# Skewed Pricing in Two-Sided Markets: An IO approach*

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**Abstract**

In two-sided markets, one widely observes skewed pricing strategies, in which the price mark-up is much higher on one side of the market than the other. Using a simple model of two-sided markets, we show that, under constant elasticity of demand, skewed pricing is indeed profit maximizing. The most elastic side of the market is used to generate maximum demand by providing it with platform services at the lowest possible price. Through the positive network externality, full participation of the high-elasticity, low-price side of the market increases market participation of the other side. As this side is less price elastic, the platform is able to extract high prices. Our skewed pricing result also carries over when analyzing the socially optimal prices. Interestingly, this leads to below-marginal cost pricing in the social optimum. We motivate the analysis by looking at the Dutch debit card system.

*Keywords:* Two-sided markets, skewed pricing, corner solution, social optimum.

*JEL Code:* G21, L10, L41

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## 1 Introduction

Credit and debit card schemes, computer operating systems, shopping malls, television networks, and nightclubs represent at first glance an odd shortlist of very different industries. Indeed, these industries serve different types of consumers, use very different technologies, and maintain quite dissimilar business arrangements and standards. Yet, firms in these so-called "two-sided" industries have adopted similar pricing strategies for solving the common problem they face -getting and keeping two sides of the market on board. To balance the demand for their product or service, these firms tend to heavily "skew" prices towards one side of the market in the sense that the price mark-up is much larger on one side than the other [1]. The main contribution of our paper is to rationalize this widely observed skewed pricing behaviour in two-sided markets, which has thus far been problematic in economic theory.

The recognition that many markets are two-sided (or multi-sided) has recently triggered a surge in the theoretic literature on the economics of two-sided markets. Two-sided markets involve two distinct types of end-users, each of whom obtains value from "transacting" or "interacting" with end-users of the opposite type. In these markets, one or several platforms enable these transactions by appropriately charging both sides of the market. As Rochet and Tirole (2004) say, platforms "court" each side of the market while attempting to make money overall. In view of our shortlist, credit and debit card payment schemes court cardholders and retailers, computer operating systems court users and application developers, shopping malls court buyers and sellers, television networks court viewers and advertisers, and nightclubs court men and women.

A key aspect of research focuses on price determination in two-sided markets. Under two-sidedness, platforms need not only choose a total price for their services, but must also choose an optimal pricing structure: the division of the total price between the two sides of the market, so as to induce participation from both sides of the market. The need for both a pricing level and a pricing structure is one of the defining characteristics that distinguishes two-sided markets from industries ordinarily studied by economists [2]. In practice, in many two-sided markets platforms often treat one side of the market as a "profit center" with high prices and the other side as a "loss leader" with low prices.

Referring again to our shortlist, credit card company American Express earned 82 percent of its revenues from merchants in 2001. In the Dutch debit card system, but also in many other European debit card schemes, consumers face zero transaction fees for using their debit cards, while retailers pay relatively high usage fees. Microsoft earns the bulk of its profits from Windows from licensing Windows to computer manufacturers or other end-users. At shopping malls, shoppers are usually not charged an entrance fee and often receive cheap car parking, while shop owners heavily pay for available rental space. Television networks, but also other media platforms like newspapers, magazines, and websites, earn a disproportionate share of their revenues from the advertisers' side of the market. For example, during the 2002 Super Bowl, the average price for a thirty-second commercial ran up to US $ 1.9 million. The fees that the media platforms collect from advertisers pay for the content that the media present to the audience. And finally, where it is legal, nightclubs often set much lower entrance fees for women than for men. Nightclub owners presumably believe that, while the

[1] See Evans (2003) for a list of examples of two-sided markets and corresponding skewed pricing strategies, see also Rochet and Tirole (2003) for existing business models in two-sided markets.

[2] Rochet and Tirole (2004) provide a formal definition of two-sidedness, and discuss conditions that make a market two-sided.

cost of servicing an additional male or an additional female is likely to be quite similar, at equal prices the bar attracts too many men, or too few women. All in all, skewed pricing is so commonly observed that it legitimates the question whether we can formalise this pricing behaviour within a standard model of two-sided markets?

Economic theory has difficulties explaining the observed skewed pricing behaviour in two-sided markets. In an elegant article, Rochet and Tirole (2003) derive a "interior" pricing result for a monopolistic two-sided market. They show that under log-concavity of the demand functions, the optimal total price is governed by a variant of the well-known monopolistic Lerner condition, and that the optimal pricing structure depends on the relative magnitude of the individual market sides' price elasticities of demand. To explain skewed pricing within their model, it must be assumed that one side of the market is far less price-elastic (or totally inelastic for a zero fee) than the other. Then, surprisingly, their interior pricing result predicts that the least elastic side of the market will be charged the lowest price. This result seems at odds with standard economic intuition where one would instead expect a lower fee for the more elastic side. In this paper we will show that under constant elasticity of demand, the side of the market which is sufficiently more elastic than the other side is kept to a minimum fee, while the other side pays a relatively high fee. The economic intuition underlying our skewed pricing result is that the most elastic side of the market is effectively subsidized by the other side so as to boost the demand for services supplied by the platform. Indeed, every agent on the high elasticity, low price side of the market will connect to the platform. Benefitting from full participation on one side, the other side is therefore also encouraged to join. However, since this side is more price-inelastic, the platform is able to extract higher prices.

Our skewed pricing result, which is mathematically characterized by a corner solution, carries over in a qualitative way when studying socially optimal prices. We prove that a central planner would choose the same corner solution, in terms of full participation of the sufficiently more elastic side of the market and recovering almost all costs from the price-inelastic side. It is further shown that when socially optimal prices are implemented, the platform operates at a loss. Not surprisingly, since socially optimal prices are in general lower than monopolistic prices, the monopolistic platform tends to undersupply its services.

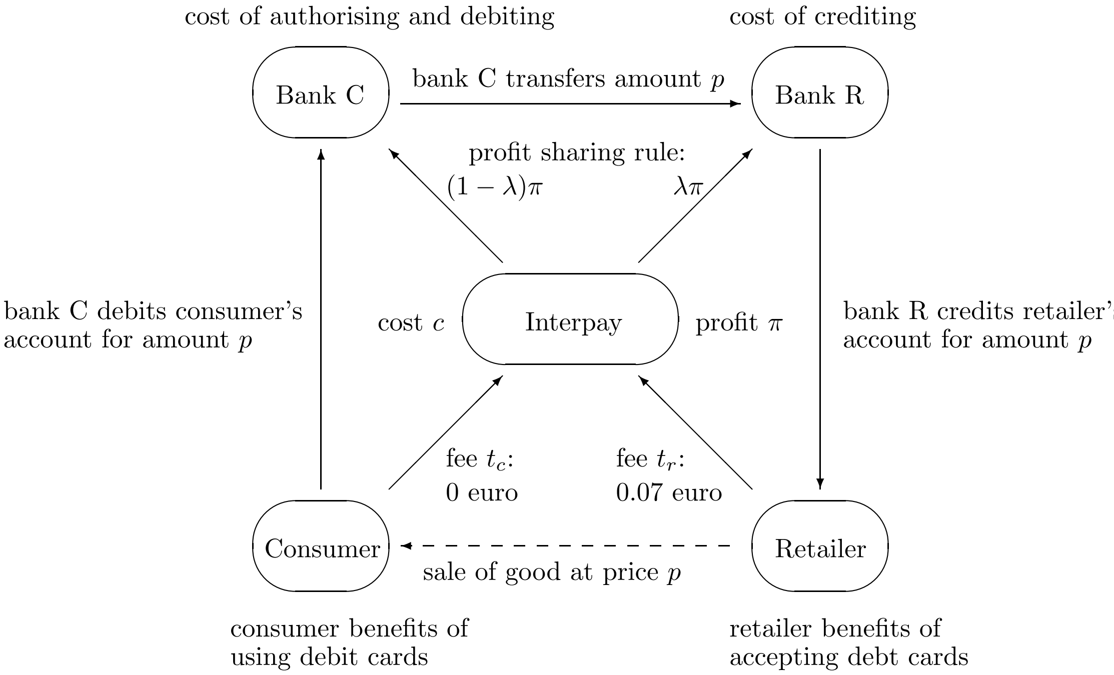
Our findings may have important bearings on antitrust issues which we will briefly touch upon. We will further illustrate our general results by analyzing the Dutch debit card system and arguing that our theoretic results better fit the observed properties of debit card pricing in the Netherlands than the results in previous theoretical papers. In fact, recent empirical studies on payment systems have also shown that consumers are quite sensitive to price changes in payment instruments. As these apparent price-elastic consumers hardly face any price for making card payments, these empirical facts support our theoretical findings.

