Inflation Targets as Focal Points*

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Abstract

We begin by assuming that the Central Bank is not always able to affect private sector expectations. Private individuals form their expectations based on all the information that is available to them. Their incentive is to forecast inflation correctly such that they negotiate wages at the right level. However the individuals also know that both the objectives of the central bank as well as the average expectation formed will affect the outcome of inflation, so they realise that they need to evaluate both actions. The main part of our paper shows that a monetary policy regime that has explicit quantitative objectives may help individuals form these expectations. However, that is only true if no great shocks affect the economy an when all other public information available is very unclear thus rendering the inflation target, the only clear piece of inflation. We derive in detail the conditions under which this is true.

JEL codes: C71, C78, E52
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1 Introduction

Modern monetary policy theory emphasizes the central role of private sector expectations in determining policy outcomes. As argued by Woodford "....insofar as it is possible for the Central Bank to affect expectations, this should be an important tool of stabilisation policy...." (Woodford 2003). It is thus widely acknowledged, that the success of maintaining a stable monetary environment depends crucially on the ability of the policy regime to control inflation expectations (Blinder et al, 2001). Evidence of that is shown by Orphanides and Williams (2004) in their analysis of US monetary policy history, where they argue that monetary policy failures are connected with changes in public sentiment about the future state of the economy. In other words, policy mistakes alone are not enough to produce long term negative effects on monetary stability.
The practice of monetary policy in the past ten to fifteen years has thus concentrated on providing institutional set-ups that provide an explicit platform of information for expectations to be formed. The main features of such institutional set-ups are

- credible institutions, mainly through independence and the pursuit of the sole objective of price stability;
- clear policy frameworks, captured by well defined intermediate policy objectives and procedures, and finally,
- transparent policy making, implemented through publication and distribution of the information set used in the decision making process (inflation forecasts, modelling strategies, well defined assumptions) and a clear demonstration of accountability (publication of minutes, regular appearance in front of parliamentary committees and regular press conferences).

Practically every monetary policy authority nowadays defines its policies according to these criteria, emphasising one or another aspect, depending on preferences. The set-up of the twelve-country Euro area, for example, has emphasised the importance of building and sustaining credibility and independence from governments, as an instrument towards low expected inflation. In the US experience instead, credibility, independence but also flexibility in following multiple objectives has helped achieve a stable monetary environment. Alternatively, inflation targeting as implemented first, by the Reserve Bank of New Zealand and then the Bank of England, and increasingly more and more banks around the world, is understood to provide clear and immediate objectives for monetary policy. Inflation targeting practitioners argue that the main advantage of an explicit numerical inflation target is its ability to provide a focal point for private sector expectations. As Mervyn King (2004, p.4) has claimed for the UK case, inflation expectations have indeed been anchored to the pre-announced target. The ability of explicit quantitative targets to tie down expectations, is also confirmed by the empirical analysis of Levin *et al* (2004), Mishkin and

Schmidt-Hebbel (2001) and more recently by Fatás et al (2004)[1].
However, in analysing various monetary policy regimes (Svensson, 1999, 2003), we observe no difference in the way we approach inflation targeting by comparison to other regimes[2]. There is thus no explicit analysis of the way the provision of a specific numerical target constitutes a better anchor for long term expectations. Is there a difference between giving a numerical target as opposed to an upper bound to desired inflation? And if there is a difference, what is the mechanism that achieves it? In our view, to be able to answer that, we need a more complex mechanism for the way expectations are formed.

The recent model put forward by Morris and Shin (2002a, 2002b) (and used in Amato *et al,* 2003 and Amato and Shin, 2003), constitutes such an attempt to identify first, how private agents form expectations based on both private as well as public information available to them, and second how policy makers (not exclusively Central Banks) can affect those expectations by providing greater or lesser information. We thus assume in this set-up that, when forming expectations, private agents care not only about their own views but also of other people's expectations as a way of confirming their own beliefs. And as this element of 'beauty contest', (based on Keynes, 1936), plays a greater role in expectations forming, signals provided by public institutions become tantamount to coordination devices. In fact Phelps (1983) noted that "...in order to reduce the price level (in relation to the accustomed trend), it is not sufficient that the central bank persuade each agent to reduce his private expectation of the money supply (in relation to the past trend) by the warranted amount. The prevalence of this expectation must be public knowledge - an accepted fact"(p.35). This therefore, implies that public information acquires a dual role -"...of conveying fundamentals information as well as serving as a focal point for beliefs" (Morris and Shin,2002a, henceforth, MS). The questions that arise following this argument is therefore, what monetary policy regimes provide better signals and in which way those signals constitute focal points. The aim of this paper is to formalise the widely believed but little analysed benefits of inflation targeting in coordinating private individuals' expectations. To this end, we use the Morris and Shin, 2002, to describe the game of monetary policy and how private sector expectations affect the final outcome. Furthermore, we employ the *Variable Universe Games* approach put forward by Bacharach (1993) to raise the issue that perceptions about policy matter in the final outcome *CHECK(
We will argue therefore, that monetary policy is a game between policy makers and private agents in which two issues are important: 1) private agents care about the beliefs of others, very much à la Morris and Shin (2002a), and 2) inflation expectations are important to the current level of inflation and therefore, to the direct interest of the Central Bank. The first point implies that the

[1] See also Leiderman and Svensson (1995) and Bernanke et al (1999) for earlier accounts of experiences with inflation targeting.

