Should Central Banks target Consumer Prices or the Exchange Rate?*

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Abstract

In this paper we consider two arguments suggesting that monetary authorities in an open economy should target output price inflation and not consumer price inflation. The first suggests that output price inflation corresponds to the distortions caused by price rigidity. The second shows how policy rules involving consumer price inflation can induce instability because of the feedback from interest rates to consumer price inflation via the exchange rate.We examine both arguments in the context of an open economy which is subject to a range of shocks. We show that both arguments remain robust, but that there is a case for the including an 'exchange rate gap' term in the authorities welfare function alongside the output gap and output price inflation.

*Very preliminary: not to be quoted without the authors permission.

1 Introduction

In all relatively open economies where the monetary authorities have an explicit inflation objective, such as the U.K., that objective involves some measure of consumer price inflation. Recently two arguments have been put forward that suggest that this is the wrong inflation measure to target. The first asserts that output price inflation, rather than consumer price inflation, more accurately reflects the welfare loss associated with changing prices. A second argues that simple policy rule based on consumer price inflation may lead to instability because of the influence of interest rates on the exchange rate, and its feedback to consumer price inflation.

The relationship between consumer and output price inflation depends on the exchange rate, and so these arguments are part of a more long-standing debate about what role the exchange rate should have in influencing monetary policy. It has frequently been argued that interest rate setting should be influenced by exchange rate movements, or the distance between the exchange rate and some concept of its equilibrium level (e.g. Wren-Lewis (1997)). This argument became intense in the UK at the end of the 1990s, as sterling appreciated substantially but inflation remained close to its target level. However adding an exchange rate target alongside output and inflation as a policy objective remains unorthodox, and some of the recent literature that explicitly derives social welfare functions suggest it is unnecessary.

In this paper we critically examine both arguments against the use of consumer price inflation as a policy objective. Section 2 outlines the welfare arguments for focusing on output price inflation. We generalise earlier analysis by explicitly introducing Uncovered Interest Parity (UIP) shocks into the model. The focus on output price inflation remains, but now there is a clear potential role for some form of exchange rate objective analogous to the output objective. Section 3 uses the same model to illustrate the instability that may emerge with simple rules involving consumer price inflation, and shows how the chances of this instability increase as the economy becomes more open and monetary policy becomes more active. Section 4 concludes.

2 Social Welfare with UIP shocks

Suppose an economy is made up of a number of identical individuals, each of which have utility

$$\max_{\{C_s, y_s\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t}[u(C_s, \xi_s) - v(y_s(z), \xi_s)] \tag{1}$$

where C is consumption, y is output for good z, ξ is a taste shock, and β is a discount rate. (The function $v(\cdot)$ embodies both the disutility of labour and the production technology.) While this or a similar assumption forms the basis of most theoretical macromodels, when analysing policy these models have traditionally postulated a quite separate objective function for policy makers, typically involving quadratic terms in output and inflation. The absence of any link between utility functions and social welfare functions is a major embarrassment for models that emphasise their microfoundations.[1]

It is therefore not surprising that Woodford's analysis Woodford (200x) showing how such welfare functions could be derived as second order approximations to (1) has been rapidly adopted in the literature. There are now a large number of papers that derive policy makers objective functions explicitly from consumers' utility. (These include, besides those papers cited below, Sutherland (2002) and Batini, Harrison, and Millard (200x)) A key point about such derivations is that they are model specific: although the utility function (1) may be fairly general, the welfare function derived as a second order approximation to (1) depends on the structure of the model. Woodford shows that if nominal inertia in the model is represented by Calvo contracts and there are sufficient government subsidies to offset monopoly distortions that remain in a zero inflation steady-state, then a second order approximation to social welfare can be represented by quadratic terms in inflation and the deviation of output from its 'natural' level.

In the closed economy analysed by Woodford, there is no distinction between output prices and consumer prices. Aoki (2002) considers a two sector model, where prices in one sector a completely flexible, and shows

[1] Wren-Lewis (2003) analyses this methodological position

that it is only inflation in the non-flexible sector that is relevant for welfare. He explicitly suggests that imported goods in an open economy are akin to the flexible price sector, and that therefore the price of imported goods should not appear in the inflation measure representing welfare. Gali and Monacelli (2002) consider a small open economy and come to the same conclusion, although the result is only demonstrated in the special case where the functions $u(\cdot)$ and $v(\cdot)$ above are logarithmic. Clarida, Gali, and Gertler (2001) use the same model, and argue that this result holds for any constant elasticity formulation of the utility functions, although the result is not formally demonstrated.

In this section we generalise the small open economy model analysed by Gali & Monacelli and Clarida et al by adding preference shocks and UIP shocks. The reason for adding UIP shocks is straightforward. One argument that is frequently invoked in favour of exchange rate targeting (and its limit, monetary union) is that markets often drive the exchange rate well away from levels implied by fundamentals, and that this has damaging effects on the economy as a whole. (For example, see Buiter and Grafe (2003) in evidence submitted to the U.K. Treasury enquiry into joining EMU.) It is therefore interesting to note whether these shocks introduce a role for the exchange rate in the welfare function, and what form that might take. In addition, allowing for preference shocks allows us to examine an alternative interpretation of sterling's appreciation in the late 1990s: that it represented the consequence of a positive private demand shock to the UK economy. We formally derive a second order approximation to social welfare for this model, keeping the form of our utility functions general (as in Woodford (200x)), but noting the implications of special cases.

To focus on the economics behind this derivation, we present the detailed derivation of the model, and the details of the second order approximation to welfare, in two appendices, and only present key relationships in the main text. In terms of log-linear deviations from a zero inflation steady state, the model of a small open economy can be represented by the following equations. For any variable X_t, $\hat{X}_t = \ln(X_t/X)$, where X is the steady state value of X_t, and we refer to this as X disequilibrium. Here we present only first order approximations for clarity, although the appendix derives some second order approximations that are required for welfare analysis.

The Phillips curve derived from the existence of Calvo contracts is given by

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda m\hat{c}_t \tag{2}$$

where π_H is output price inflation (output prices = price of domestic production = p_H), and λ is a function of model parameters (see Appendix A). Real marginal costs mc are given by

$$m\hat{c} = \hat{W} - \hat{P}_H \tag{3}$$

where w are nominal wages, and all variables are at time t..We assume that a subsidy exists in steady state to exactly offset the monopoly mark-up, so that steady state real marginal cost is unity. We also define 'natural' levels of variables as those levels that would occur in the absence of nominal inertia i.e. with perfect price flexibility. The natural level of real marginal cost is always equal to its steady state level.

The demand curve is

$$\hat{Y} = \alpha\hat{Y}^* + (1-\alpha)\widehat{C} + \alpha\eta(2-\alpha)\hat{S} \tag{4}$$

where Y is output, Y^* overseas output, C consumption, S are the terms of trade (the ratio of the price of overseas produced goods to domestically produced goods), $0 < \alpha < 1$ and η are demand curve parameters (α is the share of foreign goods in the consumption bundle.) Thus the demand for domestic output depends on domestic consumption, world output and the real exchange rate.

International risk sharing implies

$$\hat{C} = \hat{Y}^* + \sigma(1-\alpha)\hat{S} - \sigma\widehat{\zeta} \tag{5}$$

where overseas output is equal to overseas consumption (our country is small), and $\widehat{\zeta}$ is a distortion that can be related to departures from UIP.

