How Puzzling is the PPP Puzzle? An Alternative Half-Life Measure of Convergence to PPP

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Abstract

Evidence of lengthy half-lives for real exchange rates in the presence of high degree of exchange rate volatility has been considered as one of the most puzzling empirical regularities in international macroeconomics. This paper suggests that the measure of half-life used in the literature might be problematic and proposes an alternative measure which focuses on the cumulative effects of the shocks. Empirical analysis of bilateral $US real exchange rates employing the alternative half life measure produces results consistent with theory and indicates that the PPP puzzle is less pronounced than initially thought.

Keywords: PPP, Half-Life, Real Exchange Rates

JEL Codes: C12, C15, C23, F30, F31

1 Introduction

Evidence of real exchange rates lengthy half-lives in the presence of high degree of (nominal and real) exchange rate volatility has been considered as one of the most puzzling empirical regularities in international macroeconomics (see, Rogoff (1996), Obstfeld and Rogoff (2000), Taylor (2001), Taylor and Taylor (2004)). This conundrum has intrigued international economists working on real exchange rates since it seems to be at odds with the implications of sticky-price versions of both traditional and dynamic stochastic general equilibrium models of open economies, which typically imply that the half-life of a shock to the real exchange rate should be between one and two years.

The concept of half-life is not the only possible measure for assessing the speed of mean reversion or persistence in real exchange rates[1]. It has emerged, however, as the dominant measure in the literature on real exchange rates and Purchasing Power Parity (PPP). Nevertheless, more recent research questions various aspects of the half-life measure including

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[1]See, e.g., Andrews and Chen (1994) for a discussion of measures such as the spectrum at zero frequency, the sum of the autoregressive coefficients, and the largest autoregressive root.

uncertainty about point estimates (Rossi (2003)), the presence of bias associated with inappropriate aggregation across heterogeneous coefficients (Taylor (2001)), time aggregation of commodity prices (Imbs, Mumtaz, Ravn, and Rey (2004)), downward bias in estimation of dynamic lag coefficients (Choi, Mark, and Sul (2004)), and so on. In this paper we focus on the half-life measure itself and explore the possibility that the reason for the long half-lives giving rise to the PPP puzzle may be that the measure that is used in the literature is responsible for a bias towards long half-lives. In particular, the half life measures considered in the literature invariably focus on the instantaneous effects of the shock. This aspect of the measure, however, has a number of weaknesses (e.g., non-uniqueness), which we explore in detail. We consider an alternative measure of half-lives which appears to have superior properties to the one typically used in the international finance literature. This measure focuses on the cumulative effect of the shocks instead of the instantaneous effect.

When we employ our half-life measure to the real exchange rates of a set of industrialized countries the emerging half-lives are between one and two years. This is consistent with the predictions of sticky price models. Thus, the so-called PPP puzzle is less pronounced than initially thought, or even non-existent. The next section reviews briefly the literature on the PPP puzzle. Section 3 discusses the measures of half-lives and their weaknesses, and motivates the introduction of an alternative measure. We introduce the alternative definition of half-life and discuss its properties in section 4. In section 5 we apply this measure to US bilateral exchange rates. Section 6 considers the implication of non-linearities in the impulse responses and finally, section 7 concludes.

2 Motivation and review of the literature

The PPP puzzle consists in observing very high short run volatility of the real exchange rates on one hand and very slow speed of adjustment to PPP on the other. The high volatility in real exchange rates is usually expected to be explained in terms of monetary and financial shocks. The empirical measurements of the speed of adjustment to PPP, however, show that it is too slow to be compatible with such explanations. To examine the properties of real exchange rates and the persistence of their deviations from PPP, researchers employ impulse response analysis and use the concept of half-life to consider how long it takes for the impulse response to a unit shock to dissipate by half.[2]

Most of the recent accounts of half-lives in real exchange rates are associated with the empirical literature on PPP. Studies focusing on groups of industrial countries include Abuaf

[2]This definition although apparently informative is not very clear. It is usually taken to mean that the half life of the impulse response, ϕ_i, is the value of i for which $\phi_i = \phi_0/2$.

and Jorion (1991) who find that the annual half-life in ten industrial countries is 3.3 years on average. Manzur (1990) considers seven industrial countries and find that the half-lives or their real exchange rates are 5 years while Fung and Lo (1992) put the half-lives to 6.5 years for the six industrial countries they consider. Cheung and Lai (2000) put the half-lives to a range between 2 and 5 years for industrial countries but under 3 years for developing countries.[3] Higgins and Zakrajšek (1999) focus on OECD countries and WPI-based[4] real exchange rates and on a set of open economies, CPI-based[5] rates finding half-lives of 2.5 and 11.5 respectively. The influential study of Frankel and Rose (1996) who focus on very broad panels finds that the half-life is 4 years, on average, for 150 countries.

