Optimal Dynamic Taxation with Indivisible Labor

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Abstract

We analyze optimal dynamic taxation when labor supply is indivisible, as in Hansen (1985) and Rogerson (1988). Markets are complete, and an employment lottery determines who works. The consumer can buy insurance to diversify this extrinsic income uncertainty. The optimal wage tax is zero in both the short and long run only when leisure is neutral. If leisure is normal (inferior), labor should be taxed (subsidized). We further derive a HARA class of preferences, which encompasses normal and non-normal leisure. For those preferences we characterize the dynamic paths of the wage tax.

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1. INTRODUCTION

There are two ways of introducing second-best government policy in a competitive equilibrium. The first is when the government has to raise an exogenously specified amount of revenue without recourse to lump-sum taxation. The second-best tax system then minimizes the distortions.[1] The second alternative is to highlight the redistributive role of the government when individuals are heterogeneous in terms of factor ownership. The government then resorts to distortionary taxation for redistributive reasons. Under both approaches considerable research has been devoted to finding the optimal capital-income tax. The central result is typically that the optimal capital-income tax is zero in the steady state. This is the well-known Chamley-Judd result (Judd, 1985 and Chamley, 1986).[2]

Relatively less attention has been devoted to analyzing the second-best labor income taxation.[3] In particular, one may question whether the labor income tax is also zero in the steady state. If the government can accumulate capital, it could raise all necessary revenues by taxing capital and labor at the beginning of the optimization period, and set all taxes zero at the steady state. In Chamley's framework, the labor tax is indeed positive in the steady state if leisure is non-inferior (see Renström, 1999). However, both Chamley and Judd assume that labor is divisible, in the sense that the household can choose the number of hours to work. Following the work of Hansen (1985), a new stream of real business cycle literature treats labor supply as indivisible.[4] In this paper, we generalize the Chamley model to allow for indivisible labor supply (as in

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[1] In the second best the government designs an optimal dynamic tax formula at date zero, and remains precommitted to it (i.e. the government chooses all future consumer prices at date zero). If lump-sum taxation is allowed, the analysis reduces to the first-best.

[2] While Chamley (1986) takes the first view of second-best taxation where the government is just revenue raising, Judd (1985) views the government as redistributive. The zero capital-income tax is a result robust to several generalizations, see Renström (1999) and Atkeson et. al. (1999) for surveys.

[3] In a model with divisible labor, Chamley (1985) analyzes second-best wage taxation in the absence of a capital tax. His model provides useful insights about the marginal excess burden of taxation, and time inconsistency of the wage tax. He suggests in his conclusion that the exact tax policy depends on the utility function. We explore this idea further, and show the explicit linkage between the form of the utility function, and the optimal wage tax in an environment with a capital tax and indivisible labor.

[4] Hansen (1985) and Rogerson (1988) establish that indivisible labor explains aggregate fluctuations better than models with divisible labor. A subsequent literature explored further into the business cycle implications of indivisible labor (see Greenwood and Huffman, 1988, Hansen and Sargent, 1988). Mulligan (1999) points out that the tax implications are different for these two classes of models.

Hansen, 1985 and Rogerson, 1988). Contrary to Hansen-Rogerson, we allow utility to be non-separable in leisure.

Why is the issue of optimal taxation in an environment with indivisible labor worth exploring? When labor supply is indivisible, the household faces a choice between working a fixed number of hours and not working at all. Rather than deciding whether to work or not, the individual would find it optimal to randomize her decision, i.e. engaging in an employment lottery. This makes households receive different labor incomes ex post, and thus gives rise to *extrinsic* labor-income uncertainty.[5] When markets are incomplete in the sense that there is neither an insurance market nor a set of contingent-claims markets (where the household can diversify income risk), it is optimal for the government then to use corrective taxes for the missing markets.[6]

However, the role of labor-income taxation in a complete-markets environment has not been fully explored in the literature.[7] We show that the second-best labor-income tax may be positive in the steady state even when markets are complete.[8] In particular, we establish that the optimal labor-income tax is positive (negative) if leisure is normal (inferior). This is true in a steady state, as well as for a period of time out of the steady state. If leisure is normal, the income effect of a positive labor-income tax makes the individual choose a higher probability of work, which enhances the tax base. In a second-

[5] This happens even though there is no intrinsic uncertainty in the sense that preferences, endowment and technology are non-stochastic. Shell and Wright (1993) establish in a static model that this extrinsic uncertainty in labor income due to indivisible labor gives rise to a non-degenerate sunspot equilibrium when individual can randomize the labor supply.

[6] This has been explored in the literature on risk sharing. There, even without a government revenue constraint, a labor tax may be levied. The labor tax then corrects for a market failure (a missing insurance market). In a two-period setting, Hamilton (1987) demonstrates that the optimal wage tax is positive if the second period labor income is uncertain. The issue is indirectly dealt in the macroeconomics literature concerning debt non-neutrality. Chan (1983) and Barsky et al. (1986), as well as Basu (1996), examine the debt non-neutrality hypothesis when future income is uncertain. Although none of these models explicitly deals with the labor-supply decision, there is one common result: taxing future income at a flat rate would be welfare improving if markets were unable to fully insure households from future income risk.

[7] A related literature focuses on the stochastic properties of optimal taxes in economies with uncertainty arising from randomness in government expenditure, and without physical capital. See Aiyagiri et. al. (2002) and the references therein. The focus has been on tax smoothing (whether the labor tax inherits the randomness of government expenditure). As shown in Aiyagiri et. al. (2002) the answer depends on whether public-debt markets are complete. For our purposes it is important to include physical capital. There is then a possibility of having zero-tax equilibria, since the government (if it finds it optimal) can accumulate physical resources to be used for future public expenditure.