The remainder of the paper is organized as follows. Section 2 motivates our paper by analyzing the Dutch debit card model, which features a monopolistic platform processing the payments and setting the prices for both the consumers and retailers. Section 3 describes the model and derives the skewed pricing result. In section 4 we analyse socially optimal prices of the platform network. Finally, section 5 concludes.

## 2 Motivation: Debit Cards in the Netherlands

To motivate our analysis about skewed pricing in two-sided markets, we use the Dutch debit card system as an illustration. Given that we observe skewed pricing in many different

Figure 1: The "Dutch" Debit Card Model




two-sided markets, many features of this specific illustration carry over to various other situations. Over the last decade the Netherlands, and many other western economies, have shown a rapid increase in the usage of debit cards. In particular, debit card payments in the Netherlands exceeded EUR 50 billion in 2002 (around 11 percent of GDP) - with a volume of over one billions debit card transactions, more than 40 times higher than in 1991 - and are still growing rapidly [3]. Consumer participation is complete, with virtually all consumers over age 18 carrying a debit card. Usage density on the retailers' side is lower, with about 56 percent of all Dutch retailers accepting debit card payments. On average, retailers pay some 7 eurocents per debit card transaction, while consumers do not pay any such usage fee. In this sense, debit card pricing in the Netherlands is completely skewed to the retailers' side, as is the case in most other countries with a developed debit card payment infrastructure [4].

To avoid an abundance of costly bilateral agreements and procedures between financial institutions and banks, Dutch commercial banks have opted for a business model with only one debit card platform, called Interpay. Interpay acts as a central network switch facilitating the exchange of highly secured electronic payment data between end-users (consumers and

[3] See Bolt (2003) for a more elaborate description of Dutch retail payment systems.

[4] For a theoretic analysis of the Dutch debit card market, see Bolt and Tieman (2003). Usually, cardholders in the Netherlands pay a small annual fee. However, these costs may also be attributed to having checking accounts at banks, using debit cards for stored value also, and using them abroad. In our analysis, for simplicity we abstract from membership fees. See Rochet and Tirole (2003) for dealing with fixed costs in a theoretic model of two-sided markets.

retailers) and their respective banks [5]. Moreover, in the Netherlands, this platform signs up retailers to accept debit cards, and in principle it could set the prices of debit card transactions for both consumers and retailers, although currently it only charges retailers. Interpay is a private limited liability firm jointly owned by the banks. Banks get reimbursed for incurred authorisation and other costs by Interpay through a specific profit sharing rule. Interestingly, and in contrast with most other countries, the Dutch debit card system does not implement any interchange fee [6]. Figure 1 depicts a schematic representation of a debit card payment in the Dutch system and the corresponding flow of funds and fees.

Recently, Dutch retailers expressed their dissatisfaction with the current pricing strategy of Interpay, perceiving the high retailer fee as a form of abuse of market power. Indeed, the skewness of prices in the debit card industry triggered antitrust scrutiny and led to an in-depth investigation by the Dutch competition authority NMa. As a result, Interpay and the participating commercial banks were fined some 50 million euro. In addition, retailers went to court to reclaim foregone revenues in the amount of 200 million euro [7]. Both cases are still pending.

Obviously, the debit card model as described in Figure 1 is quite limited. For instance, Rochet and Tirole (2002) and Wright (2003b) have extended the model by incorporating competition among consumer and retailer banks, and strategic behaviour by retailers. The basic model as described in the next section concentrates explicitly on the links between both groups of end-users and the monopolistic platform.

# 3 The Basic Model: Monopoly Platform

Our model setup is similar to Schmalensee (2002) and Rochet and Tirole (2003). Potential gains from trade are created by transactions between two end-users, whom we will call buyers (subscript $b$) and sellers (subscript $s$). Such transactions are mediated and processed by the monopoly platform. To provide these (network) services, the platform charges buyers and sellers a transaction fee, denoted by $t_b \geq 0$ and $t_s \geq 0$. For simplicity, we abstract from any fixed periodic fees for end-users to connect to the platform. In performing its tasks, the platform incurs joint marginal costs, which are set at $c \geq 0$ per transaction. It is convenient to introduce a distinction between the pricing level, defined as the the total price $t_T = t_b + t_s$ by the platform to the two sides, and the pricing structure, referring to the allocation between $t_b$ and $t_s$ of the total price to buyers and sellers. As Evans (2003) shows, in two-sided industries the product may not exist at all if the business does not get the price structure right.

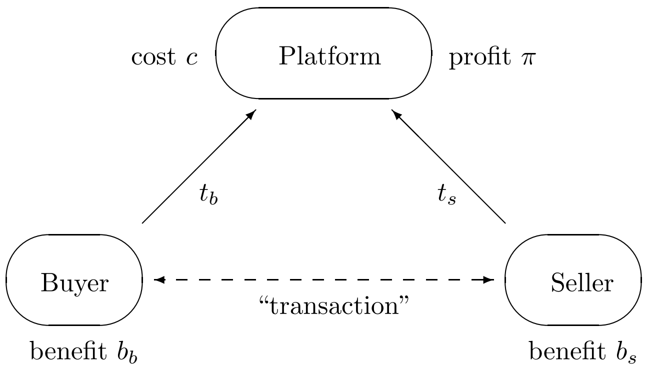
Buyers and sellers enjoy benefits when transacting on the platform. We assume that buyers are heterogeneous in the benefits $b_b$, $b_b \in [\underline{b}_b, \bar{b}_b]$, $\bar{b}_b \leq \infty$, they receive from a transaction. The probability density function of these benefits is labelled $h_b(\cdot)$ with cumulative density $H_b(\cdot)$. Similarly, sellers differ in the benefits $b_s$ associated with transacting on the platform, $b_s \in [\underline{b}_s, \bar{b}_s]$, $\bar{b}_s \leq \infty$, with probability density function $h_s(\cdot)$ and cumulative density $H_s(\cdot)$. To illustrate, in case of a debit card transaction, a buyer (or consumer) who wants to buy a good or service from a seller (or retailer) at price $p$, prefers to use his debit card whenever he gets positive benefits from using his card relative to other payment instruments, say cash. A

[5] Also in other countries such as Belgium, France, Norway, and Switzerland a nation-wide debit card network exists with only one central routing switch.

[6] In much of the theoretic analysis about card payment systems, the optimality of interchange fees has played a central role, see e.g. Gans and King (2001), Rochet and Tirole (2002), Wright (2003b).

[7] The NMa report on the Dutch debit card system can be downloaded from www.nmanet.nl

Figure 2: The monopoly platform




transaction using the debit card takes place if at the same time the seller prefers accepting the debit card payment to accepting cash.

