Measuring trend growth: how useful are the great ratios?[*]

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Abstract

Standard macroeconomic models suggest that the 'great ratios' of consumption to output and investment to output should be stationary. The joint behaviour of consumption, investment and output can then be used to measure trend output. We adopt this approach for the USA and UK, and find strong support for stationarity of the great ratios when structural breaks are taken into account. From the estimated vector error correction models, we extract multivariate estimates of the permanent component in output, and comment on trend growth rates in the 1980s and the New Economy boom of the 1990s.

Keywords: Trend growth, great ratios, structural breaks, permanent components, New Economy.
JEL Classification: E2, E3, C32, C51

1 Introduction

The trend growth rate is one of the most important variables in all of macroe-conomics. It is an essential component in making informed judgements about monetary policy, in making strategic decisions on fiscal policy, and in evaluating the effects of structural reform programmes. Yet the problem of measuring trend output is a difficult one, and there is disagreement on the best solution. In this paper, we develop and extend a relatively new approach to this question, building on the pioneering work of King, Plosser, Stock and Watson (1991), henceforth KPSW.

Conventionally, trend/cycle decompositions have been applied to a single series, output. In recent years, however, there has been growing interest in methods which bring information in other series to bear. It is often possible to give this multivariate approach an intuitively appealing justification. For example, as Cochrane (1994) and Cogley (2001) have pointed out, to the extent that consumers are forward-looking and follow the permanent income hypothesis, movements in consumption may be informative about future movements in output. Consumers are likely to have access to information about future movements in productivity that will not be available to the econometrician from the history of past output alone.

In this paper, we examine in depth one particular multivariate approach to trend measurement, introduced by KPSW. Their starting point is the observation that the "great ratios" of investment to output, and consumption to output, will be stationary processes if economies converge towards a balanced growth path. The balanced growth hypothesis implies that log consumption and log investment should each be cointegrated with log output, with unit cointegrating vectors. KPSW used this idea to evaluate the relative importance of various kind of shocks in explaining US business cycles.

A less widely noted contribution of their paper was to provide a new method of measuring the permanent component in output, based on the joint behaviour of consumption, investment and output.[1] Their strategy was to estimate a VECM and then extract the permanent component in output using a multivariate version of the Beveridge-Nelson (1981) decomposition, hereafter BN. The appeal of this procedure is that it uses recent movements in consumption and investment, as well as in output, to estimate a common permanent component. It is this aspect of the KPSW paper that we investigate and extend here.

We first explore the stationarity of the great ratios in some depth. Re-

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[1]Related ideas can be found in work by Cochrane (1994) and Fama (1992). Also note that for their reported trend/cycle decomposition, KPSW use a six variable system (p. 837).

searchers who have followed KPSW, and extended their work to other countries, have sometimes concluded that the evidence for stationarity of the great ratios is relatively weak.[2] In order to explain this, we use a standard theoretical growth model to show that the great ratios may be subject to occasional mean shifts, leading to rejections of stationarity. In our own empirical work, we implement recent multivariate tests for structural breaks, and use these to specify and test a more complex VECM that allows for structural breaks in the great ratios.

Our approach involves some additional innovations. Our sample includes the New Economy period of the 1990s, and so we have to address the recent adoption of chain-weighting for real aggregates in the US National Income and Product Accounts. We therefore modify the KPSW approach to the construction of the variables. This modification has a second advantage: our empirical strategy is consistent with the implications of a class of two sector growth models, more general than the one sector models typically analysed in this literature.

Using these ideas, we estimate a VECM for both the USA and the UK, between roughly 1955 and 2001. Formal tests indicate the importance of structural breaks within this period, for both countries. Once these breaks are incorporated, there is stronger evidence for the two cointegrating vectors predicted by the theory, using both the multivariate Johansen procedure and single-equation tests. Moreover, these results are robust to the precise choice of break dates. The evidence for unit coefficients is weaker, but the departures from unity are not large in economic terms.

Given the evidence for two cointegrating vectors, estimates of the permanent component in output can be extracted from the estimated VECM. To do this we use a multivariate permanent-temporary decomposition based on the recent work of Gonzalo and Granger (1995) and Proietti (1997). This allows changes in the permanent and transitory shocks to affect changes in the permanent component of the series. Hence the approach is more general than in the BN decomposition, in which the permanent component is a pure random walk. We use the results in Attfield (2003) to modify the Gonzalo-Granger-Proietti decomposition to incorporate structural breaks.

We then show that the multivariate approach to trend measurement is potentially illuminating. Perhaps the most interesting findings relate to the 1990s. Unlike a more standard univariate decomposition, our approach indicates that strong growth was partly due to transitory favourable shocks, as argued by Gordon (2000). Since the late 1990s, however, the joint behaviour of consumption,

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[2]See for example Clemente et al. (1999) and Serletis and Krichel (1995). In the case of the USA, using data until 1998, Evans (2000) finds that the net investment ratio is stationary, but the gross investment ratio is only trend stationary.

investment and output suggests that the common permanent component has grown rather more rapidly than output. Again contrary to a univariate decomposition, our work suggests that the improved productivity performance of the New Economy era may be sustainable.

The remainder of this paper is structured as follows. Section 2 sets out some theoretical considerations. We use a basic growth model to show that variation in structural parameters can give rise to substantial changes in the equilibrium values of the great ratios. This means that tests for stationarity of the great ratios are ultimately testing a joint hypothesis: not only convergence towards long-run balanced growth, but also parameter stability. With this in mind, we allow for structural breaks, and section 3 briefly describes the new econometric results that we use to carry out the trend/cycle decompositions in their presence. Section 4 discusses the data and our empirical strategy, including some important conceptual points raised by chain-weighted real aggregates and changes in relative prices over time. Our empirical results are then presented in the heart of the paper, sections 5, 6 and 7. Section 5 reports the strong evidence for structural breaks, section 6 our estimates of the cointegrating vectors, and section 7 our estimates of the permanent component in output. Section 8 provides some additional discussion and summarizes our main findings.

2   Theoretical considerations

Analysis of long-term movements in the great ratios is usually based on the neoclassical growth model. Within this model, if technical progress is strictly labour-augmenting and occurs at a constant rate, there will usually be a balanced growth path along which output, consumption, capital, and investment all grow at the same constant rate. This implies that the great ratios of consumption to output, and investment to output, are constant in the steady state.

As KPSW pointed out, this property has a natural analogue in models where technical progress is stochastic. When there is a stochastic steady state, the great ratios will be stationary stochastic processes. Certain endogenous growth models also imply stationarity of the great ratios, as in the stochastic version of Romer (1986) analyzed by Lau and Sin (1997). More generally, the stochastic endogenous growth model introduced by Eaton (1981) also admits an equilibrium in which all real quantities grow at the same stochastic rate.

This is perhaps not surprising, since the steady state of a growth model can be thought of as an outcome that can be sustained indefinitely. It follows from a closed economy's aggregate resource constraint ($Y = C + I$) that, if investment

and consumption are always positive, then consumption and investment can only grow at constant rates indefinitely if they both grow at the same rate as output. This also makes good sense from an economic point of view. At least in a closed economy, consumption cannot grow more quickly than output indefinitely, while under standard assumptions, it would rarely be in the interests of consumers to save an ever-increasing fraction of their income.

Hence the conclusion that the great ratios should be stationary appears fairly general, and appears to have useful empirical implications. The long-run restrictions are common to a large class of models, and impose some theoretical structure without being unduly restrictive. Structural VAR modelling based on weak long-run restrictions is often regarded as a promising research strategy, as for example in Soderlind and Vredin (1996).

The generality of the restrictions raises a puzzle, however. In the work that has followed KPSW, researchers have studied countries other than the USA, and have found that stationarity of the great ratios is frequently rejected. Sometimes, this is used as evidence against models of exogenous growth, as in the work of Serletis (1994, 1996) on Canadian data and Serletis and Krichel (1995) for ten OECD countries. Yet as we have seen, the implication that the great ratios are stationary is not unique to such models.

The puzzle can also be seen to some extent in figures 1 and 2, which plot the logs of the great ratios for the USA and UK from 1955 onwards (we discuss the data sources in more detail later). In each figure, the upper line is the log consumption ratio, and the lower line the log investment ratio.[3] The dotted line is the log ratio for each quarter; the dark line is a centred 10-year moving average. These moving averages clearly indicate long swings in the ratios over many years, and indicate that mean reversion is only occurring slowly, if at all. For the USA, there is an upwards trend in the consumption ratio in the 1980s. The finding for the UK is consistent with the work of Mills (2001), who finds that the UK ratios are highly persistent, and the evidence for stationarity mixed.

Why does mean reversion appear to be so slow? As noted above, we address this puzzle by arguing that the great ratios are likely to be subject to periodic mean shifts, or structural breaks. Our view of the ratios is that, even if the majority of shocks to them are temporary, there may be occasional permanent shocks that reflect changes in underlying parameters.

