Inflation Persistence Revisited

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Abstract

It is commonly asserted that inflation is a jump variable in the New Keynesian Phillips curve, and thus wage-price inertia does not imply inflation inertia. We show that this "inflation flexibility proposition" is highly misleading, relying on the assumption that real variables are exogenous. In a general equilibrium setting (in which real variables not only affect inflation, but are also influenced by it) the phenomenon of inflation inertia re-emerges. Under plausible parameter values, high degrees of inflation persistence (prolonged after-effects of inflation in response to temporary money growth shocks) and under-responsiveness (prolonged effects in response to permanent shocks) can arise in the context of standard wage-price staggering models.

Keywords: Inflation persistence, wage-price staggering, New Keynesian Phillips curve, nominal inertia, monetary policy, forward-looking expectations.

JEL Classifications: E31, E32, E42, E63.

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1 Introduction

According to the path-breaking contributions associated primarily with Phelps (1978) and Fuhrer and Moore (1995), staggered nominal contracts can account for price inertia, but not inflation inertia. Specifically, the contracting model of Phelps (1978) and Taylor (1980a) implies that inflation is jump variable, responding instantaneously to exogenous macroeconomic shocks. This proposition - which may be called the "inflation flexibility proposition" - implies that there is no inflation persistence (serial correlation in inflation) independent of the persistence in the shocks. The implication is widely recognized as a deficiency of the New Keynesian Phillips curve that rests on the contracting model; it cannot account for the high degree of inflation persistence commonly described by the empirical evidence. This insight has spawned a large literature that attempts to provide new explanations for inflation persistence.[1]

In this paper we revisit this debate and argue that, under staggered nominal contracts, inflation is generally not a jump variable after all. In fact, we show the standard versions of the contract model may be compatible with high degrees of inflation persistence.

Our argument may summarized as follows. In the textbook version of New Keynesian Phillips curve, current inflation (π_t) depends on expected future inflation and some real variable (x_t), such as output, the output gap, real marginal costs, employment or unemployment: $\pi_t = E_t\pi_{t+1} + ax_t$. From this, it is commonly inferred that there is no inflation persistence independent of the persistence in x_t. After all, a one-period shock to x_t affects inflation for only one period. For this argument to hold, the real variable x_t must be viewed as exogenous. But in the context of all reasonable macro models of the Phillips curve, x_t is not exogenous. Rather, inflation π_t and, say, output x_t are both endogenous. Commonly, output (or employment, etc.) depends, among other things, on real money balances (or some other relation between money and a nominal variable). And real money balances, in turn, depend on prices, whose evolution is given by the inflation rate. Once the influence of inflation on output is taken into account in a general equilibrium context, it can be shown that, under plausible assumptions, inflation responds only gradually to shocks.

The paper is organized as follows. Section 2 presents a simple model of the New Keynesian Phillips curve and obtains the standard result that, when output is exogenous, inflation is a jump variable. We then let output depend on real money balances and derive the resulting inflation persistence. Section 3 extends this model in various standard ways. Section 4 concludes.

[1] See, for example, Christiano, Eichenbaum, and Evans (2001), Dotsey, King, and Wolman (1997), Estrella and Fuhrer (1998), Fuhrer and Moore (1995), Galí, Gertler, and López-Salido (2001), Huang and Liu (2001), Mankiw and Reis (2002), Roberts (1997), and many others.

2 Inflation Persistence in a Simple Model of the Phillips Curve

We begin with a simple, standard model of staggered price setting:

$$P_t = \alpha P_{t-1} + (1-\alpha) E_t P_{t+1} + \gamma x_t, \tag{1}$$

where P_t is the price level, γ is the "demand sensitivity parameter" (a constant), $\alpha = \frac{1}{1+\beta}$, the discount factor $\beta = \frac{1}{1+r}$, and r is the discount rate. This equation clearly implies price inertia: a one-period shock to output affects the price level for many periods. The corresponding New Keynesian Phillips curve is

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (1+\beta) x_t, \tag{2}$$

where $\pi_t \equiv P_t - P_{t-1}$ is the inflation rate.[2] Note that when the interest rate is zero (so that $\alpha = 1/2$ and $\beta = 1$), this equation reduces to the standard textbook version of the Phillips curve, for which there is no long-run tradeoff between inflation and output.

The New Keynesian Phillips curve (2) is commonly thought to imply the absence of inflation persistence. Using recursive substitution, the Phillips curve can be expressed as

$$\pi_t = \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}. \tag{3}$$

Thus if output x_t is exogenous, a one-period output shock in period t cannot affect inflation beyond that period.

But, as noted, output is not exogenous. In the standard macro models, it usually depends on real money balances. So, for simplicity, we write:

$$x_t = M_t - P_t, \tag{4}$$

where M_t denotes the money supply. Substituting this equation into equation (1), we obtain the following price equation:[3]

$$P_t = \phi P_{t-1} + \theta E_t P_{t+1} + \left(\frac{\gamma}{1+\gamma}\right) M_t, \tag{5}$$

[2] Subtract αP_t from both sides of (1) to get $(1-\alpha) P_t = -\alpha (P_t - P_{t-1}) + (1-\alpha) E_t P_{t+1} + \gamma x_t$ so that $\alpha \pi_t = (1-\alpha) E_t \pi_{t+1} + \gamma x_t$; this implies the New Keynesian Phillips curve above.