Only buyers with benefits $b_b$ larger than incurred fees $t_b$ will transact on the platform. Formally, the fraction of buyers connecting to the platform is given by

$$q_b = D_b(t_b) = \Pr(b_b \geq t_b) = 1 - H_b(t_b). \tag{1}$$

Analogously, the fraction of sellers which connects to the platform is equal to

$$q_s = D_s(t_s) = \Pr(b_s \geq t_s) = 1 - H_s(t_s). \tag{2}$$

Assuming independence between $b_b$ and $b_s$, the total expected fraction of transactions processed by the platform amounts to

$$q = D(t_b, t_s) = D_b(t_b)D_s(t_s). \tag{3}$$

Further, we assume that the platform operates in a price region such that the price elasticities of (quasi-)demand exceed 1 for both sides of the market. That is, we define

$$\epsilon_i(t) = -\frac{\partial D_i}{\partial t}\frac{t}{D_i} \tag{4}$$

and assume that $\epsilon_i(t) > 1$, $i = b, s$, for every feasible fee $t \geq 0$. As will be shown below, these elasticities are of critical importance for setting optimal prices in two-sided markets. It is also convenient to make a distinction between the *joint* price mark-up, defined by the ratio $(t_T - c)/t_T$, and the *individual* (or own) price mark-up, defined by the ratio $(t_i - c)/t_i$, $i = b, s$.

Throughout the paper, we assume that one side of the market is more price-elastic than the other.

**Assumption 3.1.** *Without loss of generality we assume that the buyers' side of the market is more price-elastic. That is,*

$$\epsilon_b(t) \geq \epsilon_s(t) \text{ for every feasible fee } t \geq 0.$$

Finally, for simplicity, we exogenously fix the total number of transactions, both on and off the platform, at $N$. So, total demand for network services on the platform is given by $N \cdot D(t_b, t_s)$. Figure 2 schematically depicts the model.

### 3.1 Interior platform pricing under log-concave demand

In a two-sided market the monopoly platform must make sure that both sides of the market get on board by appropriately setting the transaction fees. Two-sidedness has strong implications for the pricing strategy of the platform, which will depend on both sides' price elasticities of demand. A key finding of price determination in two-sided markets is that the optimal prices for the two groups of end-users must balance the demand among these groups. This is seen as follows.

The monopoly platform sets its transaction fees so as to maximize total profits

$$\pi(t_b, t_s, c) = N(t_b + t_s - c)D(t_b, t_s). \tag{5}$$

Under log-concavity of the demand functions $D_b$ and $D_s$, the maximum is determined by the first-order conditions

$$\frac{\partial(\log \pi)}{\partial t_b} = \frac{1}{t_b + t_s - c} + \frac{(D_b)'}{D_b} = 0 \tag{6}$$

and

$$\frac{\partial(\log \pi)}{\partial t_s} = \frac{1}{t_b + t_s - c} + \frac{(D_s)'}{D_s} = 0. \tag{7}$$

Consider the following theorem by Rochet and Tirole (2003).

**Theorem 3.2.** *(Interior pricing)*
*i) Under log-concavity of the demand functions $D_b$ and $D_s$, the monopoly platform earns maximum profits if it sets fees*

$$(t_b^*, t_s^*) = \left( \frac{c\,\epsilon_b^*}{\epsilon_T^* - 1}, \frac{c\,\epsilon_s^*}{\epsilon_T^* - 1} \right), \tag{8}$$

*where $\epsilon_i^* = \epsilon_i(t_i^*)$, $i = b, s$, and $\epsilon_T^* = \epsilon_b^* + \epsilon_s^*$.*

*ii) The total fee $t_T^* = t_b^* + t_s^*$ is determined by the 'inverse elasticity rule' for total elasticity $\epsilon_T^* = \epsilon_b^* + \epsilon_s^*$. That is,*

$$\frac{t_T^* - c}{t_T^*} = \frac{1}{\epsilon_T^*}, \tag{9}$$

*iii) The corresponding price structure is characterized by*

$$\frac{t_b^*}{t_s^*} = \frac{\epsilon_b^*}{\epsilon_s^*}. \tag{10}$$

Equation (9) illustrates that the profit-maximising price level in a two-sided market is governed by the well-known monopolistic *Lerner* pricing rule when demand is log-concave. However, the Lerner rule applies to the total price $t_T$ and total elasticity $\epsilon_T$: The joint price mark-up is inversely related to the total elasticity (defined by the sum of the two individual elasticities). Hence, if the market as a whole gets less elastic, the total price will rise. For the two seperate sides of the market a different but related result holds. Rewriting (10) using (8)-(9) yields

$$\frac{t_i^* - (c - t_j^*)}{t_i^*} = \frac{1}{\epsilon_i^*}, \quad i, j = b, s, \ i \neq j, \tag{11}$$

which can be seen as a modified Lerner rule, where the individual price mark-up needs to be corrected for the contribution in terms of revenues from the opposite side. In addition, (10) shows that the distribution of the optimal total fee over the two sides of the market is determined by the relative magnitude of the individual market sides' elasticities. This gives rise to the next corollary that is directly derived from Theorem 3.2.

**Corollary 3.3.** *Under log-concavity, the market side with the highest price elasticity of demand is charged the highest fee. So, given Assumption 3.1, buyers pay a higher price than sellers, that is,* $t_b^* \geq t_s^*$.

Interior platform pricing gives rise to some counterintuitive results. First, as seen from Corollary 3.3, interior pricing in two-sided markets means that the most price-elastic side of the market pays the highest price, while standard economic intuition would predict the opposite [8]. Second, interior pricing cannot easily explain the widely observed skewness in pricing towards one side of the market, in the sense that in many real world settings the price mark-up is far less on one side of the market than on the other. Theorem 3.2 can only explain this empirical fact by assuming that the side which get charged the least, is far less elastic than the other side of the market. To illustrate, consumers in the Netherlands face a zero transaction fee when using their debit cards, while retailers get charged. Interior pricing would imply that consumers would be either totally inelastic, $\epsilon_b^* = 0$ in absolute terms, or much less elastic relative to retailers, since $t_b^* \to 0$ as $\epsilon_s^* \to \infty$. However, recent empirical studies have found the opposite, concluding that consumers are quite sensitive to price changes of payment services, probably more so than retailers, contradicting the results of the above theorem [9].

Together, these observations seem to indicate that log-concavity of the demand function may not be an appropriate assumption.

## 3.2 Skewed platform pricing under constant elasticity of demand

The choice of the probability distribution describing the heterogeneity among both groups of end-users determines the curvature of the demand functions. If the resulting demand

[8] For instance, in the case of multiproduct pricing with complementary products, the product with relatively inelastic demand sells for a higher price, see e.g. Tirole (1989) for optimal pricing strategies in multiproduct markets.

[9] Humphrey, Kim, and Vale (2001) reach this conclusion by empirically estimating consumers' price elasticities with respect to various payment instruments that are used in Norway.

functions are not log-concave then solving the first order conditions do not yield a global maximum but instead induce a saddlepoint solution. In this case maximum profits are found in one of the corner solutions. In a corner solution, participation of one side of the market is complete, while the pricing structure is completely "skewed" towards the other side of the market. This side of the market is then charged a high fee above joint marginal costs, whereas the side of the market with complete participation pays a low price below joint marginal costs.

Formally, a *corner solution* is characterized by player $i$ being tied down to its minimum benefit level $\underline{b}_i$ and player $j$ being charged a "two-sided" monopoly price $m_j$. Observe that in the low-price market $i$, demand is at its maximum, i.e. $D_i(\underline{b}_i) = 1$. Moreover, as we will see below, the two-sided monopoly price can even be higher than the "normal" one-sided monopoly price, as the platform needs to recover (almost) all costs from only one side of the market.