In the remainder of this section, we use a simple growth model to illustrate this dependence of the great ratios on underlying structural parameters. These

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[3]The log investment ratio is more volatile because reallocating 1% of GDP from consumption to investment has a greater proportional effect on the investment ratio, given that investment accounts for a much smaller share of GDP than consumption.
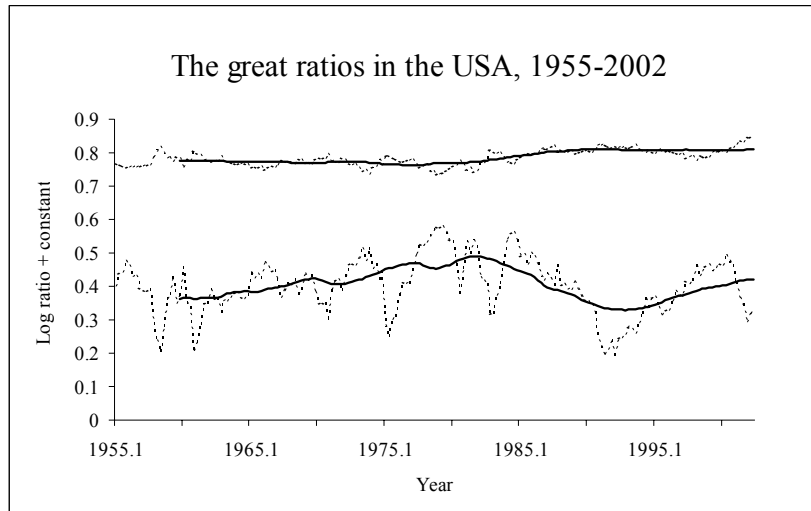
Figure 1: A plot of the log great ratios for the USA. The upper line is the log consumption ratio, and the lower line the log investment ratio. Constants have been added to the log ratios to facilitate graphing.
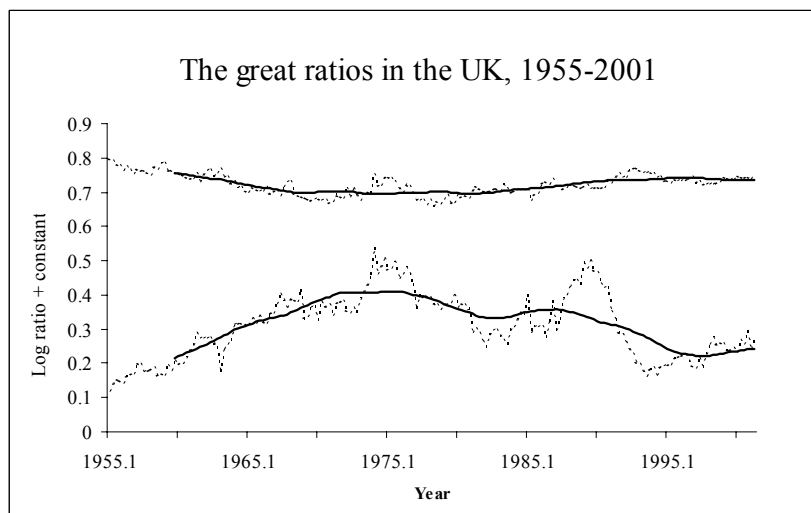

Figure 2: A plot of the log great ratios for the UK. The upper line is the log consumption ratio, and the lower line the log investment ratio. Constants have been added to the log ratios to facilitate graphing.

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parameters include the rate of technical progress, the depreciation rate, the intertemporal elasticity of substitution, the subjective discount factor, and the share of capital income in total income. We briefly document a few reasons to believe that some of these parameters have changed over time. Our analysis shows that plausible variations in these parameters can have substantial effects on the great ratios, and could therefore lead to structural breaks and spurious rejections of stationarity.[4]

Our theoretical analysis builds on the classic analysis of the stochastic growth model due to King, Plosser and Rebelo (1988), henceforth KPR, and on the widely circulated technical appendix to that paper. Since the analysis is now standard, we introduce a simplified version of it only briefly, and then investigate the sensitivity of the great ratios to changes in the underlying parameters.

We should emphasize that we adopt this model without making strong claims for its descriptive accuracy. Our aim is to highlight some of the determinants of the great ratios, and investigate the magnitudes of the associated effects, to support our overall claim that sizeable shifts in the ratios are possible.[5] The setup we adopt is perhaps the simplest interesting growth model in which the ratios are determined endogenously. It should be remembered, however, that the balanced growth restrictions are general to a much wider class of models.

The model is one of many identical agents, who each supply one unit of labour inelastically (this latter restriction is easily generalized). The representative agent seeks to maximise lifetime utility:

$$
\begin{aligned}
U &= \sum_{t=0}^{\infty} \beta^t \frac{C(t)^{1-\sigma}}{1-\sigma} \text{ if } 0 < \sigma < 1 \text{ or } \sigma > 1 \\
&= \sum_{t=0}^{\infty} \beta^t \log C(t) \text{ if } \sigma = 1
\end{aligned}
\tag{1}
$$

where the parameter σ is the inverse of the intertemporal elasticity of substitution. We will later impose a restriction on β to ensure that lifetime utility is finite.

The agents (or firms) each produce a single good using a constant returns to scale production function

$$
Y_t = F(K_t, X_t N_t)
\tag{2}
$$

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[4]As Cooley and Dwyer (1998) and Soderlind and Vredin (1996) indicate, changes in structural parameters will also have implications for the short-run dynamics. Investigation of this point is beyond the scope of the current paper, however.

[5]See Parker (1999, p. 325-326) for a related analysis. He considers the determinants of the great ratios in a continuous time Ramsey model, but does not explicitly quantify the effects.

where X_t is an index of labour-augmenting technical change that evolves over time according to

$$X_{t+1} = \gamma X_t$$

Hence the growth rate of the technology index is constant and given by $\gamma - 1$. As is standard in long-run analyses of growth models, we are basing our investigation on a deterministic steady state.

The single good can be consumed or invested, and the evolution of physical capital is hence given by

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

where δ is the rate of depreciation. The aggregate resource constraint is

$$C_t + I_t \leq Y_t \tag{4}$$

Since all agents are identical, there is no intertemporal trade. Hence each agent maximises lifetime utility (1) subject to the infinite sequences of constraints implied by equations (2)-(4). Using the arguments in King, Plosser and Rebelo (forthcoming) there will be a steady state growth path in which consumption, investment, capital and output all grow at the same rate as technology, namely $\gamma - 1$.

It can also be shown that the real rate of return on capital, net of depreciation, is given by

$$r = \frac{\gamma^\sigma}{\beta} - 1$$

We need to impose a restriction on β to ensure that lifetime utility is finite, namely

$$\beta \gamma^{1-\sigma} < 1 \tag{5}$$

which implies that the real return on capital is higher than the long-run growth rate.

It can then be shown that the ratio of gross investment to output is given by:

$$s_i = \frac{[\gamma - (1-\delta)]\,\beta\gamma^{1-\sigma}\alpha}{\gamma - \beta\gamma^{1-\sigma}(1-\delta)} \tag{6}$$

where the new parameter α denotes the share of capital income in total income. Given our assumptions, the gross investment ratio is exactly equal to the saving ratio, or one minus the ratio of consumption to output. (We will discuss this point in more detail later in this section.)

What are the implications for research on the great ratios? The first point to note is that the investment ratio depends on the long-run growth rate γ unless two conditions are met: logarithmic utility ($\sigma = 1$) and complete depreciation of capital within each period ($\delta = 1$). The second condition in particular is clearly unrealistic, and so in general the investment ratio will be a function of the rate of technical progress.[6] Any change in that rate, such as the productivity slowdown of the 1970s, has implications for the steady-state investment ratio.

The investment ratio also depends on the subjective discount factor (β), the intertemporal elasticity of substitution ($1/\sigma$), the capital share α and the depreciation rate δ. Can we say anything about the direction of these effects? If utility is logarithmic ($\sigma = 1$) it is straightforward to show that the ratio is increasing in all four remaining parameters α, β, δ and γ. If utility is not logarithmic, the analysis is less straightforward, but analytical results are still possible. As before, differentiation indicates that the investment ratio is increasing in α, β and δ.[7] The investment ratio is also increasing in the intertemporal elasticity of substitution (in other words, decreasing in σ). On the other hand, the effect of the long-run growth rate γ is ambiguous without further assumptions.

We next investigate whether the impact of parameter changes on the great ratios is likely to be quantitatively important. To gain some insight into this question, we repeatedly plot the function (6) allowing two parameters to vary and holding the other three constant at default values. We base our default parameter values mainly on the work of KPR (their Table 1) and define our parameters in quarterly terms. The default value for γ is 1.004, implying an annual growth rate of 1.6%. We set the discount factor β to 0.99 which is broadly consistent with the values implicit in KPR's simulations, and implies sensible real returns to capital for most of the combinations of γ and σ that we consider in the plots. (It also ensures that lifetime utility is finite for the parameter values we consider.) We follow KPR in setting the capital share to 0.42 and the quarterly depreciation rate to 0.025, where the latter implies annual depreciation of 10%. Our default value for σ is 2. Overall, evaluating equation (6) at these default parameter values implies an investment ratio of around 28%.

First of all, we study the effects of the trend growth rate and the utility parameter σ on the investment ratio. To do this, we vary the annual growth rate between 0% and 4%, corresponding to values of the quarterly (gross) growth rate γ between 1.00 and 1.01. We vary σ, the inverse of the elasticity of intertemporal

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[6]This long-run solution of the model treats technical progress as deterministic. In our later empirical work, we will treat the trend in output as stochastic rather than deterministic. In that case, the great ratios may be functions of the rate of drift in output.

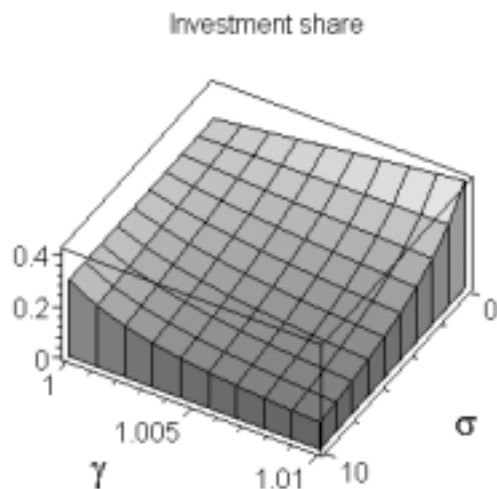[7]Some of these results make use of the parameter restriction (5).

Figure 3: Sensitivity of the investment ratio to variation in the trend growth rate and a utility parameter

substitution, between 0.001 and 10. The results are shown in Figure 3. Importantly, the exercise reveals that the steady-state investment ratio can be quite sensitive to the trend growth rate unless σ is close to unity. The direction of the effect depends on the value of σ. For values of σ below 1, the investment ratio is increasing in the trend growth rate. For higher values of σ, the investment ratio is decreasing in the trend growth rate.

We carry out similar exercises where the variation in σ is eliminated and one of the other parameters is varied within a plausible range, while setting σ to its default value, namely $\sigma = 2$. These plots are shown as Figures 4 through 6. In these figures, too, the investment ratio is clearly quite sensitive to parameter changes.

