Option pricing and spikes in volatility: theoretical and empirical analysis[*]

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Abstract

This paper considers a financial market where the asset prices and the corresponding volatility are driven by a multidimensional mixture of Wiener shocks and Poisson jumps. While implied volatility is characterized by spikes, the existing models rely on the restrictive assumption of positive jumps in volatility. To overcome this inadequacy, the present paper introduces normally distributed jumps in the log-variance process. The model proposed is able to mimic empirically observed spikes in volatility and, consequently, improves upon the existing literature as it replicates the main features of both the stock return series and the corresponding option prices. After estimating the stock returns via the Efficient Method of Moments, the expression for the valuation of a plain vanilla European call option is derived, using the no-arbitrage argument.

S&P500 option prices are used to assess quantitatively the empirical performance of the innovative features of the proposed model. The estimates indicate that spikes in volatility introduce a significant improvement in option pricing and provide evidence for stochastic jump risk premia.

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1 Introduction

Since options are derivative securities, their value is strictly tied to the value of the underlying asset. This means that an adequate option pricing model ought to have two main characteristics. On one hand, it should replicate as closely as possible the behavior of the underlying asset price and it should therefore match not only its mean and variance, but also the higher moments, such as skewness and kurtosis. On the other hand, it should also be consistent with the observed option prices, thereby replicating the implied volatility pattern.[1]

Taking the seminal Black and Scholes model as the starting point, many generalizations have been introduced in order to improve its pricing performance. These include:

i) Stochastic Volatility in order to account for negative skewness and high kurtosis in the stock return series;[2]

ii) Stochastic Volatility and jumps in the stock price process so as to improve the pricing of short term options;[3]

iii) Stochastic Volatility and jumps in both the stock price and volatility process in order to achieve the empirically observed persistence in the impact of jumps.[4]

A major issue, however, remains unexplored since none of the existing models is capable of accounting for *spikes* in the observed implied volatility. More specifically, in Duffie et al. (2000), volatility is modelled as an affine process that can jump up violently, but that subsequently cannot jump down as observed in the data. Therefore, as Eraker (2003) observes relying on the assumption of *positive jumps* in the volatility process, the Duffie et al. (2000) model can explain the abrupt increase in volatility registered on the day of the crash, but not the subsequent violent fall.

The present paper proposes a model for asset pricing that allows for spikes in volatility by constructing a new, more general log-variance model. The key aspects of the model proposed

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[1]While the Black and Scholes model predicts a flat profile for the implied volatility surface, empirical data indicate that, especially after the 1987 crash, the implied volatility for equity options strongly depends on the strike price.

[2]See e.g., Cox (1975, 1996), Hull and White (1987), Scott (1987), Wiggins (1987), Stein and Stein (1991), Heston (1993), Naik (1993), Duan (1995), Fouque, Papanicolaou and Sircar (2000), Davydov and Linetsky (2001), Detemple and Tian (2002).

[3]Bates (1996), Scott (1997), Carr et al. (2003)

[4]Duffie, Pan and Singleton (2000)

here are the following:

i) stock prices follow a mixture of Brownian motion and multivariate compensated Poisson process;

ii) the logarithm of the variance follows an Ornstein-Uhlenbeck process with jumps whose size is random and whose sign is unrestricted;

iii) the stock price can jump both alone and together with volatility.

Regarding the choice of a framework for the volatility process, I assume that the logarithm of the variance follows an Ornstein-Uhlenbeck process.[5] This choice reflects the ability of the exponential function to generate high volatility values in a very limited time. Although many generalizations of the loglinear model have been introduced, there has been so far no exploration of the possibility of Poisson jumps within this framework.[6] The closest contribution in this direction is the model proposed by Duffie et al. (2000) who introduce jumps in a Cox, Ingersoll and Ross (CIR) model for volatility (affine process). Compared to this specification, the model proposed here shows a higher flexibility, since: *i*) there are no constraints on the sign of the jumps in the volatility process; *ii*) the stock price is able to jump both alone and together with volatility, unlike in Duffie et al. (2000);[7] *iii*) the exponential function is suitable for modelling moments of market stress, because of its ability to generate extremely high volatility values. All these features enable my model to price a stock and the underlying derivatives even in the wake of a major financial crisis.

In an incomplete market setting, the technique for option pricing adopted in the present paper is the Equivalent Martingale Measure approach (as in Jeanblanc-Picque and Pontier (1990), Xue (1991), Shirakawa (1992), Bardhan and Cao (1995), Cox and Ross (1976), Harrison and Kreps (1979)) under which the expected rate of return on any asset is equal

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[5]In discrete time, the counterpart of this model can be found in the EGARCH model of Nelson (1991). Alternatively, Scott (1987) assumes that the logarithm of volatility (the square root of the variance process) follows an Ornstein-Uhlenbeck process. Other branches of the literature model volatility as an Ornstein-Uhlenbeck process where the underlying state variable is Gaussian (Wiggins (1987), Chesney and Scott (1989), Melino and Turnbull (1990)), as a CIR (Cox , Ingersoll and Ross) process with a reflecting barrier at zero where the underlying state variable is Gamma distributed (Cox , Ingersoll and Ross(1985), Bailey and Stulz (1989), Heston (1993))or as a CEV (constant elasticity of variance) process (Cox (1975), Cox and Ross (1976), Jones (2003)).

[6]Alizdeh et Al. (2002), Chacko and Viceira (1999), Gallant and Tauchen (1999) consider volatility as a multifactor diffusion model.

[7]Duffie et al. (2000) assume that the stock price can jump either by itself or together with the volatility process.

to the riskless interest rate.[8]

The model proposed in this paper is tested empirically using a two stage approach. In the first stage, I estimate the parameters of the structural model for the stock price dynamics using the Efficient Method of Moments (EMM) and employing the time series of the S&P500 stock returns . This choice reflects the fact that one can observe a part of the state vector (in this case, the stock return series), but not its corresponding volatility. This necessarily rules out estimation approaches such as maximum likelihood (MLE) or the generalized method of moments (GMM). MLE is intractable, while GMM is infeasible.[9] The EMM is based on *indirect inference*. The main idea is to replace the initial model with a more tractable, approximated one. The latter is denoted the *auxiliary model* and is a descriptive model with a large number of parameters. Following Gallant and Tauchen (2001), I evaluate the scores of the auxiliary model using the simulated series of data which derives from the structural model. In this way, I determine the moment conditions for this problem. The proposed log-variance model is capable of accommodating the linear aspect and the tail behavior of the data. In addition, the estimate of the mean of the jumps that affect volatility is negative and significantly different from zero. This result shows that the assumption of positive jumps in volatility made by Duffie et al. (2000) is too restrictive.

In the second stage, I address a specific question: are spikes in volatility an important factor in explaining option price dynamics? I answer this question by investigating whether option data show any evidence of jumps in volatility and, more specifically, if my model can mimic more adequately and eventually forecast option prices. Holding the first round of estimates fixed, I price the risk premia embedded in option prices and I estimate the risk neutral parameters. For this second application, I use the cross section data on the S&P500 call options. I find that employing a log-variance model with spikes dramatically improves the pricing performance. In addition, I find evidence for stochastic risk premia.

The outline of this paper is as follows. In section 2, I clarify the core of the entire debate. Section 3 contains the setup of the model. The relevance of the risk premia and of the binding

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[8]Another approach often used by many authors (e.g.: Bates (1998), Naik and Lee (1990), Aase (1993), Dieckmann (2002)) is based on a general equilibrium argument and explicitly links the risk premia to the preference parameters of the representative agent when the markets are incomplete.

[9]The moment restrictions lack an analytical representation in terms of the observables and unobservables

no arbitrage condition is addressed in Section 4 which also lays the Equivalent Martingale Measure approach. In Section 5, I explain the estimation methods adopted in the paper. Finally, Section 6 shows the empirical results and corresponding diagnostics and Section 7 concludes.

2   The origins of the debate

When pricing an option, the first task one faces is to value the underlying assets (such as stocks, futures or currencies) on which the option depends. In the past, the empirically observed absence of significant autocorrelations in the stock returns led to modelling them as independent random variables, or more precisely, as *random walks* in discrete time. By applying the *Invariance Principle* (Functional Central Limit Theorem), Brownian motion can be seen as the continuous time counterpart of the random walk process. In 1900, Bachelier proposed the following very simple model for stock pricing:

$$S_t = S_0 + \sigma W_t$$

where W_t is a Brownian motion process.

In 1973, a similar setting was adopted by Black and Scholes in their seminal paper for option pricing:

$$S_t = S_0 \exp\left(\mu t + \sigma W_t\right)$$

Lately, this approach has been criticized for its failure to capture important features of both stock and option price data because it relies on the restrictive assumption of independence of returns. Even though it has been observed that the stock returns are not autocorrelated, several tests have shown that non linear functions of returns are indeed autocorrelated (see Figure 1). These tests are based upon the analysis of several autocorrelation functions such as, for example:

i) the autocorrelation function of various powers of returns:

$$C_{1\tau} = corr\left(\left|r(t,\Delta\tau)\right|^{\varepsilon}, \left|r(t+\tau,\Delta\tau)\right|^{\varepsilon}\right)$$

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ii) the autocorrelation of absolute power of returns:

$$C_{2\tau} = corr\left(\ln|r(t, \Delta\tau)|, \ln|r(t + \tau, \Delta\tau)|\right)$$

iii) the correlation of returns with subsequent squared returns:

$$C_{3\tau} = corr\left(|r(t + \tau, \Delta\tau)|^2, r(t, \Delta\tau)\right)$$

for some given time lag τ.

This stylized fact, often referred to as *volatility clustering*, represents a clear violation of the independence assumption and means that large price movements are typically followed by other large movements.

Furthermore, large downward movements are usually observed more often than their upward counterparts. Translated in statistical terms, this means that the stock returns show negative skewness. Since the Black and Scholes model cannot replicate heavy tails in the distribution of returns (high finite kurtosis) and instead predicts zero skewness, it fails to capture these important empirical features of the stock returns.

As the model's ultimate goal is to price options, another way to test its empirical performance is to check how precisely it can replicate actual option prices. Given the assumptions of the Black and Scholes model, if it correctly resembled the option price behavior, the same implied volatility should characterize all otherwise identical options despite the presence of different strike prices. Figure 2, which represents the market implied volatility vs. the option moneyness, shows clearly that, for empirical implied volatilities, this is not the case.

In order to eliminate these biases, many generalizations of the Black and Scholes model have been introduced. The jumps in the return process allowed for by Merton (1976) improved the tail behavior (skewness and kurtosis). However, unlike the corresponding empirical time series characterized by volatility clustering, the resulting process for stock returns still retains the property of independent increments. Models with stochastic volatility without jumps, are capable of replicating important features such as the tail behavior of the stock returns, volatility clustering and leverage effect, and they reproduce realistic implied volatilities for long maturities. At the same time though, they fail to yield a realistic implied

volatility pattern for short maturities.[10] The latter feature can instead be easily captured by introducing jumps that reproduce realistic implied volatility smiles at short maturities.[11]

The evidence from the existing literature seems thus to suggest that the way forward lies in combining both stochastic volatility and jumps in a model for stock returns.[12] One of the most recent contributions is the model by Duffie et al. (2000) which features jumps in both volatility and stock return processes. This model can explain violent and persistent market movements with upward movements in volatility, though it cannot reproduce volatility spikes. These large market movements, far from being simple outliers, are an important characteristic of the stock return time series.

The goal of the present paper is to propose a new model for stock pricing that can replicate spikes in volatility. This model shall be estimated by the Efficient Method of Moment using stock return series. Finally, after deriving the corresponding call option prices by Monte Carlo simulation, the risk neutral parameters and the risk premia shall be evaluated by minimizing the squared deviations from the market implied volatility.

3 Security Market Model

The $\sigma-$field $\mathcal{F}_t$ represents the information that investors have at each point in time $t \in [0,1]$ with $\mathcal{F}_s \subset \mathcal{F}_t$ if $s \le t$. Suppose that $(\Omega, P, \mathcal{F})$ is the probability space for this model. More specifically, P is the probability measure which represents the investors' beliefs, Ω is the set of states of the world and $\mathcal{F} \equiv \mathcal{F}_1$ is the set of events that can be seen at the trading horizon. The filtration $\mathbf{F} \equiv \{\mathcal{F}_t; t \in [0,1]\}$ is assumed to be right continuous and P-complete. In this economy there are N stocks. The price of the stock portfolio S_t, at time t is assumed to follow a mixture of Brownian motion and multivariate compensated Poisson process. More specifically, the stock price process is right continuous (securities are traded ex-dividend) and left limited[13].

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[10]See, for example, Cox (1975, 1996), Hull and White (1987), Scott (1987), Wiggins (1987), Stein and Stein (1991), Heston (1993), Naik (1993), Duan (1995), Fouque, Papanicolaou and Sircar (2000), Davydov and Linetsky (2001), Detemple and Tian (2002).

[11]See, for example, Cox and Ross (1976) and Merton (1976).