The Euler equations from consumer optimisation are

$$\hat{W} - \hat{P}_H - \alpha\hat{S} = [\hat{y}\frac{1}{\varphi} + \hat{C}\frac{1}{\sigma} + \hat{\xi}(\frac{h}{\varphi} + \frac{g}{\sigma})] \tag{6}$$

$$\hat{C}_t = E_t[\hat{C}_{t+1}] - \sigma(r_t - E_t[\pi_{t+1}]) \tag{7}$$

where

$$\sigma = -\frac{U_C(\cdot)}{U_{CC}(\cdot)C} \tag{8}$$

$$g = \frac{U_{C\xi}(\cdot)}{U_{CC}(\cdot)C} = -\frac{U_{C\xi}(\cdot)\sigma}{U_C(\cdot)} \tag{9}$$

$$\varphi = \frac{v_y(\cdot)}{v_{yy}(\cdot)y} \tag{10}$$

$$h = \frac{v_{y\xi}(\cdot)}{v_{yy}(\cdot)y} = \frac{v_{y\xi}(\cdot)\varphi}{v_y(\cdot)} \tag{11}$$

The left hand side of (6) is the real consumer wage, and the production technology is log-linear (i.e. $y = n$). Consumer price inflation π and output price inflation π_H are related by

$$\pi_t = \pi_{H,t} + \alpha\Delta s_t \tag{12}$$

The utility function (1) represents the utility of a representative consumer who supplies labour to industry z. A benevolent policy maker will attempt to maximise the utility of all workers, and so will maximise the discounted sum of terms which at each time s will be of the form

$$U(C_s, \xi) - \int_0^1 v(y_s(z), \xi_s)dz \tag{13}$$

Taking a second order approximation of (13) around the steady state gives

$$W = U(C_s, \xi) - \int_0^1 v(y_s(z), \xi_s)dz = U_c(\cdot)C[\hat{C}(1 - \frac{g}{\sigma}\hat{\xi}) + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{C}^2] \tag{14}$$

$$- Yv_y(\cdot)[\hat{Y}(1 + \frac{h}{\varphi}\widehat{\xi}) + \frac{1}{2}(1 + \frac{1}{\psi})\hat{Y}^2 + V_z\widehat{y}(z)\frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})] + tip$$

where V_z denotes the variance across goods, and *tip* represents terms that are independent of policy (i.e. terms involving the steady state or shocks alone) and terms higher than second order (see Appendix B). The first two terms come from the utility of consumption, and represent the costs of aggregate consumption deviating from its optimal path. The next two terms represent similar magnitudes for labour supply and hence output, while the final term represents costs associated with the output of individual goods differing from average output.

As Woodford (200x) shows, the term in the variance of output across producers exists because of the distortion due to Calvo contracts and can be replaced with a quadratic term in inflation. With nominal inertia in the form of Calvo contracts, the only reason for output of individual goods to differ is that some firms change their prices while others do not. The greater is inflation, the larger the movement in relative prices, and therefore the larger the variance in output across goods. This is a particularly clear representation of one of the standard arguments for costs associated with inflation: that higher inflation is associated with a greater distortion in relative prices, and therefore a larger misallocation of production and labour across goods. However it also apparent from this derivation that it is only the relative price of domestically produced goods, and therefore output price inflation, that is relevant for welfare, because only domestic goods are involved in domestic labour supply. This is the key insight that lies behind the argument that social welfare depends on output price inflation rather than consumer price inflation. To put this another way, inflation only matters to agents as workers, not as consumers.

The assumption of Calvo contracts is important in determining the way inflation appears in the welfare function. For example, as Woodford shows, if we adapt the Calvo contract formulation such that producers who were unable to set the optimal price in the current period can index that price to observed inflation then the quadratic term becomes one in the change rather than the level of inflation.

What about the remaining terms? Although we have quadratic terms, we also have linear terms. Linear terms are problematic, because on their own they imply infinite desired values. Linear terms imply that it is not optimal for policy makers to reproduce the flexible price equilibrium (i.e. to eliminate the gap between actual levels and natural levels). The key to eliminating these linear terms is to introduce an employment subsidy which offsets the distortions due to imperfect competition and ensures that, in steady-state, the marginal utility of consumption equals the marginal disutility of the labour supply required to produce it. By assuming that this optimality condition holds in steady-state we eliminate the first-order terms in welfare (see Appendix B).

Furthermore, we can replace the terms in the preference shocks by measures of consumption and output disequilibrium at their 'natural' level. This is the level that would occur if there was no nominal inertia in the model, and no UIP shocks. In the closed economy case, when output equals consumption, then Woodford shows that we can combine quadratic terms in output disequilibrium with terms combining output disequilibrium with 'natural' levels such that we have only quadratic terms in the 'output gap', where the output gap is the difference between actual output disequilibrium and natural output, i.e. the additional output disequilibrium generated by nominal inertia.

As a result, in the closed economy we can write welfare as quadratic terms in inflation and the output gap alone.[2] In the open economy case this will not in general be possible, except in special cases, such as when utility

[2] We should also note that an objective function involving quadratic terms in the output gap and inflation is not the only possible form that results from (1). As Aoki (200x) shows, we could use Calvo contracts to replace the output gap by future inflation, producing an objective function involving terms in inflation alone. It makes less sense to replace inflation by terms in the output gap, however, because current inflation depends on all future anticipated output gaps.

is logarithmic Gali and Monacelli (2002). The Appendix shows that welfare can be written in terms of both the output and consumption gap, with the following general form

$$W = A_1(\hat{Y} - \hat{Y}^n)^2 + A_2(\hat{C} - \hat{C}^n)^2 + A_3(\hat{Y} - \hat{Y}^n)\hat{S}^n + A_4\pi_H^2 + tip \quad (15)$$

where A_i are parameters. The term multiplied by A_3 implies that if a preference shock leads to a movement in the terms of trade away from steady state under flexible prices (and no UIP shock), then there is an incentive for policy makers to move output away from the flexible price level. As a result, and unlike the closed economy case, it will not be optimal to attempt to move variables exactly to their flexible price level, a point that has been noted by Beningno and Benigno (200x) in a related context. The intuition behind this term is as follows. Although a steady state subsidy can eliminate the monopoly distortions in a closed economy, in an open economy the size of this distortion depends on the terms of trade. Any movement in the terms of trade generated by preference shocks will allow some scope for a welfare improving movement of output away from its natural level.

In the absence of UIP shocks, it is possible to replace the terms in the consumption gap by terms in the output gap, thereby reproducing the closed economy result except for the linear term just discussed. To see this, simply note that the demand curve (4) and the risk sharing condition (5) represent two equations in three unknowns (C, Y, and S), and we can subtract the natural counterparts of these equations to eliminate foreign output (as long as $\widehat{\zeta} = 0$). Thus terms in the consumption gap and terms of trade gap can always be replaced by terms in the output gap. This is the point noted by Clarida, Gali, and Gertler (2001). However, once we introduce UIP shocks, such a transformation is no longer possible. (The UIP shock, $\widehat{\zeta}$, is present for actual values but not natural levels.) Instead, if we want to eliminate the consumption gap, we obtain an equation of the form

$$\begin{aligned} W = {} & B_1(\hat{Y} - \hat{Y}^n)^2 + B_2(\hat{S} - \hat{S}^n)^2 + B_3(\hat{S} - \hat{S}^n)(\hat{Y} - \hat{Y}^n) \\ & + B_4(\hat{Y} - \hat{Y}^n)\hat{S}^n + A_4\pi_H^2 + tip \end{aligned} \quad (16)$$

with terms in the terms of trade gap. (Pappa (2002) derives a similar expression.) Quite simply, UIP shocks can lead to distortions in the pattern of consumption even if the output gap is zero (or vice versa), so we need to also target the terms of trade gap.

A key point to note about the terms of trade term is that, like output, it is in the form of deviations from the natural level i.e. the terms of trade that would occur with no nominal inertia or UIP shocks. A number of studies have experimented with simple feedback rules which include some form of exchange rate targeting, but generally not in terms of deviations from its natural level. For example, Kollmann (2002) finds that adding a quadratic term in the change in exchange rate to a feedback rule (with optimised parameters) that already includes output price inflation and output disequilibrium terms adds virtually nothing to welfare.[3] This result is interesting, because CPI inflation targeting can be roughly 'recovered' from separate terms on output price inflation and the change in the exchange rate. However our analysis suggests terms in exchange rate 'gap': the difference between actual exchange rate disequilibrium and the disequilibrium that would occur with no distortions. Not only is the dimension of this expression different from the change in the exchange rate, but a change in exchange rate term makes no attempt to allow for 'warranted' exchange rate movements i.e. natural disequilibrium. For this reason CPI inflation targeting cannot be considered as an attempt to combine the output price inflation targeting and exchange rate targeting in this welfare framework. In the recent UK context, if sterling's appreciation was the result of strong domestic demand caused by a preference shock, then the exchange rate gap term could be zero and there would be no reason for policy to resist the appreciation on this account. However, if the appreciation reprented a bubble (i.e. a UIP shock), then the exchange rate gap term would be non-zero, and our analysis suggests that policy should respond.[4]

[3] In Kollmann (2003), it is argued that exchange rate targeting may be of greater value if it helps reduce the size of UIP shocks.