Another set of studies focuses on European real exchange rates. Parsley and Wei (1995) find that the half-lives for the EMS (European Monetary System) countries is 4.25 years. The findings of Papell (1997) suggest an annual half-life of 1.9 for the European Community and of 2.8 for the EMS. Higgins and Zakrajšek (1999) indicate that the same number is 5 for Europe, when CPIs are used and 3 when WPIs are used. Finally, a number of studies focuses on single real exchange rates. For example, Frankel (1990) finds that the half-life of the dollar-pound real exchange rate is 4.6 years. Lothian and Taylor (1996) find that the corresponding numbers are 2.8 for the franc-pound and 5.9 for the Dollar-pound real exchange rate.

The literature has tried to improve upon those results by employing a number of methodological advances. A number of authors have pointed out the bias emerging from inappropriate pooling of cross sectional units, that typically biases the half-life upwards; see, e.g., Choi, Mark, and Sul (2004). Acceptance of this type of bias has not been unanimous in the literature and while Imbs, Mumtaz, Ravn, and Rey (2004) attempt to correct for it, Chen and Engel (2004) find that it is not important. Imbs, Mumtaz, Ravn, and Rey (2004) focus on the heterogeneity of the speed of convergence due to the different composition of the tradables price indexes. Nevertheless their approach in correcting this problem consists essentially in considering a different, possibly more appropriate, dataset and not in using a different methodology. Another approach points out to the temporal aggregation bias and finds that it leads to higher half-lives because it biases upwards the autocorrelation coefficient of the model (Taylor (2001)). A number of other studies focus on the uncertainty surrounding the half-life estimates. For example, Rossi (2003) constructs confidence intervals

[3]This may be consistent with research trying to explain slow convergence in terms of bandwagon effects. Bandwagon effects can send a variable away from its equilibrium thereby prolonging the convergence. The result that speed of convergence in developing countries is faster may be supportive of this view as exchange rates of developing countries are less subject to speculative currency movenents.

[4]WPI: Wholesale Price Index

[5]CPI: Consumer Price Index

that are robust to high persistence in small sample sizes and finds that their lower bound is as low as four quarters. This finding, however, has little to offer towards resolving the PPP puzzle given that the upper bounds are infinity. Kleijn and Dijk (2001) also find low half-lives from a Bayesian unobserved components model for the real interest rate. Another promising line of research considers the possibility of non-linearities in the real exchange rate process. Taylor (2001) finds that when non-linearities are taken into account the half-lives are significantly shorter.

While all those studies contribute in different ways to a better understanding of the PPP-puzzle they leave intact a major methodological aspect of half-life measurement, namely the concept of half-life itself. The measure/method used for measuring the half-lives in the literature is not the only possible that one can use. Moreover, it may not be optimal since it suffers from a number of drawbacks. For example, if the impulse response follows an oscillating pattern instead of a monotonically decaying one, then the current measure cannot adequately capture the persistence of deviations from PPP. But even with monotonically decaying impulse response functions, meaningful comparisons are frequently difficult when the series display varying rates of decay and the impulse responses cross each other.

In this paper we discuss the weaknesses that emerge from the standard definition of half-life and propose the use of an alternative definition which solves some of the problems of the standard definition such as non-uniqueness, varying rates of decay, etc. The above problems become critical when the specific measure of half-lives is employed to assess mean reversion in real exchange rates. This implies that the presence of the PPP puzzle may be sensitive to the choice of the half-life measure used. The weaknesses of the standard measure emerge because of the focus on the instantaneous concept of half-life. We propose instead a measure that is based on the cumulative effects of the impulse responses.

When the PPP real exchange rate is used as a benchmark for setting exchange rate parities or for evaluating the degree of misalignments of actual from benchmark exchange rates,[6] using the currently popular concept of half life may not be a problematic practice. When the focus is on the implications of the degree of persistence in the real exchange rates, however, this concept may not be the most appropriate. The real exchange rate puzzle that Rogoff (1996) points to is related to this aspect of real exchange rates and half-life measurement. In particular, financial and monetary shocks should imply a lower degree of persistence while real shocks (such as productivity, technology and taste shocks) should imply a high degree of persistence.

[6]Other applications include the measurement of output for international comparisons.

Actually, a number of theoretical explanations of real exchange rate persistence (e.g., bandwagon effects, non-linearities) seems to be consistent with a view of the half life based on the cumulative effects of the shocks. For example, non-linearities in the real exchange behavior may exist, emerging from transaction costs. One approach in reconciling theory with empirical facts (or explaining the PPP puzzle) is to stress the possibility of nonlinear real exchange rate behavior due to transaction costs. The presence of transaction costs makes adjustment costly and arbitrage takes place more difficultly.