[8] As expected, we find that the second-best capital-income tax is zero at the steady state.

best world, the fiscal authority can take advantage of this income effect by imposing a positive wage-income tax.

Next, we derive a HARA class of preferences (i.e. preferences with Hyperbolic Absolute Risk Aversion) with non-separable leisure to characterize the short run dynamics of the labor income tax. For these preferences we show that a single parameter determines the demand for insurance and the sign of the optimal labor tax. We find that when leisure is normal the optimal labor tax is increasing as long as capital is taxed. When leisure is inferior, and labor is subsidized, the subsidy is increasing as long as capital is taxed. We also look at two special cases of the HARA class, iso-elastic utility and negative exponential utility. We find that for (in the literature widely used) iso-elastic preferences, the optimal labor income tax is positive both in the short and long run. For the negative exponential utility, where leisure is neutral, the optimal labor-income tax is zero in the short and long run.

The paper is organized as follows. The following section presents the model. Section 3 derives the optimal-tax implications in a second-best world. Section 4 derives a class of HARA preferences and presents examples of the time path of the optimal labor tax. Section 5 concludes.

2. AN ECONOMY WITH INDIVISIBLE LABOR

2.1. *Individual Economic Behavior*

Following Hansen (1985) and Rogerson (1988), we consider an economy where labor supply is indivisible. The consumption set is restricted so that that the individuals can work either full time, h_0 or not at all. Households have access to an insurance market where they can buy insurance. In each period the household member engages in an employment lottery, choosing the probability of working, $\alpha(t)$, and the probability of not working, $(1-\alpha(t))$. This makes her wage income uncertain. She has access to an insurance market where she buys unemployment insurance, $y(t)$. The household's consumption ($c^s(t)$) and asset accumulation ($\dot{a}^s(t)$) are thus contingent on whether the household works (s=1) or not (s=2). There is no intrinsic uncertainty, which means that preferences and technology are non-stochastic. The household thus maximizes the following life-time expected utility[9]

$$(1) \quad J(a_0) \equiv \max_{c,y,\alpha} \int_0^{\infty} e^{-\theta t}\left[\alpha(t)u\left(c^1(t),1-h_0\right)+(1-\alpha(t))u\left(c^2(t),1\right)\right]dt$$

$$(2) \quad \dot{a}^1(t) = \rho(t)a(t) + \omega(t)h_0 - p(\alpha(t))y(t) - c^1(t)$$

$$(3) \quad \dot{a}^2(t) = \rho(t)a(t) + y(t) - p(\alpha(t))y(t) - c^2(t)$$

$$(4) \quad a(0) = a_0$$

where $a(t)$ equals the sum of outstanding public debt, $b(t)$, and the capital stock, $k(t)$, that earn the after-tax interest at rate $\rho(t)= (1-\tau^K(t))r(t)$, and $\omega(t)= (1-\tau^L(t))w(t)$ is the after tax wage; $r(t)$ and $w(t)$ are the rental- and wage-rates, respectively, $\tau^K(t)$ and $\tau^L(t)$ are the proportional tax rates on capital- and labor-income, respectively, and $p(\alpha(t))$ is the competitive price of insurance. The insurance company behaves competitively and maximizes the expected profit, $p(\alpha(t))y(t)-(1-\alpha(t))y(t)$, which gives rise to the zero-profit condition, $p(\alpha(t))=1-\alpha(t)$.[10]

Substituting the zero-profit condition into (2) and (3), the current value Hamiltonian of the representative household can be written as:

$$\begin{aligned}(5) \quad H= & \alpha(t)u(c^1(t), 1-h_0) + (1-\alpha(t))u(c^2(t),1) \\ & + \alpha(t)\, q^1(t)\, [\rho(t)a(t)+\omega(t)h_0-(1-\alpha(t)y(t)-c^1(t)] \\ & + (1-\alpha(t))\, q^2(t)\, [\rho(t)a(t)+ \alpha(t)y(t) -c^2(t)]\end{aligned}$$

[9] Since there is no intrinsic uncertainty, lifetime utility is the discounted sum of expected utility at each date. The household chooses a randomized labor supply, $\alpha(t)$, given the state of the economy at date t summarized by the wealth $a(t)$.

[10] The household randomizes the labor supply decision in this setting by choosing a probability of work $\alpha(t)$. A realistic description of this arrangement is that the representative household consists of a family of N members. In each period the household decides the proportion, $\alpha(t)$, of members working. The labor supply is then $\alpha(t)h_0N$. The household can buy insurance on a competitive market to fully diversify the income uncertainty arising from $(1-\alpha(t))N$ of its members not working. After choosing the probability of work, $\alpha(t)$ the household is pre-committed to it, and cannot renege. The insurance company then charges the actuarially fair premium, $1-\alpha$. This rules out adverse selection in the model. The household then realizes that, when choosing work probability, the insurance premium is a linear function of the probability.