[4] It might be objected that even if the appreciation was a consequence of strong demand (and therefore 'warranted'), it was undesirable because it hit some sections of the economy more than others. We cannot address this issue here, because all producers export an equal

One early example that does come close to trying to capture the concept of an exchange rate gap is the Target Zone proposal of Williamson and Miller (Williamson and Miller (1987), see also Currie and Wren-Lewis (1989) for an evaluation), where interest rates differentials were assigned entirely to stabilising the real exchange rate around its medium term equilibrium level (FEER), and fiscal policy was assigned to inflation stabilisation. While this particular policy assignment may no longer be on the agenda (except, perhaps, for countries within a currency union), the FEER measure of an equilibrium exchange rate is close to the idea of a natural level. In particular, the FEER is the real exchange rate that would occur if the economy was in 'internal balance', which can be interpreted as abstracting from business cycle effects generated by nominal inertia as well as UIP shocks.

While our analysis confirms the result that the monetary authorities should target output price inflation rather than consumer price inflation, it also suggests that there is an additional case for a separate exchange rate target, in the form of the exchange rate gap: the difference between the exchange rate and its natural level. The intuition behind both results is straightforward. Nominal inertia leads to the staggering of price changes, leading to variation in individual goods prices and therefore a misallocation in the distribution of production across goods. This misallocation will be directly related to the overall level of inflation. However the fact that the distortion relates to the allocation of output, rather than the pattern of consumption, means that it is output price inflation rather than consumer price inflation that is the relevant proxy for this inefficiency. The intuition behind the presence of exchange rate disequilibrium terms is that the intertemporal profile of consumption is not identical to the profile of output in an open economy, and so ensuring that the latter is at its efficient level will not guarantee the former is as well. In particular, UIP shocks can move the terms of trade away from their efficient level, which may move consumption off its optimal intertemporal path without necessarily creating output gaps.

Meade (1951) argued for targeting output price inflation rather than consumer price inflation, because of a concern that terms of trade shocks will

proportion of their output in our model.

require offsetting domestic price changes (given CPI target), and that this will be costly because domestic prices are sticky. Kara and Nelson (2002) suggest the result is vulnerable to the assumption that all imported goods are finished consumer goods that contain no domestic value added. They suggest an alternative framework, where all imported goods are intermediate goods, and they argue that this framework is more consistent with empirical evidence on aggregate pricing. It is certainly the case that the final purchase price of finished imported consumer goods contains a large proportion of domestic value added, because of wholesale and distributor margins. However, the logic of the analysis above is that the target inflation index should in fact relate to a measure of domestic value added, such as the GDP deflator. The GDP deflator will include the element that domestic retailers add to imported consumer goods prices, but not the element of price that is generated overseas.

One limitation of these welfare results is that they focus on just one cost of inflation. Business cycles only matter here because they disrupt the optimal intertemporal allocation of consumption and labour supply and they distort relative prices. The model ignores money, and therefore the cost of inflation that arises from foregone interest in holding the medium of exchange. However, Woodford (200x) adds money to the utility function and shows that it implies that a quadratic term in the level of the nominal interest rate should be present in the welfare function, because the nominal interest rate represents the cost of holding money. It does not imply a move from output price inflation to CPI targetting.

Another clear limitation of this and the other models that have been used so far is that business cycles affect all consumers/workers in a similar way. The representative agent assumption means that everyone's consumption is equally affected, and everyone keeps working, although some will be working too little or too much. There is no unemployment, and no bankruptcy. It is therefore perhaps not surprising that Woodford finds that the relative size of the output disequilibrium term is small compared to the inflation term: this reflects a point made by Lucas and Johnson (1987) that the cost of the intertemporal misallocation of consumption generated by business cycles appeared relatively small. Even without attaching additional disutility to becoming unemployed or bankrupt, the fact that business cycles leave some consumers/workers untouched while others are severely affected will increase the overall impact on welfare unless utility is linear, as Lewis (2003) clearly illustrates. However this criticism is about the relative weight to attach to the output gap relative to inflation, and does not appear to have any bearing on the choice of inflation target.

3 Stability

There has been another concern with targeting consumer price inflation that has been occasionally raised in the literature. This is that targeting consumer price inflation might be destabilising, because of the feedback from interest rates to the exchange rate. We can illustrate this point by considering the small open economy model outlined in section 2. We ignore shocks, and combine (6) with (3), and then substitute into (2) to give

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\hat{C}/\sigma + \hat{Y}/\psi + \alpha\hat{S}) \tag{17}$$

Substituting (12) into (7) gives

$$\hat{C}_t = E_t[\hat{C}_{t+1}] - \sigma(r_t - E_t[\pi_{H,t+1} + \alpha\Delta\hat{S}_{t+1}]) \tag{18}$$

We can go on to use the risk sharing equation (ignoring shocks and world output) and the demand curve to substitute for both output and the terms of trade in 17, to give

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + A\hat{C}_t \tag{19}$$

where A is a composite of structural parameters (and is positive). These last two equations represent a dynamic system in consumption and inflation. For the rest of the section all variables are in deviation from steady state form, and so we drop the ^ notation for clarity.

We consider the two most simple monetary policy rules:

forecast output inflation targeting

$$r_t = (1+m)E_t[\pi_{H,t+1}] \tag{20}$$

forecast consumer inflation targeting

$$r_t = (1+m)E_t[\pi_{H,t+1} + \alpha\Delta s_{t+1}] \tag{21}$$

Taking (20) first, and substituting gives the following dynamic system

$$\begin{bmatrix} \pi_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -A/\beta \\ \frac{m(1-\alpha)\sigma}{\beta} & 1-\frac{m(1-\alpha)\sigma}{\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} \tag{22}$$

Both variables jump, so we need two unstable roots, and we show in Appendix 33 that under reasonable conditions for m (including the 'Taylor principle') this is assured.

Turning to (21), then (18) becomes

$$(1-\frac{m\alpha}{1-\alpha})c_t = (1-\frac{m\alpha}{1-\alpha})E_t[c_{t+1}] - m\sigma E_t[\pi_{H,t+1}] \tag{23}$$

The conditions required for stability are more restrictive. In particular, we require

$$1-\frac{m\alpha}{1-\alpha} = \frac{1-\alpha(1+m)}{1-\alpha} > 0 \tag{24}$$

A similar result (for a continuous time version of a small open economy model) is demonstrated in Linnemann and Schabert (2001). Recall that α is the share of overseas goods in the consumption bundle. If α is small then this condition is likely to hold. However if α is large, then the condition could well fail with a fairly aggressive monetary policy. (For example, if $m = 1$, such that real interest rates rise by 1 point for each 1 point increase in inflation, then the system will be indeterminate for $\alpha > 0.5$.) [5] On the other hand, any reduced pass through of exchange rate changes, generated by pricing to market for example, will reduce the risk of this instability occurring.

This stability problem appears to be fairly generic: it would apply, for example, to an economy with a more traditional static IS curve, but with dynamics provided by UIP. Leith and Wren-Lewis (2003) show that it also extends to a two-country world. Their set-up involves some differences from the model above: in particular, international risk sharing is dropped and instead consumers are of the Blanchard/Yaari type, and all goods are traded internationally. This last assumption is equivalent to setting $\alpha = 1$, with the result that in their model consumer price inflation targeting by both countries will leave the system indeterminate *for any positive m.*

The intuition behind these instability results is fairly straightforward. As α becomes large, most consumer goods are produced overseas, and if the country is small these prices will be immune to the domestic nominal inertia distortion. Following a domestic demand shock, the CPI will be largely unaffected, but any increase in domestic interest rates will appreciate the exchange rate which will lower the CPI.

Even if the parameters of the model are such that complete instability is avoided, it seems likely that the feedback from interest rates to the CPI through the exchange rate may cause problems for any policy based on simple rules. For simplicity the model above postulated rules that reacted to one period ahead forecast inflation. If, in practice, policy reacts to actual inflation, then the rule may generate erratic paths for inflation. For example, consider a domestic demand shock which will raise future inflation. Markets, knowing the rule and that future inflation will rise, will anticipate future interest rate increases, and this will generate an immediate appreciation. This will lower current CPI inflation, leading the monetary authorities to lower interest rates. If underlying welfare depends on domestic inflation and output gaps, then this policy response is unhelpful. (See Leith and

[5] Estimated values of m are generally less than unity, but exercises computing the optimal value of m often result in values much larger than one (e.g. Kollman).

Wren-Lewis (2001) for some further examples.)

4 Conclusions

In this paper we have considered two arguments why monetary authorities in an open economy should not target consumer price inflation or the exchange rate, but instead target output price inflation alone. The first derives second order approximations to the authorities objective function from the representative consumer's utility, and shows that it is output price inflation, rather than consumer price inflation or the exchange rate, that appears in this objective function. The second shows how policy rules involving consumer price inflation can induce instability because of the feedback from interest rates to consumer price inflation via the exchange rate.