3 Weaknesses of half-life measures

Half life measures have been discussed in the literature for the best part of the last 20 years.[7] In the majority of papers dealing with half lives, we see that the measure is inextricably linked to the $AR(1)$ model of the form

$$y_t = \rho y_{t-1} + \epsilon_t \tag{1}$$

where y_t, $t = 1, \ldots, T$ is the process under investigation. Then, the half life is defined as

$$h = \frac{ln(1/2)}{ln(\hat{\rho})} \tag{2}$$

where $\hat{\rho}$ denotes the estimate of ρ. We will refer to this as Definition 1. In fact, this coincides with the more formal definition of the half life which is

$$h = i, \text{for which } \phi_i = \phi_0/2 \tag{3}$$

where

$$\phi_i = E(y_{t+i}|\epsilon_t = 1) - E(y_{t+i}|\epsilon_t = 0) \tag{4}$$

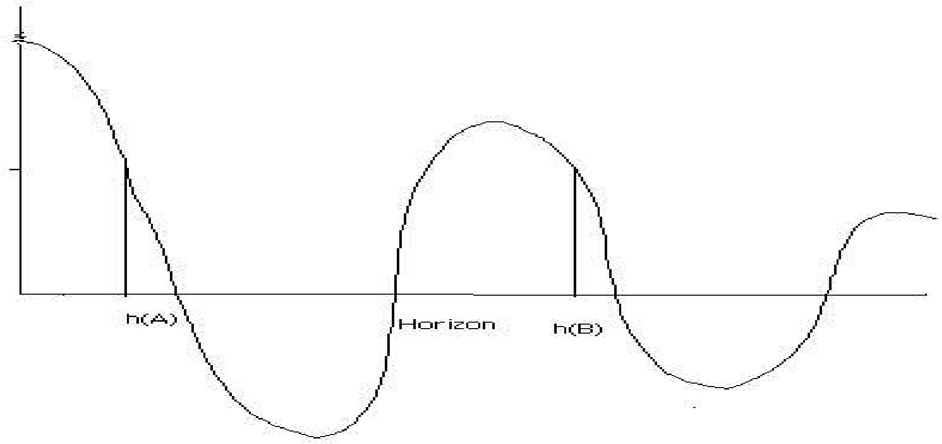
which we refer to as Definition 2 (see Mark (2001) for more details). Since for the $AR(1)$ model, $\phi_i = \rho^i$, Definition 1 follows. In what follows we will allow for non-integer i in ϕ_i.

A first objection with Definition 1 is that it does not coincide with Definition 2 for other dynamic models such as $AR(p)$, $p > 1$ or $ARMA(p, q)$ models. While obtaining the half life according to Definition 2 appears to be an easy task conceptually, the mechanics of doing so are quite complicated. As a result a number of alternative definitions based on simplifications of Definition 2 have appeared in the literature. Perhaps the most interesting one is that by Rossi (2003) which is given by

$$h = \frac{ln(1/2)b(1)}{ln(\hat{\rho})} \tag{5}$$

[7] For a recent summary see also Choi, Mark, and Sul (2004)

Figure 1:

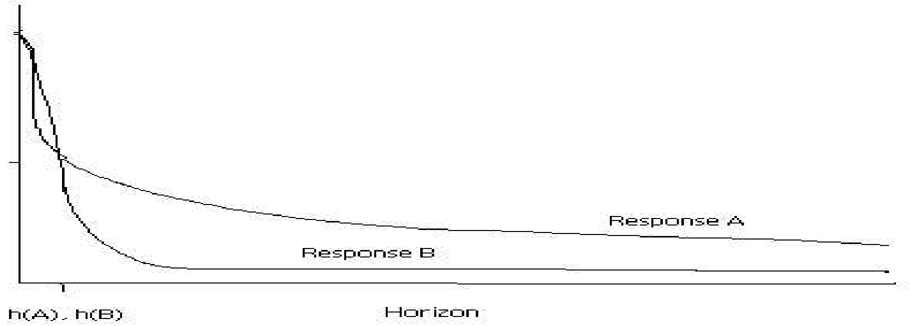


where $b(1)$ is the sum of the estimated AR coefficients of an $AR(p)$ model fitted onto the residuals of (1). This definition, referred to as Definition 1A arises out of assuming that the process generating the data is near unit root, i.e. that $\rho = 1 - c/T$ for some constant c.

Moving on to Definition 2 we have a common complaint in the literature. This complaint is that if the impulse response of a stationary series (or indeed a non-stationary series for which shocks are temporary such as, e.g., $ARFIMA(p, d, q)$ processes for $1/2 < d < 1$) is not monotonically declining then this definition does not necessarily give a unique half life as there may be multiple i for which $\phi_i = 1/2\phi_0$. In this case researchers usually resort to defining half life as either the smallest i for which $\phi_i = 1/2\phi_0$ (see, e.g. Rossi (2003)) or alternatively the largest such i (see, e.g. Ng (2003)). This is clearly problematic. In Figure 1 we illustrate the problem pictorially using a non monotonically declining impulse response. With reference to that Figure, why would $h(A)$ be preferable to $h(B)$ as a half life measure or vice versa?