We will define the "buyers' corner" solution in which the buyer is "cornered" to his minimum benefit level and the pricing structure completely skewed towards the sellers' side of the market by

$$(\underline{b}_b, m_s), \text{ where } m_s = \arg\max_{t_s} \pi(\underline{b}_b, t_s, c), \tag{12}$$

and the "sellers' corner" solution by

$$(m_b, \underline{b}_s), \text{ where } m_b = \arg\max_{t_b} \pi(t_b, \underline{b}_s, c). \tag{13}$$

We will illustrate skewed pricing in a two-sided market by imposing a density function that induces demand functions with constant elasticity. Therefore, let us assume the following probability density function describing the benefits from a transaction

$$f_{\underline{b}_i,\epsilon_i}(x) = \underline{b}_i^{\epsilon_i} \epsilon_i x^{-\epsilon_i - 1}, \quad x \in [\underline{b}_i, \infty], \quad \underline{b}_i > 0, \quad \epsilon_i > 1, \quad i = b, s. \tag{14}$$

This density function yields a demand $D_i(t)$ that is not log-concave

$$D_i(t) = \underline{b}_i^{\epsilon_i} t^{-\epsilon_i}, \quad i = b, s. \tag{15}$$

and features a constant elasticity of demand $\epsilon_i$. The induced profit function is given by

$$\pi(t_b, t_s, c) = N \underline{b}_b^{\epsilon_b} \underline{b}_s^{\epsilon_s} t_b^{-\epsilon_b} t_s^{-\epsilon_s} (t_b + t_s - c). \tag{16}$$

Similar to (8), solving the first-order conditions yields

$$t_b^* = \frac{c\,\epsilon_b}{\epsilon_b + \epsilon_s - 1}, \qquad t_s^* = \frac{c\,\epsilon_s}{\epsilon_b + \epsilon_s - 1}. \tag{17}$$

However, the second-order conditions show that this interior solution is a saddlepoint, as graphically depicted in Figure 3 for a specific numerical example. Since the interior solution does not yield maximum profits, the global maximum can be found in one of the two corner solutions. Following definitions (12)-(13), and given the constant elasticity density function (14), the buyers' and sellers' corner solutions respectively yield

$$(\underline{b}_b, m_s) = \left( \underline{b}_b, \frac{(c - \underline{b}_b)\epsilon_s}{\epsilon_s - 1} \right) \text{ and } (m_b, \underline{b}_s) = \left( \frac{(c - \underline{b}_s)\epsilon_b}{\epsilon_b - 1}, \underline{b}_s \right). \tag{18}$$

Hence, which of the two corner solutions yields maximum profits, depends on the relative magnitude of the individual market sides' elasticities. The next proposition states that for sufficiently low minimum benefit levels, profits are maximized by charging the low-elastic side of the market the highest price, whereas the high-elastic side is kept to its minimum benefit level. In particular, given Assumption 3.1, for a sufficiently high elasticity of buyers' demand, buyers pay the lowest possible price so as to ensure their complete participation. In other words, buyers' demand for platform services is at its maximum. Sellers, who are less price elastic, pay high prices [10]. Consider the following proposition.

**Proposition 3.4.** *(Skewed pricing)*
*Assume a constant elasticity distribution as specified in (14) $\underline{b}_b + \underline{b}_s \leq c$. Then there exists an $\bar{\epsilon} \geq \epsilon_s$, such that for all $\epsilon_b > \bar{\epsilon}$ the monopoly platform earns maximum profits by setting fees*

$$(t_b^M, t_s^M) = (\underline{b}_b, m_s) = \left(\underline{b}_b, \frac{(c - \underline{b}_b)\epsilon_s}{\epsilon_s - 1}\right). \tag{19}$$

*At these fees, buyers' participation is complete, that is, $D_b(t_b^M) = 1$.*

*Proof*:
First, one may verify that if a buyer is charged $\underline{b}_b$ and a seller is charged $m_s$ then platform profits are given by

$$\pi(\underline{b}_b, m_s, c) = N \frac{\underline{b}_s^{\epsilon_s} (\frac{\epsilon_s(c - \underline{b}_b)}{\epsilon_s - 1})^{1-\epsilon_s}}{\epsilon_s}. \tag{20}$$

Likewise, the sellers' corner solution yields

$$\pi(m_b, \underline{b}_s, c) = N \frac{\underline{b}_b^{\epsilon_b} (\frac{\epsilon_b(c - \underline{b}_s)}{\epsilon_b - 1})^{1-\epsilon_b}}{\epsilon_b}. \tag{21}$$

These values exist and are positive for sufficiently small minimum benefit levels, i.e if $\underline{b}_i \leq c$, $i = b, s$. Second, observe that $\pi(\underline{b}_b, m_s, c)$ is constant in $\epsilon_b$. In the limit, as $\epsilon_b \to \infty$ and holding $\epsilon_s$ constant, we compute $\bar{\pi}_b = \lim_{\epsilon_b \to \infty} \pi(\underline{b}_b, m_s, c) > 0$, and $\bar{\pi}_s = \lim_{\epsilon_b \to \infty} \pi(t_b^M, \underline{b}_s, c) = 0$. Third, we derive

$$\frac{\partial \pi(m_b, \underline{b}_s, c)}{\partial \epsilon_b} = -N \frac{(c - \underline{b}_s) \left(\frac{\epsilon_b(c - \underline{b}_s)}{\underline{b}_b(\epsilon_b - 1)}\right)^{-\epsilon_b} \log\left(\frac{\epsilon_b(c - \underline{b}_s)}{\underline{b}_b(\epsilon_b - 1)}\right)}{\epsilon_b - 1} < 0 \tag{22}$$

if and only if $\log\left(\frac{\epsilon_b(c - \underline{b}_s)}{\underline{b}_b(\epsilon_b - 1)}\right) > 0$. Some algebraic manipulations show that $\pi(m_b, \underline{b}_s, c)$ is decreasing for every $\epsilon_b > 1$ if and only if $\underline{b}_b + \underline{b}_s \leq c$. Since $\pi(m_b, \underline{b}_s, c)$ is decreasing in $\epsilon_b$ and $\pi(\underline{b}_b, m_s, c)$ is constant, and $\bar{\pi}_s < \bar{\pi}_b$, this implies that there exists an $\bar{\epsilon} \geq \epsilon_s$ such that for all $\epsilon_b > \bar{\epsilon}$ it holds that $\pi(\underline{b}_b, m_s, c) > \pi(m_b, \underline{b}_s, c)$. Hence, $(t_b^M, t_s^M) = (\underline{b}_b, m_s)$ [11]. Existence of derivative (22) requires $\underline{b}_s \leq c$ which is automatically guaranteed if $\underline{b}_b + \underline{b}_s \leq c$. Finally, by definition, $D_b(t_b^M) = D_b(\underline{b}_b) = 1$, that is, complete buyers' participation. □

[10] In a model of competition among intermediaries, Caillaud and Jullien (2003) show that providing low prices or transfers to one side of the market helps to solve the initial "chicken-and-egg" problem. They refer to this strategy as "divide-and-conquer".

[11] Note that, due to symmetry, if $\underline{b}_b = \underline{b}_s$ then $\pi(\underline{b}_b, m_s, c) = \pi(m_b, \underline{b}_s, c)$ for $\epsilon_b = \epsilon_s$. Further, one can show that $\bar{\epsilon} > \epsilon_s$ for $\underline{b}_b > \underline{b}_s$; otherwise, for $\underline{b}_b \leq \underline{b}_s$ one may choose $\bar{\epsilon} = \epsilon_s$.

The result of Proposition 3.4 provides an explanation for the earlier noted empirical fact that in many two-sided markets businesses tend to settle on pricing structures that are heavily skewed towards one side of the market. It shows that, under constant elasticity of demand, skewed pricing structures are indeed profit-maximising. The mechanism at work is that the high-elastic buyers' side of the market is used to generate demand by charging buyers the lowest possible price. Every buyer will now participate and connect to the platform. The sellers' side benefits from full buyer participation through positive network externalities and is therefore also encouraged to participate. However, since sellers are less price elastic, the platform is able to extract higher prices from them. This mechanism, which results in the most inelastic side of the market paying higher prices, confirms standard economic intuition. By contrast, the interior pricing theorem of the previous section predicts the opposite.