These findings have a clear implication: even if the majority of shocks to the great ratios are transitory, there is clearly some potential for occasional changes in parameters to shift the great ratios, in such a way that they could appear non-stationary using standard tests. In the remainder of this section, we briefly discuss the potential for changes in the relevant parameters.

The case for quite substantial changes in the trend growth rate is clear. The 1970s saw a well-documented productivity slowdown across the developed world, with intermittent improvements in performance in the following decades. More formally, Ben-David and Papell (1998, 2000) have compiled evidence of secular changes in long-term growth rates. The evidence for these changes is weaker for the UK and USA, but the 1990s have seen faster trend growth in the USA, the

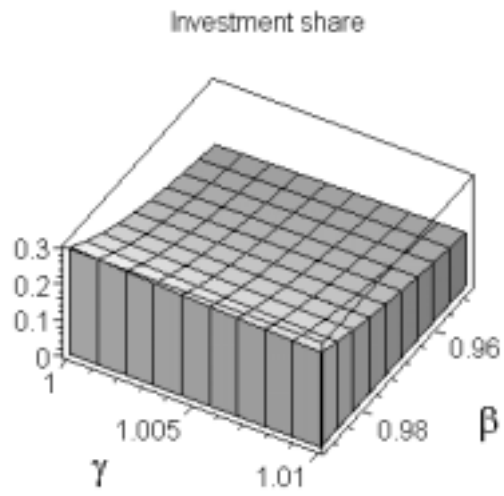Investment share

Figure 4: Sensitivity of the investment ratio to variation in the trend growth rate and the discount factor
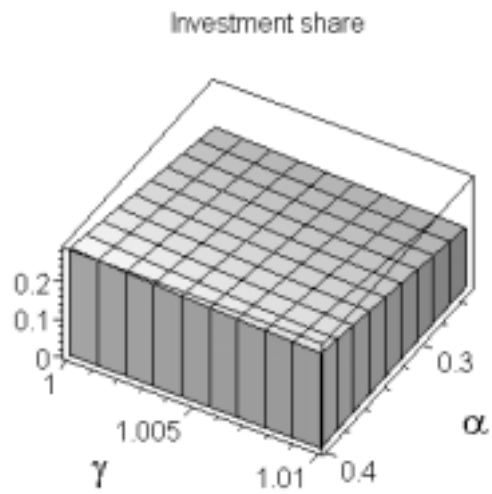
Investment share

Figure 5: Sensitivity of the investment ratio to variation in the trend growth rate and the capital share
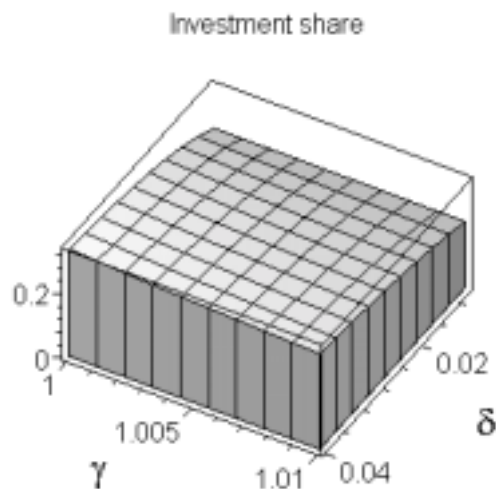
Figure 6: Sensitivity of the investment ratio to variation in the trend growth rate and the depreciation rate

evidence for which is summarized in Temple (2002).

The share of capital income has shown some variation over time in some OECD countries, as pointed out by Blanchard (1997). This has to be interpreted carefully, however, as in the above model the capital share is only constant along the long-run balanced growth path, or if the production technology is Cobb-Douglas.

Less obviously, but perhaps more importantly, one could make a strong case for a change in the rate of depreciation. Evans (2000) points out that the depreciation rate implicit in the US National Income and Product Accounts has risen substantially over time, reflecting a change in the composition of the capital stock towards equipment and away from structures. Tevlin and Whelan (2000) also show that the composition of the capital stock is tending to shift towards assets with shorter service lives, as investment in equipment (particularly computers) assumes increasing importance. The analysis above indicates that a rise in the depreciation rate will tend to raise the equilibrium ratio of gross investment to output.

It is less conventional to make a case that the 'deep' parameters relating to preferences (β and σ in this model) have changed. Even here, though, periodic shifts may be possible. Moving away from models of infinitely-lived representative agents, the constancy of preference parameters appears less persuasive in a world of overlapping generations, since different cohorts may not look exactly alike in their preferences. Although a parameter such as the discount factor may

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be roughly constant over a decade or more, we have less reason to assume this over the relatively long time span considered in this paper. Parker (1999) argues that a rise in the effective discount rate is a possible explanation for the decline in the US saving ratio after 1980.

We briefly consider one final point in relation to the great ratios, and the relationship between the theoretical framework and empirical testing. In a closed economy, the gross investment ratio is essentially the mirror image of the ratio of consumption to output, and stationarity of one ratio necessarily implies stationarity of the other. In the data we use, however, household consumption and private sector investment do not sum to private sector output, mainly because of the current account. We therefore follow previous authors, including KPSW, in looking for stationarity in both ratios. The more ambitious task, of extending the KPSW framework to open economies, is one that we are pursuing in further research.[8]

3   Permanent-temporary decompositions

In this section we describe the first part of the empirical strategy we adopt, namely permanent-temporary decompositions that incorporate the possibility of structural breaks in the cointegrating equations. These breaks have implications for the estimation of the VECM, and for the extraction of the permanent component from the estimated model. The permanent-temporary decompositions that we implement empirically use new results developed in Attfield (2003), and we briefly spell out the main details below.

As in KPSW we consider a three variable system based on consumption C_t, investment, I_t and output, Y_t (KPSW also consider larger systems). Let c_t, i_t and y_t be the natural logarithms of consumption, investment and output variables respectively, and let $x'_t = (c_t, i_t, y_t)$. We will discuss the precise construction of these series in the next section.

We first consider the case without structural breaks. If x_t is $I(1)$ then we can write the VECM as:

$$\triangle x_t = \theta_o + \theta_1 \triangle x_{t-1} + ... + \theta_k \triangle x_{t-k} + \beta\alpha' x_{t-1} + \zeta_t \qquad (7)$$

where $\triangle x_t = x_t - x_{t-1}$, ζ_t is a Gaussian error and α' is the set of cointegrating vectors. There are T observations in total.

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[8]For existing work along these lines, see DeLoach and Rasche (1998) and Mellander et al. (1992). Alternative long-run restrictions in open economies are considered by Garratt et al. (2003). Daniel (1997) uses the Johansen procedure to study international interdependence in productivity growth.

If there are structural breaks in the mean of either the VECM or the cointegrating relations, then the specification in (7) is inappropriate. It will also be inappropriate if there are shifting trends in the cointegrating equations.[9] To address this problem, suppose there are two breaks in the sample with T_1 observations in the first period, $T_2 - T_1$ observations in the second period, and $T - T_2$ observations in the third period. Johansen et al. (2000) derive a likelihood ratio test for cointegration in the presence of breaks in trend and mean at known points, and that is the test we will implement below.

The VECM with structural breaks can be written as:

$$\triangle x_t = \theta_o \Xi_t + \sum_{j=1}^{k} \theta_j \triangle x_{t-j} + \beta(\alpha', \gamma') \begin{pmatrix} x_{t-1} \\ t\Xi_t \end{pmatrix} + \sum_{i=1}^{k+1} \sum_{j=2}^{3} \kappa_{ji} D_{jt-i} + \zeta_t \quad (8)$$

where $x_t = (c_t, y_t, i_t)'$, $\theta_o = (\theta_{o1}, \theta_{o2}, \theta_{o3})$, $D_{jt} = 1$ for $t = T_{j-1}$, with $T_o = 0$, and $D_{jt} = 0$ otherwise and $\Xi_t' = (\Xi_{1t}', \Xi_{2t}', \Xi_{3t}')$ with $\Xi_{jt} = 1$ for $T_{j-1} + k + 2 \le t \le T_j$ and zero otherwise.

The $\Xi_{jt}s$ are dummies for the effective sample period for each sub-period. The $D_{jt-i}s$ have the effect of eliminating the first $k + 1$ residuals of each period from the likelihood, thereby producing the conditional likelihood function given the initial values in each period. Hence this specification allows for shifts in the intercepts of both the VECM and the cointegrating equations, although such shifts cannot be identified individually. These intercept corrections are captured in the term $\theta_o \Xi_t$. The model also allows for shifts in any time trends in the cointegrating equations, in the term $\gamma' t \Xi_t$.

Once the model (8) has been estimated, we can extract estimates of the permanent component in the series using either the BN decomposition or the generalization of it due to Gonzalo and Granger (1995) and Proietti (1997). Our implementation of these decompositions requires some new results, however, in order to incorporate structural breaks.

