Active monetary policy, passive fiscal policy and the value of public debt: some further monetarist arithmetic*

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Abstract

We consider the properties of two monetary policy rules ('strict inflation targeting', 'constant money growth rule') in an intertemporal equilibrium model with flexible prices in which monetary policy is 'active', while fiscal policy is 'passive'. Specifically, we assume that the fiscal agent takes the monetary policy rule as given and restricts itself to a policy which is consistent with a sustainable debt burden and stable steady-state dynamics. The paper shows that dynamic properties of the model economy may differ significantly between the two monetary policy rules if public debt is issued in nominal terms. Under a constant money growth rule which allows for temporary deviations of inflation from target in response to shocks there is scope for revaluations of public debt, acting as automatic stabilizers of government debt dynamics. By contrast, a policy of strict inflation targeting implements the target inflation rate also outside the steady state and precludes thereby such stabilizing revaluations. Owing to this feature, additional fiscal restraint may be needed which is not required under a constant money growth rule.

Keywords: Monetary policy, Fiscal regimes, Overlapping generations.

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1 Introduction

As pointed out in the seminal contribution by Sargent and Wallace (1981), monetary policy by itself will not always be in a position to control the evolution of the price level, unless being appropriately supported by the fiscal agent. This insight reflects that in any macroeconomic model the government's budget constraint entails contributions of both monetary and fiscal policy. In terms of the widely used terminology introduced by Leeper (1991), the Sargent-Wallace result says that for the monetary agent to be able to control 'actively' inflation the fiscal agent needs to behave 'passively' in the sense that he accepts the residual role within the government's budget constraint, taking as given the behaviour of the active agent.[1] But how to enforce a credible mix of active monetary and passive fiscal policy? This question is of particular importance in the context of the European Monetary Union which is characterized by potential coordination failures and additional incentive problems arising from 'one money, but many fiscal policies' (Uhlig, 2002). Within this context, many economists have argued that it would be desirable to subject the fiscal agent to some kind of a rule which imposes certain limits on the government's borrowing behaviour (Chari and Kehoe (1998), Sims (1999)). Evidently, the requirements of the Stability and Growth Pact prevailing in the European Monetary Union reflect such concerns.

Woodford (2001) has recently reemphasized, however, that the interaction between monetary and fiscal policy can never be one-way only. Specifically, regarding the effects of monetary policy on fiscal policy, Woodford stresses the effects of monetary policy on the real value of government debt (and the real debt service associated with it) through its effects on the price level, given that public debt is largely issued in nominal terms. Moreover, these fiscal effects of monetary policy can be potentially large, even if the traditionally considered channel, the seigniorage contribution to the government's budget, is negligible.

This paper takes the desirability of a combination of active monetary and passive fiscal policy rules for granted and investigates some of the implications of the revaluation channel of government debt for the design of such rules. The key idea is that because of this channel different monetary policy rules are likely to be associated with different government debt dynamics, restraining thereby the fiscal agent in dif-

[1] The possibility of price level adjustments which reconcile otherwise inconsistent claims of active monetary and active fiscal policy in the government's intertemporal budget constraint is the subject of the 'Fiscal Theory of the Price Level' as advanced by Woodford (1994, 1995) and Sims (1994). The equilibrium foundations of the theory are, however, subject to a substantial controversy (Buiter (2002), McCallum (2001). For summary treatments of the debate on the 'Fiscal Theory of the Price Level', see Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), Bassetto (2002). In any case, the view associated with the 'Fiscal Theory of the Price Level' makes the case for fiscal rules even more compelling.

ferent ways. To illustrate the potential strength of these differences, we present a small general equilibrium model with strong supply-side features in which the price level is fully flexible and determined according to simple 'monetarist' principles. We compare two specifications of monetary policy, both of them being consistent with identical long-run equilibria. First, we consider a policy of a 'constant money growth rule' which allows for temporary deviations of inflation from target and leaves room for revaluations of outstanding public debt as a response to supply shocks. Investigating the dynamic properties of equilibria, we show how this channel acts as an automatic stabilizer of government debt dynamics, relaxing thereby the constraint of the passive fiscal agent. Second, we look at a policy of 'strict inflation targeting' which keeps inflation always on target and preserves the real value of outstanding debt in response to shocks. As a result, this policy lacks the stabilization properties regarding government debt dynamics and we derive how this feature leads to a tighter constraint of the fiscal agent, compared with a constant money growth rule.[2]

While the main insight of this paper is largely model-independent, we organize our analysis, inspired by Sargent and Wallace, around an overlapping generations economy of the Diamond-type with two interest-bearing assets (physical capital, government bonds) and return-dominated outside money. Specifically, following the two-stage modelling strategy of Sargent and Wallace, we start out with a simple, fully tractable benchmark economy with backward-looking dynamics in which the preferences of agents are specified in a highly monetarist way, yielding a demand for real balances which is strictly proportional to contemporaneous output and displays a constant velocity.[3] We assume that monetary policy has a certain inflation target, while fiscal policy aims at a certain non-negative target value of the deficit ratio (corresponding in long-run equilibrium one-to-one to a certain debt ratio). Given such targets, the model gives rise to a unique steady state with positive levels of output and real balances as well as non-negative government debt, as long as the deficit ratio remains below a certain feasibility bound. Depending on the specific

[2] Svensson (1999) offers a framework with short-run rigidities in which 'inflation targeting' is associated with the permanent accommodation of price level changes, while 'price-level targeting' is not. Svensson (2003) further distinguishes between 'strict' and 'flexible' inflation targeting where the latter concept allows for a positive weight on output stabilization in the central bank's loss function. The framework presented here lacks the trade-offs considered by Svensson since it exclusively concentrates on features of the long run, i.e. prices are fully flexible and actual output is always identical to potential ouput. Despite the different motivation, however, the constant money growth rule considered in this paper acts like a flexible inflation targeting rule in the sense that it tolerates temporary deviations of inflation from target.

[3] Unlike the Sargent-Wallace set-up, our model accounts for variable real interest rates due to variations in the marginal product of capital. In this respect, our model is related to contributions by Schreft and Smith (1997, 1998), Espinosa-Vega and Russell (2001), and Bhattacharya and Kudoh (2002). The focus of these papers, however, is on different aspects of monetary and fiscal policy interaction.

interaction between monetary and fiscal policy, however, steady states of this type are not necessarily stable. Defining our particular specification of a passive stance of fiscal policy, we restrict the fiscal agent to deliver stable dynamics, taking as given the specification of the monetary policy rule.[4] In order to establish the different fiscal consequences associated with strict inflation targeting and a constant money growth rule we consider two scenarios.

First, as a deliberately strong example, we require the fiscal agent to maintain a constant deficit ratio not only in steady state, but in all periods. Under this assumption, under a constant money growth rule all feasible steady states are always stable. By contrast, under a policy of strict inflation targeting all feasible steady states become unstable beyond a certain threshold value of the deficit ratio, implying that the fiscal agent faces ex ante a narrower choice set than under a constant money growth rule. This different stability behaviour results from the fact that under strict inflation targeting, because of the absence of stabilizing revaluations of government debt, the economy is more vulnerable to adverse debt dynamics in response to shocks. Specifically, the severity of such dynamics depends on the initial steady-state level of the deficit ratio which, through its correspondence to the economy's debt ratio, is directly linked to the pre-shock level of the real interest rate. Hence, the lack of stabilizing revaluations of government debt is particularly harmful under a high deficit ratio, generating the possibility of unstable dynamics. Moreover, we also show that under strict inflation targeting the richer interaction between debt dynamics and crowding out effects implies that off-steady state dynamics, even when being stable, can be associated with endogenous fluctuations - a feature which cannot occur under a constant money growth rule.

Second, we discuss how the potential instability of steady states under strict inflation targeting can be removed by fiscal policy rules which maintain the same long-run target value of the deficit ratio, but allow for more flexibility outside the steady state. While upon appropriate changes in the fiscal rule all feasible steady-state deficit ratios can be stabilized, the main result remains nevertheless unaffected, i.e. the lack of stabilizing debt revaluations under strict inflation targeting may

[4] Using a Ramsey-type economy, Leeper's contribution follows the logic that 'a passive authority is constrained by consumer optimization and the active authority's actions, so it must generate sufficient tax revenues to balance the budget.' (p.136) Moreover, combinations of one actively and one passively behaving authority are shown to satisfy the condition for a unique saddlepath equilibrium. In overlapping generations economies of the Diamond-type the role of the government's budget constraint is different, since, in principal, the government may find it optimal to permanently roll over debt between members of different generations, without ever raising taxes. Moreover, in the absence of Ramsey-type transversality conditions stability properties of long-run equilibria follow a different logic. Given these structural differences between Ramsey- and Diamond-economies, we find it natural to link the concept of passive policy to the notion of (locally) stable dynamics around a certain target steady state. Related to this, see also Woodford (2001, Section 2) and Chalk (2000).

well require additional fiscal restraint which is not needed under a constant money growth rule.[5]
Following Sargent and Wallace, we then change preferences of agents in a way that the money demand-specification becomes forward-looking and depends as well on future inflation. Everything else being equal, this modification leaves all the dynamic (in)stability properties of the benchmark economy under strict inflation targeting qualitatively unaffected, since the forward-looking component remains fully predictable. By contrast, under a constant money growth rule the forward-looking component is now genuinely expectation driven. As a result, real balances turn into a forward-looking jump variable and the overall dynamics now contain both backward-looking and forward-looking elements. Despite this change, however, the stability properties of the benchmark economy remain qualitatively unaffected. Specifically, we establish that for all feasible deficit ratios steady states are dynamically approachable from fixed initial conditions in a uniquely determined and smooth manner.

The remainder of the paper is structured as follows. Section 2 introduces the benchmark model. Section 3 discusses the stability properties of this model under strict inflation targeting and a constant money growth rule. Section 4 extends the benchmark model by allowing for a forward-looking money demand specification. Finally, Section 5 offers some conclusions. Proofs not included in the main text and numerical simulation output are delegated to the appendix.

2 The benchmark model

We consider a deterministic overlapping generations economy with production in the tradition of Diamond (1965), in which one-period government bonds and physical capital act as perfect substitutes in the portfolios of agents. Deviating from Diamond, however, we present a monetary economy in which return-dominated money coexists with interest-bearing assets by means of a simple cash-in-advance constraint which applies to a certain fraction of consumption purchases.[6]

[5] The focus of this paper is not on optimal monetary and fiscal policies as investigated, for example, by Chari, Christiano, and Kehoe (1991). Related to the mechanics of the revaluation channel of government debt, papers in this tradition typically stress that unanticipated inflation should play the role of a fiscal shock absorber as long as government debt is nominal and non-state-contingent. For a careful analysis of this finding under conditions of perfect and imperfect condition, see Schmitt-Grohé and Uribe (2001). Moreover, we do not question why government debt is largely issued in nominal terms, as done in Bohn (1988).

[6] For a more detailed description of the benchmark economy established in this section, see the companion paper by von Thadden (2002) which focuses on properties of interest-rate rules. For papers which introduce money into Diamond-type overlapping generations economies by means of a cash-in-advance constraint, see, in particular, Hahn and Solow (1995) and Crettez, Michel, and Wigniolle (1999).

Let N_t denote the number of identical and two-period lived young agents in a representative period t, with $N_t/N_{t-1} = 1 + n > 1$, i.e. the population is assumed to grow over time at a constant rate. At the beginning of the representative period t, the old generation owns certain predetermined levels of the aggregate stocks of physical capital (K), nominal bonds ($\widetilde{B}$) and nominal money balances ($\widetilde{M}$), resulting from decisions undertaken in period $t-1$. To facilitate the discussion of asset dynamics below, let $k_{t-1} = K_{t-1}/N_t$, $\widetilde{b}_{t-1} = \widetilde{B}_{t-1}/N_t$, $\widetilde{m}_{t-1} = \widetilde{M}_{t-1}/N_t$ denote the predetermined stocks of real capital, nominal bonds, and nominal money balances at the beginning of period t, measured per period-t young agent. We assume that the economy inherits from period $t-1$ some positive interest factor $I_{t-1} > 1$ (relevant for interest payments on government bonds, whenever $\widetilde{b}_{t-1} > 0$) and a positive price level $p_{t-1} > 0$. In sum, using the generic index $t-1$ for the initial (or predetermined) conditions of the economy, we assume:

(A 1) Predetermined variables
In the representative period t, the economy operates subject to predetermined variables, resulting from decisions in period $t-1$:
i) $I_{t-1} > 1$, $p_{t-1} > 0$,
ii) $k_{t-1} > 0$, $\widetilde{m}_{t-1} > 0$, $\widetilde{b}_{t-1} \geq 0$.

As to be seen below, we assume that these variables are sufficiently close to a certain target steady state such that dynamics are locally governed by the force field around this steady state.