This second argument, concerning stability, may not be critical in economies with a large proportion of non-traded goods as long as monetary policy is not too aggressive. Problems of timing, where the exchange rate reacts to future expected increases in interest rates leading to changes in the CPI that may not be coincident with domestic demand pressure, are a nuisance that can be dealt with if policy avoids sticking to a fixed simple rule. However these difficulties need only be faced if policy requires stabilising consumer rather than output price inflation, which brings in the first argument.

In this paper we have shown that it does not make sense for policy makers to ignore the exchange rate and focus only on output price inflation and the output gap. In particular, UIP shocks may lead to distortions in the exchange rate which impact on the intertemporal pattern of consumption, without necessarily producing output gaps. It may therefore be appropriate for monetary authorities to target the 'exchange rate gap' as well as the 'output gap', where the exchange rate gap is the difference between the actual exchange rate and the level that would occur in the absence of UIP shocks and nominal inertia. The 'exchange rate gap' concept has many similarities to the deviation of the exchange rate from its Fundamental Equilibrium level (FEER).

However, these arguments do not influence the case for targeting output rather than consumer price inflation. In welfare terms, inflation captures the distortion in the pattern of output generated by asynchronised price adjustment caused by menu costs, and here it is output price inflation rather than consumer price inflation that is relevant. A consumer price inflation target can be thought of as the combination of an output price inflation target and a target for changes in the exchange rate. Our welfare analysis suggests targeting the exchange rate gap, and not the change in the exchange rate. There is also absolutely no reason to attach equal weight to targets for domestic inflation and any exchange rate measure. Therefore the argument that the inflation target should be output price inflation rather than consumer price inflation appears robust.

This welfare analysis of policy targets remains restricted in many ways. In particular, the representative agent assumption in these models diminishes the impact that business cycles have compared to those we actually observe. The analysis also ignores the costs of inflation associated with the medium of exchange. However, it is not at all clear why these limitations should justify the continued use of a consumer price inflation target. The intuition behind output price inflation targeting, which is that nominal inertia distorts the allocation of labour to domestic production, remains sound. The price of imported consumer goods, on the other hand, depends either on prices set elsewhere or the exchange rate, and there is no inertia in exchange rate movements. Indeed, if there are no distortions in the foreign exchange market, movements in the exchange rate (and consequent changes in consumer prices) perform a useful allocative role, and there is no need for the authorities to react to these movements.

Take the recent UK experience, for example. One argument, put forward strongly by H.M.Treasury in their recent discussion of EMU entry, is that the appreciation in sterling from 1997 to 2002 reflected the strength of UK domestic demand. Here the real appreciation was helpful in diverting overseas demand from UK goods, and it could have been dangerous to attempt to avoid this appreciation through policy. However, to the extent that this appreciation reduced CPI inflation in 1997/8, it may have led the Bank of England to under react. An alternative interpretation of sterling's appreciation is that it represented, at least in part, an exchange rate bubble.

In this case, as our paper shows, there would be a strong case for reducing interest rates to moderate the impact of this appreciation on the domestic economy. However, in this case interest rates should be lower for as long as the bubble persisted, and not just in the early years when the appreciation influenced consumer prices.

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A Deriving the Model

Home and Foreign goods

The representative household maximises (1) where the aggregate consumption bundle is given by

$$C = [(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{25}$$

where C_H and C_F are indices of consumption of domestic and foreign goods, and we drop the time subscript wherever all variables are dated at t. The parameter α is related to the share of imported goods in domestic consumption. In turn

$$C_H = \left[\int_0^1 c_H^{\frac{\epsilon-1}{\epsilon}}(z)dz\right]^{\frac{\epsilon}{\epsilon-1}}, \qquad C_F = \left[\int_0^1 c_F^{\frac{\epsilon-1}{\epsilon}}(z)dz\right]^{\frac{\epsilon}{\epsilon-1}}, \qquad \epsilon > 1 \tag{26}$$

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$c_H(z) = \left(\frac{p_H(z)}{P_H}\right)^{-\epsilon} C_H, \qquad c_F(z) = \left(\frac{p_F(z)}{P_F}\right)^{-\epsilon} C_F, \qquad z \in [0,1] \tag{27}$$

where

$$P_H = \left[\int_0^1 p_H^{1-\epsilon}(z)dz\right]^{\frac{1}{1-\epsilon}}, \qquad P_F = \left[\int_0^1 p_F^{1-\epsilon}(z)dz\right]^{\frac{1}{1-\epsilon}} \tag{28}$$

The optimal allocation of expenditures between domestic and foreign goods implies, given the law of one price implies:

$$C_H = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C, \qquad C_F = \alpha\left(\frac{P_F}{P}\right)^{-\eta} C \tag{29}$$

where the consumer price index (CPI) is:

$$P = ((1-\alpha)P_H^{1-\eta} + \alpha P_F^{1-\eta})^{\frac{1}{1-\eta}} \tag{30}$$

We can also define the home output index Y as

$$Y = [\int_0^1 y(z)^{\frac{\epsilon-1}{\epsilon}} dz]^{\frac{\epsilon}{\epsilon-1}} \tag{31}$$

Intertemporal optimisation

Each household faces the same flow budget constraint:

$$P_t C_t + E_t[R_{t,t+1} B_{t+1}] \leq B_t + W_t N_t + T_t \tag{32}$$

where B_{t+1} is the nominal payoff in period t+1 of a portfolio held at the end of period t, R is a stochastic discount factor for nominal payoffs, W is the nominal wage, N labour supply and T denotes lump sum transfers/taxes. The riskless short term interest rate, r_t, is given by

$$\frac{1}{1+r_t} = E_t(R_{t,t+1})$$

Together with transversality conditions, the budget constraint can be solved forward to yield:

$$\sum_{s=t}^{\infty} E_t(R_{t,s} P_s C_s) \leq B_t + \sum_{s=t}^{\infty} E_t(R_{t,s}[W_s N_s + T_s])$$

Assume a linear production technology, such that $N_t = y_t(z)$. The Lagrangian and first order conditions are

$$\begin{aligned} L = E_t \sum_{s=t}^{\infty} \beta^{s-t} [u(C_s^i, \xi_s) - v(y_s(z), \xi_s)] \\ - \lambda [\sum_{s=t}^{\infty} E_t(R_{t,s} P_s C_s) - B_t - \sum_{s=t}^{\infty} E_t(R_{t,s}[W_s y(z)_s + T_s]) \end{aligned}$$

$$\frac{\partial L}{\partial y_s(z)} = -\beta^{s-t} v_h(y_s, \xi_s) + \lambda R_{t,s} W_s = 0 \tag{33}$$

$$\frac{\partial L}{\partial C_s} = \beta^{s-t} u_C(C_s^i, \xi_s) - \lambda R_{t,s} P_s = 0 \tag{34}$$

The consumption first order condition can be rewritten as:

$$\beta \frac{u_C(C_{t+1}^i, \xi_{t+1})}{u_C(C_t^i, \xi_t)} \frac{P_t}{P_{t+1}} = \frac{1}{1+i_t}$$

and we can also write the familiar liesure/consumption trade off

$$\frac{v_y(y_s, \xi_s)}{u_C(C_s^i, \xi_s)} = \frac{W_s}{P_s} \tag{35}$$

Terms of trade and real exchange rate

We assume that the law of one price holds

$$\epsilon = \frac{P_F}{P_F^*} \tag{36}$$

We define terms of trade as

$$\mathcal{S} = \frac{P_F}{P_H} = \frac{P_F^*}{P_H^*} \tag{37}$$

and the real exchange rate

$$Q = \frac{\epsilon P^*}{P} \tag{38}$$

We also assume that prices are equal in equilibrium and the second economy is large so that

$$P_F^* = P^* \tag{39}$$

$$Y^* = C^* \tag{40}$$

Price setting

Price setting follows the usual Calvo set-up with $1-\theta$ of firms changing price in a given period. The log-linear pricing rule for prices changed in period t is given by,

$$\tilde{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k}\}$$

where $\mu = \log(\frac{\varepsilon}{\varepsilon-1})$ is the gross mark-up in the steady-state, and mc are nominal marginal costs. The following log-linear approximation

$$\pi_{H,t} = (1-\theta)(\tilde{p}_{H,t} - p_{H,t-1}) \tag{41}$$

allows us to write (set $p_{H,t-1} = p_{H,t} - \pi_{H,t}$ to obtain an equation for $\theta\pi_{H,t}$ and subtract $\beta\theta\pi_{H,t+1}$)

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \frac{(1-\theta)(1-\beta\theta)}{\theta} m\hat{c}_t \tag{42}$$

which is 2 in the main text.