The definition of the BN permanent component in a multivariate context is:

$$x_t^{BN-P} = x_t + \sum_{i=1}^{\infty} E_t(\Delta x_{t+i} - \mu_{\Delta x}) \quad (9)$$

as in Cochrane (1994) for example. To determine a solution for (9), write the VECM in (8) as

$$\Delta x_t = \mathcal{K}_o H_t + \sum_{j=1}^{k} \theta_j \Delta x_{t-j} + \beta v_{t-1} + \zeta_t. \quad (10)$$

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[9]We exclude a linear time trend in the VECM as it would imply a quadratic trend in the levels of the variables.

where $\mathcal{K}_o = (\theta_o, \varkappa)$ where $\varkappa$ contains the κ_{ji} vectors, and:

$$H_t = \begin{bmatrix} \Xi_t \\ D_1 \end{bmatrix}$$

where D_1 contains the $D_{jt-i}s$, and $v_{t-1} = \alpha' x_{t-1} + \gamma' t \Xi_t$. It follows that:

$$v_t = \alpha' x_t + \gamma'(t+1)\Xi_t = \alpha' \Delta x_t + \gamma' \Xi_t + v_{t-1}$$

and then:

$$v_t = \mathcal{K}_{oo} H_t + \alpha' \theta_1 \Delta x_{t-1} + \cdots + \alpha' \theta_k \Delta x_{t-k} + \left(I + \alpha' \beta \right) v_{t-1} + \alpha' \zeta_t \qquad (11)$$

where:

$$\mathcal{K}_{oo} = \left(\alpha' \theta_o + \gamma', \alpha' \varkappa \right).$$

Appending (11) to the system in (10) we have a first order stationary vector autoregression of the form:

$$z_t = A_o H_t + A_1 z_{t-1} + \Psi \zeta_t \quad t = 1, \ldots, T \qquad (12)$$

where z_t' is the $(1 \times pk + r)$ vector:

$$z_t' = (\Delta x_t', \Delta x_{t-1}', ..., \Delta x_{t-k+1}', v_t').$$

The matrices A_o and A_1 are defined as:

$$A_o = \begin{bmatrix} \mathcal{K}_o \\ 0 \\ 0 \\ \vdots \\ 0 \\ \mathcal{K}_{oo} \end{bmatrix}$$

and:

$$A_1 = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_{k-1} & \theta_k & \beta \\ I & 0 & \cdots & 0 & 0 & 0 \\ 0 & I & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 & 0 \\ \alpha' \theta_1 & \alpha' \theta_2 & \cdots & \alpha' \theta_{k-1} & \alpha' \theta_k & \alpha' \beta + I \end{bmatrix} \qquad (13)$$

and Ψ is defined as:

$$\Psi = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \\ \alpha' \end{bmatrix}.$$

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From (12) it follows that:

$$E(z_t) = \mu_z = (I - A_1)^{-1} A_o H_t$$

so that:

$$z_t - \mu_z = (I - A_1 L)^{-1} \Psi \zeta_t. \tag{14}$$

Define the matrix:

$$G = \begin{bmatrix} I_p \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Then $G' z_t$ selects out Δx_t and it follows from (12) that:

$$\Delta x_t - \mu_{\Delta x} = G'(z_t - \mu_z) = G'(I - A_1 L)^{-1} \Psi \zeta_t = C(L) \zeta_t \tag{15}$$

which is the moving average representation. Inverting $[I - A_1]$, it is straight-forward to show that[10]:

$$C(1) = G'[I - A_1]^{-1} \Psi = \theta(1)^{-1} - \theta(1)^{-1} \beta (\alpha' \theta(1)^{-1} \beta)^{-1} \alpha' \theta(1)^{-1}$$

where $\theta(1) = I_p - \sum_1^k \theta_i$.

The expectations term in equation (9) can then be written as:

$$\sum_{i=1}^{\infty} E_t(\Delta x_{t+i} - \mu_{\Delta x}) = G' A_1 [I - A_1]^{-1} (z_t - \mu_z). \tag{16}$$

Some algebra (see Attfield 2003) produces:

$$x_t^{BN-P} = C(1)\theta(L) x_t - Q\gamma'(t+1)\Xi_t + \delta_o \tag{17}$$

where $\theta(L) = I_p - \sum_{j=1}^k \theta_j L^j$; $Q = \theta(1)^{-1} \beta (\alpha' \theta(1)^{-1} \beta)^{-1}$; and $\delta_o = -C(1)\theta^*(1)\mu_{\Delta x} + Q\mu_v$ with $\theta(L) = \theta(1) + (1-L)\theta^*(L)$ and where the population means $\mu_{\Delta x}$ and μ_v of the stationary variables Δx_t and v_t can be estimated by their sample counterparts. Definitions of the multivariate BN permanent component equivalent to (17) are used by KPSW and by Cochrane (1994) for the case of no structural breaks.

As noted by KPSW, the BN decomposition is a natural one in that the permanent component represents the long-run forecast of the series. However,

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[10]Proietti [43, 1997] obtains the same result using the Kalman filter except that instead of $\Theta(1)^{-1}$ he has $(\Theta(1) - \beta\alpha')^{-1}$. It is easy to show that the two forms give exactly the same $C(1)$.

the BN specification for the permanent component has sometimes been criticised, as in Blanchard and Quah (1989) and Lippi and Reichlin (1994), because it does not contain any dynamics in the permanent and transitory shocks. To remedy this problem, Gonzalo and Granger (1995), suggest a method of decomposing the series into permanent and transitory components in which the permanent component incorporates some dynamics.

Proietti (1997) noticed that the Gonzalo-Granger decomposition can be obtained as a relatively simple extension of the BN decomposition by substituting $\theta(1)$ for $\theta(L)$. In the context of the model in (17) this gives the permanent, or stochastic trend, component as:

$$x_t^P = C(1)\theta(1)x_t - Q\gamma'(t+1)\Xi_t + \delta_o$$

which is the decomposition we use in the empirical section of the paper.

4 Our empirical strategy

This section describes our construction of the data set, including our measure of private sector output. It also explains some important conceptual issues that arise in moving between the analytically convenient one sector world of the growth models, and the more complex processes that generate the data we use. The final data set we adopt is one of quarterly, seasonally adjusted data for 1955Q1 until 2001Q2 (for the UK) or 2002Q2 (for the USA). Following KPSW, our measure of output excludes government expenditure.

For the USA, we need to take into account the recent introduction of chain-weighted quantity and price indices in the National Income and Product Accounts, and heed the important warnings of Whelan (2000a) in this regard. When real quantities are chain aggregates, the components of GDP can have unfamiliar properties. In particular, real GDP is no longer the standard sum of real components, so Y does not equal C+I+G+X-M when all variables are measured in real, chain-weighted terms. This is because real output is no longer defined as the sum of the expenditure components all evaluated at a constant set of relative prices. Although this lack of additivity may seem rather mysterious, it is a direct consequence of using the chain-weighted growth rates of the series to construct measures of levels.[11]

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[11]Whelan (2000a) provides an excellent summary of the justification for the chaining procedure, and its implications for empirical work, including the resulting lack of additivity of expenditure components. Note that chain-weighted indices are also being introduced in the UK National Accounts.

The difference between a chain quantity aggregate and the more familiar fixed-weight aggregate emerges when relative prices are changing. This can raise problems for analysing the great ratios in what appear to be real terms. It might seem obvious that one should deflate nominal consumption and nominal investment by specific price indices (a consumption deflator and an investment deflator respectively). Yet the resulting figures for real consumption and real investment will not sum to real private sector output if chain aggregates are used. Whelan (2000a) argues that ratios of real chain-aggregated series usually do not make sense.

Moreover, when relative prices are changing, it is not clear how one should interpret the ratio between a series like real consumption and real output, or whether these "real shares" are economically meaningful, even when fixed-weight methods are used. Whelan (2000a) writes:

> The inability to calculate real shares with chain-aggregated data could be viewed as a disadvantage. It is important to note, though, that even when using the fixed-weight methodology, real shares are an elusive concept. The ratio of real year-b dollar output of product i to real year-b dollar fixed-weight GDP answers the following question: "Suppose all prices had remained at year-b's level; what proportion of the total value of this year's output would have been accounted for by the output of product i?". Clearly, the answer depends on the base year chosen... (Whelan 2000a, p. 11-12).

Whelan's discussion indicates that real shares (ratios of real investment to real output, for example) are problematic concepts, even when the data are constructed using fixed weights. It is rare that one needs to know what the ratio of investment to output would have been, if only relative prices had remained at earlier values. As Whelan shows with some examples, the choice of base year can make enormous differences to the calculated shares, confirming that the empirical usefulness of real shares may be limited.

A further point suggests that one should be wary of real shares in the present context. Note that in the simple model discussed in section 2, the division of income between consumption and investment is determined by the representative consumer. The consumer's decision problem relates to the fraction of income to be devoted to investment in each period, which is a sequence of decisions about a nominal share. It would make little sense for the representative consumer to frame her decisions in terms of a fixed-weight real share, since then her choices would be affected by the base year used to compute the share.

Most empirical tests of the great ratios are based on real shares. These tests are typically motivated by one sector frameworks, in which there is no role for changing relative prices of capital goods, for example. In a one sector world, there is no substantive distinction between the nominal investment share and the ratio of real investment to real GDP. In a two sector world, in which the relative price of capital goods can change, the distinction matters. Real investment may grow at a different rate to real consumption indefinitely, and the great ratios need not be stationary when expressed in real terms.[12]

With this in mind, and motivated by recent growth models, we focus on stationarity in the *nominal* ratios. The two sector model of Greenwood et al. (1997) has the property that nominal consumption and nominal investment grow at the same rate as nominal output along a balanced growth path. The model of Whelan (2000a) has a similar property. We therefore base our empirical work on the ratios of investment to output, and consumption to output, all measured in current prices.

The assumptions which give rise to constant nominal ratios in these models are admittedly quite restrictive. This should be a warning that, in two sector models, the idea that the great ratios are stationary is less appealing than in a one sector framework. Nevertheless, our use of nominal ratios means that we are testing a more general version of the balanced growth hypothesis than previous work. To see why this approach is a generalization, note that if one is willing to accept the maintained assumptions of a one sector growth model (as in KPSW) then the distinction between real and nominal ratios is immaterial, and our approach requires no additional assumptions beyond those of previous research in this field.[13] Yet, by casting the test in terms of nominal ratios, our approach is also consistent with an existing class of two sector growth models. We do not know of any two sector models in which a declining relative price of capital goods is associated with a secular downwards trend in the nominal investment ratio.

The decision to focus on the nominal ratios also means that we can easily overcome the lack of additivity in the chain-weighted real aggregates. By using nominal series instead, we can construct a measure of nominal private sector output by subtracting nominal government expenditure from nominal output. This would not be possible when working with the variables in real terms, be-

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[12]For example, consider a simple example in which a constant share of nominal GDP is invested in each period. If the relative price of capital goods is declining, real investment grows more quickly than real consumption.

[13]To put this slightly differently, if the strong assumptions of a one sector model were genuinely met in the data, then looking at the nominal shares would give the same answers as the more conventional focus on real shares.

cause real variables constructed using chain-weighted indices cannot simply be added or subtracted in this way, but have to be reaggregated from their separate components. Hence our approach has two considerable strengths: it is potentially consistent with a broader class of models than previous research, and the construction of a measure of private sector output does not encounter the additivity problems associated with chain-weighted real aggregates.