Perhaps more fundamentally, this definition is suspect on more basic grounds. To appreciate the point consider the two impulse responses in Figure 2. They both have the same half life. Few, however, would argue that the same proportion of the shock has been dissipated for the two impulse responses. The problem seems to be that Definition 2 consid-

Figure 2:



ers only points in the impulse response in isolation and not the whole of the impulse response.

A further problem arises if we consider the case of a non-stationary process. Assume that for a non-stationary process the effect of a shock (impulse response) settles for long horizons at a non zero value which is less than half the initial effect of the shock. Perversely, this means that the half life measure according to definition 2 will be finite. Clearly, a permanent shock cannot have a finite half life. Again the failure of intuition and formal definition is due to the consideration of points in the impulse response in isolation. Then, one emerging task is to come up with an alternative definition that addresses all the above issues. We provide such a definition in the following section.

4 An alternative definition of half life

Before suggesting a possible solution to the questions raised in the previous section we should point out that no half life measure will be able to convey the informational content of an impulse response since it is only a summary statistic. Hence, there will always be cases where any half life measure will not do justice to the underlying impulse response. Nevertheless, the half-life measure has the advantage that it is readily interpreted in terms of time units and the debate on real exchange rate convergence to PPP values has been casted in terms of this measure.

The concept of half life originates from experimental sciences where it arises in a multitude of contexts. Perhaps the most widely familiar definition to laymen is taken from nuclear physics. There, it is defined as the amount of time it takes for half of the atoms in a sample of radioactive isotope to decay. Note the discrepancy with Definition 2 which taken to a physics context would define half life as the point in time at which half the amount of atoms instantaneously decay compared to the amount of atoms that instantaneously decay at the start of the decay process.

An intuitive analogy to our context then may be the following: Define the impulse response as a function of i, which we denote as $\phi(i)$ to provide a distinction in focus from standard impulse responses. Then, the half life is the point h^* at which

$$\int_0^{h^*} |\phi(i)| di = \int_{h^*}^{\infty} |\phi(i)| di \tag{6}$$

In other words, h^* is the point in time at which half the absolute cumulative effect of the shock has dissipated. We refer to this definition as Definition 3. The use of $|\phi(i)|$ rather than $\phi(i)$ solves the problem arising out of the possibility of negative as well as positive impulse responses. The use of the integral firstly guarantees uniqueness of the measure and secondly accords with the intuition behind shock dissipation. How does Definition 3 compare with, say, Definition 1? Simple algebra indicates that if the model is $AR(1)$ then Definitions 1 and 3 coincide. That is, any additional insights that the new half-life measure may provide do not come at the expense of insights that would be provided by the standard measure.

An immediate concern relates to the calculation of half life according to Definition 3. In particular, we are concerned with calculating h^* given the estimates of the coefficients of an $AR(p)$. Denote these coefficients by $\rho_1, \ldots, \rho_p$ and define the matrix coefficient of the companion form of the AR(p) model by

$$A = \begin{pmatrix} \rho_1 & \rho_2 & \ldots & \rho_p \\ 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}$$

Then, denote the ordered eigenvalues of A by $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_1$. Hamilton (1994) shows that if the eigenvalues are distinct then

$$\phi(i) = \sum_{j=1}^{p} c_j \lambda_j^i \tag{7}$$

where

$$c_j = \frac{\lambda_j^{p-1}}{\prod_{k=1,k\neq i}^{p}(\lambda_j - \lambda_k)} \tag{8}$$

Then, simple algebra implies that h^* solves the equation.

$$2\sum_{j=1}^{p}\frac{c_j\lambda_j^{h^*}}{ln(\lambda_j)} = \sum_{j=1}^{p}\frac{c_j}{ln(\lambda_j)} \tag{9}$$

Solving for h^* seems a complicated task given the form of (9) so we resort to numerical methods. In particular we use a Newton-Raphson type of algorithm to solve for h^*. Numerical methods can, of course, be used to obtain half life estimates, for any model, as long as an estimate of $\phi(i)$ exists.