International risk sharing

With complete securities markets, the Euler equation

$$\beta \frac{u_C(C^i_{t+1}, \xi_{t+1})}{u_C(C^i_t, \xi_t)} \frac{P_t}{P_{t+1}} = R_{t,t+1} \tag{43}$$

must also hold for the rest of the world, i.e.

$$\beta \frac{u_C(C^*_{t+1}, \xi_{t+1})}{u_C(C^*_t, \xi_t)} \frac{P^*_t}{P^*_{t+1}} = R^*_{t,t+1} \tag{44}$$

Perfect arbitarge would suggest

$$E_t[\frac{R^*_{t,t+1}\epsilon_{t+1}}{\epsilon_t}] = E_t[R_{t,t+1}] \tag{45}$$

which is Uncovered Interest Parity when $\frac{1}{1+i_t} = E[R_{t,t+1}]$ and $\frac{1}{1+i^*_t} = E[R^*_{t,t+1}]$. We want to introduce a distortion into UIP, such that

$$\frac{\epsilon_t \zeta_t}{E_t[\epsilon_{t+1}\zeta_{t+1}](1+r^*_t)} = \frac{1}{1+r_t} \tag{46}$$

where ζ_t is the distortion or UIP shock.The empirical importance of distortions to UIP are discussed in Kollmann (2003) and Kollmann (2002). Using this relationship with the two Euler equations gives

$$\frac{u_C(C_{t+1}, \xi_{t+1}) u_C(C^*_t, \xi^*_t)}{u_C(C^*_{t+1}, \xi^*_{t+1}) u_C(C_t, \xi_t)} \frac{\zeta_t}{\zeta_{t+1}} = \frac{Q_t}{Q_{t+1}} \tag{47}$$

where Q is the real exchange rate defined above. We assume that ξ and ξ^* are identical for simplicity: departures from this assumption add terms that are very similar to our UIP distortion. By interating we can show that home consumption will be related to world consumption, the real exchange rate, and the UIP distortion. A linearised version is shown below.

Log-linearisation

We now log linearise around a steady state, where, for any variable X_t, $\hat{X}_t = \ln(X_t/X)$, where X is the steady state value of X_t. This will enable us to derive the equations presented in Section 2 of the main text. However, for the demand curve and risk sharing, we also need second order expansions to plug into the welfare analysis.

Log-linearising the definition of consumer prices around a steady state where $P_H = P_F$ gives

$$\hat{P} = (1-\alpha)\hat{P}_H + \alpha\hat{P}_F \tag{48}$$

$$\hat{P} = \hat{P}_H + \alpha\hat{S} \tag{49}$$

Denoting inflation as $\pi_{i,t+1} = \ln(P_{i,t+1}/P_{i,t})$, then we can also write

$$\pi = \pi_H + \alpha\Delta\hat{S} \tag{50}$$

where π is CPI inflation, and π_H output price inflation (Equation 12.in the main text.) Using the first order expansion

$$U_C(C_t,\xi_t) = U_C(C_t,\xi_t)(1 + \frac{U_{CC}(C,\xi)C\hat{C}_t}{U_C(C,\xi)} + \frac{U_{C\xi}(C,\xi)\hat{\xi}_t}{U_C(C,\xi)}) \tag{51}$$

and a similar expression for v_y allows us to derive both 7 and 6 in the main text.

To derive the demand curve for home goods, note that these goods are either consumed at home or abroad, so that

$$Y = C_H + C_H^* \tag{52}$$
$$\widehat{Y} = (1-\alpha)\widehat{C}_H + \alpha\widehat{C}_H^* \tag{53}$$

to first order. From the demand curve we have

$$\widehat{C}_H - \widehat{C} = -\eta(\widehat{P}_H - \widehat{P}) = \eta\alpha\hat{S} \tag{54}$$

We can write a similar relationship for foreign consumers:

$$\widehat{C}^*_H - \widehat{C}^* = -\eta(\widehat{P}^*_H - \widehat{P}^*) \tag{55}$$

As the rest of the world is large, $C^* = Y^*$ and $P^* = P_F$, so output is given by

$$\hat{Y} = (1-\alpha)(\widehat{C}+\eta\alpha\hat{S})+\alpha(\hat{Y}^*+\eta\hat{S}) = \alpha\hat{Y}^*+(1-\alpha)\widehat{C}+\alpha\eta(2-\alpha)\hat{S} \tag{56}$$

which is 4 in the main text. We will also need a second order approximation for the demand curve in our welfare analysis. This is given by

$$\begin{aligned}
\widehat{Y}_s &= (1-\alpha)\widehat{C} + \alpha\widehat{Y}^* + \alpha\eta(2-\alpha)\widehat{\mathcal{S}} \\
&+ \frac{1}{2}\kappa\eta^2\alpha^2(2-\alpha)\widehat{\mathcal{S}}^2 + \kappa\alpha\eta\widehat{\mathcal{S}}\widehat{Y}^* - \kappa\alpha\eta(1-\alpha)\widehat{\mathcal{S}}\widehat{C} \\
&+ \kappa\frac{1}{2}(1-\alpha)\widehat{C}^2 - \kappa\frac{1}{2}\widehat{Y}^2 + \kappa\frac{1}{2}\alpha\widehat{Y}^{*2}
\end{aligned}$$

where $\kappa = 1$,but can be set to zero to eliminate second order terms.

Taking a second order expansion of the risk sharing condition gives

$$\begin{aligned}
&\widehat{C}_s = \widehat{Y}^*_s + \sigma(1-\alpha)\widehat{\mathcal{S}}_t - \sigma\widehat{\zeta}_s \\
&- \frac{1}{2}\kappa b\widehat{C}^2_s - \kappa d\widehat{C}_s\widehat{\xi}_s + \frac{1}{2}\kappa\sigma(1-\alpha)(1-\alpha(2-\eta))\widehat{\mathcal{S}}^2_t \\
&+ \frac{1}{2}\kappa b\widehat{Y}^{*2}_s - \kappa(1-\alpha)\widehat{C}_s\widehat{\mathcal{S}}_t - \kappa g(1-\alpha)\widehat{\xi}_s\widehat{\mathcal{S}}_t \\
&- \kappa g\frac{1}{2}a\widehat{\xi}^2_s - \kappa\frac{1}{2}\sigma\widehat{\zeta}^2_s + \kappa\widehat{Y}^*_s\widehat{\zeta}_s + \kappa g\widehat{\xi}^*_s\widehat{\zeta}_s + \kappa g\frac{1}{2}a\widehat{\xi}^{*2}_s + \kappa d\widehat{Y}^*_s\widehat{\xi}^*_s
\end{aligned} \tag{57}$$

where

$$\begin{aligned}
a &= 1 + \frac{u_{C\xi\xi}(C^*,1)}{u_{C\xi}(C^*,1)} = 1 + \frac{u_{C\xi\xi}(C,1)}{u_{C\xi}(C,1)} \\
b &= 1 + \frac{Cu_{CCC}(C,1)}{u_{CC}(C,1)} = 1 + \frac{C^*u_{CCC}(C^*,1)}{u_{CC}(C^*,1)} \\
d &= \frac{u_{CC\xi}(C^*,1)}{u_{CC}(C^*,1)} = \frac{u_{CC\xi}(C,1)}{u_{CC}(C,1)}
\end{aligned}$$

and σ and g are defined in the main text. Setting $\kappa = 0$ allows us to recover 5 in the main text.