[3] To derive this equation, observe that $P_t = \alpha P_{t-1} + (1-\alpha) E_t P_{t+1} + \gamma (M_t - P_t) \Rightarrow P_t = \left(\frac{\alpha}{1+\gamma}\right) P_{t-1} + \left(\frac{1-\alpha}{1+\gamma}\right) E_t P_{t+1} + \left(\frac{\gamma}{1+\gamma}\right) M_t$.

where $\phi = \frac{\alpha}{1+\gamma}$, $\theta = \frac{1-\alpha}{1+\gamma}$. The corresponding inflation equation is[4]

$$\pi_t = \phi\pi_{t-1} + \theta E_t\pi_{t+1} + \left(\frac{\gamma}{1+\gamma}\right)\mu_t + \theta v_t, \tag{6}$$

where $\mu_t \equiv M_t - M_{t-1}$ is the money growth rate and $v_t = P_t - E_{t-1}P_t$ is an expectational error.[5]

In this equation, current inflation depends on past inflation, as well as on expected future inflation, and thus the possibility of inflation persistence reemerges. The degree of persistence is of course related to the stochastic process generating the money supply. To analyze this relation, it is convenient to rewrite the price equation (5) as[6]

$$P_t = \lambda_1 P_{t-1} + \frac{\gamma}{\lambda_2(1-\alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j E_t M_{t+j}, \tag{7}$$

where λ_1 and λ_2 are the roots of equation (5):

$$\lambda_{1,2} = \frac{1 \mp \sqrt{1-4\phi\theta}}{2\theta} = \frac{1 \mp \sqrt{1-4\frac{\alpha(1-\alpha)}{(1+\gamma)^2}}}{2\left(\frac{1-\alpha}{1+\gamma}\right)} \tag{8}$$

and $0 < \lambda_1 < 1$ and $\lambda_2 > 1$. In words, prices depend on past prices and expected future money supplies. Thus different stochastic monetary processes give rise to different price dynamics. We now consider two such processes in turn:

- A temporary money growth shock: The persistent after-affects of inflation to this temporary shock we refer to as *inflation persistence*. The greater the inflation effect after the shock has disappeared, the greater is inflation persistence.

- A permanent money growth shock: Since this shock leads to a permanent change in inflation, it is desirable to have a different name for the the inflation effects. Thus the delayed inflation effects of a permanent monetary shock we call *inflation under-*

[4]To derive the inflation equation, lag eq. (5) once: $P_{t-1} = \phi P_{t-2} + \theta E_{t-1}P_t + \left(\frac{\gamma}{1+\gamma}\right)M_{t-1}$, and subtract it from (5) to get $\pi_t = \phi\pi_{t-1} + \theta E_t P_{t+1} - \theta E_{t-1}P_t + \left(\frac{\gamma}{1+\gamma}\right)\mu_t$. Now add and subtract θP_t on the right-hand side of the above to obtain the inflation staggered equation in terms of the *exogenous* growth rate of money: $\pi_t = \phi\pi_{t-1} + \theta E_t\pi_{t+1} + \left(\frac{\gamma}{1+\gamma}\right)\mu_t + \theta\left(P_t - \theta E_{t-1}P_t\right)$.

[5]The error term $v_t = P_t - E_{t-1}P_t$ is included in Roberts (1995, 1997), but ignored by Fuhrer and Moore (1995) and much of the subsequent literature. It can be shown that, in the above price staggering model, this error term does not affect the dynamic structure of inflation; it only rescales its impulse response function to a temporary monetary shock.

[6]To see this, write (5) as $(1-\lambda_1 B)(1-\lambda_2 B)E_t P_t = \frac{-\gamma B M_t}{(1-\alpha)}$, where B is the backshift operator. This gives $(1-\lambda_1 B)E_t P_t = \frac{\gamma}{\lambda_2(1-\alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j E_t M_{t+j}$ which leads to (7) since $E_t P_t = P_t$.

responsiveness. The more slowly inflation responds to a permanent shock, the more under-responsive inflation is.

Although the persistent after-effects of a temporary money growth shock and the delayed after-effects of a permanent money growth shock are two distinct phenomena, they are often, rather confusingly, both denoted by the word "persistence" in the prevailing literature.

2.1 A Temporary Money Growth Shock

Let the money growth be stationary, fluctuating randomly around its mean (μ):

$$\mu_t = \mu + \varepsilon_t, \text{ where } \varepsilon_t \sim iid\left(0, \sigma^2\right). \tag{9}$$

A positive shock ε_t represents a temporary rise in money growth or, equivalently, a sudden, permanent increase in the money supply. The money supply is a random walk: $M_t = \mu + M_{t-1} + \varepsilon_t$, so that $E_t M_{t+j} = M_t + j\mu$, for $j \geq 0$. Substituting this last expression into the price equation (7), we obtain

$$P_t = \lambda_1 P_{t-1} + (1 - \lambda_1) M_t + \frac{(1 - \lambda_1)}{(\lambda_2 - 1)}\mu. \tag{10}$$

The first difference of this equation yields the closed form rational expectations solution of inflation:

$$\pi_t = \lambda_1 \pi_{t-1} + (1 - \lambda_1)\mu + (1 - \lambda_1)\varepsilon_t. \tag{11}$$

(In the long-run $\pi = \mu$, i.e. there is no money illusion, as for the other models below.)