[2] Kuttner 2004, also alludes to this fact. The benefits of inflation targeting as a coordination device have been discussed by Hughes Hallett and Viegi, 2002, but then in the context of two policy authorities, the policies of which might have strong "spillovers".

monetary policy game is of a 'matching nature', in the sense that private agents' welfare improves when their expectations lie closer to the average. The second point in turn, implies that the Central Bank has an incentive to be transparent about its intentions in the hope that, the signal provided will be a focal point for agents to anchor their expectations at.
Both points from above imply that monetary policy can be viewed as a coordination game between the Central Bank and the public but also between the public themselves. The theory on coordination games provides valuable insight into the way that such games are resolved. For example, it is often observed that in matching games players coordinate much more frequently than by randomising (Casajus, 2000). Indeed, according to Wilson and Rhodes (1997), it is to the benefit of all actors to avoid the conflict that escalates as solutions are delayed. To achieve that, players rely heavily on salient features when deciding on their actions. And salience in this context, can be a "...social custom or convention, namely, a mode of behaviour that finds automatic acceptance" (Dixit and Skeath, 1999) and a salient item is "...one that stands out from the rest by its uniqueness in some conspicuous respect. Salience thus defined has two dimensions: conspicuousness or noticeability of some feature, and unique instantiation of this feature." (Bacharach 1993). Furthermore, Wilson and Rhodes (1997) argue that all that is required for such salience to be achieved is a signal from somebody that can be recognised as the "leader" in the game, to send a signal. The existence of a leader in a clearly defined leader-follower(s) game can thus provide such a focal point (Wilson and Rhodes, 1997). In our set up we will thus assume that the Central Bank sends a signal (of different degrees of precision), and only then do agents form expectations. The timing of the game, thus allows for it to be considered of a leader-follower sort. The precise model in which inflation is implemented is of no great relevance here as we concentrate on the way the signal affects (helps coordinate) private sector expectations.

The paper is organised as follows. In section 2, we describe how monetary policy can be described as a matching game between private sector agents. Section 3 then, provides Bacharach's (1993) framework for solving such matching games and section 4, applies it explicitly to monetary policy and specifically to an inflation targeting regime. Section 5 concludes.

2 Monetary Policy as an Information Game

The Central Bank has a standard loss function in which it chooses the rate of inflation x to minimise the distance of inflation from its target $\left(x - x^T\right)$ and close the output gap y,

$$L_{CB}|_{\xi} = E\frac{1}{2}\left[\left(x - x^T\right)^2 + y^2\right] \tag{1}$$

subject to a standard Lucas supply function, $y = x - x^e + \xi$ where ξ is a supply shock with zero mean and constant variance. Note that any Central

Bank will have an objective x^T irrespective of whether it has communicated it to the public, clearly or even at all. We assume for simplification that the CB's instrument is x. Optimisation of (1) implies that

$$x|_\xi = \frac{x^T}{2} + \frac{x^e}{2} - \frac{\xi}{2} \tag{2}$$

where x is the *ex post* inflation outcome and x^e is private sector expectations about the relevant rate of inflation. Representation (2) is of a structural form[3] in the sense that expectations are not replaced (Leitemo, 2005). Svensson (2003) argues in favour of such a representation in order to indicate that factors like judgement that contribute to the way expectations are formed but cannot always be modelled, are an important contributor to monetary policy. In a typical commitment game, where the Central Bank communicates its target, x^T, and commits to it, expectations formed by al individuals collectively are equal to the CB's objectives, $x^e = x^T$ and the ex post outcome is

$$x|_\xi = x^T - \frac{\xi}{2} \tag{3}$$

$$E(x) = x^T \tag{4}$$

The objective of this paper however, is to depart from the assumption that expectations are always equal to the objective of the Central Bank and analyse the information that is available to the individuals and how they go about interpret it when forming expectations. Every individual i will be forming an expectation of inflation, x_i and the collective outcome $x^e = \int_0^1 x_j dj$, (for a continuum of agents), is the expectation that is relevant to the central bank.