There is one clear problem with the use of nominal series. If the log of the price level is I(2), the series for logs of nominal output, consumption and investment will also inherit this I(2) property. To avoid this problem we follow Greenwood et al. (1997, p. 347) in deflating all the nominal series by the same consumption-based price deflator. We also divide all three series by the size of the population in each quarter, so our analysis is entirely based on per capita quantities, exactly as in KPSW (see their footnote 5).

With these points in mind, our construction of the data proceeds as follows for both countries. We obtain series for nominal GDP, consumption, investment, and government expenditure. We subtract government expenditure from GDP to obtain a measure of nominal private sector output. We then divide this series, and those for consumption and investment, by the implicit price deflator for personal consumption expenditure, and by population. The resulting three variables are the measures of Y, C and I that we will use for testing the stationarity restrictions implied by growth theory. We provide full details of the data sources in the data appendix.

5　The evidence for structural breaks

In this section and the next, we seek to estimate a three variable VECM for the USA and the UK using quarterly, seasonally adjusted data for the period 1955Q1 to 2002Q2 for the USA and 1955Q1 to 2001Q2 for the UK. The regressions are run over slightly shorter periods to allow for initial conditions. As noted above, for each country, each of the three series (consumption, investment and output) is defined in per capita terms and deflated by the same consumption-based price deflator. We denote the natural logarithms of these variables by lower case letters (c, i, y). All computations were carried out in GAUSS (2001).

The empirical analysis is relatively involved, and so we first provide an overview of this section and the next. Our first step is to examine the order of integration of each series. We then examine the evidence for stationarity of the great ratios without allowing for structural breaks, and show that the evidence for stationarity of both ratios is mixed at best, especially when using the

Johansen procedure.

We then investigate whether this result is due to structural breaks, using recently developed tests that identify possible break points and calculate confidence bands for the break dates. Applying these tests to our data, we find strong evidence of structural breaks. In the next section we are able to confirm that the evidence for stationary great ratios is stronger when structural breaks are taken into account, and this result is not sensitive to the precise break dates.

A necessary preliminary is to examine the order of integration of each series. For both countries, the null of a unit root cannot be rejected for any of the variables when using standard ADF tests. We also implement a more rigorous test that allows for structural breaks. For both countries, each variable was tested using the procedure of Banerjee et al. (1992) which allows for a break in the intercept (a mean shift) or a change in the slope of a deterministic trend (a trend shift).

For all the variables the null of a unit root is not rejected at conventional levels, even when allowing for structural breaks. For the USA the test statistics for c_t, i_t, and y_t, allowing for mean shifts were -4.78 (3), -4.37 (1) and -4.27 (1) using BIC to choose the lag length, reported in parenthesis. Critical values were obtained from Banerjee et al. (1992). The 5% critical value is -4.8. When allowing for trend shifts, the test statistics were -4.55 (3), -4.13 (1) and -4.12 (1). The critical value for a shift in trend is -4.48. For the weakest of these results, c_t, at any other choice of lag length from 0 to 5 the null of a unit root could not be rejected. For the UK for c_t, i_t, and y_t the test statistics allowing for mean shifts were -3.54, -3.72 and -3.66 and for trend shifts the test statistics were -2.89, -2.60 and -3.10. The BIC selected zero lags for all cases but the null of a unit root could not be rejected at any other lag length from 1 to 5 either. Hence we treat the vector x_t as $I(1)$ for both countries in the empirical work that follows.

The KPSW arguments imply that log consumption and log investment should be cointegrated with log output, with coefficients of unity in the cointegrating vectors. The simplest way to test this is to impose the unit coefficients and use single-equation unit root tests on the log ratios. These tests usually fail to reject the null of a unit root (detailed results not reported). Since our theoretical prior is that the ratios are stationary, we have also used the Kwiatkowski et al. (1992) procedure, henceforth KPSS, which tests the null of stationarity against the alternative of a unit root. For the USA, we could reject stationarity at the 5% level for the log consumption ratio with a test statistic of 0.838 but not for

the log investment ratio with a test statistic of 0.141.[14] The 5% critical value is 0.463 from KPSS (p. 166). For the UK the KPSS results are better: we could not reject stationarity for either of the great ratios with test statistics of 0.287 for the consumption ratio and 0.248 for the investment ratio.

We have also tested the stationarity hypothesis using the standard Johansen (1995) maximum likelihood procedure for estimating the cointegrating rank, again without assuming any structural breaks. We do not give all the results here but for each country we tested for cointegration in models with (i) restricted intercepts but no trends; (ii) unrestricted intercepts; (iii) unrestricted intercepts plus restricted trends.

For the USA, using the trace statistic, there was evidence for only one cointegrating vector at the 5% level under specifications (i) and (iii). For specification (ii) there was some evidence for two cointegrating vectors.[15] Under this specification the model has intercepts in the cointegrating equations only so that in (7) the intercept is $\theta_o = \beta \alpha'_o$, where α_o is the vector of intercepts in the cointegrating equations. With this model, however, the unrestricted coefficient on log output in the log investment equation is much higher than unity, at 2.46 with a standard error of 0.28. As this might suggest, a likelihood ratio test easily rejects the null of unit coefficients in the cointegrating vectors, with a test statistic of 13.34 and a p-value of 0.001.

Overall, these findings conflict with the results of KPSW, who found much stronger evidence for two cointegrating vectors with unit coefficients for the USA. Note that we are considering a more recent time period, 1955Q1-2002Q2 rather than the 1949Q1-1988Q4 period in KPSW. A time period closer to ours is considered in Bai, Lumsdaine and Stock (1998), whose sample ends in 1995Q4. They note (their footnote 11) that there is some evidence for highly persistent shifts in the share of output allocated to consumption, and possibly investment. This is consistent with our own findings from the KPSS tests, and the observed decline in the US saving ratio. These long-lived shifts in the great ratios may explain why the evidence for stationarity is relatively weak when applying the Johansen procedure to recent US data.

The results for the UK from the Johansen procedure also tend to reject stationarity of the great ratios. Under all three specifications, there was evidence for at most one cointegrating vector at the 5% level when using the trace statistic.

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[14]We also considered the results when including a time trend. This did not alter our conclusions.

[15]With one lag first difference in the VECM, as selected by BIC, the trace test statistic was 30.5 for the null of one cointegrating vector against the alternative of two, with a 5% critical value of 19.96. The critical values are from Osterwald-Lenum (1992).

This multivariate result for the UK is consistent with the work of Mills (2001), who found that the existence of two cointegrating vectors with unit coefficients could be rejected for the UK when using the Johansen procedure.[16]

It may seem surprising that the evidence for stationarity of the great ratios is not stronger. As argued previously, one reason for this result could be structural breaks in the great ratios, which make them appear non-stationary. We believe that a plausible process for the great ratios would be one in which the majority of shocks are transitory, combined with occasional mean shifts as the determinants of the ratios change.

With this in mind, we examine the case for stationarity when we adopt the generalized VECM formulation in (8) and test for cointegration allowing for structural breaks. The first step is to identify the break points in the system. There are a number of papers which suggest methods for finding break points in single equation cointegrating models, with well-known examples including Gregory and Hansen (1996) and Bai and Perron (1998). Recently Bai, Lumsdaine and Stock (1998), hereafter BLS, have provided a method for estimating confidence bands for break dates in multivariate systems. Importantly, they argue that tighter confidence bands can be obtained from a multivariate approach, and it is their technique that we emphasize here.

The BLS method assumes a system of the form of (7) with given cointegrating vectors and estimates a confidence interval for a shift in the intercept in the VECM. Their model is the same as the specification in (8) when there is one mean break and the $\kappa_{ji} = 0$ and $\gamma' = 0$. There are no trends in the cointegrating equation, and the model is similar to one with a break in a restricted intercept (that is, a model with an intercept, and shift in intercept, in the cointegrating equation only). The BLS test procedure is clearly a leading candidate for identifying structural breaks in a model such as ours, especially given that we have a strong prior on the cointegrating equations. Stationarity of the great ratios implies the following matrix of cointegrating vectors when the variables are ordered, c_t, i_t and y_t:

$$\alpha' = \left[\begin{array}{ccc} -1 & 0 & 1 \\ 0 & -1 & 1 \end{array} \right].$$

Our strategy for locating break points was to apply the BLS multivariate test over the whole period with the α matrix constrained as above, and allowing the lag length in the VECM to be selected by the BIC. Having located one break

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[16]He also presented the results of some alternative testing procedures, which provided stronger evidence for stationarity. We will show below that the Johansen procedure also indicates two cointegrating vectors, if structural breaks are incorporated.

point we then examined periods before and after the first break date, in order to locate any second structural break. We limit the number of breaks to two partly because, with the relatively small sample size available, allowing for more than two breaks would tend to blur the distinction between our null hypothesis (a stationary process with infrequent mean shifts) and a non-stationary process. This choice also simplifies the analysis, especially as we used the Johansen et al. (2000) test statistic for testing for the rank of the cointegrating space subject to structural shifts, and critical values for this test statistic are currently only available for a maximum of two breaks.

For the USA for the whole sample the BLS procedure located 1982Q1 as a possible break point with a 90% confidence region of (1979Q3, 1984Q3).[17] This break date closely coincides with the start of the long-term decline in the US saving ratio, typically dated around 1980 (Parker 1999). For the period 1955Q1 to 1978Q1, prior to the lower confidence limit for the first break, the BLS test indicated no significant break. For the period 1985Q1 to 2002Q1 there was a highly significant break at 1998Q2 with a 90% confidence region of (1997Q4, 1998Q4).[18]

As a check, we have also applied the univariate procedures due to Bai and Perron (2001). Using the full sample their SupF statistic identified two breaks, at 1976Q4 and 1989Q2, for the log investment ratio and only one break, at 1985Q1, for the log consumption ratio.[19] For the period up to 1978Q1 there were no significant breaks in the log consumption ratio but a break at 1964Q4 for the log investment ratio. Compared to the BLS procedure, the different outcomes indicate that dating structural breaks is an inexact science. We will base our later analysis on the BLS point estimates of the break dates, but will also examine robustness to alternative choices within the estimated confidence bands.