The first order conditions are (subscripts denoting partial derivatives):

$$(6) \quad \frac{\partial H}{\partial c^1(t)} = u_c(c^1(t), 1-h_0) - q^1(t) = 0$$

$$(7) \quad \frac{\partial H}{\partial c^2(t)} = u_c(c^2(t), 1) - q^2(t) = 0$$

$$(8) \quad \frac{\partial H}{\partial y(t)} = q^1(t) - q^2(t) = 0$$

Using (6), (7) and (8) it follows that

$$(9) \quad u_c(c^1(t), 1-h_0) = u_c(c^2(t), 1) = q(t)$$

which gives the optimal time paths of state-contingent consumption, $c^1(q(t))$ and $c^2(q(t))$, as functions the co-state variable $q(t)$. In other words, by buying insurance, the individual equalizes the marginal utilities across states. However, this does not necessarily imply that household will equalize consumption across states. For consumption equalization, one requires an additional restriction on the preferences that the utility function is additively separable between consumption and leisure, meaning $u_{c(1-h)}=0$. It turns out that without any such restriction on the preference, the household will not choose to have full insurance as in Hansen (1985). This can be seen from (6), (7), and (8). Since $1-h_0$ is not equal to unity, c^1 cannot equal c^2 unless $u_{c(1-h)}$ is equal to 0. Next, since $q^1(t)=q^2(t)$ it follows that the optimal asset holding decisions must satisfy:

$$(10a) \quad \dot{a}^1(t) = \dot{a}^2(t),$$

$$(10b) \quad \dot{q}(t) = (\theta - \rho(t))q(t).$$

An individual's asset accumulation is thus independent of her employment history. This implies that individuals starting with the same a_0 will have the same $a(t)$ at all t, regardless of their employment history. Substituting (10a) in (2) and (3) gives

(11) $y(t) = \omega(t)h_0 + c^2(q(t)) - c^1(q(t))$

Notice now that the household chooses full insurance if the optimal consumption bundles are such that $c^1(q(t)) = c^2(q(t))$. In the absence of any restriction on $y(t)$, the household can choose to have positive, negative or zero insurance. [11] Finally, the optimal choice of $\alpha(t)$ must be such that

(12) $$\frac{\partial H}{\partial \alpha(t)} = u(c^1(q(t)),1-h_0) - u(c^2(q(t)),1) - q\left[c^1(q(t)) - c^2(q(t)) - \omega(t)h_0\right] = 0$$

which upon the use of (11) can be rewritten as:

(12') $$u(c^2(q(t)),1) - u(c^1(q(t)),1-h_0) = q(t)y(t)\,.$$

The household chooses to buy a positive insurance, $y(t)$, if the utility gain from not working balances the utility cost of the insurance purchase.

2.2. *Production*

There is a large number of competitive firms in the economy each operating under the following constant returns-to-scale technology:

(13) $f(k(t), \alpha(t)h_0) = f_1 \cdot k(t) + f_2 \cdot \alpha(t)h_0$

2.3. *The government*

The government taxes labor and capital income to finance an exogenously specified sequence of public spending, $g(t)$, the use of which, as in Chamley (1986), is not explicitly modeled. It adjusts two tax rates, $\tau^L(t)$ and $\tau^K(t)$, continuously. The government is assumed to borrow and lend freely at the market rate of interest, $r(t)$. The government's budget constraint is, therefore, given by:

[11] One needs to be careful about the non-negativity constraint on consumption while thinking about negative unemployment benefit. $y(t)$ can be negative as long as $c^2(t)$ is non-negative. We assume interior solutions, meaning $c^2(t)>0$.

(14) $\dot{b} = r(t)b(t) - \tau^k(t)r(t)a(t) - \tau^L(t)w(t)\alpha(t)h_0 + g(t)$

with $b(0)=b_0$.

2.4. *Equilibrium*

The equilibrium is characterized by the following conditions:

(a) Facing $w(t)$, $r(t)$, $\tau^L(t)$, $\tau^K(t)$, the household chooses optimal sequences of $c(t)$, $a(t)$, $\alpha(t)$, $y(t)$ that solves the problem stated in (1), (2) and (3).[12]

(b) Given an exogenous steam of government spending, $g(t)$, the government pre-commits to a tax sequence, $\tau^L(t)$ and $\tau^K(t)$, and a debt sequence, $b(t)$, that satisfies the government budget constraint (14).

(c) Goods, labor, rental markets clear meaning

(15) $\dot{k}(t) = f(k(t), \alpha(t)h_0) - \alpha(t)c^1(t) - [1-\alpha(t)]c^2(t) - g(t)$,

(16) $\omega(t) = (1-\tau^L(t))\, f_2(k(t), \alpha(t)h_0)$,

(17) $\rho(t) = (1-\tau^K(t))\, f_1(k(t), \alpha(t)h_0)$,

Notice that the equilibrium level of employment, $h(t)$ ($\equiv \alpha(t)h_0$) is determined by the time path of the probability of work, $\alpha(t)$. The equilibrium time path of $\alpha(t)$ can be determined in two steps. First, using (12) one determines the market clearing after tax wage, $\omega(t)$ as a function of $q(t)$. Define that equilibrium wage function as:

(18) $\omega(t) = \Omega(q(t))$

Next, using (16) and (18), one can characterize the path of $\alpha(t)$ as a function of $k(t)$, $q(t)$, and $\tau^L(t)$ as follows:

(19) $\alpha(t) = \alpha(k(t), \tau^L(t), q(t))$

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[12] We assume no-Ponzi games.

Proposition 1. *In equilibrium,* $y(t)>0$, *if and only if, leisure is normal at date t;* $y(t)<0$, *if and only if, leisure is inferior at date t.*

Proof. See Appendix A.

Positive insurance demand and normality of leisure are inextricably connected.[13] To see the intuition, start from a scenario where $y(t)=0$. In this case, the individual is indifferent between work and no work (see Equation 12'). Starting from this scenario suppose initial wealth a_0 increases. A higher wealth makes the consumer value leisure more if leisure is normal. The household cannot choose hours of work in this indivisible labor world. The only choice is to decrease the probability of work and buy positive insurance in response to increase in wealth. This is why a positive insurance demand is associated with a positive utility gain from not working as in (12').[14]

3. SECOND-BEST OPTIMAL TAXATION

We now solve for the optimal tax problem for the government for this economy with indivisible labor. The government solves a Ramsey problem for pre-committed tax sequences, $\tau^L(t)$ and $\tau^K(t)$, that maximize the household's utility functional (1) subject to its own budget constraint (14), the economy wide resource constraint (15), the first-order optimality conditions (9), (10b), and (12), and a no-confiscation constraint on capital income as follows:[15]

$$\rho(t) \geq 0 \tag{20}$$

[13] In a divisible labor economy, Renström (1999) finds that non-inferiority of leisure is sufficient for wage income tax to be positive. In the indivisible-labor setting, normality is both necessary and sufficient for a positive labor-income tax.

