Openness and Inflation[*]

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Abstract

This paper develops a general equilibrium model based on a set of small open economy assumptions and derives an expression for equilibrium inflation. A standard argument is that inflation is decreasing in openness because openness alters the slope of the Phillips curve. But openness also affects the monetary authority's utility function, and this introduces an ambiguity. Inflation may rise and fall as the economy becomes more open because foreign demand for domestic production and openness interact, altering the incentives of the monetary authority.

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1 Introduction

Mundell-Fleming extensions of the Barro and Gordon (1983) model suggest there is an inverse relationship between openness and inflation. In these models expansionary monetary policy causes both an increase in domestic output and a deterioration in the terms of trade. As openness changes, the incentives the (discretionary) monetary policy maker faces change because openness alters the slope of the Phillips curve and the effect of monetary policy on output. As the degree of openness rises the Phillips curve trade-off becomes less favourable and optimal policy is less expansionary. This mechanism generates an inverse relationship between openness and inflation.[1] Although the argument is persuasive it is common to assume the monetary authority's policy decision is based on a directly postulated loss function.[2] A natural question is whether a similar openness-inflation relationship holds when households and the monetary authority optimise over a set of well defined preferences. Consistent with recent work I use an optimising model incorporating nominal rigidities to study this question where monetary policy decisions are based on a utility function.

An important part of determining the equilibrium level of inflation (and therefore it's relation to openness) are the costs and benefits of expansionary monetary policy. King and Wolman (2003) and Woodford (2003) both assume a staggered pricing structure so that the costs of expansionary policy are welfare losses associated with relative price distortions across goods. This approach has also been used by Clarida et al. (2002) in a two country model looking at policy co-ordination. Albanesi et al. (2002) take a different approach measuring the cost of expansionary policy through distortions in money demand associated with movements in the nominal interest rate when consumers have access to cash and credit goods. They adopt Svensson's (1985) timing change for cash goods so that households only have access to the beginning of period level of real balances. To simplify the exposition here I take advantage of this second approach using the beginning of period definition of real balances in the utility

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[1]Empirical investigations based on this idea have been carried out by Romer (1993), Lane (1997) and Temple (2002).

[2]One theoretical example of the ad-hoc approach is Rogoff (1985).

function implying that households make their money holding decision before their production and consumption decisions.[3] This simplification is made for two reasons. First, it allows the derivation of an explicit expression for equilibrium inflation without linear approximations. Second, the beginning of period assumption helps capture the type of results found in the ad-hoc literature allowing for some interesting comparisons. The benefits of expansionary policy in the model occur through increases in an inefficiently low level of output. This is modelled by assuming a monopolistic supply of labour and preset money wage.

Monetary policy here is the choice of the entire sequence of money growth rates, which under discretion may be re-set each period.[4] Together with the beginning of period assumption this implies the policy choice is characterised by a trade-off between increasing domestic output and reducing consumers holdings of real balances. The beginning of period assumption works because if prices rise, current real balances fall, and this leads to reduced purchasing power and a loss in welfare. As the supply of money determines the consumer price index and households have access to domestic and foreign produced goods openness affects the monetary authority's utility function directly. As firms are concerned with the GDP deflator when demanding labour services openness also affects the Phillips curve giving rise to the standard terms of trade effect emphasised in previous studies. The policy trade-off is therefore subject to two influences. Despite this it is still possible to identify how openness alters the equilibrium level of inflation.

The model shows how equilibrium inflation is a function of consumption and real money balances and that the effects of openness operate through both arguments. The more familiar channel, based around the Phillips curve, is associated with the real money balances argument. The second channel works through the utility function of the monetary authority and

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[3] As Nicolini (1998) emphasises the beginning of period specification is entirely consistent with the worker-shopper argument used to motivate both CIA and MIU models. The specification used here can also be thought of as implying a precautionary demand for money.

[4] This is somewhat different to specifying monetary policy as the choice of the nominal interest rate, although the equilibrium concept used has a similar interpretation. See Clarida et al. (1999) for a closed economy model and Clarida et al. (2001) who consider a small open economy.

is associated with the consumption argument. Openness has an unambiguous relationship with equilibrium real balances, as do real balances with inflation. Therefore, holding consumption constant, openness and inflation have the standard inverse relationship. However, consumption depends on foreign demand and different levels of foreign demand are associated with changes in the equilibrium terms of trade. An increase in foreign demand therefore creates a larger incentive for the monetary authority to inflate. This incentive disappears when the economy is closed so that if foreign demand is sufficiently high when the economy is open equilibrium inflation may fall as the economy becomes more closed. Openness has an inverse relationship with inflation when foreign demand is 'low' but when foreign demand is 'high' the standard openness-inflation result is overturned, inflation may rise and fall as the country becomes more open.

The remainder of paper is organised as follows. Section two describes the model. Section three solves the model for an arbitrary rate of money growth. Consistent with the arbitrary policy choice the Phillips curve and consumption-output relations which act as constraints on policy making decisions are derived in section four. Section five computes the (Markov) equilibrium level of inflation when the monetary authority and households play a one shot game and looks in detail at the relationship between openness and inflation. Section six concludes.

2   The Model

The structure of the model is closely related to a number of recent two country macro models that assume optimising agents, nominal rigidities and monopolistic competition. Both economies (domestic and foreign) consist of a continuum of $j \in [0, 1]$ households which supply a differentiated labour type. Households hold real money balances, nominal bonds and consume domestic and foreign goods. The consumption and price level of domestically produced goods are denoted with h-subscripts, and foreign economy equivalents with f. Foreign economy variables are denoted with asterisks. Firms choose amongst labour types and produce

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a single good and a monetary authority controls the supply of money by making lump-sum transfers directly to households. The foreign economy is assumed to be in a zero inflationary steady state and is large relative to the domestic economy. This implies the domestic economy takes conditions in the foreign economy as given and domestic exports form a negligible component of the foreign economy's consumption basket.

