Optimal Fiscal Policy Rules in a Monetary Union

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Abstract

This paper investigates the importance of fiscal policy in providing macroeconomic stabilisation in a monetary union. We use a microfounded New Keynesian model of a monetary union which incorporates persistence in inflation and non-Ricardian consumers, and derive optimal simple rules for fiscal authorities. We find that fiscal policy can play an important role in reacting to inflation and , but that not much is lost if national fiscal policy is restricted to react only to national differences in inflation and output.

Key Words: Optimal monetary and fiscal policies, Monetary union, Simple rules
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1 Introduction

In this paper we examine the potential that national fiscal policy has to help stabilise individual economies within a monetary union. While the vulnerability of monetary unions to asymmetric shocks are well known, there has been surprising little analysis of the extent to which fiscal policy can overcome these problems within the framework of the new international macroeconomics (see Lane (2001) for a survey). This is despite the fact that policy makers in potential members of the European Monetary Union have actively discussed the possibility of using fiscal policy in this way (Treasury (2003), Swedish Committee (2002)).

One advantage of using a model with clear microfoundations is that we can directly compute welfare, using a measure explicitly derived from agents utility. In addition, we can directly address the issue of solvency, and investigate the extent to which the requirement that fiscal policy ensures debt stability may or may not conflict with using fiscal policy for macroeconomic stabilisation. While our analysis does not deal directly with some of the important political economy issues involved in using fiscal policy as a countercyclical tool (see e.g. Calmfors (2003)), it should help inform that debate. In particular, one of the issues we investigate is whether there is a significant welfare cost to restricting fiscal policy to respond to differences between national and union wide inflation and output.

Our analytical framework is close to that in a recent paper by Beetsma and Jensen (2004), whose model is in turn based on a model developed in Benigno and Benigno (2000). They also look at the role of fiscal policy in a microfounded two country model of monetary union. However our analysis is more general in three important respects. First, while their representative consumers are identical across countries (and therefore consume an identical basket), we allow for some home bias in consumption, along lines that are familiar from Gali and Monacelli (2002), for example[1]. Second, while both papers embody nominal inertia in the form of Calvo contracts, we also allow for some additional inflation inertia, using a set up outlined in Steinsson (2003). This not only makes our model more realistic [2], but it also gives policy a greater potential role in influencing the dynamic response to shocks. Inflation inertia introduces a key potential instability into the economies of the union, and so a stabilising fiscal policy may become vital. Third, while consumers in Beetsma and Jensen (2004) are infinitely lived and Ricardian, we allow for non-Ricardian behaviour by adopting the constant probability of death model due to Blanchard (1985). (Blanchard/Yaari consumers are also modelled in Leith and Wren-Lewis (2001) who examine issues of stability and monetary/fiscal policy interaction in a monetary union, and Smets and Wouters (2002)). Allowing non-Ricardian behaviour is important when looking at the interrelationships between debt management and macroeconomic stabilisation. [3]

In the same manner as Beetsma and Jensen (2004), the monetary union is not open to the rest of the world, and the two member countries are big with respect to each

[1] See also Duarte and Wolman (2002).

[2] See Mankiw (2001), Benigno and Lopez-Salido (2002) among many others.

[3] In some respects our set up is more restrictive than Beetsma and Jensen (2004): for example, we assume our two economies are of equal size while they do not.

other.With these assumptions, our approach is complementary to the one in Gali and Monacelli (2004), who consider many small countries in a monetary union. In their paper each country is small, and is subject to idiosyncratic shocks. We focus on big countries, subject to asymmetric shocks. We assume that although fiscal decisions are taken independently, each fiscal authority can react to events in the other country, as well as to its own. We follow Beetsma and Jensen (2004), in that our monetary union is not open to the rest of the world.

One of the difficulties of working with a richer model is that the benevolent policy makers loss function can depart substantially from the objective function that monetary policy makers are generally assumed to follow. In this paper, therefore, we consider an alternative to our main case where the monetary authorities optimise a conventional welfare function, which is a sum of squares of deviation of inflation and output from their target values. However, as our analysis of fiscal policy is designed to be normative rather than realistic, we always optimise fiscal rules with respect to the social loss derived from individual agents utility. There is a standard problem about how to avoid linear terms in such a measure of social loss. There are three common approaches to resolving this problem. Schmitt-Grohe and Uribe (2003) following Sims (2000), abandon the linear-quadratic framework and instead work with second-order approximations to the model equations. As an alternative, Benigno and Woodford (2003), Benigno and Woodford (2004), and Sutherland (2002) assume specific policy rules, which *of themselves* remove the linear terms[4]. However, for our purposes it is more convenient to take a third approach (as in Rotemberg and Woodford (1997) and Benigno and Benigno (2000) for example), where we assume the existence of an employment subsidy, financed by lump-sum taxation, precisely of the kind necessary to remove linear terms in the measure of social loss.

2 The Model

2.1 The Setup

Our monetary union consists of two economies, labelled a and b. Each of these is inhabited by a large number of individuals and firms. Each representative individual specialises in the production of one differentiated good, denoted by z, and spends $h(z)$ of effort on its production. He consumes a consumption basket C, and also derives utility from per capita government consumption G. Private and public consumption are not perfect substitutes.

In each of the two economies the consumption basket consists of two composite goods, the domestic composite good (produced in the home country, subscripts Ha, Hb), the foreign composite good from the other open economy (produced in the foreign country, subscripts Hb, Ha). Each composite good consists of a continuum of produced goods $z \in [0, 1]$. We also assume that countries a and b are identical in all their parameters. In order not to repeat symmetric equations, we will use the index k for a single country in the union, $k \in \{a, b\}$, and use index $\bar{k}$ to denote the other country, i.e. if $k = a$ then $\bar{k} = b$, if $k = b$ then $\bar{k} = a$.

[4]Sutherland (2002) imposes a form of policy which is too specific for our purposes, while Benigno and Woodford (2003), Benigno and Woodford (2004) impose a 'timeless perspective' on policy.

Preferences of individuals are assumed to be :

$$\max_{\{C_s,h_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \left[\frac{\beta}{1+p}\right]^{s-t} [u(C_s,\xi_s)+f(G_s,\xi_s)-v(h_s(z),\xi_s)] \tag{1}$$

where we allow for taste/technology shocks ξ. Domestically produced goods may be consumed either at home or abroad:

$$y_{kt}(z) = c_{Hk,t}(z) + c^{\bar{k}}_{Hk,t}(z) + g_{Hk}(z) \tag{2}$$

where the superscript denotes the final destination of consumption goods, whose price is denominated in a different currency than that of country k. $g_k(z)$ is government consumption. Superscripts denote currency denomination, where necessary. We assume that the government in each country consumes the domestically produced good only, so $g_{Hk} = g_k$.

All goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by $\epsilon_t > 1$ (which is a stochastic elasticity[5] with mean ϵ):

$$C_{Hkt} = \left[\int_0^1 c_{Hkt}^{\frac{\epsilon_t-1}{\epsilon_t}}(z)dz\right]^{\frac{\epsilon_t}{\epsilon_t-1}} \tag{3}$$

Every household consumes both domestic and foreign goods with the elasticity of substitution between them given by $\eta > 0$. Therefore, the consumption basket in country k is

$$C_k = \left[(\alpha_d)^{\frac{1}{\eta}} C_{Hk}^{\frac{\eta-1}{\eta}} + (\alpha_n)^{\frac{1}{\eta}} C_{H\bar{k}}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

where the index t is suppressed for notational convenience, α_d is the share of consumption of domestic goods, α_n is the share of consumption of goods imported from the neighbour country (the other open economy), $k \in \{a,b\}$.

2.2 Demand: Optimal Consumption Decisions

An individual chooses optimal consumption and work effort to maximise the criterion (1) subject to the intertemporal budget constraint:

$$\sum_{s=t}^{\infty} \mathcal{E}_t(R_{t,s}P_{ks}C_{ks}) \leq \mathcal{B}_{kt} + \sum_{s=t}^{\infty} \mathcal{E}_t(R_{t,s}(1-\tau)w_{ks}(z)h_{ks}(z))$$

where $P_{kt}C_{kt} = \int_0^1 (p_{Hk}(z)c_{Hk}(z) + p_{H\bar{k}}(z)c_{H\bar{k}}(z))dz$, $\mathcal{E}_t(R_{t,s}) = \prod_{k=t}^{s-1} \frac{1}{(1+i_k)(1+p)}$, i_t is short-term interest rate and $\mathcal{B}_{kt}$ are nominal bond holdings, $k \in \{a,b\}$. Here w is the wage rate, and τ a constant tax rate on labour income. In equilibrium we assume $\pi = 0$.

[5] We make this parameter stochastic to allow us to generate shocks to the mark-up of firms.

The household optimisation problem is standard (see Appendix and Smets and Wouters (2002) for one recent example) and leads to the following two dynamic relationships for *aggregate* variables:

$$C_{kt} = \frac{1}{[\beta(1+r_t)]^{\sigma}} \left[C_{kt+1} + \frac{p}{\Phi_{kt+1}} \frac{\mathcal{A}_{kt+1}}{P_{kt+1}} \right] \tag{5}$$

$$\Phi_{kt} = 1 + \Phi_{kt+1} \frac{[\beta(1+r_t)]^{\sigma}}{(1+p)(1+r_t)} \frac{\xi_{kt}}{\xi_{kt+1}} \tag{6}$$

where $1/\Phi_{kt}$ is average propensity to consume out of total resources (nominal financial wealth and human wealth, see Appendix X), $1+r_t = (1+i_t)/(1+\pi_{t+1})$ is real interest rate, parameter σ is defined as: $\sigma = -u_C(C,1)/u_{CC}(C,1)C$.

As aggregate assets accumulate as:

$$\mathcal{A}_{at+1} = (1+i_t)(\mathcal{A}_{at} + (1-\tau)P_{Hat}Y_{at} - P_{at}C_{at}) \tag{7}$$

We denote $A_t = \mathcal{A}_t/P_{t-1}$, and linearise equations (5), (6) and (7) around the steady state (for each variable X_t with steady state value X, we use the notation $\hat{X}_t = \ln(X_t/X)$).

Equation (5), leads to the following Euler equation (intertemporal IS curve):

$$\hat{C}_{kt} = [\beta(1+i)]^{-\sigma}(\hat{C}_{kt+1} + \frac{pB}{\Phi\theta}(\hat{A}_{kt+1} - \hat{\pi}_{kt+1} - \hat{\Phi}_{kt+1})) - \sigma(\hat{\imath}_t - \hat{\pi}_{kt+1}) + \hat{\xi}_{kt} - \hat{\xi}_{kt+1} \tag{8}$$

where the average propensity to consume evolves as:

$$\frac{(1+p)(1+i)}{\beta^{\sigma}(1+i)^{\sigma}}\hat{\Phi}_{at} = \hat{\Phi}_{at+1} - (1-\sigma)(\hat{\imath}_t - \hat{\pi}_{at+1}) - \hat{\xi}_{at} + \hat{\xi}_{at+1} \tag{9}$$

here $\theta = C/Y$ is a steady state share of private consumption in Y and A is the steady state level of real assets as a share of Y.

The assets equation can be linearsied as

$$\hat{A}_{kt+1} = \hat{\imath}_t + (1+i)(\hat{A}_{kt} - \hat{\pi}_{Hkt} + \frac{(1-\tau)}{A}\hat{Y}_{kt} - \frac{\theta}{A}\left(\hat{C}_{kt} + \alpha_n \hat{S}_{k\bar{k}t}\right)) \tag{10}$$

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$c_{Hkt}(z) = \left(\frac{p_{Hk}(z)}{P_{Hk}}\right)^{-\epsilon_t} C_{Hkt} \tag{11}$$

where:

$$P_{Hkt} = \left[\int_0^1 p_{Hkt}^{1-\epsilon_t}(z)dz\right]^{\frac{1}{1-\epsilon_t}} \tag{12}$$

The optimal allocation of expenditures between domestic and foreign goods implies:

$$C_{Hk} = \alpha_d \left(\frac{P_{Hk}}{P_k}\right)^{-\eta} C_k,\ C_{H\bar{k}} = \alpha_n \left(\frac{P_{H\bar{k}}}{P_k}\right)^{-\eta} C_k \tag{13}$$

where the consumer price indexes for all countries are:

$$P_k = (\alpha_d P_{Hk}^{1-\eta} + \alpha_n P_{H\bar{k}}^{1-\eta})^{\frac{1}{1-\eta}} \tag{14}$$

We define the terms of trade S_{ab}, the nominal exchange rate E_{ab}, and the real exchange rate Q_{ab} as follows

$$S_{ab} = \frac{P_{Hb}}{P_{Ha}},\ E_{ab} = \frac{P_{Hb}}{P_{Hb}^b},\ Q_{ab} = \frac{E_b P_b^b}{P_a}. \tag{15}$$

2.3 Supply: Pricing Decisions by Firms

In order to describe price setting decisions we split firms into two groups according to their pricing behaviour, following Steinsson (2003). In each period, each firm is able to reset its price with probability $1-\gamma$, and otherwise, with probability γ, its price will rise at the steady state rate of domestic inflation. Among those firm, which are able to reset their price, a proportion of $1-\omega$ are forward-looking and set prices optimally, while a fraction ω are backward-looking and set their prices according to a rule of thumb.

Forward-looking firms are profit-maximising, they reset prices optimally, given Calvo-type constraints on price setting, that results in the following formula for $P_{Hk,t}^F$, which for convenience is written in terms of log-deviations from the steady state (see Appendix A.2):

$$\begin{aligned} \hat{P}_{Hk,t}^F &= \gamma\beta \hat{P}_{Hk,t+1}^F + \gamma\beta\pi_{Hk,t+1} \\ &+ \frac{(1-\gamma\beta)\,\psi}{\psi+\epsilon}\left(\alpha_n \hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} + (\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C})\hat{\xi}_{kt} + \hat{\eta}_{kt}\right) \end{aligned} \tag{16}$$

where $\pi_{Hk,t}$ is resulting domestic inflation in country k.

