Measuring Inflation Persistence: A Structural Time Series Approach

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Abstract

Using a structural time series approach we measure different sorts of inflation persistence allowing for an unobserved time-varying inflation target. Unobserved components are identified using Kalman filtering and smoothing techniques. Posterior densities of the model parameters and the unobserved components are obtained in a Bayesian framework based on importance sampling. We find that inflation persistence, expressed by the half life of a shock, can range from 1 quarter in case of a cost-push shock to several years for a shock to long-run inflation expectations or the output gap.

JEL Classification: C11, C22, C32, E31

Key Words: Inflation Target, State Space Model, Kalman Filter, Bayesian Analysis.

1 Introduction

It is generally accepted that over the medium to long run inflation is a monetary phenomenon, i.e. entirely determined by monetary policy. Over shorter horizons various macroeconomic shocks, including variations in economic activity or production costs, will temporarily move inflation away from the central bank's inflation target. Therefore, a profound understanding of the process generating inflation, in particular the speed of inflation adjustment in response to such shocks is of crucial importance for an inflation targeting central bank. Inflation persistence then refers to the tendency of inflation to converge slowly towards the central bank's inflation target in response to these shocks.

With respect to measuring historical inflation persistence, a common practice in empirical research is to use a univariate autoregressive (AR) time series model and measure persistence as the sum of the estimated AR coefficients (Nelson and Plosser 1982; Fuhrer and Moore 1995; Pivetta and Reis 2004). In most of these studies, inflation is found to exhibit high to very high persistence over the post-WW II period, i.e. persistence is found to be close to that of a random walk. This suggests that a central bank's task of pursuing price stability might be more complicated than if persistence were low.

Important to note, though, is that this estimated high persistence should be interpreted as a measure of unconditional inflation persistence as this literature does not take into account that the data generating process of inflation is composed of a number of distinct components, each of them exhibiting its own level of persistence. As such, there are various factors underlying measured historical inflation persistence. First, over the last four decades large changes in the monetary policy strategy of industrialised economies have occurred. This has led to permanent shifts in the inflation target[1] of central banks. Second, due to asymmetric information, sticky information or imperfect credibility, private agents' perceptions about the central bank's inflation target can differ from the true inflation target. The persistence of such deviations can be called expectations-based persistence (Angeloni et al. 2004). Third, the sluggish response of inflation to various macroeconomic shocks is likely to be related to the wage- and price-setting mechanism. If wages and prices are adjusted infrequently, they will only gradually incorporate the effects of these shocks and therefore deviations of the observed inflation rate from the perceived inflation target will persist during several consecutive periods. This kind of inflation persistence can be called intrinsic inflation persistence (Angeloni et al. 2004). Also price and wage indexation, which introduces backward-lookingness into inflation, add to intrinsic inflation persistence. Fourth,

[1] Although inflation targeting is a monetary policy strategy that only emerged in the 1990s, we will still use this framework for the 1970s and 1980s. It enables us to identify the implicit inflation target of central banks from their policy choices as well as subsequent economic outcomes.

inflation persistence is determined by the persistence of the various macroeconomic shocks hitting inflation, e.g. persistent deviations of output from its potential level. This type of inflation persistence can be called extrinsic inflation persistence (Angeloni et al. 2004).

In order to get a reliable estimate of the various types of inflation persistence, each of the above mentioned components should be taken into account explicitly when constructing the data generating process of inflation. First, permanent shifts in the central bank's inflation target lead to permanent changes in inflation. As standard AR models assume that inflation has a stable mean, these shifts induce an upward bias on measured inflation persistence (Levin and Piger 2004). In fact, this argument goes back to Perron (1990) who pointed out that the standard Dickey-Fuller unit root test is biased towards non-rejection of the unit root hypothesis if the true data generating process includes breaks in its deterministic components. Taking historical changes in the central bank's inflation target into account might not be straightforward, though. Contrary to the current conduct of monetary policy, most countries typically did not directly communicate their inflation target to the public. Second, if the central bank's inflation target is not known to private agents or if it is not fully credible, the inflation target perceived by economic agents might differ from the central bank's inflation target. In this case intrinsic and extrinsic inflation persistence should be measured as the persistence in the deviations of the actual inflation rate from the perceived inflation target rather than from the central bank's inflation target. Third, in order to estimate extrinsic persistence, the persistence in macroeconomic shocks hitting inflation should be modelled as well.