2.1 The Formation of Expectations

We thus start by arguing that that while the CB is always clear itself about what its objectives are, it is not always possible to assume that individuals form expectations that are consistent with those objectives. To analyse how

[3] Note that (2) is specific to the underlying Lucas supply function assumed but demonstrates that the outcome will be a function of both the policy the Central Bank pursues as well as what the private sector anticipates. Similarly, had the model been of the standard Neokeynesian type,

$$x_t = \beta E_t x_{t+1} + k y_t + \varepsilon_t$$
$$y_t = E_t y_{t+1} - \gamma (i_t - E_t x_{t+1}) + \eta_t$$

then the structural representation of the ex post inflation outcome would be

$$x_t = \frac{k^2}{1+k^2} x^T + \frac{1}{1+k^2} E_t x_{t+1} + \frac{\varepsilon_t}{1+k^2}.$$

In both cases the ex post outcome is a function of both the CB objective as well as the expectations of the private sector.

expectations are indeed formed, then we need to look at the information that is available to them. Typically, every individual is forming expectations based on two information sets, namely what is publicly available and therefore common to everyone and what is available to them privately. Observing (2), every individual knows that the ex post outcome of inflation x will be affected equally (given the model assumed) by the policy the Central Bank pursues to attain its objectives, as well as the average of expectations formed by the public. However, what the individual is interested in, is predicting correctly the *ex post* level of inflation such that any deviations from the final outcome is penalising. This is captured by a standard expected utility for the individual,

$$u_i\left(\mathbf{x}^e, x^T\right) \equiv \frac{1}{2} E_i (x_i - x)^2 \tag{5}$$

Note that the the subscript i in the expectations operator, indicates that the individual will be seeking to minimise her expected utility, given her own perceptions. x_i is individual i's expectation of what the state will be and x is the *ex post* inflation outcome. We use $\mathbf{x}^e$ to refer to the expectations profile over all agents. The objective of each individual i is thus to form expectations x_i which she will use in wage negotiations. Individual i, is therefore deciding on its action x_i based on the first-order condition of (5). This is

$$\arg\min_{x_i} u_i\left(\mathbf{x}^e, x^T\right) \equiv E_i\left(x\right)$$

and from (2),

$$\begin{aligned} x_i &= E_i\left(x\right) \\ x_i &= E_i\left(\frac{x^T}{2} + \frac{x^e}{2} - \frac{\xi}{2}\right) \\ x_i &= \frac{1}{2} E_i\left(x^T - \xi\right) + \frac{1}{2} E_i\left(x^e\right) \end{aligned} \tag{6}$$

The optimal action for individual i is thus a function of her interpretation of what the objective of the central Bank is and hence the policy it will pursue, the shock that will occur and finally her interpretation of what the average expectation formed by all individuals collectively. Equation (6) is not dissimilar to equation (2) of Morris and Shin (MS, 2002) in which the individual forms a view about the state θ and the average action, $\bar{a}$. The strength with which she pursues that is given by the "beauty term" parameter r, equal here to the value $\frac{1}{2}$, provided by the model. Moreover, an important difference to the MS approach is that the state x is now endogenous, in the sense of being affected by the average action, whereas in the MS approach θ is independent of $\bar{a}$. This now implies that in forming expectations x_i, individuals need to evaluate three things: the objective of the CB, the value of the shock ξ that will occur, and finally what is the common perception about inflation. It follows that if $x_i = x_j$ $\forall j$, then $x_i = x^e$ and individuals' expectations are matched. However, although

necessary, coordination between agents at any level of inflation is not sufficient; the optimal outcome occurs when agents coordinate at the objective pursued by the Central Bank. Coordination at any other expectation rate still leaves agents away from the level of inflation that the CB aims to achieve. Following MS (2002), we argue that information used by the agents is available in the form of a public signal common to all, and a private signal which is specific to each agent in the economy. These take the following form:

$$\text{Public signal:} \quad y = \left(x^T - \xi\right) + \eta \tag{7}$$

$$\text{Private signal:} \quad z_i = \left(x^T - \xi\right) + \varepsilon_i \tag{8}$$

Both η and ε_i are normally distributed with a zero mean and variance σ_η^2 and σ_ε^2 respectively. Furthermore, the two error terms are independent of x and of each other, such that $E\left(\varepsilon_i \varepsilon_j\right) = 0$ for $i \neq j$. It is worth mentioning that what is publicly available to the private sector is not only what the CB releases but all possible information that might be relevant in predicting inflation at the relevant time horizon. Based on these two types of signals, MS show that action for agent i then is

$$\begin{aligned} x_i &= \frac{2\alpha y + \beta z_i}{2\alpha + \beta} \\ &= \theta + \frac{2\alpha\eta + \beta\varepsilon_i}{2\alpha + \beta} \end{aligned} \tag{9}$$

where $\alpha = \frac{1}{\sigma_\eta^2}$ and $\beta = \frac{1}{\sigma_\varepsilon^2}$, precision for the two information sets respectively, and $\theta = \left(x^T - \xi\right)$. It follows that the expectations across all agents will then equal,

$$x^e = \int_0^1 x_j dj = \theta + \frac{2\alpha\eta}{2\alpha + \beta} \tag{10}$$

Equation (10) shows that the average expectation across all agents will be distorted by the precision of the two signals as well as the preference for the 'beauty term' r, here equal to $\frac{1}{2}$.