The rule of thumb used by a backward-looking firm to set its price $P_{Hk,t}^B$ is

$$p_{Hk,t}^B = p_{Hk,t-1}^r \Pi_{Hk,t-1}(\frac{Y_{kt-1}}{Y_{kt-1}^n})^\delta \tag{17}$$

where $P_{Hk,t-1}^r$ is the average domestic price in the previous period, $\Pi_{Hk,t} = P_{Hk,t}/P_{Hk,t-1}$ is past period growth rate of prices and Y_{kt}/Y_{kt}^n is output relative to the flexible-price equilibrium. For the economy as a whole, the price equation can be written as:

$$P_t = [\gamma(\Pi P_{t-1})^{1-\epsilon_t} + (1-\gamma)(1-\omega)(P_t^F)^{1-\epsilon_t} + (1-\gamma)\omega(P_t^B)^{1-\epsilon_t}]^{\frac{1}{1-\epsilon_t}}. \tag{18}$$

Following Steinsson (2003) and allowing for government consumption terms in the utility function, we can derive the following Phillips curve for our economy, written in terms of log-deviations from the steady state[6]:

$$\begin{aligned} \hat{\pi}_{Hkt} &= \chi\beta\hat{\pi}_{Hkt+1} + (1-\chi)\hat{\pi}_{Hkt-1} + \kappa_c \hat{C}_{kt} + \kappa_s \hat{S}_{abt} \\ &+ \kappa_{y0}\hat{Y}_{kt} + \kappa_{y1}\hat{Y}_{kt-1} + (\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C})\hat{\xi}_{kt} + \hat{\eta}_{kt} \end{aligned} \tag{19}$$

[6] The derivation is identical to the one in Steinsson (2003), amended by the introduction of mark-up shocks as in Beetsma and Jensen (2003). A detailed derivation is given in Appendix A.2.

Coefficients χ and κs are given in Appendix A.2 as functions of γ and ω and other structural parameters. Although the constant wage income tax τ has no effect on the dynamic equations for log-deviations from the flexible price equilibrium, it alters the equilibrium choice between consumption and leisure for the consumer. The Phillips curve (19) has a familiar structure where both current and past output have an effect on inflation. Its specification is derived in Steinsson (2003) and we briefly repeat this derivation in Appendix A.2, where we explain our open-economy extension. In the case when all consumers are forward-looking, i.e. $\omega = 0$, this Phillips curve collapses to the standard forward-looking Phillips curve (see Rotemberg and Woodford (1997)). If all consumers use the rule of thumb in price-setting decisions, i.e. if $\omega = 1$, this Phillips curve can be brought into the form of an 'accelerationist' Phillips curve. The presence of the term of trade in the Phillips curve is due to the fact that people consume a basket of goods but, of course, produce only domestic goods.

2.4 The Economy as a Whole

2.4.1 Aggregate Demand

Aggregate demand for country $k \in \{a, b\}$, is given by a linearised GDP identity:

$$\hat{Y}_{kt} = \theta\alpha_d\hat{C}_{kt} + \theta\alpha_n\hat{C}^{\bar{k}}_{\bar{k}t} + (1-\theta)\hat{G}_{kt} + 2\theta\eta\alpha_d\alpha_n\hat{S}_{k\bar{k}t} \tag{20}$$

The derivation of this formula is sketched in Appendix A.4. The parameter θ denotes the share of private consumption in output, so $1 - \theta$ is the share of the government sector in the economy.

2.4.2 Aggregate Supply

The Phillips curve equation (19) contains terms in the preference shock ξ. These can be replaced by consumption, output and the terms of trade at their 'natural' level (superscript n), which is the level of these variables that would occur in an economy with flexible prices and no mark-up shocks. Under flexible prices the real wage is always equal to the inverse of this mark-up, see Appendix A.2. Optimisation by consumers then implies (we assume the production function $y_t = h_t$) :

$$\frac{w_{kt}}{P_{Hkt}} = \frac{P_{kt}}{P_{Hkt}}\frac{v_y(y^n_{kt}(z), \xi_{kt})}{(1-\tau)u_C(C^n_{kt}, \xi_{kt})} = \frac{\mu^w}{\mu_t} \tag{21}$$

where $\mu_t = -(1-\epsilon_t)/\epsilon_t$ is a monopolistic mark-up and μ^w is employment subsidy for producers. Linearisation of (21) yields:

$$\hat{Y}^n_k\frac{1}{\psi} + \hat{C}^n_k\frac{1}{\sigma} + \alpha_n\hat{S}^n_{k\bar{k}} + (\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C})\hat{\xi}_k = 0 \tag{22}$$

2.4.3 Fiscal Constraint

We assume that the government buys goods (G), taxes income (with tax rate τ), and issues nominal debt $\mathcal{B}$. The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{kt+1} = (1+i_t)(\mathcal{B}_{kt} + G_{kt}P_{Hkt} - \tau Y_{kt}P_{Hkt}) \tag{23}$$

This equation can be linearised as (assuming $B_t = \mathcal{B}_t/P_{t-1}$) :

$$\hat{B}_{at+1} = \hat{\imath}_t + (1+i)(\hat{B}_{at} - \hat{\pi}_{Hat} + \frac{1-\theta}{B}\hat{G}_{at} - \frac{\tau}{B}\hat{Y}_{at}) \tag{24}$$

where B is the steady state level of real bonds as a share of Y.

There is no capital in this model, so the amount of bonds issued is equal to the amount of bonds held:

$$\mathcal{A}_{at} + \mathcal{A}_{bt} = \mathcal{B}_{at} + \mathcal{B}_{bt}$$

2.4.4 Financial Markets

We assume complete capital markets with perfect capital mobility and thus a common interest rate.

2.5 Putting things together

We now write down the final system of equations for the 'law of motion' of the out-of-equilibrium economy. We simplify notation by denoting gap variables with lower case letters: for any variable $x_t = \hat{X}_t - \hat{X}_t^n$. We can use relationship (22) to substitute out ξ–shock terms in the Phillips curve and the Euler equation, and rewrite the dynamic system in 'gap' form. (We also substitute out for consumer price inflation in terms of domestic inflation and exchange rates and denote $\nu = Bp/\Phi\theta$, $\mu = [\beta(1+i)]^{-\sigma}$.)

$$\begin{aligned} c_{at} &= \mu c_{at+1} + \mu\nu(a_{at+1} - \phi_{at+1}) - \sigma i_t + (\sigma - \mu\nu)(\alpha_d \pi_{Hat+1} + \alpha_n \pi_{Hbt+1}) && (25) \\ c_{bt} &= \mu c_{bt+1} + \mu\nu(a_{bt+1} - \phi_{bt+1}) - \sigma i_t + (\sigma - \mu\nu)(\alpha_d \pi_{Hbt+1} + \alpha_n \pi_{Hat+1}) && (26) \end{aligned}$$

$$(1+p)(1+i)\mu\phi_{at} = \phi_{at+1} - (1-\sigma)(i_t - \alpha_d \pi_{Hat+1} - \alpha_n \pi_{Hbt+1})$$
$$(1+p)(1+i)\mu\phi_{bt} = \phi_{bt+1} - (1-\sigma)(i_t - \alpha_d \pi_{Hbt+1} - \alpha_n \pi_{Hat+1})$$

$$\pi_{Ha,t} = \chi\beta\pi_{Ha,t+1} + (1-\chi)\pi_{Ha,t-1} + \kappa_c c_{at} + \kappa_{y0} y_{at} + \kappa_{y1} y_{at-1} + \kappa_D s_t + \hat{\eta}_{at} \quad (27)$$

$$\pi_{Hb,t} = \chi\beta\pi_{Hb,t+1} + (1-\chi)\pi_{Hb,t-1} + \kappa_c c_{bt} + \kappa_{y0} y_{bt} + \kappa_{y1} y_{bt-1} - \kappa_D s_t + \hat{\eta}_{bt} \quad (28)$$

$$y_{at} = (1-\theta)g_{at} + \theta\alpha_d c_{at} + \theta\alpha_n c_{bt} + \theta\eta\left(1-\alpha_D^2\right)s_t \quad (29)$$

$$y_{bt} = (1-\theta)g_{bt} + \theta\alpha_d c_{bt} + \theta\alpha_n c_{at} - \theta\eta\left(1-\alpha_D^2\right)s_t \quad (30)$$

$$s_t = s_{t-1} - \frac{1}{2}\pi_{Hat} + \frac{1}{2}\pi_{Hbt} \quad (31)$$

$$b_{at+1} = i_t + (1+i)(b_{at} - \alpha_d\pi_{Hat} - \alpha_n\pi_{Hbt} + \frac{(1-\theta)}{B}g_{at} - \frac{\tau}{B}y_{at}) + i(1-\alpha_D)s_t \quad (32)$$

$$b_{bt+1} = i_t + (1+i)(b_{bt} - \alpha_d\pi_{Hbt} - \alpha_n\pi_{Hat} + \frac{(1-\theta)}{B}g_{bt} - \frac{\tau}{B}y_{bt}) - i(1-\alpha_D)s_t \quad (33)$$

$$a_{at+1} = i_t + (1+i)(a_{at} - \alpha_d\pi_{Hat} - \alpha_n\pi_{Hbt} + \frac{(1-\tau)}{B}(y_{at} - (1-\alpha_D)s_t) - \frac{\theta}{B}c_{at}) \quad (34)$$

$$a_{bt} = b_{at} + b_{bt} - a_{at} \quad (35)$$

Equations (25) - (26) are consumption equations for each country from (8), written in terms of domestic inflation. Equations (29) and (30) are aggregate demand equations from (20). Equation (31) follows from the requirement of the fixed nominal exchange rate between countries a and b. From the system it is clear that cost-push shocks $\hat{\eta}$ are distortionary. The absence of terms in taste shocks shows that taste shocks alone have no impact on gap variables. However, as we show below, taste shocks do influence natural levels and therefore the size of the impact of cost-push shocks on welfare.

2.6 Policy Framework

In this paper, we study simple and potentially implementable fiscal rules. We postulate that fiscal authorities operate with rules in a form

$$g_{kt} = \theta_{\pi k}\pi_{kt-1} + \theta_{\pi\bar{k}}\pi_{\bar{k}t-1} + \theta_{yk}y_{kt-1} + \theta_{y\bar{k}}y_{\bar{k}t-1} + \theta_s s_{k\bar{k}t-1} + \theta_k b_{kt-1}$$

Excluding contemporary shocks or the current value of variables from the reaction function capturures to some extent lags in the operation of fiscal policy. Monetary policy, in contrast, is considered to be optimal and not subject to implementation lags, and will take into account all available information. We assume monetary policy is formulated under commitment (i.e. it is time inconsistent), but results are very similar if we assume a discretionary (time consistent) policy.

If the fiscal authorities are given such rules, and monetary authorities use some optimising policy, this leads to a stochastic equilibria that should be compared across a suitable metric. The coefficients θ are then chosen such that it would optimise the chosen welfare criterion. Clearly setting some θ to zero reduces the information set that the fiscal authorities can respond to, so worse outcomes will be achieved. In this paper we examine the magnitude of the cost of these restrictions.

Optimal simple rules are time inconsistent. A social planner, which designs such a rule for the fiscal authorities, assumes given optimal reactions of both monetary authorities and the private sector (see Currie and Levine (1985) for discussion). Thus the fiscal authorities precommit themselves to a rule. In addition, we no longer have certainty equivalence, and so optimal θ will be dependent on the assumed distribution of shocks. We examine the robustness of this choice below.

The union-wide social loss takes the form

$$\mathcal{L} = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\mathcal{U}_{as} + \mathcal{U}_{bs} \right) = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \mathcal{U}_s$$

where intra-period loss $\mathcal{W}_s$ takes the form (see Rotemberg and Woodford (1997), Beetsma and Jensen (2003), Steinsson (2003), and Appendix B to this paper for a discussion of the derivation):

$$\begin{aligned} \mathcal{U}_s &= \lambda_c \left(c_{as}^2 + c_{bs}^2 \right) + \lambda_y \left(y_{as}^2 + y_{bs}^2 \right) + \lambda_g \left(g_{as}^2 + g_{bs}^2 \right) \\ &+ \lambda_s s_{abs}^2 + \lambda_{sc} s_{abs} c_{as} - \lambda_{sc} s_{abs} c_{bs} + \lambda_\pi \left(\pi_{Has}^2 + \pi_{Hbs}^2 \right) + \mu_{\Delta\pi} \left((\Delta\pi_{Has})^2 + (\Delta\pi_{Hbs})^2 \right) \\ &+ \nu_{cn} c_{as} \left[\hat{Y}_{as}^n - \hat{C}_a^n + \nu_{cs} \hat{S}_{abs}^n \right] + \nu_{cn} c_{bs} \left[\hat{Y}_{bs}^n - \hat{C}_{bs}^n - \nu_{cs} \hat{S}_{abs}^n \right] \\ &+ \nu_{xn} y_{as} \left[\hat{C}_{as}^n - \hat{Y}_{as}^n + \nu_{xs} \hat{S}_{abs}^n \right] + \nu_{xn} y_{bs} \left[\hat{C}_{bs}^n - \hat{Y}_{bs}^n - \nu_{xs} \hat{S}_{abs}^n \right] \\ &+ \nu_{gn} g_{as} \left[\hat{Y}_{as}^n - \hat{G}_{as}^n + \nu_{gs} \hat{S}_{abs}^n \right] + \nu_{gn} g_{bs} \left[\hat{Y}_{bs}^n - G_{bs}^n - \nu_{gs} \hat{S}_{abs}^n \right] \\ &+ \nu_{sn} s_{abs} \left[\nu_{ss} \hat{S}_{abs}^n + \left(\hat{Y}_{as}^n - \hat{Y}_{bs}^n \right) - \left(\hat{C}_{as}^n - \hat{C}_{bs}^n \right) \right] \\ &+ \mu_y \left(y_{as-1}^2 + y_{bs-1}^2 \right) + \mu_{y\Delta\pi} \left(y_{as-1} \Delta\pi_{Has} + y_{bs-1} \Delta\pi_{Hbs} \right) + tip(3) \end{aligned} \tag{36}$$

There are two unconventional features of this loss function. First, terms with $\mu-$coefficients are present only because of rule of thumb price setters. The presence of these terms implies that inflation and output will be brought back to the equilibrium smoothly. Steinsson (2003) has shown that when the private sector is predominantly backward-looking, terms with weights denoted by μ dominate the loss function, and that conversely, when the private sector is forward-looking these $\mu-$terms essentially disappear. Second, the terms with weights denoted by ν arise; as Kirsanova, Leith, and Wren-Lewis (2004) discuss in detail, in an open economy with taste/technology shocks it is in general no longer optimal to exactly reproduce the flexible price equilibrium, because changes in the terms of trade alter the impact of the monopoly distortion, and this introduces 'linear in policy' terms with a ν coefficient.