A one-period shock to money growth $\varepsilon_t = 1$, $\varepsilon_{t+j} = 0$ for $j > 0$ (i.e. a permanent increase in the level of money supply) is associated with the following impulse response function of inflation:

$$R\left(\pi_{t+j}\right) = \lambda_1^j (1 - \lambda_1), \ j = 0, 1, 2, ... \tag{12}$$

Observe that the responses die out geometrically, and the rate of decline is given by the autoregressive parameter λ_1. In this context, we measure inflation persistence (σ) as the sum of the inflation effects for all periods after the shock has occurred ($t + j$, $j \geq 1$):

$$\sigma \equiv \sum_{j=1}^{\infty} R\left(\pi_{t+j}\right) = \lambda_1 \tag{13}$$

By equation (8), we see that the degree of persistence rises with the discount rate (and α) and falls with the demand sensitivity parameter γ. It can be shown that inflation has this qualitative pattern of persistence when money growth follows any stationary ARMA process.

2.2 A Permanent Money Growth Shock

Let money growth be a random walk:

$$\mu_t = \mu_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim iid\left(0, \sigma^2\right). \tag{14}$$

In this case a positive one-period shock (ε_t) represents a permanent increase in money growth (the case of a negative shock represents a sudden disinflation).

By the price equation (7) and the random walk (14), we obtain the following price dynamics:[7]

$$P_t = \lambda_1 P_{t-1} + (1-\lambda_1) M_t + \left(\frac{1-\lambda_1}{\lambda_2 - 1}\right)\mu_t, \tag{15}$$

The associated closed form rational expectations solution of inflation is

$$\pi_t = \lambda_1 \pi_{t-1} + (1-\lambda_1)\mu_t + \left(\frac{1-\lambda_1}{\lambda_2 - 1}\right)\varepsilon_t. \tag{16}$$

It can be shown that the corresponding impulse response function of inflation to the permanent unit increase in money growth is:[8]

$$R(\pi_{t+j}) = 1 - \lambda_1^j (1-\lambda_1)\left(\frac{2\alpha - 1}{\gamma}\right), \ j = 0, 1, 2, ... \tag{17}$$

In this context, we measure the responsiveness of inflation as the cumulative inflation effect of the money growth shock that arises because inflation does not adjust immediately to the new long-run equilibrium. In particular, suppose that the economy, in an initial long-run equilibrium, is perturbed by a one-period money growth shock $\varepsilon_t = 1$, $\varepsilon_{t+j} = 0$ for $j > 0$. The inflation responsiveness is the sum of the differences through time between the actual inflation rate and the new (post-shock) long-run equilibrium inflation rate:

$$\begin{aligned} \rho &\equiv \sum_{j=0}^{\infty}\left[R(\pi_{t+j}) - 1\right] \\ &= -\frac{2\alpha - 1}{\gamma} \end{aligned} \tag{18}$$

If inflation responds instantaneously to the shock, then $\rho = 0$, i.e. inflation is *perfectly responsive*. If inflation responds only gradually, so that the short-run inflation effects of the shock are less than the long-run effect, then inflation is *under-responsive* and $\rho < 0$. Finally,

[7]To see this, observe that $\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^j E_t M_{t+j} = \left(\frac{\lambda_2}{\lambda_2 - 1}\right) M_t + \frac{\lambda_2}{(\lambda_2-1)^2}\mu_t$, and $\frac{\gamma}{(\lambda_2-1)(1-\alpha)} = 1 - \lambda_1$.

[8]Since λ_1 is positive and less than unity, the long-run response of inflation is $\lim_{j\to\infty} R(\pi_{t+j}) \equiv R(\pi_{LR}) = 1$, i.e., in the long-run inflation stabilizes at the new level of money growth.

if inflation overshoots its long-run equilibrium, then ρ may be positive, making inflation *over-responsive.*

This impulse response function (17) has the following interesting implications for inflation responsiveness:

- If the discount rate r is zero (i.e. $\beta = 1$, so that $\alpha = 1/2$), then inflation is perfectly responsive. In other words, it is a jump variable, along the same lines as in the recent literature on "inflation persistence" under staggered nominal contracts.

- If the discount rate is positive (i.e. $\beta < 1$, so that $\alpha > 1/2$), then inflation is under-responsive. It gradually approaches its new equilibrium from below at a rate that depends on the autoregressive parameter λ_1.

As shown below, the first result holds only for staggered price setting, but not for staggered wage setting.