In period t firms make a production decision choosing labour, $l_{j,t}$. Nominal wages, $w_{j,t}$, are preset one period in advance so labour is demand determined. Households choose the sequences $\{C_{j,t}, M_{j,t+1}, l_{j,t}\}_{t=0}^{\infty}$ to maximise $U_j = \sum_{t=0}^{\infty} \beta^t(C_{j,t}, m_{j,t}, l_{j,t})$, where $m_{j,t} \equiv M_{j,t}/P_t$ is the beginning of period definition of real money balances and $C_{j,t}$ is consumption.[5] The monetary authority makes a lump-sum nominal money transfer $\vartheta_t = M_t(\mu_t - 1)/\mu_t$ to each household, and dropping j subscripts nominal money holdings at the beginning of period t obey,

$$M_{t+1} = M_t/\mu_t, \tag{1}$$

where μ_t is the current period inverse money growth rate.[6] Changing the MIUF argument in this way and introducing monopolistic competition with a nominal rigidity has two effects. If the lump-sum transfer from the monetary authority is larger than expected, higher inflation reduces purchasing power, whilst with a fixed nominal wage an increase in prices leads to increased output levels as labour is demand determined. When money is transferred there is a trade-off between the costs and benefits of surprise inflation which may change as openness changes because the households consume both domestic and foreign goods.

The households utility function and period t budget constraint are,

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + a\nu m_t^{(\nu-1)/\nu}/(\nu - 1) - \eta l_t^{\kappa}/\kappa \right), \tag{2}$$

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[5] This is different to the specification in Calstrom and Fuerst (2001).

[6] The domestic monetary authority does not issue bonds.

$$B_t + M_{t+1} + P_t C_t = i_{t-1} B_{t-1} + M_t + w_t l_t - \vartheta_t + \varphi_t, \tag{3}$$

where B_t are one period nominal bonds which pay gross interest $i_t > 1$, φ_t are monopoly profits and $\beta, \nu \in (0,1)$, $a, \eta > 0$, and $\kappa > 1$ are parameters. Households consume a composite of domestic and foreign produced goods and the CPI which is defined from households intratemporal consumption problem deflates nominal balances. For simplicity Cobb-Douglas consumption preferences are assumed so,

$$C_t \equiv C_{h,t}{}^n C_{f,t}{}^{1-n} / n^n (1-n)^{1-n}, \tag{4}$$

where $n \in [0,1]$, and the implied price index is $P_t \equiv P_{h,t}^n P_{f,t}^{1-n}$. Similar preferences are assumed in the foreign economy, but to formally model the small open economy assumption $n^* \to 0$, and therefore $P_t^* \to P_{f,t}^*$, where P_t^* is the foreign CPI.[7] The parameter n is taken as the measure of the domestic economy's openness, where as n falls the domestic economy is more open. For each good the law of one price holds so, $P_{a,t} = s_t P_{a,t}^*$ for $a = h, f$, where s_t is the nominal exchange rate. The CPI now links the real and nominal exchange rates.

$$q_t = s_t / P_t, \tag{5}$$

where q_t is the real exchange rate.[8] Finally an arbitrage condition relates the domestic and foreign interest rates to changes in the exchange rate (UIP) which in nominal terms is expressed as,

$$i_t / i_t^* = s_{t+1} / s_t, \tag{6}$$

where i_t^* is the foreign interest rate.

Firms use differentiated labour types to produce a single good. The production process is described by a decreasing returns to scale production function where labour is the only factor input.

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[7] $P_{f,t}^* = P_t^* = 1$ as $P_{f,t}^*$ is completely exogenous.

[8] Given these definitions $q_t = (P_{f,t}/P_{h,t})^n$, where $P_{f,t}/P_{h,t}$ is the terms of trade, τ_t, and also $P_{h,t} = s_t / q_t^{1/n}$.

$$y_{h,t} = \left(\int_0^1 l_{j,t}^{(\sigma-1)/\sigma} dj \right)^{\sigma/\alpha(\sigma-1)}, \tag{7}$$

with $\sigma, \alpha > 1$, which represent the input substitution elasticity of labour types and the returns to scale of the labour input respectively.

Two more conditions are required to describe the domestic economy. The resource constraint is,

$$y_{h,t} = C_{h,t} + C_{h,t}^*, \tag{8}$$

where $C_{h,t}^*$ is aggregate consumption of the domestic good in the foreign economy, and the national budget constraint,

$$B_t = P_{h,t} y_{h,t} - P_t C_t + i_{t-1} B_{t-1}, \tag{9}$$

such that the end of period bond level is equal to domestic output minus the rate of absorption plus interest from claims on foreign bonds.

3   Model Solution

To find the conditions under which markets clear I solve for the nominal exchange rate and current account assuming an arbitrary rate of money growth, μ_t. The solution to the firms problem maximises nominal profits subject to the production constraint, (7). Taking the firms problem in two stages cost minimisation implies conditional labour demand is $l_{j,t} = (w_{j,t}/w_t)^{-\sigma} y_{h,t}^\alpha$, where the overall wage is $w_t = \left(\int_0^1 w_{j,t}^{1-\sigma} \right)^{1/(1-\sigma)}$. Profit maximisation subject to this constraint implies final labour demand depends on the domestic price level and technology,

$$y_{h,t} = (\alpha w_t/P_{h,t})^{1/(1-\alpha)}. \tag{10}$$

The solution to the households problem maximises utility (3), subject to conditional labour demand, the period budget constraint (2) and a no-ponzi game condition. The optimal conditions for the household are,

$$w_t = \sigma\eta P_t C_t / (\sigma - 1) l_t^{1-\kappa} \tag{11}$$

$$P_{t+1}C_{t+1} = P_t C_t \beta i_t \tag{12}$$

$$m_{t+1}^{1/\nu} = aC_t / (i_t - 1). \tag{13}$$

Equation (11) determines labour supply, (12) is the consumption Euler equation and (13) expresses money demand. This specification of preferences implies that the domestic nominal interest rate depends on the domestic real interest rate. The assumption that beginning of period balances enter the utility function now plays an important role because under this timing assumption the period $t+1$ real interest rate affects the period t nominal interest rate. Thus solving for the current level of the nominal interest rate only requires the determination of r_{t+1}.