As a benchmark case we assume that the monetary authorities use union-wide social welfare function. However, as monetary policy cannot react to differences between the two economies (where there is no change in aggregate union wide variables), then this expression can be simplified to the following:

$$\begin{aligned} \mathcal{U}_s &= \lambda_c \left(\frac{c_{as} + c_{bs}}{2} \right)^2 + \lambda_y \left(\frac{y_{as} + y_{bs}}{2} \right)^2 + \lambda_g \left(\frac{g_{as} + g_{bs}}{2} \right)^2 \\ &+ \lambda_\pi \left(\frac{\pi_{Has} + \pi_{Hbs}}{2} \right)^2 + \mu_{\Delta\pi} \left(\Delta \frac{\pi_{Has} + \pi_{Hbs}}{2} \right)^2 \\ &+ \mu_y \left(\frac{y_{as-1} + y_{bs-1}}{2} \right)^2 + \mu_{y\Delta\pi} \left(\frac{y_{as-1} + y_{bs-1}}{2} \Delta \frac{\pi_{Has} + \pi_{Hbs}}{2} \right) + tip(3) \end{aligned} \tag{37}$$

This eliminates cross terms from (36). Alternatively, and equivalently, it is the closed economy version of (36).

To interpret the resulting values of the social loss, we can express them in terms of compensating consumption – the permanent fall in the steady state consumption level that would balance the welfare gain from eliminating the volatility of consumption, government spending and leisure (Lucas (1987)). As explained in Appendix C, the percentage change in consumption level, Ω, that is needed to compensate differences in welfare of two regimes with social losses $\mathcal{L}_1$ and $\mathcal{L}_2$ is given by (36):

$$\Omega = \sigma \left(1 - \sqrt{1 + \frac{(1-\beta)}{\sigma} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\mathcal{U}_{2s} - \mathcal{U}_{1s} \right)} \right) \tag{38}$$

As we note above, some aspects of this social loss function are different from the, more adhoc, loss functions traditionally assumed to drive monetary policy. We therefore also examine an alternative case where the monetary authority seek to minimise the following traditional loss function of the form:

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[(\pi_{as} + \pi_{bs})^2 + 0.5(x_{as} + x_{bs})^2 \right]. \tag{39}$$

In other words, the central bank targets union-wide consumer price inflation and output[7]. We take the value of 0.5 for the weight on output variability as a conventional value in the literature.

3 Calibration

Because of the microfounded nature of the model, there are relatively few parameters to calibrate, given in Table 1. One period is taken as equal to one quarter of a year. We set the discount factor of the private sector (and policy makers) to $\beta = 0.99$.

Our knowledge regarding inflation persistence is very insecure. All empirical studies are unanimous in concluding that an empirical Phillips curve has a significant backward-looking component. The estimates of the exact weight χ, however, differ widely. Gali and Gertler (1999), Benigno and Lopez-Salido (2002) find a predominantly forward-looking specification of the Phillips curve, while Mehra (2004) finds an extremely backward-looking specification. Mankiw (2001) argues that stylised empirical facts are inconsistent with predominantly forward-looking Phillips Curve. Therefore, we calibrate $\omega = 0.5$ for the base-line case specification, which corresponds to a forward-looking coefficient of $\chi = 0.3$ in the Phillips curve (27)-(28), but we also look at robustness to alternative values extensively below. To calibrate parameter δ we follow Stensson's procedure, which is as follows. The possible range of values for ω in the Phillips curve is $0 \leq \omega \leq 1$. As noted above, when $\omega \to 0$ it collapses to the familiar purely forward-looking specification of Woodford (2003) $\pi_t = \beta\pi_{t+1} + \kappa_c c_t + \kappa_{y0} y_t + \kappa_s s_t$, whilst when $\omega \to 1$, it collapses to $\pi_t = \pi_{t-1} + (1-\gamma)\delta x_{t-1}$, which is the accelerationist Phillips curve. The Stensson's

[7] Using consumer price inflation reflects current practice among central banks.

procedure for calibrating δ *assumes* that demand pressure in both these extreme cases is equal, i.e. it assumes that $\kappa_c + \kappa_{y0} = (1-\gamma)\delta$. This equation can then be solved to provide a value for δ. With this choice of δ total demand pressure in our general specification is independent of the number of rule-of-thumb price setters, ω, and is equal to: $\kappa = \kappa_c + \kappa_{y0} + \kappa_{y1} = (1-\gamma)(1-\gamma\beta)/\sigma\gamma$.

We follow the literature in calibrating $\gamma = 0.75$, which implies that, on average, prices (and wages) last for one year. We assume that each economy consumes 30% of imported goods. For the parameters related to fiscal policy, we calibrate the ratio of private consumption to output as 75 percent; and we assume that the equilibrium ratio of domestic debt to output is 60 percent. Then the debt accumulation equation gives us the equilibrium level of the primary surplus and the tax rate.

This calibration completely defines the coefficients of the welfare function, which are given in Table 1. It is apparent that the resulting coefficient on output stabilisation in social welfare, λ, is very small (at around 0.01) compared to the weight traditionally adopted in the monetary policy literature of around[8] 0.5. In order to compute the social loss, we calibrate the standard deviations of shocks hitting the economies as follows. We assume that the standard deviations of cost-push and taste/technology shocks are equal (in the literature a consensus number is 0.5%, see, e.g. Jensen and McCallum (2002), Bean, Nikolov, and Larsen (2002)), and all shocks are independent.

4 Results

Table 2 presents some key results for the model with Blanchard-Yaari consumers. The columns of the Table represent different forms of fiscal policy rule, where in each case the optimal parameter values are computed in the face of cost-push and taste/technology shocks. We also show the feedback parameters for optimal monetary policy in each case: however, these parameters should be interpreted with caution, because they are part of an optimal rule under commitment which also involves additional Lagrange multipliers. The first column of numbers represents the case where there is no fiscal stabilisation, although there is feedback on debt (see below). The social loss under each policy, measured in absolute loss units, is shown in the first row, while the second row computes the gain in consumption units relative to the no fiscal stabilisation column.

One important restriction placed on fiscal policy in all the cases presented in this Table, is that there needs to be some minimum feedback on own public debt to ensure solvency when monetary policy is active. Numerical simulations show that with our choice of parameters, a minimal fiscal rule of a form $g_t = -\mu b_t$ will ensure saddle path stability of the system if $\mu \gtrsim 0.027$. Obviously, if fiscal policy feeds back on other variables, then this threshold will change, but it appears to change only slightly. However in all cases we find the optimal (given the social welfare function) value of this coefficient, but as the Table shows, it is only marginally greater than this minimum value. (For further discussion of this critical feedback value, see Kirsanova and Wren-Lewis (2004), Leith and Wren-Lewis (2001), Kirsanova, Vines, and Wren-Lewis (2004)).

[8] This larger conventional value may result from the demand-driven unemployment. This phenomenon is not addressed in our model.

Key Parameters	Mnemonics	Value
Discount factor	β	0.99
Share of rule-of-thumb price-setters	ω	0.5
Proportion of agents who able to reset their price within a period	$1-\gamma$	0.25
Weight on demand pressure in the Phillips curve	κ	0.3
Share of the government sector in the economy	$1-\theta$	0.65
Steady state ratio of domestic debt to output	B/Y	0.6
Intertemporal substitution rate	σ	0.5
Elasticity of substitution between domestic and foreign goods	η	0.3
Elasticity of substitution between two domestic goods	ϵ	5.0
Production risk aversion	$1/\psi$	0.5
Share of domestic goods in consumption basket	α_d	0.7
Openness with respect to the other small open economy	α_n	0.2
Implied Parameters in system (25)-(31)	**Mnemonics**	**Value**
Tax Rate	τ	0.256
Steady state ratio of primary real surplus to output	δ_d	0.006
Weight on forward inflation in PC	χ	0.3
Weight on the country's term of trade vs. the rest of the world in AD	$2\theta\eta\alpha_d\alpha_n$	0.09

Table 1: Parameter values

The column (2w) presents what could be regarded as the other extreme benchmark case, where we allow fiscal policy to feedback on a wide range of variables: own country inflation, output and debt, the foreign country's inflation and output, and the real exchange rate. We can note that welfare is substantially improved by allowing comprehensive fiscal stabilisation (Hughes Hallett and Vines (1991), Driver and Wren-Lewis (1999) for similar results on less microfounded models). Column (2h) restricts fiscal policy to react to own country variables alone (and the terms of trade). There is no welfare cost in making this restriction.

The remaining columns look at cases where we restrict fiscal policy to react to country differentials in inflation and output. (Thus, government spending in country A reactions to the difference between output or inflation in A compared to B. They continue to react to their own debts levels.) A key result is that, as long as the reaction function contains lagged inflation, the loss involved in restricting fiscal policy to react to differences is minimal.

A comparison of the final columns of the Table show that the presence of output or the terms of trade contribute almost nothing to the welfare benefits of fiscal stabilisation. We need to remember, however, that our microfounded welfare function has a very low weight on output, and so this result may not be surprising..The sign of the terms of trade feedback implies that government spending is reduced if the price of domestic goods exported overseas falls (so s_{ab} rises). Note also the optimal response to debt remains small in all cases.

These results suggest substantial welfare gains from an active fiscal policy that reacts to inflation differentials. To understand why, we need to recall that an introduction of inflation inertia brings an important source of instability in an individual economy in a monetary union. Suppose for some reason output in one country rises and output in the other country falls, with no impact on union output. Inflation in the country with higher output will *gradually* rise because of inflation inertia. Real interest rates in that country will therefore fall, as nominal interest rates are fixed at the union level and there is no reason for monetary policy to change. (In contrast, if inflation was entirely forward-looking, it would *jump* up and then gradually fall, so the expected real interest rate would always be higher.) Lower real interest rates put further upward pressure on output and inflation. Even if instability is avoided, the adjustment mechanism is slow and *cyclical*. This is because the price level tends to overshoot: if prices are high this causes low demand and disinflation; when the price level, and demand, have returned to zero prices are still falling. This will lead to high demand in the future, which will cause a return of inflation and higher prices and so on[9]. This is illustrated by the impulse responses in Figure 1. To prevent this cyclicality requires some form of inflation control by the fiscal authorities, which is in our framework is manifested through a substantial coefficient on the inflation differential for a fiscal feedback rule. This is an important result in the light of some proposals (Treasury (2003)) which have suggested that national fiscal policy focus exclusively on output gaps, and not inflation. Our results suggest this would be severely suboptimal.

Similar factors account for why the optimal feedback on debt is small. Following a

[9] For detailed dynamic analysis of instability mechanisms in a monetary union when inflation is persistent, see Kirsanova, Vines, and Wren-Lewis (2004).

positive inflation shock, lower real interest rates would reduce the debt stock. If fiscal feedback on debt was large, this would add to government spending and further boost demand. Only a small feedback on debt is required to ensure solvency (see above), but larger feedback would aggrevate the cyclical behaviour of the economy under inflation inertia following asymmetric shocks, and this reduces welfare.

The sign on the term of trade in the optimal fiscal reaction function comes from another dynamic mechanism. The negative sign on the term of trade implies that fiscal policy is counteracting the effect of competitiveness on domestic demand. We might have viewed this competitiveness effect as inherently stabilising (a fall in the domestic price level raises the demand for domestic goods, thereby raising inflation). However this feedback process is also cyclical (Kirsanova, Vines, and Wren-Lewis (2004)). Stabilisation can be achieved more effectively be direct feedback on inflation and output, so the optimal fiscal rule tries to neutralise this competitiveness effect. As a check on this intuition, we increased the size of the demand elasticity, and we found that the optimal feedback on the terms of trade increased proportionately.

To see how robust these results are, we conducted a number of additional experiments. In Table 2, optimal policies are computed for a given distribution of shocks. Thus all the coefficients, and the welfare-consumption figures in the two top rows, correspond to the base line case with uncorrelated cost-push and preference shocks with identical standard deviations. Having obtained these optimal policy rules, we hit the economy with shocks drawn from different distributions. We have found that the maximum loss relative to the base line case is achieved when the economy is hit by asymmetric shocks. The second row with consumption percentage in Table 2 contains difference in consumption between the case of no fiscal stabilisation under uncorrelated shocks (our base case, against these shocks the optimal rule is designed) and the case when economy is hit by asymmetric shocks of similar amplitude. The uncorrelated shocks in the base case can be decomposed into a sum of symmetric and asymmetric shocks, and in this case we have only the asymmetric component. These shocks only partly removed by fiscal policy and consumption gain from this operation constitutes, for example, 1.62% for the case (2w). If we hit the economy with two sets of symmetric shocks but control it with the same rules designed to deal with uncorrelated shocks, we get 3.33% gain in consumption: monetary policy removes them successfully.