It is important to note that the under-responsiveness of inflation is closely related to the slope of the long-run Phillips curve. To see this, recall that output (or employment, etc.) depends on real money balances (in equation (4)), which (by the price equation (15)) are

$$x_t \equiv M_t - P_t = \lambda_1 (M_{t-1} - P_{t-1}) + (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t. \tag{19}$$

In the long run,

$$x_t = \left(\frac{2\alpha - 1}{\gamma} \right) \pi_t \tag{20}$$

since $\pi_t = \mu_t$ in the long run. *Observe that inflation responsiveness (18) is simply the inverse of the slope of the long-run Phillips curve.*

In the absence of time discounting ($\alpha = 1/2$), the long-run Phillips curve is vertical and inflation is a jump variable. This is an implausible, counter-factual special case, not just because there is no time discounting, but also because - as equation (19) shows - it is not just the long-run Phillips curve that is vertical; the short-run Phillips curve is vertical as well.

By contrast, in the presence of time discounting ($\alpha > 1/2$), as is well-known, the long-run Phillips curve is downward-sloping and inflation is under-responsive. The flatter is the long-run Phillips curve, the more under-responsive inflation becomes.

It is often casually asserted that, since the discount factor is close to unity in practice, the long-run Phillips curve must be close to vertical. Inspection of the long-run Phillips curve (20), however, shows this presumption to be false. As we can see, the slope of this Phillips curve depends on both the discount parameter α and demand sensitivity parameter γ. Table

1 presents the slope for various common values of α and commonly estimated values of γ:[9] It is clear that for a range of plausible parameter values the long-run Phillips curve is quite flat and, correspondingly, inflation is highly under-responsive.

Table 1: Slope of the long-run Phillips curve

			slope				
r (%)	β	α	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.07$	$\gamma = 0.10$
1.0	0.990	0.502	−2.01	−4.02	−10.1	−14.1	−20.1
2.0	0.980	0.505	−1.01	−2.02	−5.05	−7.07	−10.1
3.0	0.971	0.507	−0.68	−1.35	−3.38	−4.74	−6.77
4.0	0.962	0.510	−0.51	−1.02	−2.55	−3.57	−5.10
5.0	0.953	0.512	−0.41	−0.82	−2.05	−2.87	−4.10

3 Extensions

To gain some perspective on the determinants of inflation persistence and responsiveness, we now examine these phenomena in the context of other forms of nominal staggering.

3.1 Price Staggering and Future Demand

Whereas the price setting equation (1) is common in the literature on inflation persistence, microfoundations of staggered price setting suggest that current prices (set over periods t and $t+1$) depend not only on current demand (x_t) but also on future demand (x_{t+1}). Thus, let us consider the following price setting behavior:

$$P_t = \alpha P_{t-1} + (1-\alpha) E_t P_{t+1} + \gamma \left[\alpha x_t + (1-\alpha) E_t x_{t+1}\right]. \tag{21}$$

Substituting real money balances (4) into this equation, we obtain

$$P_t = \phi_p P_{t-1} + \theta_p E_t P_{t+1} + \left(\frac{\gamma}{1+\gamma\alpha}\right)\left[\alpha M_t + (1-\alpha) E_t M_{t+1}\right], \tag{22}$$

where $\phi_p = \frac{\alpha}{1+\gamma\alpha}$, and $\theta_p = \frac{(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)}$. In this model the lead parameter is positive under the plausible assumption that $\gamma < 1$. The sum of both the lag and lead parameters is less than one.

Expressing this difference equation as

$$P_t = \lambda_{1p} P_{t-1} + \frac{\gamma}{\lambda_{2p}(1-\gamma)(1-\alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_{2p}}\right)^j E_t \left[\alpha M_{t+j} + (1-\alpha) M_{t+1+j}\right], \tag{23}$$

[9] Taylor (1980b) estimates it to be between 0.05 and 0.1; Sachs (1980) finds it in the range 0.07 and 0.1; Gali and Gertler (1999) estimate it to be between 0.007 and 0.047; calibration of microfounded models (e.g. Huang and Liu, 2002) assigns higher values. The discount rate applies to a period of analysis which is half the contract span.

where[10]

$$\lambda_{1p,2p} = \frac{1 \mp \sqrt{1 - 4\phi_p \theta_p}}{2\theta_p} = \frac{1 \mp \sqrt{1 - 4\frac{\alpha(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)^2}}}{2\frac{(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)}}, \tag{24}$$

$0 < \lambda_{1p} < 1$, and $\lambda_{2p} > 1$, we find how price dynamics depend on the stochastic monetary process. Once again, we examine inflation persistence arising from a temporary money growth shock and inflation responsiveness arising from a permanent money growth shock.