3 A framework for Interpreting Expectations

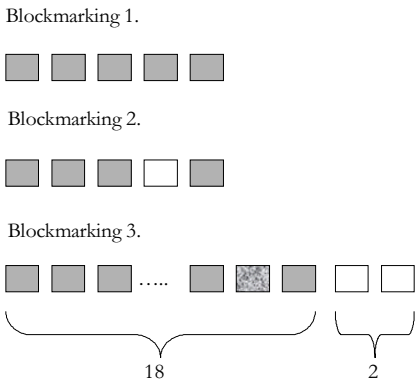
The optimal decision shown in (6) implies that individual i needs to interpret the set of choices that are available to her when deciding her options. We provide next a framework that provides this set of interpretations. This is based on Bacharach's Variable Universe Games (1993) framework, which helps describe how players evaluate their strategies to identify salient points when forming expectations in matching games[4]. The novelty of this approach is that it allows explicitly for differences in perceptions which then helps players choose

[4] Used and extended by Janssen (2001).

rationally between alternative outcomes. The framework provided shows that in matching games, the players' incentive to coordinate induces them to look for salient points. However, as salience is subject to personal interpretation, the existence of such features is not necessarily uniquely defined.

3.1 An Expected Utility Approach

The game of blockmarking is played in the following way. Two players are shown a number of wooden blocks and each has to secretly pick one. If both players pick the same block, they receive an identical pecuniary prize; otherwise they receive nothing. The author then describes three variants of the game, summarised in figure 2.



In Blockmarking 1, (B1), the players are given five identical blocks (in size, colour, shape and material). In Blockmarking 2, (B2), the same game is repeated, except now one of the five blocs is of a different colour, (white). In Blockmarking 3, (B3), players are now given 20 blocks, eighteen of which are grey and two are white. Furthermore, closer inspection of the blocks, allows players to see that the grain of the wood in just one of the grey blocks is wavy. B3 can thus be described either in terms of colour, (C) or in terms of the grain of the wood, (G). As the game is of a matching nature, it is to the players' interest to look for salient features that help achieve tacit coordination. In example B1 above, there is no clear way of differentiating between the blocks, so one is inclined to simply pick at random. At example B2 however, the difference in colour allows players to distinguish between the blocks in such a way, that it is always wise to go for the one that is white. The *unique instantiation* of the white block thus provides the two players with a focal point. Similarly in B3, if colour is the distinguishing feature that occurs to the players, they are then inclined to pick one of the white blocks, even though such action does not automatically lead to coordination. However, if a player has managed to see that not only colour but also the grain of the wood differentiates the blocks, uniqueness is again guaranteed. The difficulty now however, is that the grain

pattern of the wood is not necessarily identifiable (*conspicuous*) by (to) all players. In forming her choice therefore, having seen the difference in grain herself, player 1, needs to assess how likely her partner is to distinguish the blocks in terms of the grain as well. Bacharach's analysis shows, that if this likelihood is big, then it is to her interest to pick the grey block with the wavy pattern; otherwise she is better off picking one of the white blocks and face an, at most, 50% chance of matching the choice of her partner.
Bacharach provides a thorough proof to B3 in the appendix to his paper, but the essence of the game faced by the two players individually can be summarised as follows. In solving B3, player 1 is effectively faced with two alternative actions: $M\widetilde{h}$, mark a white block at random, or Mw, mark the grey block with the wavy grain. Furthermore, as explained above, the crucial point in this analysis is the likelihood with which player 1 believes player 2 has noticed the grain. She is thus left with the following two choices when forming her views about player 2. Either she believes that her opponent has seen the grain (and assigns probability v to that event), or she does not believe that he has seen the grain (and assigns probability $1-v$ to that event). It is reasonable to assume that if player 2 has indeed noticed the grain, then he will pick it with some non-zero probability. However, if he has not noticed the grain then he can never mark a block accordingly. From player 1's perspective therefore, her expected utility from choosing one of her two actions is the following.

Definition 1: *Both players have an identical set of feasible strategies,* $R^{+}=\{C,G\}$ *and possible actions,* $A=\left\{M\widetilde{h},Mw\right\}$. *Define* $U_1\left(x_{1,a(\bullet)},x_{2,a(\bullet)}\right)$, *player 1's utility from following action* $x_{1,a(\bullet)}$ *and player 2 following action* $x_{2,a(\bullet)}$, *for* $a\in A$ *where* $a(C)=M\widetilde{h}$ *and* $a(G)=Mw$.