Our results are also robust to replacing Blanchard-Yaari consumers with infinitely lived consumers (the setup similar to Beetsma and Jensen (2004) paper, and those papers which ignore solvency constraint). The results are very similar and therefore are not shown here. Clearly this similarity reflects the small size of the optimal feedback on debt discussed above.

Finally we present in Table 3 results where monetary policy maximises the more traditional welfare function discussed in Section 2.6. The qualitative results are very similar to the main case where monetary policy maximises social welfare. The only noticable difference is in comparing fiscal feedback on all variables with feedback on own country variables alone. Whereas there was no welfare cost in making this restriction in the main case, now there is some, although its size is not large. In addition, the behaviour of the monetary authorities is different in the cases where fiscal policy is not restricted to responding to differences. In the main case, the parameters on the reaction function were all intuitive: interest rates rise in response to a positive cost push shock, and to increases in lagged output and inflation. However, in this case the response to lagged output and inflation has the opposite sign. We need to be cautious in interpreting these parmeters, which are from a reaction function that also includes Lagrange multipliers that are derived from full optimisation under commitment, and is therefore quite different from a simple policy rule. However one interpretation of this case is that the primary role of macroeconomic stabilisation is being fulfilled by fiscal policy (government spending falls if lagged inflation or output is high). Fiscal policy is more effective in this case because monetary policy is targetting the 'wrong' objective i.e. traditional rather than social welfare. Figure 2 illustrates the movements of both instruments following a symmetric cost-push shock. It is also interesting to note that the coefficients of the monetary policy reaction function are more conventional when fiscal policy is restricted to reacting to differences, as it can no longer respond to symmetric shocks, leaving monetary policy to take the full burden of aggregate stabilisation in this case.

5 Conclusion

In this paper we have examined the potential role for fiscal policy to help stabilise individual economies within a monetary union. While the vulnerability of monetary unions to asymmetric shocks are well known, there has been surprising little analysis of the extent to which fiscal policy can overcome these problems within the framework of the new international macroeconomics. This is despite the fact that policy makers in potential members of the European Monetary Union have actively discussed the possibility of using fiscal policy in this way (Treasury (2003), Swedish Committee (2002)).

Our analysis looks at the potential welfare gains from national governments operating different forms of simple rules for fiscal policy. We find substantial welfare gains from government expenditure responding to national inflation. However there is very little welfare benefit from governments responding to other variables, including overseas variables or the terms of trade. We also find that the optimal feedback from government debt is only slightly above the minimum level required to ensure solvency. These results appear robust to the specification of consumption, the distribution of shocks, and the goals of optimal monetary policy.

These results have three important implications for the policy debate on fiscal policy in a monetary union. First, we find that the potential gains from fiscal stabilisation are large, and that these do not conflict with the requirements for debt sustainability. Second, these gains are largest when fiscal policy responds to inflation: responding to output alone (as suggested in Treasury (2003), and analysed in Dixit and Lambertini (2003)) appears severely suboptimal. Third, very little is lost if fiscal policy only responds to differences in inflation and output, along with the level of national debt. This last result is important, because it may help avoid some of the political economy concerns that have been expressed about fiscal stabilisation.

		AS	Feedback on country's variables		Feedback on differences			
		(1)	(2w)	(2h)	(2d)	(3d)	(4d)	(5d)
Blanchard-Yaari consumers and government solvency constraint								
Absolute Units of Loss		7.66	4.37	4.40	4.37	4.39	4.39	7.56
Consumption gain, %								
uncorrelated shocks[♭]		0	1.62	1.60	1.62	1.61	1.61	0.05
asymmetric shocks[†]		0	3.33	3.32	3.33	3.31	3.32	0.11
symmetric shocks[‡]		0	0.00	-0.02	0.00	0.00	0.00	0.00
Optimal Coefficients for Fiscal Policy in country a								
Inflation	π_a	0	-6.37	-12.81	-6.10	-6.47	-5.18	0
Output	x_a	0	-0.46	-1.55	-0.43	-0.66	0	0
Debt	b_a	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Inflation	π_b	0	5.82	0	6.10	6.47	5.18	0
Output	x_b	0	0.40	0	0.43	0.66	0	0
Term of Trade	s_{ab}	0	-0.44	0.26	-0.44	0	-0.64	-0.39
Optimal Commitment Solution for Monetary Policy (feedback on state variables only)								
Cost-push shock	$\frac{\eta_a+\eta_b}{2}$	10.08	10.16	12.16	10.08	10.08	10.08	10.08
Inflation	$\frac{\pi_a+\pi_b}{2}$	3.46	3.39	1.04	3.46	3.46	3.46	3.46
Output	$\frac{x_a+x_b}{2}$	0.65	0.63	-0.01	0.65	0.65	0.65	0.65
Debt	b_a, b_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Assets	a_a, a_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes:
AS – automatic stabilisers (feedback on debt only);
[♭] – expected shocks are uncorrelated, actual shocks are the same;
[†] – expected shocks are uncorrelated, actual supply shocks are perfectly positively correlated;
[‡] – expected shocks are uncorrelated, actual supply shocks are perfectly negatively correlated

Table 2: Optimal coefficients for monetary and fiscal policy. Monetary policy uses social welfare

		AS	Feedback on country's variables		Feedback on differences			
		(1)	(2w)	(2h)	(2d)	(3d)	(4d)	(5d)
Blanchard-Yaari consumers and government solvency constraint								
Absolute Units of Loss		8.65	4.55	5.30	5.36	5.38	5.38	8.55
Consumption gain, %								
uncorrelated shocks[♭]		0	2.01	1.65	1.62	1.61	1.61	0.05
asymmetric shocks[†]		0	3.30	3.18	3.30	1.28	1.28	0.11
symmetric shocks[‡]		0	0.78	0.17	0.00	0.00	0.00	0.00
Optimal Coefficients for Fiscal Policy in country a								
Inflation	π_a	0	-13.76	-12.95	-6.10	-6.47	-5.17	0
Output	x_a	0	-4.43	-2.77	-0.43	-0.67	0	0
Debt	b_a	-0.03	-0.04	-0.03	-0.03	-0.03	-0.03	-0.03
Inflation	π_b	0	1.47	0	6.10	6.47	5.17	0
Output	x_b	0	-2.85	0	0.43	0.67	0	0
Term of Trade	s_{ab}	0	-0.31	0.09	-0.44	0	-0.64	-0.39
Optimal Commitment Solution for Monetary Policy (feedback on state variables only)								
Cost-push shock	$\frac{\eta_a+\eta_b}{2}$	0.79	1.88	7.76	0.79	0.79	0.79	0.79
Inflation	$\frac{\pi_a+\pi_b}{2}$	0.54	-7.06	-2.92	0.54	0.54	0.54	0.54
Output	$\frac{x_a+x_b}{2}$	0.03	-4.88	-1.44	0.03	0.03	0.03	0.03
Debt	b_a, b_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Assets	a_a, a_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes:
AS – automatic stabilisers (feedback on debt only);
♭ – expected shocks are uncorrelated, actual shocks are the same;
† – expected shocks are uncorrelated, actual supply shocks are perfectly positively correlated;
‡ – expected shocks are uncorrelated, actual supply shocks are perfectly negatively correlated

Table 3: Optimal coefficients for Monetary and Fiscal Policy. Monetary Policy uses Traditional Welfare.

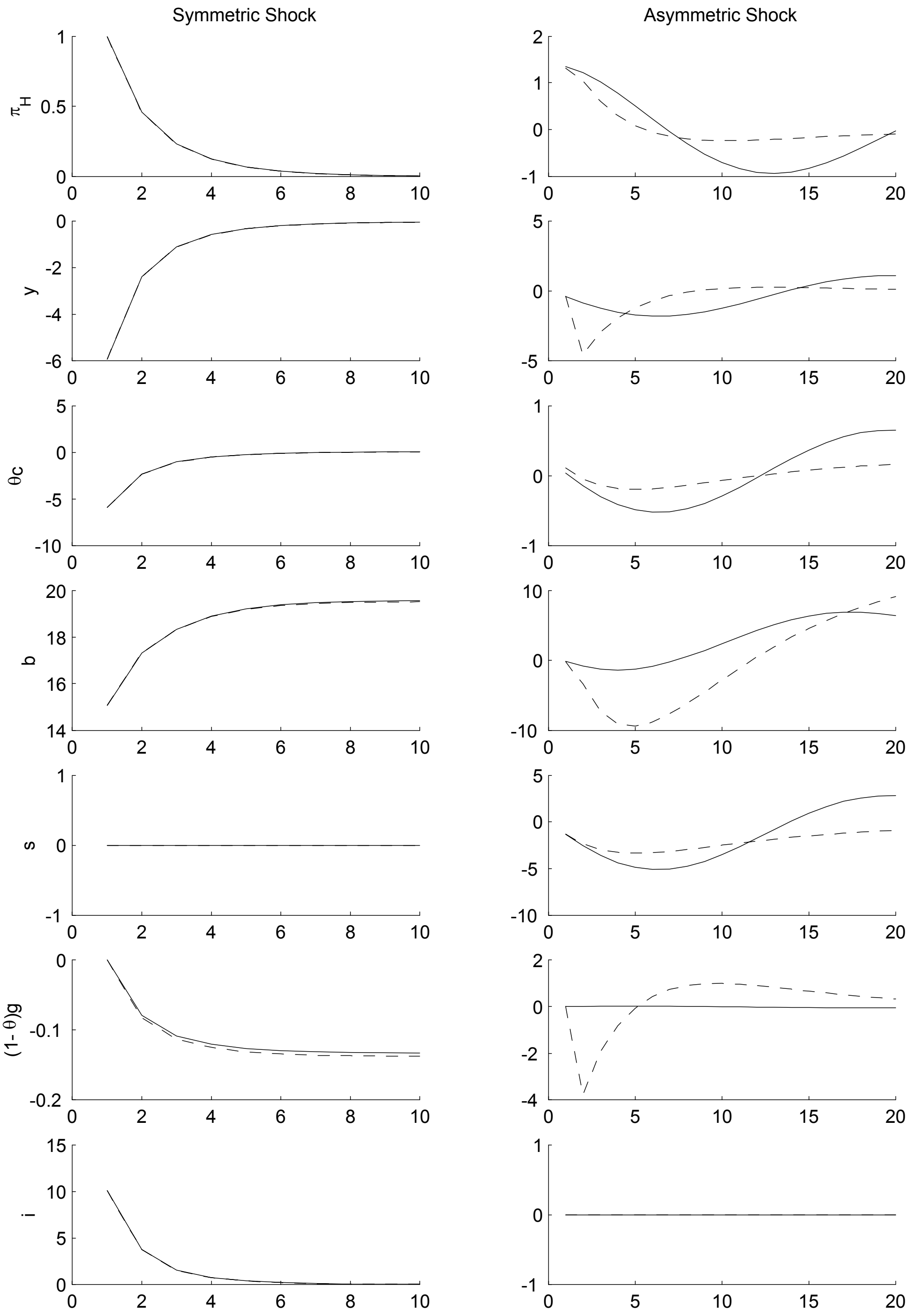

Figure 1: Monetary authorities use social welfare. Solid line denotes automatic stabilisers, dashed line denotes the case where fiscal authorities feed back on all variables (2w).

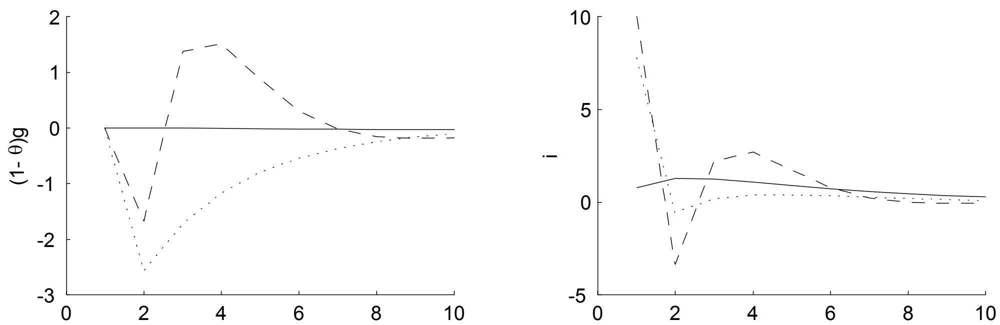

Figure 2: Movements of fiscal and monetary instruments to stabilise symmetric cost-push shock. Monetary authorities use traditional objectives. Solid line denotes the case where fiscal authorities do not attempt to stabilise symmetric shocks, dashed line denotes the case where fiscal authorities feed back on all variables (case (2w)) and dotted line denotes the case there the fiscal authorities are restricted to feed back on home country variables only (case (2h)).

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A Dynamic System

A.1 Derivation of Consumption Equation

A.1.1 Individual Relationships

To derive the first order conditions for household optimisation problem we write Lagrangian for household in country k as

$$L = \mathcal{E}_t \sum_{v=t}^{\infty} \left[\frac{\beta}{1+p}\right]^{v-t} [u(C^s_{kv}, \xi_{kv}) + f(G_{kv}, \xi_{kv}) - v(h^s_{kv}(z), \xi_{kv})]$$
$$- \lambda[\sum_{v=t}^{\infty} \mathcal{E}_t(R_{t,v} P_{kv} C^s_{kv}) - \mathcal{A}^s_{kt} - \sum_{v=t}^{\infty} \mathcal{E}_t(R_{t,v} \int_0^1 (1-\tau) w_{kv}(z) h^s_{kv}(z) dz + T^s_{kv})]$$

so the first-order conditions are:

$$\frac{\partial L}{\partial h^s_{kv}(z)} = -\left[\frac{\beta}{1+p}\right]^{v-t} v_h(h^s_{kv}, \xi_{kv}) + \lambda R_{t,v}(1-\tau) w_{kv}(z) = 0 \tag{40}$$

$$\frac{\partial L}{\partial C^s_{kv}} = \left[\frac{\beta}{1+p}\right]^{v-t} u_C(C^s_{kv}, \xi_{kv}) - \lambda R_{t,v} P_{kv} = 0 \tag{41}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{v=t}^{\infty} \mathcal{E}_t(R_{t,v} P_{kv} C^s_{kv}) - \mathcal{A}^s_{kt} - \sum_{v=t}^{\infty} \mathcal{E}_t(R_{t,v} \int_0^1 (1-\tau) w_{kv}(z) h^s_v(z) dz + T^s_{kv}) = 0 \tag{42}$$

Divide the second FOC by itself, taken one step ahead and obtain:

$$\frac{u_C(C^s_{kv+1}, \xi_{kv+1})}{u_C(C^s_{kv}, \xi_{kv})} = \frac{1}{\beta} \frac{1}{1+r_{kv}} \tag{43}$$

where we defined real interest rate as: $1 + r_v := (1+i_v) / (1+\pi_{kv+1})$.

