Information and Communication Technologies in a Multi-Sector Endogenous Growth Model*

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Abstract

This paper aims to account for the potential effect of the Information and Communication Technologies (ICT) upon growth. It studies a three sector economy, where one sector produces ICT, one uses ICT and one does not use ICT. The benefits from the ICT goods come in terms of falling prices of the ICT using sector's good. This good is used for the production of all types of intermediate capital varieties. Their falling prices provide incentives for investment in all sectors using these capital varieties. Therefore, the non ICT using sector will experience sustained growth driven by capital accumulation, despite the fact that it does not use directly the ICT goods. Although there is unbalanced growth across the three sectors, there exists a balanced growth steady-state path for the aggregate economy. Along this path there is no structural change.

JEL Classification: O40, O41

Keywords: multi-sector economy, endogenous growth, balanced growth path, Information and Communication Technologies

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1 Introduction

Current research on economic growth puts more emphasis on examining the sources of growth at the industry level. The advantage of this perspective is that it allows the identification of both the growth generating industries and the mechanism through which growth is delivered at the aggregate level. Figure 1 in Appendix D shows the employment shares (in terms of total hours worked) for three sectors that add up into the private economy of the USA, over the period 1979-2001. These correspond to the sector producing ICT goods, the sector using ICT capital goods intensively (ICT using) and the sector that uses ICT capital non intensively (non ICT using). There are two interesting features in these data. The employment shares appear constant along time. The share of the ICT producing sector is very small (3%), while this sector is suspected to be the main source of current USA growth. Further to this, the size of the ICT sector in the economy (ICT using and ICT producing) is much smaller compared to that of the non ICT sector. Hence, there is no straightforward explanation about how ICT progress delivers higher output growth for the aggregate economy. The objective of the model below is to illustrate the importance of the interaction among industries for the aggregate economic performance. The interest lies in understanding the conditions for the existence of a balanced growth steady-state path and examining whether they allow for any systematic shifts of resources across different sectors.

The motivation for this paper comes from the empirical literature that studies the USA economy over the past thirty years (Jorgenson et al., 2004, Oliner and Sichel, 2002). These studies use data from the USA at the three-digit ISIC level and perform a detailed growth accounting exercise that identifies the ICT producing sector as the source of growth, in spite of its small value added and employment shares. Complementary growth accounting exercises (ECB Working Paper, 2002, GGDC report, 2003) investigate the sources of USA and EU growth by looking at three aggregate sectors: one that produces ICT, one that uses ICT and one that does not use ICT (the difference between an ICT using and a non ICT using industry is the degree of ICT capital use out of total capital use). These studies confirm the high productivity growth in the ICT producing sector and point out the important gains in productivity for all sectors, and in particular for the ICT using industries. The most important source of USA growth has been the accumulation of ICT and non ICT capital. The incentives for ICT capital accumulation come from the dramatic price declines of the ICT goods.

In order to develop intuition about how the price declines of ICT goods can have an impact on aggregate labour productivity, one may think of the following example: In the USA economy, intensive ICT users are mainly the services' industries and industries producing equipment. Suppose that an ICT producing industry develops a new microprocessor. This chip will be used in the production of new office equipment that will be of higher quality and available at a lower price. The office chairs that will be part of this production will become available for financial institutions, as well as for hairdressers. Hence, despite the fact that the hairdressers do not use directly ICT, they benefit from every new generation of advances as long as this lowers their cost of equipment. The inter-industry transactions that allow for such mechanism to go through is the main focus of this paper.

In the model presented below, the ICT producing sector is the technology producing sector. By construction this sector becomes the engine of growth. The economy consists of two more sectors: the ICT using and the non ICT using. The model does not allow for intensities of ICT use other than the extreme ones. Despite being unrealistic, this assumption does not affect the main properties of the solution path. On the contrary, it underlies the linkages through which growth is delivered from the ICT producing sector to the entire economy.

The main structure of the model is based on Romer's (1990) model of endogenous growth. The advantage of the latter is that it illustrates the transactions between the sector producing technology and the sector using it, explaining how the growth in technology is delivered to the final good sector. The model below applies this framework to the economy of the ICT era. However, by retaining the main framework of Romer's model, the model applies to any large-scale technological advances. At the same time, Romer's model is extended by introducing the non ICT using sector that is using only technologically obsolete intermediate capital goods. This additional sector aims to account for the fact that for a long period after the introduction of new large scale technologies, some productive industries cannot make use of them. Therefore, these industries cannot fully benefit from the improving potentials that the new technologies are offering. However, they can coexist in equilibrium with industries that use fully the new technologies and that in effect have stronger growth. In the model of this paper, the most important transaction is that the non ICT using sector gets capital goods in terms of the output produced by the ICT using sector.

This transaction lies in the core of the analysis, because it aims to explain the mechanism that generates growth that is based on sustained capital deepening in all sectors of the economy. The mechanism which is described in the empirical literature is the following: as the prices of ICT goods go down, this creates incentives to invest in these goods. At the same time, as the cost for the ICT using industries is driven down, this generates further price declines and incentives to invest. The model below shows how this mechanism is compatible with a steady-state equilibrium at the aggregate level. In addition, this mechanism is effective irrespectively of the size of the ICT producing and using sector. Hence, there is no contradiction delivered from Figure 1.

The steady-state of the model economy exists under plausible restrictions upon the resources of the economy and the intertemporal and intratemporal substitutability among the different consumption goods available. The conditions on preferences are necessary due to the multiplicity of the consumption goods. These conditions give the hint that there is no structural change along the economy's balanced growth path, which is consistent with the picture delivered from Figure 1.

In effect, this model deviates from the structural change literature. In the context of the ICT economy, the USA data do not reveal significant systematic shifts of resources across sectors. Therefore, the theoretical model does not need to allow for any along the balanced growth path. In contrast, in Ngai and Pissarides (2004), there is exogenous growth while the sectors are being grouped according to whether they produce capital or consumption goods. Thus, it is plausible that their conditions on a balanced growth path are different to the present ones. Closer to the analysis of the model below is Acemoglu and Guerrieri (2004). This paper allows for an endogenous growth setup and puts emphasis on capital deepening driven growth. However, their results on unbalanced growth are heavily dependent on the assumption of different factor intensities across sectors, while their objective is to deliver a model that would encompass the existence of a steady-state balanced growth path in the limit, with structural change and unbalanced growth at the industry level. The focus of the model below is to understand how the use of different capital goods across sectors might still allow for aggregate growth, as long as the prices of all capital goods are driven down by technological progress. The intensity of capital use is allowed to be the same across sectors.

Moreover, this model deviates from the recent theoretical literature that deals with the impact of ICT upon growth. The ICT have been associated with multiple puzzles from the economists' viewpoint regarding the USA economy (Quah, 2001). The "paradox" of the low productivity and TFP growth of the 1970s and 1980s attracted most of the attention. This puzzle seems to have reached its resolution, as long as ever since the 1990s, USA economy exhibited high labour productivity and TFP growth that accelerated in the mid-1990s. This cycle of the USA economy that started with the early introduction of the ICT era in the mid 1970s has been explained at the theoretical level mostly through the literature of General Purpose Technologies (GPT). The economic historians were the first to draw the analogy between the ICT and the great inventions of the past, such as combustion engine, electricity, railways, that pioneered the first and second industrial revolutions (David, 1991, David and Wright, 1999). The features of a GPT, as given by Lipsey, Bekar, Carlaw (1998), are: "Wide scope for improvement and elaboration; Applicability across a wide range of uses; Potential for use in a wide variety of products and processes; Strong complementarities with existing or potential new technologies". Several empirical studies find supportive evidence for the GPT hypothesis, i.e. that the use of ICT goods involves important externalities for the ICT intensive industries (Jorgenson et al., 2004, Oliner and Sichel, 2002, Triplett and Bosworth, 2002, Oulton et al., 2003).

Having as a starting assumption that ICT are GPT, Helpman and Trajtenberg (1998) developed a model to explain how the arrival of a new GPT is expected to generate cycles, where labour productivity first lowers and then increases as the new steady-state is reached. Their explanation relies on a learning process developed around the adoption of the new technology. One sector after the other starts investing real resources in making its production process compatible with the use of the new GPT. A critical mass of new applications needs to take place so that a sector permanently switches to the new technology. Until this threshold is reached, the aggregate productivity growth appears low, because real resources are spent in R&D rather than in the production of consumption and capital goods. The model below does not aim to explain either the cycle involved in the introduction of a new large scale technology, or the adoption process of a new technology. Instead, it means to show how unbalanced growth at the disaggregate level, caused by the lack of adoption of a new essential technology, can still be consistent with balanced growth at the aggregate level.

Despite being mostly a theoretical exercise upon a multi-sector endogenous growth model, this paper still claims to capture some interesting features of the USA economy. Following the framework and the techniques in standard empirical literature, this paper presents data at the three-digit ISIC level from the USA. The industries reported in the original database were grouped to form the analogues of the three sectors of the model. The main descriptive statistics of the series of interest were examined and contrasted to the model's implications. There is no supportive evidence towards an important structural change undergoing in the USA economy caused by the ICT era. Also, by illustrating the importance of the aggregation over different sectors in the economy, the model adds to the explanations on the puzzle regarding why TFP growth has always been very small during the ICT era.

Section 2 presents the model. Section 3 analyses the conditions for the existence of a unique steady-state and explores its properties. Section 4 presents some evidence from data from the USA economy over the period 1979-2001 that support the predictions of the theoretical model. Section 5 concludes.

2 The Model

2.1 Production Side

The model examines a three sector economy. There are two final goods sectors in the economy: the ICT using and the non ICT using one. The third sector is the ICT producing sector, which performs R&D and comes up with new ICT goods. There is one more industry, which produces the two types of intermediate capital varieties. It may be considered as vertically integrated with the ICT producing sector. In what follows, the numeraire is the non ICT using output.

2.1.1 ICT Producing Sector

The engine of growth is the sector which produces new ICT "ideas", $\dot{N}$. The ICT producing sector employs a fraction u_N of the human capital resources, H. The production exhibits economies of scale. The motivation for the producing externalities is that there is learning-by-doing: as the production size increases, more new production ideas and practices become available. The exogenous productivity is given by parameter λ. In order to have sufficient incentives to innovate, this sector operates as a monopoly[1].

$$\dot{N} = \lambda(u_N H)N \tag{1}$$

2.1.2 ICT Using Sector

The ICT using sector absorbs a fraction u_1 of the total human capital resources and employs an array of N intermediate capital goods $\{x_1(j)\}_{j\in[0,N]}$,

[1]The alternative more general specification: $\dot{N} = \lambda(u_N H)N^{\phi}$, implies that for constant $\frac{\dot{N}}{N}$, u_N will be monotonically changing along time. This pattern is not empirically relevant. Besides, this case does not allow for the existence of a CGP altogether as it does not allow constant ICT production growth to coexist with constant aggregate output and capital growth.

so as to produce good Y_1. By construction, this variety of intermediate goods is expanding along time. This sector is perfectly competitive[2].

$$Y_1 = (u_1 H)^{1-\alpha} \int_0^N x_1^{\alpha}(j)dj \tag{2}$$

The final good is used for either consumption, or the production of capital goods (whether or not compatible to ICT). For simplicity, both forms of capital depreciate fully within every period.

$$Y_1 = c_1 + K_0 + K_1 \tag{3}$$

2.1.3 Non ICT Using Sector

The non ICT using sector employs a fraction, u_0, of the total human capital resources and combines it with the sector specific intermediate capital varieties to produce final good, Y_0. The capital varieties, $\{x_0(i)\}_{i\in[0,A]}$, are fixed in number along time. This stands for the assumption that this sector is not using ICT capital. Instead, it uses capital that is compatible with technologies that have already fulfilled their maximum range of applications. This sector is perfectly competitive[3].

$$Y_0 = (u_0 H)^{1-\alpha} \int_0^A x_0^{\alpha}(i)di \tag{4}$$

The final good is used only for consumption purposes.

$$Y_0 = c_0 \tag{5}$$

2.1.4 Intermediate Capital Varieties Market

There is a fixed number, A, of firms that produce intermediate capital varieties that are only used by the non ICT using sector. At the same time, there is an expanding number, N, of firms that produce intermediate capital varieties that are exclusively used by the ICT using sector. In both cases, the only input is a unit of the final output of the ICT using sector. Every firm in this "sector" has infinite horizon monopolistic rights that come from

[2] Allowing both sectors to use both ICT and non ICT capital at different intensities, would not change the main features of the equilibrium.

[3] Allowing for a different capital intensity in this sector would not affect the features of the equilibrium, while complicating the analytical expressions. The simplifying assumption of setting it equal to that of the ICT using sector is also used for the sake of stressing out more the differences across the two sectors that stem from the type of the capital used.

exploiting a patent. The price of every such patent equals the present discounted value of the firm's entire stream of profits. The firms operate under monopolistic competition.

In particular, a firm that produces the non ICT using capital variety i, has a market value at time t, $V_0^t(i)$, which equals the present discounted value of its future stream of profits. The discount factor depends on the market interest rate, $r(t)$. The profits are given as the gap between the firm's revenues and costs. The unit cost of production equals the price, p_1, of the ICT using final good. Given its market power, the firm selects its price $\hat{p}_0(i)$, while taking into account the demand it faces from the non ICT using final goods producers.

$$V_0^t(i) = \int_t^{\infty} e^{-\int_0^t r(\tau)d\tau} \left(\hat{p}_0(i)x_0(i) - p_1 x_0(i)\right) dt \tag{6}$$

A firm that produces the ICT using capital variety j, has a market value at time t, $V_1^t(j)$. The output will be priced at $\hat{p}_1(j)$ taking into account the demand from the ICT using final good producers. The unit cost is again p_1.

$$V_1^t(j) = \int_t^{\infty} e^{-\int_0^t r(\tau)d\tau} \left(\hat{p}_1(j)x_1(j) - p_1 x_1(j)\right) dt \tag{7}$$

In equilibrium, the markets of the two types of capital varieties should clear out.

$$K_0 = \int_0^A x_0(i)di \tag{8}$$

$$K_1 = \int_0^N x_1(j)dj \tag{9}$$

2.1.5 Human Capital Market

The human capital stock is fixed along time. Its market is perfectly competitive. The market clearing condition requires that all resources are allocated across all three sectors that use human capital.

$$1 = u_0 + u_1 + u_N \tag{10}$$

2.2 Consumer Side

2.2.1 Households

There is a continuum of identical households of size one. The representative household gains utility from its consumption of ICT using or non ICT using good. A general framework of joint CES and CRRA preferences allows both intertemporal and intratemporal substitution to come into play (the intratemporal and intertemporal elasticities of substitution are constant along time and equal to $\frac{1}{1-\epsilon}$, and $\frac{1}{\sigma}$ respectively).

$$u(c_0, c_1) = \frac{\left([\theta c_0^{\epsilon} + (1-\theta)c_1^{\epsilon}]^{\frac{1}{\epsilon}}\right)^{1-\sigma} - 1}{1-\sigma}; \theta \in (0,1), \epsilon < 1, \sigma > 0 \qquad (11)$$

The human capital is uniformly delegated across all agents in the economy, so that each of them holds H. At every period, the households' income comes from the wage, $w_H(t)$, that they earn from supplying their human capital and from the interest rate, $r(t)$, that they receive on their total asset holdings, $S(t)$. The only means of savings available are the assets of the capital producing firms. In effect, their budget constraint takes the form:

$$\dot{S} = rS + w_H H - c_0 - p_1 c_1 \qquad (12a)$$

3 Steady-State Analysis

3.1 Existence of Steady-State

A Constant Growth Path (CGP) is a steady-state equilibrium path along which the ICT production stock, N, the aggregate value of output, $Y = Y_0 + p_1 Y_1$, capital, $K = p_1 K_0 + p_1 K_1$, and consumption, $C = c_0 + p_1 c_1$, grow at a constant rate. The conditions that allow for the existence of such an equilibrium path will be investigated under the framework of the social planner's economy, since it is more insightful regarding what drives the relevant results. In addition, the Pareto Optimum solution provides a useful benchmark for the competitive equilibrium. All proofs are given in Appendix A.