2.1 Production

In every period, young agents offer inelastically a labour supply of $e = 1$ units of labour. Old agents have a zero labour endowment. The production process of the economy exhibits standard features of a neoclassical growth model. Specifically, one unit of output (Y) can be equally consumed or invested, and, when invested, it can be transformed into one unit of physical capital to be used one period later. Aggregate output in period t is given by $F(K_{t-1}, N_t)+(1-\delta)K_{t-1}$, where the function $F(.,.)$ has constant returns to scale and $\delta \in (0, 1)$ denotes the rate at which capital depreciates during the production process. Let $y_t = f(k_{t-1}) \equiv F(K_{-1}, 1)$, where $k_{t-1} = K_{t-1}/N_t$ describes the capital-labour ratio per young agent. Input and output markets are characterized by perfect competition. Let ρ_t and w_t denote the rental rate and the wage rate to capital and labour in period t, respectively:

$$\rho_t = f'(k_{t-1}) \tag{1}$$
$$w_t = f(k_{t-1}) - f'(k_{t-1}) \cdot k_{t-1} = w(k_{t-1}) \tag{2}$$

We assume that all factor incomes out of $f(k)$ are subject to a proportional and constant tax at rate $\tau \in (0,\,1)$. Accordingly, the net of tax return factor associated with a unit of capital, invested in period t and with pay-off in period $t+1$, is

$$R_t = 1 - \delta + (1-\tau) \cdot f'(k_t) = R(k_t) \tag{3}$$

The function $f(k)$ has properties which are satisfied by a Cobb-Douglas function of the form $y_t = \phi \cdot k_{t-1}^{\alpha}$, where $\phi > 0$ denotes a shift parameter for the productivity level:

(A 2) Technology is described by the function $y_t = f(k_{t-1}) = \phi \cdot k_{t-1}^{\alpha}$ with:
i) $f(k) \geqslant 0,\;\; f'(k) > 0,\;\; f''(k) < 0$ for $k \geqslant 0$; **(ii)** $\lim_{k\to 0} f'(k) = \lim_{k\to 0} \frac{f(k)}{k} = \infty$, **iii)** $\lim_{k\to\infty} f'(k) = \lim_{k\to\infty} \frac{f(k)}{k} = 0$; **(iv)** $w(k_{t-1}) = (1-\alpha)\cdot f(k_{t-1})$, with $\alpha \in (0,1)$.

2.2 Government

Let $\widetilde{G}_t$ denote nominal aggregate government consumption, yielding no utility for the private sector, and write the nominal primary deficit ($\widetilde{D}^p$) in period t as: $\widetilde{D}^p_t = \widetilde{G}_t - \tau \cdot p_t \cdot Y_t$. The nominal flow budget constraint of the government in period t can then be summarized as:[7]

$$\widetilde{D}^p_t = \widetilde{B}_t - I_{t-1}\widetilde{B}_{t-1} + \widetilde{M}_t - \widetilde{M}_{t-1}. \tag{4}$$

Equation (4) simply says that the nominal primary deficit can be funded from emitting additional amounts of bonds or money, with the return differential between these two outside assets being given by the nominal interest rate. To express (4) in real terms on a per capita basis, we use $\widetilde{d}^p_t = \widetilde{D}^p_t / N_t$ and introduce the notation $b_t = \widetilde{b}_t / p_t$, $m_t = \widetilde{m}_t / p_t$, $d^p_t = \widetilde{d}^p_t / p_t$, yielding

$$d^p_t = (1+n)\cdot b_t - I_{t-1}\cdot \frac{p_{t-1}}{p_t} \cdot b_{t-1} + (1+n)\cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1}. \tag{5}$$

2.3 Problem of the representative agent

Preferences of agents are time separable and of the type

$$U(c^y_t, c^o_{t+1}) = u(c^y_t) + \beta \cdot u(c^o_{t+1}),$$

where $u(c^y_t)$ and $u(c^o_{t+1})$ denote, respectively, the utility of an agent (born in t) from consumption in his youth and his old age, while β is a constant discount

[7] For simplicity, interest earned on governement bonds is assumed to be not taxed. However, this assumption could be easily relaxed.

factor, $\beta \in (0,1)$. When making consumption and portfolio choices, young agents take all prices and return rates as given. Let asset demands of the representative agent be denoted by $\hat{}$-variables. Specifically, in addition to first-period consumption c_t^y, agents choose real money balances $\widehat{m}_t$ (with real return factor $R_t^m = p_t/p_{t+1}$), government bonds $\widehat{b}_t$ (with real return factor $I_t \cdot p_t/p_{t+1}$), and holdings of physical capital $\widehat{k}_t$ (with real return factor R_t). Since government bonds and capital are considered to be perfect substitutes, the latter two return rates need to be identical. For money to be valued in equilibrium, we specify the second-period flow utility $u(c_{t+1}^o)$ as a composite index which represents the consumption of cash goods (c_{CA}) and credit goods (c_{CR}). To keep the benchmark model particularly simple, we assume that i) savings are a constant fraction of first-period disposable income and ii) the elasticity of substitution between cash and credit goods is unity. As we will show in the remainder of this section, these assumptions suffice to generate a simple quantity theory demand for money which is strictly proportional to current income and displays a constant velocity, similar to part one of Sargent and Wallace (1981). To this end, preferences of the representative agent are given by:

(A 3) Preferences of the representative agent**:**

$$U = \ln c_{CR,\,t}^y + \beta \cdot [z \cdot \ln_{CR,\,t+1}^o + (1-z) \cdot \ln_{CA,\,t+1}^o], \qquad z \in (0,1).$$

Accordingly, the decision problem of a young agent in period t amounts to finding values $\{c_{CR,\,t}^y,\, c_{CR,\,t+1}^o,\, c_{CA,\,t+1}^o,\, \widehat{m}_t,\, (\widehat{b}_t + \widehat{k}_t)\}$ which maximize

$$\begin{aligned} max:\ & \ln c_{CR,\,t}^y + \beta \cdot [z \cdot \ln_{CR,\,t+1}^o + (1-z) \cdot \ln_{CA,\,t+1}^o] \\ & + \vartheta_t \cdot (R_t^m \cdot \widehat{m}_t + R_t \cdot (\widehat{b}_t + \widehat{k}_t) - c_{CR,\,t+1}^o - c_{CA,\,t+1}^o) \\ & + \zeta_t \cdot (R_t^m \cdot \widehat{m}_t - c_{CA,\,t+1}^o) \\ & + \nu_t \cdot [(1-\tau) \cdot w_t - c_{CR,\,t}^y - \widehat{m}_t - \widehat{b}_t - \widehat{k}_t], \end{aligned} \tag{6}$$

with the multipliers ϑ_t, ζ_t, ν_t being associated with the budget constraint in $t+1$, the additional cash constraint which applies to purchases of cash goods in $t+1$, and the budget constraint in t, respectively.
Cash and credit goods are assumed to be homogenous in production such that they are sold to consumers at identical prices. However, from the perspective of consumers, there is an opportunity cost associated with the use of money whenever the nominal interest rate on bonds is positive, i.e. $I_t > 1$. To restrict the analysis of (6) to interior solutions with a strictly binding cash-in-advance constraint, we assume from now on

$$I_t = R_t \cdot \frac{p_{t+1}}{p_t} > 1, \tag{7}$$

implying that money holdings in period t will always be identical to the expected nominal value of cash purchases in period $t+1$. Under this assumption there exist uniquely defined asset demands $\widehat{m}_t$ and $(\widehat{b}_t + \widehat{k}_t)$ and, reflecting the assumption of perfect substitutability, the mix between $\widehat{b}_t$ and $\widehat{k}_t$ is indeterminate at the individual level. Differentiating (6) with respect to $\{c^y_{CR,\,t},\, c^o_{CR,\,t+1},\, c^o_{CA,\,t+1},\, \widehat{m}_t,\, (\widehat{b}_t + \widehat{k}_t)\}$ and eliminating the multipliers gives the system of equations:

$$c^y_{CR,\,t} = \frac{1}{1+\beta} \cdot (1-\tau) \cdot w_t \tag{8}$$

$$\widehat{m}_t + \widehat{b}_t + \widehat{k}_t = \frac{\beta}{1+\beta} \cdot (1-\tau) \cdot w_t \tag{9}$$

$$\frac{\frac{\partial U}{\partial c^o_{CA,\,t+1}}}{\frac{\partial U}{\partial c^o_{CR,\,t+1}}} = \frac{1-z}{z} \cdot \frac{c^o_{CR,\,t+1}}{c^o_{CA,\,t+1}} = \frac{R_t}{R^m_t} = I_t \tag{10}$$

$$c^o_{CA,\,t+1} = R^m_t \cdot \widehat{m}_t \tag{11}$$

$$c^o_{CR,\,t+1} = R_t \cdot (\widehat{b}_t + \widehat{k}_t) \tag{12}$$

The feature of a constant savings rate in equations (8) and (9) follows from the log-specification of preferences. Equation (10) shows that in an interior optimum the marginal rate of substitution between cash and credit goods needs to be equal to the price ratio as perceived by consumers, which in turn is naturally given by the nominal inflation factor. Combining (10)-(12) gives a simple portfolio relationship between real balances and interest-bearing assets, reflecting our assumption to restrict the elasticity of substitution between cash and credit goods to unity:

$$\widehat{m}_t = \frac{1-z}{z} \cdot (\widehat{b}_t + \widehat{k}_t). \tag{13}$$

2.4 Intertemporal equilibrium conditions

In a competitive equilibrium, agents take prices, return rates, and the tax rate τ as given and choose in all periods quantities which are individually optimal and, at the aggregate level, consistent with the economy's resource constraint and the government's budget constraint. Specifically, a competitive equilibrium needs to satisfy:

Definition *Given the predetermined variables listed in (A 1), a competitive equilibrium consists of a tax rate* τ and *a sequence of prices* $\{p_t,\, I_t,\}$, *nominal asset supplies* $\{\widetilde{B}_t,\, \widetilde{M}_t\}$ *and quantities* $\{k_t,\, c^y_{CR,t},\, c^o_{CR,t},\, c^o_{CA,t},\, g_t\}$ *such that in all periods: i) the markets of government bonds, physical capital, money, and output clear, re-*

spectively, according to:[8]

$$\widehat{b}_t = \frac{\widetilde{B}_t}{N_t \cdot p_t} = (1+n) \cdot b_t \tag{14}$$

$$\widehat{k}_t = \frac{K_t}{N_t} = (1+n) \cdot k_t \tag{15}$$

$$\widehat{m}_t = \frac{\widetilde{M}_t}{N_t \cdot p_t} = (1+n) \cdot m_t \tag{16}$$

$$f(k_{t-1}) = c^y_{CR,t} + \frac{c^o_{CR,t} + c^o_{CA,t}}{1+n} + g_t + (1+n) \cdot k_t - (1-\delta) \cdot k_{t-1}, \tag{17}$$

ii) the budget constraint of the government (5) is satisfied,
iii) labour and capital are competitively paid according to (1) and (2),
iv) consumption plans of agents are optimal under price taking behaviour according to (8)-(12),
v) return rates satisfy (7).

Using (14)-(16) within (1)-(13) it is possible to derive the following set of intertemporal equilibrium conditions which describe compactly the evolution of the economy over time:

$$I_t = R(k_t) \cdot \frac{p_{t+1}}{p_t} > 1 \tag{18}$$

$$d^p_t + R(k_{t-1}) \cdot b_{t-1} = (1+n) \cdot b_t + (1+n) \cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1} \tag{19}$$

$$k_t + m_t + b_t = \frac{c}{1+n} \cdot f(k_{t-1}) \tag{20}$$

$$m_t = (1-z) \cdot \frac{c}{1+n} \cdot f(k_{t-1}), \tag{21}$$

where $c = \frac{\beta}{1+\beta} \cdot (1-\tau) \cdot (1-\alpha) \in (0,1)$ and (21) results from substituting the equilibrium version of (13) into (20). The dynamical system (18)-(21) has a transparent structure. The inequality (18) ensures that the cash-in-advance constraint is binding and that all three assets are valued in equilibrium. Equation (19) restates the budget constraint of the government. Equation (20) represents a standard accumulation equation of Diamond-type overlapping generation economies. Owing to

[8] Equation (17) uses the notation $g_t = \widetilde{G}_t/(p_t \cdot N_t)$. Note that the labour market clears by assumption at the full employment level according to equation (2). Moreover, one can verify that the market clearing conditions (14)-(17) are by Walras' Law not independent, i.e. if one assumes within (1)-(12) that the markets for bonds, capital, and money clear, the output market needs to be in equilibrium as well.

the assumption of a constant savings rate, (20) is fully backward-looking, similar to the dynamics of a textbook Solow-model.[9] Finally, equation (21) is a simple version of the quantity theory of money with constant velocity. In other words, by imposing a unit elasticity of substitution between cash goods and credit goods we have removed all forward-looking features of money demand and established a money market equilibrium with strong 'monetarist' features in which, for a given output level, the price level and the nominal money stock are proportional to each other. Evidently, this leads to very tractable dynamics. As we show in a separate analysis in section 4, our main findings derived below remain qualitatively unaffected, however, if we relax this assumption.