In the recent literature, shifts in the central bank's inflation target are accounted for in three different ways. First, O'Reilly and Whelan (2004) and Pivetta and Reis (2004) use rolling regressions to allow for shifts in the mean of inflation over different subsamples. By lowering the subsample size, the number of breaks that can occur is reduced. Still, the authors cannot reject the hypothesis that the sum of the AR coefficients equals 1. Second, Levin and Piger (2004), Gadzinski and Orlandi (2004) and Bilke (2004) estimate an AR process allowing for discrete breaks in the mean of the inflation process. Without accounting for possible shifts, Levin and Piger (2004) report a persistence parameter for the United States GDP deflator of 0.92 over the period 1984Q1-2003Q4. Once a structural break is allowed for, persistence drops to 0.36. Third, Cogley and Sargent (2001, 2003), and Benati (2004) estimate time-varying AR coefficients conditional on a time-varying mean, which is specified as a random walk process. They find evidence that the AR coefficients of inflation have dropped considerably over the last decade.

With respect to these recent contributions to the literature, the following drawbacks should be stressed. First, rolling regressions do not entirely rule out the possibility that a shift occurred in a specific subsample, especially when shifts are frequent. Moreover, this approach has limits in terms of degrees of freedom. Second, capturing shifts in monetary policy by allowing for a time-varying mean inflation rate, either by adding discrete breaks or a random walk process to the AR model, is inappropriate if the perceived inflation target differs from the central bank's inflation target. As this difference is not accounted for in these models, the persistence in the deviation of the perceived inflation target from the central bank's inflation target is implicitly restricted to equal the average of intrinsic and extrinsic inflation persistence.

This paper uses a structural time series approach to model the data generating process of inflation in the euro area[2] and the United States. Given the various sources of inflation persistence, structural time series models are particularly suited as in these models a time series can be decomposed into a number of distinct components, each of them being modelled explicitly. We pursue both a univariate and a multivariate approach. In both approaches, intrinsic inflation persistence is measured as the persistence of the deviations of inflation from the perceived inflation target. In contrast to the current literature, this allows for expectations-based persistence in response to shocks to the inflation target. Expectations-based persistence is incorporated by modelling the perceived inflation target as an AR process around the central bank's inflation target, the latter being modelled as a random walk. Kozicki and Tinsley (2003) use a similar model to disentangle permanent and transitory monetary policy shifts. Contrary to these authors, in the multivariate model we explicitly decompose output into potential output and a business cycle component. In this way we can estimate extrinsic inflation persistence in response to shocks to the business cycle.

As the univariate and the multivariate model both include a number of unobserved components, they are cast in a linear Gaussian state space representation. This enables identification of the unobserved components from the observed data using Kalman filtering and smoothing techniques. The unknown parameters are estimated in a Bayesian framework, exploiting information from both the sample data and previous studies estimating similar models. Posterior densities of the model parameters and the unobserved components are obtained using importance sampling.

The results indicate that intrinsic inflation persistence is not close to that of a random walk, i.e. the half life of a cost-push shock is only one quarter in both the euro area and the United States. The observed high degree of aggregate post-WW II inflation persistence stems from the other three components driving inflation. First, credible changes in the

[2] Although the euro area did not exist for the larger part of our data sample (1970Q2-1998Q4), we use synthetic data aggregating the national data (Fagan, Henry and Mestre 2005). As such, we implicitly assume that the euro area was an economy with a homogeneous monetary policy over the entire sample.

central bank's inflation target induce permanent changes in observed inflation. Second, expectations only adjust slowly in response to changes in the inflation target. The half life of shocks to the perceived inflation target is 9 and 16 quarters in the euro area and the United States respectively. This indicates that the dissipation of changes in the policy target is typically slower than in case of temporary shocks. Third, given the relative high persistence of shocks to the output gap, the half life of such shocks to inflation amounts to 13 quarters in the euro area and to 19 quarters in the United States. This extrinsic persistence explains why inflation may deviate from the perceived inflation target during several consecutive periods.