We need to deal with two cases:

Case 1: Player 2 always marks a block according to colour, either because he has not seen the grain himself, or because he believes his partner has not. Then player 1' expected utility is

$$E_1U(M\widetilde{h},M\widetilde{h})=(1-v)U_1\left(M\widetilde{h},M\widetilde{h}\right)+vU_1\left(M\widetilde{h},M\widetilde{h}\right)$$
$$E_1U(Mw,M\widetilde{h})=(1-v)U_1\left(Mw,M\widetilde{h}\right)+vU_1\left(Mw,M\widetilde{h}\right)$$

We normalise next $U(x_1=x_2)=1$, and calculate the expected utilities:

$$E_1U(M\widetilde{h},M\widetilde{h})=(1-v)\frac{1}{2}U(x_1=x_2)+v\frac{1}{2}U(x_1=x_2)=\frac{1}{2}$$
$$E_1U(Mw,M\widetilde{h})=(1-v)*0+v*0=0$$

This implies that $E_1U(M\widetilde{h},M\widetilde{h})>E_1U(Mw,M\widetilde{h})$ and therefore player 1 has an incentive to match her partner's action by also picking a white block at random.

Case 2: Player 2 now, marks a block based on the grain when he has noticed it. Otherwise, he marks a block according to colour. Then expected utility for player 1 is now

$$E_1U\left[M\widetilde{h},\left(M\widetilde{h}\text{ or }Mw\right)\right]=(1-v)U_1\left(M\widetilde{h},M\widetilde{h}\right)+vU_1\left(M\widetilde{h},Mw\right)$$
$$E_1U\left[Mw,\left(M\widetilde{h}\text{ or }Mw\right)\right]=(1-v)U_1\left(Mw,M\widetilde{h}\right)+vU_1\left(Mw,Mw\right)$$

and therefore

$$E_1U\left[M\widetilde{h},\left(M\widetilde{h}\text{ or }Mw\right)\right]=(1-v)\frac{1}{2}U\left(x_1=x_2\right)+v*0=\frac{1-v}{2}$$
$$E_1U\left[Mw,\left(M\widetilde{h}\text{ or }Mw\right)\right]=(1-v)*0+vU\left(x_1=x_2\right)=v$$

It follows that,

$$E_1U\left[Mw,\left(M\widetilde{h}\text{ or }Mw\right)\right]>E_1U\left[M\widetilde{h},\left(M\widetilde{h}\text{ or }Mw\right)\right],\ \ iff\ \ v>\frac{1}{3}.$$

But between the two cases, the necessary and sufficient condition for player 1 to decide to mark a block according to the grain, is

$$E_1U\left[Mw,\left(M\widetilde{h}\text{ or }Mw\right)\right]>E_1U(M\widetilde{h},M\widetilde{h})\Longleftrightarrow$$
$$v>\frac{1}{2} \tag{11}$$

In other words, the balance of reasons favours marking the block with the wavy grain, only if v is a large enough number by comparison to $\frac{1}{m}$ where m is the number of white blocks. Bacharach argues therefore, that the relative rarity of the white blocks, captured here by $\frac{1}{2}$, is pulling against the conspicuousness v of the grain pattern, as the less rare the white blocks, the more likely the player is to pick the wavy grey block.

3.2 Is Monetary Policy a Variable Universe Game?

The point that is crucial to Bacharach's analysis is the fact that players have particular ways of perceiving the game and the framing of the game available to them individually, is not necessarily available to other players as well. Before deciding on a possible action therefore (e.g. picking the block with the wavy grain,¨provided they have seen it), they have to form a view on how likely the other player is to have noticed the grain, as well as the colour, as a possible distinguishing feature. Evaluating that is therefore, necessary before picking on a strategy, implying that having noticed the grain for oneself is not sufficient to pick it. Any player therefore, needs to assess whether their own beliefs as to

what is conspicuous to them is also conspicuous to others. If I have not seen the grain then, the optimal strategy to go for is to pick a yellow block at random. If on the other hand, I have seen the difference in grain, then I form a view about how likely the other person is of having seen the grain as well. I pick the wavy block only if, given my assessment of this likelihood, the expected value of doing so is greater than the expected value of picking a yellow block at random.