As agents can only be surprised in the initial period, markets clear in all future periods and for periods $t \geq 1$ the monopolistic supply of labour equals the demand for labour. From (10) and (11) the supply of goods takes the form,

$$y_{h,t} = \left(\alpha\eta\sigma C_t q_t^{(1-n)/n} / (\sigma - 1)\right)^{1/(1-\kappa\alpha)} \quad \forall t \geq 1. \tag{14}$$

The second condition that describes the real side of the economy is the goods market equilibrium relation. This is derived by combining the resource constraint (8), the price index and the demand conditions, $C_{h,t}^* = g^* s_t / P_{h,t}$ and $C_{h,t} = nP_t C_t / P_{h,t}$, where $g^* = n^* C^*$ if $g_t^* = g^*$ $\forall t$.[9] As the price ratio is related to the real exchange rate uniquely,

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[9] Whilst $n^* \to 0$, $P^* C^*$ is large, and from the assumptions $P^* = 1$.

$$y_{h,t} = n q_t^{(1-n)/n} C_t + g^* q_t^{1/n} \ \forall t, \tag{15}$$

where an increase in g^* can be viewed as an exogenous increase in foreign demand.[10] These
two equations form an implicit good-supply system for $y_{h,t}$, q_t, and C_t which for periods $t \geq 1$
is self-contained implying the real interest rate is at it's steady state value, $r_t = 1/\beta \ \forall t \geq 1$.[11]
Given $r_{t+1} = 1/\beta$ then $C_{t+1} = \overline{C}$ and so it is possible to solve for the monetary sector by
transforming (13) into a difference equation involving real money balances by multiplying
through by the nominal money supply and substituting out the real interest rate for it's
steady state value.

$$m_{t+1} = \mu_{t+1} \beta \left(1 + a \overline{C} m_{t+2}^{-1/\nu} \right) m_{t+2}, \tag{16}$$

where $\mu_{t+1} = M_{t+1}/M_{t+2}$. As m_{t+1} is non-predetermined when money growth is constant
at μ a saddle path condition holds so that real balances jump to their steady state value for
periods $t \geq 1$. Going back to (13) it is apparent that the timing of private agents decisions
over money balances simplifies the interaction between the real and monetary sectors relative
to a model with the end of period balances specification and that the interest rate dynamics
are trivial.[12]

Remark 1 *Following a permanent money shock the nominal interest rate jumps immediately
to it's steady state value, $i_t = 1/\beta\mu \ \forall t$.*

The fixed money wage in period t does not affect the behaviour of the nominal interest rate
in period t and this rules out any exchange rate dynamics.

To describe the behaviour of the current account to money shocks take the period t national
budget constraint (9) and I iterate forward. Applying a no-ponzi game condition yields the
national intertemporal budget constraint,

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[10]If $C^* \simeq y_f$, where y_f is foreign income, then $g^* \ (= n^* y_f)$ approximates foreign demand.

[11]The derivation of this result uses the real UIP condition, $r_t/r_t^* = q_{t+1}/q_t$, the real consumption Euler
equation and the assumption $\beta = \beta^*$.

[12]See Cooke (2002) for further discussion.

$$i_{-1}B_{-1} = -\sum_{t=0}^{\infty} (i_0 i_1 .....i_{t-1})^{-1} \left(g_t^* s_t - P_t C_t (1-n) \right), \tag{17}$$

where B_{-1} is the initial bond stock and $i_0 i_1 .....i_{t-1} \equiv 1$ when $t = 0$. Setting the initial level of debt equal to zero, noting $i_t = 1/\beta\mu$ $\forall t$, using the nominal consumption Euler equation (12), the nominal UIP condition (6) and accounting for the initial conditions as $s_t = (1/\mu)^t s_0$, and $P_t C_t = (1/\mu)^t P_0 C_0$ I rewrite (17) as,

$$0 = \beta\mu \left(P_0 C_0 (n-1) + g^* s_0 \right) / (1 - \beta), \tag{18}$$

where $\sum_{t=0}^{\infty} \beta^{t+1} = \beta/(1-\beta)$. Equation (18) itself implies that the initial level of the nominal exchange rate is $s_0 = P_0 C_0 (1-n)/g^*$ and that the real version of this condition by (5) is $q_0 = C_0 (1-n)/g^*$. This holds for all periods and by substitution into the goods market condition (15) a zero current account condition holds.

Remark 2 *A zero current account holds in all periods (zero trade balance), $P_{h,t} y_{h,t} = P_t C_t$ $\forall t$.*

If the interest rate is constant and at the steady state in all periods and in the initial period the bond stock is assumed to be zero, in all future periods the bond stock will remain at zero. Thus a zero current account in the initial period implies a zero current account in all periods, and following a money shock the economy does not run a current account imbalance. The real side of the economy is in a steady state in periods $t \geq 1$ because of the type of rigidity assumed and therefore $y_{h,t} = \overline{y}_h$ $\forall t \geq 1$, where an upper-bar denotes the natural rate. Given there is a zero current account in these periods there is an explicit solution for the natural rate of output which can be derived by again appealing to the good-supply system.

Remark 3 *The natural rate of output is below the competitive level and depends on preferences and technology, $\overline{y}_h = \left((\sigma - 1) / \sigma\alpha\eta \right)^{1/\kappa\alpha}$. It does not depend on the openness parameter, n.*