[12]Bates (1996), Scott (1997), Carr et al. (2003)

[13]One can assume, without loss of generality, that the left limit exists and is finite
$S_{nt^-} = \lim_{u \to t} S_{nu}$
Consequently, the jump of the stock price at time t will be

Besides the bond $B_t = e^{-rt}$, there is a portfolio of risky assets and a stochastic volatility component

$$d \ln S_t = \mu dt + \sqrt{V_t} \left[\beta_{12} dW_{2t} + \sqrt{1 - \beta_{12}^2} dW_{1t} \right] \tag{1}$$

$$+ \sqrt{1 - \psi_{33}^2} \int_{R \backslash \{0\}} \zeta_1 (\Gamma) P_1 (d\Gamma, dt) + \psi_{33} \int_{R \backslash \{0\}} \zeta_2 (\Gamma) P_2 (d\Gamma, dt)$$

where the stochastic part of the corresponding volatility follows the law

$$V_t = \exp(U_t)$$

$$dU_t = \left(\mu^U + \alpha_{22} U_t \right) dt + \beta_{20} dW_{2t} + \int_{R \backslash \{0\}} \zeta_3 (\Gamma) P_2 (d\Gamma, dt) \tag{2}$$

where $\zeta_1 (\Gamma) \sim N(\psi_{11}, \psi_{12}^2)$, $\zeta_2 (\Gamma) \sim N(\psi_{13}, \psi_{23}^2)$ and $\zeta_3 (\Gamma) \sim N(\psi_{21}, \psi_{22}^2)$.

$\int_{R \backslash \{0\}} \zeta_i (\Gamma) P_j (d\Gamma, dt) - \mu_{\zeta_i} \lambda_j dt$ for $i = 1, 2, 3$ and $j = 1, 2$ are the compensated Poisson random measures. More specifically, $\int_{R \backslash \{0\}} \zeta_i (\Gamma) P_j (d\Gamma, dt)$ counts the number of jumps with random size $\zeta_i (\Gamma)$ in the set $R \backslash \{0\}$ during the small time interval dt. $P_j (d\Gamma, dt) = 1$ just whenever the jump event of size $\zeta_i (\Gamma)$ happens, $P_j (d\Gamma, dt) = 0$ in all the other cases.

The intuition behind these two equations is very simple. The stock price S_t is allowed to vary not only over time, but also in response to two kinds of shocks:

i) *diffusive shocks* such as W_{1t}, W_{2t} which affect the stock price gradually and by small amounts. Any diffusive shock affecting the volatility process impacts the stock price process through the "weak leverage effect" β_{12}.

ii) *Poisson driven shocks* represented by the random measures P_1 and P_2 which account for abrupt and huge changes in the stock price. P_1 represents the number of jumps, with stochastic size ζ_1, experienced by the stock return over the time interval $(0, t]$. P_2 plays the same role as P_1, but it regulates the time varying impact exerted on the stock price by any shock to the corresponding volatility. In this case, the jump size is ζ_2. We are thus in the position to replicate the abnormal market movements taking place when volatility is affected

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$\Delta S_{nt} = S_{nt} - S_{nt^-}$

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by huge shocks which are transmitted to the stock price through the "strong leverage effect" ψ_{33}.

4   Risk neutral pricing

Following the Black and Scholes (1973) and Merton (1973) approach in order to price derivative securities, the only assumption one needs to make about agents' preferences is non-satiation (agents prefer more to less). Therefore, the price of a derivative security must be the same regardless of the agents' risk tolerance. This means that, in order to rule out any arbitrage opportunity, a risk averse economy must price an option exactly in the same way as a risk neutral economy. In particular, in a risk neutral setting, the expected rate of return of any asset must be equal to the riskless interest rate r^* (Cox and Ross (1976)):

$$
\begin{aligned}
E_t^Q \left[\frac{dS_t}{S_t} + \delta dt \right] &= r^* dt \\
E_t^Q \left[\frac{dS_t}{S_t} \right] &= (r^* - \delta)\, dt \\
&= r dt
\end{aligned}
$$

where $E_t^Q\left[.\right]$ is the expectation at time t taken with respect to the probability measure Q adjusted to be consistent with risk neutrality, $r \equiv r^* - \delta$ and δ is the constant dividend rate. In more detail, the main portfolio is transformed as follows:

$$
\begin{aligned}
d \ln S_t = {} & \left[r - \frac{1}{2} V_t - \widetilde{\overline{\gamma}}_1 \lambda_1 - \widetilde{\overline{\gamma}}_2 \lambda_2 \right] dt \\
& + \sqrt{V_t} \left[\beta_{12} d\widetilde{W}_{2t} + \sqrt{1 - \beta_{12}^2} d\widetilde{W}_{1t} \right] \\
& + \sqrt{1 - \psi_{33}^2} \int_{R\backslash\{0\}} \widetilde{\zeta}_1\left(\Gamma\right) \widetilde{P}_1\left(d\Gamma, dt\right) + \psi_{33} \int_{R\backslash\{0\}} \widetilde{\zeta}_2\left(\Gamma\right) \widetilde{P}_2\left(d\Gamma, dt\right)
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
d \ln V_t = {} & \left(\mu^U + \alpha_{22} \ln V_t - \beta_{20} \vartheta_2 + (\lambda_2 - \phi_3)\overline{\gamma}_3 - \widetilde{\overline{\gamma}}_3 \lambda_2 \right) dt \\
& + \beta_{20} d\widetilde{W}_{2t} + \int_{R\backslash\{0\}} \widetilde{\zeta}_3 P_2\left(d\Gamma, dt\right)
\end{aligned}
$$

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$$d\widetilde{W}_{1t} = dW_{1t} + \vartheta_1(V_t)dt$$

$$dW_{2t} = dW_{2t} + \vartheta_2(V_t)dt$$

$$\int_{R\setminus\{0\}} \left(\widetilde{\gamma}_1 \widetilde{P}_1 \left(d\Gamma, dt \right) - \widetilde{\overline{\gamma}}_1 \widetilde{\lambda}_1 dt \right) \tag{4}$$

$$= \int_{R\setminus\{0\}} \left((\gamma_1 P_1 \left(d\Gamma, dt \right) - \overline{\gamma}_1 \lambda_1 dt) + \overline{\gamma}_1 \phi_1(\Gamma) dt \right)$$

$$\int_{R\setminus\{0\}} \left(\widetilde{\gamma}_2 \widetilde{P}_2 \left(d\Gamma, dt \right) - \widetilde{\overline{\gamma}}_2 \widetilde{\lambda}_2 dt \right) \tag{5}$$

$$= \int_{R\setminus\{0\}} \left((\gamma_2 P_2 \left(d\Gamma, dt \right) - \overline{\gamma}_2 \lambda_2 dt) + \overline{\gamma}_2 \phi_2(\Gamma) dt \right)$$

$$\int_{R\setminus\{0\}} \left(\widetilde{\gamma}_3 \widetilde{P}_2 \left(d\Gamma, dt \right) - \widetilde{\overline{\gamma}}_3 \widetilde{\lambda}_2 dt \right) \tag{6}$$

$$= \int_{R\setminus\{0\}} \left((\gamma_3 P_2 \left(d\Gamma, dt \right) - \overline{\gamma}_3 \lambda_2 dt) + \overline{\gamma}_3 \phi_3(\Gamma) dt \right)$$

where $\widetilde{\zeta}_1 \sim N(\widetilde{\psi}_{11}, \widetilde{\psi}_{12}^2)$, $\widetilde{\zeta}_2 \sim N(\widetilde{\psi}_{13}, \widetilde{\psi}_{23}^2)$ and $\widetilde{\zeta}_3 \sim N(\widetilde{\psi}_{21}, \widetilde{\psi}_{22}^2)$, $\widetilde{P}_1 \sim Poisson(\widetilde{\lambda}_1)$ and $\widetilde{P}_2 \sim Poisson(\widetilde{\lambda}_2)$ and where $\vartheta_1(V_t)$ and $\vartheta_2(V_t)$ are the risk premia that compensate the investor for bearing the diffusive risks W_{1t} and W_{2t}. $\phi_1(\Gamma_t)$ $\phi_2(\Gamma_t)$ and $\phi_3(\Gamma_t)$ are the jump risk premia that are meant to compensate the investor for facing the risk of abrupt changes of the stock price and are a function of some general stochastic process Γ. We elaborate on these terms below. The original sources of randomness (Brownian and Poisson driven shocks) are now transformed in order to embed proper risk premia as proper adjustment for risk neutrality.

Q is the probability measure under which $\widetilde{W}_{1t}$ and $\widetilde{W}_{2t}$ are Brownian motions, while $\widetilde{P}_{1t}$ and $\widetilde{P}_{2t}$ are the Poisson processes respectively. In a more formal manner, Harrison and Kreps (1979) show that, under Q, the discounted stock price is a martingale:

$$E_t^Q \left[\frac{S_T}{B_T} \right] = \frac{S_t}{B_t}.$$

The relation between the initial probability measure P and the risk adjusted counterpart (Equivalent Martingale Measure) is regulated by the Radon-Nikodyn derivative

$$\eta_t = E^P \left[\frac{dQ}{dP} \middle| F_t \right]$$

where

$$
\begin{aligned}
\eta_t \ = \ & 1 - \int_0^t \eta_{u^-} \left(\vartheta_1(V_u) dW_{1u} + \vartheta_2(V_u) dW_{2u} \right) \\
& - \int_0^t \int_{R \backslash \{0\}} \left(\Phi_1 \left(\Gamma \right) P_1 \left(d\Gamma, du \right) - \phi_1 \left(\Gamma \right) du \right) \\
& - \int_0^t \int_{R \backslash \{0\}} \left(\Phi_2 \left(\Gamma \right) P_2 \left(d\Gamma, du \right) - \phi_2 \left(\Gamma \right) du \right) \\
& - \int_0^t \int_{R \backslash \{0\}} \left(\Phi_3 \left(\Gamma \right) P_2 \left(d\Gamma, du \right) - \phi_3 \left(\Gamma \right) du \right)
\end{aligned}
\tag{7}
$$

This change of probability measure is accomplished by the Girsanov theorem. As pointed out by Harrison and Kreps (1979), this change in the probability measure consists of a redistribution of probability mass such that the expected rate of return of every asset becomes equal to the riskless rate of interest r_t and the set of events which initially had positive probability remains unchanged[14]. The main point of this probability transformation is to ensure that all arbitrage opportunities are ruled out. This goal is reached by assuming that η_t embeds as many risk premia as the number of sources of risk present in this economy.[15] $\phi_1(\Gamma)$, $\phi_2(\Gamma)$ and $\phi_3(\Gamma)$ simultaneously compensate the investor for the jump size uncertainty and the jump timing uncertainty. In order to ensure that no arbitrage opportunities are possible, the whole set of risk premia must respect the following condition:

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[14]Following Harrison and Kreps (1979), the equivalent martingale measure is a probability measure Q on (Ω, F) such that

i) P and Q are equivalent. This means that the null sets of P and Q coincide or, in other terms, that $P(B) = 0$ if and only if $Q(B) = 0$ for any $B \in F$.

ii) The Radon-Nikodym derivative $\eta = \frac{dQ}{dP}$ is such that $E\left(\eta^2\right) < \infty$.

iii) The discounted stock price $\widetilde{S}_t \equiv \frac{S_t}{B_t}$ is a martingale over the fields $\{F_t\}$ with respect to Q or, in other terms, $E^Q \left[\widetilde{S}_s \middle| F_t \right] = \widetilde{S}_t$ for any $s \geq t$.

[15]The explicit expression for η_t is derived in Appendix F.

$$
\begin{aligned}
r &= r^* - \delta = \tag{8} \\
&= \mu + \frac{1}{2}V_t + \overline{\gamma}_1 \left(\lambda_1 - \phi_1\right) + \overline{\gamma}_2 \left(\lambda_2 - \phi_2\right) \\
&\quad - \sqrt{V_t}\left(\beta_{12}\vartheta_2 + \sqrt{1 - \beta_{12}^2}\vartheta_1\right)
\end{aligned}
$$

where $V_t = f(\vartheta_2(V_t), \phi_3(\Gamma))$ and δ is the dividend rate.

Equation (1.8) is simply the translation in mathematical terms of the principle that the expected rate of return of the stock under consideration is equal to the riskless interest rate r^* minus the constant dividend rate δ.