For the UK, the different procedures are in much closer agreement on possible break dates. For the whole sample the BLS procedure located a significant break point at 1990Q3 with 90% confidence region of (1987Q4, 1993Q2). For the sample period up to and including 1986Q1 there was a break at 1963Q3 with 90% confidence region (1962Q4, 1964Q2).[20] The period after 1993Q2 is

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[17]The Sup-W and lExp-W test statistics were 11.77 (12.58) and 3.96 (3.63) where the 10% critical values are in brackets. The lExp-W test statistic is significant at the 10% level, while the Sup-W statistic is approaching significance at the 10% level. Critical values were obtained by a simulation similar to those implemented in BLS.

[18]The Sup-W and lExp-W test statistics were 29.18 (18.39) and 11.56 (6.10) which are both significant at the 1% level.

[19]The Bai and Perron test procedures produce a battery of test statistics which are not reported here but can be obtained from the authors.

[20]For the first break the Sup-W and lExp-W test statistics were 15.65 (14.44) and 4.74 (4.43).

too short to investigate structural breaks, so we take 1963Q3 and 1990Q3 as our candidate break dates. These results are strongly reinforced by applying the simpler univariate procedures of Bai and Perron (2001) to the full sample: their SupF statistic identified two breaks, at 1963Q3 and 1990Q3 for the log consumption ratio, and 1963Q4 and 1991Q2 for the log investment ratio.

Tables 1(a) and 1(b) summarise the point estimates and confidence bands for the break dates, as obtained by the BLS procedure, for the two economies.

Table 1(a). Break Points for the USA		
90% Lower bound	Break Point 1	90% Upper bound
1979Q3	1982Q1	1984Q3
90% Lower bound	Break Point 2	90% Upper bound
1997Q4	1998Q2	1998Q4

Table 1(b). Break Points for the UK		
90% Lower bound	Break Point 1	90% Upper bound
1962Q4	1963Q3	1964Q2
90% Lower bound	Break Point 2	90% Upper bound
1987Q4	1990Q3	1993Q2

6 Estimates of the cointegrating vectors

The previous section has highlighted the possibility of mean shifts in the great ratios, reflected in breaks in the cointegrating equations. In this section, we will test for cointegration allowing for the structural breaks identified above and listed in Table 1. Our main result is that, allowing for these breaks, the Johansen procedure indicates the presence of two cointegrating vectors. The evidence for the unit coefficients implied by the balanced growth restriction is weaker, but the departures from unity are small in economic terms. When we impose unit coefficients and apply KPSS tests, again allowing for structural breaks, we fail to reject the null hypothesis that the great ratios are stationary. Finally, we are able to show that our results are robust to alternative choices of break dates.

We begin with the break points identified by the multivariate tests, and listed in Table 1 above. We test for cointegration in the presence of these two structural breaks using the recent results of Johansen et al. (2000). They derive the distribution of the trace test statistic for the rank of the cointegrating space in a model such as equation (8). They also calculate the weights for the

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For the second break we obtained Sup-W and lExp-W test statistics of 15.14 (14.44) and 3.55 (4.43). These are all significant at the 5% level except for 3.55 which is close to the 10% critical value of 3.63.

estimated response surface to enable critical values to be easily calculated from a $\Gamma-$ distribution.

We assume a model with intercept shifts only so that the model in (8) becomes:

$$\triangle x_t = (\theta_o + \beta\alpha_o')\Xi_t + \sum_{j=1}^{k}\theta_j\triangle x_{t-j} + \beta\alpha'x_{t-1} + \sum_{i=1}^{k+1}\sum_{j=2}^{3}\kappa_{ji}D_{jt-i} + \zeta_t.$$

where α_o' is the vector of intercepts in the cointegrating equations. If $\theta_o = 0$ then there are intercepts in the cointegrating equations only. If $\theta_o \neq 0$ and $\alpha_o' \neq 0$ there is an unrestricted intercept in the VECM of the form:

$$(\theta_o + \beta\alpha_o')\Xi_t$$

so that θ_o and α_o' cannot be identified.

Both these models are consistent with the theory developed in section 2 but the model with $\theta_o = 0$ allows us to identify and estimate the coefficients on the breaks in the cointegrating equations. A likelihood ratio test of the model with $\theta_o = 0$ against the model with $\theta_o \neq 0$ resulted in a chi square statistic, with 3 degrees of freedom, of 10.5 for the USA and 10.4 for the UK. We cannot reject the null that $\theta_o = 0$ at the 1% level but we can at the 5% level. With this in mind, we first present results with $\theta_o = 0$, allowing us to obtain estimates of the shift parameters α_o', and then present results for the case $\theta_o \neq 0$.

Using the model with restricted intercepts, we obtained the results in Table 2. Note that for the multivariate cases BIC and AIC were consistent in selecting one lag of first differences in the VECM (that is, two lags in levels).

Table 2. Tests of rank allowing for structural breaks					
USA			UK		
Hypothesis	Test Statistic	p-value	Hypothesis	Test Statistic	p-value
r = 0	112.46	0.00	r = 0	91.88	0.00
r ≤ 1	40.63	0.01	r ≤ 1	36.22	0.01
r ≤ 2	14.33	0.07	r ≤ 2	10.99	0.23

Allowing for the two break points selected previously, the likelihood rank test statistic rejects one cointegrating vector in favour of two and the null of two vectors is not rejected, for either the USA or the UK. From now on, we assume that the rank of the cointegrating space is two.

Without any loss of generality we can interpret the first vector as a consumption equation and the second as an investment equation. With rank two, we can normalise two coefficients in the two equations. The first candidates are obviously the coefficients on c_t and i_t, normalised at -1. Since our focus is on

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the great ratios, it is natural to normalize the coefficient on log consumption in the investment equation (and log investment in the consumption equation) to zero. The cointegrating equations are therefore:

$$
\begin{aligned}
v_{1t} &= -c_t & +\alpha_{13}y_t & +\gamma_{11} & +\gamma_{12} & +\gamma_{13} \\
v_{2t} &= & -i_t & +\alpha_{23}y_t & +\gamma_{21} & +\gamma_{22} & +\gamma_{23}
\end{aligned}
$$

where the γs represent the intercepts for the three periods defined by the two break points.

The hypothesis that the great ratios are stationary (allowing for two mean shifts) implies the unit coefficients restriction $\alpha_{13} = \alpha_{23} = 1$. We have examined this null hypothesis for both countries using likelihood ratio tests. The switching algorithm technique of Doornik (1995) is used to estimate the restricted models and calculate asymptotic standard errors. The likelihood ratio test statistics were 7.4 (p-value 0.03) for the USA and 6.52 (p-value 0.04) for the UK, with two degrees of freedom. This implies that the hypothesis of unit coefficients is rejected at the 5% level for both countries, although not at the 1% level. We show below that the departures from unity are relatively small in economic terms, for many possible break dates.[21]

We have also carried out KPSS single-equation tests of stationarity on the great ratios, imposing unit coefficients in α and allowing for the structural breaks indicated above.[22] We cannot reject the null of stationarity at the 5% level for any of the cases considered. For the USA the test statistics were 0.114 for the log consumption ratio and 0.180 for the log investment ratio (5% critical value 0.181), while for the UK the same two test statistics were 0.071 and 0.052 (5% critical value 0.173). Hence the KPSS results for the USA are much stronger when including structural breaks, as before we could reject stationarity of the log consumption ratio. The results are also stronger for the UK, since the test statistics are further away from rejecting the null than for the case without breaks.

Therefore, our main result is that, provided one allows for occasional mean shifts, there is evidence consistent with stationarity of the great ratios. Log consumption and log investment are each cointegrated with log output. Although the evidence for unit coefficients is weaker, the departures from unity are small in

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[21]There are certain break dates for which the coefficients are not significantly different from unity. The justification for these break dates would inevitably be slightly arbitrary, however. In the interests of overall rigour we have preferred to emphasize the point estimates of break dates indicated by the BLS procedure.

[22]To obtain the critical values where we have breaks in intercepts we simulated the model with 50000 replications on the null hypothesis (great ratios stationary with breaks in trends corresponding to those we have selected for each data set and corresponding sample sizes).

economic terms. This implies that shocks to the ratios are predominantly transitory, consistent with the long-run predictions of the various models discussed in sections 2 and 4 above.

We now present detailed estimates of the consumption and investment equations for the two countries. Tables 3 and 4 show the results for the USA and UK. Note that all the broken intercepts are significant, confirming the importance of structural breaks in the cointegrating equations. Tables 3 and 4 also report the Box-Ljung statistics, which are calculated from the residuals for the VECM equations for consumption and investment, with the number of terms equal to $\sqrt{T}$. In most cases, the results are consistent with the null hypothesis that the equation disturbances are white noise, although with a rejection at the 10% level for the USA consumption equation.