Clearly, under skewed pricing, buyers are charged below joint marginal cost, whereas sellers face a high own price mark-up. In particular, the two-sided monopoly price can even be higher than the "normal" one-sided monopoly price which would occur if the inelastic side of the market is treated in isolation. To see this, let us assume a simple additive cost structure $c = c_b + c_s$, where $c_i$, $i = b, s$, denotes the individual costs on the buyers' and sellers' side of the market when processing a single transaction. Then, the one-sided monopoly price for sellers would equal

$$t_s^N = \arg\max_{t_s} \; N(t_s - c_s)D_s(t_s) = \frac{c_s \epsilon_s}{\epsilon_s - 1}. \tag{23}$$

We state the following proposition without proof.

**Proposition 3.5.** *Under constant elasticity of demand, the two-sided monopoly fee $t_s^M$ for sellers is higher than the one-sided monopoly price $t_s^N$ if the buyers' minimum benefit level $\underline{b}_b$ is sufficiently small. That is, $t_s^M \geq t_s^N$ if and only if $\underline{b}_b \leq c_b$.*

Hence, if in the low-price (buyers) market the contribution to total revenues is less than the own individual marginal costs $c_b$, then, in order to compensate for this negative margin on one side of the market, the two-sided monopoly price exceeds the normal one-sided monopoly price on the other, high-price (sellers) side of the market.

Our results potentially have important bearings on antitrust issues. In antitrust analysis, high mark-ups raise concerns of abuse of market power. However, this traditional antitrust logic ceases to hold in two-sided markets [12]. The fact that benefits and costs arise jointly in the two sides of the market effectively means that there is no direct economic relation between price and cost on either side of the market. Any change in demand or cost on either side of the market will affect both the total pricing level and pricing structure. Hence, it is generally not possible to examine price effects on one side of a market without considering the corresponding effect on the other side and the resulting feedback effects between them. Therefore, in antitrust matters, one cannot examine prices on either side of the market in isolation. Rather, to analyze market power one should investigate whether total price is significantly above joint marginal cost.

In particular, the skewed pricing result of Proposition 3.4 allows a nice interpretation of the current pricing practice in the Dutch debit card market where consumers pay no transaction fee for using their debit cards but retailers do. The reasoning is as follows. First, recent empirical analysis has shown that consumers are quite sensitive to price changes of payment

[12] Wright (2003a) discusses eight fallacies that can arise from using convential wisdom from one-sided markets in two-sided industries. See Evans (2003) for antitrust implications of pricing in two-sided markets.

Table 1: Monopoly profits under a constant elasticity distribution

| | | Interior | Corner solutions | |
|---|---|---|---|---|
| | | | Buyer | Seller |
| | | $(t_b^*, t_s^*)$ | $(\underline{b}_b, m_s)$ | $(m_b, \underline{b}_s)$ |
| **Price**: | | | | |
| buyer fee | $t_b$ | 0.046 | 0.020 | 0.069 |
| seller fee | $t_s$ | 0.034 | 0.081 | 0.018 |
| total fee | $t_T$ | 0.080 | 0.101 | 0.087 |
| **Demand** (in %): | | | | |
| buyer demand | $D_b(t_b, t_s)$ | 8.4 | 100.0 | 2.4 |
| seller demand | $D_s(t_b, t_s)$ | 25.5 | 3.7 | 100.0 |
| total demand | $D(t_b, t_s)$ | 2.1 | 3.7 | 2.4 |
| **Profit**: | | | | |
| total | $\pi(t_b, t_s, c)$ | 0.32 | 1.35 | 0.56 |
| per transaction ($\times 10^{-3}$) | $\pi_q(t_b, t_s, c)$ | 15.2 | 36.7 | 23.0 |
| **Welfare**: | $W(t_b, t_s, c)$ | 1.41 | 4.20 | 1.77 |

*Parameters:* $\epsilon_b = 3$, $\epsilon_s = 2.2$, $\underline{b}_b = 0.02$, $\underline{b}_s = 0.018$, $c = 0.064$ ($c_b = c_s = 0.032$), $N = 1000$.

services, see e.g. Humphrey, Kim, and Vale (2001). Second, retailers often complain that due to competitive pressures they are "forced" to facilitate debit card services. Retailers cannot afford to sell "no" to their customers. At the same time, they do not see many payment alternatives. Hence, retailers may be assumed to be much less price elastic in their demand for debit card services than consumers. Following Proposition 3.4, this might explain why the pricing structure in the debit card market is so heavily skewed towards the retailers' side of the market [13]. Indeed, as mentioned previously, the high retailer fee triggered antitrust controversy, resulting in an in-depth investigation by the Dutch competition authorities. On the basis of this investigation a heavy penalty of about 50 million euro was imposed on the Dutch debit card platform and participating banks for setting excessive tariffs for retailers. However, it is questionable whether the Dutch antitrust authority fully took notice of the two-sided nature of the debit cards market and its economic consequences.

For the monopoly platform, the recouping of costs from the sellers' side of the market actually leads to positive profits, which are increasing when the minimum benefit level on the buyers' side of the market is lowered. To see this, we look at total net profits per transaction $\pi_q(\cdot, \cdot, \cdot)$, which amount to

$$\pi_q(t_b^M, t_s^M, c) = t_b^M + t_s^M - c = \frac{(c - \underline{b}_b)\epsilon_s}{\epsilon_s - 1} - (c - \underline{b}_b). \tag{24}$$

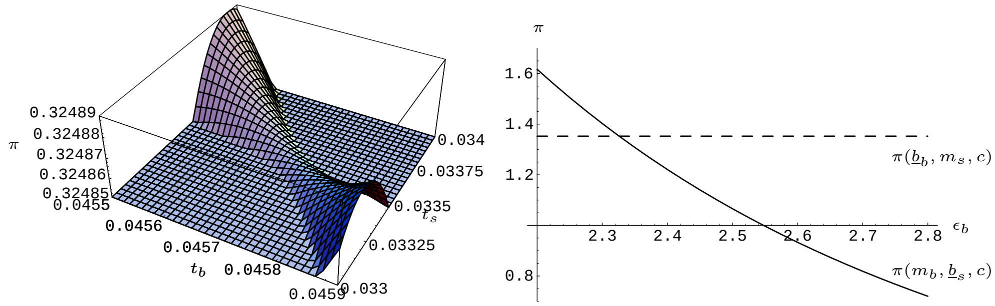
Hence, positive profit margins are realized for sufficiently small minimum benefit levels in the low-price buyers' market, i.e.

$$\pi_q(t_b^M, t_s^M, c) \geq 0 \quad \text{if and only if} \quad \underline{b}_b \leq c, \tag{25}$$

which always holds under the conditions of Proposition 3.4. Moreover, $\partial \pi_q(t_b^M, t_s^M, c)/\partial \underline{b}_b < 0$. That is, lower minimum benefit levels on one side of the market dampen the fee on this

[13] It is noted however that our model specification does not support a zero price, since $\underline{b}_i > 0$, $i = 1, 2$.

Figure 3: Profits under a constant elasticity distribution




*Explanatory note*: The left panel shows that the interior solution (0.0457,0.0335) is a saddlepoint (profits below 0.32485 are surpressed); the right panel shows both "corner" profit functions as a function of $\epsilon_b$, and verifies $\bar{\epsilon} \approx 2.33$.

side of the market, but increase the fee levied on the other side of the market. This latter effect dominates, so that profits per transaction increase when minimum benefit level in the low-price market decreases.

The following example illustrates our findings.

**Example**: Monopoly profits
We set the following parameter values: $c = 0.064$ ($c_b = c_s = 0.032$), $\underline{b}_b = 0.02$, $\underline{b}_s = 0.018$, $\epsilon_b = 3$, $\epsilon_s = 2.2$, and $N = 1000$, in accordance with the assumptions in Proposition 3.4. Prices, demans, and profits for the parameter values can be found in Table 1.