[14] The insurance market is added to obtain insights about the relationship between normality of leisure and the optimal tax, as well as deriving preference structures for which leisure is normal. One can alternatively construct an environment with contingent-claims markets as in Shell and Wright (1993), and derive the same equilibrium allocation as a sunspot equilibrium. A proof of this equivalence is available from the authors upon request.

[15] No such confiscation constraint is relevant for labor income taxation because if labor income is confiscated by the government it is optimal for the household to set $\alpha(t)=0$ which means no production. On the other hand, in principle, the capital income can be confiscated and the government can eventually own all the capital to run production.

Using (16) and (17), and the CRS property of the production function, the government's budget constraint, (14), can be rewritten as:

(21) $$\dot{b}(t) = \rho(t)\, b(t) + \rho(t)\, k(t) + \alpha(t)\omega(t)\, h_0 - f\left(k(t), \alpha(t) h_0\right) + g(t)$$

We may write the government's current value Hamiltonian as follows (ignoring the time indices from now on):

(22) $$\begin{aligned} H^g &= \alpha u(c^1, 1-h_0) + (1-\alpha) u(c^2, 1) + \mu\{\rho b + \rho k + \alpha\omega h_0 - f(k, \alpha h_0) + g\} \\ &+ \lambda\left\{f(k, \alpha h_0) - \alpha c^1 - (1-\alpha) c^2 - g\right\} + \psi(\theta - \rho) q + \nu\rho \end{aligned}$$

In principle, the government faces the states, k, b and q, and chooses the controls ρ and τ^L. For algebraic convenience, we pose the government's problem as follows. The government chooses the controls ρ and α. Then using the equilibrium sequence of α as in (19), one can determine the optimal labor tax, τ^L. [16] Denoting $u^1=u(c^1,1-h_0)$ and $u^2=u(c^2,1)$, the first-order conditions facing the government are as follows:

(23) $$\frac{\partial H^g}{\partial \alpha} = u^1 - u^2 + \mu[\omega h_0 - f_2 . h_0] + \lambda\left[f_2 h_0 + c^2 - c^1\right] = 0$$

(24) $$\begin{aligned} &\frac{\partial H^g}{\partial q} = -\dot{\psi} + \theta\psi \\ &\Rightarrow \dot{\psi} = \rho\psi - \alpha u_c^1 c_q^1 - (1-\alpha) u_c^2 c_q^2 - \mu\alpha\Omega'(q) h_0 + \lambda\left[\alpha c_q^1 + (1-\alpha) c_q^2\right] \end{aligned}$$

(25) $$\frac{\partial H^g}{\partial \rho} = \mu(b+k) - \psi q + \nu = 0$$

(26) $$\frac{\partial H^g}{\partial k} = \mu(\rho - f_1) + \lambda f_1 = \theta\lambda - \dot{\lambda}$$

[16] It is straightforward to verify that for given k and q, $\alpha'(\tau^L)<0$ and hence $\alpha(\cdot)$ can be inverted with respect to τ^L.

(27) $$\frac{\partial H^g}{\partial b} = \mu(\rho - \theta) = -\dot{\mu}$$

3.1. *The optimal capital-income tax*

One may now establish, as in Judd (1985) and Chamley (1986), that the optimal capital-income tax is zero in steady state, also in the our economy with indivisible labor. This can be verified as follows. At steady state the individual's marginal utility of consumption is constant over time, implying, by (10b), that $\theta=\rho$. This in turn implies, by (27) that μ is constant in the steady state. With μ being constant (and q constant) equation (23) implies that also λ is constant in the steady state. Setting the time derivative of λ to zero in (26), and using $\theta=\rho$, gives $(\rho-f_1)(\lambda-\mu)=0$ which holds if and only if $f_1=\rho$ because $(\lambda-\mu)>0$. We summarize this result in terms of the following proposition.

Proposition 2. *If the indivisible labor economy converges to a steady state under the second-best tax program, then in the steady state the optimal capital-income tax is zero,* $\tau^K=0$.

3.2. *The optimal labor-income tax*

Our primary interest in this paper is to explore carefully the optimal labor-tax implications. The problem is complicated by the fact that in an economy with indivisible labor and lottery, the consumer has the option to transfer consumption not only across dates but also across states. Whether in the second best, the government should tax wage income crucially depends on the household's risk preference, which will determine its demand for insurance. The following proposition states a key result about the relationship between the household's demand for insurance and the second best wage taxation.

Proposition 3. *Under the second-best tax program, from the date when the no-confiscation constraint* (20) *on capital ceases to bind, the optimal wage-income tax is positive (negative) if the household's demand for insurance, y, is positive (negative).*

Proof. See Appendix B.

Thus, there is a direct link between the sign of y and the sign of the labor-income tax. The question arises whether the optimal labor-income tax is zero when y=0, the case when leisure is neutral. This cannot be directly inferred from Proposition 3, because it is based on an equilibrium wage equation subject to the condition that y is non-zero.[17] We next analyze a benchmark case when the optimal labor income tax is zero.