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The labour market becomes more competitive as $\sigma \to \infty$ ($\bar{y}_h \to (1/\alpha\eta)^{1/\kappa\alpha} \equiv y_h^{pc}$) and thus it might appear that the presence of monopolistic competition gives the monetary authority an incentive to increase output by making a larger than expected money transfer. Importantly though households have access to two goods (domestic and foreign) so it does not necessarily follow that the perfect competition and social planners' outcomes are the same. The social planners' problem for this economy can be stated in full (dropping t subscripts) as,

$$\max_{C_h, C_f, l} U = \ln\left(C_h^n C_f^{1-n} / \left(n^n (1-n)^{1-n}\right)\right) - \eta l^\kappa / \kappa \tag{19}$$

$$s.t.$$

$$y_h = l^{1/\alpha} \tag{19a}$$

$$y_h = C_h + C_h^* \tag{19b}$$

$$C_h^* = \tau C_f \tag{19c}$$

$$C_h^* = \tau g^* \tag{19d}$$

Equation (19) is the utility function without the real money balances term and with the consumption sub-index replacing C, (19a) is the production function, (19b) is the resource constraint, (19c) represents a balanced trade condition and (19d) is the export demand function. From (19c) and (19d) for any arbitrarily given value of g^* and C_f the only possible solutions for τ and C_h^* are for them both to equal zero. Relaxing the assumption that C_f is given $C_f = g^*$, but in this case (19c) and (19d) no longer uniquely determine τ_t and C_h^* as functions of g^* and C_f. Proceeding with the idea that C_f is pegged to g^* it is possible to substitute foreign goods consumption, C_f, directly into the utility function and drop the final constraint (19d). By imposing $C_h^* = \tau = 0$ the problem is reduced still further by the elimination of (19c) and then C_h^* from the resource constraint. Substituting the remaining constraint into the maximand,

$$\max_{C_h} U = \ln\left(C_h^n C_f^{1-n} / (n^n (1-n)^{1-n})\right) - \eta C_h^{\alpha\kappa} / \kappa, \tag{20}$$

where the first order condition is,

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$$n/C_h - \eta\alpha C_h^{\alpha\kappa-1} = 0. \tag{21}$$

Re-substituting the transformed resource constraint into the first order condition gives the desired result.

Remark 4 *The solution to the social planners' problem depends on exogenous parameters and in particular on the degree of openness, n. The social planners' output level is given by,* $\widetilde{y}_h = (n/\alpha\eta)^{1/\kappa\alpha}$.

There are two comments which need to be made concerning this remark. First, although $\tau = 0$ is a special type of result which derives from the specific assumption made over consumer preferences, there is an intuitive explanation. Foreign consumption of the domestic good (i.e. exports) and the terms of trade do not affect domestic utility, and so as negative values are excluded, from the viewpoint of the social planner, their optimal values are zero. The fact that it is feasible to drive these values to zero is a consequence of the particular unit-elastic demand functions implied by the model. In this case, domestic labour goes entirely into raising C_h as opposed to C_h^*, because only domestic consumption raises welfare. Under these conditions the social planners outcome is a corner solution.

The second point turns out to be important when considering optimal monetary policy. In the open economy, when the representative agent has a choice over two goods, the social planners problem and the perfectly competitive market outcome diverge. In a closed economy these values would be the same because taking $n \to 1$ in the planners outcome or taking $\sigma \to \infty$ in the market outcome both imply $y_h = (1/\alpha\eta)^{1/\kappa\alpha}$. The reason for this divergence follows similar lines to an optimal tariff argument. If a country is large enough in world markets it can impose a tariff on imports to alter the terms of trade, which increases welfare. The tariff reduces the overall volume of trade in the world and generates production and consumption costs, but by improving the terms of trade a moderate tariff can produce benefits that outweigh these costs. In trade theory the small open economy assumptions usually imply that both the domestic and foreign prices of goods are taken as given by the domestic economy, but

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the Mundell-Fleming assumptions employed here imply that the domestic economy has power over the domestic price level because it exports a specialised output. In a decentralised economy no individual private agent is able to affect the terms of trade despite monopolistic power over the wage rate, but the social planner does have this ability. The social planner therefore effectively coordinates the behaviour of all private agents to the detriment of the foreign economy, increasing welfare by improving the terms of trade. This is why there is a divergence between the planning and competitive outcomes.[13]

4   The Phillips Curve and Consumption-Output Relationship

To express the monetary authority's problem it is first necessary to derive constraints consistent with an arbitrary policy choice. The constraints consist of a Phillips curve and a consumption-output relationship. The Phillips curve is constructed by using the good-supply system (15) and (14), and the zero trade balance condition. The Phillips curve is defined in terms of actual and expected real money balances instead of inflation, where expected real money balances are $m_t^e = M_t/P_t^e$. For households, choosing a value for m_t^e is equivalent to choosing the inverse of the money wage, w_t, which is fixed in period t, as expected real balances are simply the expected price level normalised by the beginning of period (and therefore predetermined) nominal money supply. Deciding on w_t requires a forecast of the CPI and so in this model m_t and m_t^e play the role actual and expected inflation would do in an ad-hoc model.

Profit maximising firms make supply decisions based on the nominal wage deflated by the domestic price level. Using the definitions of the price index and terms of trade there is an alternative supply relation that depends on the CPI level and the terms of trade explicitly,[14]

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[13] Although the optimal tariff argument may seem a little strange in this context it is an application of a beggar-thy-neighbour argument, such as also stressed in the recent analysis of Tille (2001).

[14] It should be realised that $q_t^{1/n} = \tau_t$.

$$y_{h,t} = \left(\alpha \tau_t^{(1-n)} \left(w_t / P_t \right) \right)^{1/(1-\alpha)}. \tag{22}$$

The first stage in deriving the Phillips curve is to introduce expectations in (22). In the current period wages are set so the expected real wage is a function of the expected price level. It is straightforward to show that the current period output level is related to the actual price level, the expectation of the price level and the natural rate of output, and as actual and expected prices can be normalised by the beginning of period t level of nominal money, the difference between the actual and expected level of real money balances determines the difference between the current level of output and the natural rate. The goods equation (15) and the zero trade balance condition relate the terms of trade to the level of output, and finally to the labour input using the production function. Using this it is possible to derive a Phillips curve type-relation which does not explicitly depend on the terms of trade,

$$y_{h,t} = (m_t / m_t^e)^{1/(n-\alpha)} \overline{y}_h. \tag{23}$$

For any incorrect forecast of real balances (i.e. $m_t \neq m_t^e$) the difference between current output and it's natural rate changes as the degree of openness changes. Therefore as openness changes the trade-off the monetary authority faces when setting optimal policy also changes. Lower n, which is an increase in the degree of openness, reduces $1/(n-\alpha)$ in absolute value (recall $n - \alpha < 0$) and thus a given reduction in m_t has a smaller effect on domestic output. This type of relationship is stressed in previous studies because the terms of trade are really what affect the slope of the Phillips curve. When expectations are correct the current level of output equals it's natural rate level and this makes clear how a change in monetary policy can increase output. Agents expect the full output level to prevail in each period, or rather that $m_t = m_t^e$ and if the monetary authority increases the transfer, $m_t < m_t^e$ for the period wages are fixed then $y_{h,t} > \overline{y}_h$. One important difference to note is that because period t nominal money holdings are predetermined when thinking about a monetary expansion in period t it

does not mean that M_t has increased, rather that M_{t+1} has increased and this produces the increase in P_t. This explains why surprise inflation in period t unambiguously reduces current real money balances.