The table indeed shows that the interior solution of Rochet and Tirole (2003) does not yield maximum profits for the monopoly platform when demand derives from a constant-elasticity distribution. The interior solution $(0.046, 0.034)$ is a saddlepoint as clearly depicted in the left panel of Figure 3. The right panel of Figure 3 shows both "corner" profit functions $\pi(\underline{b}_b, m_s, c)$ and $\pi(m_b, \underline{b}_s, c)$ as a function of $\epsilon_b$. We know that $\pi(\underline{b}_b, m_s, c)$ is constant in $\epsilon_b$ whereas $\pi(m_b, \underline{b}_s, c)$ is decreasing in $\epsilon_b$. We numerically verify that $\bar{\epsilon} \approx 2.33$. Hence, following Proposition 3.4, for an $\epsilon_b = 3.0 > 2.33$, profits are maximized when buyers are tied down to their minimum benefit level $\underline{b}_b = 0.02$ and sellers being charged a high two-sided monopoly price $t_s^M = m_s = 0.081$.

Also, from the table, total demand is highest in the buyers' corner solution even though it has the highest total price. This stresses the importance of the pricing structure in two-sided markets. Complete buyers' participation boosts total demand, which at the same time allows a higher price to be extracted from the relatively inelastic sellers. As a result, around 4% of all payment transactions are executed by debit card. Moreover, from (25) it follows that positive profit margins are realised, since $\underline{b}_b = 0.02 \leq c = 0.064$. But note that on the buyers' side operational losses are incurred since the charged fee cannot make up for the individual costs, that is, $t_b^M = \underline{b}_b = 0.02 < c_b = 0.032$. Hence, the two-sided monopoly fee for the sellers is higher than the one-sided monopoly price, that is, $t_s^M = 0.081 > 0.059 = t_s^N$. ■

# 4 Social Welfare

In the model, the network platform enjoys monopolistic power in setting prices for both buyers sellers. It is well known that a monopolistic price-setter distorts the market by reducing supply to raise prices, inducing a dead weight loss. In a two-sided market where network externalities play an important role, the underlying economics get more complicated.

In our model, total (expected) social welfare that is generated from platform services is equal to consumer plus retailer (expected) benefits minus costs, conditional upon buyers' and sellers' participation in the network. Formally, total social welfare is described by

$$W(t_b, t_s, c) = N(\beta_b(t_b) + \beta_s(t_s) - c)D(t_b, t_s), \tag{26}$$

where $\beta_b(t_b)$ denotes the (conditional) expected benefit to a buyer, defined by

$$\beta_b(t_b) = \mathbb{E}(b_b | b_b \geq t_b) = \frac{\int_{t_b}^{\bar{b}_b} x h_b(x) dx}{1 - H_b(t_b)}, \tag{27}$$

and, similarly for a seller,

$$\beta_s(t_s) = \mathbb{E}(b_s | b_s \geq t_s) = \frac{\int_{t_s}^{\bar{b}_s} x h_s(x) dx}{1 - H_s(t_s)}. \tag{28}$$

We will examine the social welfare function (26) for the constant elasticity probability distribution used in the previous section. Two questions of particular interest arise. First, how do socially optimal prices and demand compare to the prices set by the monopolistic platform? Second, under socially optimal prices, is the platform profitable or does it operate at a loss? As it turns out that the platform will lose money in the social optimum, we subsequently investigate optimal social welfare under a balanced budget constraint (i.e. Ramsey pricing).

Substituting the density function $f_{\underline{b}_b, \epsilon_b}(x) = \underline{b}_b^{\epsilon_b} \epsilon_b x^{-\epsilon_b - 1}$, $x \in [\underline{b}_b, \infty]$, $i = b, s$, in social welfare function (26) leads to

$$W(t_b, t_r, c) = N \frac{\underline{b}_b^{\epsilon_b} \underline{b}_r^{\epsilon_r} t_b^{-\epsilon_b} t_s^{-\epsilon_s} (\epsilon_b(\epsilon_s - 1)t_b + \epsilon_r(\epsilon_b - 1)t_r - (\epsilon_b - 1)(\epsilon_s - 1)c)}{(\epsilon_b - 1)(\epsilon_s - 1)}. \tag{29}$$

As was the case in the previous section, $W(t_b, t_s, c)$ exhibits an interior saddle point solution. Analogous to the result in Proposition 3.4, the next proposition states that, for a sufficiently high elasticity of buyers' demand, the global social optimum is found at the corner solution where buyers are charged the lowest admissible fee so that the whole buyers' market is captured. In contrast, sellers, who are more price-inelastic, are confronted with higher socially optimal prices.

To see this, let us first define both corner solutions

$$(\underline{b}_b, w_s), \text{ where } w_s = \arg\max_w W(\underline{b}_b, w, c) = c - \frac{\underline{b}_b \epsilon_b}{\epsilon_b - 1}, \tag{30}$$

and

$$(w_b, \underline{b}_b), \text{ where } w_b = \arg\max_w W(w, \underline{b}_s, c) = c - \frac{\underline{b}_s \epsilon_s}{\epsilon_s - 1}. \tag{31}$$

Consider the following proposition.

**Proposition 4.1.**
*Assume a constant elasticity distribution as specified in (14) and* $\underline{b}_s \leq \frac{1}{\epsilon_s}(\epsilon_s - 1)(c - \underline{b}_b)$*. Then there exists an* $\tilde{\epsilon} \geq \epsilon_s$ *such that for all* $\epsilon_b > \tilde{\epsilon}$ *social welfare is maximized when the platform implements fees*

$$(t_b^W, t_s^W) = (\underline{b}_b, w_s) = \left(\underline{b}_b, c - \frac{\underline{b}_b \epsilon_b}{\epsilon_b - 1}\right). \tag{32}$$

*At these fees, buyers' participation is complete, i.e.* $D_b(t_b^W) = 1$*.*

*Proof*:
First, one may verify that[14]

$$W(\underline{b}_b, w_s, c) = N \frac{\underline{b}_s^{\epsilon_s}(c - \frac{\epsilon_b \underline{b}_b}{\epsilon_b - 1})^{-\epsilon_s}(\epsilon_b(c - \underline{b}_b) - c)}{(\epsilon_s - 1)(\epsilon_b - 1)} \tag{33}$$

and

$$W(w_b, \underline{b}_s, c) = N \frac{\underline{b}_b^{\epsilon_b}(c - \frac{\epsilon_s \underline{b}_s}{\epsilon_s - 1})^{-\epsilon_b}(\epsilon_s(c - \underline{b}_s) - c)}{(\epsilon_s - 1)(\epsilon_b - 1)}. \tag{34}$$

These values exist and are positive for sufficiently small minimum benefit levels, i.e if $\underline{b}_i < \frac{1}{\epsilon_i}(\epsilon_b - 1)c$, $i = b, s$. In the limit, as $\epsilon_b \to \infty$ and holding $\epsilon_s$ constant, we compute

$$\bar{W}_b = \lim_{\epsilon_b \to \infty} W(\underline{b}_b, w_s, c) = N \frac{\underline{b}_s^{\epsilon_s}(c - \underline{b}_b)^{1-\epsilon_s}}{\epsilon_s - 1} > 0 \tag{35}$$

and, if $\underline{b}_s \leq \frac{1}{\epsilon_s}(\epsilon_s - 1)(c - \underline{b}_b)$,

$$\bar{W}_s = \lim_{\epsilon_b \to \infty} W(w_b, \underline{b}_s, c) = 0. \tag{36}$$