Following Cox and Ross (1976), after neutralizing any source of risk by compensating the investor with the appropriate risk premia, one can price any derivative security in the way a risk neutral economy would do. Therefore, the option price is only the expected value of the total payoffs discounted at the riskless rate r^* minus the constant dividend rate δ:

$$
C_t = e^{-r(T-t)} E_t^Q \left[Max\left\{S_T\left(\xi_1, \xi_2\right) - K, 0\right\}\right]
$$

where

$$
\begin{aligned}
S_T = S_0 \exp\Bigg\{ &\int_0^T \left[r - \frac{1}{2}V_u - \widetilde{\overline{\gamma}}_1\lambda_1 - \widetilde{\overline{\gamma}}_2\lambda_2\right] du \\
&+ \int_0^T \sqrt{V_u}\left[\beta_{12}d\widetilde{W}_{2u} + \sqrt{1 - \beta_{12}^2}d\widetilde{W}_{1u}\right] \\
&+ \sqrt{1 - \psi_{33}^2}\int_0^T\int_{R\backslash\{0\}}\widetilde{\zeta}_1\left(\Gamma\right)\widetilde{P}_1\left(d\Gamma, du\right) + \psi_{33}\int_0^T\int_{R\backslash\{0\}}\widetilde{\zeta}_2\left(\Gamma\right)\widetilde{P}_2\left(d\Gamma, du\right)\Bigg\}
\end{aligned}
$$

$$
\xi_1 = \begin{array}{l}
\left(\alpha_{10}, \beta_{12}, \psi_{33}, \psi_{11}, \psi_{12}, \psi_{13}, \psi_{23}, \psi_{13},\right. \\
\left.\alpha_{20}, \alpha_{22}, \beta_{20}, \psi_{21}, \psi_{22}, \lambda_1, \lambda_2\right)
\end{array}
$$

is the vector of the parameters of the stock price and volatility processes and

$$
\xi_2 = \left(\vartheta_2(V_t), \widetilde{\psi}_{11}, \widetilde{\psi}_{12}, \widetilde{\psi}_{13}, \widetilde{\psi}_{23}, \widetilde{\psi}_{13}, \widetilde{\psi}_{21}, \widetilde{\psi}_{22}, \widetilde{\lambda}_1, \widetilde{\lambda}_2\right)
$$

is the vector of the risk adjusted parameters.

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The entire analysis is carried out in an incomplete market setting where the sources of randomness outnumber the traded assets. The lack of an expression describing all the risk premia in this economy and the non-uniqueness of the equivalent martingale measure are overcome by means of the empirical estimation of those premia.

5   Estimation Method

In principle, this model could be estimated using maximum likelihood estimation (MLE) and Semi Non Parametric (SNP) methods. Since volatility is a latent variable, MLE would be too demanding and intensive from the computational point of view. Indeed, volatility ought to be integrated out of the likelihood function and the dimension of this integral would be as large as the number of observations in the time series. For similar reasons, SNP procedures are also not easily implemented. Monte Carlo simulation methods allow us to evaluate the GMM criterion when, as in this case, a closed form specification of the moment conditions is not available. These methods show a greater flexibility as they can be easily used to estimate a wide range of different models and, at the same time, they provide useful diagnostics about the model specification.

Other possible approaches which involve the use of semi-nonparametric procedures are very problematic to use when volatility is unobservable.[16]

A possible alternative to this approach would involve the use of both series of data, the stock return series and the cross section of option prices at the same time. However, while this technique appears more intuitive for its ability to supply directly the estimates of the risk premia, it is vulnerable to the critique that the Efficient Method of Moments (EMM) estimates based on a multidimensional auxiliary model (see Duffee and Stanton 2001) suffer from poor finite sample properties. Moreover, the use of return and option data at the same time is computationally so intensive that its implementation typically involves the use of very short data sets. An example can be found in Chernov and Ghysels (2000) who estimate the Heston model by EMM using stock and option data at the same time. The multidimensional approach is also employed by Pan (2001) who estimates a square root model with jumps by GMM, by Eraker who evaluates the Duffie et al. model by the Markov Chain Monte Carlo

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[16]See, for example, Hansen (1995), Ait-Sahalia (1996), Jiang and Knight (1997) and Johannes (1999).

(MCMC) technique and Jones (2000) who chooses a CEV (Constant Elasticity of Variance) model. Alternatively, the unidimensional approach (where only the stock return series is used) is chosen by Andersen et al. (2002) and Chernov, Gallant, Ghysels and Tauchen (2003) who use the EMM to estimate the parameters of many possible models for the stock return process. Eraker et al. (2003), instead, estimate the Duffie et al. model by the MCMC technique.

A two stage method will be adopted in this paper. In the first stage, the parameters of the structural model for stock pricing will be estimated using the Efficient Method of Moments. The choice of this method is due to the complex form of the structural model which shall be replaced by an approximated counterpart (*auxiliary model*) which is easier to handle. This auxiliary model has mainly a descriptive function and does not have an interpretation in terms of the structural model. It usually contains a very large number of parameters for purposes of calibration. As the number of these parameters increases, the auxiliary model gives a good approximation for the distribution of the data with the potential to reach asymptotic efficiency. In the second stage, holding the estimates of the structural parameters fixed, this paper estimates the risk premia embedded in the call option prices so as to minimize the implied volatility mean squared residuals.

5.1 Stage one: estimating the stock return and volatility parameters by EMM

In the initial stage, the parameters of the stock price and volatility processes will be estimated via the Efficient Method of Moments. Specifically, the vector of parameters to be estimated at this point is (see equations [1] and [2]):

$$\xi_1 = \begin{array}{l} (\alpha_{10}, \beta_{12}, \psi_{33}, \psi_{11}, \psi_{12}, \psi_{13}, \psi_{23}, \psi_{13}, \\ \alpha_{20}, \alpha_{22}, \beta_{20}, \psi_{21}, \psi_{22}, \lambda_1, \lambda_2) \end{array}$$

Since the stochastic volatility is unknown, this latent variable must be integrated out in the computation of the loglikelihood. In this case, the dimension of the integral is the same as the sample size. Therefore, the direct evaluation of the likelihood function is either computationally intensive or infeasible. This is the main reason why several authors (see Gallant and Tauchen (1996), Pan (2002), Chernov, Gallant, Ghysels and Tauchen (1999), Ander-

sen et al., (2000)) have employed EMM, thus avoiding direct estimation of the likelihood function.

The key aspect of this method is the efficiency of the standard GMM (Generalized Method of Moments). This efficiency is associated with a careful choice of the moment conditions based upon a detailed analysis of the main features of the observed data. As a result, the EMM is a methodology for the estimation and analysis of non linear systems of partially observable variables. This method is based on the simulation of the state vector.

The initial step consists in choosing an appropriate transition density called *auxiliary model*, which is a close approximation of the data generating process. The parameters of this density are estimated by QMLE (Quasi Maximum Likelihood Estimation). The score of this model represents the score generator for EMM.

5.1.1 Choice of the auxiliary model

The EMM is based on *indirect inference*. The main idea is to replace the initial model with an approximated one that is more tractable. This model, denoted the *auxiliary model*, is a descriptive model with a large number of parameters. In particular, its parameters do not have any structural interpretation, but are only used for calibration purposes. As their number increases, asymptotic efficiency is reached.

Following Gallant and Tauchen (2001), the auxiliary model is derived by using the so-called SNP (Semi-Non-Parametric) approach. This method is considered to lie halfway between the parametric and non parametric inference procedures, since classical parametric estimation is applied to models with truncated series expansions. The main purpose of this section is to find an auxiliary model that closely approximates the density of the data. The corresponding density function is then approximated using a Hermite expansion, whose leading term is a standard Gaussian density. The higher order terms of this expansion will accommodate any deviation from Gaussianity as, for example, high kurtosis and negative skewness.

Let

$$y_t = 100[\ln(S_{1t}) - \ln(S_{1t-1})]$$

be the daily stock return for our estimation problem and $\{y_1, y_2, ..., y_n\}$ the data set

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available and characterize as

$$x_{t-1} \equiv \{y_{t-L}, y_{t-L+1}, ..., y_{t-1}\}$$

the L lagged values of the realization of the time series $\{y_t\}_{t=-\infty}^{\infty}$.

Denote by H the finite dimensional Euclidean space where the likelihood functional is characterized. The likelihood can therefore be written as

$$\left[\prod_{t=1}^{n} p\left(y_t | x_{t-1}, \xi_1\right)\right] \int p\left(y, x_0, \xi_{10}\right) dy$$

where

$$p\left(y_t | x_{t-1}, \xi_1\right) = \frac{p\left(y_t, x_{t-1}, \xi_1\right)}{\int p\left(y, x_{t-1}, \xi_1\right) dy}$$

and ξ_1 is the parameter vector of this model.

Following Gallant and Tauchen (2001), by expanding $[p\left(y, x_{t-1}\right)]^{\frac{1}{2}}$ in an Hermite series and deriving the transition density of the truncated expansion, it is possible to calculate the transition density $f_K\left(y_t | x_{t-1}\right)$ where

$$y_t = Rz_t + \mu_{x_{t-1}}$$

and R is an upper triangular matrix

$$
\begin{aligned}
vech(R_{x_{t-1}}) &= \rho_0 + \sum_{i=1}^{L_r} P_i \left| y_{t-1-L_r+i} - \mu_{x_{t-2-L_r+i}} \right| + \\
&\quad + \sum_{i=1}^{L_g} diag(G_i) vech(R_{x_{t-2-L_g+i}})
\end{aligned}
$$

and

$$\mu_x = b_0 + Bx_{t-1}$$

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is the location function where b_0 is a vector and B is a matrix. The resulting standardized residual will be

$$z_t = R^{-1} \left(y_t - b_0 - B x_{t-1} \right)$$

with corresponding density function

$$h_K \left(z_t | x_{t-1} \right) = \frac{\left[P \left(z_t, x_{t-1} \right) \right]^2}{\int \left[P \left(u, x_{t-1} \right) \right]^2 \phi \left(u \right) du}$$

where $P \left(z_t, x_{t-1} \right)$ is the Hermite polynomial with rectangular expansion

$$P \left(z_t, x_{t-1} \right) = \sum_{j=0}^{K_z} \sum_{i=0}^{K_x} a_{ij} \left(x_{t-1} \right)^i z_t^j$$

where $a_0 = 1$ in order to have identification. $P \left(z_t, x_{t-1} \right)$ is a polynomial in z of degree K_z whose coefficients are polynomials of degree K_x. K_z is the order of the polynomial expansion that allows for deviations of the tails of the distribution from the Normal density. In the extreme case that $K_z = 0$, this density is simply the Normal density. $\phi \left(. \right)$ is the standard normal density and the normalization term

$$\int \left[P \left(u, x_{t-1} \right) \right]^2 \phi \left(u \right) du$$

is such that the SNP density integrates to one. Using this SNP model, it is possible to derive the conditional density of y_t as

$$f_K \left(y_t | x_{t-1} \right) = \frac{h_K \left[R_x^{-1} \left(y_t - \mu_x \right) | x_{t-1} \right]}{\det \left(R_x \right)}$$

The Hermite expansion consists of a polynomial in z (which represents the innovation) multiplied by the standard Gaussian density. The flexibility of this model is the main reason why this might be considered as the best choice in order to approximate the data generating process. In fact, if $K_z = 0$, then this density function is just a standard Gaussian density and any deviation from that, if any, can be taken care of just by allowing for $K_z > 0$.

In the case that the coefficients a_i are considered not as functions of x_{t-1} but as constants, the density function of the innovation will be

$$h_K(z_t) = \frac{\left[\sum_{i=0}^{K_z} a_i z_t^i\right]^2 \phi(z_t)}{\int \left[\sum_{i=0}^{K_z} a_i u^i\right]^2 \phi(u)\, du}$$

this density will generate a Gaussian VAR if $K_z = 0$, while any departure from Gaussianity will be accommodated just by setting $K_z > 0$.

Moreover, in order to model an important aspect of the data such as the presence of conditional heteroschedasticity, it will be useful to assume that these coefficients a_i are actually functions of x_{t-1}:

$$a_i(x_{t-1}) = \sum_{j=0}^{K_x} a_{ij} x_{t-1}^j$$

This further generalization introduces a nonlinear conditional shape variation with x_{t-1}.

The conditional density of the innovations in this case will be

$$h_K(z_t) = \frac{\left[\sum_{i=0}^{K_z}\left(\sum_{j=0}^{K_x} a_{ij} x_{t-1}^j\right) z_t^i\right]^2 \phi(z_t)}{\int \left[\sum_{i=0}^{K_z}\left(\sum_{j=0}^{K_x} a_{ij} x_{t-1}^j\right) u^i\right]^2 \phi(u)\, du}$$

The only restriction, in this case, is that the dimension of a (the parameter vector of the auxiliary model) is greater or equal to the dimension of the parameter vector of the structural model ξ_1.