We begin with a **temporary money growth shock.** When money growth follows the stationary process (9), the rational expectations solution of (23) is

$$P_t = \lambda_{1p} P_{t-1} + (1 - \lambda_{1p}) M_t + \kappa (1 - \lambda_{1p}) \mu, \tag{25}$$

where $\kappa = \lambda_{2p} / (\lambda_{2p} - 1) - \alpha$. Consequently inflation is given by

$$\pi_t = \lambda_{1p} \pi_{t-1} + (1 - \lambda_{1p}) \mu + (1 - \lambda_{1p}) \varepsilon_t. \tag{26}$$

Observe that this inflation dynamics equation has the same form as the corresponding equation (11) in the previous model. Thus, the impulse response function $R(\pi_{t+j}) = \lambda_{1p}^j (1 - \lambda_{1p})$, $j = 0, 1, 2, ...$, has the same form as well. Inflation persistence now is simply λ_{1p}. The magnitude of this autoregressive parameter is all that differentiates the inflation responses in the two models.

Now consider a **permanent money growth shock**. When money growth follows the random walk process (14), the rational expectations solution of (23) is

$$P_t = \lambda_{1p} P_{t-1} + (1 - \lambda_{1p}) M_t + \kappa (1 - \lambda_{1p}) \mu_t, \tag{27}$$

First differencing the above gives the following inflation equation:

$$\pi_t = \lambda_{1p} \pi_{t-1} + (1 - \lambda_{1p}) \mu_t + \kappa (1 - \lambda_{1p}) \varepsilon_t. \tag{28}$$

Once again, this equation has the same form as its counterpart (28) in the previous model. As above, inflation is perfectly responsive when $\alpha = 1/2$, it is under-responsive when $\alpha > 1/2$, and the degree of under-responsiveness is inversely related to the slope of the long-run Phillips curve.

[10] It can be shown that $(1 - \lambda_{1p}) = \frac{\gamma}{(\lambda_{2p}-1)(1-\gamma)(1-\alpha)}$.

3.2 Wage Staggering

Consider the following common wage staggering model:

$$W_t = \alpha W_{t-1} + (1-\alpha) E_t W_{t+1} + \gamma x_t. \tag{29}$$

Assuming constant returns to labor, the price is a constant mark-up over the relevant wages:

$$P_t = \frac{1}{2}(W_t + W_{t-1}). \tag{30}$$

Substitution of the price mark-up (30) and real money balances (4) equations into the wage setting equation (29) gives

$$W_t = \phi_w W_{t-1} + \theta_w E_t W_{t+1} + \left(\frac{2\gamma}{2+\gamma}\right) M_t, \tag{31}$$

where $\phi_w = \frac{2\alpha-\gamma}{2+\gamma}$, $\theta_w = \frac{2(1-\alpha)}{2+\gamma}$. We can write the above second order difference equation as

$$W_t = \lambda_{1w} W_{t-1} + \frac{\gamma}{\lambda_{2w}(1-\alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_{2w}}\right)^j E_t M_{t+j}, \tag{32}$$

where $\lambda_{1w,2w} = \frac{1 \mp \sqrt{1-4\phi_w\theta_w}}{2\theta_w}$, $0 < \lambda_{1w} < 1$, and $\lambda_{2w} > 1$.

In this context, consider the inflation effects of a **temporary money growth shock**. We substitute the money growth stochastic process (9) into (32) to obtain the wage dynamics equation:

$$W_t = \lambda_{1w} W_{t-1} + (1-\lambda_{1w}) M_t + \left(\frac{1-\lambda_{1w}}{\lambda_{2w}-1}\right)\mu. \tag{33}$$

Insert this wage dynamics equation into the price mark-up eq. (30) to obtain the price dynamics equation:

$$P_t = \lambda_{1w} P_{t-1} + \frac{1}{2}(1-\lambda_{1w}) M_t + \frac{1}{2}(1-\lambda_{1w}) M_{t-1} + \left(\frac{1-\lambda_{1w}}{\lambda_{2w}-1}\right)\mu. \tag{34}$$

Therefore, inflation is given by

$$\pi_t = \lambda_{1w}\pi_{t-1} + (1-\lambda_{1w})\mu + \frac{1}{2}(1-\lambda_{1w})\varepsilon_t + \frac{1}{2}(1-\lambda_{1w})\varepsilon_{t-1}. \tag{35}$$

The responses of inflation to a period-t unit money growth shock are:

$$\begin{aligned} R(\pi_t) &= \frac{1}{2}(1-\lambda_{1w}), && (36) \\ R(\pi_{t+j}) &= \frac{\lambda_{1w}^{j-1}}{2}\left(1-\lambda_{1w}^2\right), \; j=1,2,3,... \end{aligned}$$

Thus inflation persistence is

$$\sigma = \frac{1+\lambda_{1w}}{2}. \tag{37}$$

Now turning to the inflation effects of a **permanent change in money growth**, the rational expectations solution of the model gives the following dynamics equation:

$$W_t = \lambda_{1w}W_{t-1} + (1-\lambda_{1w})M_t + \left(\frac{1-\lambda_{1w}}{\lambda_{2w}-1}\right)\mu_t, \tag{38}$$

$$P_t = \lambda_{1w}P_{t-1} + (1-\lambda_{1w})M_t + \frac{1}{2}(1-\lambda_{1w})\left(\frac{2-\lambda_{2w}}{\lambda_{2w}-1}\right)\mu_t + \frac{1}{2}\left(\frac{1-\lambda_{1w}}{\lambda_{2w}-1}\right)\mu_{t-1}, \tag{39}$$

$$\pi_t = \lambda_{1w}\pi_{t-1} + (1-\lambda_{1w})\mu_t + \frac{1}{2}(1-\lambda_{1w})\left(\frac{2-\lambda_{2w}}{\lambda_{2w}-1}\right)\varepsilon_t + \frac{1}{2}\left(\frac{1-\lambda_{1w}}{\lambda_{2w}-1}\right)\varepsilon_{t-1}. \tag{40}$$