For simplicity, we assume some particular utility function

$$u(C^s_{kv}, \xi_{kv}) = \frac{(C^s_{kv})^{1-1/\sigma}}{1-1/\sigma} \xi_{kv}$$

so equation (43) now becomes:

$$\frac{C^s_{kv+1}}{C^s_{kv}} = \left[\frac{1}{\beta} \frac{1}{1+r_{kv}}\right]^{-\sigma} \frac{\xi_{kv+1}}{\xi_{kv}}$$

Therefore

$$C^s_{kv} = C^s_{kt} \frac{C^s_{kv}}{C^s_{kt}} = C^s_t \prod_{m=0}^{v-t-1} (\frac{C^s_{k,t+m+1}}{C^s_{k,t+m}}) = C^s_t \prod_{m=0}^{v-1-t} \left[\frac{1}{\beta} \frac{1}{1+r_{k,t+m}}\right]^{-\sigma} \frac{\xi_{k,t+m+1}}{\xi_{k,t+m}}$$

$$P_{kv} = P_{kt} \frac{P_{kv}}{P_{kt}} = P^s_{kt} \prod_{m=0}^{v-t-1} (\frac{P_{k,t+m+1}}{P_{k,t+m}}) = P_{kt} \prod_{m=0}^{v-t-1} (1+\pi_{k,t+m+1})$$

We have for an *individual* consumption and wealth of a generation born at time s :

$$P_{kt}C_{kt}^s + \sum_{v=t+1}^{\infty} \mathcal{E}_t(R_{t,v}P_{kv}C_{kv}^s) = P_{kt}C_{kt}^s + P_{kt}C_{kt}^s \sum_{v=0}^{\infty} \mathcal{E}_t(R_{t,t+v+1}\frac{P_{k,t+1+v}}{P_{kt}}\frac{C_{k,t+1+v}^s}{C_{kt}^s})$$

$$= P_{kt}C_{kt}^s + P_{kt}C_{kt}^s \sum_{v=1}^{\infty} \frac{\beta^{\sigma v}}{(1+p)^v} \prod_{m=0}^{v-1} (1+r_{k,t+m})^{\sigma-1} \frac{\xi_{k,t+m+1}}{\xi_{k,t+m}} = P_{kt}C_{kt}^s\Phi_{kt}$$

where

$$\Phi_{kt} = 1+\sum_{v=1}^{\infty} \frac{\beta^{\sigma v}}{(1+p)^v} \prod_{m=0}^{v-1} (1+r_{k,t+m})^{\sigma-1} \frac{\xi_{k,t+m+1}}{\xi_{k,t+m}} = 1+\frac{\beta^{\sigma}}{(1+p)\,(1+r_t)^{1-\sigma}}\frac{\xi_{k,t+1}}{\xi_{k,t}}\Phi_{k,t+1} \tag{44}$$

and from the last FOCs it follows that:

$$P_{kt}C_{kt}^s = \frac{1}{\Phi_{kt}}(\mathcal{A}_{kt}^s + \mathcal{H}_{kt}^s) \tag{45}$$

Where nominal human capital $\mathcal{H}_{kt}^s$ is:

$$\mathcal{H}_{kt}^s := \sum_{v=t}^{\infty} \mathcal{E}_t(R_{t,v}\int_0^1 (1-\tau)w_{kv}(z)h_{kv}^s(z)dz + T_{kv}^s)$$

A.1.2 Aggregate Relationships

We aggregate all relationships accrues all generations. The size of total population at time t

$$\frac{p}{(1+p)} \sum_{s=-\infty}^{t} (\frac{1}{1+p})^{t-s} = 1.$$

Therefore,

$$C_{kt}^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}C_{kt}^s, \quad \mathcal{A}_{kt}^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}\mathcal{A}_{kt}^s$$

and we define aggregate nominal human capital as:

$$\mathcal{H}_{kt}^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)}(\frac{1}{1+p})^{t-s} \sum_{v=t}^{\infty} R_{t,v}\{\int_0^1 (1-\tau)w_{kv}(z)h_{kv}^s(z)dz + T_{kv}^s\}$$

$$= \sum_{v=t}^{\infty} R_{t,v}((1-\tau)Y_{kv}P_{kv} + T_{kv}^a)$$

where

$$Y_{kv}P_{kv} = \sum_{s=-\infty}^{t} \frac{p}{(1+p)}(\frac{1}{1+p})^{t-s} \int_0^1 w_{kv}(z)h_{kv}^s(z)dz, \quad T_{kv}^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}T_{kv}^s$$

Aggregating relationship (45) yields:

$$\Phi_{kt} P_{kt} C^a_{kt} = \mathcal{A}^a_{kt} + \mathcal{H}^a_{kt}.$$

We now derive a dynamic Euler equation for aggregate consumption. We note that

$$\begin{aligned}
\mathcal{A}^a_{kt+1} &= \frac{1}{\mathcal{E}_t(R_{t,t+1})} \frac{1}{1+p} \sum_{s=-\infty}^{t} \frac{p}{(1+p)} (\frac{1}{1+p})^{t-s} (\mathcal{A}^s_{kt} - P_{kt} C^s_{kt} + \int_0^1 (1-\tau) w_{kt}(z) h^s_{kt}(z) dz + T^s_{kt}) \\
&= \frac{1}{\mathcal{E}_t(R_{t,t+1})} \frac{1}{1+p} (\mathcal{A}^a_{kt} + (1-\tau) P_{kt} Y_{kt} + T^a_{kt} - P_{kt} C^a_{kt}) \\
\mathcal{H}^a_{kt+1} &= \sum_{v=t+1}^{\infty} R_{t+1,v} (1-\tau) Y_{kv} P_{kv} = \frac{R_{t,t+1}}{R_{t,t+1}} \sum_{v=t+1}^{\infty} R_{t+1,v} (1-\tau) Y_{kv} P_{kv} \\
&= \frac{1}{R_{t,t+1}} (\sum_{v=t}^{\infty} R_{t,v} (1-\tau) Y_{kv} P_{kv} - R_{t,t} (1-\tau) Y_{kt} P_{kt}) = \frac{1}{R_{t,t+1}} (\mathcal{H}^a_{kt} - (1-\tau) Y_{kt} P_{kt})
\end{aligned}$$

Therefore

$$\begin{aligned}
\Phi_{kt+1} P_{kt+1} C^a_{kt+1} &= \mathcal{A}^a_{kt+1} + \mathcal{H}^a_{kt+1} = \mathcal{A}^a_{kt+1} + \frac{1}{\mathcal{E}_t(R_{t,t+1})} (\mathcal{H}^a_{kt} - (1-\tau) Y_{kt} P_{kt}) \\
&= \mathcal{A}^a_{kt+1} + \frac{1}{\mathcal{E}_t(R_{t,t+1})} (\Phi_{kt} P_{kt} C^a_{kt} - \mathcal{A}^a_{kt} - (1-\tau) Y_{kt} P_{kt}) \\
&= \mathcal{A}^a_{kt+1} + \frac{1}{\mathcal{E}_t(R_{t,t+1})} (\Phi_{kt} P_{kt} C^a_{kt} - \mathcal{A}^a_{kt+1} - P_{kt} C^a_{kt}) \\
&= -p \mathcal{A}^a_{kt+1} + \frac{1}{\mathcal{E}_t(R_{t,t+1})} P_{kt} C^a_{kt} (\Phi_{kt} - 1) \\
&= -p \mathcal{A}^a_{kt+1} + \frac{1}{\mathcal{E}_t(R_{t,t+1})} P_{kt} C^a_{kt} \frac{[\beta(1+r_{kt})]^\sigma}{(1+p)(1+r_{kt})} \frac{\xi_{kt+1}}{\xi_{kt}} \Phi_{kt+1}
\end{aligned}$$

From where

$$C^a_{kt} = [\beta(1+r_{kt})]^{-\sigma} \left[C^a_{kt+1} + \frac{p}{\Phi_{kt+1}} \frac{\mathcal{A}^a_{kt+1}}{P_{kt+1}} \right] \frac{\xi_{kt}}{\xi_{kt+1}} \tag{46}$$

A.2 Price-setting decisions

Pricing behaviour is taken as in Rotemberg and Woodford (1997) and Steinsson (2003). Households are able to reset their price in each period with probability $1-\gamma$ in which case they re-contract a new price P^F_H. For the rest of the household sector the price will rise at the steady state rate of domestic inflation $\overline{\Pi}_H$ with probability γ:

$$P_{Hkt} = \overline{\Pi}_{Hk} P_{Hkt-1}$$

Those who recontract a new price (with probability $1-\gamma$), are split into backward-looking individuals and forward-looking individuals, in proportion ω, such that the aggregate index of prices set by them is

$$P^\times_{Hkt} = (P^F_{Hkt})^{1-\omega} (P^B_{Hkt})^\omega \tag{47}$$

Backward-looking individuals set their prices according to the rule of thumb:

$$P^{B}_{Hkt} = P^{\times}_{Hkt-1}\Pi_{Hkt-1}(\frac{Y_{kt-1}}{Y^{n}_{kt-1}})^{\delta} \tag{48}$$

where

$$\Pi_{Hkt} = \frac{P_{Hkt}}{P_{Hkt-1}}$$

and Y^{n}_{kt} is the efficient level of output.

We define log deviations from the steady state domestic price levels for both types of price-setters as:

$$\hat{P}^{B}_{Hkt} = \ln\frac{P^{\bar{k}}_{Hkt}}{P_{Hkt}}, \hat{P}^{F}_{Hkt} = \ln\frac{P^{F}_{Hkt}}{P_{Hkt}}$$

A.2.1 Forward-looking price-setters

From the first order conditions (40) and (41) it follows that:

$$\frac{v_h(h_{ks}(z), \xi_{ks})}{(1-\tau)u_C(C^{i}_{ks}, \xi_{ks})} = \frac{w_{ks}(z)}{P_s} \tag{49}$$

so nominal wage is defined as:

$$w_{kt}(z) = \frac{v_y(y_{kt}(z), \xi_{kt})}{(1-\tau)u_C(C^{i}_{kt}, \xi_{kt})}P_{kt}$$

Production possibilities are specified as follows:

$$y_{kt}(z) = h_{kt}(z)$$

The cost of supplying a good is given as $Cost(z) = \frac{1}{\mu^w}w_{ks}(z)h_{ks}(z) = \frac{1}{\mu^w}w_{ks}(z)y_{ks}(z)$. Where we assume some labour subsidy μ^w. We do not assume any other taxes and the labour cost is the only cost.

Each producer understands that sales depend on demand, which is a function of price, intra-temporal consumption optimisation implies

$$y_{ks}(z) = \left(\frac{p_{Hk}(z)}{P_{Hk}}\right)^{-\epsilon_t} Y_k.$$

Maximisation of expected profit requires the solution of:

$$\max_{p_{Hkt}(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t}R_{t,s}\left[p_{Hkt}(z)y_{ks}(z) - \frac{1}{\mu^w}w_{ks}(z)y_{ks}(z)\right]$$

$$\max_{p_{Hkt}(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t}R_{t,s}P^{\epsilon}_{Hks}Y_{ks}\left[p^{1-\epsilon}_{Hkt}(z) - \frac{1}{\mu^w}w_{ks}(z)p^{-\epsilon}_{Hkt}(z)\right]$$

which implies the following first order condition:

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} Y_{ks} \left(\frac{p_{Hkt}(z)}{P_{Hks}} \right)^{-\epsilon_t} [p_{Hkt}(z) - \frac{\mu_t}{(1-\tau)} M_{kt}(z)]$$

where $\mu_t = -\frac{\epsilon_t}{1-\epsilon_t}$, $M_{kt}(z)$ is marginal cost and $R_{t,s}$ is discount factor. This condition holds for both flexible and fixed price equilibria. However, for the fixed price equilibrium the nominal marginal cost is a function of price, set at the period t. Substituting for the nominal marginal cost, we get a final equation for the optimal $p_{Hkt}(z) = p^f_{Hkt}(z)$

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} Y_{ks} \left(\frac{p^f_{Hkt}(z)}{P_{Hks}} \right)^{-\epsilon_t} \left[p^f_{Hkt}(z) - \frac{\frac{\mu_t P_{ks}}{(1-\tau)} v_y(\left(\frac{p^f_{Hkt}(z)}{P_{Ha,s}} \right)^{-\epsilon_t} Y_{ks}, \xi_{ks})}{u_C(C_{ks}, \xi_{ks})} \right] \quad (50)$$

where τ is constant wage income tax. The linearisation of the equation (**??**) can be found in Rotemberg and Woodford (1997) for the closed economy case. We briefly repeat it here for the open economy.

First of all, each term in the price-setting first order conditions (**??**) is the product of two terms, the term in curly brackets and the term in square brackets. The term in the square brackets vanishes in the equilibrium so its deviations from the equilibrium are of first order. Therefore, all products of it with the first term will be higher than of first order, unless the first term is taken at its equilibrium level, which is $(\gamma\beta)^{s-t}$, up to some constant multiplier.