5.1.2 Estimation of the parameters of the auxiliary model

The parameter vector a will be estimated by the QMLE method. Hence, $\widehat{a}_{QMLE}$ will be such that

$$\frac{1}{n}\sum_{t=0}^{n} \frac{\partial}{\partial a} \ln f_K(y_t \mid x_{t-1}, \widehat{a}_n) = 0$$

At this point, it is useful to characterize the score function of the auxiliary model whose role will be crucial for the next steps:

$$s_f(Y_t, \widehat{a}_n) \equiv \frac{\partial}{\partial a_n} \ln f_K(y_t \mid x_{t-1}, \widehat{a}_n)$$

Following Gallant and Long (1997), a consistent estimator of the asymptotic covariance matrix of the sample score vector may be obtained by the following formula:

$$\widehat{V}_n = \frac{1}{n} \sum_{t=1}^{n} s_f\left(Y_t, \widehat{a}_n\right) s_f\left(Y_t, \widehat{a}_n\right)'.$$

5.1.3 Simulated Method of Moments

Fixing the parameter vector ξ_1, it is possible to simulate a series of data by using the structural model

$$\widehat{Y}_T\left(\xi_1\right) = \left\{\widehat{y}_1\left(\xi_1\right), \widehat{y}_2\left(\xi_1\right), \ldots\ldots\ldots, \widehat{y}_T\left(\xi_1\right)\right\}$$

Evaluating the score functions at this simulated series of data and keeping the parameters of the auxiliary model fixed at $\widehat{a}_{QMLE}$, the moment conditions for this problem will be

$$m_T\left(\xi_1, \widehat{a}_n\right) \equiv \frac{1}{T} \sum_{t=1}^{T} s_f\left(\widehat{Y}_t\left(\xi_1\right), \widehat{a}_n\right)$$

where

$$s_f\left(\widehat{Y}_t\left(\xi_1\right), \widehat{a}_n\right) \equiv \frac{\partial}{\partial a_n} \ln f_K\left(\widehat{y}_t \middle| \widehat{x}_{t-1}\left(\xi_1\right), \widehat{a}_n\right)$$

The EMM estimator $\widehat{\xi}_{1n}$ will be such that

$$\widehat{\xi}_{1n} = \min\left\{m_T\left(\xi_1, \widehat{a}_n\right) \widehat{V}_n^{-1} m_T\left(\xi_1, \widehat{a}_n\right)'\right\}$$

It has been shown (see Gallant and Tauchen (1996), Gallant and Long (1997), Tauchen (1997)) that if the auxiliary model closely approximates the true data generating process, then

$i)$ the QMLE becomes a sufficient statistic;

$ii)$ the efficiency of the EMM is close to that of the MLE.

5.1.4 Diagnostics

The main tool for evaluating the capability of the model to mimic the salient features of the data is represented by the *test for overidentifying restrictions*

$$m_N\left(\widehat{\xi}_1, \widehat{a}\right)' \widehat{I}_N^{-1} m_N\left(\widehat{\xi}_1, \widehat{a}\right) \longrightarrow \chi^2\left(l_a - l_{\xi_1}\right)$$

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under the null hypothesis that the structural model is the "true data generating process" where l_a is the length of the vector of parameters of the auxiliary model and l_{ξ_1} is the length of the vector of parameters of the structural model. In case of rejection of the model specification, the individual elements of the score vector may provide useful information regarding the dimensions in which the structural model fails to replicate the main features of the data. Another powerful tool for testing the model is represented by the t-statistics of the individual elements of the score vector $m_N\left(\widehat{\xi}_1, \widehat{a}\right)$. High values of these statistics for a given parameter mean that the structural model is unable to account for that specific parameter of the auxiliary model

$$\widehat{t}_N = \left\{ diag\left[\widehat{I}_N\right] \right\}^{-\frac{1}{2}} \sqrt{N} m_N\left(\widehat{\xi}_1, \widehat{a}\right)$$

In general, a value of this statistics higher than 2 indicates the failure to fit the corresponding score.

5.2 Stage two: estimating the risk premia for jumps and diffusive shocks contained in the volatility process

The main purpose of this section is to estimate the risk neutral parameters and the risk premia embedded in the call option prices. For this purpose, I minimize *MSE of the B&S implied volatility.* More specifically, it is possible to express the call option price as a function of the future stock price and the strike price

$$C_t = e^{-r(T-t)} E_t^Q\left[Max\left\{ S_T\left(\widehat{\xi}_{1_{EMM}}, \xi_2\right) - K, 0 \right\}\right]$$

where

$$\widehat{\xi}_{1_{EMM}} = \begin{array}{c} (\alpha_{10}, \beta_{12}, \psi_{33}, \psi_{11}, \psi_{12}, \psi_{13}, \psi_{23}, \psi_{13}, \\ \alpha_{20}, \alpha_{22}, \beta_{20}, \psi_{21}, \psi_{22}, \lambda_1, \lambda_2)_{EMM} \end{array}$$

is the vector of estimates from the first stage, while ξ_2 contains the risk premium for diffusive shocks in the volatility process and the risk adjusted parameters

$$\xi_2 = \left(\vartheta_2(V_t), \widetilde{\psi}_{11}, \widetilde{\psi}_{12}, \widetilde{\psi}_{13}, \widetilde{\psi}_{23}, \widetilde{\psi}_{13}, \widetilde{\psi}_{21}, \widetilde{\psi}_{22}, \widetilde{\lambda}_1, \widetilde{\lambda}_2 \right)$$

The jump risk premia can be derived from equations (5), (6) and (7) in Section 4.

ξ_2 is estimated by minimizing the *MSE of the B&S implied volatility* as in Broadie, Chernov and Johannes (2004)

$$IVMSE(\xi_2) = \frac{1}{n} \sum_{i=1}^{n} (\sigma_i - \sigma_i(\xi_2))^2$$

where $\sigma_i = BS^{-1}(C_i, T_i, K_i, S, r)$ and $\sigma_i(\xi_2) = BS^{-1}(C_i(\xi_2), T_i, K_i, S, r)$ with BS^{-1} indicating the inverse of the B&S call option formula. More specifically, σ_i is the B&S implied volatility series observed in the market, while $\sigma_i(\xi_2)$ is the implied volatility series derived from the model simulation. The choice of this objective function appears to be the most natural and intuitive given that the main purpose of the present paper is to find a model that can replicate the observed spikes in Implied Volatility. Minimizing this specific loss function is also particularly interesting considering the widespread convention of quoting option prices in terms of volatility. Moreover, this specific objective function allows me to avoid the heteroschedasticity problem that is related to other possible choices.[17]

6   Empirical Results

In this section, I initially describe the data set I am using and explain the way in which I estimated the auxiliary model; in the second part, I proceed to present my estimation results and the corresponding statistical tests.

6.1   Stage one: estimation of the stock return parameters by EMM

The data consist of 4299 daily observations from January 3, 1980 to December 31, 1996 on the percentage return

$$y_t = 100[\ln(S_t) - \ln(S_{t-1})]$$

where S_{1t} is the S&P500 index.[18] Since this series presents a mild autocorrelation while the series of squared returns is quite persistent (see Fig.1), I used the augmented Dickey-Fuller

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[17]See Christoffersen and Jacobs (2004) for a comment on the possible objective functions.

[18]The same data set is used by Andersen, Benzoni and Lund (2003) while they also propose estimates for a longer time frame (January 2, 1953 to December 31, 1996). Eraker, Johannes and Polson (2003) estimate affine models using the S&P 500 index data from January 2, 1980 to December 31, 1996.

statistics to test for the presence of unit roots and I strongly rejected the null hypothesis (see Table 1). This autocorrelation, which might be caused by nonsynchronous trading, has not been prefiltered as in Andersen et al. (2002) because it might be a key factor in the second stage of the empirical application where I estimate the risk premia embedded in the option prices.

As already stated, the first step is to choose an auxiliary model whose main parameters are the following:

L_u	number of lags in the location function μ_x
L_g	number of lags in $vech(R_{x_{t-1}})$
L_r	number of lags in $vech(R_{x_{t-1}})$
L_p	number of lags in the x_{t-1} part of the polynomial $P(z_t, x_{t-1})$
K_z	degree of the polynomial $P(z_t, x_{t-1})$
K_x	degree of the polynomial $P(z_t, x_{t-1})$

Following Chernov, Gallant, Ghysels and Tauchen (2003), I choose the values of these parameters that minimize the BIC (Schwarz or Bayes information criterion). The final non-linear-non-parametric auxiliary model I select is characterized by

$$L_u = 1 \quad L_g = 1 \quad L_r = 1 \quad L_p = 1 \quad K_z = 8 \quad K_x = 1$$

This is a GARCH(1,1) process with an eighth-degree Hermite expansion as a non-parametric error density function.[19]

The EMM estimation is based on the simulation of the return sequence and variance process. Using the standard Euler discretization scheme, my simulation involves a sampling frequency of one step per day as well as daily scaling for the parameters ($dt = 1$). More specifically, the EMM estimation of my model is based on two simulations of $75,000$ sample paths for the stock returns and for the stochastic factor which drives the volatility process.[20] The initial $5,000$ observations are eliminated in order to avoid the impact of the initial values.

Table 2 shows the parameter estimates with the corresponding t-ratios (based on Wald type standard errors) in parenthesis for the models belonging to the log-variance class while

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[19]ABL (2002) use an EGARCH(1,1), $K_z(8) - K_x(0)$ auxiliary model after prefiltering the data.

[20]This is the minimum number of simulations necessary in order to have a stable objective function in presence of jumps and in order to obtain robust results (see Gallant and Tauchen 2003)

Table 3 reports the corresponding results for the affine models. For each model, the value of the χ^2 test for overidentifying restrictions is provided. Figure 3 and Figure 4 report the quasi t-ratios of individual SNP scores.

6.1.1 Black and Scholes model

In order to have an initial benchmark, I estimate the Black and Scholes model (see Table 2).[21] Considering the value of the χ^2 test, this model is clearly rejected. As one clearly sees, the rejection reflects this model's inability to satisfy any of the SNP moment conditions as indicated by the values of the quasi t ratios on the individual SNP scores (see Figure 3). This model is thus unable to fit the tail behavior of the S&P 500 stock returns, as it violates the moment conditions associated with the Hermite polynomial coefficients. It also scores poorly in replicating the GARCH volatility persistence and the AR nature of the data. My findings are in line with Gallant, Hsieh and Tauchen (1997).

6.1.2 Stochastic volatility (log-variance) model (SV1)

This specification, focusing on the logarithm of the variance process, represents a variation of Scott (1987) who models the logarithm of volatility (square root of the variance) instead.

Analyzing the empirical results (see Table 2), the value of the χ^2 statistics drops dramatically, as the negative and highly significative "leverage effect" coefficient β_{12} makes the model capable of replicating the negative skewness of the data.[22] The model is nevertheless rejected, as explained easily by checking the t ratios of the SNP scores (see Figure 3). The model fails to capture the tail behavior (excess kurtosis) of the data controlled by the Hermite polynomial moment conditions. Moreover, it cannot accommodate the linear aspect of the data, as the t statistics on the moment conditions associated with the AR parameters are quite large.

6.1.3 Stochastic volatility (square root) model (SV2)

This model was first proposed by Cox, Ingersoll and Ross (1985) and Heston (1993).

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[21]The estimate of α_{10} is in line with ABL (2002) while my estimate of β_{10} (0.92767) is higher than their 0.7176 and more in line with the sample volatility (0.9623853)

[22]All the parameter estimates are very close to the ones proposed by ABL(2003).

The χ^2 test for overidentifying restrictions (see Table 3) indicates that the model is rejected by the data because of its inability to capture their tail tickness and their AR behavior as indicated by the t ratios on the SNP moment conditions (see Figure 4).[23]

6.1.4 Stochastic volatility (log-variance) model with jumps in the return process (SV1J)

The model which incorporates both jumps and stochastic volatility dramatically improves upon the previous specification. The χ^2 test for overidentifying restrictions (see Table 2) indicates that the model is not rejected by the data at a significance level of 5%; however, this result does not hold as the test becomes more demanding with a significance level of 10%.[24] The reason can be easily found in the t ratios on the specific moment conditions (see Figure 3). This model fails in fact to capture the linear aspect of the data as the high t statistics on one of the moment conditions associated with the AR parameters testifies.

6.1.5 Stochastic volatility (square root) model with jumps in the return process (SV2J)

This model has been introduced by Bates (1996) and Scott (1997). This model is rejected by the data as indicated by the χ^2 test for overidentifying restrictions (see Table 3). More specifically, it fails to mimic all the salient features of the data as indicated by the t ratios on the SNP moment conditions (see Figure 5).[25]

6.1.6 Stochastic volatility (log-variance) model with jumps in the return and in the variance processes (SV1CIJ)

In this specification, I allow for contemporaneous and independent jumps in the stock price and in the volatility processes, the correlation between contemporaneous jumps being regulated by the leverage effect coefficient ψ_{33}. The χ^2 test for overidentifying restrictions

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[23]All the parameter estimates are very close to their counterparts in EJP(2003), the slight differences are definetly due to their choice of a different data set (January 2, 1980 to December 31, 1999).

[24]The estimated leverage effect (β_{12}) is much lower than the estimate proposed by ABL (2003). This difference might be due to their transformation of the data. By the same token, my estimates of the jump size parameters (ψ_{11} and ψ_{12}) are much higher ($\psi_{11} = -1.75806$ vs ABL (2003) -0.000235445 and $\psi_{12} = 1.63595$ vs ABL (2003) 0.0217)

[25]The only parameter estimate which substantially differs from the EJP(2003) counterpart is the standard deviation of the jump size (0.698 vs. theirs 4.072).

indicates that this model is not rejected by the data at the 10% significance level. Checking the magnitudes of the quasi t ratios on the moment conditions reveals that the success of the model is due to its capability to simultaneously accommodate the linear aspect and the tail behavior of the data. Moreover, it credibly mimics the moment conditions relative to the GARCH volatility persistence.