Proposition 1 *The necessary and sufficient condition for the existence a CGP with N, Y, and K growing at constant rates is that the allocation of human capital in the ICT producing sector is constant, i.e. $\dot{u}_N = 0$. In order for the aggregate consumption, C, to grow at constant rate, the preferences need to exhibit unit intratemporal elasticity of substitution, i.e. $\epsilon = 0$. Along the unique CGP there is no reallocation of resources, $\dot{u}_0 = \dot{u}_1 = \dot{u}_N = 0$.*

The requirement on constant allocation of human capital in the ICT producing sector is straightforward. Since the only direct input in the ICT production is the human capital, then the result follows immediately.

The growth of aggregate output has three potential sources: capital accumulation, labour and TFP growth. When considering the growth expressed in terms of the good chosen as the numeraire, the TFP growth is just absorbed by the relative prices of the two final goods (due to the CRTS in their production). Capital accumulation is itself enforced by the falling capital prices. The latter result from the increased availability of intermediate ICT capital varieties. Therefore, constant growth for the aggregate final good is delivered, if the ICT production stock grows at a constant rate. Since this also implies constant allocation of human capital in the final goods' sectors, any reallocation between them should not affect the aggregate final goods' sector. Because the capital stock absorbs a constant fraction of output, the same reasoning goes through when looking at the aggregate capital growth rate.

The explanation on the requirement upon the preferences is more involved. The case of $\frac{1}{1-\epsilon} > 1$, is the one where consumers respond a lot to differences in prices by substituting one good for the other. Within this context, since the relative price of the ICT using good is monotonically falling along time, the consumers would like to substitute their consumption of non ICT good with that of ICT using good. This implies that the expenditure share of the non ICT using good would be decreasing along time. In order for the marginal rate of substitution (MRS) to be equal to the marginal rate of transformation (MRT), the share of human capital in the non ICT using sector should be also decreasing along time

However, in order for the market clearing condition to be satisfied along time, the consumption of the non ICT using sector needs to grow at the same rate as the production in that sector. Yet, the implied rate of growth for the allocation of human capital in this sector would be lower than the one required from the static optimization condition. The only case that they would be equal is if consumers were indifferent regarding the timing of their consumption (otherwise, they would be willing to reduce by more their current consumption of the non ICT using good, which would imply that part of its production would remain unavailable). That cannot be a steady-state equilibrium. Hence, a condition is required that would put restrictions on the consumers' intratemporal substitution patterns. The assumption of unit intratemporal elasticity of substitution implies constant expenditure shares. This allows the consumption growth rates to follow the production ones. Constant MRT along time implies that there will be no systematic shift of resources from one sector to the other.

3.2 Features of the Steady-State

In what follows, the focus is back on the competitive equilibrium steady-state. Given the result from Proposition 1, the steady-state of the competitive equilibrium is tracked down after imposing unit intratemporal elasticity of substitution and constant human capital shares. The details are given under Proposition 2. The most interesting static equilibrium results involve the following:

$$p_1 = \left(\frac{A}{N}\right)^{1-\alpha} \tag{13}$$

$$\hat{p}_0 = \hat{p}_1 = \frac{p_1}{\alpha} \tag{14}$$

Condition (13) shows that the relative price of the ICT using good is falling along time at a rate of growth which is proportional to the rate of expansion of the ICT capital. As condition (14) shows, the prices of all capital varieties fall at the same rate as the price of the final ICT using good. Therefore, the productivity gain of the non ICT using sector comes only indirectly. This sector is using a fixed number of capital varieties along time, but these varieties become cheaper and cheaper relative to the non ICT using final good. The falling prices generate increased demand for the existent capital varieties. Capital deepening is the only source of growth in this sector. At the same time, ICT using sector benefits from more varieties of capital becoming available along time. The benefits from more varieties complement those from cheaper varieties delivering faster growth for this sector relative to the non ICT using one.

$$\frac{u_0}{u_1} = \frac{(1-\alpha^2)\theta}{1-(1-\alpha^2)\theta} \tag{15}$$

Condition (15) comes from equating MRS to MRT and using the market clearing conditions. It shows the relative shares of human capital in the two final good sectors. Given that there is no reallocation of resources along the steady-state, the relative shares should remain constant along time. Their ratio depends on the expenditure share of the non ICT using good, as long as it affects the marginal utility of consumption. It also depends on the output elasticity of capital, α, since it affects the capital-labour substitution. The same parameter also specifies the size of the mark-up, $\frac{1-\alpha}{\alpha}$, that the capital producers enjoy.

The following Proposition summarizes the dynamic equilibrium results.

Proposition 2 *For preferences that satisfy: $\sigma > 0$ and $\epsilon = 0$, along the competitive equilibrium CGP the following are true*[4]*:*

The growth rate of every sector and of the aggregate economy is proportional to the endogenous growth rate of the ICT producing sector, g_N^c:

$$\begin{aligned}
\frac{\dot{Y}_0}{Y_0} &= \frac{\dot{c}_0}{c_0} = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \alpha g_N^c \\
\frac{\dot{Y}_1}{Y_1} &= \frac{\dot{c}_1}{c_1} = \frac{\dot{K}_1}{K_1} = \frac{\dot{K}_0}{K_0} = g_N^c \\
g_N^c &= g_N^c(\theta, \rho, \sigma, \alpha, \lambda; H)
\end{aligned}$$

The human capital allocations are constant and given as a function of all parameters of the model and the human capital stock:

$$u_z^c = u_z^c(\theta, \rho, \sigma, \alpha, \lambda; H) \; ; z = \{0, 1, N\}$$

Given the static optimization conditions described above, the features of the dynamic optimization conditions follow immediately. In particular, the ICT using sector will be growing at the same rate as the ICT producing sector, since this sector fully benefits from any advances in the ICT production, both in terms of capital deepening and in terms of TFP increase. On the contrary, the non ICT using sector grows only due to sustained investment driven by the fact that non ICT capital is becoming cheaper over time.

When the numeraire is the non ICT using good, the growth of the aggregate economy is pinned down by the output growth of the non ICT using sector. The effect of any output growth from the ICT using sector above the output growth of the non ICT using sector, is completely cancelled out by the growth rate of the relative prices. The economy is along a balanced growth path, where the underlying sectors experience different growth rates, but the aggregate consumption to output and capital to output ratios are constant. The consumption to output and capital to output ratios are also constant within every sector, but different across sectors. The growth rate of

[4] The conditions for an interior solution are an endogenously determined lower bound for human capital resources, $\bar{H}(\theta, \alpha, \lambda, \rho)$, and an upper bound for the intertemporal elasticity of substitution, $\frac{1}{\bar{\sigma}(\theta,\alpha,\lambda,\rho;H)}$.

the economy is a function of the preference and production parameters and the available human capital stock.

The condition for positive growth ensures that human resources are sufficiently high for the economy to have incentives to direct part of its real resources into R&D.

The condition in order for the TVC to be satisfied in the limit, i.e. that real resources are still driven into the two final goods' sectors, requires that the intertemporal elasticity of substitution is not too high. The threshold depends on the parameters of the model and the economy's human capital resources. Growth takes place when people substitute current consumption with future one. The incentives to do so depend on the gap between the real interest rate in consumption units and the subjective discount rate. While the market interest rate is constant, the real interest rate in consumption units changes over time as long as the consumption goods' prices change due to the sustained growth. The level of the interest rate in consumption units, as well as the responsiveness of the consumers to the payoffs in terms of interest rate predominately depend upon the intertemporal elasticity of substitution. The resource constraint requires that consumption does not grow faster or slower than the output. Hence, the elasticity should not be too high.

Corollary 3 *This economy does not exhibit transition dynamics.*

This result may easily be explained and proved through the CGP equilibrium of the social planner's economy. The production economy is driven entirely by the externalities in the ICT producing sector. In particular, given the employment in this sector, there are constant returns to any additional unit of ICT production stock. Since the ICT production stock constitutes the unique state of the economy, there are no diminishing returns to the ICT production stock that would allow for smooth transition of the economy towards the steady-state.

The transition dynamics could be delivered by a slowly depreciating physical capital, since the final goods' production functions retain the property of diminishing returns to capital. Yet, that would increase the nonlinearity of the model and would require numerical solution methods. This case is displayed and discussed in Appendix B.

Corollary 4 *The growth rate of the economy will be higher and the shares of human capital in the two final goods' sectors will be lower, the more patient* *the agents in the economy are (the lower ρ is) and the more productive the ICT producing sector is (the higher λ is). The effect of a higher output elasticity of capital (α), or of the expenditure share of the non ICT using good (θ) is ambiguous and depends on the values of different parameters of the model.*

Patient agents would be more willing to put up current consumption for future one. Under this framework, the savings take the form of resources being directed towards the ICT producing sector. This is because as asset holdings increase, they drive interest rates down and patent prices up. This enables higher growth in the long run since it provides incentives for higher ICT production growth. An increased productivity in the ICT producing sector would have the same effect. It would increase the marginal product of the human capital in this sector, and thus would attract more human capital resources. The incentives to produce more ICT would come from higher patent prices, that would result both from the increased productivity and the reduced interest rate.

The comparative statics following an increased preference towards the non ICT using consumption good is more involved. On the one hand, since the marginal utility of consumption goes up in this sector, there are forces to increase resources in its production. On the other hand, reducing the resources from the ICT producing sector implies that the rate of growth of the economy would fall. So is the rate at which the price of the non ICT using good increases relative to the ICT good. This implies that there would be a force that reduces consumption growth in the non ICT using sector, since it reduces the gap between the interest rate in consumption units and the subjective discount rate. For unit intertemporal elasticity of substitution, this second effect is eliminated because the market interest rate coincides with the interest rate in consumption units. Hence, higher θ would imply lower growth rate and resources being driven out of the ICT using and producing sector and into the non ICT using sector.

Same reasoning applies for the case that the output elasticity of capital would increase. On the one hand, this would reduce the mark-up that the capital producers enjoy, and thus would increase the production of capital and output. The effect of capital accumulation process upon growth would become stronger. On the other hand, since the human capital share in output would fall, this would reduce the incentive for growth as it would mitigate the gap between interest rate in consumption units and the subjective discount rate. Again, for unit intertemporal elasticity of substitution, the second effect is eliminated and thus the result of the increase in the output elasticity of capital would be an increase in the share of human capital in the ICT producing sector and a decrease of it in the final goods' sectors.

4 Supportive Evidence

This section uses data from the "60-Industry Database", which is constructed by the Groningen Growth and Development Centre (GGDC) and provides data on employment and value added for 57 industries in the USA economy, for the period 1979-2001[5]. Data on personal consumption expenditures by type of expenditure over the same period, come from the Bureau of Economic Analysis[6]. The source of the interest in the USA economy is twofold. First, this economy fits closely the theoretical framework in terms of the structure of the input/output markets and the degree of government intervention. Second, there are multiple empirical studies describing the behavior of USA economy after the mid-70s and the possible linkages between economic growth and the ICT production and use (Jorgenson et al. 2004, Gordon 2002, Oliner and Sichel, 2002, European Central Bank, 2002, Groningen Growth and Development Centre, 2003)[7].

The industries are grouped into three major sectors: ICT producing, ICT using and non ICT using. An industry is classified as ICT producing according to the OECD definition of the ICT manufacturing sector, including also the software producing industry. The criterion used for classifying an industry as ICT using is its degree of ICT capital intensity. In particular, the average share of the ICT capital out of total capital compensation for an industry over the period 1979-2001 needs to exceed the average share that is observed in the aggregate economy over the same period[8]. Details on

[5]This dataset has been developed by the Groningen Growth and Developemnt Centre and is based on the STAN OECD database and official USA governement sources.

[6]Since this dataset is for 103 industries, the appropriate mapping with the 57 industies was made following the ISIC Rev. 3 system.

[7]Another benefit from focusing on USA data is that the official data were readily adjusted into using a hedonic deflator system, so as to account better for the benefits arising from the ICT production and use. The deflators provided in the GGDC database come from official BEA data (harmonising of the deflators for other countries in the dataset does not affect USA data). There is still discussion in the literature whether hedonic deflators are adequate. For an exposition see OECD "Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes: Special Application to Information Technology Products", Triplett J. (2004).

[8]Data provided by GGDC for 26 industries. The appropriate mapping with the 57 industies was made following the ISIC Rev. 3 system.

the industries in each major sector and the descriptive statistics of the main series are provided in Appendix C.

According to the theoretical exercise, the ICT using sector is the sector fueling the economy with intermediate capital varieties, while the non ICT using sector is mainly a consumption producing sector. To check whether the resulting grouping of sectors supports this, the BEA "Use Table" of the "Benchmark 1997 Input-Output Table" was used to calculate the use shares of the commodities of the three sectors. The use considered is "total intermediates", "personal consumption" and "fixed investment", or the former two alone. The ICT producing sector turns out as a clearly capital/intermediate producing sector, followed by the ICT using sector. This Table 1 in Appendix C[9]. All the figures discussed below are presented in Appendix D.

In Figures 1 and 2 one may look at the aggregate economy from two different perspectives, revealing different dynamics along time. In either case, aggregate private economy is split into three sectors. Figure 2 identifies three sectors corresponding to whether the good comes from agriculture (ISIC: 01-05), manufacturing (ISIC: 10-45) or services' (ISIC: 50-95) industries. Figure 1 identifies three sectors with respect to whether there is production, intensive use or non intensive use of ICT capital. In spite of the short time span of the data, the trends reported in the structural change literature (Ngai and Pissarides, 2004) are striking in Figure 2. In particular, over the period 1979-1995, the share of agriculture in total hours worked is relatively constant around 4%, that of services increases from 54% to 63% and that of manufacturing falls from 41% to 33%[10]. On the contrary, over the same period, the shares of the ICT producing, ICT using and non ICT using sector remain considerably constant. The share of the ICT producing sector is fairly constant around 4%, that of ICT using changes from a minimum of 24% to a maximum of 26% and that of non ICT using sector changes from 73% to 71%. The reason that the focus is on the period before 1995, is that the empirical literature suggests that in 1995 there has been a structural break in most of the series of interest.