Linearising the term in square brackets yields:

$$\ln \left[\frac{p^f_{Hkt}(z)}{P_{Hks}} - \frac{\mu_t}{(1-\tau)} \frac{v_y(\left(\frac{p^f_{Hkt}(z)}{P_{Hks}} \right)^{-\epsilon_t} Y_{ks}, \xi_{ks})}{u_C(C_{ks}, \xi_{ks})} \right]$$

$$= \hat{p}^f_{Hkt} - [\sum_{m=1}^{s-t} \hat{\pi}_{Hkt+m} + \frac{1}{\psi}\hat{Y}_{ks} + \frac{1}{\sigma}\hat{C}_{ks} + \alpha_n \hat{S}_{k\bar{k}s}$$

$$- \frac{\epsilon_t}{\psi}\{\hat{p}^f_{Hkt} - \sum_{m=1}^{s-t} \pi_{Hkt+m}\} + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{ks} + \hat{\eta}_{ks}]$$

where $\hat{S}_{k\bar{k}} = \frac{P_{H\bar{k}}}{P_{Hk}}$ are two terms of trade and

$$\sigma = -\frac{u_C(C,1)}{u_{CC}(C,1)C}, \quad \psi = \frac{v_y(Y,1)}{v_{yy}(Y,1)Y}$$

We solve out this equation for prices and, using the fact that $\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}\sum_{m=1}^{s-t}\pi_{Hkt+m} = \frac{1}{1-\gamma\beta}\sum_{m=1}^{\infty}(\gamma\beta)^m \pi_{Hkt+m}$ we obtain the following formula for the forward-looking individuals:

$$\hat{p}^f_{Hkt} = \sum_{m=1}^{\infty}(\gamma\beta)^k \pi_{Hkt+m} + \frac{(1-\gamma\beta)}{1+\frac{\epsilon}{\psi}} \sum_{s=t}^{\infty}(\gamma\beta)^{s-t}[\alpha_n \hat{S}_{k\bar{k}s} \quad (51)$$

$$+ \frac{1}{\psi}\hat{Y}_{ks} + \frac{1}{\sigma}\hat{C}_{ks} + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{ks} + \hat{\eta}_{ks}]$$

Here we also used the fact that the linearisation of the similar equation for the flexible price equilibrium helps to get rid of shocks and write down the optimisation equation in terms of gaps with natural levels for output and consumption. Here $\alpha_n \hat{S}_{k\bar{k}}$ comes in as the result of the wedge between consumption of the CPI basket and the production of domestic goods and different prices set on them. The constant tax rate, τ, does not enter the final formula when written in log-deviations from equilibrium (see Benigno and Benigno (2000) for similar derivation).

This can be rewritten in a quasi-differenced form as:

$$\begin{aligned} \hat{p}^f_{Hkt} &= \gamma\beta\hat{p}^f_{Hkt+1} + \gamma\beta\pi_{Hkt+1} \\ &+ \frac{1-\gamma\beta}{1+\frac{\epsilon}{\psi}}\left(\alpha^a_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{kt} + \hat{\eta}_{kt}\right) \end{aligned} \tag{52}$$

A.2.2 Rule of thumb price-setters and Phillips curve

The rule of thumb price-setters use formula (48) to set the new price. The linearisation of this equation (using (47)) straightforwardly yields:

$$\hat{P}^b_{Hkt} = (1-\omega)\ln\frac{P^f_{Hkt-1}}{P_{Hkt-1}} + \omega\ln\frac{P^b_{Hkt-1}}{P_{Hkt-1}} - \ln\Pi_{Hkt} + \ln\Pi_{Hkt-1} + \delta\ln(\frac{Y_{kt-1}}{Y^n_{kt-1}})$$

so we have the following equations

$$\begin{aligned} \hat{P}^b_{Hkt} &= (1-\omega)\hat{P}^f_{Hkt-1} + \omega\hat{P}^b_{Hkt-1} - \pi_{Hkt} + \pi_{Hkt-1} + \delta y_{kt-1} \\ \pi_{Hkt} &= \frac{(1-\gamma)}{\gamma}((1-\omega)\hat{P}^f_{Hkt} + \omega\hat{P}^b_{Hkt}) \\ \hat{p}^f_{Hkt} &= \gamma\beta\hat{p}^f_{Hkt+1} + \gamma\beta\pi_{Hkt+1} + \frac{1-\gamma\beta}{1+\frac{\epsilon}{\psi}}[\alpha_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} \\ &+ (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{kt} + \hat{\eta}_{kt}] \end{aligned}$$

Doing manipulations similar to Steinsson (2003) (A.1)-(A.6) we eliminate $\hat{P}^b_{Hkt}$ and $\hat{p}^f_{Hkt}$ and obtain the following specification of the Phillips curve

$$\begin{aligned} \pi_{Hkt} &= \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}\beta\pi_{Hkt+1} + \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\pi_{Hkt-1} \\ &+ \frac{(1-\gamma)\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta\hat{Y}_{kt-1} - \frac{(1-\gamma)\gamma\beta\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta\hat{Y}_{kt} \\ &+ \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi}{(\gamma+\omega(1-\gamma+\gamma\beta))(\psi+\epsilon)}[\alpha_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} \\ &+ (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{kt} + \hat{\eta}_{kt}] \end{aligned} \tag{53}$$

Substituting taste/technology shock from (22) we come to the form written in gaps:

$$\begin{aligned}\pi_{Hkt} &= \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}\beta\pi_{Hkt+1} + \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\pi_{Hkt-1} \\ &+ \frac{(1-\gamma)\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta y_{kt-1} - \frac{(1-\gamma)\gamma\beta\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta y_{kt} \\ &+ \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi}{(\gamma+\omega(1-\gamma+\gamma\beta))(\psi+\epsilon)}[\alpha_n s_{k\bar{k}t} + \frac{1}{\psi}y_{kt} + \frac{1}{\sigma}c_{kt} + \hat{\eta}_{kt}]\end{aligned} \tag{54}$$

Note that when $\omega = 0$ then the Phillips curve collapses to the standard forward-looking specification:

$$\pi_{Hkt} = \beta\pi_{Hkt+1} + \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}\left(\alpha_n s_{k\bar{k}t} + \frac{1}{\psi}y_{kt} + \frac{1}{\sigma}c_{kt} + \hat{\eta}_{kt}\right)$$

When $\omega = 1$ then the Phillips curve takes the specification

$$\pi_{Hkt} = \frac{\gamma\beta}{(1+\gamma\beta)}\pi_{Hkt+1} + \frac{1}{1+\gamma\beta}\pi_{Hkt-1} - \frac{(1-\gamma)}{1+\gamma\beta}(\gamma\beta\delta y_{kt} - \delta y_{kt-1}). \tag{55}$$

This equation was obtained by integrating and can contain extra solutions. We are looking for solution without forward looking components, as suggested by initial formula (48). Such a solution exists and can be written in the form of accelerationist Phillips curve:

$$\pi_{Hkt} = \pi_{Hkt-1} + (1-\gamma)\delta y_{kt-1}. \tag{56}$$

Finally, (53) is a linear combination of forward-looking specification and rule of thumb specification:

$$\begin{aligned}\pi_{Hkt} &= \frac{\omega(1+\gamma\beta)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}(\frac{\gamma\beta}{(1+\gamma\beta)}\pi_{Hkt+1} \\ &+ \frac{1}{1+\gamma\beta}\pi_{Hkt-1} - \frac{(1-\gamma)}{1+\gamma\beta}(\gamma\beta\delta y_{kt} - \delta y_{kt-1})) \\ &+ \frac{\gamma(1-\omega)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}(\beta\pi_{Hkt+1} + \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}z_{kt}).\end{aligned} \tag{57}$$

where we need to substitute (55) with (56) before doing numerical simulations:

$$\begin{aligned}\pi_{Hkt} &= \frac{\omega(1+\gamma\beta)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}(\pi_{Hkt-1} + (1-\gamma)\delta y_{kt-1}) \\ &+ \frac{\gamma(1-\omega)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}(\beta\pi_{Hkt+1} + \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}z_{kt}).\end{aligned} \tag{58}$$

Finally, (58) can be rewritten as

$$\pi_{Hkt} = \chi\beta\pi_{Hkt+1} + (1-\chi)\pi_{Hkt-1} + \kappa_c c_{kt} + \kappa_{y0}y_{kt} + \kappa_{y1}y_{kt-1} + \kappa_s s_{k\bar{k}t}$$

where

$$\chi = \frac{\gamma(1-\omega)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}, \ \kappa_c = \frac{(1-\omega)(1-\gamma\beta)(1-\gamma)\psi}{(\gamma(1-\omega)+\omega(1+\gamma\beta))(\psi+\epsilon)\sigma}$$
$$\kappa_{y0} = \frac{(1-\omega)(1-\gamma\beta)(1-\gamma)}{(\gamma(1-\omega)+\omega(1+\gamma\beta))(\psi+\epsilon)}, \ \kappa_{y1} = \frac{\omega(1+\gamma\beta)(1-\gamma)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}\delta$$
$$\kappa_s = \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi(1-\alpha_d)}{(\gamma+\omega(1-\gamma+\gamma\beta))(\psi+\epsilon)}$$

A.3 Steady State

In symmetric steady state with zero inflation and prices normalised to one, the following relationships should hold:

$$A = (1+i)(A+(1-\tau)Y-C) \tag{59}$$
$$B = (1+i)(B+G-\tau Y) \tag{60}$$
$$\Phi C = A+H \tag{61}$$
$$H = \frac{(1-\tau)(1+i)(1+p)}{(1+i)(1+p)-1}Y \tag{62}$$
$$C = \theta Y \tag{63}$$

In order to obtain relationship (62) we compute steady state human capital as net present value of steady state income, accounting for mortality rate. Equations (59) and (60) are consistent with that $Y = C+G$. We assume that the steady state private consumption constitute share θ of the steady state income, so the government consumption is $G = (1-\theta)Y$.

A steady state value for Φ is:

$$\Phi = \left(1-\frac{[\beta(1+i)]^{\sigma}}{(1+p)(1+i)}\right)^{-1}$$

We substitute this into relationships (59)–(63), and simplify them to obtain:

$$(\theta - \left(1-\frac{[\beta(1+i)]^{\sigma}}{(1+p)(1+i)}\right)\frac{(1+i)(1+p)(1-\tau)}{(1+i)(1+p)-1})Y = \left(1-\frac{[\beta(1+i)]^{\sigma}}{(1+p)(1+i)}\right)A \tag{64}$$

$$A = B = -\frac{(1+i}{i})(1-\tau-\theta)Y \tag{65}$$

Thus, in a steady state $A = B$, that can be found from equation (65), if we know θ and τ. Alternatively, as the budget is not balanced in equilibrium, so there is some steady state level of the government debt, $B = A = BY$, and equation (65) can be used to find the steady state level of tax rate, which ensures this steady state level of debt, given the interest rate. Equation (64) is equation for i, the steady state level of interest rate; it has a unique solution and in equilibrium $1+i > 1/\beta$.

A.4 Aggregate Demand

Aggregation implies:

$$Y_k = C_{Hk} + C^{*\bar{k}}_{Hk} + G_{Hk} \tag{66}$$

its linearisaion yields:

$$\hat{Y}_{ks} + \frac{1}{2}\hat{Y}^2_{ks} = \theta\alpha_d(\hat{C}_{Hks} + \frac{1}{2}\hat{C}^2_{Hks}) + \theta\alpha_n(\hat{C}^{*\bar{k}}_{Hks} + \frac{1}{2}\hat{C}^{*\bar{k}2}_{Hks}) + (1-\theta)(\hat{G}_{Hks} + \frac{1}{2}\hat{G}^2_{Hks})$$

On the other hand,

$$C_k = C_{Hk} + C_{H\bar{k}} \tag{67}$$

and its linearisation yields:

$$\hat{C}_k = \alpha_d(\hat{C}_{Hk} + \frac{1}{2}\hat{C}^2_{Hk}) + \alpha_n(\hat{C}_{H\bar{k}} + \frac{1}{2}\hat{C}^2_{H\bar{k}}) - \frac{1}{2}\hat{C}^2_k$$

Now we substitute consumption into the aggregate demand and obtain:

$$\begin{aligned}\hat{Y}_k &= \theta\hat{C}_k + (1-\theta)(\hat{G}_{Hk} + \frac{1}{2}\hat{G}^2_{Hk}) + \theta\frac{1}{2}\hat{C}^2_k - \frac{1}{2}\hat{Y}^2_k \\ &+ \theta\alpha_n(\hat{C}^{*\bar{k}}_{Hk} - \hat{C}_{H\bar{k}}) + \frac{1}{2}\theta\alpha_n(\hat{C}^{*\bar{k}2}_{Hk} - \hat{C}^2_{H\bar{k}})\end{aligned}$$

Now, the *exact* relationships between consumption are

$$C_{Hk} = \alpha_d \left(\frac{P_{Hk}}{P_k}\right)^{-\eta} C_k, \tag{68}$$

We also have *exact* price indexes

$$P_k = (\alpha_d P^{1-\eta}_{Hk} + \alpha_n P^{1-\eta}_{H\bar{k}})^{\frac{1}{1-\eta}} \tag{69}$$

We linearise them and substitute in aggregate demand and assume symmetric countries. We finally obtain the linear aggregate demand relationship:

$$\hat{Y}_k = \theta\alpha_d\hat{C}_k + \theta\alpha_n\hat{C}^*_{\bar{k}} + (1-\theta)\hat{G}_{kt} + 2\theta\eta\alpha_d\alpha_n\hat{S}_{k\bar{k}},$$

and its second-order version:

$$\begin{aligned}\hat{Y}_k &= \theta\alpha_d\hat{C}_k + \theta\alpha_n\hat{C}_{\bar{k}} + (1-\theta)\hat{G}_k + 2\theta\eta\alpha_d\alpha_n\hat{S}_{k\bar{k}} \\ &+ \frac{1}{2}\theta\alpha_d\hat{C}^2_k + \frac{1}{2}\theta\alpha_n\hat{C}^2_{\bar{k}} - \frac{1}{2}\hat{Y}^2_k + \frac{1}{2}(1-\theta)\hat{G}^2_k + \theta\alpha_d\alpha_n\eta\hat{S}\hat{C}^*_{\bar{k}} + \theta\alpha_d\alpha_n\eta\hat{S}_{k\bar{k}}\hat{C}_k\end{aligned} \tag{70}$$

A.5 Risk sharing condition

We derive it for the case of infinitely-lived consumers only.