The main findings resulting from the EMM estimation of my model are the following. It is worth noting that the only parameters which are not significantly different from zero are ψ_{12} (the standard deviation of the size of the independent jumps in the stock process) and ψ_{13} (the mean of the size of the contemporaneous jumps in the stock process). Moreover, $\widehat{\psi}_{21}$, the estimate of the mean of the size of the jumps in volatility, is negative and significantly different from zero. This result clearly contradicts the assumption of positive jumps in volatility made by Duffie et al.(2000).

6.1.7 Stochastic volatility (square root) model with jumps in the return and in the volatility processes (SV2IJ)

Although the χ^2 test for overidentifying restrictions shows that the SVJ model is not rejected by the data, the t ratios on the specific moment conditions reveal very clearly that this model fails to accommodate the GARCH volatility persistence behavior of the S&P500 returns. This problem is completely solved by my new model. In fact, not only the p-value of the χ^2 test is higher than in any other model, but the t ratios on the SNP moment conditions also show that this new model can mimic all the salient aspects of the data.

6.2 Option pricing implications

Although this first stage of estimation already shows remarkably powerful results in favor of the model proposed in this paper, the ongoing debate in the recent literature on this topic calls for further investigation of the jumps in the volatility process. To be more precise, Andersen et al. (2002) show that the SVJ model is not rejected by the S&P500 stock return data, thus, implicitly, they do not find evidence for jumps in volatility; nevertheless, they recognize that this model cannot account for violent market movements such as the October 1987 crisis. Moreover, these results can be due to their choice of prefiltering the data using a MA(1) model for the S&P500 daily returns in order to accommodate their

mild serial correlation. Chernov , Ghysels, Gallant and Tauchen (CGGT) (2003) choose instead not to prefilter the data, because, by doing so, some important features necessary for pricing options might be removed. Their empirical application does not point toward a specific choice between a model with or without jumps in volatility. While Eraker (2003) and Eraker et al. (2003) find strong evidence for jumps in volatility, Pan (2003) does not. More recently, Broadie, Chernov and Johannes (BCJ) (2004) stress that option data provide strong evidence supporting jumps in volatility. Therefore, in order to place my contribution in this debate, I investigate whether option data show any evidence of jumps in volatility and, more specifically, if the new model I propose can mimic more adequately and eventually forecast option prices. First of all, it is worth noting that the B&S market implied volatility for the ten years period (1987-1997) is characterized by spikes. As Eraker (2003) points out, this feature cannot be captured by the Duffie et al. model which allows only for positive jumps in volatility. This is not an issue for the model I propose, since no restrictions are imposed on the sign of the jumps in the volatility process. Following the two stage procedure proposed by Benzoni (2002) and adopted by BCJ (2004), I hold the parameter estimates, obtained in the EMM stage, fixed in order to estimate the jump and diffusion risk premia of the volatility process embedded in the option prices.

6.3 Stage two: estimation of the risk premia for jumps and diffusive shocks contained in the volatility process

In this stage I address a specific question: are spikes in volatility an important factor in explaining option price dynamics?

I answer this question by comparing the performance of a model which only allows for positive jumps in volatility proposed by Duffie et al. (2000) (*SV2IJ*) versus a model where volatility can suddenly increase as easily as it can violently fall (*SV1CIJ*). The last specification is meant to reflect the implied volatility dynamics observed over the entire ten year period 1987-1997 (see Figure 5).

6.3.1 Estimating the implied volatility dynamics

The experiment I conduct involves the use of option price data for the 1987 period for the in-sample estimates of the parameter vector

$$\xi_2 = \left(\vartheta_2(V_t), \widetilde{\psi}_{11}, \widetilde{\psi}_{12}, \widetilde{\psi}_{13}, \widetilde{\psi}_{23}, \widetilde{\psi}_{13}, \widetilde{\psi}_{21}, \widetilde{\psi}_{22}, \widetilde{\lambda}_1, \widetilde{\lambda}_2\right)$$

which includes the premium for diffusive risk in volatility and the risk neutral parameters.[26]

Once ξ_2 is estimated, the jump risk premia can be priced using equations (5), (6) and (7) in Section 4. The data relative to 1988 are used in order to evaluate the out-of-sample performances of the two models (SV1CIJ and SV2IJ).

Stochastic volatility (log-variance) model with jumps in the return and in the variance processes (SV1CIJ)   The model I propose performs better than the existing ones both in sample and out of sample (see Table 4, Table 6 and Figure 8). My findings differ from the existing ones in two main ways: first of all, the premium for diffusive risk in volatility $\vartheta_2(V_t)$ is significantly different from zero (see Table 4).[27] Secondly, I find evidence for stochastic jump premia (see Figure 6).[28]

Stochastic volatility (square root) model with jumps in the return and in the volatility processes (SV2IJ)   The assumption of positive jumps in volatility (as in Duffie et al. 2000) turns out to be too restrictive when pricing options. More specifically, this model is not capable of replicating the main features of the market implied volatility, as shown by both the in sample and out of sample performances (see Table 5, Table 6 and Figure 9). BCJ (2004) estimate the Duffie et al. (2001) model in the case of contemporaneous jumps in the stock price and in the volatility process; in line with their results, I find that the premium for diffusive risk in volatility is statistically insignificant even when independent jumps are allowed. However, unlike in BCJ (2004), I find evidence for stochastic jump risk premia.

7   Implied volatility curves

Figure 6 shows the implied volatility smiles for the SV1CIJ and SV2IJ models and market implied volatility data for the randomly selected day, January 7 1987. Those curves are

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[26]For more details, see section 5.2

[27]BCJ (2004), while estimating the DPS (2000) model with contemporaneous jumps in the stock process and in volatility, find that this premium is statistically insignificant.

[28]BCJ (2004) find evidence for time varying, deterministic jump risk premia.

based on the call option prices which derive from the simulation of the corresponding model. For each maturity, the market implied volatility (IV) is derived as a term of comparison. I selected a random trading day from the entire sample (January 7, 1987). The graphs clearly show that spikes in volatility play a key role in explaining the behavior of IV as a function of the moneyness (strike to spot price ratio). More specifically, when there are 9 days to maturity, the SV1CIJ model performs slightly better than the SV2IJ counterpart for in the money (ITM) call options. The behavior of the two models is almost the same for at the money (ATM) and out of the money (OTM) options. In the case of 44 days to maturity, the SV1CIJ model performs much better than the SV2IJ counterpart especially for in-the-money and at-the-money call options. The behavior of the two models overlaps for the out-of-the-money options. For higher maturities, again the SV1CIJ model does a better job in replicating the behavior of the market IV.

8   Conclusions

Market implied volatility is characterized by spikes which cannot be replicated by existing models. The present paper aims to fill the gap in the current literature by proposing a new model for option pricing which allows for spikes in implied volatility. The two key aspects of the model proposed here are the following:

i) the logarithm of the variance follows an Ornstein-Uhlenbeck process with jumps whose size is random and whose sign is unrestricted;

ii) it incorporates two scenarios where the stock price can jump both alone and together with volatility.

Unlike the existing literature, the main characteristic of this volatility function is the absence of restrictions in the paths it can follow. Its driving factor can either rise or fall rapidly, while the entire process always remains positive due to the use of an exponential function.

This new model is then tested following a two step procedure. The results show an improvement over the existing literature in pricing stock portfolios and the corresponding options. The estimate of the mean of the jumps that affect volatility is negative and significantly different from zero. This result shows that the assumption of positive jumps in

volatility made by Duffie et al. (2000) is too restrictive.

Time series and cross section S&P500 option prices are used to assess quantitatively the empirical performance of the innovative features of the proposed model. The estimates indicate that spikes in volatility introduce a significant improvement in option pricing and provide evidence for stochastic jump risk premia.[29]

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[29]BCJ (2004) find evidence for time varying, deterministic jump risk premia.

Table 1

Augmented Dickey-Fuller test for a unit root in the percentage return series

Dickey-Fuller test for unit root Number of obs = 4298

————- Interpolated Dickey-Fuller ————

Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t) -62.120	-3.430	-2.860	-2.570

* MacKinnon approximate p-value for Z(t) = 0.0000

❋ ❋❋

Augmented Dickey-Fuller test for a unit root in the squared percentage return series

Dickey-Fuller test for unit root Number of obs = 4298

————- Interpolated Dickey-Fuller ————

Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t) -58.786	-3.430	-2.860	-2.570

* MacKinnon approximate p-value for Z(t) = 0.0000

Table 2 EMM estimates for the sample period 1/ 3/1980 - 12/ 31/1996: loglinear model.

SV1 Models $\quad d\ln S_t = \alpha_{10}dt + \sqrt{V_t}\left[\beta_{12}dW_{2t} + \sqrt{1-\beta_{12}^2}dW_{1t}\right]$
$+\sqrt{1-\psi_{33}^2}\int_{R\backslash\{0\}}\zeta_1\left(\Gamma\right)P_1\left(d\Gamma,dt\right) + \psi_{33}\int_{R\backslash\{0\}}\zeta_2\left(\Gamma\right)P_2\left(d\Gamma,dt\right)$
$\zeta_2 \sim N(\psi_{13},\psi_{23}^2) \quad \zeta_1 \sim N(\psi_{11},\psi_{12}^2) \quad P_1 \sim Poisson(\lambda_1)\ \ P_2 \sim Poisson(\lambda_2)$
$V_t = \exp(U_t)\ dU_t = (\alpha_{20}+\alpha_{22}U_t)\,dt + \beta_{20}dW_{2t} + \int_{R\backslash\{0\}}\zeta_3\left(\Gamma\right)P_2\left(d\Gamma,dt\right)\ \zeta_3 \sim N(\psi_{21},\psi_{22}^2)$

parameter	BS	SV1	SV1J	SV1CIJ
α_{10}	0.06670 [37.78]	0.046913 [5.14874]	0.06359 [9.7740]	0.06083 [5.05158]
α_{20}		-0.016023 [-5.1551]	-0.01741 [-21.6589]	-0.01975 [-4.8051]
α_{22}		-0.02987 [-25.634]	-0.03190 [-16.3033]	-0.05711 [-6.8572]
β_{10}	0.92767 [32.8282]			
β_{12}		-0.44965 [-32.9623]	-0.05387 [-15.0947]	-0.27036 [-15.0535]
β_{20}		0.20664 [26.4605]	0.20862 [31.8522]	0.23563 [10.0198]
ψ_{11}			-1.75806 [-6.5088]	-3.79328 [-3.9009]
ψ_{12}			1.63595 [7.64080]	2.84778 [1.8652]
ψ_{21}				-1.20728 [-2.8043]
ψ_{22}				1.12440 [4.83]
ψ_{13}				0.13162 [1.89]
ψ_{23}				0.91416 [7.4911]
ψ_{33}				-0.82133 [-6.81598]
λ_{10}			0.00678 [2.3331]	0.0045 [3.2472]
λ_{20}				0.01205 [3.71899]
χ^2 test [d.f.]	99.3733 [20]	34.9 [17]	21.7998 [14]	9.0135 [8]
10% critical value	28.41	24.77	21.06	13.36
5% critical value	31.41	27.59	23.68	15.51

Table 3 EMM estimates for the sample period 1/ 3/1980 - 12/ 31/1996: affine model.