Proposition 4. *If the preferences of the consumer are such that the consumer chooses to buy zero insurance* (*meaning* y=0), *the optimal labor-income tax is zero at all dates.*

Proof. Plugging (12') into (23) and using (11), one obtains,

$$f_2 - \omega = \frac{(q-\lambda)y}{(\lambda-\mu)h_0} \tag{29}$$

It immediately follows that when y=0, f_2=ω which means τ^L=0. ‖

To summarize, the optimal labor tax is zero when leisure is neutral, and this is not just a steady-state result. The labor income tax is positive from the date when the non-confiscation constraint does not bind, if and only if leisure is normal. The issue arises whether for normal leisure, the labor income tax rises or falls over time. It is difficult to characterize the exact time path of the labor tax without fully specifying the preferences. In the next section, we derive a class of HARA preferences for which it is possible to characterize the exact short run path of the labor tax.

4. DYNAMICS OF THE LABOR TAX: A PARAMETRIC EXAMPLE

Consider a class of preferences for which insurance demand is proportional to the wage income as follows:

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[17] To see this clearly note that the proof of Proposition 3 rests on the fact that $\Omega'(q)=-y/qh_0$, which holds when y is non-zero.

(30) $$y = (1-\pi)\omega h_0$$

where π is a parameter which can be either positive, negative or zero. It turns out that the parameter π pins down alternative preference structures for which leisure may be normal, neutral or inferior.

4.1. *Full insurance*

The benchmark case of full insurance arises π=0. We have the following result.

Proposition 5. *Necessary and sufficient for* π=0 *is that the utility function* $u(c,1\text{-}h_0)$ *is additively separable in leisure*: $u_{c(1\text{-}h)}$=0.

Proof. If π=0, from (11) and (30) it follows that $c^1 = c^2$. Next, note that when $c^1 = c^2$, the only way marginal utilities can be equalized in (9) is by setting $u_{c(1\text{-}h)}$=0. This proves necessity. Next we prove the sufficiency. If $u_{c(1\text{-}h)}$=0, then from (9) it follows that $c^1 = c^2$, which upon plugging in (11) and (30), gives π=0. ‖

The additive separable leisure in the utility function is widely used in the literature. When the utility function is additively separable in leisure, it implies that leisure is a normal good. In this case, evidently labor is taxed as per Proposition 2. However, the converse is not true. For normality of leisure, it is not necessary that $u_{c(1\text{-}h)}$=0. We now derive a wide range of preferences involving non-separable leisure that belongs to a family of preferences known as the Hyperbolic Absolute Risk Aversion (HARA) class. Such a family of preferences can be derived when $\pi \neq 0$.

4.2. *HARA preferences*

We shall establish now that for $\pi \neq 0$, the derived HARA class can encompass various possibilities: (a) $0 < \pi < 1$, partial insurance; (b) $\pi = 1$ no insurance; (c) $\pi > 1$, negative insurance, and (d) $\pi<0$, over-insurance.

Plugging (30) into (11) to eliminate ω we get

$$y = \frac{1 - \pi}{\pi}(c^1 - c^2) \tag{31}$$

Plugging (31) into (12') gives

$$u^1 - u^2 + q\frac{1 - \pi}{\pi}(c^1(q) - c^2(q)) = 0 \tag{32}$$

where $u^1 = u(c^1(q), 1 - h_0)$ and $u^2 = u(c^2(q), 1)$. Equation (32) holds for all preferences for which $\pi \neq 0$.[18] We wish to find the class of preferences for which π is constant (i.e. independent of *q*). We have the following lemma.

Lemma 1. *It is necessary that the class of preferences for which* $\pi \neq 0$, *satisfies the following condition*:

$$u_c^1 c_q^1 - u_c^2 c_q^2 + (1 - \pi)(c^1 - c^2) = 0 \tag{33}$$

where u_c^s *and* c_q^s *denote the derivatives of u and c in state s*={1,2} *w.r.t. c and q, respectively.*

Proof. Define (32) as the implicit function $J(q,\pi) = 0$. Using the implicit function theorem,

$$\frac{\partial \pi}{\partial q} = -\frac{J_q}{J_\pi} \tag{34}$$

For (34) to be zero, it is necessary that either $J_q = 0$ or $J_\pi = \infty$. However, $J_\pi = \infty$ would mean

$$-q(c^1 - c^2)/\pi^2 = \infty \tag{35}$$

q being the marginal utility of consumption cannot be infinity at the optimum. Thus for (35) to hold, π must equal zero, which violates the restriction that $\pi \neq 0$. Thus it is necessary that $Jq = 0$, which yields (33). ‖

Lemma 2. *For* $J_q=0\ \forall\ q$, *it is necessary that the utility function is of the following Hyperbolic Absolute Risk Aversion (HARA) form*:

$$(36)\qquad u(c_s, 1-h^s) = D^s - \frac{B^s}{\pi}(A+(1-\pi)c_s)^{\pi/(\pi-1)}$$

for $s=\{1,2\}$. B^s *and* D^s *are constants (possibly dependent of h), with* $B>0$, *and A is a constant independent of h.*

Proof. Taking the derivative of (9) w.r.t. q gives $c_q^s(q) = 1/u_{cc}^s$, $s=\{1,2\}$. Plugging this into (33), we get

$$(37)\qquad \frac{u_c^1}{u_{cc}^1} + (1-\pi)c^1 = \frac{u_c^2}{u_{cc}^2} + (1-\pi)c^2$$

Since preferences are state independent (i.e. expected utility) the left- and right-hand sides must equal the same constant, (say $-A$), i.e.

$$(38)\qquad \frac{u_c^s}{u_{cc}^s} = -A - (1-\pi)c^s$$

for $s=\{1,2\}$. This means that A and π cannot be state dependent (i.e. dependent of h). Integrating (38) twice (see, Appendix C for derivation) yields (36). ‖

We next show that for the class of HARA preferences (36), the constant D is state independent (i.e. independent of leisure).