2.5 Long-run targets of monetary and fiscal policy

As it stands, the system (18)-(21) is not yet fully specified, since it lacks a description of monetary and fiscal policy. To start out with, we pin down the long-run behaviour of the system by making two assumptions. First, monetary policy has a constant inflation target (π). Second, to provide a long-run anchor for the evolution of government debt, fiscal policy is assumed to target a constant deficit ratio (χ), which corresponds in steady state one-to-one to a certain debt-ratio. Specifically, going back to (4), we assume that the nominal steady-state budget deficit is a constant fraction of nominal output:

$$\widetilde{D}_t^p + (I-1)\cdot \widetilde{B}_{t-1} - (\widetilde{M}_t - \widetilde{M}_{t-1}) = \widetilde{B}_t - \widetilde{B}_{t-1} = \chi \cdot p_t \cdot f(k) \cdot N_t, \qquad (22)$$

$$\Leftrightarrow \chi = [1 - \frac{1}{(1+\pi)(1+n)}] \cdot \frac{(1+n)\cdot b}{f(k)} \approx (n+\pi)\cdot \frac{(1+n)\cdot b}{f(k)}, \qquad (23)$$

where we use the fact that in steady state $k_t = k$, $b_t = b$, $I_t = I = R(k)\cdot(1+\pi)$, and $\widetilde{B}_t/(p_t \cdot N_t \cdot f(k)) = (1+n)\cdot b/f(k)$ denotes the (end-of-period) steady-state debt ratio. Note that the particular notion of the government's deficit maintained in equation (22) implies that in the special situation of a balanced budget-rule ($\chi = 0$) the sum of the nominal primary deficit and of interest payments on government bonds, measured net of seigniorage revenue, must be equal to zero. To rationalize this notion of the government's deficit, it is easy to verify that it is in line with the well-known budgetary arithmetic of the European Stability and Growth Pact. Assuming exogenously, for example, a growth rate of $n = 0.03$ and an inflation target of $\pi = 0.02$, (23) says that the debt ratio can be approximately stabilized

[9] Under the assumption that the interest elasticity of savings is sufficiently small, (20) can be rationalized as a short-cut to escape the technically more tedious case of a flexible, partly forward-looking savings decision $s_t = s(R(k_t), w(k_{t-1}))$, as studied in detail by Galor and Ryder (1989). For related papers which also follow this short-cut, see, in particular, Schreft and Smith (1997, 1998).

at 60% if the steady-state deficit ratio is set at 3%, as predicted by the well-known 'Maastricht-arithmetic'.[10]

Within certain feasibility limits, the arithmetic in (23) is consistent with steady-state constellations which are equally characterized by fiscal deficits or surpluses and price inflation or deflation, and any such constellation can be used to study revaluation effects of government debt. Since the focus of this paper is not on establishing optimal target values of monetary and fiscal policy, no additional insights emerge in the following analysis if one goes through all these combinations. Instead, to ease the exposition, we find it convenient to restrict the set of monetary and fiscal target values at the outset simply to non-negative numbers:

(A 4) Long-run targets of monetary and fiscal policy: $\chi \geq 0$, $\pi \geq 0$

From a steady-state perspective, (23) can be used to replace (19). Define

$$\overline{\chi} = z \cdot c \cdot (1 - \frac{1}{(1+n)(1+\pi)}) \in (0,1). \tag{24}$$

Then substituting out for m and b, the system (18)-(21) reduces in steady state to

$$R(k) \cdot (1+\pi) > 1 \tag{25}$$

$$\frac{f(k)}{k} = \frac{1+n}{zc(1-\frac{\chi}{\overline{\chi}})}, \tag{26}$$

with m and b being recursively determined by the steady-state versions of (20) and (21). Invoking (A 2), (25)-(26) has at best a unique steady state, as graphed in Figure 1. In line with the idea of active monetary policy, we assume that the monetary agent is free to choose an inflation target ($\pi \geq 0$), while we restrict the passive fiscal agent to a choice of χ which, taking as given the inflation target, is consistent with $k > 0$ (and, hence, $y > 0$). As to be seen from (26), $\overline{\chi} = \overline{\chi}(\pi)$ defines for any particular inflation target an upper feasibility bound for the steady-state deficit ratio, i.e. $\overline{\chi}(\pi)$ establishes the upper bound for the amount of steady-state debt which can be rolled over between generations. In particular, the fact that this bound depends itself positively on steady-state inflation reflects that the fiscal agent might have an incentive to challenge the inflation target in order to relax his own constraint, as long as he is not disciplined by a mix of active monetary and passive fiscal policy.[11] In (26), higher steady-state deficit ratios lead to lower levels of the

[10] This particular type of 'arithmetic' depends on the appropriate book-keeping treatment of the seigniorage term in the budget constraint of the government. In the companion paper von Thadden (2002), taking a 'flow-of-funds' perspective, the deficit ratio is not defined net of seigniorage contributions, leading to a different budgetary arithmetic.

[11] In steady-state comparison, a higher inflation rate leads to higher seigniorage income which in turn provides the additional revenue to stabilize a higher debt ratio over time.

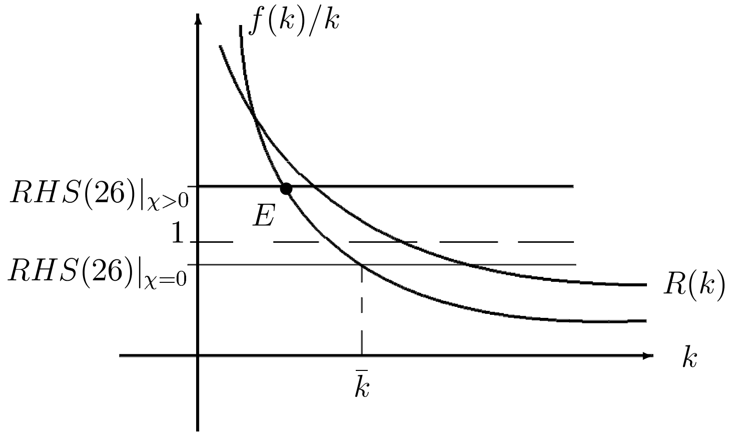


Figure 1: *Existence of a steady state ($\chi > 0$)*

steady-state capital intensity (and of the steady-state capital stock) because of the crowding out of physical capital through government bonds. Hence, the inequality (25) will be satisfied for all $\chi \in [0, \overline{\chi})$, if it is satisfied at $\chi = 0$. To this end, we evaluate $R(k)$ at $\chi = 0$ (using from (A 2), $f'(k) = \alpha f(k)/k$)) and make the mild restriction on structural parameters:

(A 5) $\frac{1+\beta}{\beta} \frac{\alpha}{1-\alpha} \frac{1+n}{z} - \delta > 0.$

From (A 5) follows directly:

Proposition 1 *(Existence of a steady state)*
Consider an inflation target $\pi \geq 0$. Then, if $\chi \in [0, \overline{\chi})$, there exists a unique steady state, satisfying (25) and (26), with $k > 0$, $m > 0$, $b \geqslant 0$, $I > 1$.

Remark: Figure 1 shows a steady state with a strictly positive deficit ratio, i.e. $\chi \in (0, \overline{\chi})$, denoted by E. Equation (26) has at $\chi = 0$ the solution $\overline{k}$, which is independent of π. Assumption (A 5) ensures $R(\overline{k}) > 1$, implying $I(k) > 1$ for any $\pi \geqslant 0$ and $\chi \in [0, \overline{\chi})$.[12]

Steady states of this type, however, do not need to be stable, and the restriction to deliver stable dynamics is the second key element of our definition of passive fiscal policy to which we turn in the following section.

[12] According to (A 5), $R(\overline{k}) > 1$ will be satisfied if the structural parameters, ceteris paribus, tend to increase the real return factor (R) by means of a low depreciation rate and a low level of capital formation (through a low propensity $(1-\alpha) \cdot \beta/(1+\beta)$ to save out of factor income $f(k)$ and a low share of credit goods (z) in consumption). If (A 5) is violated, χ needs to respect $\chi \in (\underline{\chi}, \overline{\chi})$, with $\underline{\chi} > 0$ determining the minimum crowding out effect which is required to establish $I(k) > 1$.

3 Dynamic properties of the benchmark economy: strict inflation targeting vs. constant money growth

The long-run position of the economy summarized in Proposition 1 can be supported by various monetary and fiscal policy rules with potentially different stability properties, depending on the off-steady state dynamics implied by these rules.[13] To rule out the possibility of unstable dynamics, however, we assume in the following that fiscal policy takes the specification of monetary policy as given and accepts the residual role, whenever necessary, to stabilize the local dynamics of the steady states established in Proposition 1. Using this particular notion of active monetary and fiscal policy, the main purpose of this section is to show that different monetary policy rules imply different restrictions on the behaviour of the fiscal agent, depending on whether the monetary agent follows outside the steady state a policy of strict inflation targeting or a constant money growth rule.

To link the initial conditions summarized in (A 1) to the steady state analysis, assume that the economy has been in steady state up to period $t-1$ and experiences at the beginning of period t, before all activities with index t start, a one-time temporary productivity shock, $\phi_t \neq \phi$. Specifically, to illustrate the following findings, we consider an adverse productivity shock ($\phi_t < \phi$), implying that the output in period t is below its steady state level. Monetary policy can react to this shock according to two different rules. First, we consider a policy of strict inflation targeting which is characterized by the commitment to keep from period t onwards the inflation rate at its steady state level π. Alternatively, we consider a policy of a constant money growth rule which accommodates the shock. According to this latter policy, the aggregate nominal money supply grows from period t onwards at the constant steady state rate μ, with $1+\mu = (1+\pi)\cdot(1+n)$. In sum, we compare:

(A 6) Monetary policy rules:
i) *Strict inflation targeting*: $p_t/p_{t-1} = 1+\pi$
ii) *Constant money growth rule:* $\widetilde{M}_t = (1+\mu)\cdot\widetilde{M}_{t-1} \Leftrightarrow \widetilde{m}_t = \frac{1+\mu}{1+n}\cdot\widetilde{m}_{t-1}$

Given the realization of ϕ_t, 'strict inflation targeting' and a 'constant money growth rule' are two special cases of deterministic money supply rules. In the first case the money supply is adjusted flexibly to maintain a constant inflation rate. In the second case the money supply follows a constant growth rate, making thereby the

[13] Although our analysis is not conducted in terms of a game, we loosely use the term 'rule' in order to indicate that all monetary and fiscal actions are assumed to be perfectly and costlessly foreseen by the private sector, i.e. government behaviour does not suffer from a a discretionary bias.

inflation rate endogenous. Evidently, along the balanced growth path (i.e. in the absence of the productivity shock) the realizations of p_t and $\widetilde{m}_t$ are identical under the two rules, keeping $m_t = m$ constant over time.
Finally, to complete the description of off-steady-state behaviour, we assume, in a first step, that the fiscal agent follows a policy which keeps the deficit ratio from t onwards constant at the steady state value χ :

(A 7) Fiscal policy rule:

$$\chi \cdot f(k_{t-1}) = (1+n) \cdot b_t - \frac{p_{t-1}}{p_t} \cdot b_{t-1}. \tag{27}$$

As we show in the next two subsections, the key difference between strict inflation targeting and a constant money growth rule is the different valuation of previously emitted nominal government debt in the government's budget constraint, i.e. the term $p_{t-1} \cdot b_{t-1}/p_t$. Equation (27) offers a deliberately rigid specification of the off-steady state behaviour of fiscal policy which strongly amplifies the dynamic effects of different valuations of public debt over time, illustrating thereby the mechanics of the revaluation channel of government debt in a particularly strong way. Notice, however, that we allow in Section 3.3 for a more flexible version of (27) and show that this change leaves the qualitative nature of the following findings unaffected.

3.1 Dynamics under strict inflation targeting

Starting out from (18)-(21) and with monetary and fiscal policy being specified by part i) of (A 6) and (A 7), dynamics of the economy are locally governed by

$$\chi \cdot f(k_{t-1}) = (1+n) \cdot b_t - \frac{1}{1+\pi} \cdot b_{t-1} \tag{28}$$

$$k_t + m_t + b_t = \frac{c}{1+n} \cdot f(k_{t-1}) \tag{29}$$

$$m_t = (1-z) \cdot \frac{c}{1+n} \cdot f(k_{t-1}), \tag{30}$$

where (28) replaces (19) and the economy is assumed to be sufficiently close to the steady state such that (18) is satisfied as a strict inequality. Importantly, one infers from (28) that strict inflation targeting precludes revaluations of government debt, i.e. the real value of outstanding government debt emitted in the previous period $(b_{t-1}/(1+\pi))$ is stabilized at its pre-shock steady state level. To establish the stability behaviour of (28)-(30) we substitute out for m_t, yielding a two-dimensional system in k_t and b_t :

$$\chi \cdot f(k_{t-1}) = (1+n) \cdot b_t - \frac{1}{1+\pi} \cdot b_{t-1} \tag{31}$$

$$k_t = \frac{zc}{1+n} \cdot f(k_{t-1}) - b_t. \tag{32}$$

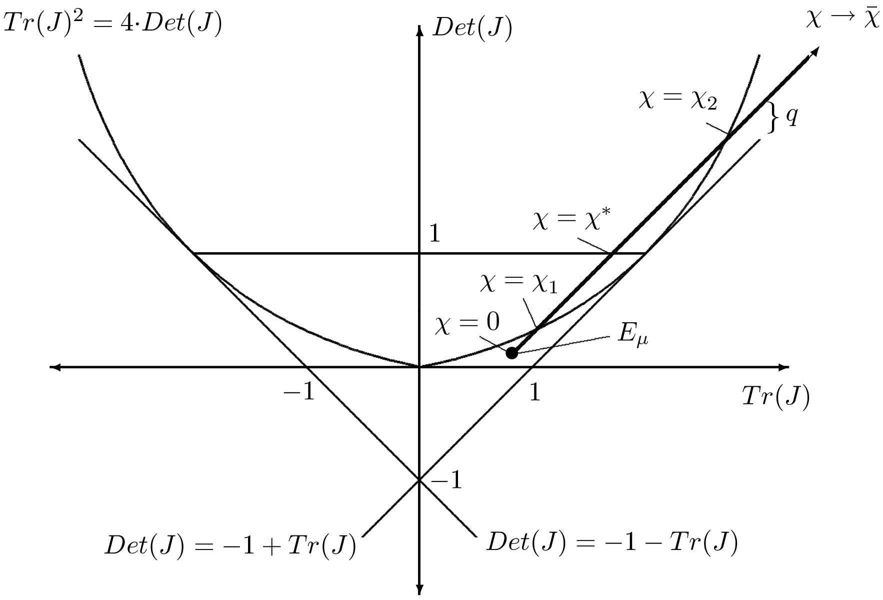


Figure 2: *Dynamic behaviour of strict inflation targeting vs. constant money growth*

The dynamics of (31)-(32) are entirely backward-looking since, according to (A 1), both k_{t-1} and b_{t-1} are predetermined variables.

Proposition 2 *(Local dynamics: stability)*
There exists a unique $0 < \chi^* < \overline{\chi}$ *such that for*
i) $\chi \in [0, \chi^*)$ *dynamics are locally stable,*
ii) $\chi \in (\chi^*, \overline{\chi})$ *dynamics are locally unstable.*

Proof: see appendix.