The analogy with monetary policy is as follows. The central bank has an objective for inflation which is captured by (1). This implies that the CB will be aiming to set inflation equal to x^T. However this objective is not necessarily common knowledge to all agents that form the private sector but is instead subject to interpretation affected by the way the CB communicates its objectives to the public, or indeed by its ability to achieve this objective as indicated by its past (credibility) . Following the signal therefore, the objective of the central bank is not conspicuous to all.
This is true for any monetary policy regime. Private individuals will then be weighing private versus public information as indicated in (9) and deciding accordinginly. We argue that a Central Bank that provides a quantitative target differs to a Central Bank that does not. Now the individual is faced with two options: either pursue the action as indicated in (9) in which the inflation target is internalised and judged just like any other piece of public information, or alternatively, driven by her desire to coordinate, she may adopt the inflation target and fix her expectations on it. To do this however, she needs to have sufficient confidence that others will coordinate at that level two. Just as in the blockmarking game, the fact that a target is provided is not sufficient for the individual to coordinate at t; what is required further is for the individual to have enough people will think the same way. In that respect my interpretation of the way the target is perceived by others is key to my decision. So while there is no uncertainty as to what strategies are available to people, there is uncertainty as to how these strategies are interpreted.

4 The Role of Inflation Targets

The provision of a quantitative objective has the following implication. Every individual is faced with two actions, a_i; either to weigh all information and thus follow the strategy suggested by Morris and shin, x_i, or simply form an expectation equal to the quantitative level of inflation announced by the Central Bank. By analogy the same applies for the average inflation expectation, such that the MS action is $\bar{a}$ and x^T respectively for the two alternatives. Following the utility of each individual, and given that each individual has the option of two strategies, a_i or x^T Let $u_i\left(x_i, x^e\right)$ denote the utility of individual i following her action a_i, given average action $\bar{a}$.

$$u_i\left(a_i, \bar{a}\right) \equiv \left(x_i - x\right)^2 \tag{12}$$

Following the announcement of an inflation target, then individual i is faced with

two options. Either she weighs the public information available against her own private information and thus follows $a_i = x_i$ as suggested by MS, where

$$x_i = x^T - \xi + \frac{2\alpha\eta + \beta\varepsilon_i}{2\alpha + \beta} \tag{13}$$

or simply ignores all information and follows the target, such that her action is

$$a_i = x^T \tag{14}$$

If individuals follow collectively the action suggested by Morris and Shin, $\bar{a} = x^e$ then,

$$\bar{a} = x^e = x^T - \xi + \frac{2\alpha\eta}{2\alpha + \beta} \tag{15}$$

For a continuum of agents the inflation outcome is affected by the average action such that

$$\begin{aligned} x|_\xi &= \frac{x^T}{2} + \frac{x^e}{2} - \frac{\xi}{2} \\ &= x^T - \xi + \frac{\alpha\eta}{2\alpha + \beta} \end{aligned} \tag{16}$$

By contrast when all individuals follow the target then the inflation outcome is

$$x^e = x^T$$

and the average level of inflation is by consequence

$$\begin{aligned} x|_\xi &= \frac{x^T}{2} + \frac{x^T}{2} - \frac{\xi}{2} \\ &= x^T - \frac{\xi}{2} \end{aligned} \tag{17}$$

Following these four possible actions, we calculate next the payouts for individual i.
$a_i = x_i, \bar{a} = x^e$

$$\begin{aligned} u_1(a_i, \bar{a}) &\equiv E(a_i - x)^2 \\ &= \frac{\alpha}{(2\alpha + \beta)^2} + \frac{\beta}{(2\alpha + \beta)^2} = \frac{\alpha + \beta}{(2\alpha + \beta)^2} \end{aligned} \tag{18}$$

$a_i = x^T, \bar{a} = x^e$

$$\begin{aligned} u_1\left(x^T, \bar{a}\right) &\equiv E(a_i - x)^2 \\ &= \sigma_\xi^2 + \frac{\alpha}{(2\alpha + \beta)^2} \end{aligned} \tag{19}$$

$a_i = x_i, \bar{a} = x^T$

$$\begin{aligned} u_1\left(a_i, x^T\right) &\equiv E(a_i - x)^2 \\ &= \frac{1}{4}\sigma_\xi^2 + \frac{4\alpha+\beta}{(2\alpha+\beta)^2} \end{aligned} \tag{20}$$

$a_i = x^T, \bar{a} = x^T$

$$\begin{aligned} u_1\left(x^T, x^T\right) &\equiv E(a_i - x)^2 \\ &= \frac{1}{4}\sigma_\xi^2 \end{aligned} \tag{21}$$

We summarise the payout matrix in normal formal.