From the first-order conditions it follows

$$\beta \frac{u_C(C_{t+1})}{u_C(C_t)} \frac{P_{at}}{P_{at+1}} = \frac{1}{1+i_t}, \beta \frac{u_C(C^*_{bt+1})}{u_C(C^*_{bt})} \frac{P^*_{bt} E_t}{P^*_{bt+1} E_{t+1}} = \frac{1}{(1+i_t)} \tag{71}$$

divide one by another to get

$$\frac{u_C(C^*_{bt}, \xi_{bt})}{Q_t u_C(C_{at}, \xi_{at})} = \frac{u_C(C^*_{bt+1}, \xi_{bt+1})}{u_C(C_{at+1}, \xi_{at+1}) Q_{t+1}} \tag{72}$$

We can iterate this forward to obtain:

$$\frac{u_C(C^*_{bt}, \xi_{bt})}{Q_t u_C(C_{at}, \xi_{at})} = \frac{u_C(C^*_{bt+m}, \xi_{bt+m})}{u_C(C_{at+m}, \xi_{at+m}) Q_{t+m}} = \vartheta_{t+m}(C_{jt+m}, \xi_{jt+m}, Q_{t+m}) \tag{73}$$

where m is large.

We want formula (73) would be written in terms of term of trade, not the real exchaneg rate. Using $Q = (\alpha^b_n + \alpha^b_d S^{1-\eta})^{\frac{1}{1-\eta}} (\alpha^a_d + \alpha^a_n S^{(1-\eta)})^{-\frac{1}{1-\eta}}$ we obtain:

Substitute linearised consumption and term of trade into (73) and get:

$$\begin{aligned} \hat{C}_a &= \hat{C}^*_b + \sigma(\alpha_d - \alpha_n)\hat{S} + \frac{1}{2}\sigma(\alpha_d - \alpha_n)^2 \hat{S}^2 \\ &- \frac{1}{2} b\hat{C}_a^2 - d\hat{C}_a \hat{\xi}_a + \frac{1}{2} b \hat{C}_b^{*2} - (\alpha_d - \alpha_n)\hat{C}_a \hat{S} - (\alpha_d - \alpha_n) g\hat{\xi}_a \hat{S} \\ &- g(\hat{\xi}_a - \hat{\xi}^*_b) - \frac{1}{2} ga\hat{\xi}_a^2 + \frac{1}{2} ga \hat{\xi}_b^{*2} + d\hat{C}^*_b \hat{\xi}^*_b - \hat{\Theta}_{t+m} \end{aligned} \tag{74}$$

Here $\hat{\Theta}_{t+m}$ can be treated as shock, because by sufficient iterating forward, we make C_{kt+m} Q_{t+m} close to terminal conditions, which are explicitely defined for jump variables C_{kt+m}, and relative prices. We can make m as big as we want, and if we deal with small shocks, then for large m all the terms are heavily discounted and their impact can be made as small as required.

A.6 Useful relationsips

We can find $\hat{S}_{k\bar{k}}$ from the risk sharing conditions and substitute it into aggregate demand:

$$\begin{aligned} \hat{Y}_k &= \theta\alpha_d \left(1 + \frac{2\alpha_n \eta}{\sigma(\alpha_d - \alpha_n)}\right) \hat{C}_k + \theta\alpha_n \left(1 - \frac{2\alpha_d \eta}{\sigma(\alpha_d - \alpha_n)}\right) \hat{C}_{\bar{k}} + (1-\theta)\hat{G}_k \\ &- \theta\alpha_d\alpha_n\eta(\alpha_d - \alpha_n)\hat{S}^2_{k\bar{k}} + \theta\alpha_d \left(\frac{\alpha_n \eta b}{\sigma(\alpha_d - \alpha_n)} + \frac{1}{2}\right) \hat{C}_k^2 + \theta\alpha_n \left(\frac{1}{2} - \frac{\alpha_d \eta b}{\sigma(\alpha_d - \alpha_n)}\right) \hat{C}^2_{\bar{k}} \\ &+ \frac{2\theta\alpha_d\alpha_n\eta d}{\sigma(\alpha_d - \alpha_n)} \hat{C}_k \hat{\xi}_k + \frac{2\theta\alpha_d\alpha_n\eta(\alpha_d - \alpha_n)}{\sigma(\alpha_d - \alpha_n)} g\hat{\xi}_k \hat{S}_{k\bar{k}} - \frac{1}{2}\hat{Y}_k^2 + \frac{1}{2}(1-\theta)\hat{G}_k^2 \\ &+ \frac{2\theta\alpha_d\alpha_n\eta g}{\sigma(\alpha_d - \alpha_n)}(\hat{\xi}_k - \hat{\xi}_{\bar{k}}) + \frac{1}{2}\frac{2\theta\alpha_d\alpha_n\eta ga}{\sigma(\alpha_d - \alpha_n)}\hat{\xi}_k^2 - \frac{1}{2}\frac{2\theta\alpha_d\alpha_n\eta ga}{\sigma(\alpha_d - \alpha_n)}\hat{\xi}^2_{\bar{k}} - \frac{2\theta\alpha_d\alpha_n\eta d}{\sigma(\alpha_d - \alpha_n)}\hat{C}_{\bar{k}}\hat{\xi}_{\bar{k}} \\ &+ \theta\alpha_d\alpha_n\eta\left(1 + \frac{2}{\sigma}\right)\hat{C}_k \hat{S}_{k\bar{k}} + \theta\alpha_d\alpha_n\eta\hat{S}_k \hat{C}_{\bar{k}} + \frac{2\theta\alpha_d\alpha_n\eta}{\sigma(\alpha_d - \alpha_n)}\hat{\Theta}_{t+m} \end{aligned}$$

We can take sum of them and obtain the following formula

$$
\begin{aligned}
\left(\hat{C}_a+\hat{C}_b\right) &= \frac{1}{\theta}\left(\hat{Y}_a+\hat{Y}_b\right)-\frac{(1-\theta)}{\theta}\left(\hat{G}_a+\hat{G}_b\right)-\frac{1}{2}\left(\hat{C}_a^2+\hat{C}_b^2\right) \\
&+\frac{1}{2\theta}\left(\hat{Y}_a^2+\hat{Y}_b^2\right)-\frac{1}{2}\frac{(1-\theta)}{\theta}\left(\hat{G}_a^2+\hat{G}_b^2\right)+2\alpha_d\alpha_n\eta\left(\alpha_d-\alpha_n\right)\hat{S}_{ab}^2 \\
&-2\alpha_d\alpha_n\eta\frac{g}{\sigma}\left(\hat{\xi}_a-\hat{\xi}_b\right)\hat{S}_{ab}-\frac{2}{\sigma}\alpha_d\alpha_n\eta\left(\hat{C}_a-\hat{C}_b\right)\hat{S}_{ab}+tip
\end{aligned}
\tag{75}
$$

where tip are terms intependent of policy (including $\hat{\Theta}_{t+m}$). We use this formula later in the text.

A.7 Government expenditures in flexible price equilibrium

As aggregate demand relationships and risk sharing condition always hold, they are identities so we can differentiate them with respect to government expenditures, to obtain relationships which will be valid along the solution to dynamic system.

Differentiate the aggregate demand relationships (with rspect to G_a, and the other two equations can be obtained using symmetry of economies):

$$
\begin{aligned}
\frac{\partial Y_a}{\partial G_a} &= \alpha_d\left(\frac{P_{Ha}}{P_a}\right)^{-\eta}\frac{\partial C_a}{\partial G_a}+\alpha_n\left(\frac{P_{Ha}^{*b}}{P_b^{*b}}\right)^{-\eta}\frac{\partial C_b}{\partial G_a} \\
&+\eta\alpha_n\alpha_d S_{ab}^{-\eta}\left(C_a\left(\frac{P_{Ha}}{P_a}\right)^{1-2\eta}+C_b\left(\frac{P_{Ha}^{*b}}{P_b^{*}}\right)^{1-2\eta}\right)\frac{\partial S_{ab}}{\partial G_a}+1
\end{aligned}
\tag{76}
$$

$$
\begin{aligned}
\frac{\partial Y_b^{*b}}{\partial G_a} &= \alpha_n\left(\frac{P_{Hb}}{P_a}\right)^{-\eta}\frac{\partial C_a}{\partial G_a}+\alpha_d\left(\frac{P_{Hb}^{*b}}{P_b^{*b}}\right)^{-\eta}\frac{\partial C_b}{\partial G_a} \\
&-\alpha_n\eta\alpha_d S_{ab}^{-2+\eta}\left(C_a\left(\frac{P_{Hb}}{P_a}\right)^{1-2\eta}+C_b\left(\frac{P_{Hb}^{*b}}{P_b^{*}}\right)^{1-2\eta}\right)\frac{\partial S_{ab}}{\partial G_a}
\end{aligned}
\tag{77}
$$

Differentiation of the risk sharing condition yields:

$$
\begin{aligned}
\frac{\partial C_{bt}^{*}}{\partial G_a} &= \left(\left(\alpha_d(\alpha_d^b S_{ab}^{(1-\eta)}+\alpha_n^b)^{-1}-(\alpha_d^a+\alpha_n^a S_{ab}^{1-\eta})^{-1}\alpha_n\right)S_{ab}^{-\eta}\frac{\partial S_{ab}}{\partial G_a}\frac{u_C(C_{at})}{u_{CC}(C_{bt}^{*})}+\frac{u_{CC}(C_{at})}{u_{CC}(C_{bt}^{*})}\frac{\partial C_{at}}{\partial G_a}\right) \\
&\times\vartheta_{ab}(\alpha_d S_{ab}^{(1-\eta)}+\alpha_n)^{\frac{1}{1-\eta}}(\alpha_d+\alpha_n S_{ab}^{1-\eta})^{-\frac{1}{1-\eta}}
\end{aligned}
\tag{78}
$$

A labour market equilibrium condition () is also an identity along the dynamic path of adjustment. Its differention yields:

$$
\frac{1-\tau}{\mu}u_{CC}(C_a,\xi_a)\frac{\partial C_a}{\partial G_a}=\frac{P_a}{P_{Ha}}v_{yy}(Y_a,\xi_a)\frac{\partial Y_a}{\partial G_a}+v_y(Y_a,\xi_a)\left(\frac{P_a}{P_{Ha}}\right)^{\eta}\alpha_{nSab}^{-\eta}\frac{\partial S_{ab}}{\partial G_a}\tag{79}
$$

$$
\frac{1-\tau}{\mu}u_{CC}(C_b,\xi_b)\frac{\partial C_b}{\partial G_a}=\frac{P_b}{P_{Hb}}v_{yy}(Y_b,\xi_b)\frac{\partial Y_b}{\partial G_a}-v_y(Y_b,\xi_a)S_{ab}^{-2+\eta}\left(\frac{P_b}{P_{Hb}^{b}}\right)^{\eta}\alpha_n\frac{\partial S_{ab}}{\partial G_a}\tag{80}
$$

These five relationships can be solved with respect to five unknowns: $\frac{\partial S_{ab}}{\partial G_a}, \frac{\partial Y_a}{\partial G_a}, \frac{\partial Y_b}{\partial G_a}, \frac{\partial C_a}{\partial G_a}, \frac{\partial C_b}{\partial G_a}$ as functions of C_a, C_b, S_{ab}.