Proposition 2 says that local dynamics of (31)-(32) become explosive beyond a certain threshold value of the deficit ratio χ^*, i.e. not all feasible steady-state values of χ can be stabilized by the combination of strict inflation targeting and a rigid fiscal rule as given by (27), restricting thereby the fiscal agent in his choice of χ beyond the requirements of Proposition 1. Moreover, as an aside, one can show that dynamics, even when being stable, may well be subject to endogenous volatility:

Proposition 3 *(Local dynamics: monotone vs. fluctuating adjustment)*
There exists a pair $\{\chi_1,\chi_2\}$, *with* $0 < \chi_1 < \chi^* < \chi_2 < \overline{\chi}$ *such that for*
i) $\chi \in [0, \chi_1)$ *dynamics are monotone,*
ii) $\chi \in (\chi_1, \chi_2)$ *dynamics are fluctuating,*
iii) $\chi \in (\chi_2, \overline{\chi})$ *dynamics are monotone.*

Proof: see appendix.

Figure 2 illustrates Propositions 2 and 3. Specifically, Figure 2 plots the determinant and the trace of the Jacobian matrix associated with the two-dimensional system in k_t and b_t as a function of χ, $\chi \in [0, \overline{\chi})$. As derived in the appendix, the pairs of the determinant and the trace resulting from this comparative statics exercise are simply lined up along the straight line drawn in bold type, which crosses the stability regions as summarized in Propositions 2 and 3.

Propositions 2 and 3 reveal that in (31) and (32) standard (i.e. Solow-type) accumulation dynamics interact with portfolio composition dynamics induced by the government budget constraint (31) in a rich way. To get a better intuitive grasp of these findings, one infers from the steady-state version of (31) that for given values of n and π the steady-state level of b is subject to a 'Laffer-type' effect, i.e. b is a hump-shaped function of the deficit ratio χ. At $\chi = 0$, evidently $b = 0$. As χ rises, $f(k)$ falls because of the crowding out of capital, implying that b has a unique maximum and reaches zero again as $\chi \to \overline{\chi}$ (since $k \to 0$). Hence, any feasible long-run level of b is generically associated with a low (χ^L) and a high (χ^H) level of the deficit ratio. The dynamic implications of this structure become transparent if one combines (31) and (32), yielding

$$k_t = \frac{zc - \chi}{1+n} \cdot f(k_{t-1}) - \frac{b_{t-1}}{1+\mu}. \tag{33}$$

Figure 3 depicts the two steady-state combinations of (33) consistent with some particular level of $b > 0$.[14] To rationalize the stability findings of Proposition 2, consider the effects of a one-time temporary productivity shock ($\phi_t < \phi$), implying a downward shift of the production function for one period.[15] At χ^L, there is little crowding out of capital and the marginal productivity of capital is 'low', leading to 'standard' accumulation dynamics, i.e. on impact the output and ensuing savings losses in response to the shock are 'small' and over time the economy returns to the steady state as capital formation recovers. By contrast, at the steady state associated with χ^H the production function is 'steep' because of strong crowding out effects (i.e. it crosses the 45°-degree line from below), leading to large output losses on impact and destabilizing dynamics.

According to Proposition 3, as the system transits from stable to unstable dynamics, there exists an interval of χ where the system fluctuates. The reason for this finding is that the composition of savings changes towards government bonds as χ rises. Depending on the particular value of χ, this portfolio composition effect leads

[14] At the maximum level of b, the two curves depicted in Figure 3 coincide and have a unique tangency point with the line $k_t = k_{t-1}$.

[15] In Figure 3, the RHS of equation (33) consists of two branches, since χ^L and χ^H are associated with different slopes of the production function.

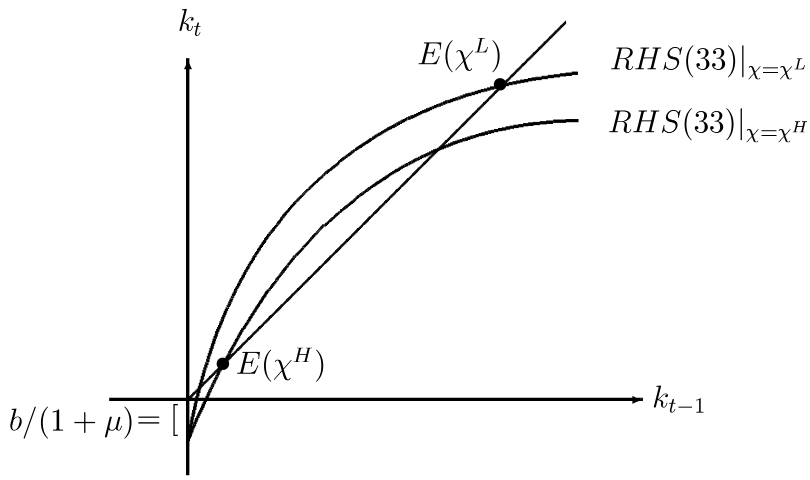


Figure 3: *Stable vs. unstable dynamics under strict inflation targeting*

to different adjustment speeds of k and b. This is illustrated, in the appendix, in Simulations $1-5$ which report impulse-response patterns as χ moves from 0 to $\overline{\chi}$. If $\chi = 0$, dynamics are stable and entirely standard, since government bonds never enter the picture. As χ rises, deviations of k from the steady state become more persistent due to the smaller share of k in overall savings. According to the budget rule (31), this implies that the deficit remains for a longer time span below its steady state level, leading in turn to a persistent decline in b. Specifically, if $\chi \in (\chi_1, \chi^*)$, as reported in Simulation 3, the delayed recovery of k triggers a sufficiently offsetting, persistent decline in b and induces stable, endogenous fluctuations around the steady state. If $\chi \in (\chi^*, \chi_2,)$, as reported in Simulation 4, the timing is such that these fluctuations reinforce each other over time and generate explosive dynamics. Finally, if $\chi \in (\chi_2, \overline{\chi})$ the adverse impact effect on output and capital formation is sufficiently strong to dominate the portfolio composition effect induced by the budget constraint, leading to monotonically diverging dynamics (Simulation 5).[16]

3.2 Dynamics under a constant money growth rule

Under a constant money growth rule, the relationship $\widetilde{m}_t = \frac{1+\mu}{1+n} \cdot \widetilde{m}_{t-1} = (1+\pi) \cdot \widetilde{m}_{t-1}$ established in part ii) of (A 6) can be used to express the government's budget constraint as

$$\chi \cdot f(k_{t-1}) = (1+n) \cdot b_t - \frac{1}{1+\pi} \cdot \frac{m_t}{m_{t-1}} \cdot b_{t-1}, \tag{34}$$

[16]In terms of Figure 3, $\chi^H \to \overline{\chi}$ implies $b \to 0$, i.e. if one rethinks the analysis in terms of a conventional Solow-framework $\chi^H \to \overline{\chi}$ essentially selects the zero-activity steady state with unstable, monotone dynamics.

implying that the real value of outstanding government debt is no longer stabilized at its pre-shock steady state level, but rather depends on the realization of m_t. Accordingly, local dynamics are now governed by the system (29), (30), and (34). Again, upon substituting out for m_t, it is straightforward to represent these three equations by means of a two-dimensional, fully backward-looking system in k_t and b_t with initial conditions k_{t-1} and b_{t-1}. As derived in the appendix, the stability behaviour of this system, however, is different from the findings established in Propositions 2 and 3 :

Proposition 4 *(Local dynamics)*
Assume $\chi \in [0, \overline{\chi})$. Then dynamics are always locally stable and display monotone adjustment behaviour.

Proof: see appendix.

Proposition 4 says that all feasible steady-state values of χ can be stabilized by the combination of a constant money growth rule and a rigid fiscal rule as given by (27). To see intuitively why the stability behaviour is so different under strict inflation targeting and a constant money growth rule, consider again the experiment that in period t output is below trend because of an adverse productivity shock ($\phi_t < \phi$). According to (30), under both i) strict inflation targeting and ii) a constant money growth rule real balances m_t will be lower on impact by an identical amount, as one infers also from Simulations $1-5$. Under a constant money growth rule, however, the rise in the price level p_t must be stronger because of the more generous, non-state-contingent nominal money supply, leading to a stronger devaluation of the outstanding nominal debt $\widetilde{b}_{t-1}$. Hence, for a given deficit ratio χ fewer new bonds need to be emitted, leading to lower liabilities of the government in the future. To put it differently, a constant money growth rule acts like an automatic stabilizer of the debt dynamics of the economy, since it accommodates in 'bad' times a rise of the price level which triggers a devaluation of government debt.[17] This in turn reduces the necessity to emit new bonds, thereby leading to less crowding out and more physical capital formation. Correspondingly, if the economy's initial position results from a positive productivity shock ($\phi_t > \phi$), a constant money growth rule tolerates that the price level will be temporarily below trend, implying a revaluation of government debt and a self-stabilizing reduction of capital formation. As a general feature, this built-in-mechanism of a constant money growth rule is strong enough to stabilize all feasible steady-state deficit ratios. Moreover, it also

[17] Evidently, from a more general perspective, elements of the tax system which are not inflation-neutral have qualitatively similar automatic stabilization properties within the government's budget constraint.

precludes the possibility of endogenously arising volatility during the adjustment towards the steady state.[18]
By contrast, as derived above, a policy of strict inflation targeting lacks the stabilizing effects of the revaluation channel of government debt and dynamics become explosive if the initial debt exposure (measured by χ) is sufficiently high. Note that in the special case of $\chi = 0$ (implying $\widetilde{b}_{t-1} = 0$) under both strict inflation targeting and a constant money growth rule the revaluation channel of government debt is switched off. We establish in the appendix that in this special case the expressions for the trace and the determinant are identical, implying an identical stability behaviour (for an illustration, see Simulation 1). Under a constant money growth rule the trace and the determinant are always given by these particular expressions (denoted by E_μ in Figure 2), independently of χ. By contrast, under strict inflation targeting dynamics depend on the magnitude of χ, and as the debt exposure rises debt dynamics eventually become unstable.

3.3 Alternative specifications of fiscal policy

The strong result of potentially unstable dynamics under strict inflation targeting as summarized in Proposition 2 results from the fact that, according to (A 7), not only monetary but also fiscal policy attempts to maintain its long-run target outside the steady state. We show in this section that this result can be overcome if strict inflation targeting is combined with a fiscal policy which accepts temporary deviations of χ_t from χ with the aim to deliver stable dynamics for all $\chi \in [0, \overline{\chi})$. This additional flexibility in the fiscal rule is needed as a substitute for the self-stabilizing revaluation channel of government debt which is absent under strict inflation targeting. Thus, this section establishes the simple insight that the stabilization of all steady states consistent with Proposition 1 requires under strict inflation targeting additional fiscal measures which are not needed under a constant money growth rule.
Under the specification (A 7), fiscal policy is slightly procyclical, since, to keep the deficit ratio χ_t constant when output is below trend, the deficit itself must also be lower than in steady state.[19] However, purely on stability grounds, this procyclical

[18] In terms of the experiment conducted in Figure 3, the key difference is that equation (33) now changes to

$$k_t = \frac{zc - \chi}{1+n} \cdot f(k_{t-1}) - \frac{1}{1+\mu} \cdot \frac{m_t}{m_{t-1}} \cdot b_{t-1},$$

implying that the considered negative productivity shock in period t on impact not only reduces the slope of the production function, but, through the relationship $m_t = (1-z)\frac{c}{1+n} \cdot f(k_{t-1})$, also shifts the intercept upwards.

[19] Remember that the tax rate τ is assumed to be constant. Hence, adjustments in the (primary) deficit occur in our model through variations of government spending g_t.

element is not strong enough, as shown in Proposition 2, to stabilize a steady state with a high debt burden ($\chi > \chi^*$) under strict inflation targeting. Alternatively, we consider now a more strongly procyclical fiscal policy which fixes the (end-of-period) steady-state debt ratio, to be denoted by θ, over time:

(A 7a) Fiscal policy rule:

$$\widetilde{B}_t = \theta \cdot p_t \cdot f(k_{t-1}) \cdot N_t \Leftrightarrow (1+n) \cdot b_t = \theta \cdot f(k_{t-1}) \tag{35}$$

Using (35), the budget constraint in period t can be rewritten as

$$d_t = \theta \cdot f(k_{t-1}) - \frac{1}{1+\pi} b_{t-1}, \tag{36}$$

where d_t denotes the real per capita deficit. In contrast to (A 7), a temporary reduction in output now leads, on impact, to a below-trend deficit ratio ($\chi_t < \chi$), implying stronger adjustments of the primary deficit. Combining (35) and (36) with the system (29)-(30), one obtains a fully recursive system with one-dimensional dynamics in k_t which are given by the equation

$$k_t = (\frac{zc - \theta}{1+n}) \cdot f(k_{t-1}). \tag{37}$$

Assessing the local dynamics of (37) by a linear approximation around the steady state yields the expression $dk_t = \alpha \cdot dk_{t-1}$, $\alpha \in (0,1)$. Thus, dynamics are now stable independently of the magnitude of χ, since the role of the fiscal shock absorber is now fully played by procyclical adjustments in the deficit.

Similar to growth models, the focus of our model is exclusively on supply-side-features. This makes it difficult to rationalize within our model the notion of anti-cyclical fiscal policy, as typically suggested by models which allow for short-run frictions and Keynesian demand features. Despite this shortcoming of our model, consider for the sake of the argument the following flexible deficit rule which has an anticyclical component (through ψ) and leaves room for autonomous actions a_t.

(A 7b) Fiscal policy rule:

$$\chi_t = \chi - \psi(\frac{y_t - y}{y}) + a_t, \quad \psi > 0. \tag{38}$$

In the light of (38), one can easily imagine a path of fiscal policy which accommodates on impact a negative output shock by a temporary rise in the deficit ratio and spreads the required fiscal adjustments out over the following periods by appropriate contractionary choices of a_t.

In any case, the 'need' for the additional adjustment measures implied by alternative fiscal rules like (35) or (38) originate in our model exclusively from the lack of self-stabilizing revaluations of government debt under strict inflation targeting. Correspondingly, purely on stability grounds, amendments as given by (35) or (38) are not required under a constant money growth rule.