Figure 3a gives the value added at current prices (millions of USA dollars expressed in logarithmic units) of the ICT producing, ICT using and non ICT using sectors over the period 1979-2001. The rates of growth are roughly the

[9]The shares are reported for the three aggregate sectors (ICT using, non ICT using and ICT producing), as well as for the services and manufacturing industries within each major sector.

[10]The employment share for manufacturing is considerably higher than that usually reported. The difference comes from using data only for the private economy and using total employment in terms of hours worked rather than in number of persons engaged in production.

same[11]. The non ICT using sector delivers on average 71% of total value added over the period 1979-1995. The ICT using and the ICT producing deliver on average 24% and 4% respectively. The share of value added of the non ICT using sector out of the two final goods' sectors is on average 75% over this period. Given the predictions of the model, this should be equal to the share of this sector in hours out of total hours used in the two final goods' sectors. This is equal to 74% from the data. Also, the static optimization condition (15), implies that the share of the value added of the non ICT using sector out of total final goods' value added, is equal to $\theta(1-\alpha^2)$. From the BEA data, the average expenditure share of the non ICT good is 87% over the same period. Using the expenditure, hours and value added share estimates, the implied from the model output elasticity of capital is between 0.39 and 0.38[12].

Next, Figure 4a gives the value added at constant prices. The aggregation over the different industries within each major sector is based on using Törnqvist weights[13]. In contrast to Figure 3a, it is prevalent that the growth is stronger in the ICT producing sector, followed by the two final goods' sectors. The ICT using sector is picking up in growth especially after 1995[14].

Moreover, according to the model, when evaluating the growth rate of the final good in terms of the non ICT using good, the aggregation should give the growth rate of output of the non ICT using good. This is because the negative price effect upon the growth rate of the value of the ICT using good completely cancels out the positive output effect in that sector. Figure 5 shows together the output and price effects. Over the period 1980-1995 these two effects clearly move in the opposite direction and have approximately the same size. In fact, the series that comes from their aggregation does not have a statistically different from zero mean. After 1995 though, these two effects move together. This allows the aggregate output growth to be higher than the growth rate of the output in the non ICT using sector.

Finally, the model delivers an implication regarding the TFP growth rate for the aggregate economy. The TFP growth is the part of the growth rate

[11]The same picture is delivered from Figure 3b. The displayed series are the annual (exponential) growth rates (no filtering).

[12]Despite being significantly different to the benchmark estimate of $\alpha = 1/3$, it is a plausible estimate for the USA capital share. The difference might arise from the difficulties in mapping between ISIC Rev 3 and NAICS.

[13]The Törnqvist aggregation method is based on weighting each industry's exponential annual growth rate with a two-period average of its share in aggregate value added. After computing the growth rate, the implied quantity index was derived, with the normalisation that it is equal to 100 in 1995. This index was next used to derive the series in levels and logarithmic levels.

[14]Same picture is delivered by contrasting Figures 3b and 4b.

in aggregate output which is not explained by the contribution of labour or capital input. Using the production functions for the two final goods' sectors and the predictions of the market equilibrium of the model, the Solow residual is equal to $(1-\alpha)\frac{p_{Y_1}Y_1}{p_{Y_0}Y_0+p_{Y_1}Y_1}g_N^c$[15]. This means that the TFP growth is only a fraction of the growth of the ICT producing sector and this fraction depends on the output elasticity of labour and the value added share of the ICT using sector. The former appears due to the labour augmenting role of the technology of expanding varieties, while the latter appears because the ICT using sector is the only sector which benefits from ICT capital goods beyond capital deepening. For a labour share of 70% and a share of ICT using sector of 26%, only 18% of the growth of the production ICT translates into TFP growth[16].

However, all results should be treated with precaution. First, because the mapping of the model onto the data is not straightforward[17]. Second, even with hedonic pricing, USA data are still susceptible to noise regarding the derived value added deflator. Third, Törnqvist aggregation lacks the "dual"‘property, i.e. a Törnqvist price index does not imply a Törnqvist quantity index and vice versa. Fourth, given the small sample, standard time series stationarity and structural change analysis indicates that most of the series of interest are non stationary and/or have multiple breaks.

5 Conclusions

This paper has developed a simple theoretical framework, which provides insight into how multiple sectors of different growth potentials interact within an economy in a way that allows for a CGP at the aggregate level. Along

[15] If real GDP, $\tilde{Y}$, aggregates the final good production, then its implied growth rate is: $\frac{\dot{\tilde{Y}}}{\tilde{Y}} = \frac{p_{Y_0}Y_0}{p_{Y_0}Y_0+p_{Y_1}Y_1}\frac{\dot{Y}_0}{Y_0} + \left(1-\frac{p_{Y_0}Y_0}{p_{Y_0}Y_0+p_{Y_1}Y_1}\right)\frac{\dot{Y}_1}{Y_1}$. Using the results under Proposition 2:

$$\textbf{SolowResidual} \equiv \frac{\dot{\tilde{Y}}}{\tilde{Y}} - \frac{p_{Y_0}Y_0}{p_{Y_0}Y_0+p_{Y_1}Y_1}\left[(1-\alpha)\left(\frac{\dot{\overline{u_0H}}}{u_0H}\right)+\alpha\frac{\dot{K}_0}{K_0}\right]$$
$$-\left(1-\frac{p_{Y_0}Y_0}{p_{Y_0}Y_0+p_{Y_1}Y_1}\right)\left[(1-\alpha)\left(\frac{\dot{\overline{u_1H}}}{u_1H}\right)+\alpha\frac{\dot{K}_1}{K_1}\right]$$
$$=\left(1-\frac{p_{Y_0}Y_0}{p_{Y_0}Y_0+p_{Y_1}Y_1}\right)(1-\alpha)\frac{\dot{N}}{N}$$

[16] Using the implication of the model that the growth rate of the ICT using sector reflects the ICT technology growth, the resulting own estimate of the TFP series is higher than the estimates from Jorgenson et al. (2004).

[17] For example, the output measure is value added rather than gross output, or the measured value added for the ICT production includes production of both old and new ICT stock.

this path, growth is sustained endogenously. The source of growth is the ICT production. The market economy provides the incentives for ICT production through a patent system that protects monopoly rights. Despite the fact that only one sector is using the capital goods which are compatible with the new technologies, the benefits from their use spread throughout the economy. This is due to capital deepening, which is caused by continuously falling capital prices. The source of TFP growth in the economy is the expanding variety of potential applications of the ICT goods, which stems from the externalities present in the ICT production.

Along the steady-state CGP, there exists no reallocation of real resources and the size of every sector is fixed along time. The growth of the ICT using sector reflects the rate of growth of the ICT producing sector, since this sector uses capital varieties that follow the advances of the ICT production stock. The rate of growth of the non ICT using sector is equal only to a fraction of the ICT production growth. This is because this sector uses obsolete technologies that are available at falling prices, due to the advances in their production. These are the advances associated with the use of ICT technologies. At the aggregate level, any positive output effect is completely cancelled out by a negative price effect. In effect, the growth rate of the aggregate output is equal to the growth rate of the production of the good chosen as the exchange unit. The aggregate consumption to capital and output to capital ratios are constant along time. These ratios are also constant for the disaggregated sectors, but different across sectors.

Data from the USA economy, when mapped to the theoretical structure, reveal considerably constant sectoral shares out of the total hours worked. They support that the output effect at every point in time and along time is cancelled out by a negative price effect. This shows up in that the growth rates of the output at current prices are the same across sectors. The volume of the ICT using production grows faster than the non ICT using one. The ICT producing sector is the source of productivity growth, when considering the pervasiveness of the ICT goods and the sector's contribution to aggregate growth despite the small value added and employment shares of this sector.

Nevertheless, the empirical literature suggests that the USA economy does not appear to be along a balanced growth path during the past three decades, while the interest lies mostly in understanding the acceleration in growth after mid-1990s and its consequences. The numerical exercise presented in Appendix B of this paper provides a flavour of the out of steady-state dynamics of the social planner's economy. Future research may involve solving for the dynamics of the competitive equilibrium, while calibrating the behavior of the USA economy during the 1990's.

This however implies that the productivity of the ICT sector is still to be regarded exogenous. A possible extension for the model of this paper is to scrutinize the incentives to produce ICT, which will endogenize the productivity of this sector. This requires to reconcile the theory of endogenous economic growth with the theory of industrial organization regarding the goods that exhibit network externalities. The difficulty of bridging this gap is that the expectations in a market of network goods imply a special demand for these goods that results in multiple possible equilibria (Katz and Shapiro, 1986, Economides, 1996).

That framework would be more appropriate also to account for cross-country differences with respect to the use of ICT technologies and the effect on their aggregate economic performance. Otherwise, it would be interesting to consider more closely the key implication of the current model, i.e. that the falling relative price of the ICT using goods drives capital deepening and economic growth. An idea to be explored empirically is that the cross-country differences arise mainly from differences in the relative prices of the ICT using goods.

References

[1] Acemoglu, D. and V. Guerrieri (2005). "Capital Deepening and Non-Balanced Economic Growth", *Work in Progress*

[2] Aghion, P. and P. Howitt (1998). "On the Macroeconomic Effects of Major Technological Change", in Helpman, E., eds., General Purpose Technologies and Economic Growth, MIT Press, Cambridge.

[3] Albers, R. and F. Vijselaar (2002). "New Technologies and Productivity Growth in the Euro Area", *ECB Working Paper*, No. 122, European Central Bank, Frankfurt.

[4] Basu, S., J. G. Femald, N. Oulton and S. Srinivasan (2003). "The Case of the Missing Productivity Growth: Or, Does Information Technology explain Why Productivity Accelerated in the United States But the United Kingdom", *NBER Working Papers*, No. 10010, National Bureau of Economic Research, October.

[5] David, P. A. (1991). "Computer and Dynamo: The Modern Productivity Paradox in a Not-Too-Distant Mirror", in Technology and Productivity, OECD, Paris.

[6] David, P. A. and G. Wright (1999). "General Purpose Technologies and Surges in Productivity: Historical Reflections on the Future of the ICT Revolution", *Manuscript*.

[7] Economides, N. (1996). "The Economics of Networks", *International Journal of Industrial Organization*, Vol. 14, No. 6, 673-699.

[8] Gordon, R. J. (2002). "Technology and Economic performance in the American Economy, *NBER Working Papers* , No. 8771, National Bureau of Economic Research, January.

[9] Helpman, E. and M. Trajtenberg (1998). "A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies", in Helpman, E., eds., General Purpose Technologies and Economic Growth, MIT Press, Cambridge.

[10] Helpman, E. and M. Trajtenberg (1998). "Diffusion of General Purpose Technologies", in Helpman, E., eds., General Purpose Technologies and Economic Growth, MIT Press, Cambridge, 1998.

[11] Inkaar R., M. O'Mahony and M. Timmer (2003). "ICT and Europe's Productivity Performance. Industry-level Growth Account Comparisons with the United States", Research Memorandum GD-68, Groningen Growth and Development Centre.

[12] Jones, C. J. (2001). "Sources of U.S. Economic Growth in a World of Ideas", *American Economic Review*

[13] Jorgenson, D. W., C. A. Ho and K.J. Stiroh (2004). "Growth of U.S. Industries and Investments in Information Technology and Higher Education", forthcoming in Corrado C., Haltiwanger J. and Sichel D., eds., Measuring Capital in a New Economy, Chicago, University of Chicago Press

[14] Katz, M. and C. Shapiro (1986). "Technology Adoption in the Presence of Network Externalities", *Journal of Political Economy*, 94, 822-841.

[15] Lipsey, R. G., C. Bekar and K. Carlaw (1998). "What Requires Explanation?", in Helpman, E., eds., General Purpose Technologies and Economic Growth, MIT Press, Cambridge, 1998.

[16] Mulligan, C. B. and X. Sala-i-Martin (1993). "Transitional dynamics in Two-Sector Models of Endogenous Growth", *Quarterly Journal of Economics*, 108, 737-773.

[17] Ngai, L. R. and C. A. Pissarides (2004). "Structural Change in a Multi-Sector Model of Growth", *CEP/CEPR Discussion Paper*

[18] OECD (2003). "ICT and Economic Growth, Evidence from OECD Countries, Industries and Firms", Paris.

[19] Oliner, S. D. and D. E. Sichel (2002). "Information Technology and Productivity: Where are We Now and Where are We Going?", *Economic Review*, Federal Bank of Atlanta, 87(3), Quarter Three, 15-44.

[20] O' Mahony M. and B. van Ark, eds, "EU Productivity and Competitiveness: An Industry Perspective. Can Europe Resume the Catching-up Process?", Office for Official Publications of the European Communities, Luxemburg, 2003.

[21] Quah, D. (2002) "Technology Dissemination and Economic Growth: Some Lessons for the New Economy", Technology and the New Economy, Chapter 3, MIT Press, Cambridge

[22] Romer, P.M. (1990). "Endogenous Technological Change", *The Journal of Political Economy*, Vol. 98, No. 5 : S71-S102.

[23] Triplett, J. E. and B. B. Bosworth (2002). "Baumol's Disease Has Been Cured: IT and Multifactor Productivity in US Services Industries", paper prepared for Brookings Workshop on Services Industry Productivity, Brookings Institution, Washington DC.

6 Appendix A: Analytical Proofs

Proof of Proposition 1

The social planner solves economy's dynamic optimization problem, having controls over: $\{c_0, c_1, x_1(i), x_0(j), u_0, u_1\}$, $\forall i \in [0, A]$, $\forall j \in [0, N]$. The unique state variable is:N.

$$
\begin{aligned}
\mathcal{H} &= e^{-\rho t} \frac{\left([\theta c_0^{\epsilon} + (1-\theta) c_1^{\epsilon}]^{1/\epsilon} \right)^{1-\sigma} - 1}{1-\sigma} + \kappa \left[(u_0 H)^{1-\alpha} \int_0^A x_0^{\alpha}(i) di - c_0 \right] \\
&\quad + \mu \left[(u_1 H)^{1-\alpha} \int_0^N x_1^{\alpha}(j) dj - \int_0^A x_0(i) di - \int_0^N x_1(j) dj - c_1 \right] \\
&\quad + \nu \left[(1 - u_0 - u_1) \lambda H N \right]
\end{aligned} \tag{16}
$$

The FOCs from the maximization problem (16) are the following[18]:

$$
\begin{aligned}
\frac{\partial \mathcal{H}}{\partial c_0} &= 0 \Rightarrow e^{-\rho t} E^{1-\epsilon-\sigma} \theta c_0^{\epsilon-1} - \kappa = 0 && (17) \\
\frac{\partial \mathcal{H}}{\partial c_1} &= 0 \Rightarrow e^{-\rho t} E^{1-\epsilon-\sigma} (1-\theta) c_1^{\epsilon-1} - \mu = 0 && (18) \\
\frac{\partial \mathcal{H}}{\partial x_0(i)} &= 0, \forall i \Rightarrow \kappa \left[\alpha \left(u_0 H \right)^{1-\alpha} x_0^{\alpha-1}(i) \right] - \mu = 0, \forall i && (19) \\
\frac{\partial \mathcal{H}}{\partial x_1(j)} &= 0, \forall j \Rightarrow \mu \left[\alpha \left(u_1 H \right)^{1-\alpha} x_1^{\alpha-1}(j) - 1 \right] = 0, \forall j && (20) \\
\frac{\partial \mathcal{H}}{\partial u_0} &= 0 \Rightarrow \kappa \left[(1-\alpha) \frac{Y_0}{u_0} \right] - \nu \lambda H N = 0 && (21) \\
\frac{\partial \mathcal{H}}{\partial u_1} &= 0 \Rightarrow \mu \left[(1-\alpha) \frac{Y_1}{u_1} \right] - \nu \lambda H N = 0 && (22) \\
-\dot{\nu} &= \frac{\partial \mathcal{H}}{\partial N} && (23) \\
&= \mu \left[(u_1 H)^{1-\alpha} x_1^{\alpha}(N) - x_1(N) \right] + \nu (1 - u_1 - u_0) \lambda H
\end{aligned}
$$

The TVC is the following:

$$
\lim_{T \to \infty} \left[\nu(T) N(T) \right] = 0 \tag{24}
$$

[18] Let $E \equiv [\theta c_0^{\epsilon} + (1-\theta) c_1^{\epsilon}]^{1/\epsilon}$ be the composite index of consumption.