5 Reassessing the PPP puzzle: The case of the US Real Exchange Rates

The bulk of the literature on the PPP puzzle has focused on bilateral US real exchange rates and we focus on them to consider the implications of using the proposed alternative measures. We investigate the half life of quarterly US real exchange rates using both the proposed and available half life definitions. We consider the exchange rates of France, Germany, Italy, Japan, Spain and the UK. We construct the bilateral real exchange rate q of the i-th currency against the US Dollar at time t as $q_{i,t} = s_{i,t} + p_{j,t} - p_{i,t}$, where $s_{i,t}$ is the corresponding nominal exchange rate (*i-th* currency units per one US dollar), $p_{j,t}$ the price level in the United States, $p_{i,t}$ the price level of the *i-th* country, and variables are in logs. That is, a rise in $q_{i,t}$ implies a real appreciation of the US Dollar against the *i-th* currency. Data are quarterly, spanning from 1957Q1 to 1998Q4. We use the average quarterly nominal exchange rates and the price levels are consumer price indices (not seasonally adjusted). All data are from the International Monetary Fund's *International Financial Statistics* in CD-ROM.

Any meaningful discussion about convergence to PPP requires to ensure first that the real exchange rates do not contain a unit root. We use two procedures to consider whether the series in question are stationary. The first is the Chang (2002) univariate unit root test based on nonlinear IV estimation and considers the case of a constant, a trend and 4 lag augmentations. The second is a procedure based on panel unit root tests discussed in Chortareas and Kapetanios (2004). This procedure enables unit root inference for the individual cross sectional units in a panel. The univariate unit root test shows that the US real exchange rates with respect to France, Japan, the UK, and Italy are stationary at the 5% significance level. This evidence is reinforced by the findings of the above mentioned panel data methodology, which suggests stationarity for all real exchange rates considered. So, we include in our analysis the German and Spanish real exchange rates as well.[8] We note that in half-life analysis, results from unit root tests are usually discounted anyway as many standard unit root tests have low power. We estimate an AR(1) model to construct a half life measure according to Definition 1 and we refer to it as 'Traditional Half-Life' (THL) with AR(1). We also estimate an AR(p) model for each series and use that to get half life measures according to definitions 1A and 3 to which we refer respectively as THL-AR(p) and 'Alternative Half Life' measure (AHL) . Table 1 presents the chosen lags (using Akaike's information criterion) and the estimated half lives, where the AHL measure has been obtained numerically. It is clear that AHL provides plenty of evidence that the half life puzzle identified repeatedly in the literature is due to an inappropriate definition of half life. THL-AR(p) incorporates an assumption that the series is highly persistent (near unit root). Hence, it is not surprising that it produces the highest half life measure of the three.

Table 1

Country	Lag	THL-AR(1)	THL-AR(p)	AHL
France	2	3.282	8.689	1.603
Germany	4	4.649	11.399	1.786
Japan	2	4.167	11.172	1.620
UK	7	2.103	5.981	1.162
Italy	4	2.951	7.208	1.146
Spain	3	3.882	10.217	1.726

To further analyze these real exchange rate half lives and confirm the intuitive appeal of the AHL definition we plot the impulse responses implied by the AR(p) and AR(1) models in Figure 3.

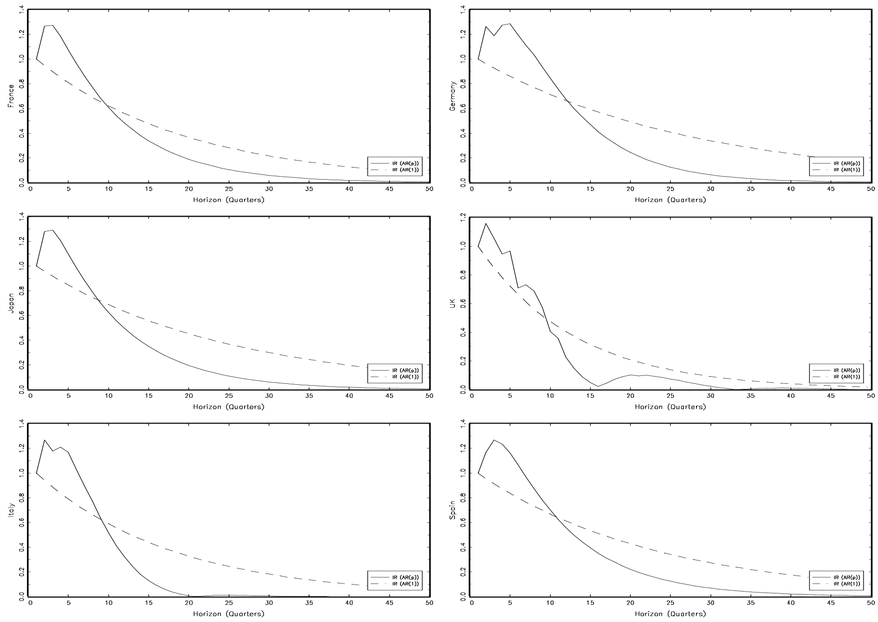
The impulse responses cross each other at around 0.55, implying that a half life measure according to Definition 2 would be close to that provided by THL-AR(1). In fact, for the case of the UK the two impulse responses cross at a point uncannily close to 0.5. For that case the AR(p) and AR(1) model would give equal half life measures according to Definition 2. In general, however, few people would claim that the shock dissipates equally fast for any pair of impulse responses in Figure 3. Hence, the use of AHL looks increasingly justified, both on theoretical and empirical grounds.