The second constraint links current consumption and domestic output. In a closed economy consumption equals output when markets clear but here the zero trade balance condition implies $P_{h,t}y_{h,t} = P_tC_t$, where $P_{h,t}$ and P_t are endogenous. The relationship between these two variables is a transformation of the terms of trade, but the terms of trade are uniquely related, via the good-supply equations, to the level of output. Thus,

$$C_t = ((1-n)/g^*)^{n-1}\, y_{h,t}^n. \tag{24}$$

Taking the limit of (24) as $n \to 1$, then $C_t \to y_{h,t}$, as in the closed economy. Both (23) and (24) together determine the second constraint the monetary authority takes into account when setting policy. The interesting feature of this second constraint is that it contains both the openness parameter and the foreign demand measure. Re-expressing (24) as $\tau_t = ((1-n)/g^*)\, y_{h,t}$ the terms of trade must deteriorate when there is a money shock and as this relationship depends on openness and foreign demand the ratio $((1-n)/g^*)$ determines whether monetary shocks affect output with less of greater force.[15]

5 Optimal Policy

The focus of this section is on the optimal policy choice when the monetary authority and households play a one shot game.[16] Under this assumption the monetary authority's choice over money growth is equivalent to a choice over real balances. A sketch of this argument and a more general statement of the monetary authority's problem is presented in the Ap-

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[15] $C_t = \tau_t^{n-1} y_{h,t}$

[16] The outcome of such a game is commonly referred to as a Markov equilibrium.

pendix.[17] The key results examined here are; how the rate of inflation varies with (i) the degree of competition in the labour market, (ii) foreign demand, (iii) openness and (iv) how the openness-inflation relationship is itself dependent on foreign demand.

5.1 The Monetary Authority's Problem

The monetary authority maximises utility subject to three constraints; the Phillips curve, (23), the consumption-output relationship (24), and as with the social planner, the production constraint, (7). In full (dropping t subscripts),

$$\max_{m} U \;=\; \ln C + a\nu m^{(\nu-1)/\nu}/(\nu-1) - \eta l^{\kappa}/\kappa \tag{25}$$

$$s.t.$$

$$y_h \;=\; l^{1/\alpha} \tag{25a}$$

$$l \;=\; (m/m^e)^{\alpha/(n-\alpha)}\,\bar{l} \tag{25b}$$

$$C \;=\; ((1-n)/g^*)^{n-1}\,y_h^n, \tag{25c}$$

with m^e given. From inspection the monetary authority's problem is very similar to the social planners' problem, except the constraints (25b) and (25c) are the result of private optimising behaviour, and the monetary authority chooses real money balances, not consumption and labour supply.[18] The solution to this problem describes a reaction function, which makes real balances an implicit function of expected real balances, and writing this out with real money balances on the left hand side,

$$m\left(am^{(\nu-1)/\nu} + (\alpha\eta\widetilde{y}_h^{\kappa\alpha}/(n-\alpha))\right)^{(\alpha-n)/\kappa\alpha} = m^e\left(\alpha\eta\bar{y}_h^{\kappa\alpha}/(n-\alpha)\right)^{(\alpha-n)/\kappa\alpha}. \tag{26}$$

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[17] An extended discussion of this idea is also provided by Ireland (1997).

[18] It is worth noting at this point that much of the recent literature on optimal policy exploits a second-order approximation to the utility function which assumes the welfare from liquidity services is negligible, see for example Woodford (2003) ch.6. This approach is also given as a justification for assuming something similar to an ad-hoc loss function.

Equation (26) also makes use of the definitions of the natural rate of output and the social planners' output level. Solving for equilibrium real balances requires the private agents reaction function which is given by the long-run Phillips curve. Thus to obtain the equilibrium level of real balances I set $m = m^e = m^*$, which gives,

$$m^* = \left(\alpha\eta \left(\widetilde{y}_h^{\kappa\alpha} - \overline{y}_h^{\kappa\alpha}\right) / a \left(\alpha - n\right)\right)^{\nu/(\nu-1)}. \tag{27}$$

This solution raises a significant point, because if the natural rate of output exceeds the optimal level, then the level of real money balances that obtains in equilibrium is undefined. In terms of the parameters of the model the condition $\widetilde{y}_h > \overline{y}_h$, required for a well defined equilibrium, can be equivalently expressed as $n > (\sigma - 1)/\sigma \equiv \delta$ and because the social planners' output level is determined from the structural relationships of the model it cannot be guaranteed this condition holds. In a closed economy, $n \rightarrow 1$ implies $\widetilde{y}_h \rightarrow (1/\alpha\eta)^{1/\kappa\alpha}$, such that the socially optimal level of output is always greater than the monopolistic natural rate, and the level of equilibrium real balances is always positive. To further understand why there is the possibility an undefined level of real balances it is possible to depict the optimisation problem for the monetary authority in (m, y_h) space. From (23) the slope of the Phillips curve is negative because $\alpha > n$. Rearranging the utility function, imposing the consumption-output relation and holding utility constant shows the slope of the indifference curves depend on the level of output,

$$\partial m/\partial y_h = -m^{1/v} \left(n - \alpha\eta y_h^{\kappa\alpha}\right) / a y_h. \tag{28}$$