Third, it is straightforward to show that

$$\frac{\partial W(\underline{b}_b, w_s, c)}{\partial \epsilon_b} = -N \frac{\underline{b}_s^{\epsilon_s}(c - \frac{\epsilon_b \underline{b}_b}{\epsilon_b - 1})^{-\epsilon_s}}{(\epsilon_b - 1)^2} < 0 \tag{37}$$

and

$$\frac{\partial W(w_b, \underline{b}_s, c)}{\partial \epsilon_b} = N \frac{\underline{b}_b^{\epsilon_b}(c - \frac{\epsilon_s \underline{b}_s}{\epsilon_s - 1})^{-\epsilon_b}(\epsilon_s(c - \underline{b}_s) - c)\left((\epsilon_s - 1)\log\left(\frac{(\epsilon_s - 1)\underline{b}_s}{\epsilon_s(c - \underline{b}_s) - c}\right) - 1\right)}{(\epsilon_s - 1)(\epsilon_b - 1)^2} < 0, \tag{38}$$

for $\underline{b}_s \leq \frac{1}{\epsilon_s}(\epsilon_s - 1)(c - \underline{b}_b)$. So, both value functions are monotonically decreasing, but converge to different limit points. Hence, because $\bar{W}_b > \bar{W}_s$, there exists an $\tilde{\epsilon} \geq \epsilon_s$ such that for every $\epsilon_b > \tilde{\epsilon}$ we have that $W(\underline{b}_b, w_s, c) > W(w_b, \underline{b}_s, c)$. Therefore, maximum social welfare is realized at fees $(t_b^W, t_s^W) = (\underline{b}_b, w_s)$ for sufficiently large $\epsilon_b$. Finally, by definition, $D_b(\underline{b}_b) = 1$. □

[14] Note that due to symmetry $W(\underline{b}_b, w_s, c) = W(w_b, \underline{b}_s, c)$ for $\underline{b}_b = \underline{b}_s$ and $\epsilon_b = \epsilon_s$.

Proposition 4.1 shows that the socially optimal price structure does not qualitatively differ from the optimal price structure in the monopolistic case. A central planner would choose the same corner solution as the monopoly platform. So, in a social optimum, pricing is completely skewed towards the sellers' side of the market and hence all costs (net from the buyers' minimum benefit contribution) are recovered from this side. Buyers are tied down to their minimum benefit levels, so that participation is complete on their side. Again, the antitrust implications are apparent: To balance the demand for platform services in a socially optimal way, one will always observe a non-neglible price gap between the sellers' and buyers' price. Sellers might mistakenly perceive the resulting own price mark-up as a consequence of abuse of market power by the platform.

Quantitatively, it is easy to show that for sufficiently small buyers' minimum benefit levels, the socially optimal sellers' fee is lower than the monopolistic sellers' fee. More precise,

$$t_s^W \leq t_s^M \quad \text{if and only if} \quad \underline{b}_b \leq c. \tag{39}$$

So, although the skewed pricing structure is optimal from both a social welfare and a monopoly point of view, a monopolistic platform will nevertheless distort the market by charging suboptimally high tariffs to sellers. This unambiguously leads to underprovision of network services, i.e. $D(\underline{b}_b, t_s^M) < D(\underline{b}_b, t_s^W)$.

To assess the profitability of the platform under social optimal pricing, we focus again on the net profits per transaction

$$\pi_q(t_b^W, t_s^W, c) = t_b^W + t_s^W - c = -\frac{\underline{b}_b}{\epsilon_b - 1}, \tag{40}$$

which is smaller than zero, as we have assumed that $\epsilon_b > 1$. That is,

**Proposition 4.2.**
*Under socially optimal fees $(t_b^W, t_s^W)$, the platform sets prices below marginal costs and hence runs at a loss, i.e.*

$$t_b^W + t_s^W < c \quad \Rightarrow \quad \pi_q(t_b^W, t_s^W, c) < 0. \tag{41}$$

Thus in two-sided markets, we have the striking result that a setup with constant unit costs and zero fixed costs leads to below-marginal cost pricing [15]. The result derives from high social benefits of having sellers trading on the platform. These high benefits outweigh the negative contribution to social welfare in the form of setting total prices below joint marginal costs. This socially optimal tradeoff between benefits and costs thus results in negative profits for the platform.

The high social benefits of sellers' participation stem from the positive network externality induced by complete buyers' participation. This can easily be seen from substituting $t_b = \underline{b}_b$, $\beta_b(\underline{b}_b) = \mathbb{E}(b_b)$, and $D_b(\underline{b}_b) = 1$, into social welfare function (26), which then reduces to

$$N\left(\mathbb{E}(b_b)D_s(t_s) + \int_{t_s}^{\infty} x h_s(x)dx - cD_s(t_s)\right). \tag{42}$$

[15] Bolt and Tieman (2003) show that this loss under socially optimal fees also occurs under log-concave demand functions, when applying a uniform distribution. In a somewhat different setup of two-sided markets, see Armstrong (2004) for a similar result where socially optimal prices are set below marginal costs.

Table 2: Social welfare under a constant elasticity distribution

| | | Interior Solution | Corner solutions | |
|---|---|---|---|---|
| | | | Buyer | Seller |
| | | | $(\underline{b}_b, m_s)$ | $(m_b, \underline{b}_s)$ |
| **Price**: | | | | |
| buyer fee | $t_b$ | 0.030 | 0.020 | 0.031 |
| seller fee | $t_s$ | 0.018 | 0.034 | 0.018 |
| total fee | $t$ | 0.048 | 0.054 | 0.049 |
| **Demand** (in %): | | | | |
| buyer demand | $D_b(t_b, t_s)$ | 28.3 | 100.0 | 26.9 |
| seller demand | $D_s(t_b, t_s)$ | 96.6 | 24.7 | 100.0 |
| total demand | $D(t_b, t_s)$ | 27.3 | 24.7 | 26.9 |
| **Profit**: | | | | |
| total | $\pi(t_b, t_s, c)$ | -4.2 | -2.5 | -4.0 |
| per transaction ($\times 10^{-3}$) | $\pi_q(t_b, t_s, c)$ | -15.2 | -10.0 | -15.0 |
| **Welfare**: | $W(t_b, t_s, c)$ | 4.2 | 7.0 | 4.2 |

*Parameters:* $\epsilon_b = 3$, $\epsilon_s = 2.2$, $\underline{b}_b = 0.020$, $\underline{b}_s = 0.018$, $c = 0.064$ $(c_b = c_s = 0.032)$, $N = 1000$.

Optimizing the sellers' price by taking the derivative with respect to $t_s$ and equating to zero yields

$$N\left(-\mathbb{E}(b_b)h_s(t_s) - t_s h_s(t_s) + c h_s(t_s)\right) = 0. \tag{43}$$

or

$$t_s + \mathbb{E}(b_b) = c. \tag{44}$$

Hence, in the case of two-sided markets, the standard "price equals marginal cost equation" is augmented by a term which arises from the positive network externality [16]. In other words, equating marginal costs and marginal revenues on the sellers' side, requires adjusting the socially optimal sellers' fee to reflect the positive externality of complete buyers' participation, as measured by their average benefit of platform services. However, as this average benefit which accrues to sellers is larger than the price buyers pay, that is, $\mathbb{E}(b_b) = \epsilon_b \underline{b}_b/(\epsilon_b - 1) > \underline{b}_b$, costs cannot be fully recovered. The resulting negative profit per transaction is equal to

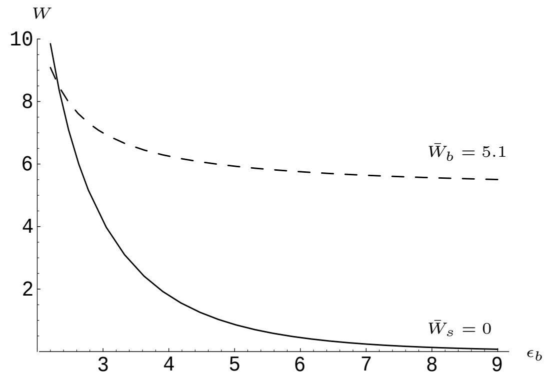
$$t_b^W + t_s^W - c = \underline{b}_b - \mathbb{E}(b_b) = -\frac{\underline{b}_b}{\epsilon_b - 1} < 0.$$

These losses incurred by the platform in the social optimum could warrant compensation through a goverment subsidy or cross subsidization from other sources of income. In the case of payment networks, imposing an interchange fee may also alleviate the problem (see e.g. Wright (2003b) and Rochet and Tirole (2002)).