6 Relaxing the linearity assumption when constructing impulse responses

Recent work in the macroeconometric literature has been moving away from the paradigm of stationary linear processes, usually parametrized using the Box-Jenkins framework of

[8] Detailed results for the stationarity tests are available upon request.

Figure 3: Impulse Responses from AR models



ARMA models Our previous analysis was contingent on such a framework. Such work includes unit root nonstationary processes and nonlinear processes. Focusing on covariance stationary processes, the increased focus on nonlinearity has been productive in a number of ways. Firstly, nonlinear models have been shown to provide a superior fit to a number of macroeconomic series. Secondly, impulse response analysis has illuminated a number of issues such as asymmetry for economic phenomena such as the business cycle.

Work on impulse responses for nonlinear processes has been carried out by Koop, Pesaran, and Potter (1996) and Potter (2000) . That body of work is firmly set in a parametric context even though the underlying ideas can easily extend to nonparametric contexts. Therein, lies a possibly serious issue concerning the validity of impulse response analysis. Once the restrictive assumption of linearity has been relaxed, the choice of the nonlinear model becomes paramount. It is clear that misspecification of the model can lead to equally if not greater inferential problems compared to restricting the analysis to linear models.

Unfortunately, model selection in a nonlinear world is much more difficult compared to the same task in the ARMA framework. The main difficulty lies in actually defining the space of parametric models to consider. The problem appears intractable given the infinity of parametric nonlinear models that can be used to fit a time series. A possible way out is provided by nonparametric analysis. In particular, in this section we will argue that obtaining an impulse response from a nonparametric analysis may provide useful information on such issues as the persistence of series. Of course a nonparametric analysis has serious costs. Firstly, it is clearly inefficient compared to the true parametric model. This is a well known cost which we will not comment upon further. Secondly, the nonparametric analysis we suggest will be based on the Wold representation of a covariance stationary stochastic process. As Potter (2000) argued using such a representation may obscure interesting local features such as asymmetry. Nevertheless, as the Wold representation is valid even for nonlinear processes, the obtained impulse response will be informative for global features such as persistence.

Our suggestion in more detail, is as follows. Let us extend the specification of the model by assuming that

$$y_t = f(y_{t-1}, \ldots, y_{t-p}; v_t; \theta) \tag{10}$$

where v_t is an i.i.d. zero mean process with finite variance and θ is a vector of parameters. Nevertheless, the form of f is not known and is difficult to retrieve. Additionally, since we do not assume additivity of the error term, it is not clear how one can obtain impulse responses using nonparametric regression analysis. Nevertheless, as long as y_t is covariance stationary, the following Wold representation exists

$$y_t = \sum_{i=1}^{\infty} c_i u_{t-i} \tag{11}$$

where u_t is white noise. Note that $u_t \neq v_t$ is not i.i.d. As Potter (2000) states, impulse response analysis using this representation may obscure local features such as asymmetry to shocks. Nevertheless, global features such as the persistence of the process will still be correctly represented. The only genuine nonparametric alternative to the Wold representation is the use of a Volterra expansion of the form

$$y_t = \sum_{i=0}^{\infty} c_i v_{t-i} + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{ij} v_{t-i} v_{t-j} + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} c_{ijk} v_{t-i} v_{t-j} v_{t-k} + \ldots$$

This is clearly a hopelessly overparametrized representation of little practical use. We suggest estimation of the Wold representation and use of the estimated c_i as impulse responses. To carry out estimation we use the algorithm suggested in the proof of Theorem 2.10.1 of Fuller (1986) which proves the existence of the Wold representation. This algorithm is equivalent to estimation of the infinite AR representation of y_t and use of the residual of that as an estimate of u_t. More specifically, the infinite AR representation given by

$$y_t = \sum_{i=1}^{\infty} d_i y_{t-i} + u_t$$

is guaranteed to exist as long as $\sum_i i^s c_i < \infty$ for some $s > 1$ by, e.g., Hannan and Kavalieris (1986) . Then, we estimate

$$y_t = \sum_{i=1}^{p_T} d_i y_{t-i} + u_t$$

Then, $\hat{u}_t$ and its lags are used as a regressor in

$$y_t = \sum_{i=1}^{p_T} c_i \hat{u}_{t-i} + z_t$$

to get estimates of c_i, denoted $\hat{c}_i$. As long as $p_t \to \infty$ then $\hat{c}_i$ is consistent for c_i. Once $\hat{c}_i$ are obtained a nonparametric estimate of the half life can be easily obtained too using (6).[9]