When $\overline{y}_h > \widetilde{y}_h$, $\partial m/\partial y_h$ is positive. These conditions provide a somewhat familiar looking diagram.
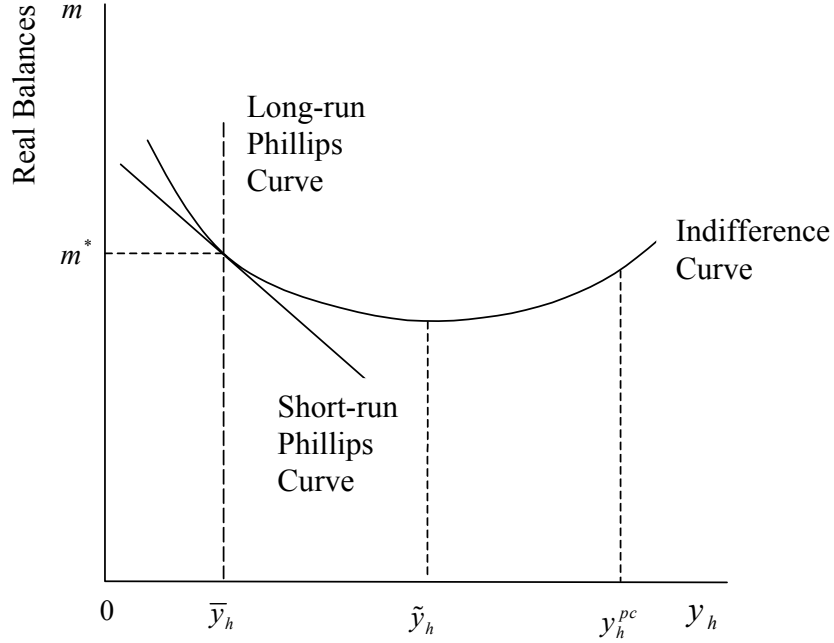
Figure One: Monetary Authority's Optimization Problem

The short-run Phillips curve only allows for tangency points on the negatively sloped portion of the monetary authority's indifference curves, where the natural rate of output is below the social optimum. A natural rate of output above the social optimum will not characterise a feasible equilibrium, something clearly shown by (28) because if $\delta = n$, then $\partial m / \partial y_h = 0$.

The final step in solving for optimal inflation is to equate equilibrium real balances with steady state real balances. In the steady state real balances are stationary so $m_t = m \; \forall t$ and inflation is equal to the money growth rate, μ. From the money demand function $m = \left(a\overline{C}\beta / (\pi - \beta) \right)^{\nu}$, where π denotes gross inflation and by substituting in (27) the discretionary rate of inflation, denoted π^*, can be written in the compact form,

$$\pi^* = \beta + \underbrace{a\beta\overline{C}\left(m^*\right)^{-1/\nu}}_{bias}. \tag{29}$$

Inflation is now clearly split between the Friedman rule level of inflation and the bias term.

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The Friedman level of inflation in this economy is simply $\pi^* = \beta$ and is the outcome when the monetary authority has access to a commitment technology. The basic intuition for this result is that there is a wedge generated between the private and social marginal cost of holding money and when $i > 1$ this generates an inefficiency. If there were no opportunity cost to holding money this inefficiency would disappear, but this requires that inflation equal the inverse of the real interest rate, which is given by β. The bias term is decreasing in real balances and increasing in consumption. The first part representing the consumption-output relationship (24) is such that the bias contains both the openness and foreign demand parameters. The second term is the equilibrium level of real balances, given by (27). The bias component split is important because, as I will argue, the real balances part captures the essence of the ad-hoc approach, whilst consumption, which overturns many of the results in the older literature, appears from the use of micro-foundations.

5.2 Inflation

In this section I examine how the distortion in the labour market, the degree of openness and foreign demand alter the equilibrium rate of inflation. The analysis focuses on the case where $\widetilde{y}_h$ exceeds $\overline{y}_h$. Rewriting (29) in terms of exogenous parameters alone using (24) and (27),

$$\pi^* = \beta + a\beta \underbrace{(\delta/\eta\alpha)^{n/\kappa\alpha} \left((1-n)/g^*\right)^{n-1}}_{\equiv \overline{C}} \underbrace{\left((n-\delta)/a(\alpha-n)\right)^{1/(1-\nu)}}_{\equiv (m^*)^{-1/v}}. \qquad (30)$$

Taking the derivative of (30) with respect to $\delta \in [0, n]$ shows that there are two competing effects from the distortion on the level of inflation,

$$\partial\pi^*/\partial\delta = (\pi^* - \beta)\left((n/\alpha\kappa\delta) - (1/(n-\delta)(1-\nu))\right). \qquad (31)$$

When the monopolistic distortion is high a reduction in the distortion (i.e. a higher δ) raises the rate of inflation as the marginal cost of inflation in utility terms falls, via a leisure effect, and the term $n/\alpha\kappa\delta$ dominates the right hand side of equation (31). As the distortion continues to rise there is a second effect through real money balances and the term $1/(n-\delta)(1-\nu)$

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dominates. Higher distortion levels imply lower equilibrium demand for real balances so that for a given rate of inflation an increase in the distortion lowers holdings of real balances. As holdings of real balances lower, the associated cost of inflation is higher and thus the monetary authority will alter it's policy to reflect this additional cost. As the elasticity of labour demand continues to rise $(\delta \to n)$ inflation continues to fall and so these two effects give rise to a non-monotonicity result stressed in Neiss (1999). Only in a closed economy will the rate of inflation approach the Friedman rule as the labour market becomes more competitive. In this case, the leisure effect is simply a consumption effect as $C = y_h = l^{1/\alpha}$. Hence the consumption term is what really overturns the monotonic relation in Barro and Gordon (1983).