B Second Order Approximation

A second order Taylor expansion of a function of two variables $f(x, y)$ around the point (x_0, y_0) is given by

$$\begin{aligned} f(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) \\ + \frac{1}{2}f_{xx}(x_0,y_0)(x-x_0)^2 + \frac{1}{2}f_{yy}(x_0,y_0)(y-y_0)^2 \\ + f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + O[(x,y)^3] \end{aligned}$$

Expanding $U(C_s, \xi_s)$ around $U(C, 1)$ gives

$$\begin{aligned} U(C_s,\xi) = U(\cdot) + U_C(\cdot)(C_s - C) + U_\xi(\cdot)(\xi_s - 1) + \frac{1}{2}U_{CC}(\cdot)(C_s - C)^2 \\ + \frac{1}{2}U_{\xi\xi}(\cdot)(\xi_s - 1)^2 + U_{C\xi}()(C_s - C)(\xi - 1) + O[(C,\xi)^3] \end{aligned} \tag{58}$$

where utility and its derivatives are all valuated at $(C, 1)$. If we collect those terms that are independent of policy and terms of order three or more as *tip*, then this can be rewritten as

$$U(C_s,\xi) = U_C(\cdot)(C_s - C) + \frac{1}{2}U_{CC}(\cdot)(C_s - C)^2 + U_{C\xi}(\cdot)(C_s - C)(\xi_s - 1) + tip \tag{59}$$

We are interested in log linear deviations from steady state, so we use the following second order approximation:

$$\begin{aligned} Let \quad y = e^x, \ln y = x \\ e^x = e^{x_0}[1 + (x - x_0) + \frac{1}{2}(x - x_0)^2 + O[(x - x_0)^3] \\ so \quad y = y_0[1 + \ln(\frac{y}{y_0}) + \frac{1}{2}\ln(\frac{y}{y_0})^2 + O[\ln(\frac{y}{y_0})^3]] \\ y - y_0 = y_0[\ln(\frac{y}{y_0}) + \frac{1}{2}\ln(\frac{y}{y_0})^2 + O[\ln(\frac{y}{y_0})^3]] \end{aligned}$$

Let us denote, for any y, $\hat{y} = \ln(y/y_0)$. Applying this gives

$$\begin{aligned} U(C_s, \xi) &= U_C(\cdot)C(\hat{C} + \frac{1}{2}\hat{C}^2) + \frac{1}{2}U_{CC}(\cdot)C^2(\hat{C} + \frac{1}{2}\hat{C}^2)^2 \\ &+ U_{C\xi}(\cdot)C(\hat{C} + \frac{1}{2}\hat{C}^2)\xi(\hat{\xi} + \frac{1}{2}\hat{\xi}^2) + tip \\ &= U_C(\cdot)C(\hat{C} + \frac{1}{2}\hat{C}^2) + \frac{1}{2}U_{CC}(\cdot)C^2\hat{C}^2 + U_{C\xi}(\cdot)C\hat{C}\hat{\xi} + tip \end{aligned} \tag{60}$$

We introduce two pieces of notation. σ is the intertemporal elasticity of substitution, and g is the percentage variation in consumption required to keep the marginal utility of consumption constant after a unit preference shock, where

$$\sigma = -\frac{U_C(\cdot)}{U_{CC}(\cdot)C} \tag{61}$$

$$g = \frac{U_{C\xi}(\cdot)}{U_{CC}(\cdot)C} = -\frac{U_{C\xi}(\cdot)\sigma}{U_C(\cdot)} \tag{62}$$

Using this notation implies

$$\begin{aligned} U(C_s, \xi_s) &= U_C(\cdot)C(\hat{C} + \frac{1}{2}\hat{C}^2) - \frac{1}{2}\frac{U_C(\cdot)}{\sigma}C\hat{C}^2 - U_C(\cdot)\frac{g}{\sigma}C\hat{C}\hat{\xi} + tip \\ &= U_c(\cdot)C[\hat{C} + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{C}^2 - \frac{g}{\sigma}\hat{C}\hat{\xi}] + tip \end{aligned} \tag{63}$$

Applying the same procedure to $v(y_s(z), \xi_s)$ gives

$$v(y_s(z), \xi_s) = v_y(\cdot)y[\hat{y}(z) + \frac{1}{2}(1 + \frac{1}{\psi})\hat{y}(z)^2 + \frac{h}{\psi}\hat{y}(z)\hat{\xi}] + tip \tag{64}$$

where

$$\varphi = \frac{v_y(\cdot)}{v_{yy}(\cdot)y} \tag{65}$$

$$h = \frac{v_{y\xi}(\cdot)}{v_{yy}(\cdot)y} = \frac{v_{y\xi}(\cdot)\varphi}{v_y(\cdot)} \tag{66}$$

Integrating over goods

$$\int_0^1 v(y_s(z),\xi_s)dz = Yv_y()\int_0^1 [\widehat{y}_s(z) + \frac{1}{2}(1+\frac{1}{\psi})\widehat{y}(z)^2 + \frac{h}{\varphi}\widehat{y}(z)\widehat{\xi}]dz + tip \tag{67}$$

$$= Yv_y()[E_z\widehat{y}(z) + \frac{1}{2}(1+\frac{1}{\psi})[(E_z\widehat{y}(z))^2$$
$$+ V_z\widehat{y}(z)] + \frac{h}{\varphi}\widehat{\xi}_s E_z\widehat{y}(z)] + tip$$

where the expectation $E_z x = \int xdz$ and the variance $V_z = \int (x - E_z x)^2 = \int x^2 dz - E_x x$.

We can define an index of aggregate demand Y as

$$Y = [\int_0^1 y(z)^{\frac{\epsilon-1}{\epsilon}} dz]^{\frac{\epsilon}{\epsilon-1}} \tag{68}$$

where ϵ is the elasticity of demand. Taking a Taylor expansion of this yields

$$\widehat{Y} = E_z\widehat{y}(z) + \frac{1}{2}(1-\frac{1}{\epsilon})V_z\widehat{y}(z) + O(|\xi|^3) \tag{69}$$

We can use this to eliminate terms in $E_z\hat{y}(z)$, noting that $(E_z\hat{y}(z))^2 = \hat{Y}^2 + O(|\xi|)^3$, and that $V_z\hat{y}(z)\hat{\xi}$ is $O(|\xi|)^3$. This gives

$$\int_0^1 v(y_s(z),\xi_s)dz = Yv_y(\cdot)[\hat{Y} - \frac{1}{2}(1-\frac{1}{\epsilon})V_z\widehat{y}(z) \tag{70}$$
$$+ \frac{1}{2}(1+\frac{1}{\psi})[\hat{Y}^2 + V_z\widehat{y}(z)] + \frac{h}{\varphi}\widehat{\xi}\hat{Y}] + tip$$
$$= Yv_y(\cdot)[\hat{Y}(1+\frac{h}{\varphi}\widehat{\xi}) + \frac{1}{2}(1+\frac{1}{\psi})\hat{Y}^2 + V_z\widehat{y}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip$$

We can now combine the two calculations to give

$$U(C_s,\xi) - \int_0^1 v(y_s(z),\xi_s)dz = W = U_c(\cdot)C[\hat{C}(1-\frac{g}{\sigma}\hat{\xi}) + \frac{1}{2}(1-\frac{1}{\sigma})\hat{C}^2] \tag{71}$$
$$- Yv_y(\cdot)[\hat{Y}(1+\frac{h}{\varphi}\widehat{\xi}) + \frac{1}{2}(1+\frac{1}{\psi})\hat{Y}^2$$
$$+ V_z\widehat{y}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip$$

Until now, these manipulations have only used one model property, which was the definition of the output index from the demand curve. As such, the analysis so far also applies to a closed economy, and is identical to that in Woodford (200x) and Steinsson (2002). We have four types of term in this expression: those involving level deviations in C and Y, those combining these deviations with shocks, quadratic terms in deviations in C and Y, and a term in the variance of output across goods. This last term can be related to the variance of individual prices using the demand curve: i.e.by taking logs of

$$y(z) = \left(\frac{p_H(z)}{P_H}\right)^{-\epsilon} Y \tag{72}$$

(which comes from adding consumption across home and overseas) it follows that

$$V_z\widehat{y}(z) = \epsilon^2 V_x \hat{p}(z) \tag{73}$$

so that (71) becomes

$$\begin{aligned} W = U_c(\cdot)C[\hat{C}(1 - \frac{g}{\sigma}\hat{\xi}) + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{C}^2] - Yv_y(\cdot)[\hat{Y}(1 + \frac{h}{\varphi}\widehat{\xi}) \\ + \frac{1}{2}(1 + \frac{1}{\psi})\hat{Y}^2 + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})] + tip \end{aligned} \tag{74}$$

Woodford shows that, with Calvo contracts, this last term can be related to the aggregate level of output price inflation squared.