4 Extension: A forward-looking money-demand specification

This section relaxes the simplifying assumption of a money demand with constant velocity. To this end, we remove in (A 3) the assumption of a unit elasticity between cash goods and credit goods. Instead, cash and credit goods are now specified as gross substitutes to ensure that real balances depend not only on current period output, but also on the expected path of future prices. We maintain, however, the assumption of a constant savings rate, and, to further simplify the analysis, we normalize the constant savings rate (s) to $s = 1$.[20] Deviating from the log-specification summarized in (A 3) we now assume instead:

(A 3a) Preferences of the representative agent:

$$U = \frac{1}{1-\varepsilon} \cdot [z \cdot (c^o_{CR,t+1})^{1-\varepsilon} + (1-z) \cdot (c^o_{CA,t+1})^{1-\varepsilon}] \quad \varepsilon \in (0,1),\ z \in (0,1).$$

As derived in the companion paper von Thadden (2002), this leads to a generalized money demand of the form

$$m_t = \frac{1}{1 + \widehat{z} \cdot I_t^{\frac{1-\varepsilon}{\varepsilon}}} \cdot \frac{c}{1+n} \cdot f(k_{t-1}), \qquad \text{with: } \widehat{z} = (z/(1-z))^{1/\epsilon}, \tag{39}$$

which, owing to the gross substitutability assumption ($\varepsilon \in (0,1)$), implies that real balances depend negatively on the nominal interest rate. Note that in the limiting case of $\varepsilon = 1$, (39) turns into (21) as discussed above. We invoke again the constant deficit rule (A 7). In sum, the system of intertemporal equilibrium conditions reads as:

$$I_t = R(k_t) \cdot \frac{p_{t+1}}{p_t} > 1 \tag{40}$$

$$\chi \cdot f(k_{t-1}) = (1+n) \cdot b_t - \frac{p_{t-1}}{p_t} \cdot b_{t-1} \tag{41}$$

$$k_t + m_t + b_t = \frac{c}{1+n} \cdot f(k_{t-1}) \tag{42}$$

$$m_t = \frac{1}{1 + \widehat{z} \cdot I_t^{\frac{1-\varepsilon}{\varepsilon}}} \cdot \frac{c}{1+n} \cdot f(k_{t-1}), \tag{43}$$

[20] With $s = 1$, the discount factor β can also be normalized to $\beta = 1$, without loss of generality.

where now $c = (1-\tau)\cdot(1-\alpha) \in (0,1)$. Note that the only qualitative difference to the system discussed in Section 3 is the forward-looking component of the money demand specification in (43). Investigating (40)-(43), the purpose of this section is to show that the inclusion of the forward-looking component in (43) leaves the findings of Section 3 qualitatively unaffected.
As a first step, to establish an existence result for a unique steady state under either monetary policy rule along the lines of Proposition 1, let in (43)

$$B_t = \frac{1}{1 + \widehat{z}\cdot[R(k_t)\cdot\frac{p_{t+1}}{p_t}]^{\frac{1-\varepsilon}{\varepsilon}}}. \tag{44}$$

Substituting out for m and b in (40)-(43) and using (44) gives the steady-state conditions

$$R(k)\cdot(1+\pi) \;>\; 1 \tag{45}$$

$$\frac{f(k)}{k} \;=\; \frac{1+n}{c(1-B(k)) - \frac{\chi\cdot(1+n)(1+\pi)}{(1+n)(1+\pi)-1}}. \tag{46}$$

Owing to the relationship between real balances and the (real) interest rate, the right-hand-side of (46), in terms of Figure 1, is now upwardsloping. Let $\overline{\chi}^F$ denote the upper bound for the deficit-ratio under a forward-looking money demand, with:

$$\overline{\chi}^F = c(1 - \frac{1}{(1+n)(1+\pi)}) \in (0,1).$$

Intuitively, $\overline{\chi}^F$ is larger than $\overline{\chi}$ established in (24) since the demand for real balances is no longer invariant to changes in the interest rate. Thus, a rise in the real interest rate induced by the crowding out effect of a higher deficit ratio χ leads now, for any given inflation target, to a reduction in real balances. This partly offsets the adverse effects on capital formation and raises the maximum amount of debt the economy can sustain over time.

Proposition 5 *(Existence of a steady state)*
Consider an inflation target $\pi \geq 0$. Let $\overline{k}$ denote the unique solution of (46) at $\chi = 0$ and assume $R(\overline{k}) > 1$. Then, if $\chi \in [0, \overline{\chi}^F)$, there exists a unique steady state, satisfying (45) and (46), with $k > 0$, $m > 0$, $b \geqslant 0$, $I > 1$.

Remark: If $\varepsilon \to 1$, $[1 - B(k)] \to z$, as discussed above. For $0 < \varepsilon < 1$, $B(0) \to 0$, $B(\infty) \to B^{\max} \in (0,1)$, and $B'(k) > 0$, implying that, in terms of Figure 1, the RHS of (46) rises in k. The LHS of (46) falls in k, and (46) has a unique, positive solution in k if $\chi \in [0, \overline{\chi}^F)$. As in Proposition 1, $R(\overline{k}) > 1$ ensures that (45) holds for all $\pi \geq 0$ and $\chi \in [0, \overline{\chi}^F)$.

We now turn to an assessment of the local dynamics of steady states of this type under strict inflation targeting and a constant money growth rule.

4.1 Dynamics under strict inflation targeting

Strict inflation targeting makes the forward-looking element in (43) fully predictable, since $I_t = R(k_t) \cdot (1+\pi)$. Because of this feature, local steady-state dynamics remain two-dimensional and fully backward-looking, as in Section 3.1, and the stability properties of Propositions 2 and 3 remain qualitatively unaffected. Specifically, the economy still reaches all the stability regions indicated in Figure 2 upon variations of χ, $\chi \in [0, \overline{\chi}^F)$. The only (technically tedious) difference is that the expressions for the trace and the determinant are no longer linked in a linear way. As derived in the appendix, local dynamics are given by:

Proposition 6 *(Local dynamics: stability)*
There exists a unique $0 < \chi^* < \overline{\chi}^F$ *such that for*
i) $\chi \in [0, \chi^*)$ *dynamics are locally stable,*
ii) $\chi \in (\chi^*, \overline{\chi})$ *dynamics are locally unstable.*

Proof: see appendix.

Similarly, one obtains:

Proposition 7 *(Local dynamics: monotone vs. fluctuating adjustment)*
There exists a pair $\{\chi_1, \chi_2\}$, *with* $0 < \chi_1 < \chi^* < \chi_2 < \overline{\chi}$ *such that for*
i) $\chi \in [0, \chi_1)$ *dynamics are monotone,*
ii) $\chi \in (\chi_1, \chi_2)$ *dynamics are fluctuating,*
iii) $\chi \in (\chi_2, \overline{\chi})$ *dynamics are monotone.*

Proof: see appendix.

Propositions 6 and 7 simply generalize Propositions 2 and 3 to a more flexible specification of the elasticity of substitution ε. As we show in the next section, a similar generalization is not possible under a constant money growth rule.

4.2 Dynamics under a constant money growth rule

Under a constant money growth rule, with $I_t = (1+\pi) \cdot R(k_t) \cdot m_t/m_{t+1}$, the current interest rate depends on the realization of next period's real balances (m_{t+1}) which is going to differ from m outside the steady state . Hence, the additional forward-looking component is genuinely expectation driven. Because of this feature, real balances turn into a forward-looking jump variable and the overall dynamics now contain both backward-looking and forward-looking elements. Accordingly, the question emerges whether the alleged 'gains' of a constant money growth rule in terms of the stabilizing influence on government debt dynamics survive the inclusion

of a forward-looking feature which might act, for example, as a source of indeterminacy. Algebraically, the answer to this question comes at the high cost that now one has to analyze the three-dimensional system (41)-(43) of non-linear first-order difference equations in which k and b act as predetermined variables, while m acts as a forward-looking jump variable. While in general the analytics of the dynamic structure of this system are rather involved, we offer in this section explicit solutions for two important special constellations. Then, in a second step, we extend these solutions to the general case by means of appropriate numerical output. Drawing on these two sources, we establish that the stability properties of the benchmark economy remain qualitatively unaffected in the sense that all feasible steady-state deficit ratios are dynamically approachable from fixed initial conditions in a smooth manner. Moreover, irrespective of the presence of the forward-looking component in (43), the fundamentals of the economy are strong enough to rule out indeterminate adjustment behaviour.
First, we consider the constellation of a balanced budget ($\chi = 0$), implying that the initial steady state is characterized by a zero debt stock ($b = 0$). Under this particular assumption, government debt dynamics can never take off. Correspondingly, the dynamics of the entire system have a recursive structure according to which the dynamics of b are stable and independent of a subsystem in k and m. As shown in the appendix, one can derive:

Proposition 8 *(Local dynamics under a balanced budget rule)*
Assume $\chi = 0$. *Then, eigenvalues are given by* $\lambda_1 = \frac{1}{(1+\pi)(1+n)} \in (0,1)$, $\lambda_2 \in (0,1), \lambda_3 > 1$, *implying that the steady state is dynamically approachable in a uniquely determined and smooth manner.*

Proof: see appendix.

Note that according to Proposition 8 the number of stable (unstable) eigenvalues matches the number of predetermined (forward-looking) variables. Thus, the inclusion of the forward-looking money demand specification does not lead to indeterminate dynamics, generalizing thereby the 'well-behaved' benchmark finding of Proposition 4. Second, one can show that the root structure as given by Proposition 8 remains qualitatively unchanged as the deficit ratio χ approaches the upper feasibility limit $\overline{\chi}^F$.

Proposition 9 *(Local dynamics as* χ *approaches the feasibility limit* $\overline{\chi}^F$*)*
Assume $\chi \to \overline{\chi}^F$. *Then, for* χ *sufficiently close to* $\overline{\chi}^F$, *eigenvalues are given by* $\lambda_1 \in (0,1)$, $\lambda_2 \in (0,1), \lambda_3 > 1$.

Remark: As χ moves towards $\overline{\chi}^F$, k falls because of the crowding out effect. In the limit, $k \to 0$. To establish Proposition 9, we approximate the dynamics of (41)-(43) in a forward-looking manner, $d\,x_t = J^{-1} \cdot d\,x_{t+1}$, where $x_t = (k_t,\, b_t,\, m_t)'$. We proof in the appendix that, as $\chi \rightarrow \overline{\chi}^F$, $Det(J^{-1}) \rightarrow 0$. As we show, this implies $\lambda_3^{-1} \rightarrow 0 \Leftrightarrow \lambda_3 \rightarrow \infty$, and by continuity of all entries of J^{-1} in χ one can establish the pattern of eigenvalues summarized in the proposition. For the detailed proof, see appendix.

Again, this finding generalizes Proposition 4 in the sense that the revaluation channel of government debt, when combined with an appropriate choice of m_t, is able to smoothly stabilize the model economy even for high degrees of debt exposure, as long as the feasibility limit is respected. Evidently, this result is in contrast to the dynamics under strict inflation targeting established in Section 4.1.
Table 1 (reported in the appendix) indicates the continuity of the findings established in the last two propositions for the entire interval $\chi \in [0, \overline{\chi}^F)$. As a benchmark, Table 1a considers a model economy along the lines of Section 3 and reproduces stability patterns for strict inflation targeting and a constant money growth rule which are consistent with Propositions $2-4$. In the simulations underlying Tables 1b and 1c all parameters have been kept constant, except for the money demand parameter ε, which we now allow to be less than unity. Note that under a constant money growth rule the stability pattern of eigenvalues as established in Propositions 8 and 9 holds for all reported values of $\chi \in [0, \overline{\chi}^F)$.
We conclude with two observations. First, Tables 1b and 1c reveal that for a policy of strict inflation targeting the inclusion of the forward-looking component has a stabilizing influence in the sense that the critical instability value χ^* depends positively on the interest elasticity of money demand (i.e. negatively on ε). Intuitively, this finding reflects that a rise in the interest elasticity of money demand moderates adverse crowding out effects on capital formation at any given level of the interest rate, since, ceteris paribus, agents will be more willing to replace real balances by interest-bearing assets in their portfolios. Second, from a calibration perspective, the two-period overlapping generations model without bequest motive has the unpleasant feature that the steady-state capital stock needs to be reproduced every period from savings out of wage income. For this reason, the feasibility bounds $\overline{\chi}$ and $\overline{\chi}^F$ will be implausibly low for reasonable values of the economy's inflation rate π and growth rate n. Table 1d 'corrects' for this feature and allows for a deliberately high value of n. As to be expected, this change in the parametrization raises $\overline{\chi}^F$ and induces a higher instability value χ^* under strict inflation targeting. Moreover, this change reduces the persistence of adjustment dynamics under a constant money growth rule, i.e. the larger of the two stable roots is less close to a unit root.

5 Conclusion

Following the seminal paper by Sargent and Wallace (1981), there is a large literature which stresses that the active control of inflation through the monetary agent requires that the fiscal agent accepts the residual role within the government's budget constraint, taking passively as given the behaviour of the monetary agent. For any particular inflation target, the fiscal implications of such a role assignment are likely to be quite different, however, depending on the details of the monetary policy rule. Specifically, as recently stressed by Woodford (2001), as long as government debt is issued largely in nominal terms, different monetary policy rules are likely to imply different government debt dynamics through rule-specific valuations of government debt, constraining thereby the fiscal agent in different ways.

Against this background, this paper offers a simple analytical framework to study effects of monetary policy on the valuation of outstanding government debt from a dynamic general equilibrium perspective which takes the desirability of a mix of active monetary and passive fiscal policy as given. The main idea of this paper is to illustrate that monetary policy may indeed constrain fiscal policy in rather different ways, depending on whether monetary policy accepts stabilizing revaluations of government debt or not. More specifically, using a monetary growth model with flexible prices, we compare the properties of two stylized monetary policy rules which have identical steady-state properties but require different actions out of steady state. First, we consider a policy of a constant money growth rule which allows for temporary deviations of inflation from target. As a result, there is scope for revaluations of public debt in response to shocks, and these revaluations are shown to act as automatic stabilizers of government debt dynamics. Second, we consider a policy of strict inflation targeting which implements the target inflation rate also outside the steady state. Essentially, such a policy fixes the value of government debt in real terms and precludes thereby stabilizing revaluations. As we show, this feature implies that additional fiscal restraint may be needed under strict inflation targeting which is not required under a constant money growth rule.