A second result of interest concerns the impact of foreign demand on domestic inflation. It is clear from inspecting (30) that as $n \in [0, 1]$ when foreign demand for domestic production increases, so does domestic inflation. Recalling that foreign demand appears in the domestic goods market condition (15) then the positive relation with foreign demand has some intuitive appeal. A rise in foreign demand improves the equilibrium terms of trade, implying a one unit increase in output produces a bigger increase in consumption via (24), and thus a larger increase in utility. The temptation to inflate is therefore greater and this increases the equilibrium inflation bias. What is perhaps more significant is that foreign demand is exogenous from the viewpoint of the monetary authority. Thus, although it is popular to construct arguments that it is possible to reduce the inflation bias via institutional arrangements, once there is an exogenous increase in foreign demand the rate of domestic inflation will rise, exacerbating any bias problems already present in the economy.

The major question to address is whether or not the model predicts an inverse relationship between openness and inflation. The measure of openness is given by n, but inspecting (30) it is clear that n's relationship with inflation is potentially ambiguous. One reason for this is that it is not simply the Phillips curve that alters as openness changes. Remark four in section three demonstrates that the optimal level of output changes as openness changes, and this is associated with the set of indifference curves, not the Phillips curve. A significant

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preliminary comment is that the relationship between openness and equilibrium real balances is unambiguous. Taking the derivative of (27) with respect to n,

$$\partial m^*/\partial n = \nu(\alpha - \delta)m^*/(\nu - 1)(\alpha - n)(n - \delta). \tag{32}$$

As $n > \delta$, and $\nu < 1$, (32) is negative. To demonstrate how this relates to inflation, first recall from (29), that holding consumption constant, $\partial \pi^*/\partial m^* < 0$. Given this relationship the effect of openness via real balances alone is positive and accounting for the level of real balances alone in (29) a more open economy has a lower rate of inflation. Thus the real balance effect and the relationship between openness and inflation in previous studies such as Rogoff (1985) and Romer (1993) is analogous to the real balance effect and the relationship between the distortion and inflation in Barro and Gordon (1983), because in both cases the real balances term serves to capture the main message in the ad-hoc literature. To fully describe the reaction of inflation to changes in openness it is necessary to again consider changes in steady-state consumption. Taking the two parts of consumption separately, see again (30), $(\delta/\eta\alpha)^{n/\kappa\alpha}$ is decreasing in n because $\delta/\eta\alpha \equiv \overline{y}_h < 1$, and this counteracts the effect of real balances. The term $((1-n)/g^*)^{n-1}$ increases or decreases with openness depending on foreign demand, g^*. The magnitude of foreign demand therefore determines an important component of the relationship between openness and inflation, as this parameter cannot be pinned down. Before thinking about this in more detail it is useful to plot the inflation-openness-distortion relationship by assuming reasonable parameter values; below $\beta = 0.96$, $\nu = 0.1$, $\alpha = 2$, $a = 1$, $\eta = 2$, $\kappa = 2$. I set $g^* = 100$ and $g^* = 1/100$.
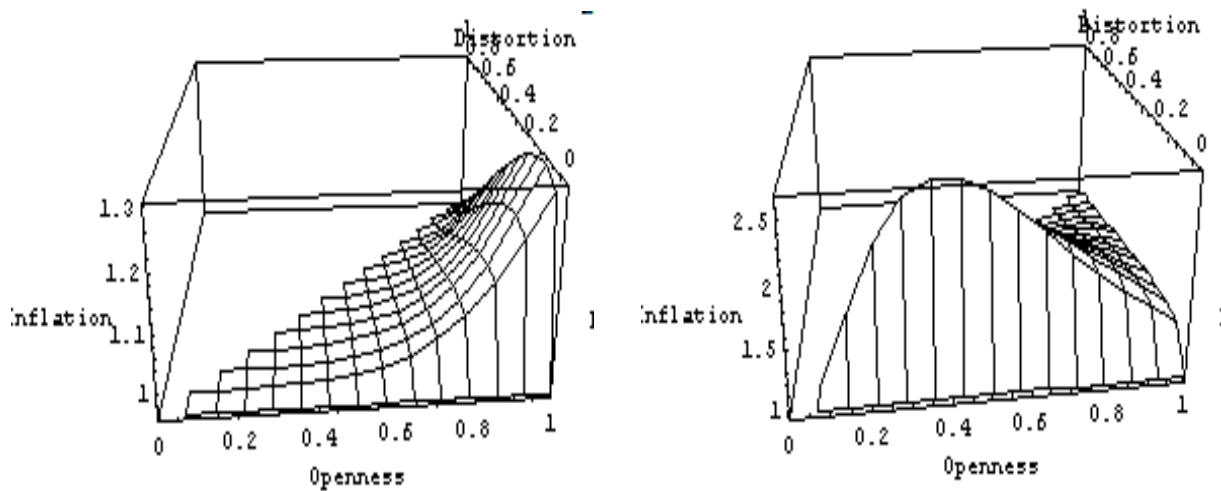
Figure Two: Openness and Inflation when $g^* = 100$ and $g^* = 1/100$

As $n \in [0, 1]$ rises the economy is more closed and as $\delta \in [0, n]$ rises the labour market is more competitive, so when $\delta = n = 1$, the Friedman rule is optimal and $\pi = 0.96$.[19] From the plots the previous results become clearer. In the space where $n > \delta$ a real balance effect operates as the distortion changes. When $n = 1$, as the distortion rises, inflation rises due to the leisure effect but eventually falls due to the real balance effect. As the domestic economy is more open the relative strength of the real balance effect increases. From the diagram a reduction in n lowers the optimal level of output and therefore only when $n = 1$, is the leisure effect at 'full' strength. Conversely when $n \to 0$ there is no leisure effect as a higher distortion cannot induce the level of output to fall without entering the region where inflation is not defined. For every point in the plot where $g^* = 100$, inflation is higher, except when the economy is closed, as then the curves coincide.