Table 1: Expected Payouts for Player 1

$a_i \quad \setminus \bar{a}$	x^e	x^T
x_i	$\frac{\alpha+\beta}{(2\alpha+\beta)^2}$	$\frac{1}{4}\sigma_\xi^2 + \frac{4\alpha+\beta}{(2\alpha+\beta)^2}$
x^T	$\sigma_\xi^2 + \frac{\alpha}{(2\alpha+\beta)^2}$	$\frac{1}{4}\sigma_\xi^2$

Based on Bacharach's approach, we need to deal with two cases regarding the interpretation of :

Case 1: First we assume that all players interpret the 'universe' in the same way, in that they consider the inflation target as an added piece of public information. Player 1 assumes that Player 2 now, picks the target when he views it in the sense intended by the Central Bank. Otherwise, he optimises subject to all information available to him. Expected utility for player 1 of pursuing either of her two options, is therefore,

$$\begin{aligned} E\left\{u_1\left[a_i, \left(\bar{a} \text{ or } x^T\right)\right]\right\} &= (1-v)u_1\left(a_i, \bar{a}\right) + vu_1\left(a_i, x^T\right) \\ E\left\{u_1\left[x^T, \left(\bar{a} \text{ or } x^T\right)\right]\right\} &= (1-v)u_1\left(x^T, \bar{a}\right) + vu_1\left(x^T, x^T\right) \end{aligned}$$

and therefore

$$\begin{aligned} E\left\{u_1\left[a_i, \left(\bar{a} \text{ or } x^T\right)\right]\right\} &= (1-v)\frac{\alpha+\beta}{(2\alpha+\beta)^2} + v\left[\frac{1}{4}\sigma_\xi^2 + \frac{4\alpha+\beta}{(2\alpha+\beta)^2}\right] \\ &= (1-v)\frac{\alpha+\beta}{(2\alpha+\beta)^2} + v\left[\frac{1}{4}\sigma_\xi^2 + \frac{4\alpha+\beta}{(2\alpha+\beta)^2}\right] \\ E\left\{u_1\left[x^T, \left(\bar{a} \text{ or } x^T\right)\right]\right\} &= (1-v)\left[\sigma_\xi^2 + \frac{\alpha}{(2\alpha+\beta)^2}\right] + v * \frac{1}{4}\sigma_\xi^2 \end{aligned}$$

It follows that,

$$E\left\{u_1\left[x^T,\left(\bar{a}\text{ or }x^T\right)\right]\right\}<E\left\{u_1\left[a_i,\left(\bar{a}\text{ or }x^T\right)\right]\right\},\ \ iff$$
$$\sigma_\xi^2<\frac{\beta+v4\alpha}{(1-v)(2\alpha+\beta)^2} \tag{22}$$

or in other words, it is sufficient for the degree of confidence that player 1 has about the others following the signal to be above a certain number before, she picks the target. Notice how if the precision of the public signal reduces, then it becomes more difficult to satisfy this condition.

Case 2: However, Player 1 may have to do with the fact that others do not interpret the target in the way intended by the Central Bank and therefore, prefer to simply weigh public against private information and thus follow the action recommended by Morris and Shin. This may be either because they do not understand or believe the intentions of the Central Bank, or because they believe others do not understand or believe the intentions of the Central Bank. This achieves an average expectation for inflation equal to $\bar{a}$. . Then player 1' expected utility of following either of her two options is:

$$E\left[u_1(a_i,\bar{a})\right]=(1-v)u_1\left(a_i,\bar{a}\right)+vu_1\left(a_i,\bar{a}\right)$$
$$E\left[u_1(x^T,\bar{a})\right]=(1-v)u_1\left(x^T,\bar{a}\right)+vu_1\left(x^T,\bar{a}\right)$$

Based on Table 1, then these are

$$E\left[u_1(a_i,\bar{a})\right]=(1-v)\frac{\alpha+\beta}{(2\alpha+\beta)^2}+v\frac{\alpha+\beta}{(2\alpha+\beta)^2}=\frac{\alpha+\beta}{(2\alpha+\beta)^2}$$
$$E\left[u_1(x^T,\bar{a})\right]=(1-v)\left[\sigma_\xi^2+\frac{\alpha}{(2\alpha+\beta)^2}\right]+v*\left[\sigma_\xi^2+\frac{\alpha}{(2\alpha+\beta)^2}\right]=\sigma_\xi^2+\frac{\alpha}{(2\alpha+\beta)^2}$$

It follows that

$$E\left[u_1(x^T,a_i)\right]<E\left[u_1(a_i,a_i)\right]\quad iff$$
$$\sigma_\xi^2<\frac{\beta}{(2\alpha+\beta)^2} \tag{23}$$

Note that if (23) holds, then (22) is also satisfied, so that the former is the necessary and sufficient condition for player 1 to pick the target.

4.1 Inflation Targeting as a dominant strategy

Note that if (23) holds, then based on Table 1 following the target becomes the dominant strategy in that the individual will always choose to form inflation

expectations according to the inflation target. For this to be true therefore, it is important that the supply shock is smaller than a given ratio. Figure 2 shows that this condition is very stringent in the sense that inflation targeting is dominant only if shocks are very small in size. Indeed, if the economy is hit by very big shocks then the provision of a target does not help agents coordinate at the level intended by the Central Bank.