As it stands, our approach suffers from the fact that we do not establish optimal programs of monetary and fiscal policy. Similarly, our paper abstracts entirely from credibility issues. Extensions of the model along these dimensions would be important. Yet, given our particular assumption of overlapping generations with finite decision horizons, such extensions raise additional questions which are beyond the scope of this paper. Despite these shortcomings, the largely descriptive findings established in this paper do indicate, however, that the revaluation channel of government debt can be of considerable importance for the dynamic properties of an otherwise standard monetary growth model.

Appendix

Proof of propositions 2 and 3:
Consider the system (31)-(32) derived in the main text:

$$\chi f(k_{t-1}) = (1+n)b_t - \frac{1}{1+\pi} b_{t-1} \tag{47}$$

$$k_t = \frac{zc}{1+n} f(k_{t-1}) - b_t. \tag{48}$$

Approximating the local dynamics of (47) and (48) around the steady state by a first-order Taylor expansion gives

$$\begin{bmatrix} d\,k_t \\ d\,b_t \end{bmatrix} = J_\pi \cdot \begin{bmatrix} d\,k_{t-1} \\ d\,b_{t-1} \end{bmatrix} = \begin{bmatrix} \frac{zc-\chi}{1+n} f'(k) & -\frac{1}{(1+n)(1+\pi)} \\ \frac{\chi}{1+n} f'(k) & \frac{1}{(1+n)(1+\pi)} \end{bmatrix} \cdot \begin{bmatrix} d\,k_{t-1} \\ d\,b_{t-1} \end{bmatrix},$$

$$Det(J_\pi) = \frac{zc}{(1+n)(1+\pi)} \frac{f'(k)}{1+n},$$

$$Tr(J_\pi) = (zc-\chi)\frac{f'(k)}{1+n} + \frac{1}{(1+n)(1+\pi)},$$

where $Det(J_\pi)$ and $Tr(J_\pi)$ denote the determinant and the trace of the Jacobian matrix J_π, respectively.
i) Using $f'(k) = \alpha \frac{f(k)}{k}$ within the steady state relationship (26), one easily establishes: at $\chi = 0$, $f'(k) = \alpha \frac{1+n}{zc} > 0$; for $\chi \in [0, \overline{\chi})$, $f'(k)$ rises continuously in χ; as $\chi \to \overline{\chi}$, $f'(k) \to \infty$. Moreover, note that $(1+n)(1+\pi) > 1$ implies $\overline{\chi} < zc$. Hence, $Det(J_\pi) > 0$, $Tr(J_\pi) > 0$.
ii) Combining the terms describing $Det(J_\pi)$ and $Tr(J_\pi)$ one verifies that the following relationship holds:

$$Det(J_\pi) = -1 + Tr(J_\pi) + q,$$

where $q = (1-\alpha)\frac{(1+n)(1+\pi)-1}{(1+n)(1+\pi)} \in (0,1)$, and q is independent of χ.
iii) At $\chi = 0$, $Det(J_\pi) = \frac{\alpha}{(1+n)(1+\pi)} \in (0,1)$, $Tr(J_\pi) = \alpha + \frac{1}{(1+n)(1+\pi)}$, implying $\lambda_1 = \alpha \in (0,1)$, $\lambda_2 = \frac{1}{(1+n)(1+\pi)} \in (0,1)$.
iv) for $\chi \in [0, \overline{\chi})$, $Det(J_\pi)$ and $Tr(J_\pi)$ rise continuously in χ, where the latter claim follows from inspecting the rearranged trace-term $Tr(J_\pi) = \alpha \frac{zc-\chi}{zc(1-\frac{\chi}{\overline{\chi}})} + \frac{1}{(1+n)(1+\pi)}$.
v) As $\chi \to \overline{\chi}$, $Det(J_\pi) \to \infty$, $Tr(J_\pi) \to \infty$.
Upon combining i)-v), one obtains Figure 2. In particular, raising χ within the interval $[0, \overline{\chi})$ amounts in terms of Figure 2 to a continuous movement along the line $Det(J_\pi) = -1 + Tr(J_\pi) + q$ in north-east direction, implying the pattern of eigenvalues established in Propositions 2 and 3. □

Proof of proposition 4:
The relevant equation system is now given by (29), (30), and (34). To obtain again a two-dimensional system of first-order difference equations in k_t and b_t, we substitute (30) in (34) and (29), yielding:

$$\chi\frac{1+n}{zc}(k_t+b_t)(k_{t-1}+b_{t-1}) = (1+n)b_t(k_{t-1}+b_{t-1}) - \frac{k_t+b_t}{(1+\pi)}b_{t-1} \quad (49)$$

$$k_t = \frac{zc}{1+n}f(k_{t-1}) - b_t. \quad (50)$$

To approximate the local dynamics around the steady state we calculate

$$\begin{bmatrix} d\,k_t \\ d\,b_t \end{bmatrix} = J_\mu \cdot \begin{bmatrix} d\,k_{t-1} \\ d\,b_{t-1} \end{bmatrix} = \begin{bmatrix} \zeta_{11} & \zeta_{12} \\ \zeta_{21} & \zeta_{22} \end{bmatrix} \cdot \begin{bmatrix} d\,k_{t-1} \\ d\,b_{t-1} \end{bmatrix},$$

$$\zeta_{11} = \alpha + \frac{\chi}{\bar{\chi}}\frac{1}{(1+n)(1+\pi)}$$

$$\zeta_{12} = -\frac{1}{(1+n)(1+\pi)}(1-\frac{\chi}{\bar{\chi}})$$

$$\zeta_{21} = \alpha\frac{\frac{\chi}{\bar{\chi}}}{1-\frac{\chi}{\bar{\chi}}} - \frac{\chi}{\bar{\chi}}\frac{1}{(1+n)(1+\pi)}$$

$$\zeta_{22} = \frac{1}{(1+n)(1+\pi)}(1-\frac{\chi}{\bar{\chi}}) = -\zeta_{12},$$

where in deriving the $\zeta-$terms we use the steady-state relationships $b/(k+b) = \frac{\chi}{\bar{\chi}}$ and $f'(k) = \alpha\frac{1+n}{zc(1-\frac{\chi}{\bar{\chi}})}$. One easily verifies:

$$Tr(J_\mu) = \alpha + \frac{1}{(1+n)(1+\pi)} \quad (51)$$

$$Det(J_\mu) = \frac{\alpha}{(1+n)(1+\pi)}, \quad (52)$$

implying the eigenvalues $\lambda_1 = \alpha \in (0,1)$, $\lambda_2 = \frac{1}{(1+n)(1+\pi)} \in (0,1)$. Hence, λ_1 and λ_2 are independent of χ and coincide with the eigenvalues of the $\pi-$regime in the special case of $\chi = 0$, as established in step iii) of the previous proof. □

Proof of propositions 6 and 7:
The proof follows closely the proof of propositions 2 and 3. Consider the equations (41)-(44). Substituting out for m_t, one now obtains the system in k_t and b_t :

$$\chi f(k_{t-1}) = (1+n)b_t - \frac{1}{1+\pi}b_{t-1}$$

$$k_t = (1-B(k_t))\frac{c}{1+n}f(k_{t-1}) - b_t,$$

where we use in (44) that under the $\pi-$rule B_t simplifies to

$$B_t = B(k_t) = \frac{1}{1+\widehat{z}\cdot[R(k_t)\cdot(1+\pi)]^{\frac{1-\varepsilon}{\varepsilon}}}.$$

Then, local dynamics around the steady state are approximately governed by

$$\begin{aligned}
\begin{bmatrix} d\,k_t \\ d\,b_t \end{bmatrix} &= J_\pi^F \cdot \begin{bmatrix} d\,k_{t-1} \\ d\,b_{t-1} \end{bmatrix} = \begin{bmatrix} \frac{(1-B)c-\chi}{1+n+cB_k f(k)} f'(k) & -\frac{1}{[1+n+cB_k f(k)](1+\pi)} \\ \frac{\chi}{1+n} f'(k) & \frac{1}{(1+n)(1+\pi)} \end{bmatrix} \cdot \begin{bmatrix} d\,k_{t-1} \\ d\,b_{t-1} \end{bmatrix}, \\
Det(J_\pi^F) &= \frac{(1-B)c}{[1+n+cB_k f(k)](1+\pi)} f'(k), \\
Tr(J_\pi^F) &= \frac{(1-B)c-\chi}{[1+n+cB_k f(k)](1+\pi)} f'(k) + \frac{1}{(1+n)(1+\pi)}, \qquad \text{with :} \\
B_k &= -\frac{(1-\varepsilon)(1-\tau)}{\varepsilon} \frac{\widehat{z}\cdot I(k)^{\frac{1-\varepsilon}{\varepsilon}}}{[1+\widehat{z}\cdot I(k)^{\frac{1-\varepsilon}{\varepsilon}}]^2} \frac{f''(k)}{I(k)} > 0.
\end{aligned}$$

i) The steady state relationship (46) implies $c(1-B) > \chi\frac{(1+\pi)(1+n)}{(1+\pi)(1+n)-1} > \chi$. Hence, $Det(J_\pi^F) > 0$ and $Tr(J_\pi^F) > 0$ for $\chi \in [0, \overline{\chi}^F)$.
ii) Combining the terms describing $Det(J_\pi)$ and $Tr(J_\pi)$ one now obtains:

$$\begin{aligned}
Det(J_\pi^F) &= -1 + Tr(J_\pi^F) + q^F(k), \qquad \text{with :} \\
q^F(k) &= \frac{(1+n)(1+\pi)-1}{(1+n)(1+\pi)}[1-\alpha\frac{1+n}{1+n+cB_k f(k)}] \in (0,1),
\end{aligned}$$

implying $Det(J_\pi^F) > -1 + Tr(J_\pi^F)$.
iii) At $\chi = 0$, using $f'(k) = \alpha\frac{f(k)}{k}$ at (46), it is possible to establish $Det(J_\pi^F) = \alpha\frac{1+n}{[1+n+cB_k f(k)](1+\pi)}\frac{1}{(1+n)(1+\pi)} \in (0,1)$, $Tr(J_\pi^F) = \alpha\frac{1+n}{[1+n+cB_k f(k)](1+\pi)} + \frac{1}{(1+n)(1+\pi)}$, ensuring $\lambda_1 = \alpha\frac{1+n}{[1+n+cB_k f(k)](1+\pi)} \in (0,1)$, $\lambda_2 = \frac{1}{(1+n)(1+\pi)} \in (0,1)$.
iv), v): note that the logic of the proof of Propositions 2 and 3 remains valid as long as the term $\frac{B_k f(k)}{f'(k)}$ is a continuous function of χ and tends towards 0 as $\chi \to \overline{\chi}^F$, implying $k \to 0$. This condition would trivially be satisfied, for example, if $f''(k)$ is constant. However, this condition is also satisfied in the case of a Cobb-Douglas-function $f(k) = k^\alpha$, which has $f'''(k) \neq 0$. To see this, note that $\frac{B_k f(k)}{f'(k)}$ can be rearranged to

$$\frac{B_k f(k)}{f'(k)} = \frac{(1-\varepsilon)(1-\tau)(1-\alpha)}{\varepsilon} \frac{\widehat{z}\cdot I(k)^{\frac{1-\varepsilon}{\varepsilon}}}{[1+\widehat{z}\cdot I(k)^{\frac{1-\varepsilon}{\varepsilon}}]^2} \frac{1}{(1-\tau)\alpha + (1-\delta)k^{1-\alpha}} > 0,$$

which is a continuous function for $\chi \in [0, \overline{\chi}^F)$ and approaches 0 as $\chi \to \overline{\chi}^F$. Hence, as $\chi \to \overline{\chi}^F$, $Det(J_\pi^F) \to \infty, Tr(J_\pi^F) \to \infty, (Tr(J_\pi^F) - Det(J_\pi^F)) \to 1 - q^F(0)$, and

$q^F(0) \in (0,1)$. Combining i)-v) implies the pattern of eigenvalues established in Propositions 6 and 7. □