Optimality implies that the real wage for good i (in terms of consumer prices) is given by

$$rw(y, C, \xi) = \frac{v_y(y, \xi)}{u_C(C, \xi)} \tag{75}$$

Log linearising (we can safely ignore second order terms, for reasons that become clear below) gives

$$r\hat{w} = [\hat{y}\frac{1}{\varphi} + \hat{C}\frac{1}{\sigma} + \alpha\hat{S} + \hat{\xi}(\frac{h}{\varphi} + \frac{g}{\sigma})] \tag{76}$$

An identical expression occurs in the closed economy, except that the term in S obviously disappears. Suppose we assume, following the literature, that there exists a tax that exactly offsets the effects of monopoly on the real wage, such that the steady state real wage is unity. If prices are flexible, then this will remain the case whatever $\hat{\xi}$, so $r\hat{w} = 0$. Let us denote as $\hat{X}^n$ the disequilibrium in X that would occur if there were no distortions in the economy: specifically, prices were fully flexible and there were no UIP shocks, and refer to these terms as natural levels. Clearly if the model is log-linear, all natural variables can be expressed as functions of $\hat{\xi}$ and $\hat{Y}^*$, world output disequilibrium.

In a closed economy, $\hat{C} = \hat{Y}$, so welfare can be further simplified as

$$W = U_c(\cdot)C[-\hat{C}\frac{g}{\sigma}\hat{\xi} - \hat{C}\frac{h}{\varphi}\widehat{\xi} - \frac{1}{2}(\frac{1}{\sigma} + \frac{1}{\varphi})\hat{C}^2 - V_z\widehat{y}(z)\frac{1}{2}(\frac{1}{\varphi} + \frac{1}{\epsilon})] + tip \tag{77}$$

and so we can eliminate the terms in $\hat{\xi}$ to give

$$\begin{aligned} W &= U_c(\cdot)C[\hat{C}\hat{C}^n(\frac{1}{\sigma} + \frac{1}{\varphi}) - \frac{1}{2}(\frac{1}{\sigma} + \frac{1}{\varphi})\hat{C}^2 - V_z\widehat{y}(z)\frac{1}{2}(\frac{1}{\varphi} + \frac{1}{\epsilon})] + tip \\ &= U_c(\cdot)C[-\frac{1}{2}(\frac{1}{\sigma} + \frac{1}{\varphi})(\hat{C} - \hat{C}^n)^2 - V_z\widehat{y}(z)\frac{1}{2}(\frac{1}{\varphi} + \frac{1}{\epsilon})] + tip \end{aligned} \tag{78}$$

First we use (75) to replace $\hat{\xi}$ by terms in $\hat{C}^n$, while the second line notes the relationship between the $\hat{C}^2$ and $\hat{C}\hat{C}^n$ terms to simplify in terms of the 'consumption gap' $\hat{C} - \hat{C}^n$, bearing in mind that terms in $\hat{C}^n$ alone can be added to tip (as they are only functions of the shock). Thus policy can increase welfare in two ways: by reducing the variance of output (across goods) and keeping output/consumption close to its natural level.

In an open economy we use the same procedure. It is instructive (but in general illegitimate) to first consider a log-linear version of the model. This is illegitimate because it ignores second order terms which will survive on substitution into the expression for welfare, and we subsequently look at the correct second order case. Consider first the terms in the level of consumption and output deviations alone:

$$U_c(\cdot)C\hat{C} - Yv_y(\cdot)\hat{Y} \tag{79}$$

We need to eliminate these first order terms. To first order, we can write an expression relating the log deviation in output and consumption as

$$\hat{C} = \Phi\hat{Y} + \hat{X} \tag{80}$$

where $\Phi = \frac{(1-\alpha)}{[(2-\alpha)\frac{\eta}{\sigma}\alpha+(1-\alpha)^2]}$ is derived from (4) and (5) and X is a combination of exogenous variables and shocks given by $X = (1-\Phi)\widehat{Y}^* + (1-(1-\alpha)\Phi)\sigma\widehat{\zeta}$. Substituting into the previous expression gives

$$U_c()C(\Phi\hat{Y} + \hat{X}) - Yv_y()\hat{Y} \tag{81}$$

An optimal subsidy that eliminates the monopolistic distortions implies $U_c()C\Phi = Yv_y$, and as $\hat{X}$ is tip, these first order terms are eliminated.

We can now examine a special case of the open economy model, where utility is logarithmic, which is the case considered by Gali and Monacelli (2002). We then have $\sigma = 1, \varphi = -1, g = h = 0$. In this case all the terms in (71) are eliminated except the final term. The open economy set-up is therefore identical to the closed economy case.

In the more general case, (71) becomes

$$\begin{aligned} W &= U_c(\cdot)C\Phi[-(\hat{Y} + \frac{\hat{X}}{\Phi})\frac{g}{\sigma}\hat{\xi} + \frac{1}{2\Phi}(1 - \frac{1}{\sigma})\hat{C}^2] \\ &- Yv_y(\cdot)[\hat{Y}\frac{h}{\varphi}\widehat{\xi} + \frac{1}{2}(1 + \frac{1}{\psi})\hat{Y}^2 + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})] + tip \\ &= Yv_y(\cdot)[\frac{1}{2\Phi}(1 - \frac{1}{\sigma})\hat{C}^2 - \frac{1}{2}(1 + \frac{1}{\psi})\hat{Y}^2 \\ &- \hat{Y}\hat{\xi}(\frac{g}{\sigma} + \frac{h}{\varphi}) + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})] + tip \end{aligned} \tag{82}$$

We can again use (75) to replace $\hat{\xi}$ with natural levels:

$$W = Yv_y(\cdot)[\frac{1}{2\Phi}(1-\frac{1}{\sigma})\hat{C}^2 - \frac{1}{2}(1+\frac{1}{\psi})\hat{Y}^2 + \hat{Y}(\hat{Y}^n\frac{1}{\varphi} + \hat{C}^n\frac{1}{\sigma} + \alpha\hat{S}^n) + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip$$
$$= Yv_y(\cdot)[\frac{1}{2\Phi}(1-\frac{1}{\sigma})\hat{C}^2 - \frac{1}{2}(1+\frac{1}{\psi})\hat{Y}^2 + \hat{Y}\hat{Y}^n\frac{1}{\varphi} + \hat{C}\hat{C}^n\frac{1}{\Phi\sigma} + \alpha\hat{Y}\hat{S}^n + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip \quad (83)$$

where the term in $\hat{X}\hat{C}^n$ is tip. Noting that $\widehat{Y}_s^2 = (\widehat{Y} - \widehat{Y}^n)^2 + 2\widehat{Y}\widehat{Y}^n + tip$ and similarly for consumption, we can rewrite as

$$W = Yv_y(\cdot)[\frac{1}{2\Phi}(1-\frac{1}{\sigma})(\hat{C}-\hat{C}^n)^2 - \frac{1}{2}(1+\frac{1}{\psi})(\hat{Y}-\hat{Y}^n)^2 + \hat{Y}\hat{Y}^n + \hat{C}\hat{C}^n\frac{1}{\Phi} + \alpha\hat{Y}\hat{S}^n + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip$$
$$= Yv_y(\cdot)[\frac{1}{2\Phi}(1-\frac{1}{\sigma})(\hat{C}-\hat{C}^n)^2 - \frac{1}{2}(1+\frac{1}{\psi})(\hat{Y}-\hat{Y}^n)^2 - \hat{Y}(\hat{Y}^n - \hat{C}^n) + \alpha\hat{Y}\hat{S}^n + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip \quad (84)$$

Using first order approximations we can note that $\widehat{Y}^n - \widehat{C}^n = [\eta(2-\alpha) - \sigma(1-\alpha)]\alpha\widehat{\mathcal{S}}^n$, so finally we have

$$W = v_y(\cdot)[\frac{1}{2\Phi}(1-\frac{1}{\sigma})(\hat{C}-\hat{C}^n)^2 - \frac{1}{2}(1+\frac{1}{\psi})(\hat{Y}-\hat{Y}^n)^2 - (\hat{Y}-\hat{Y}^n)\hat{S}^n\alpha(\eta(2-\alpha) - \sigma(1-\alpha) - 1) + \epsilon^2 V_z\hat{p}(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})] + tip \quad (85)$$

The term in $(\hat{Y} - \hat{Y})\hat{S}^n$ is interesting. Assuming that it is negative (which will be true if $0 > \sigma$ and $\eta > 1$), it suggests that if a shock moves the terms of trade in a positive direction (i.e. $\hat{S}^n > 0$, which is a real depreciation), then policy makers have an incentive to depress output below its natural level. The optimal policy is no longer to move the economy to its flexible price (natural) level. This property of an open economy has been noted in related contexts by Beningno and Benigno (200x) and Pappa (2002).