To illustrate the social welfare effects quantitatively, we revisit the example.

[16] It is straightforward to show that in the corresponding one-sided specification of our model, the expectation term $\mathbb{E}(b_b)$ drops out, so that prices equal marginal costs again in the one-sided market.

Figure 4: Social welfare in the corner solutions




*Explanatory note:* The dotted line represents $W(\underline{b}_b, w_s, c)$ and the straight line represents $W(w_b, \underline{b}_s, c)$ as a function of $\epsilon_b$.

**Example** (continued): Social welfare
As before, we set the following parameter values: $c = 0.064$ ($c_b = c_s = 0.032$), $\underline{b}_b = 0.020$, $\underline{b}_s = 0.018$, $\epsilon_b = 3$, $\epsilon_s = 2.2$, and $N = 1000$. Table 2 summarizes the results for the diferent solutions.

First, from the table it is immediately clear that the interior solution does not yield maximum social welfare. Maximum social welfare is achieved at the corner solution in which the buyers' fee is set at the minimum benefit level. Figure 4 shows social welfare in both corner solutions $W(\underline{b}_b, w_s, c)$ and $W(w_b, \underline{b}_s, c)$ as a function of $\epsilon_b$. The limit points are calculated as $\bar{W}_b = N\underline{b}_s^{\epsilon_s}(c - \underline{b}_b)^{1-\epsilon_s}/(\epsilon_s - 1) = 5.1$ and $\bar{W}_s = 0$. Numerically, we verify that $\tilde{\epsilon} \approx 2.32$, so that for every $\epsilon_b > 2.32$ we have that the buyers' corner solution leads to maximum profits. Second, following condition (39), as $\underline{b}_b = 0.020 \leq c = 0.064$, socially optimal prices will be lower than the prices set by the monopolistic platform, inducing higher demand. In fact, socially optimal demand for network services is almost seven times higher than demand in the monopolistic case. Clearly, network services are undersupplied in the latter case. Third, by Proposition 4.2, the platform incurs operational losses by implementing the socially optimal fees. ■

## 4.1 Ramsey pricing: Balanced budget fees

As shown above, for plausible parameter values, the network will operate at a loss by implementing socially optimal prices. Alternatively, we could optimize social welfare $W(t_b, t_s, c)$ under a balanced budget condition $t_b + t_s = c$. In this constrained Ramsey setting, it is no surprise that maximum social welfare with a balanced budget is again found at one of the corner solutions, depending on the relative magnitude of the elasticities. In particular, it can be shown that there exists an $\hat{\epsilon} \geq \epsilon_s$ such that for all $\epsilon_b > \hat{\epsilon}$, it holds that

$$t_b^{BB} = \underline{b}_b, \qquad t_s^{BB} = c - \underline{b}_b. \tag{45}$$

Table 3: Social welfare and balanced budget prices

| | | Social Optimum (Unconstrained) | Ramsey (Balanced budget) |
|---|---|---|---|
| **Price**: | | | |
| buyer fee | $t_b$ | 0.020 | 0.020 |
| seller fee | $t_s$ | 0.034 | 0.044 |
| total fee | $t_T$ | 0.054 | 0.064 |
| **Demand** (in %): | | | |
| buyer demand | $D_b(t_b, t_s)$ | 100.0 | 100.0 |
| seller demand | $D_s(t_b, t_s)$ | 24.7 | 14.0 |
| total demand | $D(t_b, t_s)$ | 24.7 | 14.0 |
| **Profit**: | | | |
| total | $\pi(t_b, t_s, c)$ | -2.5 | 0.0 |
| per transaction ($\times 10^{-3}$) | $\pi_q(t_b, t_s, c)$ | -10.0 | 0.0 |
| **Welfare**: | $W(t_b, t_s, c)$ | 7.0 | 6.5 |

*Parameters:* $\underline{b}_b = 0.020$, $\underline{b}_s = 0.018$, $c = 0.064$ ($c_b = c_s = 0.032$), $N = 1000$; $\epsilon_c = 3$, $\epsilon_s = 2.2$.

Hence, if buyers are sufficiently price elastic, then sellers end up paying for all the cost of the network. Again, a result similar to (39) holds in the sense that the Ramsey price for the seller is lower than the monopoly fee if buyers' minimum benefits levels are sufficiently small. That is, in the constant elasticity case,

$$t_s^{BB} \leq t_s^M \quad \text{if and only if } \underline{b}_b \leq c. \tag{46}$$

The continued example illustrates the impact of implementing balanced budget fees compared to socially optimal fees.

**Example** (continued): Balanced budget
As before, the parameter values are $c = 0.064$ ($c_b = c_s = 0.032$), $\underline{b}_b = 0.020$, $\underline{b}_s = 0.018$, $N = 1000$. We impose $\epsilon_b = 3$ and $\epsilon_s = 2.2$. Table 3 gives the results for the constant elasticity distribution.

As the total fee is higher than in the unconstrained social optimum, implementing balanced budget fees induces a drop in total demand for platform services, in this case from 24.7% to 14%. By definition, there is also a drop in social welfare, but in this example rather limited one: welfare drops just 7% from 7.0 to 6.5. By definition, the platform breaks even when implementing balanced budget fees. Further, observe that the balanced budget fee for the seller is lower than the two-sided monopolistic fee, analogous to (46), but higher than the sellers' fee in the unconstrained social optimum, in order to balance the budget. ■

# 5 Conclusions

Many real world two-sided markets have adopted skewed pricing strategies, in which the price mark-up is much higher on one side of the market than the other. Using a simple of two-sided markets, we show that under constant elasticity of demand these skewed pricing strategies are indeed profit maximizing. The underlying mechanism is a positive network externality: By keeping prices low, demand is stimulated on the more elastic side of the market, thus creating increased benefits of market participation on the other, less elastic, side. On this side, the platform can subsequently extract profits by setting high prices.

In a qualitative sense, this skewed pricing result carries over to the social optimum. We prove that a central planner would choose the same corner solution, in terms of full participation of the more elastic buyers' side of the market and recovering costs from the price-inelastic sellers' side. However, we show that the social planner will price below marginal costs, leading to an underrecovery of costs and hence an operational loss for the platfom. This result is driven by the positive externality on the sellers, exerted by full buyers' participation. The contribution of this positive externality to social welfare leads the social planner to increase sellers' participation by setting the sellers' price below marginal cost. As this low price is not compensated on the buyers' side, where the lowest admissible price is set to induce full participation, the result is an operational loss for the platform. As an alternative to a loss-making social optimum, we investigate Ramsey pricing, i.e. social welfare optimization under a balanced budget constraint. By definition, Ramsey pricing results in a zero profit. In addition, our analysis seems to indicate that having a platform operate under a balanced budget restriction results in only a limited social welfare loss.

We illustrated our general results by analysing the Dutch debit card system and argue that our theoretic results fit the observed properties of debit card pricing in the Netherlands well. Dutch retailers pay a positive transaction fee for accepting debit card payments, while the consumers are not charged per transaction. When we assume that retailers are much less price elastic in their demand for debit card services than consumers, as was found in recent empirical studies on payment systems, this may explain the pricing structure in the debit card market. Hence, these empirical findings support our theoretical results.

In our view, the antitrust consequences of skewed pricing in two-sided markets is a challenging avenue for further research. In particular, the effects on skewed pricing strategies of platform competition, when more than one platform is active in the market, seems a natural extension of our current research. We plan to go down this road of inquiry in the near future.

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