Preliminaries to the proofs of propositions 8 and 9:
Consider again the system (41)-(44). Using (42) in (43) and updating (41) and (42) one obtains the three-dimensional system of first order-difference equations in k_t, b_t and m_t :

$$\chi \cdot f(k_t) = (1+n) \cdot b_{t+1} - \frac{1}{1+\pi}\frac{m_{t+1}}{m_t} \cdot b_t \tag{53}$$

$$k_{t+1} + m_{t+1} + b_{t+1} = \frac{c}{1+n} \cdot f(k_t) \tag{54}$$

$$m_t = A(k_t, m_t, m_{t+1}) \cdot (k_t + b_t), \tag{55}$$

where

$$A(k_t, m_t, m_{t+1}) \equiv \frac{B_t}{1-B_t} = \frac{1}{\widehat{z}} \cdot I_t^{\frac{\varepsilon-1}{\varepsilon}}$$

$$I_t = R(k_t) \cdot \frac{m_t}{m_{t+1}} \cdot (1+\pi),$$

with $A_{k_t} > 0$, $A_{m_t} = -A_{m_{t+1}} < 0$. As discussed in the main text, the system (53)-(55) has two predetermined variables (k, b) and one jumping variable (m). Local dynamics can be assessed from the approximate system

$$\begin{bmatrix} d\,k_{t+1} \\ d\,b_{t+1} \\ d\,m_{t+1} \end{bmatrix} = J_\mu^F \cdot \begin{bmatrix} d\,k_t \\ d\,b_t \\ d\,m_t \end{bmatrix}$$

with the roots of the Jacobian matrix J_μ^F to be denoted by λ_1, λ_2 and λ_3. However, it turns out that the algebraic analysis much simplifies upon inspecting instead the *forward-looking* dynamics

$$\begin{bmatrix} d\,k_t \\ d\,b_t \\ d\,m_t \end{bmatrix} = J_\mu^{F,-1} \cdot \begin{bmatrix} d\,k_{t+1} \\ d\,b_{t+1} \\ d\,m_{t+1} \end{bmatrix} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix} \cdot \begin{bmatrix} d\,k_{t+1} \\ d\,b_{t+1} \\ d\,m_{t+1} \end{bmatrix},$$

$$\omega_{11} = \omega_{12} = \omega_{13} = \frac{1+n}{cf'(k)} > 0$$

$$\omega_{21} = \frac{1}{D} \cdot \{\chi(1 - A_{m_t}(k+b)) - \frac{A + A_{k_t}(k+b)}{f'(k)} \frac{1}{1+\pi}\frac{b}{m}\}$$

$$\omega_{22} = \frac{1}{D} \cdot \{(\chi - c)(1 - A_{m_t}(k+b)) - \frac{A + A_{k_t}(k+b)}{f'(k)} \frac{1}{1+\pi}\frac{b}{m}\}$$

$$\omega_{23} = \frac{1}{D} \cdot \{[\frac{c}{(1+n)} - \frac{A + A_{k_t}(k+b)}{f'(k)}] \frac{1}{1+\pi}\frac{b}{m}$$

$$
\begin{aligned}
& \quad +\chi(1-A_{m_t}(k+b))\} \\
\omega_{31} &= \frac{1}{D}\cdot\{A\chi - \frac{A+A_{k_t}(k+b)}{f'(k)}\frac{1}{1+\pi}\} \\
\omega_{32} &= \frac{1}{D}\cdot\{(\chi-c)A - \frac{A+A_{k_t}(k+b)}{f'(k)}\frac{1}{1+\pi}\} \\
\omega_{33} &= 1+\frac{1}{D}\cdot\{A\chi + \frac{c}{(1+n)(1+\pi)} - \frac{A+A_{k_t}(k+b)}{f'(k)}\frac{1}{1+\pi}\} \\
D &= \frac{c}{(1+n)(1+\pi)}[A\frac{b}{m} - (1-A_{m_t}(k+b))] < 0 \\
A\frac{b}{m} &= \chi\cdot\frac{(1+n)(1+\pi)}{(1+n)(1+\pi)-1}\cdot\frac{1}{(1-B)c} \in [0,1) \\
k+b &= \frac{(1-B)c}{1+n}f(k),
\end{aligned}
$$

where $J_\mu^{F,-1} \equiv (J_\mu^F)^{-1}$, with roots $\widetilde{\lambda}_1 = 1/\lambda_1$, $\widetilde{\lambda}_2 = 1/\lambda_2$ and $\widetilde{\lambda}_3 = 1/\lambda_3$.

Proof of Proposition 8 :
Consider the special case of $\chi = 0$. Then, $b = 0$ (and m, $k > 0$). Hence,

$$
\begin{aligned}
\omega_{21} &= \omega_{23} = 0 \\
\omega_{22} &= (1+n)(1+\pi) \\
\omega_{31} &= -\frac{1}{D}\cdot\{\frac{A+A_{k_t}k}{f'(k)}\frac{1}{1+\pi}\} \\
\omega_{32} &= -\frac{1}{D}\cdot\{cA + \frac{A+A_{k_t}k}{f'(k)}\frac{1}{1+\pi}\} \\
\omega_{33} &= \frac{1+n}{cf'(k)}\cdot\frac{A+A_{k_t}k}{1-A_{m_t}k} - \frac{A_{m_t}k}{1-A_{m_t}k} > 0 \\
D &= -c\frac{1-A_{m_t}k}{(1+n)(1+\pi)} < 0,
\end{aligned}
$$

implying that $\widetilde{\lambda}_1 = (1+n)(1+\pi) > 1$ is a root of $J_\mu^{F,-1}$. Accordingly, it is possible to factor the third-order characteristic equation associated with $J_\mu^{F,-1}$

$$
\widetilde{\lambda}^3 - Tr(J_\mu^{F,-1})\cdot\widetilde{\lambda}^2 + q\cdot\widetilde{\lambda} - Det(J_\mu^{F,-1}) = 0,
$$

$$
q = \omega_{11}\omega_{22} + \omega_{11}\omega_{33} + \omega_{22}\omega_{33} - \omega_{12}\omega_{21} - \omega_{13}\omega_{31} - \omega_{23}\omega_{32}
$$

as

$$
[\widetilde{\lambda} - (1+n)(1+\pi)]\cdot[\widetilde{\lambda}^2 + p_1\widetilde{\lambda} + p_2] = 0, \quad \text{with:}
$$

$$p_1 = -\frac{1+n}{cf'(k)} - \omega_{33} < 0$$
$$p_2 = -\frac{1+n}{cf'(k)} \cdot \frac{A_{m_t}k}{1-A_{m_t}k} > 0.$$

Using $f'(k) = \frac{\alpha(1+n)}{c(1-B)} = \frac{\alpha(1+A)(1+n)}{c}$ at $\chi = 0$, substituting yields

$$1 + p_1 + p_2 = \frac{\alpha - 1 - \frac{A_{k_t}k}{1+A}}{\alpha(1-A_{m_t}k)} < 0,$$

implying $\widetilde{\lambda}_2 > 1$, $\widetilde{\lambda}_3 \in (0,1)$. Thus, $\lambda_1 = \frac{1}{(1+n)(1+\pi)} \in (0,1)$, $\lambda_2 \in (0,1)$, $\lambda_3 > 1$. $\square$

Proof of Proposition 9 :
Consider the dynamic properties of the system as $\chi \to \overline{\chi}^F$, implying $k \to 0$. Let $\overline{\omega}_{ij}$ denote ω_{ij} evaluated at $\chi = \overline{\chi}^F$, for $i,j = 1,2,3$. Then, $\overline{\omega}_{11} = \overline{\omega}_{12} = \overline{\omega}_{13} = 0$, and $J_\mu^{F,-1}$ ceases to be invertible. To calculate the remaining $\overline{\omega}_{ij}-$terms, we establish the limits:
i) $\overline{A} = \frac{\overline{B}}{1-\overline{B}} = 0$ (using $\overline{B} = 0$);
ii) $\overline{A\frac{b}{m}} = 1$;
iii) $[\overline{A\frac{b}{m}} - (1 - \overline{A_{m_t}(k+b)})] = \frac{\varepsilon-1}{\varepsilon}$, using $A_{m_t}(k+b) = \frac{\varepsilon-1}{\varepsilon}$ =constant;
iv) $\overline{\frac{A_{k_t}(k+b)}{f'(k)}} = 0$, where we use

$$\frac{A_{k_t}(k+b)}{f'(k)} = \phi_1 \cdot I(k)^{\frac{\varepsilon-1}{\varepsilon}} \cdot \frac{1-B}{(1-\delta)k^{1-\alpha} + (1-\tau)\alpha}$$
$$\phi_1 = \frac{(1-\varepsilon)(1-\alpha)(1-\tau)c}{\varepsilon(1+n)\widehat{z}} > 0.$$

v) $\overline{\frac{A_{k_t}(k+b)}{f'(k)}\frac{b}{m}} = \frac{(1-\varepsilon)(1-\alpha)c}{\varepsilon\alpha(1+n)} > 0$.
Inserting these expressions, it tuns out that the terms $\overline{\omega}_{21}$ and $\overline{\omega}_{31}$ are constants and $Det(\overline{J_\mu^{F,-1}}) = 0$, ensuring that one of the roots $\widetilde{\lambda}_i \to 0$ as $\chi \to \overline{\chi}^F$, while the other two roots are ultimately governed by the $2x2-$submatrix

$$\overline{J} = \begin{bmatrix} \overline{\omega}_{22} & \overline{\omega}_{23} \\ \overline{\omega}_{32} & \overline{\omega}_{33} \end{bmatrix}.$$

Specifically, using i)-v) one can verify: $\overline{\omega}_{22} = \frac{1}{1-\varepsilon} + \frac{1-\alpha}{\alpha}$, $\overline{\omega}_{33} = 1 - \frac{\varepsilon}{1-\varepsilon}$, $\overline{\omega}_{23} \cdot \overline{\omega}_{32} = -\frac{\varepsilon}{1-\varepsilon}[\frac{\varepsilon}{1-\varepsilon} + \frac{1-\alpha}{\alpha}]$. Accordingly, $Tr(\overline{J}) = 2 + \frac{1-\alpha}{\alpha} > 0$, $Det(\overline{J}) = 1 + \frac{1-\alpha}{\alpha} > 1$, $[Tr(\overline{J})]^2 - 4 \cdot Det(\overline{J}) = (\frac{1-\alpha}{\alpha})^2 > 0$. Moreover, $Det(\overline{J}) + 1 - Tr(\overline{J}) = 0$, and, upon explicitly calculating this expression before taking limits, one can show $Det(J) + 1 - Tr(J) \downarrow 0$, as $\chi \to \overline{\chi}^F$. By continuity of all expressions in χ this implies $\widetilde{\lambda}_1 > 1$, $\widetilde{\lambda}_2 > 1$, $\widetilde{\lambda}_3 \in (0,1)$ as $\chi \to \overline{\chi}^F$, or alternatively, $\lambda_1 \in (0,1)$, $\lambda_2 \in (0,1)$, $\lambda_3 > 1$. $\square$

Simulation output

Table 1a: 'Strict quantity theory of money' ($\varepsilon = 1$, **i.e. log-preferences**)
$\pi = 0.02$, $n = 0.04$, $\tau = 0.3$, $\alpha = 0.2$, $\phi = 10$, $z = 0.5$, $\delta = 0.5$, $s = 1$, $\overline{\chi} = 0.016$
λ_i^π : roots under strict inflation targeting,
λ_i^μ : roots under a constant money growth rule.

$\boldsymbol{\chi}$	$\boldsymbol{\lambda}_1^\pi$	$\boldsymbol{\lambda}_2^\pi$	$\boldsymbol{\lambda}_1^\mu$	$\boldsymbol{\lambda}_2^\mu$	$\boldsymbol{\lambda}_3^\mu$
0	0.94	0.2	0.94	0.2	·
0.01	0.90	0.56	0.94	0.2	·
0.012	$0.86 + 0.2i$	$0.86 - 0.2i$	0.94	0.2	·
0.013	$0.97{+}0.21i$	$0.97{-}0.21i$	0.94	0.2	·
0.0135	$1.07 + 0.2i$	$1.07 - 0.2i$	0.94	0.2	·
0.014	1.24	1.19	0.94	0.2	·
0.015	2.82	1.03	0.94	0.2	·

Table 1b: 'Weakly forward-looking money demand' ($\varepsilon = 0.95$)
Except for ε, all parameters as in Table 1a. $\overline{\chi}^F = 0.032$

$\boldsymbol{\chi}$	$\boldsymbol{\lambda}_1^\pi$	$\boldsymbol{\lambda}_2^\pi$	$\boldsymbol{\lambda}_1^\mu$	$\boldsymbol{\lambda}_2^\mu$	$\boldsymbol{\lambda}_3^\mu$
0	0.94	0.20	0.94	0.20	40
0.01	0.90	0.52	0.94	0.20	41
0.012	$0.81 + 0.1i$	$0.81 - 0.1i$	0.95	0.20	43
0.013	$0.89 + 0.2i$	$0.89 - 0.2i$	0.95	0.21	45
0.0135	$0.94 + 0.2i$	$0.94 - 0.2i$	0.95	0.21	48
0.014	$1.02 + 0.2i$	$1.02 - 0.2i$	0.95	0.21	48
0.015	1.34	1.14	0.95	0.21	52
0.018	8.4	1.01	0.97	0.20	480

Table 1c: 'Strongly forward-looking money demand' ($\varepsilon = 0.5$)
Except for ε, all parameters as in Table 1a. $\overline{\chi}^F = 0.032$

$\boldsymbol{\chi}$	$\boldsymbol{\lambda}_1^\pi$	$\boldsymbol{\lambda}_2^\pi$	$\boldsymbol{\lambda}_1^\mu$	$\boldsymbol{\lambda}_2^\mu$	$\boldsymbol{\lambda}_3^\mu$
0	0.94	0.20	0.94	0.22	3
0.01	0.93	0.30	0.95	0.23	4
0.02	0.83	0.69	0.98	0.23	9
0.025	$0.98 + 0.2i$	$0.98 - 0.2i$	0.99	0.22	16
0.026	$1.06 + 0.2i$	$1.06 - 0.2i$	0.99	0.21	20
0.028	1.62	1.09	0.99	0.21	31
0.03	3.36	1.02	0.99	0.20	65

Table 1d: 'Weakly forward-looking money demand ($\varepsilon = 0.95$) **under high growth** ($n = 0.1$)**'**
Except for n, all parameters as in Table 1b. $\overline{\chi}^F = 0.061$

$\boldsymbol{\chi}$	$\boldsymbol{\lambda}_1^\pi$	$\boldsymbol{\lambda}_2^\pi$	$\boldsymbol{\lambda}_1^\mu$	$\boldsymbol{\lambda}_2^\mu$	$\boldsymbol{\lambda}_3^\mu$
0	0.89	0.20	0.89	0.20	40
0.01	0.86	0.30	0.89	0.20	40
0.02	0.63	0.76	0.89	0.20	42
0.025	$0.86 + 0.3i$	$0.86 - 0.3i$	0.90	0.20	45
0.028	$1.11 + 0.3i$	$1.11 - 0.3i$	0.90	0.20	51
0.03	1.89	1.10	0.91	0.20	62
0.034	7.59	1.02	0.94	0.20	386

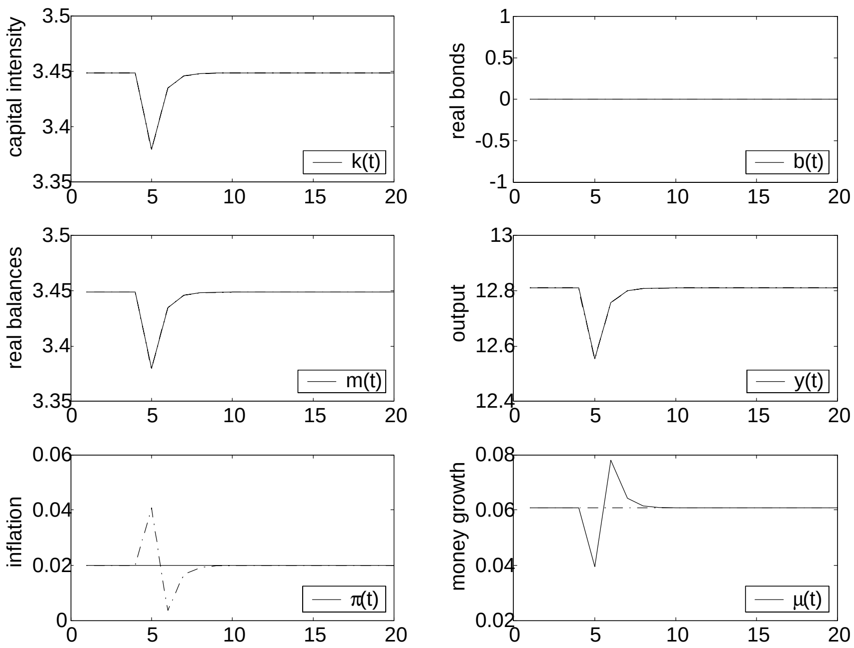
Impulse response patterns for varying values of χ

In simulations $1-5$ reported below, all parameters as in Table 1 a, i.e. we consider a backward-looking, 'strict quantity theory of money'– regime as discussed in Section 3. The economy is in the first four periods in steady state. At the beginning of period 5, the economy is hit by a one-time temporary productivity shock ($\phi_5 = 0.98 < \phi = 1$), depressing steady-state output in period 5 by 2%.