Equations (17) and (18) give[19]:

$$\frac{c_1}{c_0} = \left(\frac{1-\theta}{\theta}\frac{\kappa}{\mu}\right)^{\frac{1}{1-\epsilon}} \tag{25}$$

$$\frac{\dot{c}_0}{c_0} = \frac{1}{\sigma}\left[-\rho - (1-\psi(t))\frac{\dot{\kappa}}{\kappa} - \psi(t)\frac{\dot{\mu}}{\mu}\right] \tag{26}$$

$$\frac{\dot{c}_1}{c_1} = \frac{\dot{c}_0}{c_0} + \frac{1}{1-\epsilon}\left(\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\mu}}{\mu}\right) \tag{27}$$

Equations (19) and (20) give the each sector's demand function for intermediate goods. The model implies symmetry across each type of capital good:

$$x_0 = \left(\frac{\kappa}{\mu}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}(u_0 H) \tag{28}$$

$$x_1 = \alpha^{\frac{1}{1-\alpha}}(u_1 H) \tag{29}$$

Equations (21) and (22) equate the value of marginal product of human capital across all sectors:

$$\kappa\left[(1-\alpha)\frac{Y_0}{u_0}\right] = \nu\lambda H N \tag{30}$$

$$\mu\left[(1-\alpha)\frac{Y_1}{u_1}\right] = \nu\lambda H N \tag{31}$$

Using (30) and (31), equation (23) gives the growth rate of the shadow price of the state variable N:

$$-\frac{\dot{\nu}}{\nu} = \lambda H\,(1-u_0) \tag{32}$$

Inserting equations (28) and (29) back into the production functions gives the following implicit production functions:

$$Y_0 = \left(\frac{\kappa}{\mu}\right)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} A(u_0 H) \tag{33}$$

$$Y_1 = \alpha^{\frac{\alpha}{1-\alpha}} N(u_1 H) \tag{34}$$

Using these into (30) and (31) while dividing by parts, gives the relative shadow prices of the two final goods in levels and growth rates, given by

[19] Let $\psi(t) \equiv \frac{(1-\beta(t))(1-\epsilon-\sigma)}{1-\epsilon}$, where $\beta(t) \equiv \frac{\theta c_0^{\epsilon}}{\theta c_0^{\epsilon}+(1-\theta)c_1^{\epsilon}}$.

equations (35) and (36) respectively. Equation (37) comes from substituting (34) back into (31) and differentiating both sides with respect to time.

$$\frac{\kappa}{\mu} = \left(\frac{N}{A}\right)^{1-\alpha} \tag{35}$$

$$\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\mu}}{\mu} = (1-\alpha)\frac{\dot{N}}{N} \tag{36}$$

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{\nu}}{\nu} \tag{37}$$

By (25) it follows that $\beta(t) = \frac{\theta c_0^{\epsilon}}{\theta c_0^{\epsilon}+(1-\theta)c_1^{\epsilon}} = \frac{c_0}{c_0+\frac{\mu}{\kappa}c_1}$. The resource constraints in each sector imply $c_0 = Y_0$ and $c_1 = Y_1 - K_0 - K_1$, while the technology side of the economy implies from the demand functions for capital varieties that: $K_0 = \alpha\left(\frac{N}{A}\right)^{1-\alpha} Y_0$, $K_1 = \alpha Y_1$ and $\frac{Y_1}{Y_0} = \left(\frac{N}{A}\right)^{1-\alpha}\frac{u_1}{u_0}$. Combining all thses relations while equating the MRT to the MRS implies the following relation between human capital allocations and expenditure shares:

$$\frac{u_0}{u_1} = \frac{(1-\alpha)\beta(t)}{1-(1-\alpha)\beta(t)} \tag{38}$$

A steady-state CGP is an equilibrium that satisfies the FOCs, the market clearing conditions and the TVC together with the CGP requirements. By the law of motion of the state variable N, the necessary and sufficient condition for constant g_N, is that $\dot{u}_N = 0$. Given the resource constraint: $1 = u_N + u_1 + u_0$, this condition can alternatively take the form[20]:

$$g_1 = -g_0\frac{u_0}{u_1} \tag{39}$$

The next condition comes from the requirement that aggregate output grows at a constant rate. This condition coincides with the one for constant growth rate for the value of aggregate capital stock. The growth rates for the production in the two sectors are derived from (33) and (34), together with (35):

$$\frac{\dot{Y}_0}{Y_0} = \alpha g_N + g_0 \tag{40}$$

$$\frac{\dot{Y}_1}{Y_1} = g_N + g_1 \tag{41}$$

[20] Let $\frac{\dot{u}_s}{u_s} = g_s$ for $s = \{0,1\}$

These will be combined with (35), (36) and (33), (34):

$$\frac{\dot{Y}}{Y} = \frac{Y_0}{Y}\frac{\dot{Y}_0}{Y_0} + \frac{\frac{\mu}{\kappa}Y_1}{Y}\left(\frac{\dot{Y}_1}{Y_1} + \frac{\dot{\mu}}{\mu} - \frac{\dot{\kappa}}{\kappa}\right) = \alpha g_N$$

The last equation is derived by using the steady-state condition (39). Hence, along a CGP for Y, u_N has to be constant.

The growth rate of the aggregate consumption is:

$$\frac{\dot{C}}{C} = \frac{c_0}{C}\frac{\dot{c}_0}{c_0} + \frac{\frac{\mu}{\kappa}c_1}{C}\left(\frac{\dot{c}_1}{c_1} + \frac{\dot{\mu}}{\mu} - \frac{\dot{\kappa}}{\kappa}\right) = \frac{\dot{c}_0}{c_0} + (1-\beta(t))(1-\alpha)\frac{\epsilon}{1-\epsilon}g_N$$

Relations (26), (36) and (32) are used to get the last part of the equation. By imposing the for constant $\frac{\dot{C}}{C}$ and solving it with respect to g_0:

$$g_0 = (1-\sigma)(1-\alpha)^2\frac{\epsilon}{1-\epsilon}\beta(t)(1-\beta(t))\frac{u_N}{u_0}g_N \tag{42}$$

Equation (42) sets a condition on the evolution of the share of human capital in the non ICT sector along time[21]. The static optimization condition (38) together with the CGP condition (39) give another expression on g_0:

$$g_0 = \frac{\frac{d}{dt}(\beta(t))}{\beta(t)} = -\frac{\epsilon}{1-\epsilon}(1-\alpha)g_N(1-\beta(t)) \tag{43}$$

Conditions (42) and (43) should be jointly satisfied. Equating their LHSs and using that $u_N = 1 - \frac{u_0}{\beta(t)(1-\alpha)}$, gives an expression for the share of human capital in the non ICT using sector: $u_0 = \frac{\sigma-1}{\sigma}(1-\alpha)\beta(t)$. Given the solution path for u_0, (38), (36) and (32) imply a growth rate for the consumption of the non ICT using good:
$\frac{\dot{c}_0}{c_0} = \frac{1}{\sigma}\left[-\rho + \lambda H\left(\frac{\alpha}{\sigma} + \frac{\sigma-1}{\sigma}(1-(1-\alpha)\beta(t))\right) + \frac{(1-\sigma-\epsilon)}{1-\epsilon}(1-\alpha)g_N\right]$.

This candidate solution should satisfy the market clearing conditions along time. It should be true that: $\frac{\dot{c}_0}{c_0} = \frac{\dot{Y}_0}{Y_0} = \frac{\omega}{1-\alpha}\frac{\lambda H}{\sigma} + g_0$. This implies a third expression on the evolution of the share of human capital in the non ICT using sector:

$$g_0 = -\frac{\rho}{\sigma} - \frac{\epsilon}{1-\epsilon}(1-\alpha)\frac{\lambda H}{\sigma}(1-\beta(t)) \tag{44}$$

Equation (44) may only be reconciled with (43), if $\rho = 0$. This contradicts the original assumptions. Therefore, there is not any CGP for preferences with $\sigma > 0$ and $\epsilon < 1$.

[21] Since an interior solution implies $\beta(t) \in (0,1)$, $u_s \in (0,1)$, $\forall s \in \{0,1,N\}$ and $1-\epsilon > 0$, then $sgn(g_0) = sgn(1-\sigma)$.

For unit intertemporal elasticity of substitution ($\sigma = 1$, while $\epsilon \neq 0$), the instantaneous utility function is: $u(c_0, c_1) = \frac{1}{\epsilon}\ln\left[\theta c_0^{\epsilon} + (1-\theta)c_1^{\epsilon}\right]$. The above analysis remains the same and the FOCs will be those derived for the general case, in the limit of $\sigma = 1$. The FOCs with respect to the consumption goods result in (25) and:

$$\begin{aligned}
\frac{\dot{c}_0}{c_0} &= -\rho - \frac{\dot{\kappa}}{\kappa}\left[1 + \frac{\epsilon}{1-\epsilon}(1-\beta(t))\right] + \frac{\dot{\mu}}{\mu}\frac{\epsilon}{1-\epsilon}\left(1-\beta(t)\right) \\
\frac{\dot{c}_1}{c_1} &= \frac{\dot{c}_0}{c_0} + \frac{1}{1-\epsilon}\left(\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\mu}}{\mu}\right)
\end{aligned}$$

The aggregate consumption growth rate is:

$$\frac{\dot{C}}{C} = -\rho - \frac{\dot{\kappa}}{\kappa}$$

The CGP requirement boils down to $\frac{d}{dt}\left(-\frac{\dot{\kappa}}{\kappa}\right) = \frac{d}{dt}\left(-\frac{\dot{\nu}}{\nu}\right) = -\lambda H \dot{u}_0 = 0$. Thus, if there exists any CGP, it will not allow for any reallocation. At the same time, given condition (38), (43) implies $\frac{d}{dt}\left(\beta(t)\right) = -\frac{\epsilon}{1-\epsilon}(1-\alpha)\beta(t)(1-\beta(t))g_N = 0$. An interior solution should imply $\beta(t) \in (0,1)$ and $g_N > 0$ in steady-state. Hence, it is necessary that the preferences exhibit also unit intratemporal elasticity of substitution, i.e. $\epsilon = 0$. Unit intertemporal elasticity of substitution alone is not sufficient to allow for a CGP.

For unit intratemporal elasticity of substitution, along with a general intertemporal substitution pattern ($\epsilon = 0$ and $\sigma \neq 1$), the instantaneous utility function is: $u(c_0, c_1) = \frac{\left(c_0^{\theta} c_1^{1-\theta}\right)^{1-\sigma} - 1}{1-\sigma}$. The FOCs with respect to the two consumption goods imply:

$$\begin{aligned}
\frac{c_1}{c_0} &= \frac{1-\theta}{\theta}\frac{\kappa}{\mu} && (45)\\
\frac{\dot{c}_0}{c_0} &= \frac{1}{\sigma}\left[-\rho - \frac{\dot{\kappa}}{\kappa}\left(1-(1-\sigma)(1-\theta)\right) - \frac{\dot{\mu}}{\mu}(1-\sigma)(1-\theta)\right] && (46)\\
\frac{\dot{c}_1}{c_1} &= \frac{\dot{c}_0}{c_0} + \left(\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\mu}}{\mu}\right) && (47)
\end{aligned}$$

The aggregate consumption growth rate is:

$$\frac{\dot{C}}{C} = \frac{\dot{c}_0}{c_0} = \frac{1}{\sigma}\left[-\rho - \frac{\dot{\kappa}}{\kappa} + (1-\sigma)(1-\theta)(1-\alpha)g_N\right]$$

For the same reason as above, the condition for constant aggregate consumption growth rate excludes reallocation of resources along the steady-state, i.e. $\dot{u}_0 = \dot{u}_1 = \dot{u}_N = 0$. In this setting, condition (43) is automatically satisfied since $\beta(t) = \theta$, i.e. because the expenditure share of the non ICT using good is time invariant. Condition (42) is satisfied because $\epsilon = 0$. Therefore, the only condition left in order to pin down the CGP is (44).