BS model $\qquad d\ln S_t = \alpha_{10}dt + \beta_{10}dW_{1t}$

SV2 Model $\quad N_t^y \sim Poisson(\lambda_{10}) \quad N_t^v \sim Poisson(\lambda_{20}) \quad \xi^v \sim \exp(\psi_{13})$

$d\ln S_t = \alpha_{10}dt + \sqrt{V_{t-}}dW_{1t} + dJ^y \qquad \xi^y \sim N(\psi_{11}, \psi_{12}^2)$

$dV_t = \alpha_{20}\left(\alpha_{22} - V_{t-}\right)dt + \sqrt{V_{t-}}\beta_{20}\left[\beta_{12}dW_{1t} + \sqrt{1-\beta_{12}^2}dW_{2t}\right] + dJ^v$

parameter	BS	SV2	SV2J	SV2IJ
α_{10}	0.06670 [37.78]	0.058 [9.981]	0.05537 [5.1541]	0.07238 [5.12157]
α_{20}		0.03828 [3.57119]	0.04936 [1.79703]	0.06278 [14.93309]
α_{22}		0.70203 [10.3787]	0.54402 [8.3145]	0.47929 [13.1494]
β_{12}		-0.10570 [-1.47636]	-0.43776 [-2.4735]	-0.23624 [-2.70949]
β_{10}	0.92767 [32.8282]			
β_{20}		0.120144 [13.27315]	0.11609 [4.73288]	0.1 [2.8272]
ψ_{11}			-2.10765 [-10.5544]	-1.761021 [-3.05230]
ψ_{12}			0.69822 (0.37092) [1.8823]	0.71992 (0.727699) [0.9893]
ψ_{13}				1.03010 (2.4599) [0.41874]
λ_{10}			0.01128 [11.28]	0.013107 [1.60941]
λ_{20}				0.00724 [3.874836]
χ^2 test [d.f.]	99.3733 [20]	53.9184 [17]	36.47 [14]	19.7858 [12]
10% critical value	28.41	24.77	21.06	18.55
5% critical value	31.41	27.59	23.68	21.03

Table 4 **In sample pricing errors obtained by minimizing the IVMSE and parameter estimates (t ratios are reported in parenthesis): logvariance model**

(SV1CIJ) $d \ln S_t = \left[r - \frac{1}{2} V_t - \widetilde{\overline{\gamma}}_1 \lambda_1 - \widetilde{\overline{\gamma}}_2 \lambda_2 \right] dt + \sqrt{V_t} \left[\beta_{12} d\widetilde{W}_{2t} + \sqrt{1 - \beta_{12}^2} d\widetilde{W}_{1t} \right]$

$+ \sqrt{1 - \psi_{33}^2} \int_{R \backslash \{0\}} \zeta_1 \left(\Gamma \right) \widetilde{P}_1 \left(d\Gamma, dt \right) + \psi_{33} \int_{R \backslash \{0\}} \widetilde{\zeta}_2 \left(\Gamma \right) \widetilde{P}_2 \left(d\Gamma, dt \right)$

$\widetilde{\overline{\gamma}}_1 \equiv \exp \left(\sqrt{1 - \psi_{33}^2} \left(\mu_{\widetilde{\zeta}_1} + \frac{1}{2} \sigma_{\widetilde{\zeta}_1}^2 \right) \right) - 1 \quad \widetilde{\overline{\gamma}}_2 \equiv \exp \left(\psi_{33} \left(\mu_{\widetilde{\zeta}_2} + \frac{1}{2} \sigma_{\widetilde{\zeta}_2}^2 \right) \right) - 1$

$d \ln V_t = \left(\mu^U + \alpha_{22} \ln V_t - \beta_{20} \vartheta_2 + (\lambda_2 - \phi_3) \overline{\gamma}_3 - \widetilde{\overline{\gamma}}_3 \lambda_2 \right) dt + \beta_{20} d\widetilde{W}_{2t} + \int_{R \backslash \{0\}} \widetilde{\zeta}_3 P_2 \left(d\Gamma, dt \right)$

$\overline{\gamma}_3 \equiv \exp \left(\mu_{\zeta_3} + \frac{1}{2} \sigma_{\zeta_3}^2 \right) - 1 \quad \widetilde{\overline{\gamma}}_3 \equiv \exp \left(\mu_{\widetilde{\zeta}_3} + \frac{1}{2} \sigma_{\widetilde{\zeta}_3}^2 \right) - 1$

$d\widetilde{W}_{1t} = dW_{1t} - \vartheta_1(V_t) dt \quad d\widetilde{W}_{2t} = dW_{2t} - \vartheta_2(V_t) dt$

$\widetilde{P}_1 \sim Poisson(\widetilde{\lambda}_1) \quad \widetilde{P}_2 \sim Poisson(\widetilde{\lambda}_2)$

$\widetilde{\zeta}_1 \sim N(\widetilde{\psi}_{11}, \widetilde{\psi}_{12}^2) \quad \widetilde{\zeta}_2 \sim N(\widetilde{\psi}_{13}, \widetilde{\psi}_{23}^2) \quad \widetilde{\zeta}_3 \sim N(\widetilde{\psi}_{21}, \widetilde{\psi}_{22}^2)$

parameter	estimate
$\widetilde{\psi}_{11}$	-5.739 [-3.7758]
$\widetilde{\psi}_{12}$	1.9647 [0.39525]
$\widetilde{\psi}_{13}$	0.08844 [0.1713]
$\widetilde{\psi}_{23}$	0.9097 [0.5576]
$\widetilde{\psi}_{21}$	-0.42257 [-3.668]
$\widetilde{\psi}_{22}$	2.8227278 [24.481]
$\widetilde{\lambda}_1$	0.0081 [7.977]
$\widetilde{\lambda}_2$	0.02078 [20.985]
ϑ_2	-0.0260 [-3.82]
IVMSE with jump risk premia	0.0039925
IVMSE without jump risk premia	0.019811

Table 5 In sample pricing errors obtained by minimizing the IVMSE and parameter estimates (t ratios are reported in parenthesis): affine model (Duffie et al. 2000).

$$d \ln S_t = \left[r - \frac{1}{2} V_t - \left(e^{\mu_{\xi^y} + \frac{1}{2} \sigma_{\xi^y}^2} - 1 \right) \lambda_{\xi^y} \right] dt$$
$$+ \sqrt{V_{t-}} d\widetilde{W}_{1t} + \widetilde{\xi}^y d\widetilde{N}^y$$

$$dV_t = \left[\alpha_{20} \left(\alpha_{22} - V_{t-} \right) + \left(\lambda_{\xi^v} - \phi_{\xi^v} \right) \mu_{\xi^v} \right] dt$$
$$- \sqrt{V_{t-}} \beta_{20} \left[\beta_{12} \vartheta_1 + \sqrt{1 - \beta_{12}^2} \vartheta_2 \right] dt$$
$$+ \sqrt{V_{t-}} \beta_{20} \left[\beta_{12} d\widetilde{W}_{1t} + \sqrt{1 - \beta_{12}^2} d\widetilde{W}_{2t} \right] + \widetilde{\xi}^v d\widetilde{N}^v - \widetilde{\mu}_{\xi^v} \widetilde{\lambda}_{\xi^v} dt$$

$$d\widetilde{W}_{1t} = dW_{1t} - \vartheta_1(V_t) dt$$

$$d\widetilde{W}_{2t} = dW_{2t} - \vartheta_2(V_t) dt$$

$$\left(e^{\widetilde{\xi}^y} - 1 \right) d\widetilde{N}^y - \left(e^{\widetilde{\mu}_{\xi^y} + \frac{1}{2} \widetilde{\sigma}_{\xi^y}^2} - 1 \right) \widetilde{\lambda}_{\xi^y} dt$$
$$= \left(e^{\xi^y} - 1 \right) dN^y - \left(e^{\mu_{\xi^y} + \frac{1}{2} \sigma_{\xi^y}^2} - 1 \right) \left(\lambda_{\xi^y} - \phi_{\xi^y} \right) dt$$
$$\widetilde{\xi}^v d\widetilde{N}^v - \widetilde{\lambda}_{\xi^v} \widetilde{\mu}_{\xi^v} dt$$
$$= \xi^v dN^v - \mu_{\xi^v} \left(\lambda_{\xi^v} - \phi_{\xi^v} \right) dt$$

$$\widetilde{N}_t^y \sim Poisson(\widetilde{\lambda}_y) \quad \widetilde{N}_t^v \sim Poisson(\widetilde{\lambda}_y) \quad \widetilde{\xi}^y \sim N(\widetilde{\mu}_y, \widetilde{\sigma}_y^2) \quad \widetilde{\xi}^v \sim \exp(\widetilde{\mu}_v)$$

parameter	estimate
$\widetilde{\mu}_y$	-2.76102 [-4.5504]
$\widetilde{\sigma}_y$	11.71993 [8.7505]
$\widetilde{\mu}_v$	8.0301 [1.7076]
$\widetilde{\lambda}_y$	0.013107 [3.5721]
$\widetilde{\lambda}_v$	0.0723955 [8.314]
ϑ	0.001 [0.0078751]
IVMSE with jump risk premia	0.017529
IVMSE without jump risk premia	0.030024

Table 6 Out of sample pricing errors using the data on European Call options on the S&P500 quoted in 1988