Lemma 3. *For* (32) *and* (36) *to hold, it is necessary that that* $D^1 = D^2$.

[18] Note that $\pi=0$ cannot be nested under this case, which means additively separable utility function does not belong to our derived class of preferences.

Proof. Take the derivative of (36)

(39) $$u_c(c^s, 1-h^s) = B^s(A+(1-\pi)c^s)^{1/(\pi-1)}$$

Then, by using (9), we find c^s as functions of q:

(40) $$c^s = \frac{1}{1-\pi}\left(\frac{q}{B^s}\right)^{\pi-1} - \frac{A}{\pi-1}$$

Inserting (40) into (36) gives

(41) $$u^s = D^s - (B^s)^{1-\pi} q^{\pi}/\pi$$

Plugging (40) and (41) into (32), it follows that D^1=D^2. ‖

Since in the preference class (36), only B^s and c^s can be state dependent, it must be the case that B^s is a function of leisure. Using this insight, define $B^s = \phi^h$, where h can take two possible states 0 and h_0. Our next task is to characterize the precise restrictions on π, which generates leisure as a "good" (with positive marginal utility) in the utility function. We are ready to state a key proposition.

Proposition 6. *Necessary and sufficient for insurance to be a constant fraction of the after-tax wage income is that the preferences belong to the following class:*

(42) $$u(c, 1-h) = D - \frac{\phi^h}{\pi}(A+(1-\pi)c)^{\pi/(\pi-1)}$$

where A and D are constants. ϕ^h *is a function of leisure as follows:*

(a) *If* $\pi>0$, *then* $\phi^h=\phi(h)$, *with* $\phi'(h)>0$ *and* $\phi''(h)>0$.

(b) *If* $\pi<0$, *then* $\phi^h=\phi(1-h)$, *with* $\phi'(1-h)>0$ *and* $\phi''(1-h)<0$.

Proof. This can be verified immediately by observing that (a) and (b) hold if, and only if, $u_{(1-h)}(c,1-h)>0$ and, $u_{(1-h)(1-h)}(c,1-h)<0$. ‖

4.3. *The case of zero labor-income taxation*

In the next step we derive a subclass of preferences from the HARA class for which labor should not be taxed at the second-best optimum. This is a special case when leisure is neutral.

Proposition 7. *If the utility function is of the following exponential class*

$$u(c,1-h)=D-\phi^h e^{-c} \tag{43}$$

where D is a constant, and $\phi^h=\phi(h)$, *with* $\phi'(h)>0$ *and* $\phi''(h)>0$, *then the optimal labor-income tax is zero at all dates.*

Proof. Recall from Proposition 3 that when the individual takes no insurance, the optimal wage tax is zero. In the present context, $y=0$ if $\pi=1$ (see equation 30). Take limits of (42) as $\pi\rightarrow1$ (which requires setting *A* equal to unity) and apply l'Hôpital's rule to obtain (43). ‖

4.4. *Why does normality of leisure mean a positive tax on labor?*

We have found that whether labor should be taxed or not depends on whether the individual demands positive insurance or not, which in turn depends on whether leisure is normal or inferior. A relationship thus exists between normality of leisure and the insurance demand via the individual's preferences. To see this connection clearly, differentiate (19) with respect to $\tau^L(t)$, (keeping $k(t)$ constant), to obtain the following useful decomposition of a change in tax rate, $\tau^L(t)$ on work effort, $\alpha(t)$:

$$(44)\qquad \left.\frac{d\alpha(t)}{d\tau^L(t)}\right|_{k(t)} = \frac{\partial\alpha(t)}{\partial\tau^L(t)} + \frac{\partial\alpha(t)}{\partial q(t)}\cdot\frac{\partial q(t)}{\partial\tau^L(t)}$$

The first term in (44) represents the compensated labor-supply response when the tax rate changes (the substitution effect). If $q(t)$ is held constant, it follows from (18) that $\omega(t)$ is also constant and it is straightforward to verify from (16) that the first term is: $f_2/[(1-\tau^L)f_{22}\cdot h_0]$, and is negative. This substitution effect thus captures the distortionary effect of the wage-income tax. As far as this substitution effect is concerned, a higher labor tax lowers labor supply and thus lowers the tax base.

The second term in (44) reflects the income effect of a change in the wage tax rate, which works through the effect of $\tau^L(t)$ on $\alpha(t)$ via its effect on $q(t)$. When $\tau^L(t)$ is higher, it lowers the permanent income of the household, thus lowering consumption in both states for given $\alpha(t)$. The marginal utility of consumption, $q(t)$, thus rises, which means $\partial q(t)/\partial\tau^L(t) > 0$. A loss of permanent income (represented by higher $q(t)$) would boost the labor supply, $\alpha(t)$, if leisure is a normal good. Next, verify that (44) can be rewritten as: [19]

$$(44')\qquad \left.\frac{d\alpha(t)}{d\tau^L(t)}\right|_{k(t)} = \frac{f_2}{[1-\tau^L(t)]f_{22}h_0} - \frac{(1-\pi)f_2}{f_{22}q(t)h_0}\frac{\partial q(t)}{\partial\tau^L(t)}$$

The normality of leisure (π<1) thus makes the income effect of a higher wage tax positive. This positive income effect tends to increase the tax base ($w(t)\alpha(t)h_0$) when $\tau^L(t)$ rises, which countervails the distortionary effect of a higher $\tau^L(t)$. The government can thus tax labor more in those economies. On the other hand, if π>1, leisure is inferior. The income effect is then negative, reinforcing the distortionary effect of wage taxation. In this case, the government should subsidize labor.