The case of $\sigma = 1$ and $\epsilon = 0$, which corresponds to instantaneous utility: $u(c_0, c_1) = \theta \ln c_0 + (1-\theta)\ln c_1$, is only a special case of the $\epsilon = 0$ case. *Q.E.D.*

Proof of Proposition 2

Regarding first the production side: The final good producers are price takers. Therefore, their demand for capital comes by equating the value of marginal product of every capital variety to its price:

$$\frac{\partial Y_0}{\partial x_0(i)} = \alpha (u_0 H)^{1-\alpha} x_0^{\alpha}(i) = \hat{p}_0(i), \forall i \tag{48}$$

$$p_1 \frac{\partial Y_1}{\partial x_1(j)} = \alpha (u_1 H)^{1-\alpha} x_1^{\alpha}(i) = \hat{p}_1(j), \forall j \tag{49}$$

The producers of the capital varieties are functioning under monopolistic competition. In the absence of dynamic decision variables, they maximise their profits by choosing their price and production at every period.

$$\pi_0 = \max_{\hat{p}_0(i), x_0(i)} \{\hat{p}_0(i) x_0(i) - p_1 x_0(i); s.t.(48)\}$$

$$\pi_1 = \max_{\hat{p}_1(i), x_1(i)} \{\hat{p}_1(i) x_1(i) - p_1 x_1(i); s.t.(49)\}$$

The solutions to these programs are:

$$x_0 = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{p_1}\right)^{\frac{1}{1-\alpha}} (u_0 H) \tag{50}$$

$$x_1 = \alpha^{\frac{2}{1-\alpha}} (u_1 H) \tag{51}$$

$$\hat{p}_0 = \hat{p}_1 = \frac{p_1}{\alpha}$$

The model delivers symmetry among the varieties of each type of capital goods. The implied profit flows for every period is:

$$\pi_0 = \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{p_1}\right)^{\frac{\alpha}{1-\alpha}} (u_0 H) \tag{52}$$

$$\pi_1 = p_1 \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} (u_1 H) \tag{53}$$

These producers enter the market upon getting a "blueprint" that makes their products compatible with one of the old technologies, A, or one of the new ones, N. With well defined property rights, the cost that each producer needs to assume in order to acquire a blueprint is equal to the present discounted value of his future stream of profits. The guess is that along the steady-state the interest rate is constant. This is to be verified later. In this case, the value function of each type of firm becomes:

$$rV_0 = \pi_0 \tag{54}$$

$$rV_1 = \pi_1 \tag{55}$$

Since the human capital market is perfectly competitive, there exists a wage,w_H, that clears out the market. This wage is equal to the value of marginal product of human capital in all three sectors:

$$\frac{\partial Y_0}{\partial(u_0 H)} = (1-\alpha)\left(\frac{1}{p_1}\right)^{\frac{\alpha}{1-\alpha}} A\alpha^{\frac{2\alpha}{1-\alpha}} = w_H \tag{56}$$

$$p_1 \frac{\partial Y_1}{\partial(u_1 H)} = p_1(1-\alpha)N\alpha^{\frac{2\alpha}{1-\alpha}} = w_H \tag{57}$$

$$p_N \frac{\partial \dot{N}}{\partial(u_N H)} = \frac{\pi_1}{r}\lambda N = \frac{1}{r}p_1\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}(u_1 H)\ \lambda N\ = w_H \tag{58}$$

In (58), the price of the output of the ICT producing sector, p_N, is equal to the market value of a firm that produces intermediate capital varieties which are compatible to ICT. The market value is given by (55) and (53).

Equating (56) and (57):

$$p_1 = \left(\frac{A}{N}\right)^{1-\alpha} \tag{59}$$

$$\frac{\dot{p}_1}{p_1} = -(1-\alpha)\, g_N \tag{60}$$

Equating (57) and (58):

$$r = \alpha\lambda u_1 H \tag{61}$$

Regarding the consumer side: The households solve the following dynamic problem by choosing $\{c_0, c_1\}$:

$$\mathcal{H} = e^{-\rho t}\frac{\left(c_0^{\theta}c_1^{1-\theta}\right)^{1-\sigma} - 1}{1-\sigma} + \lambda\left[rS + w_H H - c_0 - p_1 c_1\right]$$

The solution to this problem gives the standard conditions:

$$\frac{c_1}{c_0} = \frac{1-\theta}{\theta}\frac{1}{p_1} \tag{62}$$

$$-\frac{\dot{\lambda}}{\lambda} = r \tag{63}$$

These imply:

$$\frac{\dot{c}_0}{c_0} = \frac{1}{\sigma}\left[r-\rho-(1-\sigma)(1-\theta)\frac{\dot{p}_1}{p_1}\right] \quad (64)$$

$$\frac{\dot{c}_1}{c_1} = \frac{\dot{c}_0}{c_0}-\frac{\dot{p}_1}{p_1} \quad (65)$$

Searching for a CGP, the conditions are the same as under Proposition 1. Since $\frac{\dot{Y}}{Y}=\frac{\dot{K}}{K}=\alpha g_N$, the condition for steady-state is that the share of human capital in the ICT production sector is constant. Also, since $\frac{\dot{C}}{C}=\frac{1}{\sigma}\left[r-\rho+(1-\sigma)(1-\theta)(1-\alpha)g_N\right]$, the requirement for CGP for consumption is that the interest rate is constant. This verifies the original guess. From (61), the conclusion is that there exists no reallocation of real resources along the steady-state path.

Along the CGP the market clearing conditions need to be satisfied at every point in time. Together with the FOCs, this implies constant consumption to output ratios in every sector. Equating (64) to $\frac{\dot{Y}_0}{Y_0}=\alpha g_N$ while using (61) and the ICT production function:

$$u_1=\frac{\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\theta)}{\alpha+\sigma\alpha-(1-\sigma)(1-\theta)}-\frac{\sigma\alpha-(1-\sigma)(1-\theta)}{\alpha+\sigma\alpha-(1-\sigma)(1-\theta)}u_0 \quad (66)$$

At the same time, within every period the market clearing conditions impose that: $c_0=Y_0$ and $c_1=Y_1-K_1-K_0$, while the demand functions for capital varieties imply that: $\frac{1}{\alpha}K_1=\alpha Y_1$ and $\frac{p_1}{\alpha}K_0=\alpha Y_0$. In addition, $\frac{Y_1}{Y_0}=\left(\frac{N}{A}\right)^{1-\alpha}\frac{u_1}{u_0}$. Therefore, the static optimization conditions imply:

$$\frac{u_0}{u_1}=\frac{\theta(1-\alpha^2)}{1-\theta(1-\alpha^2)} \quad (67)$$

Solving the system of the last two equations:

$$u_1^c = \frac{(1-\theta(1-\alpha^2))\left[\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)\right]}{\alpha(1-\theta(1-\alpha^2))+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)} \quad (68)$$

$$u_0^c = \frac{\theta(1-\alpha^2)\left[\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)\right]}{\alpha(1-\theta(1-\alpha^2))+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)} \quad (69)$$

$$g_N^c = \lambda H\left[\frac{\alpha(1-\theta(1-\alpha^2))-\frac{\rho}{\lambda H}}{\alpha(1-\theta(1-\alpha^2))+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)}\right] \quad (70)$$

In order to ensure that this is indeed an interior solution, it is sufficient to check that $u_1^c>0$ and $g_N^c>0$[22]. The condition for $g_N^c>0$ requires that

[22]Note that $u_1^c>0$ iff $u_0^c>0$ and $g_N^c>0$ iff $u_N^c>0$

$\alpha(1-\theta(1-\alpha^2))-\frac{\rho}{\lambda H}>0$ and $\alpha(1-\theta(1-\alpha^2))+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)>0$. The condition for $u_1^c>0$ requires that $\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)>0$ and $\alpha(1-\theta(1-\alpha^2))+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)>0$. These combine to provide two necessary and sufficient conditions: $H>\frac{1}{\lambda\alpha(1-\theta(1-\alpha^2))}$ and $\sigma>\frac{(1-\theta)(1-\alpha)-\frac{\rho}{\lambda H}}{\alpha+(1-\theta)(1-\alpha)}$.

The implied intertemporal elasticity of substitution is non-negative, we need that $H>\frac{\rho}{\lambda(1-\alpha)(1-\theta)}$. If $\theta(1-\alpha)\left[1-\alpha(1+\alpha)\right]\leq 1-2\alpha$, then $\lambda H>\frac{1}{\alpha(1-\theta(1-\alpha^2))}\geq\frac{\rho}{(1-\alpha)(1-\theta)}$ and it is sufficient that $H>\frac{1}{\lambda\alpha(1-\theta(1-\alpha^2))}$. If instead $\theta(1-\alpha)\left[1-\alpha(1+\alpha)\right]>1-2\alpha$ then it is sufficient that $\lambda H>\frac{\rho}{(1-\alpha)(1-\theta)}$.

Alternatively, the interior solution may be satisfied for $\alpha(1-\theta(1-\alpha^2))-\frac{\rho}{\lambda H}<0$ and $\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\theta)(1-\alpha)<0$. For reasonable parameterisation this case is not relevant, while it generates perverse results for the comparative statics' analysis and the comparison to the first best from an economic point of view. *Q.E.D.*

Proof of Corollary 1

The system of the FOCs, of the two resource constraints and the TVC is solved without imposing the steady-state CGP conditions.

Along any equilibrium path, the following are true: From $c_0=Y_0$ it follows that $\frac{\dot{c}_0}{c_0}=\frac{\dot{Y}_0}{Y_0}=\alpha g_N+g_0$. From $c_1=Y_1-K_1-K_0$, where $K_1=\alpha Y_1$ and $K_0=\alpha\left(\frac{N}{A}\right)^{1-\alpha}Y_0$, it follows that $\frac{\dot{c}_1}{c_1}=\frac{\dot{Y}_1}{Y_1}=g_N+g_0$. From the static optimization: $u_0=\frac{\theta(1-\alpha)}{1-\theta(1-\alpha)}u_1$ it follows that $g_0=g_1=g$. Using the latter with the resource constraint: $1=u_0+u_1+u_N$, it follows that: $\dot{u}_N=-g(1-u_N)$.

Given that in the steady-state $\tilde{c}_0=\frac{c_0}{N^\alpha}$ and $\tilde{c}_1=\frac{c_1}{N}$ remain constant, it is useful to define the composite consumption index: $E\equiv\tilde{c}_0^\theta\tilde{c}_1^{1-\theta}$. Its growth rate is equal to:$\frac{\dot{E}}{E}=\theta\left(\frac{\dot{c}_0}{c_0}-\alpha g_N\right)+(1-\theta)\left(\frac{\dot{c}_1}{c_1}-g_N\right)$. Since $\frac{\dot{c}_1}{c_1}=\frac{\dot{c}_0}{c_0}-(1-\alpha)g_N$, it follows that $\frac{\dot{E}}{E}=\frac{\dot{c}_0}{c_0}-\alpha g_N$. Using the results above regarding the growth rate of consumption implied by the market clearing conditions:

$$\frac{\dot{E}}{E}=g \tag{71}$$

At the same time, using (46) and (47), together with (36):

$$\frac{\dot{E}}{E}=\frac{1}{\sigma}\left[-\rho-\frac{\dot{\nu}}{\nu}+\left((1-\sigma)(1-\theta)-1\right)(1-\alpha)g_N\right]-\alpha g_N$$

Dynamic equation (32), and $\frac{u_0}{u_1}=\frac{\theta(1-\alpha)}{1-\theta(1-\alpha)}$ together with $1=u_0+u_1+u_N$:

$$\frac{\dot{E}}{E} = -\frac{\rho}{\sigma} + \frac{\lambda H}{\sigma}\left(1 - \theta(1-\alpha)\right)\left(1 - \sigma u_N\right) \tag{72}$$

Equating (71) and (72):

$$g = -\frac{\rho}{\sigma} + \frac{\lambda H}{\sigma}\left(1 - \theta(1-\alpha)\right)\left(1 - \sigma u_N\right)$$

Therefore:

$$\dot{u}_N = (1 - u_N)\left[\frac{\rho}{\sigma} - \frac{\lambda H}{\sigma}\left(1 - \theta(1-\alpha)\right)\left(1 - \sigma u_N\right)\right] \tag{73}$$

This first order non-linear differential equation in the share of human capital in the ICT producing sector summarizes completely the dynamics of the economy. Its solution is[23]:

$$t - \int^{u_N(t)} \frac{1}{\frac{\lambda H}{\sigma}\left[\frac{\rho}{\lambda H} - (1 - \sigma z)\left(1 - \theta(1-\alpha)\right)\right](1 - z)} dz + F = 0 \tag{74}$$

Solved for an initial condition: $u_N(0) = u_N^*$, i.e. starting from the steady-state, the solution is $u_N(t) = u_N^*$. This implies that $\dot{u}_N = g = 0$. In turn, this implies that there are no transition dynamics, as long as g_N, $\frac{c_0}{Y_0}$, $\frac{c_1}{Y_1}$, and $\frac{c_0}{p_1 K_0}$, $\frac{c_1}{K_1}$ always remain constant in and out of steady-state. Note also that (73) ,may be rewritten as:

$$\dot{u}_N = (1 - u_N)\left(1 - \theta(1-\alpha)\right)\left(u_N - u_N^*\right) \tag{75}$$

From (75), if the economy deviated from steady-state position then there would be no forces to restore the steady-state. The dynamics would imply that for $u_N > u_N^*$, $\dot{u}_N > 0$, $\forall t$ and vice-versa. *Q.E.D.*

Proof of Corollary 2

Let $\pi = \alpha\left(1 - \theta(1-\alpha^2)\right) + \sigma\alpha - (1-\sigma)(1-\alpha)(1-\theta) > 0$.

Effect of a change in λ:

$$\begin{aligned}
\frac{\partial u_1^c}{\partial \lambda} &= \frac{\left(1 - \theta(1-\alpha^2)\right)\frac{\rho}{H}}{\pi}\left(-\frac{1}{\lambda^2}\right) < 0 \\
\frac{\partial u_0^c}{\partial \lambda} &= \frac{\theta(1-\alpha^2)}{1 - \theta(1-\alpha^2)}\frac{\partial u_1^c}{\partial \lambda} < 0 \\
\frac{\partial g_N^c}{\partial \lambda} &= \frac{\alpha\left(1 - \theta(1-\alpha^2)\right)H}{\pi} > 0
\end{aligned}$$

[23] The differential equation is a quadrature. Let F be some arbitrary constant of integration.

Change in ρ:

$$\begin{aligned}
\frac{\partial u_1^c}{\partial \rho} &= \frac{(1-\theta(1-\alpha^2))}{\pi}\left(\frac{1}{\lambda H}\right) > 0 \\
\frac{\partial u_0^c}{\partial \rho} &= \frac{\theta(1-\alpha^2)}{1-\theta(1-\alpha^2)}\frac{\partial u_0^c}{\partial \rho} > 0 \\
\frac{\partial g_N^c}{\partial \rho} &= -\frac{1}{\pi} < 0
\end{aligned}$$

Change in θ:

$$\begin{aligned}
\frac{\partial g_N^c}{\partial \theta} = & \frac{\lambda H}{\pi^2}\left\{-\alpha(1-\alpha^2)\left[\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\alpha)(1-\theta)\right]\right. \\
& \left.-(1-\sigma)(1-\alpha)\left[\alpha\left(1-\theta(1-\alpha^2)\right)-\frac{\rho}{\lambda H}\right]\right\}
\end{aligned}$$

Given the condition for interior solution, the first term is negative. The second term will be also negative iff $\sigma \leq 1$. Also:

$$\frac{\partial\left(\frac{u_0}{u_1}\right)}{\partial \theta} = \frac{1-\alpha^2}{\left[1-\theta(1-\alpha)\right]^2} > 0$$

For $\sigma = 1$:

$$\begin{aligned}
\frac{\partial g_N^c}{\partial \theta} &= \frac{\lambda H}{\pi^2}\left[-\alpha(1-\alpha^2)\left(\frac{\rho}{\lambda H}+\alpha\right)\right] < 0 \\
\frac{\partial u_1^c}{\partial \theta} &= -\frac{\alpha(1-\alpha^2)}{\pi^2}\left(\frac{\rho}{\lambda H}+\alpha\right) < 0
\end{aligned}$$

Change in α:

$$\begin{aligned}
\frac{\partial g_N^c}{\partial \alpha} = & \frac{\lambda H}{\pi^2}\left\{\left[1-\theta(1-\alpha^2)+2\theta\alpha^2\right]\left[\frac{\rho}{\lambda H}+\sigma\alpha-(1-\sigma)(1-\alpha)(1-\theta)\right]\right. \\
& \left.-(1-\theta(1-\sigma))\left[\alpha\left(1-\theta(1-\alpha^2)\right)-\frac{\rho}{\lambda H}\right]\right\}
\end{aligned}$$

The first term is positive. The second term is always negative. The final effect is ambiguous. Also:

$$\frac{\partial\left(\frac{u_0}{u_1}\right)}{\partial \alpha} = \frac{-2\theta\alpha}{\left[1-\theta(1-\alpha)\right]^2} < 0$$

For $\sigma = 1$:

$$\begin{aligned}
\frac{\partial g_N^c}{\partial \alpha} &= \frac{\lambda H}{\pi^2}\left[\frac{\rho}{\lambda H}\left(2-\theta(1-\alpha^2\right)+2\theta a^2)+2\alpha^3\theta\right] > 0 \\
\frac{\partial u_1^c}{\partial \alpha} &= -\theta\frac{2-\theta(1-\alpha^2)}{\pi}\left[\frac{\rho}{\lambda H}(1+\alpha^2)+2\alpha^3\right] < 0
\end{aligned}$$

Q.E.D.