The implications for monetary policy are as follows. In a stable inflation regime, where the central bank's inflation target does not change and where the public perception about this inflation target is well anchored, inflation persistence is relatively low. In the case monetary policy would again give rise to unstable inflation, it would afterwards be very hard to disinflate due to the slow adjustment of inflation expectations in response to changes in the inflation target. With natural rate misperceptions (Orphanides and Williams 2004) this might not be straightforward to avoid.

2 A structural time series approach

In this section, we present a structural time series model for inflation which takes into account (i) possible shifts in the central bank's inflation target, (ii) expectations-based persistence, (iii) intrinsic persistence and (iv) extrinsic persistence. The model is identified both in a univariate and a multivariate set-up. The univariate approach relies on time series data for inflation only. In the multivariate model, we add information contained in real output and the central bank's key interest rate. Using a variant of the macroeconomic model of Rudebusch and Svensson (1999), this allows us to impose more economic structure on the identification process. The advantage of the univariate over the multivariate model is that its relative simplicity reduces the risk of specification errors. The state space representation of both models is given in section 3.

2.1 Baseline structural model

The baseline structural model is given by:

$$\pi^T_{t+1} = \pi^T_t + \eta_{1t}, \tag{1}$$

$$\pi^P_{t+1} = E_{t+1}\pi^T_{t+1}, \tag{2}$$

$$\pi_t = (1-\sum\nolimits_{i=1}^{q}\varphi_i)\pi^P_t + \sum\nolimits_{i=1}^{q}\varphi_i L^i \pi_t + \beta_1 z_{t-1} + \varepsilon_{1t}, \quad \sum\nolimits_{i=1}^{q}\varphi_i < 1, \tag{3}$$

where π_t^T is the central bank's inflation target, π_t^P is the perceived inflation target, π_t is the observed inflation rate and z_t is the output gap, i.e. the percentage deviation of real output from potential output. L is the lag operator so that $L^i\pi_t = \pi_{t-i}$. ε_{1t} and η_{1t} are mutually independent zero mean white noise processes.

Equation (1) specifies π_t^T as a random walk process, i.e. shifts in the central bank's inflation target are assumed to be permanent. These shifts can be thought of as representing (i) changes in the central bank's preferences over alternative inflation outcomes (Andolfatto, Hendry and Moran 2002) or (ii) an implicit change in the inflation target of the central bank created by misperceptions about the natural rate of different real variables (Orphanides and Williams 2004).

Shifts in π_t^T are unlikely to be passed on to inflation expectations immediately. Castelnuovo, Nicoletti-Altimari and Rodriguez-Palenzuela (2003) present data on long-run inflation expectations. These suggest that in the aftermath of shifts in monetary policy, convergence towards the new equilibrium evolves smoothly over time. In the literature, this is often attributed to asymmetric information and signal extraction, sticky information or imperfect credibility. The source of asymmetric information on behalf of the private agents can be due to a lack of knowledge about the central bank's inflation target (Kozicki and Tinsley 2003) or uncertainty about the central bank's preferences of inflation over real activity (Cukierman and Meltzer 1986; Tetlow and von zur Muehlen 2001). If private agents have to extract information about the central bank's inflation target from a monetary policy rule, the signal-to-noise ratio of this policy rule determines the uncertainty faced by private agents in disentangling transitory and permanent policy shocks and therefore also the speed at which they recognise permanent policy shocks (Erceg and Levin 2003). Further, even if the central bank clearly announces a new inflation target, it can take quite some time before the new policy target is incorporated into long-run inflation expectations of private agents (Castelnuovo et al. 2003). This might be due to costs of acquiring information and/or re-optimisation (Mankiw and Reis 2002). Summing up, private agents must form expectations about the inflation target π_t^T. Therefore, equation (2) introduces the perceived inflation target π_t^P, which captures the private agents' beliefs about the central bank's inflation target π_t^T.

