aligned}$$

and A is defined in connection with (4). It can be computed that the eigenvalues of Q are 0 and 1.

We first consider whether there is a unique MSV REE to this system. Let a perceived law of motion (PLM) be

$$y_t = a + bw_t$$

and compute forecasts

$$E_t^* y_{t+1} = a + bFw_t.$$

Substituting the PLM and the forecast into (28) yields the equations

$$\begin{aligned} Ma &= Aa \\ Mb &= AbF + P \end{aligned}$$

to compute a and b. It is easily verified that these equations have a unique solution.

Next, consider E-stability. The first step is to consider the temporary equilibrium for given forecasts, i.e. the equations

$$My_t = A(a + bFw_t) + Pw_t,$$

where $y_t = a^* + b^* w_t$, are to be solved for the actual law of motion (ALM). For a^* and b^* we have the equations

$$\begin{aligned} Ma^* &= Aa, \\ (I \otimes M)vecb^* &= (F \otimes A)vecb + vecP, \end{aligned}$$

where the equation for b^* has been vectorized. (Here $\otimes$ is the Kronecker product of two matrices.) The equation determining a^* has either a unique solution or a continuum of solutions since the rank of M is 1. Correspondingly, the rank of $I \otimes M$ is 2. This means that there is either no ALM for a given PLM or the ALM is not unique. The analysis of E-stability must thus be restricted to those PLM that lead to a solution for a^* and b^*. However, in this case the non-uniqueness of the ALM present a further difficulty as the E-stability differential equations are then not well-defined.

We can analyze E-stability only in partial manner in which, for example, only one component of a^* and a take values other than the MSV REE values while the other component is kept at the REE value. Likewise for $vecb^*$ and $vecb$ we can consider E-stability only in the limited sense that just two components of these vectors are not at the REE values.

For a^* and a we then have the E-stability differential equations

$$\begin{aligned} \frac{da_1}{d\tau} &= a_1^* - a_1 = \frac{A_{11}}{M_{11}} a_1 - a_1, \\ \frac{da_2}{d\tau} &= a_2^* - a_2 = \frac{A_{22}}{M_{22}} a_2 - a_2, \end{aligned}$$

where we use the notation $M = (M_{ij})$, $A = (A_{ij})$ for the elements of the two matrices. We have

$$\frac{A_{11}}{M_{11}} = -\frac{\beta}{1 + \lambda\varphi} < 0, \frac{A_{22}}{M_{22}} = \frac{\beta(\beta + \lambda\varphi)}{1 + \beta + \lambda\varphi} < 1,$$

so that these two differential equations are locally stable.

For $vecb^*$ and $vecb$ it is easily seen that the system is block diagonal since

$$I \otimes M = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}.$$

This form implies that we can have one component from each block to deviate from the REE to define the E-stability equations. Moreover,

$$F \otimes M = \begin{pmatrix} \mu & \mu\varphi & 0 & 0 \\ \lambda\mu & \mu(\beta+\lambda\varphi) & 0 & 0 \\ 0 & 0 & \rho & \rho\varphi \\ 0 & 0 & \lambda\rho & \rho(\beta+\lambda\varphi) \end{pmatrix},$$

so that the E-stability differential equations for the first block are

$$\begin{aligned} \frac{db_{11}}{d\tau} &= \left(-\frac{\mu\beta}{1+\lambda\varphi} - 1\right) b_{11} + other, \\ \frac{db_{21}}{d\tau} &= \left(\frac{\mu\beta(\beta+\lambda\varphi)}{1+\beta+\lambda\varphi} - 1\right) b_{21} + other, \end{aligned}$$

where "other" refers to constant terms that do not affect stability. Both of these differential equations are stable. Finally, for b_{21} and b_{22} we get the same differential equations, except that ρ replaces μ in the matrices. This completes the proof of E-stability.

B Appendix 2: CIP II results when h=4

Here we present results on determinacy and E-stability with CIP rule II when $h = 4$. Table 6 shows the values of γ and θ for which indeterminacy holds for each MSV solution corresponding to the model with CIP rule I. The total number of stationary MSV solutions with CIP Rule II ranges from 3 to 7 in Table 6 and *all* of these solutions are E-unstable.

Table 6. Values for which all MSV solutions are E-unstable when h=4 with CIP Rule II

γ	θ
$\{.1,..,.9\}$	.3,.6,.7,.8,.9
$\{.3,..,.9\}$	.5
.4,.5,.7,.8,.9	.4
$\{.2,..,.9\}$	.2
$\{.3,..,.6\}$	.1
.1,.2,.3	.4
.8	.6,.9
.9	.7,.8,.9

Parameter values for which determinacy and E-stability obtain for this unique MSV solution are given by $\{\gamma, \theta\} = \{.1, .1\}, \{.1, .2\}, \{.2, .1\}$.

When $\theta = .1$, and $\gamma = \{.7, .8, .9\}$, we have indeterminacy with one MSV solution and determinacy with the other solution; however, all these solutions are E-unstable. When $\{\gamma, \theta\} = \{.6, .4\}$, we have determinacy and E-stability corresponding to the first MSV solution and indeterminacy (with 3 stationary solutions) corresponding to the other MSV solution and all of the latter solutions are E-unstable. Finally, for the pairs $\{\gamma, \theta\} = \{.8, .7\}, \{.8, .8\}, \{.9, .6\}$, there were 3 MSV solutions corresponding to the original model with CIP rule I. Each of these solutions are indeterminate and E-unstable with CIP Rule II. This shows that indeterminacy and E-instability continues to be a robust feature when $h = 4$.

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