7 Appendix B: Depreciation of the capital stock

This exercise explores the implications of the model when allowing for geometric depreciation of the ICT capital stock, K_1[24]. The ICT using sector good is to be used for consumption, for the production of new capital stock-ICT and non ICT, and for the depreciation needs of the ICT capital stock. In effect, now the state of the economy is summarized by the ICT capital stock (since it may be preserved along time) and the level of ICT production stock, $\{K_1, N\}$. The optimal control problem below is solved for the the social planner's equilibrium, when there is control over:$\{c_0, c_1, K_0, u_0, u_1\}$[25]:

$$\begin{aligned}
\mathcal{H} &= \frac{\left(c_0^\theta c_1^{1-\theta}\right)^{1-\sigma}-1}{1-\sigma}+\kappa\left[(u_0H)^{1-\alpha}A^{1-\alpha}K_0^\alpha-c_0\right] \\
&\quad +\mu\left[(u_1H)^{1-\alpha}N^{1-\alpha}K_1^\alpha-c_1-K_0-\delta K_1\right]+\nu\left[\lambda(1-u_0-u_1)HN\right]
\end{aligned}$$

The standard FOCs of equating the MRS to the relative prices:

$$\frac{c_1}{c_0}=\frac{1-\theta}{\theta}\frac{\kappa}{\mu} \tag{76}$$

By setting the marginal product of the non ICT capital equal to its cost:

$$\frac{\kappa}{\mu}=\frac{K_0}{\alpha Y_0} \tag{77}$$

[24] It can be extended for the case of both ICT and non ICT capital intermediates depreciating over time. The analytical part of the solution does not indicate that the main features of the numerical solution would change.

[25] The advantage of solving for the social planner's equilibrium is that it preserves the features of the implied dynamics of the competitive equilibrium, while being more straightforward to handle analytically (and to check that the TVCs are satisfied).

By equating returns to human capital across sectors:

$$\frac{\kappa}{\mu} = \frac{Y_1}{Y_0}\frac{u_0}{u_1} \tag{78}$$

Finally, the implied growth rates of the shadow prices for the capital and the ICT stock:

$$-\frac{\dot{\mu}}{\mu} = \alpha\frac{Y_1}{K_1} - \delta \tag{79}$$

$$-\frac{\dot{\nu}}{\nu} = \lambda H(1-u_0) \tag{80}$$

The CGP solution is defined again as the steady-state equilibrium path along which the two state variables, as well as the aggregate output, consumption and capital (in terms of the non ICT using good) grow at constant rates. Along this path the TVCs need to be satisfied.

The TVCs will be satisfied, when the shadow prices, as well as the state variables grow at constant rates. For N to grow at a constant rate the condition is that: $\dot{u}_0+\dot{u}_1 = 0$. For ν to grow at a constant rate, the condition is that: $\dot{u}_0 = 0$. Therefore, the TVC on the value of ICT production in the limit implies that there is no reallocation along the steady-state: $\dot{u}_0 = \dot{u}_1 = \dot{u}_N = 0$[26]. The condition for constant growth rate of μ is: $g_{Y_1} = g_{K_1}$. But then, the production function of the ICT using sector together with the requirement of $\dot{u}_1 = 0$, imply that: $g_{Y_1} = g_N$. Furthermore, by (77) and (78): $K_0 = Y_1\frac{u_1}{u_0}$. Constant allocations along the steady-state imply: $g_{K_0} = g_{Y_1}$. Hence, from the law of motion for ICT capital, $\frac{\dot{K}_1}{K_1} = \frac{Y_1}{K_1} - \frac{c_1}{K_1} - \delta - \frac{K_0}{K_1}$, it follows that $\frac{\dot{K}_1}{K_1}$ is constant only if $g_{c_1} = g_{K_1}$. Finally, the non ICT using sector production function together with the resource constraint in this sector imply that along the steady-state: $g_{c_0} = g_{Y_0} = \alpha g_N$. To conclude, the following are true along the steady-state time path:

$$\begin{aligned} g_{u_0} &= g_{u_1} = g_{u_N} = 0 \\ g_{c_1} &= g_{K_1} = g_{K_0} = g_{Y_1} = g_N \\ g_{c_0} &= g_{Y_0} = \alpha g_N \end{aligned}$$

Given the features of the steady-state, the model may be rewritten in terms of the variables that will remain constant along the CGP. In particular,

[26] This argument is only a shortcut of the the outcome outlined and proved under Proposition 1. Adding physical depreciation does not affect the implication of this TVC.

following the procedure described in Mulligan and Sala-i-Martin (1993), the system of FOCs is redefined in terms of one "state-like" variable, $\left\{k_1 \equiv \frac{K_1}{N}\right\}$, and five "control-like" variables, $\left\{k_0 \equiv \frac{K_0}{N}, \omega_0 \equiv \frac{c_0}{N^\alpha}, \omega_1 \equiv \frac{c_1}{N}, u_0, u_1\right\}$. Nevertheless, as expected from the FOCs and the interaction between the two final sectors, the dynamics of this economy may be entirely determined by looking at the law of motion of the unique "state-like" variable (the ICT capital stock adjusted by the level of technology) and the control variables of the ICT using (human capital share and consumption adjusted for the level of technology). This comes from the following:

The resource constraint of the non ICT using good implies ω_0 as a function (k_0, u_0):

$$\omega_0 = (u_0 H)^{1-\alpha} A^{1-\alpha} k_0^\alpha \tag{81}$$

The static efficiency condition that equates the MRS to the MRT determines k_0 and u_0 as a function of (k_1, u_1, ω_1):

$$k_0 = \frac{\alpha\theta}{1-\alpha}\omega_1 \tag{82}$$

$$u_0 = \frac{\theta}{1-\theta}\omega_1 H^{\alpha-1} u_1^{-\alpha} k_1^\alpha \tag{83}$$

Therefore, the differential equations that completely summarize the dynamics are the following:

$$\frac{\dot{k}_1}{k_1} = (u_1 H)^{1-\alpha} k_1^{\alpha-1} - \left(1 + \frac{\alpha\theta}{1-\theta}\right)\frac{\omega_1}{k_1} - \delta - \lambda H\left(1 - \frac{\theta}{1-\theta}\omega_1 H^{\alpha-1} u_1^{-\alpha} k_1^\alpha - u_1\right) \tag{84}$$

$$\frac{\dot{u}_1}{u_1} = \frac{1-\alpha}{\alpha}\left[\delta + \lambda H\left(1 - \frac{\theta}{1-\theta}\omega_1 H^{\alpha-1} u_1^{-\alpha} k_1^\alpha\right)\right] + \lambda H u_1 - \left(1 + \frac{\alpha\theta}{1-\theta}\right)\frac{\omega_1}{k_1} \tag{85}$$

$$\begin{aligned}\frac{\dot{\omega}_1}{\omega_1} \quad = \quad & \frac{1}{\sigma}\left\{-\rho + \alpha\left[\alpha(u_1 H)^{1-\alpha} k_1^{\alpha-1} - \delta\right] + (1-\alpha)\lambda H u_1 + \right. \\ & \left. +(1-\sigma)(1-\theta)(1-\alpha)\left[\alpha(u_1 H)^{1-\alpha} k_1^{\alpha-1} - \delta - \lambda H u_1\right]\right\} + \\ & +(1-\alpha)\left[\alpha(u_1 H)^{1-\alpha} k_1^{\alpha-1} - \delta - \lambda H u_1\right] - \lambda H\left(1 - \frac{\theta}{1-\theta}\omega_1 H^{\alpha-1} u_1^{-\alpha} k_1^\alpha - u_1\right)\end{aligned} \tag{86}$$

The system of (84), (85) and (86) does not allow for an analytical solution, since it is highly non-linear. The steady state and the comparative statics are solved numerically in MATLAB for a reasonable parameterization of the model, when using the "time elimination" algorithm, which was proposed by Mulligan and Sala-i-Martin (1993). The parameter values are picked so as to match the data of the USA economy over the period 1995-2001 (GGDC and BEA databases; details in Appendix C). The following table summarizes:

parameters	A	H	α	ρ	σ	λ	θ	δ
values	1	1	0.3	0.028	2.5	1.311	0.849	0.213

Regarding the choice of the parameter values: In order to normalize the units of the model, both the original technology index and the assumed fixed human capital stock are set equal to one. The choice of the output elasticity of capital is standard in the growth accounting literature (e.g. Mankew, Romer, Weil, 1992). The choice of the time preference parameter comes from micro data study in Attanasio et al, 1989. The inverse of the intertemporal elasticity of substitution is consistent with the estimated intertemporal elasticity of substitution for shareholders, from the micro data study in Attanasio et al., 2002.

According to the steady-state properties of the model economy, the growth rate of the ICT using sector should reflect the growth rate of the ICT producing sector. Hence, taking the growth rate of the ICT using sector, dividing by the employment share of the ICT producing sector and getting the average over the years 1995-2001 provides the calibrated productivity of the ICT producing sector (given the normalization for the human capital stock, this calibration captures the interaction of the productivity of the ICT producing sector with the human capital stock).

The parameter that weights the preference towards the non ICT using good should be equal in equilibrium to the non ICT using good expenditure share. This can be calculated using the data on Personal Consumption Expenditures by Type of Expenditure, 1979-2001, available from BEA. Out of the private sector expenditures alone, the expenditures for ICT using goods were distinguished from the non ICT using ones. The expenditure share is calibrated by the average share over the period 1995-2001.

Finally, the depreciation rate for the ICT capital stock is taken from data that BEA has published in "The Survey of Current Business", 1997, on depreciation rates of various assets. The calibrated depreciation rate is the average depreciation rate of the ICT capital assets[27].

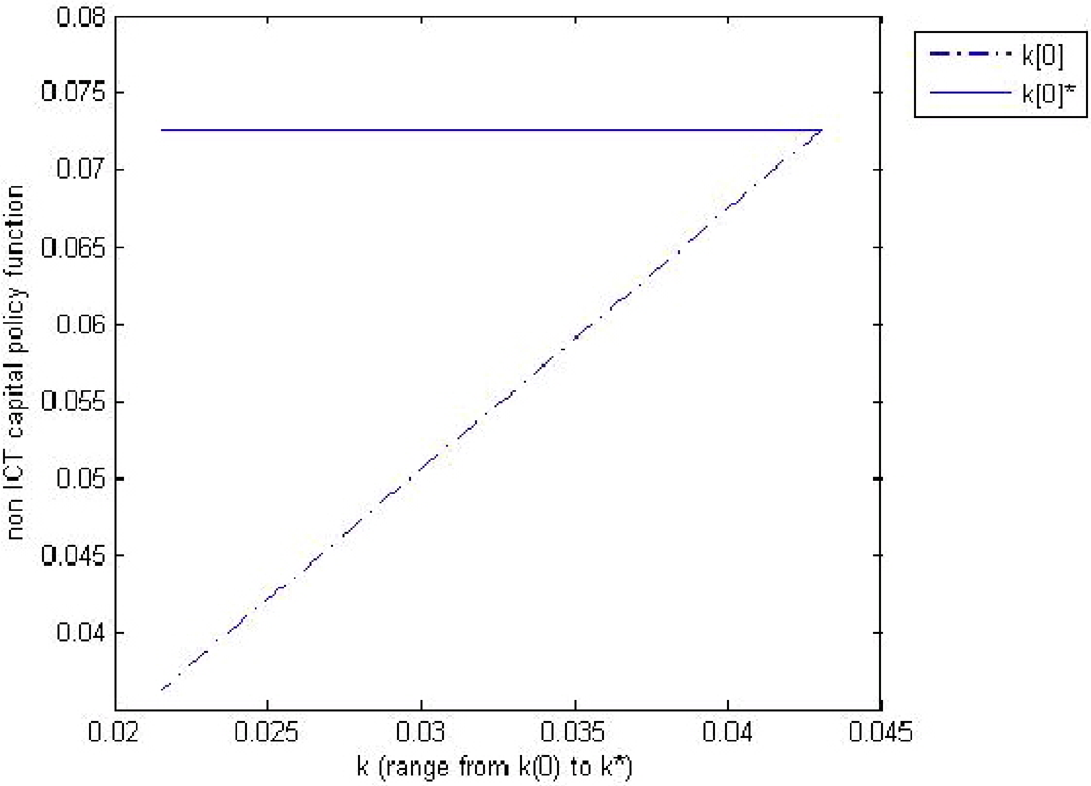
[27] These include: "Office, computing and accounting machinery", "Communications equipment", "Electronic components and accessories", "Computers and peripheral equipment", "Instruments, Photocopy and related equipment"

The steady-state is the fixed point of the homogeneous non-linear system of: $\frac{\dot{k}_1}{k_1} = \frac{\dot{u}_1}{u_1} = \frac{\dot{\omega}_1}{\omega_1} = 0$. The implied comparative statics of the social planner's equilibrium are summarized in the Table below[28]. As was the case with fully depreciating ICT capital stock, when the agents become more patient (when the time preference rate or the intertemporal elasticity of substitution decrease) the direction of change of the main variables of interest is the same to the case that the productivity of the ICT producing sector increases. In particular, the real resources are driven into the ICT producing sector enabling higher growth. On the other hand, an increased preference for the non ICT using goods (i.e. increased non ICT using good expenditure share) will drive resources out of the ICT producing and using sector and into the non ICT using sector, thus delivering lower growth. An increase in the capital intensity implies the opposite effects to the case of increased preference for non ICT using good (in all variables apart from the ICT using consumption to ICT production stock ratio). Finally, a higher depreciation rate of the ICT capital assets corresponds again to more resources driven into ICT producing and using sector, and thus to higher growth.