It can be shown that the responses through time of inflation to a period-t permanent unit money growth shock are:

$$\begin{aligned} R(\pi_t) &= 1-\frac{1}{2}\left[(1-\lambda_{1w})\left(\frac{2\alpha-1}{\gamma}\right)+\frac{1+\lambda_{1w}}{2}\right] < 1, && (41) \\ R(\pi_{t+j}) &= 1-\lambda_{1w}^{j-1}\frac{\left(1-\lambda_{1w}^2\right)}{2}\left(\frac{2\alpha-1}{\gamma}-\frac{1}{2}\right), \; j=1,2,..., \\ \lim_{j\to\infty} R(\pi_{t+j}) &= 1. \end{aligned}$$

As for the price staggering model, inflation responsiveness is $\rho \equiv -\frac{2\alpha-1}{\gamma}$. By this measure, again, inflation is perfectly responsive when the discount rate is zero ($\alpha = 1/2$) and under-responsive when the discount rate is positive ($\alpha > 1/2$). However, in neither case does inflation jump immediately to its long-run equilibrium value. Specifically, the instantaneous (period-t) response of inflation is to undershoot both when $\alpha > 1/2$ and $\alpha = 1/2$. In period-1 inflation can either remain below its new equilibrium level, if $\frac{2\alpha-1}{\gamma} > \frac{1}{2}$, or overshoot if $\frac{2\alpha-1}{\gamma} < \frac{1}{2}$. Since $0 < \lambda_{1w} < 1$, period-2 onwards inflation converges to its equilibrium in a geometric fashion.[11]

Finally, we consider a wage staggering model in which the nominal wage depends not only

[11] It can be shown that this model can generate inflation undershooting when the interest rate is greater than 5% and/or the demand sensitivity parameter γ is lower than 0.05.

The reason why our measure of inflation responsiveness, ρ, is zero when $\alpha = 1/2$, even though inflation does not jump to its long-run equilibrium value, is that the inflation effects sum to zero through time.

on current demand (x_t) but also on future demand (x_t), along the lines originally proposed by Taylor (1980a):

$$W_t = \alpha W_{t-1} + (1-\alpha) E_t W_{t+1} + \gamma \left[\alpha x_t + (1-\alpha) E_t x_{t+1}\right], \tag{42}$$

It is straightfoward to show that the associated impulse response functions of inflation to a temporary and permanent money growth shock have the same functional forms as in the previous model. The only difference between the impulse response functions of the two wage staggering models (29) and (42) lies in the autoregressive root of their rational expectations dynamic equations.[12]

4 Concluding Remarks

It is commonly asserted that inflation is a jump variable in the New Keynesian Phillips curve, and thus wage-price inertia does not imply inflation inertia. We have shown that this "inflation flexibility proposition" is a partial equilibrium result, relying on the assumption that real variables are exogenous. In a general equilibrium setting (in which real variables not only affect inflation, but are also influenced by it) the phenomenon of inflation inertia re-emerges.

To avoid confusion, we have used the term "inflation persistence" to denote inflation inertia in the presence of a *temporary* money growth shock; whereas inflation inertia in the presence of a *permanent* money growth shock we have called "inflation under-responsiveness". Table 2 summarizes our results on inflation persistence, over the four macro models. *PS*-(x_t) stands for the price staggering model in which prices depend only on current demand; *PS*-(x_t, x_{t+1}) is the model in which prices also depend on future demand. *WS*-(x_t) and *WS*-(x_t, x_{t+1}) represent the corresponding wage staggering models. The responses to a temporary money growth shock are divided into a "current" response (the impact effect, $R(\pi_t)$), a "future" response (the sum of the future effects, which is our measure of inflation persistence σ), and the the "total" response (the sum of the current and future responses, $\tau \equiv R(\pi_t) + \sigma$). We see that a temporary money growth shock always has prolonged after-effects on inflation (regardless of whether the discount rate is zero or positive, or whether there is price or wage staggering).