[19] To see this, insert (30) into (A.5) and use (16) to obtain $\partial\alpha(t)/\partial q(t) = -(1-\pi)f_2/f_{22}\,q(t)\,h_0$.

4.5. *Should labor be taxed when the utility function is isoelastic in consumption?*

Consider now a special case of the HARA class, equation (42), when A=0. We have the following lemma concerning the elasticity of interstate substitution (call it σ hereafter).

Lemma 4. *For a specific class of HARA utility functions with* A=0, *the elasticity of interstate substitution* (σ) *is given by* $1-\pi$.

Proof. By definition,

$$\sigma = \frac{d\ln\left(\dfrac{c^2}{c^1}\right)}{d\ln\left(\dfrac{u_c^1}{u_c^2}\right)} \tag{45}$$

Using (42) gives $\sigma = 1-\pi$. ‖

The utility function thus becomes isoelastic in consumption, and the elasticity of interstate substitution in consumption is uniquely characterized by π. Furthermore, we must have $\pi<1$ when A=0, otherwise positive marginal utility, $u_c>0$, is violated. This immediately implies that leisure is normal when the utility function is isoelastic in consumption, and thus labor must be taxed.

To summarize, the HARA class of preferences (42) embraces a variety of utility functions, and thus covers a wide range of optimal taxation schemes. These cases include: (a) isoelastic utility in consumption (A=0), in which case the optimal labor tax is positive, (b) negative exponential utility function (π=1), where the optimal labor tax is zero, and (c) quadratic when π=2 (where $A > \max_t c(t)$), in which case the optimal labor tax is negative (i.e. a subsidy).

4.6. *Transitional dynamics of the wage-income tax in the HARA case*

We shall now analyze the transitional dynamics of labor income tax for the range of preferences discussed earlier. Using (29) one obtains

(46) $$\frac{\tau^L}{1-\tau^L} = (1-\pi).\frac{q-\lambda}{\lambda-\mu}$$

then taking the time derivative of (46) using (10b), (26), and (27), one obtains:

(47) $$\frac{d}{dt}\frac{\tau^L}{1-\tau^L} = (1-\pi)\frac{d}{dt}\frac{q-\lambda}{\lambda-\mu} = (1-\pi)(f_1-\rho)\frac{q-\mu}{\lambda-\mu}$$

Appendix C outlines the steps in deriving the second equality. Based on the second equality in (46), and the fact that q-μ>0, and λ-μ>0, we have the following proposition.

Proposition 8. *If π<1 labor is taxed (at least from the date at which the non-confiscation constraint does not bind), and the wage tax is increasing as long as capital is taxed. If π=1 labor is always untaxed. If π >1 labor is subsidized (at least from the date at which the non-confiscation constraint does not bind), and the subsidy is increasing as long as capital is taxed.*[20]

5. CONCLUSION

In this paper, we address the issue of optimal wage taxation in a dynamic complete-markets setting. The issue of optimal labor-income taxation is a relatively ignored area of research in dynamic tax theory. We find that the second-best labor-tax depends crucially on the degree of complementarity between consumption and leisure. If leisure is a not neutral, there is scope for government intervention in the form of labor taxation or subsidy in a complete market environment. We investigated this question in a fairly general setting with preferences allowing non-separability between consumption and leisure. Our conclusion is that labor should be taxed if leisure is normal. The optimal wage tax is zero only for a

[20] When π=1, the non-confiscation constraint is binding for a period $t\in[0,T^*)$, and capital income is taxed at 100%. From T^* and onwards, capital is untaxed, and the economy is at its steady state level. T^* is a function of the present value of the stream of g_t, $t\in[0,\infty)$ discounted at the rate θ.

watershed case where leisure is neutral. The case of a wage subsidy arises in an implausible scenario where leisure is inferior, implying that the individual buys negative unemployment insurance. The results obtained here provide useful guidance in designing optimal tax policy in a dynamic environment. A useful extension of this paper would be to investigate similar issues in a third-best environment where the government may not necessarily commit to a specific tax design.

APPENDIX A

Proof of Proposition 1. First note that leisure in an indivisible labor economy is normal if $\frac{\partial \alpha(t)}{\partial a_0}$ <0, which means the household chooses a lower probability of work when its wealth is higher. Next observe that

(A.1) $$\frac{\partial \alpha(t)}{\partial a_0} = \frac{\partial \alpha(t)}{\partial q(t)} . \frac{\partial q(t)}{\partial a_0}$$

Using (10a), one obtains

(A.2) $$\frac{\partial q(t)}{\partial a_0} = \exp\left(\int_0^{\infty} (\theta - \rho(s)) ds \right) \frac{\partial q_0}{\partial a_0}$$

Next note by the application of Envelope property of the value function $J(a_0)$ in (1) that $J'(a_0) = q_0$. By strict concavity of the value function ($J''(a_0) < 0$), it follows that $\frac{\partial q_0}{\partial a_0} < 0$. Thus from (A.2), it follows that $\frac{\partial q(t)}{\partial a_0} < 0$. From (A.1), it means that

(A.3) $$sign\left(\frac{\partial \alpha(t)}{\partial a_0} \right) = -sign\left(\frac{\partial \alpha(t)}{\partial q(t)} \right)$$

Using (12) and (16), define the following implicit function (time indices suppressed)

(A.4) $$G(q, \alpha) = U(c^1(q), 1 - h_0) - U(c^2(q), 1) + q.\left[(1 - \tau^L). f_2(k, \alpha h_0) h_0 + c^2(q) - c^1(q) \right]$$