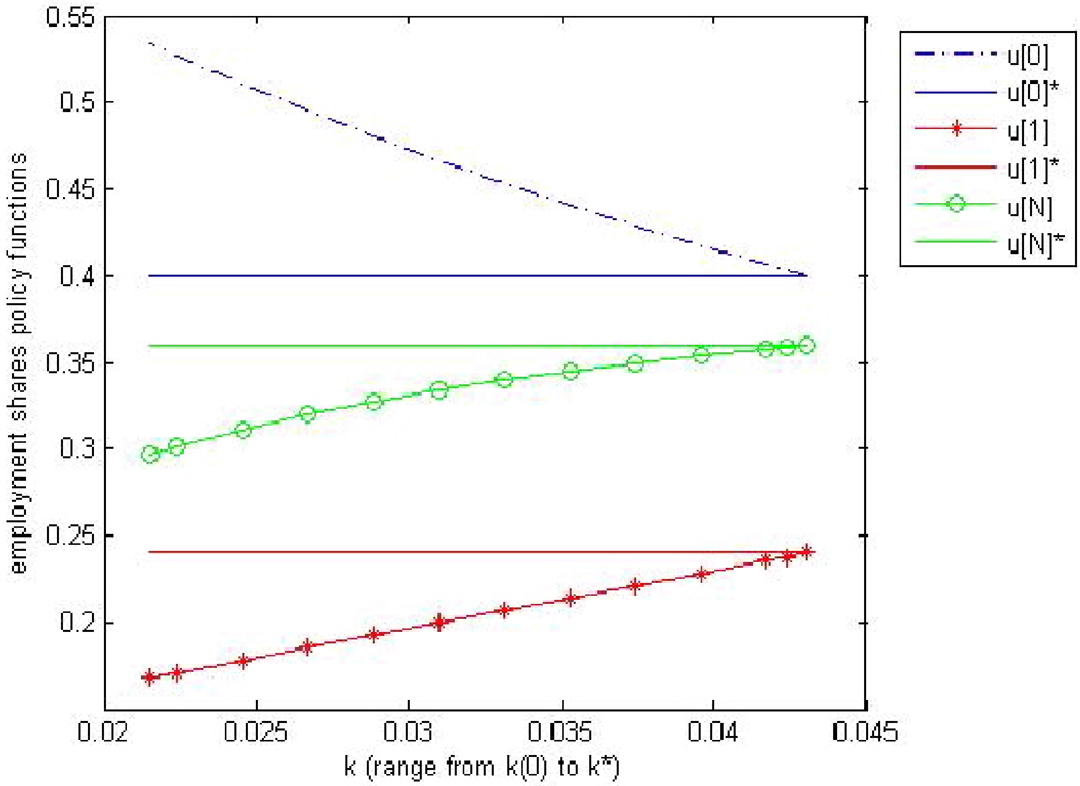
	k_1	k_0	u_1	u_0	u_N	ω_1	ω_0
Baseline	0.043	0.073	0.240	0.400	0.360	0.042	0.240
$\delta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\lambda \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\theta \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\sigma \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\rho \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\alpha \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$

One step further is to solve numerically for the policy functions. Then, the entire dynamics of the social planner's economy may be figured out, when the initial condition (initial "state-like" value) is specified. The three figures below give the policy functions of the "control-like" variables of the model economy for a range of the "state-like" variable $[k_1(0), k_1^*]$ (where $k_1(0) = \frac{k_1^*}{2}$, and k_1^* is the steady-state value).

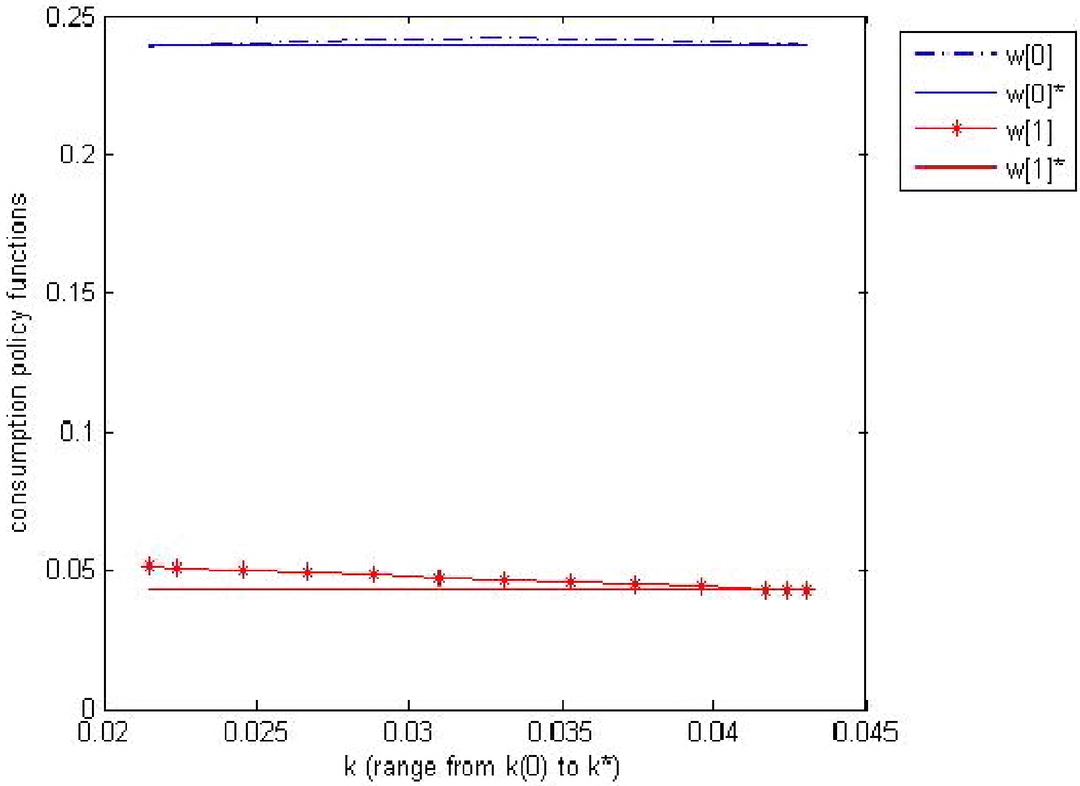
[28] The comparative statics exercise involves increasing the baseline parameter values by 10%, ceteris paribus.



0.08
0.075
0.07
0.065
0.06
0.055
0.05
0.045
0.04
non ICT capital policy function
0.02
0.025
0.03
0.035
0.04
0.045
k (range from k(0) to k*)
k[0]
k[0]*

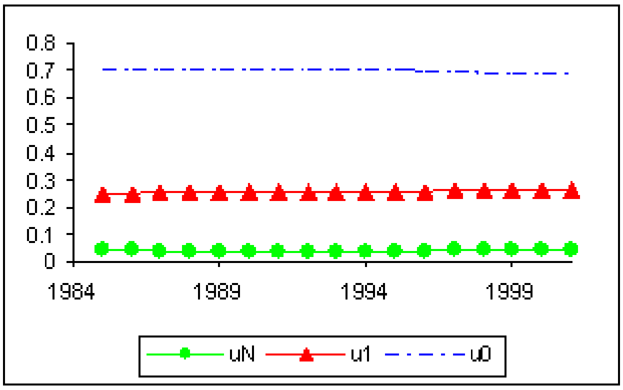


0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
employment shares policy functions
0.02
0.025
0.03
0.035
0.04
0.045
k (range from k(0) to k*)
u[0]
u[0]*
u[1]
u[1]*
u[N]
u[N]*

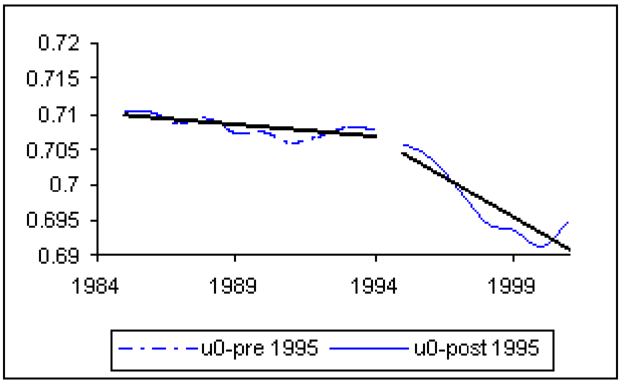


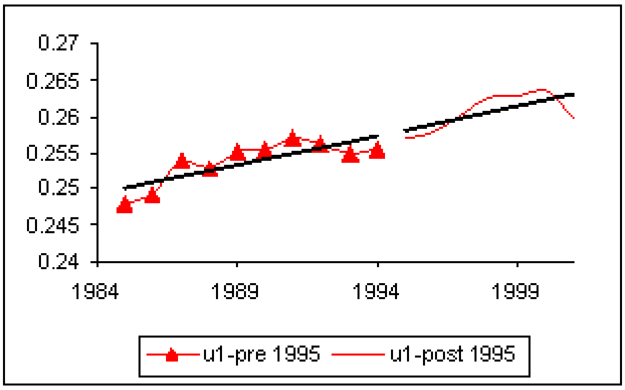
The recent empirical literature identifies 1995 as being a threshold year in the USA economic experience, after which economic growth accelerated. The cause is regarded to be an increase in the productivity of the ICT producing industries. Therefore, one may consider the USA economy being along a transition path towards its steady-state, along which it experienced a structural change.

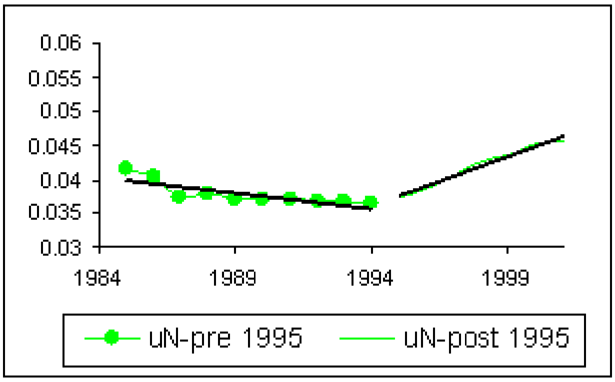
The data that are easier to follow and compare with the predictions of the theoretical model, are those of the employment shares. In order to avoid accounting for the second oil shock (1979) and its aftermath, Figure 1 of Appendix D is repeated below for the years 1985-2001. As was pointed out before, the shares out of hours worked of every major sector, do look constant along time.



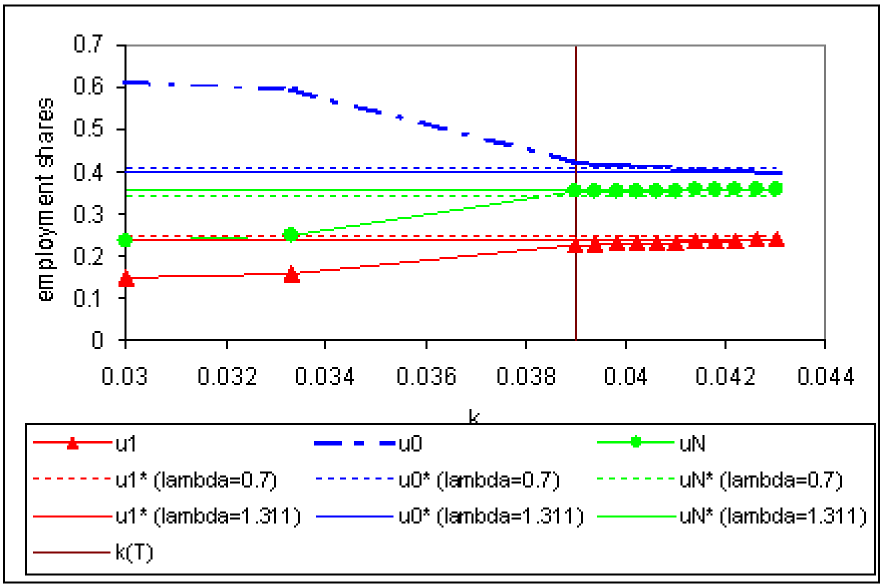
In order to investigate possible changes in the evolution of the series, each series is presented separately in the three figures below, together with a fitted line trend before and after 1995. The share in the non ICT using sector is monotonically falling with a downward jump in 1995. The opposite is the case for the ICT using sector, but with more moderate changes along time. The picture is less clear for the ICT producing sector, yet there is an upward jump in 1995.







In terms of the model economy, higher ICT producing productivity corresponds to an increase in parameter λ[29]. Given the policy functions derived above, a trajectory of the social planner's economy that would be close to that of the data, is that of an economy that starts moving towards its steady-state starting from $k_1(0) < k_1^*$, and experiences the high ICT producing productivity in period T, when the "state-like" is $k_1(T)$, where $k_1(0) < k_1(T) < k_1^{*'} < k_1^*$. The resulting pattern in the employment shares is given in the graph below:



[29] In order to get a value for the size of λ for the pre-1995 period, the same procedure was used as above (λ is the mean of the series that comes from dividing annual output growth of the ICT using sector with the employment share of the ICT producing sector). The estimated value is $\lambda = 0.731$. The numerical results are for $\lambda = 0.7$.

By construction, this graph illustrates the choices of the social planner in the hypothetical setting outlined above. It is interesting to note how the actual data compare to the choices of the social planner, and what is the implication for the ICT production growth rate. The social planner would allocate a great proportion of the human capital into R&D for any level of productivity in ICT production. He would react to the higher ICT producing productivity by increasing the employment in the sector even more. This, interacted with the higher productivity implies an increase of the ICT production growth rate by 23.2 pp. On the contrary, the data imply only a 2.7pp increase in the ICT production growth rate as a result of the enhanced productivity of ICT production. Also, the data indicate that human capital is increased in both the ICT producing and using sectors after 1995, while the aggregate volume of reallocations in the human capital market is of the order of 1pp, compared to 1.7pp in the social planner's economy.

		u_0	u_1	u_N	g_N
social planner	1985-1994; $\lambda = 0.700$	40.8	24.9	34.3	24.0
	1995-2001; $\lambda = 1.311$	40.0	24.0	36.0	47.2
	change	-0.8	-0.9	1.7	23.2
data	1985-1994; $\lambda = 0.700$	70.8	25.4	3.8	2.8
	1995-2001; $\lambda = 1.311$	69.8	26.0	4.2	5.5
	change	-1.0	0.6	0.4	2.7

8 Appendix C: Data Summary

ICT producing sector[30]*:*

Office machinery (30), Insulated wire (313), Electronic valves and tubes (321), Telecommunication equipment (322), Radio and television receivers (323), Computer and related activities (72), Scientific instruments (331)

ICT using sector:

Other electrical machinery and apparatus nec (31-313), Other instruments (33-331), Mechanical engineering (29), Wholesale trade and commission trade, except of motor vehicles and motorcycles (51), Financial intermediation, except insurance and pension funding (65), Insurance and pension funding, except compulsory social security (66), Activities auxiliary to financial intermediation (67), Renting of machinery and equipment (71), Research and development (73), Communications (64), Other business activities (50%(741-3+749))

non ICT using:

[30] ISIC codes, Rev.3, in the parentheses.