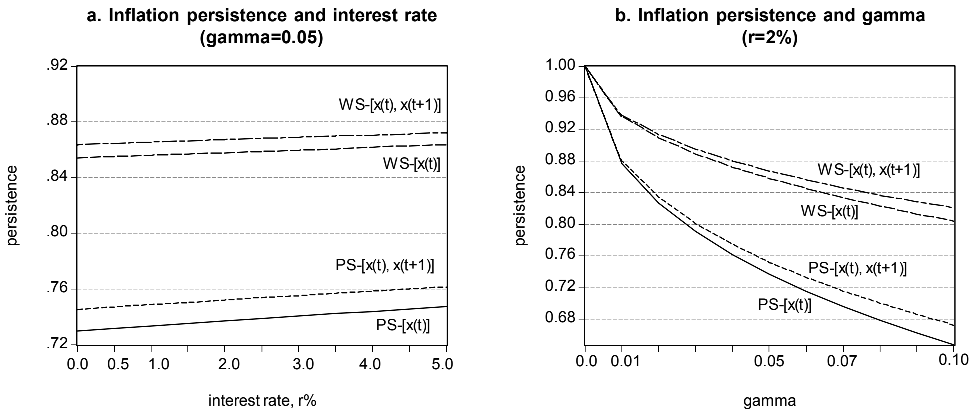
[12] For the above Taylor model, it can be shown that $\lambda_{1w} = \frac{1-\sqrt{1-4\phi_w\theta_w}}{2\theta_w}$, $\phi_w = \alpha\left(\frac{2-\gamma}{2+\gamma}\right)$, $\theta_w = (1-\alpha)\left(\frac{2-\gamma}{2+\gamma}\right)$, and $0 < \lambda_{1w} < 1$. For a detailed analysis of this model see Karanassou, Sala and Snower (2004).

Table 2: Inflation persistence after a shift in the money supply

	autoregressive coefficient λ			responses		
Models	$\lambda = \frac{1-\sqrt{1-4\phi\theta}}{2\theta}$	ϕ	θ	Current $R(\pi_t)$:	Future σ :	Total τ :
PS-(x_t)		$\frac{\alpha}{1+\gamma}$	$\frac{1-\alpha}{1+\gamma}$	$1-\lambda$	λ	1
PS-(x_t, x_{t+1})		$\frac{\alpha}{(1+\gamma\alpha)}$	$\frac{(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)}$	$1-\lambda$	λ	1
WS-(x_t)		$\frac{2\alpha-\gamma}{2+\gamma}$	$\frac{2(1-\alpha)}{2+\gamma}$	$\frac{1}{2}(1-\lambda)$	$\frac{1}{2}(1+\lambda)$	1
WS-(x_t, x_{t+1})		$\frac{\alpha(2-\gamma)}{2+\gamma}$	$\frac{(1-\alpha)(2-\gamma)}{2+\gamma}$	$\frac{1}{2}(1-\lambda)$	$\frac{1}{2}(1+\lambda)$	1

The dependence of inflation persistence on the discount rate r and the demand sensitivity parameter γ, for our four macro models, are pictured in Figures 1. Observe that, for given values of r and γ, there is more inflation persistence (a) under wage staggering than under price staggering and (b) when nominal variables depend on both present and future demands than when they depend on present demands alone. Furthermore, note that variations in the demand sensitivity parameter over the frequently estimated range have a strong effect on inflation persistence, whereas the discount rate (over the standard range) has a relatively weak effect.[13]

Figures 1



Tables 3a and 3b summarize our results on inflation responsiveness for the four macro models. The impact effect of a permanent money growth shock is denoted by $R(\pi_t)$ and the future effects by $R(\pi_{t+j})$, $j \geq 1$. The degree of inflation responsiveness ρ has been shown to be the inverse of the slope of the long-run Phillips curve. This measure of responsiveness is zero (denoting perfect responsiveness) when the discount rate is zero ($\alpha = 1/2$) and negative (denoting under-responsiveness) when the discount rate is positive ($\alpha > 1/2$). However, this

[13] Since the demand sensitivity parameter (γ) is assumed positive and non zero, the unit value of persistence in Figure 1b for $\gamma = 0$ represents a limiting case $\left(\text{i.e., } \lim_{\gamma\to 0}\sigma = 1\right)$.

does not imply that inflation necessarily jumps to its long-run value whenever the discount rate is zero. On the contrary, we have seen that under staggered wage setting inflation is never a jump variable, regardless of the discount rate.

Table 3a: Inflation responsiveness , $\alpha > 1/2$

Models	$R(\pi_t)$	$R(\pi_{t+j})$ $_{j=1,2,...}$	ρ
PS-(x_t)	<1	<1	$-\left(\frac{2\alpha-1}{\gamma}\right)$
PS-(x_t, x_{t+1})	<1	<1	$-\left(\frac{2\alpha-1}{\gamma}\right)$
WS-(x_t)	<1	?	$-\left(\frac{2\alpha-1}{\gamma}\right)$
WS-(x_t, x_{t+1})	<1	?	$-\left(\frac{2\alpha-1}{\gamma}\right)$

In all models, when $\alpha > 1/2$, the immediate response of inflation is to undershoot $(R(\pi_t) < 1)$. In the wage staggering (the bottom two rows in Table 3a), inflation will continue to undershoot its equilibrium after period-t if $\frac{2\alpha-1}{\gamma} > \frac{1}{2}$. Otherwise, inflation overshoots in period 1 and then gradually converges (from above) to its new equilibrium.