Table 3(a). USA consumption equation estimates

Variable	c_t	y_t	$intercept_1$	$intercept_2$	$intercept_3$
Cointegrating Coefficients	-1	1	-0.191	-0.153	-0.117
Estimated Standard Errors	-	-	(0.007)	(0.008)	(0.016)
			$Box\text{-}Ljung(13\ ) = 19.79,\ pval = 0.10$		

Table 3(b). USA investment equation estimates

Variable	i_t	y_t	$intercept_1$	$intercept_2$	$intercept_3$
Cointegrating Coefficients	-1	1	-1.935	-2.011	-2.030
Estimated Standard Errors	-	-	(0.052)	(0.065)	(0.129)
			$Box\text{-}Ljung(13) = 11.86,\ pval = 0.54$		

Table 4(a). UK consumption equation estimates

Variable	c_t	y_t	$intercept_1$	$intercept_2$	$intercept_3$
Cointegrating Coefficients	-1	1	-0.223	-0.276	-0.230
Estimated Standard Errors	-	-	(0.010)	(0.006)	(0.009)
			$Box\text{-}Ljung(13) = 17.21,\ pval = 0.19$		

Table 4(b). UK investment equation estimates

Variable	i_t	y_t	$intercept_1$	$intercept_2$	$intercept_3$
Cointegrating Coefficients	-1	1	-1.436	-1.273	-1.408
Estimated Standard Errors	-	-	(0.046)	(0.027)	(0.042)
			$Box\text{-}Ljung(13) = 12.17,\ pval = 0.51$		

We now address the key issue of sensitivity to alternative break dates. Our main results are robust to most possible combinations of breaks within the confidence bands estimated by the BLS procedure. For the USA the confidence bands were 1979Q3 to 1984Q3 and 1997Q4 to 1998Q4, which implies $21 \times 5 = 105$ possible combinations of two break dates (assuming just one within each band). For 104 of these 105 possible combinations, there was evidence for two cointegrating vectors at the 5% level. The exception was the combination of 1979Q4 and 1997Q4 where the two cointegrating vectors were significant only at the 7% level. Across all 105 combinations, the mean coefficient on log output in the consumption equation ranges between 0.94 and 0.99 with a mean of 0.97, and the mean coefficient on log output in the investment equation ranges between 1.14 and 1.41, with a mean of 1.27. Hence, for most break dates, the departures from unit coefficients are relatively small in economic terms.

The results for the UK are also quite robust. The confidence bands for the UK were 1962Q4 to 1964Q2 and 1987Q4 to 1993Q2, implying $7 \times 23 = 161$ possible combinations. Of these, 79 combinations yielded two cointegrating vectors at the 5% level, 31 between the 5% and 10% levels, 33 between 10% and 20% and only 18 greater than 20%, with the weakest result (1964Q2, 1992Q4) found at one extreme of the confidence bands. Across all 161 combinations, the mean coefficient on log output in the consumption equation ranges between 1.01 and 1.11 with a mean of 1.06, while the mean coefficient on log output in the investment equation ranges between 0.84 and 1.48 with a mean of 1.19.

We turn now to the case with $\theta_o \neq 0$. This corresponds to an unrestricted matrix of breaking intercepts in the VECM, which can be interpreted as breaking drift terms in the processes generating the variables $c_t, i_t,$ and y_t and breaking intercepts in the two cointegrating equations (with the same break dates for the drift terms and the intercepts). In many respects, the results for the unrestricted model are more robust. Table 5 gives the results of tests for the rank of the cointegrating space. The critical values and p-values are obtained from Theorem 3.3 in Johansen *et al* (2000).

Table 5. Tests of Rank - Unrestricted Model					
USA			UK		
Hypothesis	Test Statistic	p-value	Hypothesis	Test Statistic	p-value
r = 0	45.89	0.00	r = 0	45.51	0.00
r ≤ 1	19.87	0.00	r ≤ 1	21.07	0.00
r ≤ 2	2.18	0.13	r ≤ 2	0.43	0.51

Allowing for the two break dates selected previously, the likelihood rank test statistic rejects one cointegrating vector in favour of two and the null of

two vectors is not rejected, for either the USA or the UK. As for the restricted model, we can conclude that there are two cointegrating equations. As before, we normalise these to be consumption and investment equations. A likelihood ratio test of the null of unit coefficients on output in both equations resulted in a test statistic of 6.08 (p-val = 0.05) for the USA and 7.83 (p-val = 0.02) for the UK. Although unit coefficients are rejected at the 5% level, the departures from unity are usually small in economic terms, as we document below. Using the Box-Ljung statistic the hypothesis of white noise errors in the VECM is not rejected at conventional levels.

The sensitivity of the unrestricted model to the choice of break dates is similar to that found for the restricted case. For the USA, we again consider the confidence intervals 1979Q3 to 1984Q3 and 1997Q4 to 1998Q4. For all the $21 \times 5 = 105$ possible combinations, there was evidence for two cointegrating vectors at the 5% level. Across all combinations, the mean coefficient on log output in the consumption equation was 0.98 and ranged between 0.96 and 0.99. Less favourably, the mean coefficient on log output in the investment equation was 1.20 and ranged between 1.12 and 1.26.

For the UK we again consider the two confidence intervals 1962Q4 to 1964Q2 and 1987Q4 to 1993Q2. For the $7 \times 23 = 161$ possible combinations, 125 had two cointegrating vectors at the 5% level, 12 between the 5% and 10% levels, and only 24 failed to have two cointegrating vectors at the 10% level. Again, the weakest results are at the very border of the confidence bands. In the consumption equation, the mean coefficient on log output was 1.06, ranging between 1.02 and 1.13 across all 161 break combinations in the confidence bands. In the investment equation, the results were again less strong, with much greater sensitivity of the output coefficient to precise break dates: the mean coefficient on log output was 1.10, ranging between 0.59 and 1.38.

We end our discussion of these results by noting an important dimension in which they are less robust. A case could be made for the inclusion of time trends in the cointegrating equations, perhaps as a way to capture slow evolution of the determinants of the ratios, for example. Such trends are often significant when included in the cointegrating equations. Their role could only be temporary, however, given that the ratios are bounded above and below, and including them tends to weaken the results.

In the model with time trends, it is still possible to find evidence for unit cointegrating vectors, but this result is highly sensitive to the specific break dates. Moreover, the break dates that give the strongest results always lie outside the confidence bands identified by the formal tests for structural breaks.

We therefore report only the results which exclude time trends from the cointegrating equations. We should note that for our central purpose, extracting the permanent component in output as in the next section, our findings are quite similar even if trends are included in the cointegrating equations.

7   The permanent component in output

Using the methods described in section 3, we can now extract the permanent component in the series from the estimated VECM. For this purpose, we use a more general model with unrestricted broken intercepts. This is not significantly different from one with restricted broken intercepts, and gives smoother estimates of the stochastic trend. Having extracted the permanent component in output in this way, we will then compare it with the permanent component implied by a standard univariate decomposition.

First, we note that our estimated model gives rise to discontinuities in the permanent component, corresponding to the dates of structural breaks. To some extent, these discontinuities can be seen as artifacts generated by the assumption of sharp, discrete structural breaks, rather than more gradual changes in parameters that are harder to deal with statistically.[23] This means that the breaks in the permanent component should not be interpreted too literally, and it is the long-term patterns that are of most interest.

Extending the analysis of section 3 above, the presence of discontinuities can be seen as follows. The permanent component for the model with no trends in the cointegrating equations, $\gamma' = 0$, but shifts in an unrestricted intercept in the VECM is given by:

$$x_t^P = C(1)\theta(1)x_t + \delta_o$$

where $\delta_o = -C(1)\theta^*(1)\mu_{\triangle x} + Q\mu_v$. Integrating the model in (10) we obtain the solution for x_t as:

$$x_t = x_o + C(1)\sum_{i=1}^{t} \zeta_i + \mu_{\triangle x}t + S_t$$

where x_o is an intial value for the process and where $S_t = C^*(L)\zeta_t-$ see, for comparison, Johansen (1995, ch. 5). Substituting for x_t in the permanent

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[23]See the introduction to Hansen (2001), who writes "While it may seem unlikely that a structural break could be immediate, and might seem more reasonable to allow for a structural change to take a period of time to take effect, we most often focus on the simple case of an immediate structural break for simplicity and parsimony".

component yields:

$$x_t^P = C(1)\theta(1)x_o + C(1)\sum_{i=1}^{t}\zeta_i + C(1)\theta(1)\mu_{\triangle x}t + C(1)\theta(1)S_t + C(1)\theta(1)\delta_o$$

since $C(1)\theta(1)C(1) = C(1)$. It is easily deduced that $\mu_{\triangle x} = C(1)\mathcal{K}_o H_t$ where:

$$H_t = \begin{bmatrix} \Xi_t \\ D_1 \end{bmatrix}$$

and $\mathcal{K}_o = (\theta_o, \varkappa)$ as in section 3 so it follows that the slope of the linear time trend in the permanent component will depend upon the sub-period of the sample through the break dummies in Ξ_t. Further, it is easily deduced that:

$$\begin{aligned} \delta_o &= -C(1)\theta^*(1)\mu_{\triangle x} + Q\mu_v \\ &= -[C(1)\theta^*(1)C(1) + Q(\alpha'\theta(1)^{-1}\beta)^{-1}\theta(1)^{-1}]\mathcal{K}_o H_t \end{aligned}$$

so that intercepts for each of the sub-periods will also differ through $\mathcal{K}_o H_t$.

We now present our estimates of the permanent component, before comparing them with a simple univariate decomposition. The estimates are shown in Figure 7, which plots the log of the output series (the dotted lines) and the permanent components (the solid lines) for both the USA and the UK. The permanent components are those based on the multivariate Gonzalo-Granger-Proietti decomposition for each country.

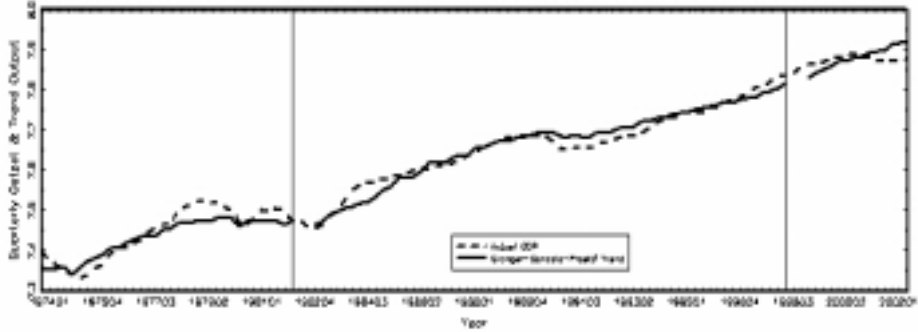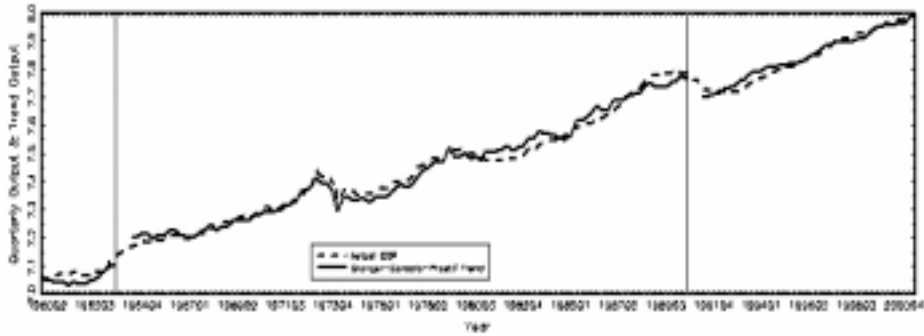Fig.7(a) USA Actual and Trend Log Real Output Per Capita

Fig.7(b) UK Actual and Trend Log Real Output Per Capita

Given the empirical method we have adopted, based on relatively weak long-run restrictions, our interest is more in the long-term pattern of the permanent component than in the short-run disparity between output and the permanent component. In other words, this procedure may be poorly suited to measuring potential output, not least because it makes no use of unemployment or inflation data, and also because of the restrictions embodied in the Gonzalo-Granger-Proietti decomposition. The great ratios approach may nevertheless be quite informative about long-term shifts in the behaviour of the permanent component, given that this component is being identified using the joint behaviour of consumption, investment and output.