Agriculture (01), Forestry (02), Fishing (05), Mining and quarrying (10-14), Food, drink & tobacco (15-16), Textiles (17), Clothing (18), Leather and footwear (19), Wood & products of wood and cork (20), Pulp, paper & paper products (21), Printing & publishing (22), Mineral oil refining, coke & nuclear fuel (23), Chemicals (24), Rubber & plastics (25), Non-metallic mineral products (26), Basic metals (27), Fabricated metal products (28), Motor vehicles (34), Building and repairing of ships and boats (351), Aircraft and spacecraft (353), Railroad equipment and transport equipment nec (352+359), Furniture, miscellaneous manufacturing; recycling (36-37), Electricity, gas and water supply (40-41), Construction (45), Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel (50), Retail trade, except of motor vehicles and motorcycles;repair of personal and household goods (52), Hotels & catering (55), Inland transport (60), Water transport (61), Air transport (62), Supporting and auxiliary transport activities; activities of travel agencies (63), Real estate activities (70), Private households with employed persons (95), Other business activities (50%(741-3+749))

Table 1

	non ICT using			*ICT using*			*ICT producing*		
shares of commodities' use	MAN	SER	TOTAL	MAN	SER	TOTAL	MAN	SER	TOTAL
Intermediates	57.0	48.7	52.8	45.4	57.3	54.1	53.2	25.4	49.8
Consumption	25.4	48.8	37.2	8.6	8.6	30.7	10.0	20.6	11.3
Fixed Investment	17.7	2.5	10.0	46.0	46.0	15.1	36.8	54.0	38.9
Intermediates	69.2	49.9	58.7	84.0	59.7	63.8	84.1	55.2	81.5
Consumption	30.8	50.1	41.3	16.0	40.3	36.2	15.9	44.8	18.5

Source: BEA, Benchmark Input Output Table, 1997.

Table 2

		Average			*St Deviation*		
series		1979-01	1979-95	1995-01	1979-01	1979-95	1995-01
share of total hours worked	ICT producing	0.039	0.038	0.042	0.003	0.002	0.003
	non ICT using	0.708	0.712	0.698	0.009	0.006	0.005
	ICT using	0.253	0.251	0.261	0.007	0.006	0.003
share of value added	ICT producing	0.047	0.044	0.056	0.007	0.005	0.003
	non ICT using	0.697	0.712	0.659	0.031	0.020	0.014
	ICT using	0.255	0.244	0.285	0.024	0.015	0.012
value added at current prices (in logs)	ICT producing	12.109	11.866	12.752	0.564	0.436	0.176
	non ICT using	14.812	14.656	15.227	0.369	0.293	0.110
	ICT using	13.804	13.582	14.390	0.504	0.379	0.172
value added at constant prices (in logs)	ICT producing	11.467	11.002	12.612	0.978	0.680	0.379
	non ICT using	14.871	14.791	15.067	0.173	0.127	0.066
	ICT using	14.122	14.004	14.404	0.235	0.135	0.139
value added deflator growth rate	ICT producing	-0.071	-0.063	-0.095	0.031	0.031	0.022
	non ICT using	0.033	0.036	0.022	0.023	0.026	0.006
	ICT using	0.037	0.047	0.015	0.025	0.020	0.017
labour productivity growth rate	ICT producing	0.133	0.139	0.126	0.042	0.046	0.031
	non ICT using	0.017	0.017	0.018	0.015	0.015	0.014
	ICT using	0.017	0.007	0.037	0.022	0.016	0.020
expenditure shares	non ICT using	0.864	0.869	0.849	0.013	0.011	0.005
	ICT using	0.136	0.131	0.151	0.013	0.011	0.005

Source: All series from GGDC "60 Industry Database". Expenditure shares from the BEA NIPA Table on ""Personal Consumption Expenditures by Type of Expenditure", 2004.

9 Appendix D: Figures

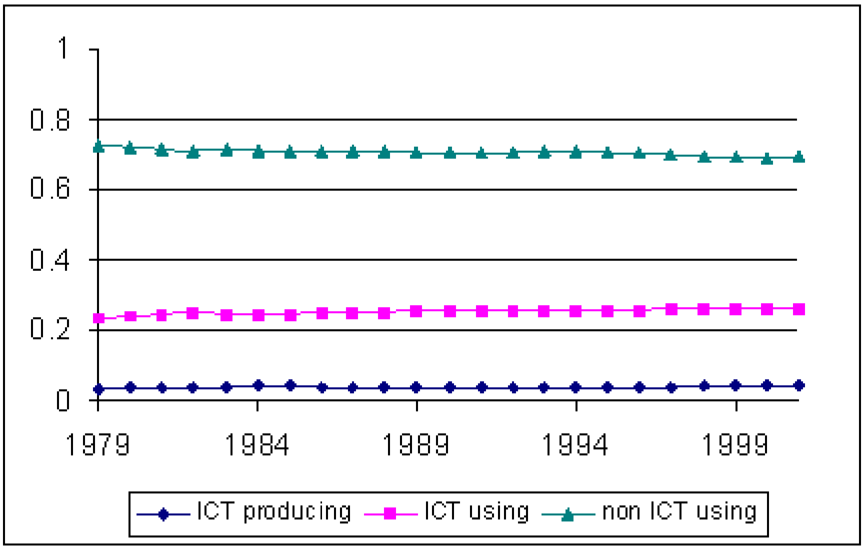


Figure 1: Shares out of total hours worked

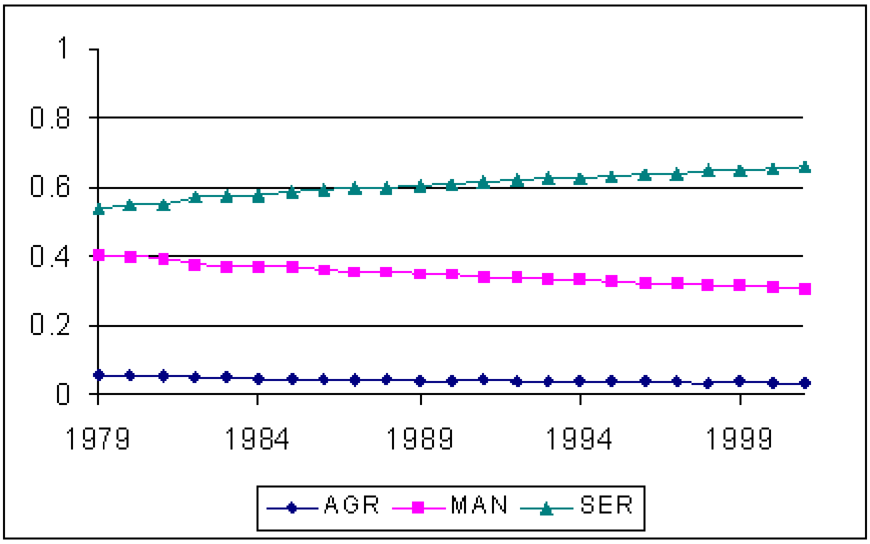


Figure 2: Shares out of total hours worked

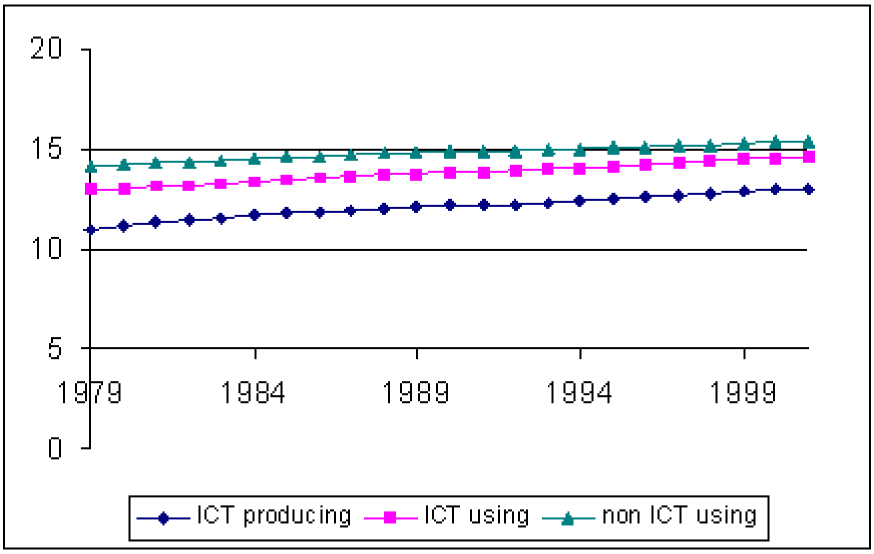


Figure 3a: Value added at current prices (units in logs, millions of USA$)

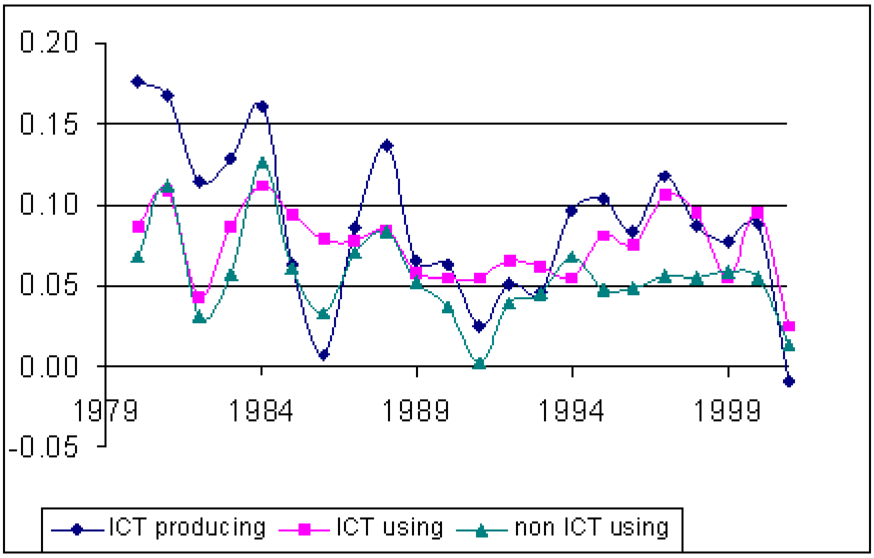


Figure 3b: Growth rate in value added (current prices)

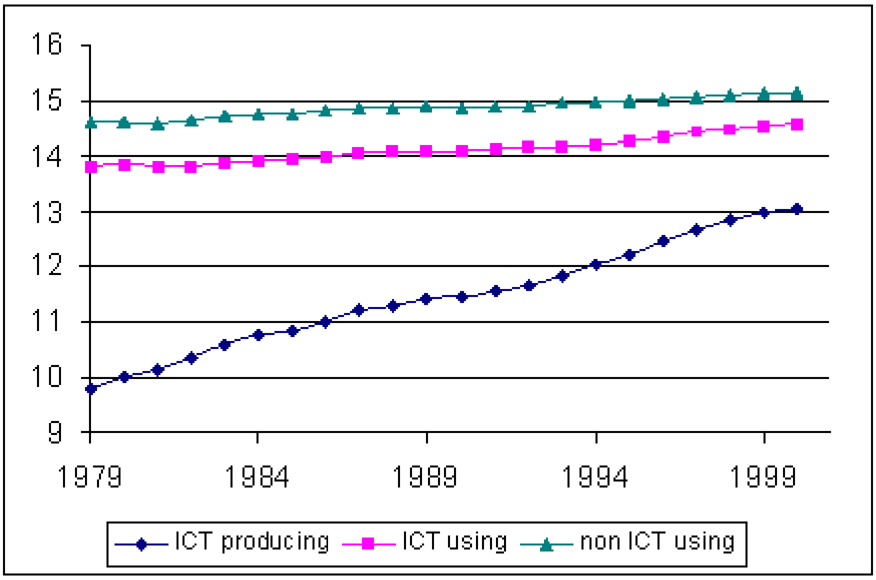


Figure 4a: Value added at constant prices (quantity index normalised to 100 for 1995)

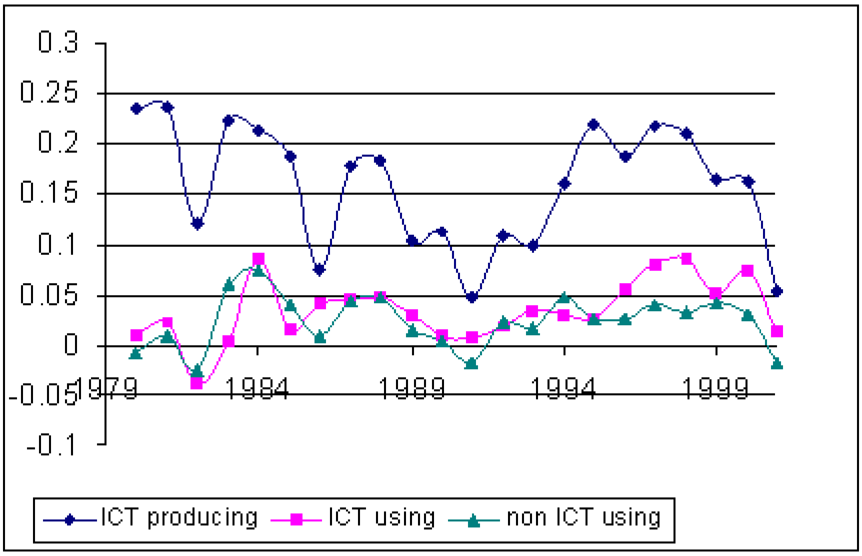


Figure 4b: Growth rate in value added (constant prices)

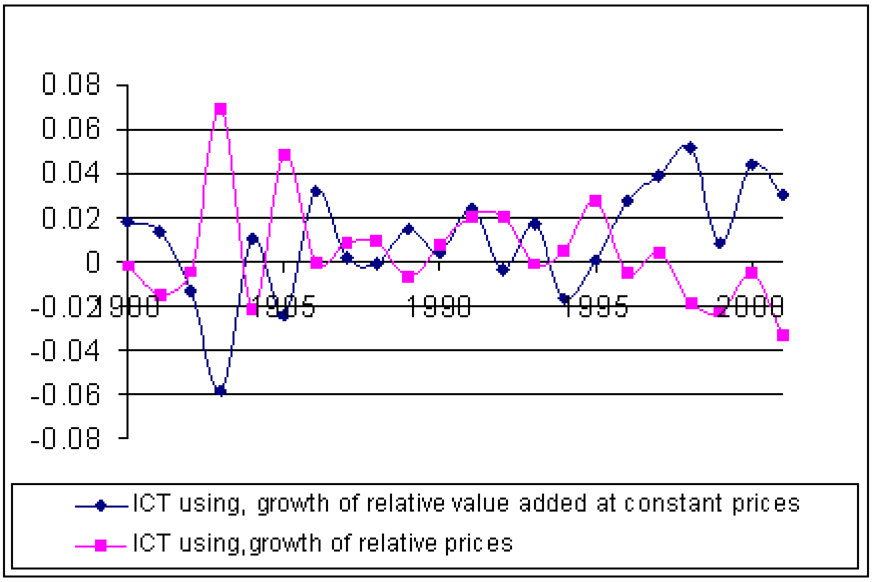


Figure 5: Quantity vs. price effect (numeraire: non ICT using good)