When $\alpha = 1/2$ (see Table 3b), the inflation generated by the price staggering models is a jump variable and both the short- and long-run Phillips curves are vertical. In other words, there is no inflation "persistence" and the monetary policy has no real effects in the economy. With wage staggering, when $\alpha = 1/2$, inflation responsiveness remains zero but inflation does not immediately jump to its new value. Initially inflation undershoots, and then it overshoots before it starts approaching its new equilibrium. The net effect is zero and so $\rho = 0$. Thus, the Phillips curve is downwards sloping in the short-run and becomes vertical in the long-run.

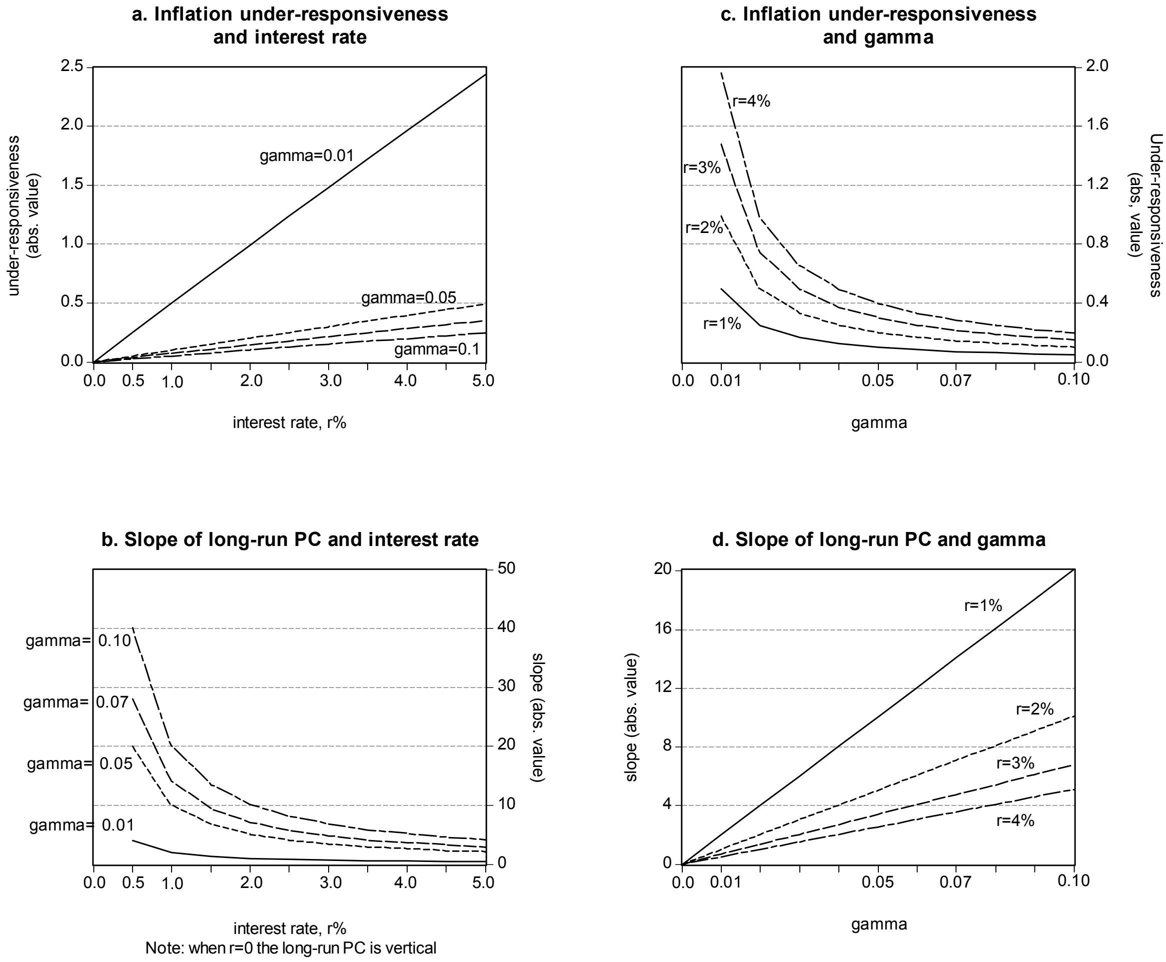
Table 3b: Inflation responsiveness , $\alpha = 1/2$

Models	$R(\pi_t)$	$R(\pi_{t+j})$ $_{j=1,2,...}$	ρ	PC (short-run)
PS-(x_t)	1	1	0	vertical
PS-(x_t, x_{t+1})	1	1	0	vertical
WS-(x_t)	<1	>1	0	non-vertical
WS-(x_t, x_{t+1})	<1	>1	0	non-vertical

Figure 2a pictures the relation between inflation under-responsiveness (in absolute value terms) and the interest rate; Figure 2b is the corresponding relation between the slope of the long-run Phillips curve and the interest rate. Along the same lines, Figures 2c and 2d show how inflation under-responsiveness and the slope of the long-run Phillips curve depend on the demand sensitivity parameter γ.

These results have one common thrust: the inflation flexibility proposition is highly misleading. Under plausible parameter values, high degrees of inflation persistence and under-responsiveness may arise in the context of standard wage-price staggering models.

Figures 2



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