Additionally, a socially optimal fiscal policy should aim to maximise union-wide social welfare, subject to static constraints (aggregate demand, risk sharing, labour market equilibrium condition). As dirivatives of these constraints all equal to zero (along the dynamic solution) then

$$\frac{\partial}{\partial G_a}[u(C_a,\xi_a)+f(G_a,\xi_a)-v(Y_a,\xi_a)+u(C_b,\xi_b)+f(G_b,\xi_b)-v(Y_b,\xi_b)]$$
$$=u_C(C_a,\xi_a)\frac{\partial C_a}{\partial G_a}+f_{G_a}(G_a,\xi_a)-v_y(Y_a,\xi_a)\frac{\partial Y_a}{\partial G_a}+u_C(C_b,\xi_b)\frac{\partial C_b}{\partial G_a}-v_y(Y_b,\xi_b)\frac{\partial Y_b}{\partial G_a}=0 \tag{81}$$

We can substitute formulae for $\frac{\partial Y_a}{\partial G_a}, \frac{\partial Y_b}{\partial G_a}, \frac{\partial C_a}{\partial G_a}, \frac{\partial C_b}{\partial G_a}$. The resulting formula can be linearised around steady state to yield:

$$g_{k\xi}\hat{\xi}_k+g_{\bar{k}\xi}\hat{\xi}_{\bar{k}}=\hat{G}^n_k+g_{ks}\hat{S}^n_{k\bar{k}}+g_{kc}\hat{C}^n_k+g_{\bar{k}c}\hat{C}^n_{\bar{k}}+g_{ky}\hat{Y}^n_k+g_{\bar{k}y}\hat{Y}^n_{\bar{k}} \tag{82}$$

in the flexible-price equilibrium, labelled with superscript ^{n}. In this formula

$$\begin{aligned}
g_{k\xi} &= 1+\frac{\sigma\left(\psi+\theta\sigma-\theta\alpha_n\left(\psi+\theta\sigma\right)-2\alpha_n\alpha_d\theta\left(\sigma-\eta\right)\left(1-\theta\right)\right)}{(\theta\sigma+\psi)(-4\alpha_d\theta\left(\sigma-\eta\right)\left(1-\alpha_d\right)+\psi+\theta\sigma)}\\
g_{\bar{k}\xi} &= -\frac{2\theta\sigma\alpha_n\alpha_d\left(\sigma-\eta\right)\left(1-\theta\right)+\theta\alpha_d\left(\psi+\theta\sigma\right)}{(\theta\sigma+\psi)(-4\alpha_d\theta\left(\sigma-\eta\right)\alpha_n+\psi+\theta\sigma)}\\
g_{ks} &= -\frac{2\sigma\alpha_d\alpha_n\theta\eta}{(-4\theta\alpha_d\alpha_n\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\
g_{kc} &= \frac{\theta\sigma\alpha_n(2\theta\left(\sigma-\eta\right)\alpha_d-\left(\psi+\theta\sigma\right))}{(\theta\sigma+\psi)(-4\theta\alpha_d\alpha_n\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\
g_{\bar{k}c} &= \frac{\theta\sigma\alpha_d(2\theta\left(\sigma-\eta\right)\alpha_n-\left(\psi+\theta\sigma\right))}{(\theta\sigma+\psi)(-4\theta\alpha_d\alpha_n\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\
g_{ky} &= \frac{\sigma(\psi+\theta\sigma-2\theta\left(\sigma-\eta\right)\alpha_d\alpha_n)}{(\theta\sigma+\psi)(-4\alpha_d\alpha_n\theta\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\
g_{\bar{k}y} &= -\frac{2\theta\sigma\alpha_n\alpha_d(\sigma-\eta)}{(\theta\sigma+\psi)(-4\alpha_d\alpha_n\theta\left(\sigma-\eta\right)+\psi+\theta\sigma)}
\end{aligned}$$

B Social loss function

The one-period utility function can be obtained by linearisation of one-period utility function in (1) up to the second-order terms (we assume symmetry):

$$\begin{aligned}
\mathcal{W}_a + \mathcal{W}_b = {} & u_C(C,1)C[\hat{C}_a + \hat{C}_b + \frac{1}{2}(1-\frac{1}{\sigma})\left(\hat{C}_a^2 + \hat{C}_b^2\right)] \\
& + \frac{u_{C\xi}(C,1)}{u_C(C,1)}\left(\hat{C}_a\hat{\xi}_a + \hat{C}_b\hat{\xi}_b\right) + G f_G(G)[\left(\hat{G}_a + \hat{G}_b\right) \\
& + \frac{1}{2}(1-\frac{1}{\sigma_g})\left(\hat{G}_a^2 + \hat{G}_b^2\right) + \frac{f_{G\xi}(G,1)}{f_G(G,1)}\left(\hat{G}_a\hat{\xi}_a + \hat{G}_b\hat{\xi}_b\right)] \\
& - Y v_y(Y,1)[\left(\hat{Y}_a + \hat{Y}_b\right) + \frac{v_{y\xi}(y,1)}{v_y(y,1)}\left(\hat{Y}_a\hat{\xi}_a + \hat{Y}_b\hat{\xi}_b\right) \\
& + \frac{1}{2}(1+\frac{1}{\psi})\left(\hat{Y}_a^2 + \hat{Y}_b^2\right) + \frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})\left(var_z\hat{y}_a(z) + var_z\hat{y}_b(z)\right)] + tip
\end{aligned} \tag{83}$$

where

$$\sigma_g = -\frac{f_G(G,1)}{f_{GG}(G,1)G}, \qquad \psi = \frac{v_Y(Y,1)}{v_{YY}(Y,1)Y}.$$

We need to find an expression for $\frac{v_y(Y)}{u_C(C,1)C}$ and $\frac{f_G(G)}{u_C(C,1)C}$. The first condition follows from the steady state condition

$$\frac{v_h(h_s,\xi_s)}{u_C(C,1)} = \frac{1-\tau}{\mu} \tag{84}$$

and in order to derive the second expression we closely follow Beetsma and Jensen (2003). The second steady state relationship is

$$\frac{f_G(G)}{u_C(C,1)} = \frac{\frac{1-\tau}{\mu} + \sigma\frac{\theta}{\psi}}{\left(1+\sigma\frac{\theta}{\psi}\right)} \tag{85}$$

and it is derived in Additional Appendix from optimality condition for the fiscal authorities.

We now need to derive a formula for $var_z\hat{y}(z)$, along the lines in Rotemberg and Woodford (1997) and Steinsson (2003). This leads to the formula (for country a):

$$\begin{aligned}
var_z\hat{y}_{at}(z) = {} & \frac{\epsilon^2}{(1-\gamma\beta)}\left(\frac{\gamma}{1-\gamma}\pi_{Hat}^2 + \frac{\omega}{(1-\omega)}\frac{1}{(1-\gamma)}(\Delta\pi_{Hat})^2\right. \\
& \left. + \frac{\omega}{(1-\omega)}(1-\gamma)\delta^2 y_{at-1}^2 + \frac{2\omega}{(1-\omega)}\delta y_{at-1}\Delta\pi_{Hat}\right)
\end{aligned}$$

We now substitite consumption from formula (75) into (83) and, using (84) and (85), obtain:

$$\begin{aligned}
\mathcal{W}_a+\mathcal{W}_b &= U_c(C,1)C[\frac{1}{\theta}\left(1-\frac{1-\tau}{\mu}\right)\left(\hat{Y}_a+\hat{Y}_b\right)-\frac{1}{2}\frac{1}{\sigma}\left(\hat{C}_a^2+\hat{C}_b^2\right) \\
&+\frac{(1-\theta)}{\theta\left(1+\sigma\frac{\theta}{\psi}\right)}\left(\frac{1-\tau}{\mu}-1\right)\left(\hat{G}_{Ha}+\hat{G}_{Hb}\right) \\
&+\frac{1}{2}\frac{1}{\theta}\left(1-\frac{1-\tau}{\mu}(1+\frac{1}{\psi})\right)\left(\hat{Y}_a^2+\hat{Y}_b^2\right)+2\alpha_n\eta\alpha_d\left(\alpha_d-\alpha_n\right)\hat{S}_{ab}^2 \\
&-\frac{2}{\sigma}\alpha_n\eta\alpha_d\left(\hat{C}_a-\hat{C}_b\right)\hat{S}_{ab}-\frac{2\alpha_n\eta\alpha_d}{\sigma}g\left(\hat{\xi}_a-\hat{\xi}_b\right)\hat{S}_{ab} \\
&+\frac{1}{2}\frac{(1-\theta)}{\theta}\left(\frac{\frac{1-\tau}{\mu}+\sigma\frac{\theta}{\psi}}{\left(1+\sigma\frac{\theta}{\psi}\right)}(1-\frac{1}{\phi})-1\right)\left(\hat{G}_{Ha}^2+\hat{G}_{Hb}^2\right) \\
&-\frac{g}{\sigma}\hat{C}_a\hat{\xi}_a-\frac{g}{\sigma}\hat{C}_b\hat{\xi}_b-\frac{1-\tau}{\theta\mu}\frac{h}{\psi}\hat{Y}_a\hat{\xi}_a-\frac{1-\tau}{\theta\mu}\frac{h}{\psi}\hat{Y}_b^*\hat{\xi}_b^* \\
&-\frac{(1-\theta)}{\theta}\frac{\frac{1-\tau}{\mu}+\sigma\frac{\theta}{\psi}}{\left(1+\sigma\frac{\theta}{\psi}\right)}\frac{g}{\phi}\hat{G}_a\hat{\xi}_a-\frac{(1-\theta)}{\theta}\frac{\frac{1-\tau}{\mu}+\sigma\frac{\theta}{\psi}}{\left(1+\sigma\frac{\theta}{\psi}\right)}\frac{g}{\phi}\hat{G}_b\hat{\xi}_b \\
&-\frac{1-\tau}{\theta\mu}V_z\hat{y}_a(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})-\frac{1-\tau}{\theta\mu}V_z\hat{y}_b(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})]+tip(3)
\end{aligned}$$

It is clear that the same subsidy $\frac{1-\tau}{\mu}=1$ that eliminates linear term in output also eliminates linear term in government expenditures, so welfare can be simplified to:

$$\begin{aligned}
\mathcal{W}_a+\mathcal{W}_b &= -U_c(\cdot)C\frac{1}{\theta}[\frac{\theta}{2\sigma}\left(\left(\hat{C}_a-\hat{C}_a^n\right)^2+\left(\hat{C}_b-\hat{C}_b^n\right)^2\right) \\
&+\frac{1}{2\psi}\left(\left(\hat{Y}_a-\hat{Y}_a^n\right)^2+\left(\hat{Y}_b-\hat{Y}_b^n\right)^2\right)+\frac{(1-\theta)}{2\sigma}\left((G_a-G_a^n)^2+(G_b-G_b^n)^2\right) \\
&-2\alpha_n\eta\alpha_d\left(\alpha_d-\alpha_n\right)\left(\hat{S}_{ab}-\hat{S}_{ab}^n\right)^2+\frac{2\theta\alpha_n\eta\alpha_d}{\sigma}\left(\hat{S}_{ab}-\hat{S}_{ab}^n\right)\left(\hat{C}_a-\hat{C}_a^n\right) \\
&-\frac{2\theta\alpha_n\eta\alpha_d}{\sigma}\left(\hat{S}_{ab}-\hat{S}_{ab}^n\right)\left(\hat{C}_b-\hat{C}_b^n\right) \\
&-\frac{\theta}{\sigma\left(\psi+\sigma\right)}\hat{C}_a\left[\sigma\hat{Y}_a^n-\sigma\hat{C}_a^n+\alpha_n\left(\psi\sigma-2\eta\alpha_d\left(\psi+\sigma\right)\right)\hat{S}_{ab}^n\right] \\
&-\frac{\theta}{\sigma\left(\psi+\sigma\right)}\hat{C}_b\left[\sigma\hat{Y}_b^n-\sigma\hat{C}_b^n-\alpha_n\left(\psi\sigma-2\left(\psi+\sigma\right)\eta\alpha_d\right)\hat{S}_{ab}^n\right] \\
&+\frac{1}{(\psi+\sigma)}\hat{Y}_a\left[\hat{Y}_a^n-\hat{C}_a^n-\alpha_n\sigma\hat{S}_{ab}^n\right]+\frac{1}{(\psi+\sigma)}\hat{Y}_b\left[\hat{Y}_b^n-\hat{C}_b^n+\alpha_n\sigma\hat{S}_{ab}^n\right] \\
&-\frac{(1-\theta)}{(\psi+\sigma)}\hat{G}_a\left[\hat{Y}_a^n-G_a^n+\psi\alpha_n\hat{S}_{ab}^n\right]-\frac{(1-\theta)}{(\psi+\sigma)}\hat{G}_b\left[\hat{Y}_b^n-G_b^n-\psi\alpha_n\hat{S}_{ab}^n\right] \\
&-\frac{2\theta\eta\alpha_n\alpha_d}{(\psi+\sigma)}\hat{S}_{ab}\left[2\left(\left(\alpha_d-\alpha_n\right)\left(\psi+\sigma\right)+\alpha_n\psi\right)\hat{S}_{ab}^n+\left(\hat{Y}_a^n-\hat{Y}_b^n\right)-\left(\hat{C}_a^n-\hat{C}_b^n\right)\right] \\
&+V_z\hat{y}_a(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})+V_z\hat{y}_b(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})]+tip(3)
\end{aligned}$$

where we substituted natural rates for taste/technology shocks, using formula (22). This is formula (36) in the main text.

C Compensating Consumption

Having computed the social loss in stochastic equilibrium for an optimal policy, we can give an interpretation of losses in terms of 'real world' variables. This optimal policy results in stochastic volatility $\mathcal{W}$ of the key variables and steady state level of consumption C . We now find percent reduction in steady-state consumption under the benchmark policy that makes household as well off as under our optimal policy. This benchmark policy is with no stochastic volatility, but results in a new steady state level of consumption of $C + \Omega C$. We determine the percentage change in consumption Ω such that we have the same level of welfare under both policies. A form of utility function is not assumed known, but $u_C(C,1)/u_{CC}(C,1)C = -\sigma$ in the steady state.

Formula (**??**) shows that the level of the welfare (to a second order approximation) of a social planner in a monetary union of two identical countries can be written as

$$
\begin{aligned}
L &= \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(2u(C,1) + 2f(G,1) - 2v(Y,1) - u_C(C,1)C\mathcal{U}_s\right) \\
&= \frac{2}{1-\beta} \left(u(C,1) + f(G,1) - v(Y,1)\right) - Cu_C(C,1)\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\mathcal{U}_s
\end{aligned}
$$

where $\mathcal{U}$ is intra-period value of the social welfare function and C, G and Y refer to steady state levels of consumption, government spending and output. Under the benchmark policy there is no volatility, $\mathcal{U}_s \equiv 0$, so:

$$
\begin{aligned}
L_0 &= \frac{2}{1-\beta} \left(u(C+\Omega C,1) + f(G,1) - v(Y,1)\right) \\
&= \frac{2}{1-\beta} \left(u(C,1)C\Omega \left(1 - \frac{\Omega}{2\sigma}\right) + u(C,1) + f(G,1) - v(Y,1)\right) + o\left(\Omega C\right)^3
\end{aligned}
$$

An individual will be indifferent between these two policies when

$$
\Omega \left(1 - \frac{\Omega}{2\sigma}\right) + \frac{(1-\beta)}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\mathcal{U}_s = 0
$$

which is an equation for Ω. The relevant solution is:

$$
\Omega = \sigma \left(1 - \sqrt{1 + \frac{(1-\beta)}{\sigma}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\mathcal{U}_s}\right) \tag{86}
$$

We find $\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\mathcal{U}_s$ using the procedure outlined in Currie and Levine (1993), see the working paper version of this paper for details (**?**).