Model	IVMSE
SV1CIJ	0.0024584
SV2CJ	0.0059667

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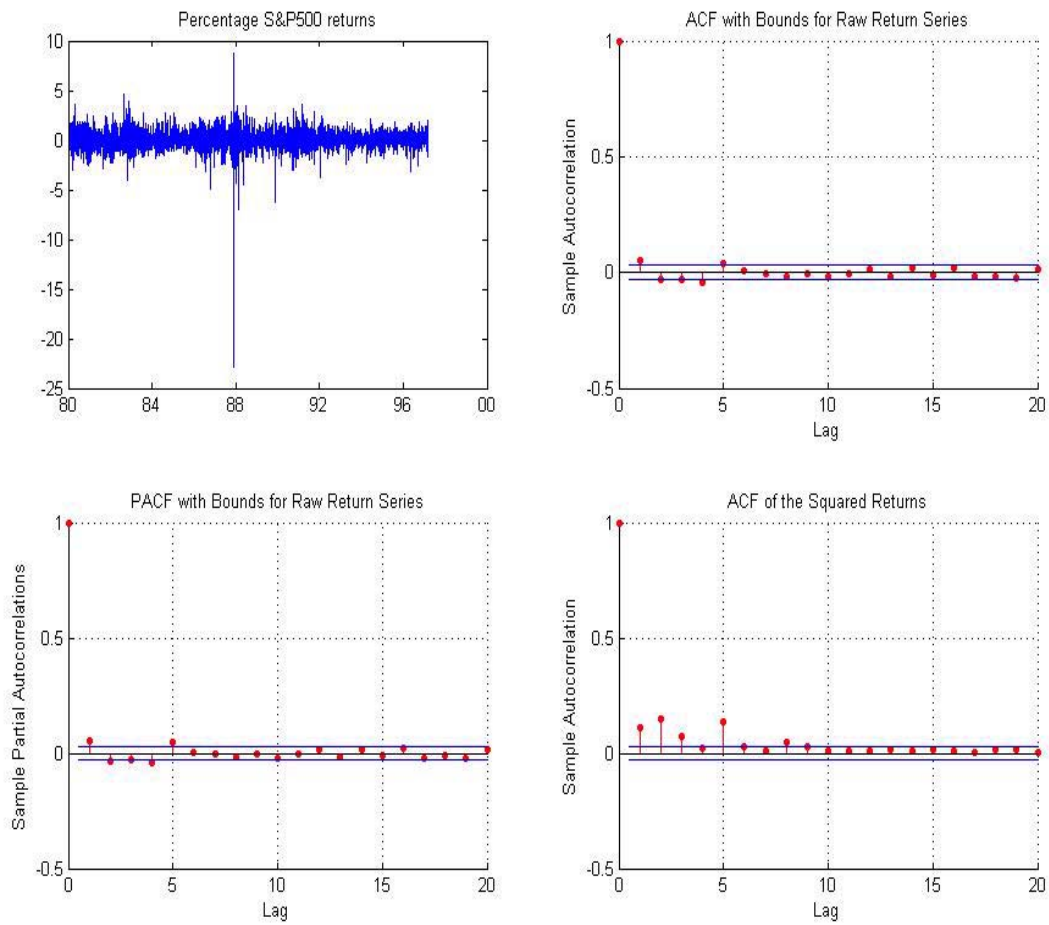
Figure 1: Autocorrelation functions on the percentage return series and of the squared percentage returns.
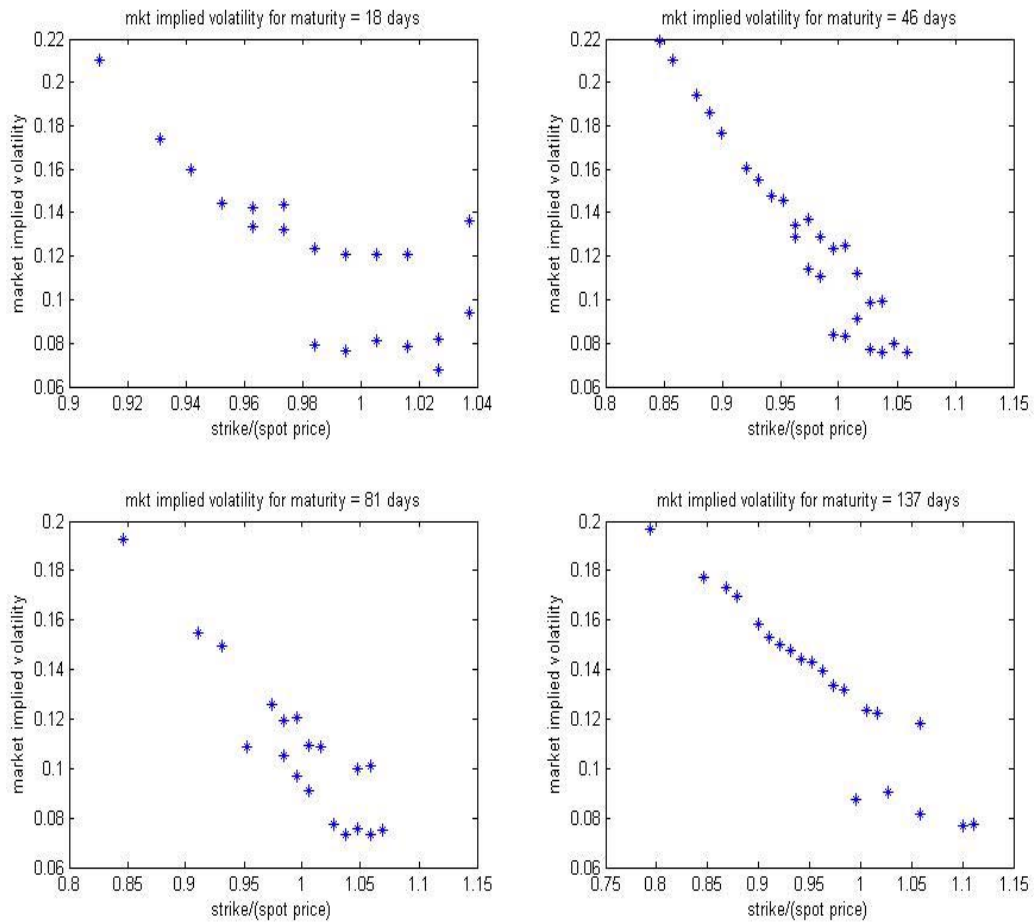
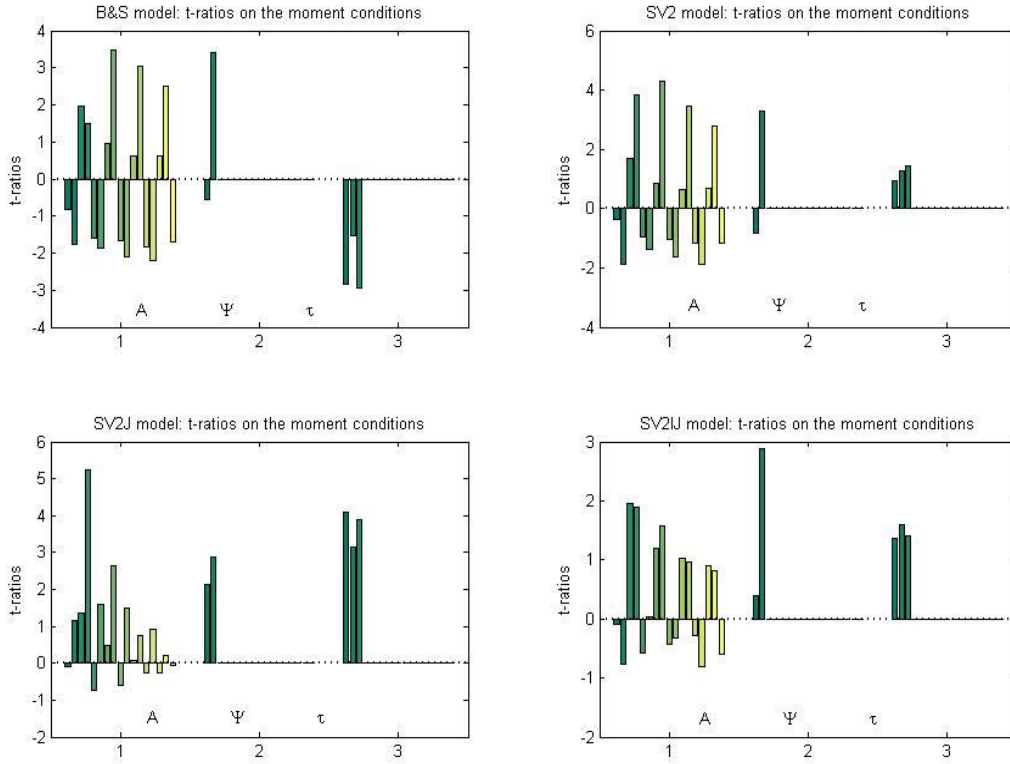Figure 2: Market implied volatilities for the randomly selected day October 31, 1994.
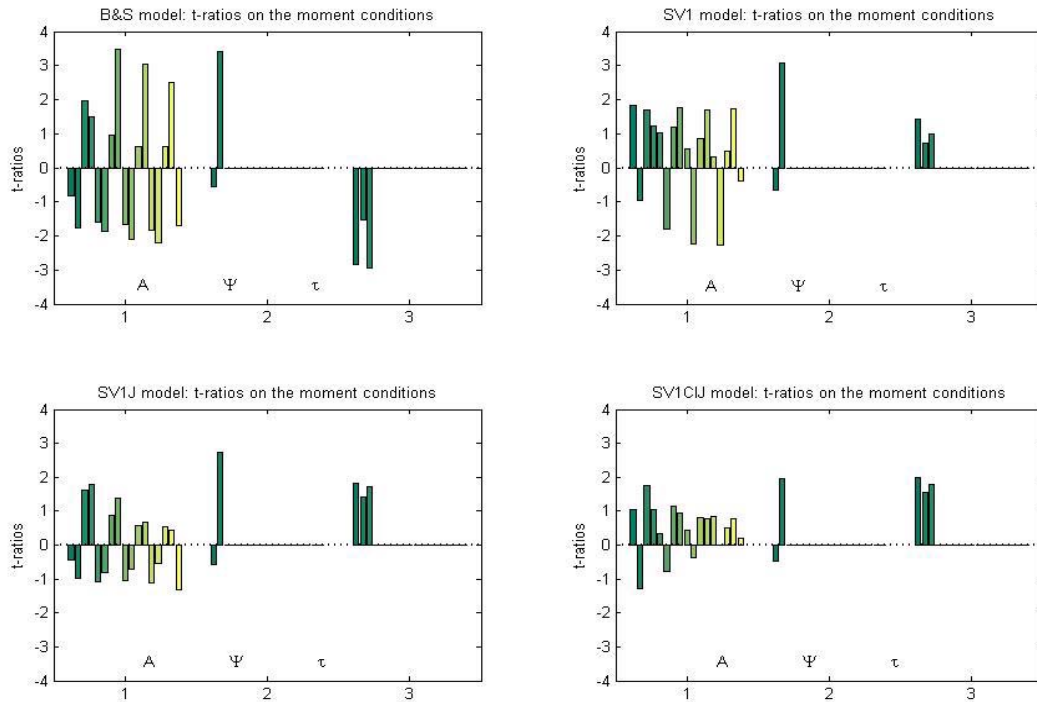
Figure 3: Log variance models: t-ratios on the SNP moment conditions (auxiliary model). A level of any of these ratios higher than 2 indicates the failure of the structural model to capture that specific aspect of the data. The moment conditions labeled with "A" control the ability to fit the tail thickness of the data (kurtosis). The moment conditions labeled with "Psi" are linked to the AR characteristic of the data. Finally, the moment conditions related to the GARCH parameters are labeled with "Tau".

Figure 4: Affine models: t-ratios on the SNP moment conditions (auxiliary model). A level of any of these ratios higher than 2 indicates the failure of the structural model to capture that specific aspect of the data. The moment conditions labeled with "A" control the ability to fit the tail thickness of the data (kurtosis). The moment conditions labeled with "Psi" are linked to the AR characteristic of the data. Finally, the moment conditions related to the GARCH parameters are labeled with "Tau".