To evaluate the new method we have carried out a small Monte Carlo experiment. We consider two nonlinear models: A threshold autoregressive (TAR) model and an exponential smooth transition autoregressive (ESTAR) model. The first is given by

$$y_t = \gamma_1 I(|y_{t-1}| < r) y_{t-1} + \gamma_2 I(|y_{t-1}| \geq r) y_{t-1} + \epsilon_t$$

and the second by

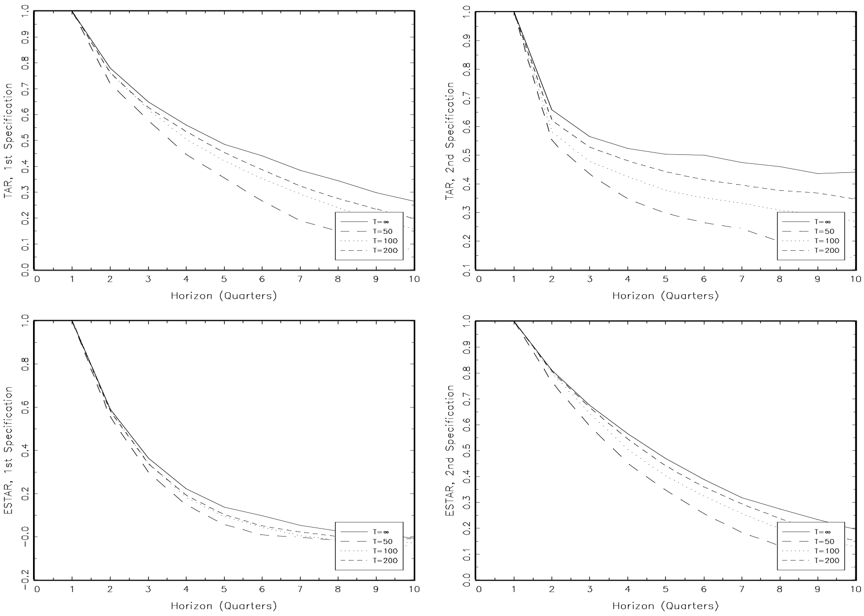
$$y_t = \delta_1 y_{t-1} + \delta_2 (1 - e^{-y_{t-1}^2}) y_{t-1} + \epsilon_t$$

We also consider two specifications for each. These are $(\gamma_1, \gamma_2, r) \in \{(1, 0.6, 3), (1.2, 0.7, 4)\}$ for the TAR model and $(\delta_1, \delta_2) \in \{(0.95, -0.4), (1.4, -0.6)\}$ for the ESTAR model. All specifications are highly persistent. Figure 4 reports the true ($T = \infty$) and the average estimated impulse responses for a horizon of up to 10 periods. The true response is obtained by using a sample of 10000 observations. We see that for persistent nonlinear processes, the estimates $\hat{c}_i$ are downward biased mirroring the downward bias of AR coefficient estimates for persistent AR processes. In order to avoid this problem we introduce a bootstrap procedure to estimate the bias of the $\hat{c}_i$. More specifically, we have considered the moving block bootstrap (see, e.g., Davison and Hinkley (1997)) to estimate the bias. Result on the average estimated impulse responses using the bootstrap are reported in Figure 5. We thus see that the bootstrap helps in that respect removing the bias even for samples of 50 observations.

We have carried out the above computations for the series we considered in the previous section up to horizon 25 setting $p_T = 25$. Longer lags are inadvisable given the size of the sample. In any case experimentation with longer lags did not lead to substantially different results. The block bootstrap is implemented with block size 30 and 199 bootstrap replications. Results on the nonparametric impulse responses are presented in Figure 6. Estimates of the nonparametric half lives are given in Table 2 where column AHL-NL corresponds to

[9] Note that to obtain values for $\phi(i)$ for non-integrer i, in (6), we simply use linear interpolation.

Figure 4: Monte Carlo results



the nonlinear definition of the alternative half-life measure and AHL-NLBC to the nonlinear definition of the alternative half-life measure corrected for biases using the bootstrap. It is again clear that both of these measures give significantly shorter half-life estimates than the standard definition.