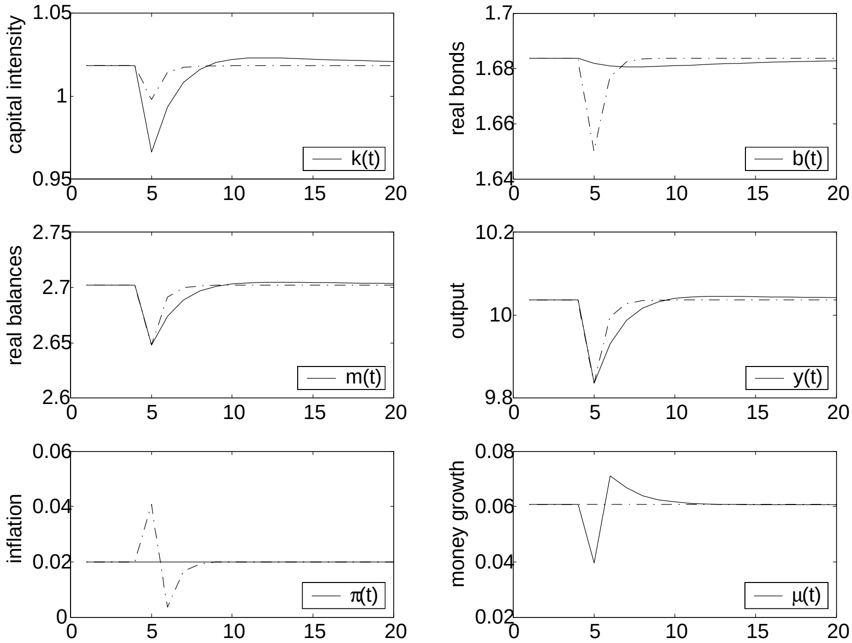
Strict inflation targeting: solid line

Constant money growth rule: broken line
(unless outcomes coincide with strict inflation targeting)

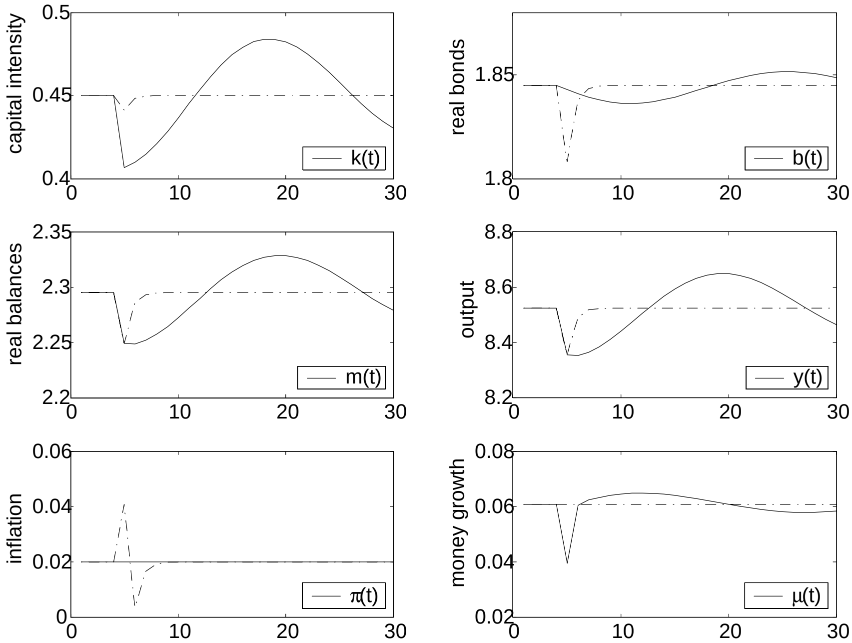
Simulation 1 (Balanced budget, $\chi = 0$)



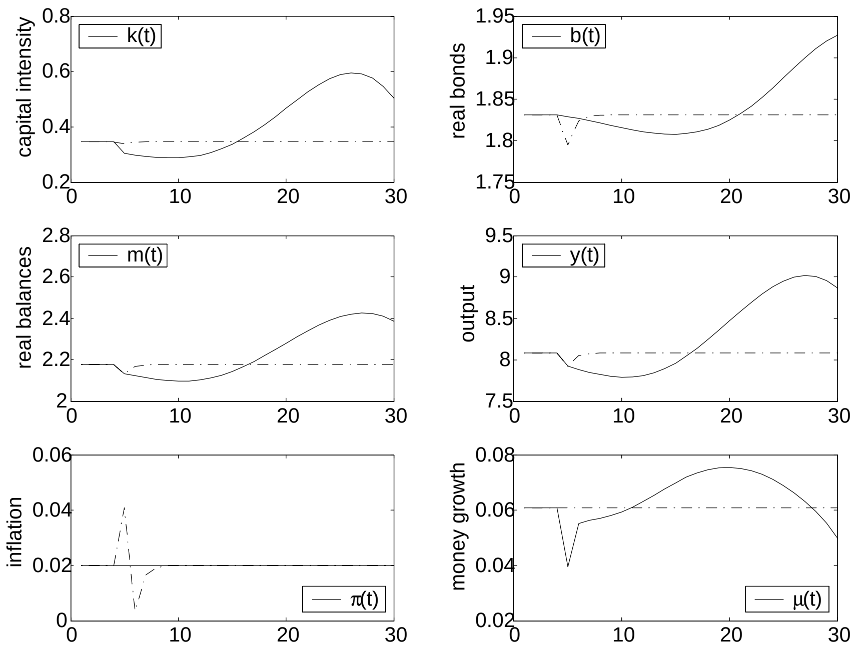
Simulation 2 ($\chi = 0.01$)



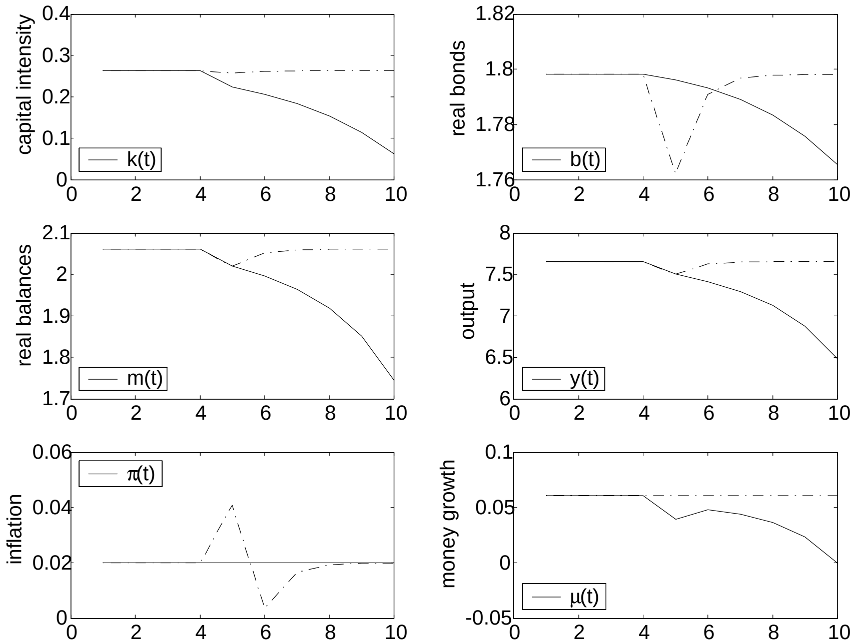
Simulation 3 ($\chi = 0.013$)



Simulation 4 ($\chi = 0.0135$)



Simulation 5 ($\chi = 0.014$)



References

[1] Bassetto, M., A game-theoretic view of the fiscal theory of the price level, *Econometrica*, 70, 6, 2167-95, 2002.

[2] Bhattacharya, J. and Kudoh, N., Tight money policies and inflation revisited, *Canadian Journal of Economics*, 35, 2, 185-217, 2002.

[3] Bohn, H., Why do we have nominal government debt?, *Journal of Monetary Economics,* 21, 127-40, 1988.

[4] Buiter, W., The fiscal theory of the price level: a critique, *Economic Journal,* 112, 459-80, 2002.

[5] Carlstrom, C. and Fuerst, T., The fiscal theory of the price level, *Federal Reserve Bank of Cleveland Economic Review*, Q1, 22-32, 2000.

[6] Chalk, N., The sustainability of bond-financed deficits: an overlapping generations approach, *Journal of Monetary Economics*, 45, 293-328, 2000.

[7] Chari, V., Christiano, L., and Kehoe, P., Optimal fiscal and monetary policy: some recent results, *Journal of Money, Banking, and Credit*, 23, 519-546, 1991.

[8] Chari, V. and Kehoe, P., On the needs for fiscal constraints in a Monetary Union, mimeo, 1998.

[9] Christiano, L. and Fitzgerald, T., Understanding the fiscal theory of the price level, mimeo, 2000.

[10] Crettez, B., Michel, P., and Wigniolle, B., Cash-in-advance constraints in the Diamond overlapping generations model: neutrality and optimality of monetary policies, *Oxford Economic Papers*, 51, 431-52, 1999.

[11] Diamond, P., National debt in a neoclassical growth model, *American Economic Review*, 55/5, 1126-50, 1965.

[12] Espinosa-Vega, M. and Russell, S., The long-run real effects of monetary policy: Keynesian predictions from a neoclassical model, mimeo, Federal Reserve Bank of Atlanta, 2001.

[13] Galor, O. and Ryder, H., Existence, uniqueness, and stability of equilibrium in an overlapping generations model with productive capital, *Journal of Economic Theory*, 49, 360-75, 1989.

[14] Hahn, F. and Solow, F., Perfectly flexible wages, in: *A critical essay on modern macroeconomic theory*, Chapter 2, Blackwell Publishers, 1995.

[15] Leeper, E., Equilibria under active and passive monetary and fiscal policies, *Journal of Monetary Economics*, 27, 129-47, 1991.

[16] McCallum, B., Indeterminacy, bubbles, and the fiscal theory of the price level, *Journal of Monetary Economics*, 47, 19-30, 2001.

[17] Sargent, T. and Wallace, N., Some unpleasant monetarist arithmetic, *Federal Reserve Bank of Minneapolis Quarterly Review*, 1-17, Fall, 1981.

[18] Schmitt-Grohé, S. and Uribe, M., Optimal fiscal and monetary policy under imperfect competition, CEPR Discussion Paper, No. 2688, 2001.

[19] Schreft, S. and Smith, B., Money, banking, and capital formation, *Journal of Economic Theory*, 73, 157-82, 1997.

[20] Schreft, S. and Smith, B., The effects of open market operations in a model of intermediation and growth, *Review of Economic Studies*, 65, 519-550, 1998.

[21] Sims, C. A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy, *Economic Theory*, 4, 381-99, 1994.

[22] Sims, C., The precarious fiscal foundations of EMU, mimeo, 1999.

[23] Svensson, L., Price-level targeting versus inflation targeting: a free lunch?, *Journal of Money, Credit, and Banking*, 31, 3, 277-295, 1999.

[24] Svensson, L., Monetary policy and real stabilization, NBER Working Paper, No. 9486, 2003.

[25] von Thadden, Leopold, Monetary and fiscal policy rules in a model with capital accumulation and potentially non-superneutral money, Economic Research Centre of the Deutsche Bundesbank, Discusssion Paper 22/02, 2002.

[26] Uhlig, H., One money, but many fiscal policies in Europe: what are the consequences?, CEPR Discussion paper, No. 3296, 2002.

[27] Woodford, Monetary policy and price level determinacy in cash-in-advance economy, *Economic Theory*, 345-380, 1994.

[28] Woodford, M., Price-level determinacy without control of a monetary aggregate, *Carnegie-Rochester Conference Series on Public Policy*, 43, 1-46, 1995.

[29] Woodford, M., Fiscal requirements for price stability, *Journal of Money, Credit, and Banking*, 33, 3 669-728, 2001.