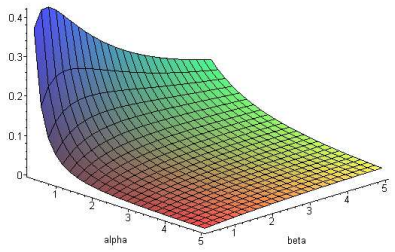


Figure 1: Inflation Targeting as a dominant Strategy

Moreover, figure 2 also shows that if public information is very imprecise (α is low) then the provision of an inflation target becomes helpful, in the sense that the condition becomes easier to satisfy. In that respect the numerical targets become substitutes for imprecise information; in the absence of concrete information then the provision of one clear inflation target, becomes the only unequivocal piece of public information. And that is true irrespective of the interpretation parameter, v.

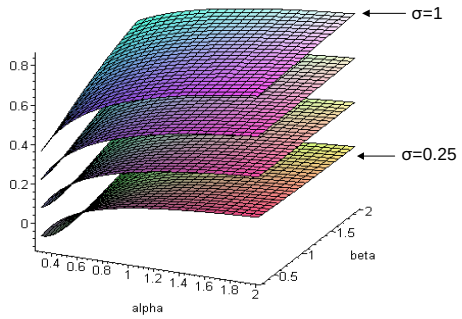
4.2 Expectations formation as a matching game

However if (23) is not satisfied, i.e., $\sigma_\xi^2 > \frac{\beta}{(2\alpha+\beta)^2}$ then, from Table 1, individual's optimal action requires matching the average actions. But this time for inflation targeting to produce higher expected utility, condition (22) must be satisfied and by consequence the value of the interpretation parameter v is relevant. Condition (22) can be re-written as

$$\sigma_\xi^2 < \frac{\beta + v4\alpha}{(1-v)(2\alpha+\beta)^2}$$

$$\frac{(2\alpha+\beta)^2 \sigma_\xi^2 - \beta}{4\alpha + (2\alpha+\beta)^2 \sigma_\xi^2} < v \tag{24}$$

Figure 3 plots condition (24) in the α and β space for four different values of the supply shock ($0 < \sigma_\xi^2 < 1$).



There are two interesting features that arise from figure 3. First, it is the case that as the variance of the shock increases, then (24) becomes more difficult to satisfy. This is consistent with what is mentioned above when inflation targeting is a dominant strategy for individual i, namely that in the presence of large shocks, inflation targeting is redundant in its role as a coordinator of expectations. Second, as public information becomes less and less clear, the provision of a clear and unique quantitative signal helps relax the stringency of the condition In both cases from above, the role of private information is deemaphsised in that it does not impose a constraint on either (23)or (24) to hold. This is demonstrated in figure 4 for the latter.

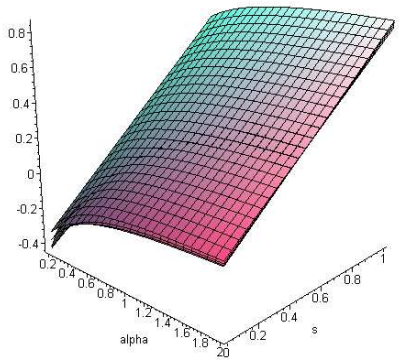


Figure 2: The role of private information

5 Conclusions

Any private individual forms expectations of inflation which it uses to negotiate wages with. Our paper concentrates on the way these expectations might be formed in the presence of a Central Bank that pursues its own objectives. We begin our analysis by arguing that it is not always possible for a monetary policy authority to assume that it can affect private expectations in the way that they will match its own intentions. Private individuals rely on information that is available to them publicly (and thus common to everyone) and information that might be unique to them individually. Monetary policy then, becomes an information game, in which private individuals base their decision on a combination of all information available, corrected for their respective degree of precision. As the level of expectations affects the final outcome of inflation, the private sector needs to deduce both what the objective of the Central Bank is, as well as what everybody else's intentions are. The latter point implies that coordinated expectations are preferable.

Further to that we then argue that Central Banks that announce a very precise quantitative target, may benefit in the form of coordinated private sector expectations at the level of its objective. We describe the conditions for which this happens and discover that inflation targeting does indeed achieve better coordination, first when the supply shocks expected are small, in other words the economy is stable and no big shocks are anticipated to push the economy off its main path, and second, when public information fails in all other respects to provide the private sector with clear signals as to what the level of inflation relevant to them, is going to be. It is in the sense that we argue that inflation targets are substitutes for poor otherwise, public information. Naturally, as we show above it is not sufficient for any individual to view this quantitative signal as a satisfactory substitute. She must have a high enough degree of confidence that all other agents do too.

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