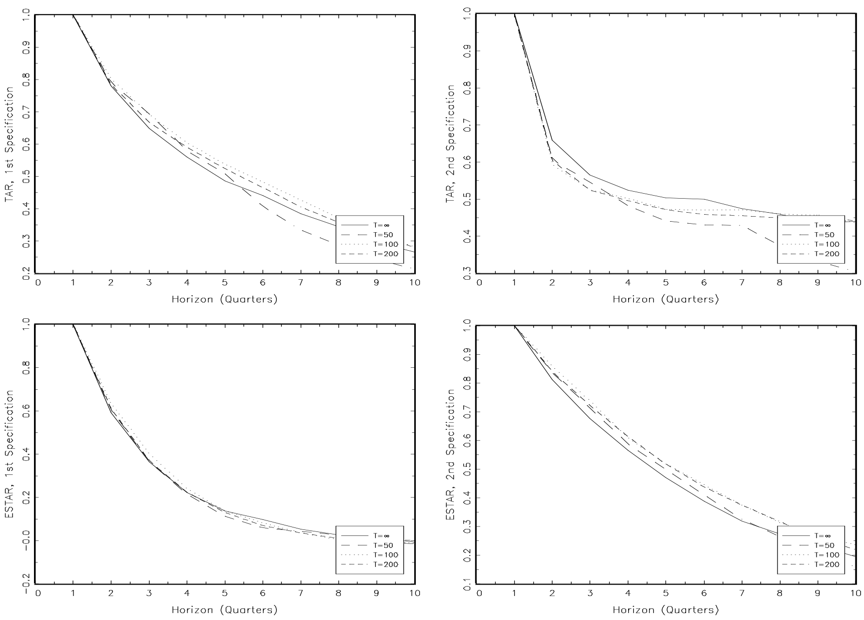
Table 2

Country	AHL-NL	AHL-NLBC
France	1.617	1.865
Germany	2.173	2.132
Japan	2.909	2.635
UK	1.742	1.929
Italy	1.587	1.871
Spain	2.394	2.307

7 Conclusion

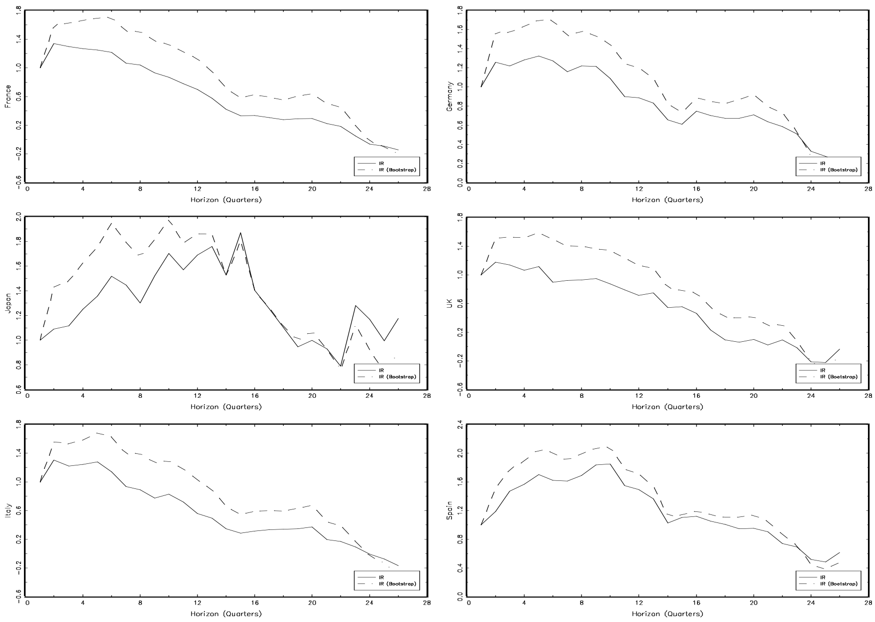
We revisit the PPP puzzle focusing on the half-life measure of the speed of real exchange rate convergence to PPP. We find that the choice of methodology for measuring half lives is not innocuous to the results that one obtains, and this has in turn implications for the degree to which the process of real exchange rates convergence to PPP can be considered puzzling.

Figure 5: Monte Carlo results for the bootstrap



The incompatibility of the observed lengthy half-lives with high degrees of exchange rate volatility is considered one of the major puzzles in international macroeconomics. While the consensus in the literature has been that the half-lives are between 3 and 5 years, more recent analyzes that adopt newer methodologies find evidence of considerably shorter half-lives. Particular emphasis has been placed on the uncertainty surrounding the half-life estimates (e.g., Rossi (2003)), and on the role of non-linearities (e.g., Taylor (2001)). Notwithstanding those developments, however, the literature still relies on an instantaneous concept of half-life. We suggest that this concept suffers from a number of drawbacks and we propose the use of an alternative measure that displays superior properties. This measure is focusing on the cumulative effect of the impulse responses. The resulting half-lives for a number of major currencies against the US dollar are well below two years and therefore are consistent with the predictions of sticky price models of exchange rate determination. Moreover, we provide half lives measures correcting for possible biases that emerge from nonlinearities. Our findings are robust to the possibility that the real exchange rate follows a non-linear process. Of course, the definition suggested in this paper follows straightforwardly from the one used in experimental sciences. To our knowledge, however, this is the first attempt to use such a measure in the PPP-puzzle debate. Its superior properties allow for a new perspective -and possibly a solution- to the PPP puzzle. In particular, our results indicate that the PPP puzzle may not be so puzzling if a more appropriate half-life measure is used.

Figure 6: Nonparametric Impulse Responses



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