Using the implicit function theorem and (9) one obtains, (A.5) $$\frac{\partial\alpha(t)}{\partial q(t)} = -\frac{G_q}{G_\alpha} = -\frac{y(t)}{q(t)(1-\tau^L(t))\, f_{22}.h_0^2}$$

From (A.3) and (A.5), it follows that $\dfrac{\partial\alpha(t)}{\partial a_0}$ is negative, zero or positive if and only $y(t)$ is positive, negative or zero. ‖

APPENDIX B

Proof of Proposition 2. Plugging (12) into (23) and using (11), one obtains,

(B.1) $$f_2 - \omega = \frac{(q-\lambda)y}{(\lambda-\mu)h_0}$$

Premultiply (24) by q and exploiting the fact that $\Omega'(q) = [c^1 - c^2 - \omega h_0]/qh_0$, one obtains

(B.2) $$q\dot{\psi} = q\rho\psi + (\lambda - q)[\alpha c_q^1 q + (1-\alpha) c_q^2 q] + \mu\alpha\left[\omega h_0 - c^1 + c^2\right]$$

Next note that

(B.3) $$\frac{d}{dt}(\psi q) = \dot{\psi} q + \psi\dot{q} = \dot{\psi} q + \psi(\theta - \rho) q$$

Plugging (B.3) into (B.2) gives

(B.4) $$\frac{d}{dt}(\psi q) = \theta\psi q + (\lambda - q)\left[\alpha c_q^1 q + (1-\alpha) c_q^2 q\right] + \mu\alpha\left[\omega h_0 - c^1 + c^2\right]$$

Next note that

(B.5) $$\frac{d}{dt}(\mu a) = \dot{\mu} a + \mu\dot{a}$$

Plugging (27) into (B.5)

(B.6) $$\frac{d}{dt}(\mu a) = \mu(\theta - \rho)a + \mu\dot{a}$$

Using (2), (3) and (10a), the household's budget constraint can be rewritten as:

(B.7) $$\dot{a} = \rho a + \alpha\omega h_0 - \alpha c^1 - (1-\alpha)c^2$$

which after plugging into (B.6) gives

(B.8) $$\frac{d}{dt}(\mu a) = \theta\mu a + \mu\left[\alpha\omega h_0 - \alpha c^1 - (1-\alpha)c^2\right]$$

Next noting that a=b+k, rewrite (25) as

(B.9) $$\nu = \psi q - \mu a\,.$$

Taking the time derivative of (B.9), one obtains

(B.10) $$\dot{\nu} = \frac{d}{dt}(\psi q) - \frac{d}{dt}(\mu a)$$

Using (B.4) and (B.8) in (B.10) one obtains

(B.11) $$\dot{\nu} = \theta\nu + (\lambda - q)\left[\alpha c_q^1 q + (1-\alpha)c_q^2 q\right] + \mu c^2$$

Chamley (1986) shows that the confiscation constraint, (20) cannot be binding forever. In our case, if it is binding forever, consumption falls to zero in both states. Suppose that it ceases to bind at date t_1. Since ν is the multiplier associated with the confiscation constraint, (20), this implies that

(B.12) $$\nu(t) = \dot{\nu}(t) = 0$$

for $t \geq t_1$. Plugging (B.12) into (B.11) and simplifying terms, we get:

$$\text{(B.13)} \quad \frac{q-\lambda}{\mu} = \frac{c^2}{\left[\alpha c_q{}^1 q + (1-\alpha) c_q{}^2 q\right]}$$

for $t \geq t_1$.

Plugging (B.13) into (B.1), one obtains

$$\text{(B.14)} \quad \frac{\lambda-\mu}{-\mu}(f_2-\omega) = \frac{-c^2 y}{\left[\alpha c_q^1 q + (1-\alpha) c_q^2 q\right] h_0}$$

Since $\lambda>0$ and $\mu<0$ and $c_q{}^1<0$, and $c_q{}^2<0$ (by concavity of u), it follows that the sign of $(f_2\text{-}\omega)$ is the same as the sign of y. Hence, the labor income tax is positive (negative) when $y>(<)\ 0$. ‖

APPENDIX C

Derivation of equation (36). Inverting (38) gives

$$\text{(C.1)} \qquad u_{cc}^s / u_c^s = -1/[A + (1-\pi) c^s]$$

for s={1,2}. Or equivalently

$$\text{(C.2)} \quad \frac{d \ln(u_c^s)}{d c^s} = -\frac{1}{1-\pi} \frac{d \ln(A + (1-\pi) c^s)}{d c^s}$$

Integrating both sides with respect to c^s gives

(C.3) $$\ln(u_c^s) = M^s - \frac{1}{1-\pi}\ln(A+(1-\pi)c^s)$$

where M^s is any constant, possibly dependent on s.

Taking exponents of both sides gives

(C.4) $$u_c^s = B^s(A+(1-\pi)c^s)^{-1/(1-\pi)}$$

where B^s=exp$\{M^s\}$, and consequently is positive.

Integrating both sides with respect to c^s finally gives

(C.5) $$u^s = D^s - \frac{B^s}{\pi}(A+(1-\pi)c^s)^{-\pi/(1-\pi)}$$

where D^s is any constant, possibly dependent on s. ‖

Derivation of equation (46). Note that

(C.6) $$\frac{d}{dt}\left[\frac{q-\lambda}{\lambda-\mu}\right] = \left[\frac{(dq/dt)-(d\lambda/dt)}{\lambda-\mu}\right] - \left[\frac{q-\lambda}{\lambda-\mu}\right].\left[\frac{(d\lambda/dt)-(d\mu/dt)}{\lambda-\mu}\right]$$

Next plug in (10b), (26) and (27) into the right hand side of (C.6) to obtain (47). ‖

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