The plots also show that for low levels of foreign demand the relationship between openness and inflation is unambiguously negative. The first thing to recall is that higher foreign demand improves the equilibrium terms of trade so that when foreign demand is low the terms of trade are relatively unfavourable. In this case the effect of real balances gives the

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[19] Or rather that the steady state interest rate is 4.2%.

standard result; inflation rises as the economy becomes more closed. When foreign demand is 'high' and the terms of trade are improved the real balances effect is only dominant when the economy is relatively open; as the economy becomes closed it is possible that inflation falls. Examining steady state consumption more closely it becomes clear why the key to the ambiguity between openness and inflation is the level of foreign demand. Consider the function $f(n, g^*) \equiv ((1-n)/g^*)^{n-1}$, which determines the relative price of domestic consumption to domestic output, $\overline{C}/\overline{y}_h^n$. An obvious point of note is the behaviour of the function as n reaches extreme values; $f(1, g^*) = 1$ and $f(0, g^*) = g^*$, so when the economy is closed and market clearing implies $C = y_h$, foreign demand plays no role in determining inflation.[20] As before with an improved terms of trade there is a greater incentive to inflate, but when the economy is closed this incentive disappears. The incentive is magnified in a more open economy and therefore if foreign demand is very high, as the economy becomes closed, inflation will fall because once the economy is closed any level of foreign demand is consistent with the same level of inflation. Therefore it is possible that inflation will fall as the economy becomes more closed because this second effect changes the equilibrium terms of trade, which alters the incentives of the monetary authority.

It is possible to draw some interesting points from the analysis. Even in this stylised model it does not always appear to be true that an inverse relationship holds. Accounting for real balances alone gives the standard result, but once changes in the steady state consumption are allowed for, it is possible to overturn this. Changing the level of foreign demand alters the openness-inflation relationship because of a second effect from the equilibrium terms of trade which operates through the monetary authority's utility function. In the case when foreign demand is 'low' for the parameter space where inflation is defined the inverse relationship between openness and inflation holds. Increasing foreign demand the results are overturned, where as initially the economy becomes more closed the rate of domestic inflation rises, after a certain point it begins to fall. This suggests that the small open economy openness-inflation relationship is more dependent on foreign demand than it might at first seem. An ad-hoc

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[20]Note $f_n(n, g^*) = 0$ when $g^* = (1-n)\exp$.

model, by not accounting for the role of consumption (or rather only accounting for the real balances part of equilibrium inflation), misses a major part of the openness-inflation story.

6   Conclusion

In this paper I develop a general equilibrium model based on a set of small open economy assumptions to analyse the inflation bias. I test the idea that in a more open economy the rate of inflation is lower and find openness alters inflation via two mechanisms. First, there is the effect of the Phillips curve; when a country is more open the slope of the Phillips curve is steeper, increasing the inflation cost and reducing the output gain from surprise inflation. There is also a second effect in the model because the socially optimal level of output depends on openness and this helps pin down the position of the monetary authority's set of indifference curves. The results suggest that inflation is inversely related to openness when accounting for real balances alone, but that for a full analysis of inflation it is necessary to account for steady state consumption, and this depends on foreign demand. When foreign demand is low the inverse relationship holds, but when foreign demand is sufficiently high inflation rises and falls with openness.

A   General Statement of the Monetary Authority's Problem

Households make forecasts over all future money growth rates, $\mu_t, \mu_{t+1}, ..$ before they observe μ_t. In the most general case these forecasts can be described by a postulated function,

$$\mu_t^e = \phi\left(\mu_{t-1}, \mu_{t-2}, ..\mu_0\right) \ \forall t. \tag{A1}$$

This function gives one step-ahead forecasts, but private agents need to forecast all future μ_{t+i}'s to make current decisions and therefore it is necessary to 'chain' together all values of μ_t^e to describe the full forecast.

$$_t\mu_{t+i}^e = \phi_{t,t+i}\left(\mu_{t-1}, \mu_{t-2}, ..\mu_0\right) \ \forall t, i, \tag{A2}$$

where $_t\mu_{t+i}^e$ is the forecast of μ_{t+i} made at the beginning of period t. Given this forecasted sequence the next step is to derive the value of m_t the household will actually choose. To do this plug each value for (A2) into the saddle path solution for (16), which makes current real balances a function of the entire sequence of money growth rates. Denote this,

$$m_t = \psi(\mu_{t-1}, \mu_{t-2}, ..\mu_0). \tag{A3}$$

This is the monetary authority's forecasting constraint. The monetary authority also needs to know what equilibrium real balances will be given any choice path for money growth. Using (A3) in the Phillips curve (23), the consumption-output relation (24) and the period t version of the difference equation (16) where m_{t+1} is set equal to m_{t+1}^e (what matters in determining money demand in period t is agents expectations about money demand in period $t+1$) gives a second constraint, denoted,

$$m_t^e = j(\mu_t; \psi(\mu_{t-1}, \mu_{t-2}, ..\mu_0), \psi(\mu_t, \mu_{t-1}, ..\mu_0)). \tag{A4}$$

Call this the real balances constraint. Given these two constraints the full statement of the monetary authority's problem is,

$$\max_{\{\mu_t\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + a\nu m_t^{(\nu-1)/\nu} / (\nu - 1) - \eta l_t^{\kappa}/\kappa \right) \tag{A5}$$

$$s.t.$$

$$y_{h,t} = l_t^{1/\alpha}$$

$$l_t = (m_t/m_t^e)^{1/(n-\alpha)} \bar{l}$$

$$C_t = ((1-n)/g^*)^{n-1} y_{h,t}^n$$

$$m_t = \psi(\mu_{t-1}, \mu_{t-2}, ..\mu_0)$$

$$m_t^e = j(\mu_t; \psi(\mu_{t-1}, \mu_{t-2}, ..\mu_0), \psi(\mu_t, \mu_{t-1}, ..\mu_0)).$$

A rational expectations equilibrium exists when the forecasting constraint is consistent with the real balances constraint.

There is a very simple relationship between this statement of the problem and (25) in the main text. When there is no history dependence the household forecasts over the money growth rate are not a set of functions but a sequence of values, $\mu_0, \mu_1, ..$ and likewise the counterpart to (A3) are the values $m_0, m_1, ...$ In this case each money growth rate only affects the corresponding level of real balances within the period and therefore the monetary authority can equivalently maximise utility with respect to m_t. The rational expectations equilibrium obtains when $m_t = m_t^e \; \forall t$ and then the discretionary equilibrium is stationary.

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