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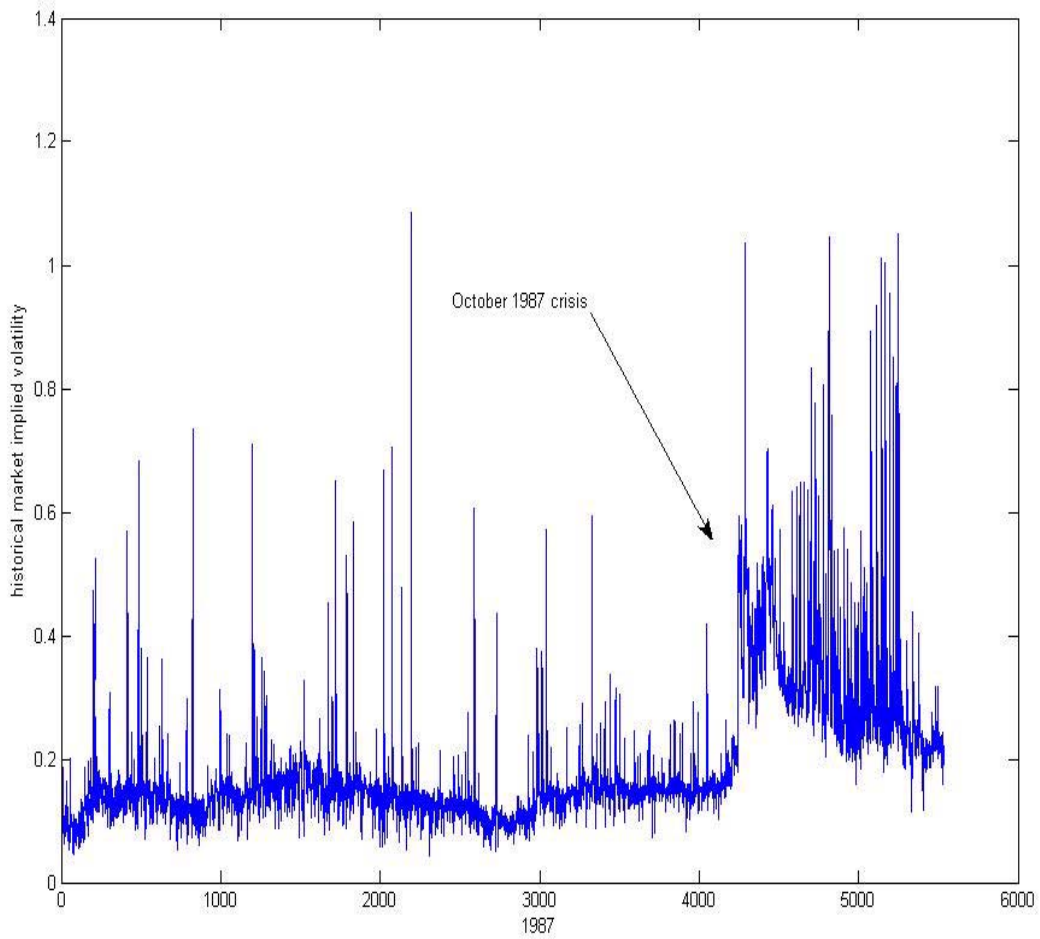
Figure 5: Spikes in the historical market implied volatility series.
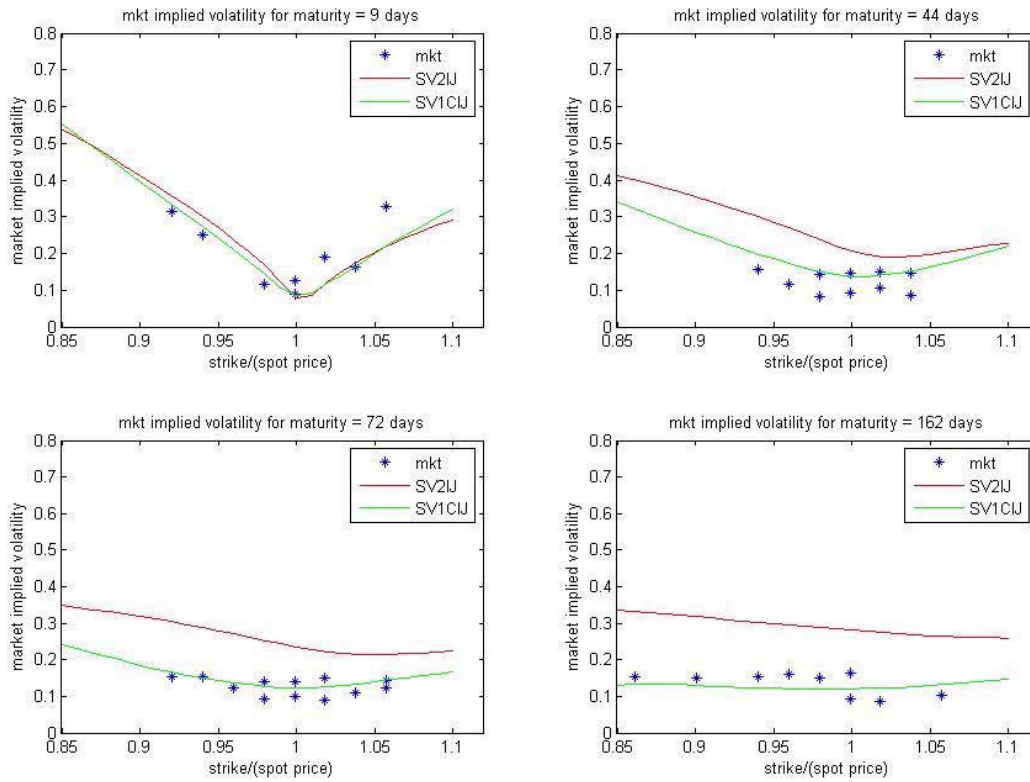
Figure 6: Implied volatility smiles for the SV1CIJ and SV2IJ models and market implied volatility data for the randomly selected day, January 7 1987
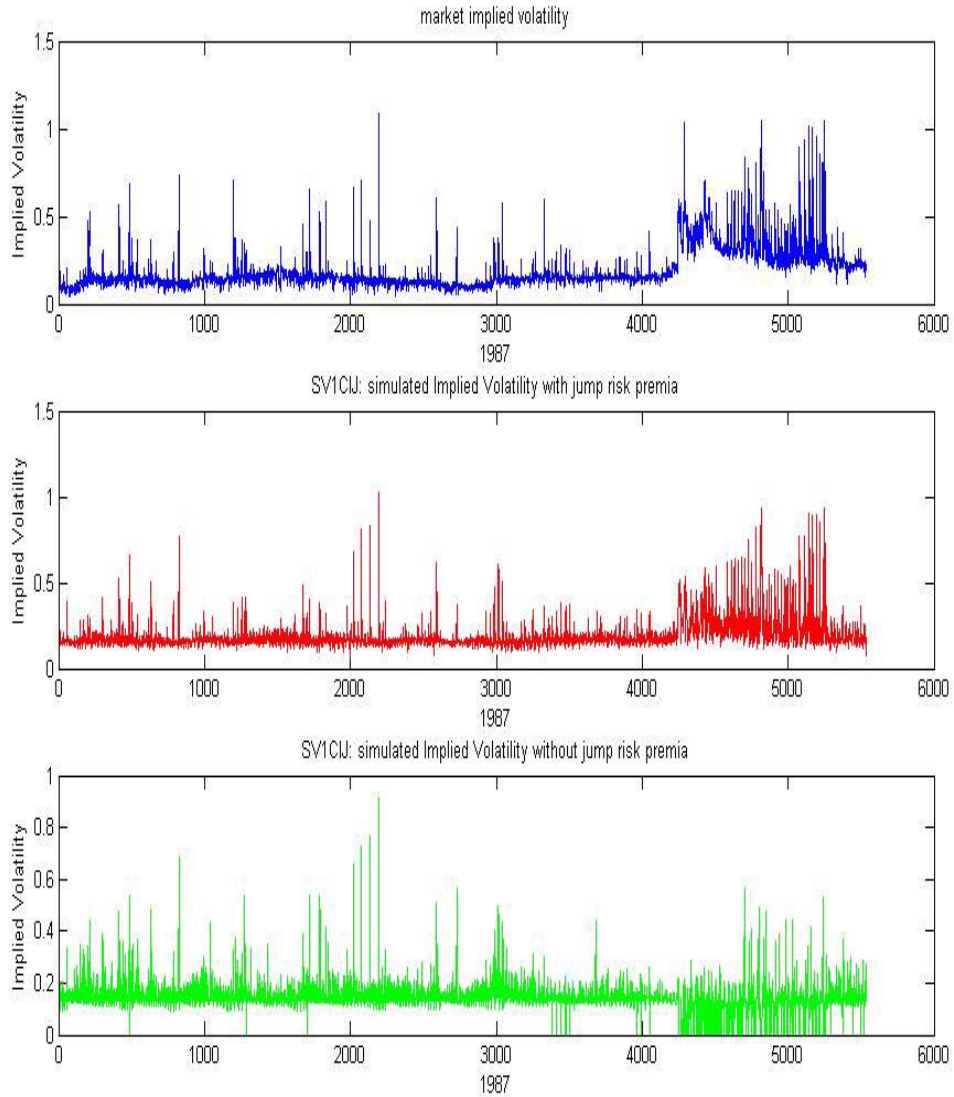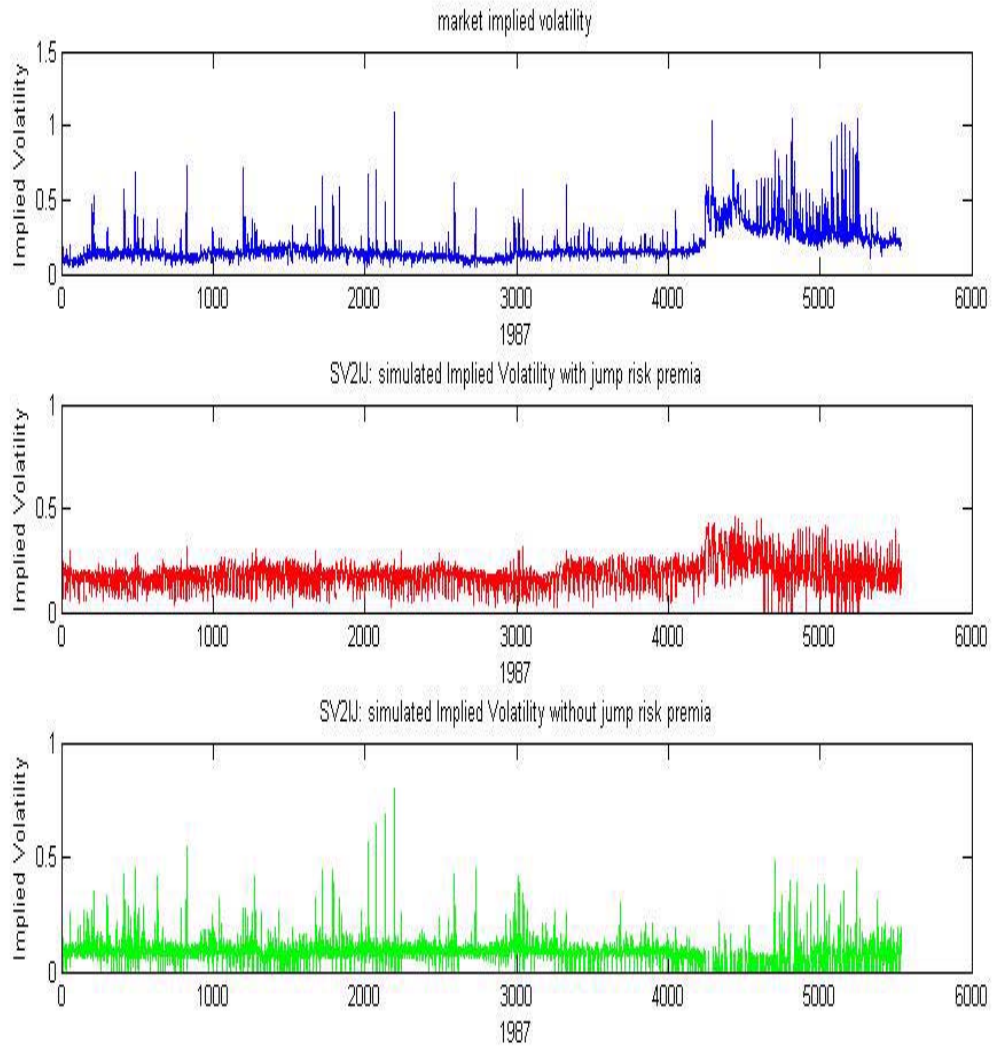
Figure 7: The top panel represents the market implied volatility series for the year 1987. The second panel contains the simulated IV series when the affine model (SV1CIJ) is adopted and jump risk premia are allowed. The case when these premia are set equal to zero is represented in the last panel.

Figure 8: The top panel represents the market implied volatility series for the year 1987. The second panel contains the simulated IV series when the affine model (SV2IJ) is adopted and jump risk premia are allowed. The case when these premia are set equal to zero is represented in the last panel.
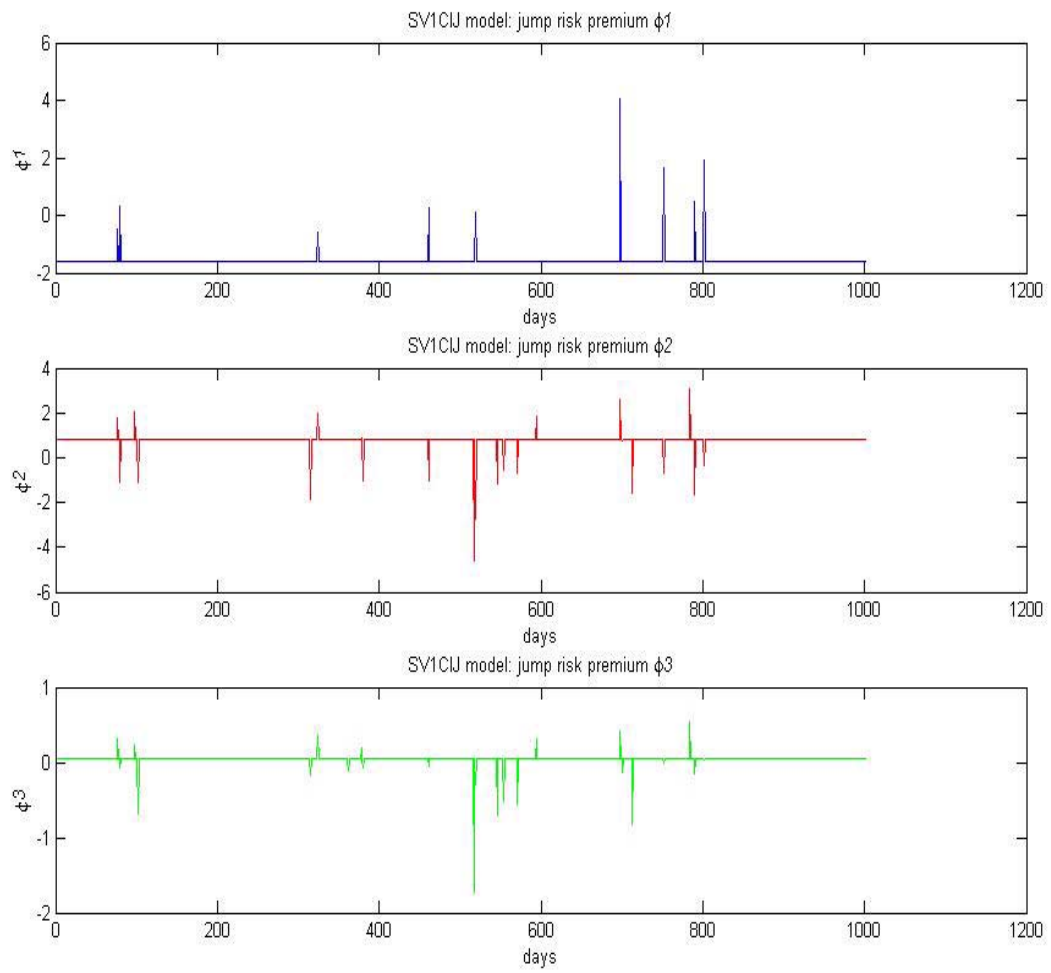
Figure 9: Log-variance model: jump risk premia (see equations 5, 6 and 7 in Section 4)
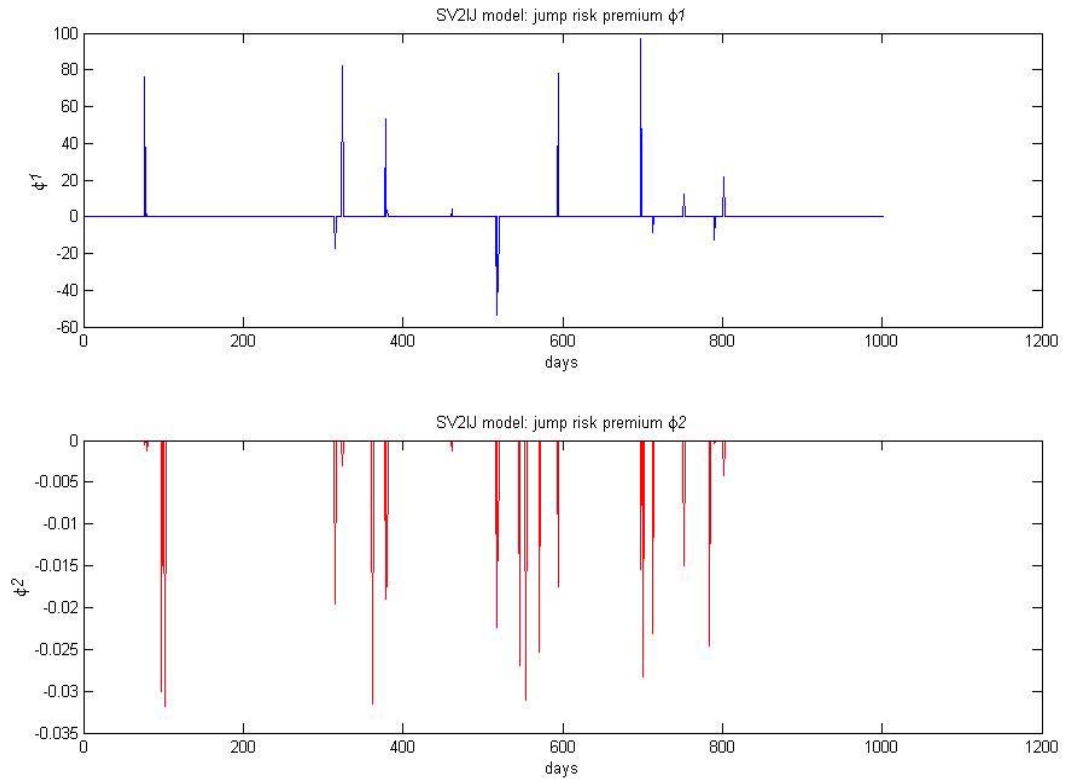
Figure 10: Affine model with independent jumps in the stock process and in the volatility process: jump risk premia.