We look first at the case of the USA (the upper panel) beginning in the mid-1970s. Here we see that the permanent component of output grew very slowly in the 1970s, consistent with the much-discussed productivity slowdown that revealed itself over the course of the decade. The growth of the permanent component is more rapid in the 1980s. The results for the 1990s are of especial interest, since they appear to reflect the massive New Economy boom of this period. Our analysis clearly indicates that the rate of output growth observed in the 1990s was higher than the rate of growth of the permanent component, reflecting favourable transitory shocks. For the period after the structural break of the late 1990s, however, this is no longer true, and the permanent component has grown more rapidly than output. Hence, the joint behaviour of consumption, investment and output is consistent with the idea that trend growth has continued to be strong.

The results for the UK (the lower panel) are shown for the 1960s onwards, and are less striking than for the USA. The permanent component varies in a similar way to observed output, and indicates that the multivariate approach is relatively uninformative in the case of the UK. To investigate this further, we now compare the VECM decomposition with a simple univariate trend, for both countries.

For the univariate trend we adopt the Beveridge-Nelson decomposition.[24] It is worth noting that interest in the BN decomposition has recently been increased by the work of Morley, Nelson and Zivot (2003). For one class of unobserved component models, they estimate the correlation between trend and cycle disturbances and show it to be quite close to minus one, the figure implicit in the BN decomposition. Such a finding can be given an economic interpretation if productivity shocks are an important source of fluctuations. A positive shock

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[24]An obvious extension would be to consider the univariate Granger-Gonzalo-Proietti decomposition, and we are pursuing this in further work.

to productivity will increase trend output, but this implies output will be below
trend for a transitory period. Hence innovations to the trend are negatively
correlated with cyclical innovations, as the BN decomposition assumes.[25]

Figure 8 compares our multivariate trend, with structural breaks, to a uni-
variate trend based on the BN decomposition for output. The use of BIC sug-
gested only one lag in first differences for both countries. As is often found, the
permanent component identified by the BN decomposition is almost indistin-
guishable from actual output, implying that most of the variation in output is
driven by permanent shocks. In figure 8 the univariate trend is based on 8 lags
of first differences and still appears to account for most of the fluctuations in
actual output shown in figure 7.

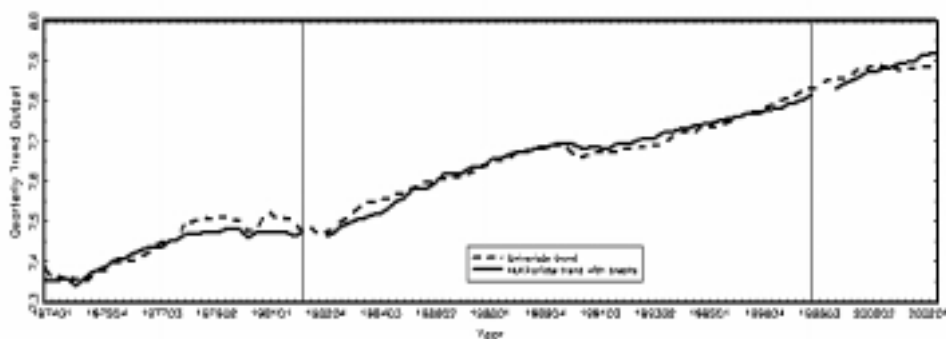Fig. 8(a) Comparison of Univariate and Multivariate Trends for USA Log Real Output Per Capita.

Fig. 8(b) Comparison of Univariate and Multivariate Trends for UK Log Real Output Per Capita.
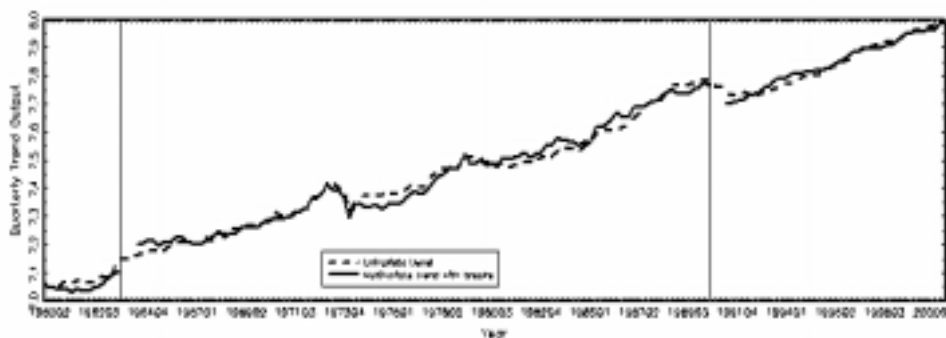
Figure 8 clearly reveals the potential consequences of a multivariate approach
to trend measurement. For the USA (the upper panel) the pattern observed ear-
lier is much less clear in the univariate decomposition (the dotted line). This
univariate decomposition does not highlight the slow trend growth of the 1970s,
or the above-trend growth of the mid-1990s, as clearly as the multivariate ap-
proach. Overall, it is clear that the multivariate approach can offer some useful

[25]Note, however, that a range of models are consistent with correlated trend/cycle innova-
tions. See Proietti (2002) for further discussion.

insights into the evolution of the permanent component, and could provide a useful complement to univariate trend analysis.

8 Conclusions

We have taken as our starting point the work of KPSW, one of the most widely cited papers in recent research on macroeconometrics. The central focus of KPSW was on the relative importance of different forms of shocks in explaining short-run fluctuations. Their examination of the joint behaviour of consumption, investment and output also offers a new approach to measuring trend growth, based on extracting the common permanent component from a multivariate system.

In this paper, we have investigated this possible application. As we have discussed at length, there are some reasons to be sceptical that the great ratios will revert to constant means, since the equilibrium ratios are functions of parameters that may vary over time (including the trend growth rate). This may explain why previous researchers have often rejected the hypothesis that the great ratios are stationary.

Sometimes, researchers have used rejection of stationarity in the great ratios as evidence against models of exogenous growth. The problem with this argument is that many other models would also yield a balanced growth path in which the great ratios are stationary. One resolution to this puzzle is to acknowledge that empirical testing of theoretical models inevitably involves some strong auxiliary assumptions, notably parameter constancy, and so it is really a joint hypothesis that is being tested. The rejection of the long-run restrictions implied by the model may sometimes represent a failure of these auxiliary assumptions, rather than the hypotheses about economic behaviour that are built into the model.

We find that, when allowing for two structural breaks, the evidence for two cointegrating vectors in the USA and UK is much stronger. Our estimates provide some support for the balanced growth hypothesis. There are various ways in which our analysis could be refined further. An alternative treatment of the government sector, or a careful distinction between population and employment, might yield better results even in the absence of structural breaks. More fundamentally, there are two promising avenues for developing richer empirical frameworks. First, the analysis of two sector models may imply balanced growth paths that have unfamiliar properties. Secondly, there is an obvious case for extending the analysis to an open economy context, and we are pursuing this line

of research in further work.

These extensions seem worthwhile, because the multivariate approach to trend measurement clearly has some potential. Our multivariate permanent-temporary decompositions yield some interesting findings, especially for the USA. Perhaps most revealing are the results for the 1990s. The joint behaviour of consumption, investment and output indicates that strong growth was partly due to transitory favourable shocks. More recently, however, output has grown rather more slowly than the permanent component, suggesting that recent improvements in performance could be sustainable. This is consistent with recent evidence that labour productivity has continued to grow strongly despite the recession.

9 Data

For the USA, the data series were downloaded from the Bureau of Economic Analysis website on 23 October 2002. The data are seasonally adjusted (SA) and expressed as annual rates, with the exception of population, which is measured mid-period. The output figures are for GDP, while government expenditure corresponds to government consumption and gross investment. The price index we use is the implicit price deflator for personal consumption expenditure.

For the UK, the data series were constructed from the following series in the Economic Trends Annual Supplement (2001): Households Final Consumption Expenditure, current prices, code ABJQ; Households Final Consumption Expenditure, 1995 prices, code ABJR; GDP at market prices, 1995 prices, code ABMI; GDP at market prices per capita, 1995 prices, code IHXW; Gross Domestic Product at market prices, current prices, code YBHA; Government Final Consumption Expenditure, current prices, code NMRP; Gross Fixed Capital Formation, current prices, code NPQS. Note that for the UK, for data availability reasons, our measure of government expenditure corresponds to government consumption, and government investment is included in our measure of investment.

The price index, p, is obtained from the ratio ABJQ/ABJR. A population series, N, is obtained from ABMI/IHXW. Real per capita consumption is then defined as $C_t = \text{ABJQ}/(p*N)$, real per capita investment as $I_t = \text{NPQS}/(p*N)$ and real per capita private sector output, $Y_t = (\text{YBHA-NMRP})/(p*N)$. Note that C_t and I_t do not sum to Y_t because of the current account, and small discrepancies due to consumption of non-profit institutions, inventory adjustments, and measurement errors in the national accounts statistics.

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