In the absence of UIP shocks, it is possible to rewrite terms in the consumption gap as terms in the output gap alone. (As $\hat{Y}^*$ is the same for both actual and natural levels, then with $\widehat{\zeta} = 0$ we have $\hat{C} - \hat{C}^n = \Phi(\hat{Y} - \hat{Y}^n)$.) This is the point made by Clarida, Gali, and Gertler (2001). However the implication that they draw that the open economy case is isomorphic to the closed economy also depends on eliminating the $\hat{Y}\hat{S}^n$ term by some means. When UIP shocks are present, it is no longer possible to replace the consumption gap by the output gap. Instead, eliminating the consumption gap introduces terms in both the output gap and the terms of trade gap. (Alternatively, the output gap could be replaced by terms in the consumption gap and the terms of trade gap. In either case we use the demand curve i.e. $(1-\alpha)(\hat{C} - \hat{C}^n) + \alpha\eta(2-\alpha)(\hat{S} - \hat{S}^n) = (\hat{Y} - \hat{Y}^n)$). In both cases, we introduce cross terms that combine the terms of trade gap with either the output or consumption gap (see Pappa (2002)).

We now repeat this process using second order approximations to the key relationships in the model. We replace 80 with the following expression, which can be derived by combining the second order aggregate demand and risk sharing conditions outlined in Appendix A.

$$\begin{aligned}\widehat{C} &= \Phi\widehat{Y} + \frac{1}{2}\Phi\kappa\widehat{Y}^2 \\ &+ \frac{1}{2}\Phi\kappa(2-\alpha)\alpha\eta(1-2\alpha)\widehat{\mathcal{S}}^2 - \frac{1}{2}\kappa\frac{\Phi}{\Phi_b}\widehat{C}_s^2 \\ &- \kappa d\Phi\frac{\alpha\eta(2-\alpha)}{\sigma(1-\alpha)}\widehat{C}_s\widehat{\xi}_s - \Phi\frac{\alpha\eta(2-\alpha)}{\sigma(1-\alpha)}\kappa\rho\widehat{\xi}_s\widehat{\mathcal{S}}_t(1-\alpha) - \Phi\kappa\alpha\eta\widehat{\mathcal{S}}\widehat{Y}^* \\ &+ \Phi\kappa\alpha\eta(1-\alpha)\widehat{\mathcal{S}}\widehat{C} - \Phi\frac{\alpha\eta(2-\alpha)}{\sigma(1-\alpha)}\kappa(1-\alpha)\widehat{C}_s\widehat{\mathcal{S}}_t + tip\end{aligned} \tag{86}$$

where

$$\Phi_b = \frac{\sigma(1-\alpha)}{[b(2-\alpha)\eta\alpha + \sigma(1-\alpha)^2]} \tag{87}$$

The variable $\kappa = 1$, but can be set to zero to recover the first order case. We then obtain the following expression for welfare

$$
\begin{aligned}
W = & \frac{1}{2}\kappa\widehat{Y}^2 - \frac{1}{2}(1+\frac{1}{\psi})\widehat{Y}_s^2 - \frac{1}{2}\kappa\frac{1}{\Phi_b}\widehat{C}_s^2 + \frac{1}{2}\frac{1}{\Phi}(1-\frac{1}{\sigma})\widehat{C}_s^2 \\
& + \frac{1}{2}\kappa(2-\alpha)\alpha\eta(1-2\alpha)\widehat{\mathcal{S}}^2 - \kappa\alpha\eta(2-\alpha)(d+\frac{g}{\sigma})\widehat{\mathcal{S}}_t\widehat{\xi} \\
& - \kappa\alpha\eta\widehat{\mathcal{S}}\widehat{Y}^* + \kappa\alpha\eta[(1-\alpha) - \frac{(2-\alpha)}{\sigma}]\widehat{C}\widehat{\mathcal{S}} \\
& + \widehat{Y}_s(\frac{u_{C\xi}(C,1)}{u_C(C,1)}\widehat{\xi}_s - \frac{v_{y\xi}(y,1)}{v_y(y,1)}\widehat{\xi}_s) - \frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})var_z\widehat{y}_s(z) + tip
\end{aligned} \tag{88}
$$

By observing the terms multiplied by κ, we can see that adding second order terms has introduced additional terms in $\widehat{C}_s^2, \widehat{\mathcal{S}}^2, \widehat{\mathcal{S}}_t\widehat{\xi}, \widehat{\mathcal{S}}\widehat{Y}^*$,and $\widehat{C}\widehat{\mathcal{S}}$. The key conclusions discussed in the main text therefore continue to hold.

C Stability Analysis

We consider the two most simple monetary policy rules:

forecast output inflation targeting

$$r_t = (1+m)E_t[\pi_{H,t+1}] \tag{89}$$

forecast consumer inflation targeting

$$r_t = (1+m)E_t[\pi_{H,t+1} + \alpha\Delta s_{t+1}] \tag{90}$$

Taking (89) first, and substituting gives the following dynamic system

$$\begin{bmatrix}\pi_{t+1}\\ c_{t+1}\end{bmatrix} = \mathbf{B}\begin{bmatrix}\pi_t\\ c_t\end{bmatrix} \tag{91}$$

where $\mathbf{B} = \begin{bmatrix}\frac{1}{\beta} & -A/\beta \\ \frac{m(1-\alpha)\sigma}{\beta} & 1-\frac{m(1-\alpha)\sigma}{\beta}\end{bmatrix}$. Both variables jump, so we need two unstable roots. However since we are operating in discrete time, we need to assess whether or not the eigenvalues are real or complex. To do so consider the trace,

$$tr_B = \frac{1+\beta}{\beta} - \frac{m(1-\alpha)\sigma A}{\beta}$$

and determinant,

$$|\mathbf{B}| = \frac{1}{\beta} > 1$$

Since the determinant is positive the three conditions we need to satisfy for determinacy are as follows,

$$|\mathbf{B}| > 1, \;\; |\mathbf{B}| - tr_B > -1, \text{ and } |\mathbf{B}| + tr_B > -1$$

The first of these conditions is already satisfied. The second is given by,

$$|\mathbf{B}| - tr_B = \frac{m(1-\alpha)\sigma A}{\beta} - 1 > -1 \; iff \; m > 0$$

In other words the rule must satisfy the Taylor principle. Finally, the third condition is given by

$$|\mathbf{B}| + tr_B = \frac{2+\beta - m(1-\alpha)\sigma A}{\beta} > -1$$

This condition is only satisfied if,

$$m < \frac{2\beta+2}{(1-\alpha)\sigma A}$$

In other words, although we must satisfy the Taylor principle, in the case of a rule defined in terms of expected future inflation determinacy requires that the interest rate response to future inflation cannot be too great. It should be noted that this restriction on the aggressiveness of monetary policy would not apply if the rule was responding to observed rather than expected inflation.

Turning to (90), then (91) becomes

$$(1 - \frac{m\alpha}{1-\alpha})c_t = (1 - \frac{m\alpha}{1-\alpha})E_t[c_{t+1}] - m\sigma E_t[\pi_{H,t+1}] \tag{92}$$

so that the NKPC can be substituted into the consumption Euler equation to obtain,

$$\begin{bmatrix} \pi_{t+1} \\ c_{t+1} \end{bmatrix} = \mathbf{C} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} \tag{93}$$

where $\mathbf{C} = \begin{bmatrix} \frac{1}{\beta} & -A/\beta \\ \frac{\sigma m}{\beta(1-\frac{m\alpha}{1-\alpha})} & 1-\frac{\sigma m A}{\beta(1-\frac{m\alpha}{1-\alpha})} \end{bmatrix}$. Here the conditions for determinacy are given by,

$$|\mathbf{C}| = \frac{1}{\beta} > 1$$

$$|\mathbf{C}| - tr_C = \frac{\sigma m A}{\beta(1-\frac{m\alpha}{1-\alpha})} - 1 > 0 \; iff \; \frac{m}{(1-\frac{m\alpha}{1-\alpha})} > 0$$

and finally,

$$|\mathbf{C}| + tr_C = \frac{2+\beta}{\beta} - \frac{\sigma m A}{\beta(1-\frac{m\alpha}{1-\alpha})} > -1$$

which is satisfied for,

$$\frac{m}{(1-\frac{m\alpha}{1-\alpha})} < \frac{2\beta+2}{\sigma A}$$

The key thing to note about this result is that for sufficiently large values of α, and values of m which would satisfy determinacy in the case of output price inflation targeting, defining the rule in terms of CPI inflation may result in indeterminacy. In fact, as $\alpha \to 1$, any value of m which is satisfies the conditions for determinacy in the case of output price inflation targeting will result in indeterminacy in the case of CPI inflation targeting.