The expectations operator in equation (2) is operationalised by modelling π_{t+1}^P as a weighted average of $\pi_t^P$and π_{t+1}^T,

$$\pi_{t+1}^P = (1-\delta)\pi_t^P + \delta\pi_{t+1}^T + \eta_{2t}, \quad 0 < \delta \leq 1, \tag{4}$$

where η_{2t} is a zero mean white noise process. The weighting parameter δ can be interpreted as being the information updating parameter λ in a variant of the sticky-information model of Mankiw and Reis (2002) or as being proportional to the Kalman gain parameter k_g in the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002).[3] Consequently, δ measures the speed with which changes in the central bank's inflation target affect long-run inflation expectations of private agents, i.e. δ measures expectations-based persistence. If δ is one, a shift in the central bank's inflation target is immediately and completely passed on to inflation expectations. This would be the case if the central bank's inflation target is perfectly known to all private agents and immediately credible. The smaller δ, the slower expectations respond to a shift in the central bank's inflation target.[4] In the sticky-information model of Mankiw and Reis (2002), δ decreases in the cost of acquiring information and/or the cost of re-optimising prices in response to a shift in the central bank's inflation target. In the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002), δ increases in the signal-to-noise ratio of the monetary policy rule, i.e. the lower the uncertainty about whether monetary policy signals reflect transitory rather than permanent policy changes, the faster private agents will react to these signals by updating their inflation expectations.[5]

Note that shocks to the perceived inflation target, η_2, only have a short-run impact on π^P. These shocks should be interpreted as misperceptions of private agents about the central bank's inflation target, due to for instance noise in the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002). Shocks to the central bank's inflation target, η_1, have a unit long-run impact on π^P, i.e. π^T is the long-run equilibrium inflation rate. This is consistent with the generally accepted feature that long-run inflation is a purely monetary phenomenon.

Equation (3) is a Phillips curve, relating the observed inflation rate π_t to the perceived inflation target π_t^P, q lags of inflation and the lagged output gap z_{t-1}. The perceived inflation target π_t^P is the inflation rate consistent with the private agents' inflation expectations. Therefore, it serves as the medium-run inflation anchor. Both business cycle shocks, reflected in the output gap z_{t-1}, as well as cost-push shocks, measured by ε_{1t}, hitting inflation induce temporary deviations of π_t from π_t^P. The sluggish adjustment of π_t in response to cost-push shocks ε_{1t} is measured by the sum of the AR coefficients, $\sum_{i=1}^{q} \varphi_i$. This intrinsic inflation persistence is likely to be related to price- and wage-setting mechanisms, e.g. price and wage indexation. The sluggish adjustment of π_t in response to business cycle shocks is determined, besides the intrinsic inflation persistence, by the persistence of the output gap

[3] See Appendix A for more details on how equation (4) can be derived from these two models.

[4] We do not allow δ to take a value of 0, as in this case π_t^P does not react to monetary policy shocks, i.e. monetary policy is not credible. Note that this restriction does not imply that all monetary policy actions are fully credible. Rather, only credible shifts in the central bank's inflation target are included in η_{1t}.

[5] Equation (4) does not distinguish between these two theories, neither excludes that δ is a weighted average of k_g and λ, which could be the case if reality is a mixture of both theories.

z_t in response to business cycle shocks. The latter source of inflation persistence can be called extrinsic inflation persistence.

Note that equation (3) does not impose that the observed inflation series is additively composed of the perceived inflation target and a temporary component. Rather, shifts in π_t^P are only slowly passed on to observed inflation, with the speed of convergence being determined by the degree of intrinsic inflation persistence. In this way, we assume that in case of a shift in the perceived inflation target the structural determinants for intrinsic persistence, e.g. price and wage indexation, are present in addition to the determinants of expectations-based persistence, e.g. sticky or imperfect information.

2.2 Univariate identification

In a first step, we use time series data on inflation only to estimate the model specified in equations (1)-(4). Given the limited information set, the baseline model is simplified in two respects. First, we set $\beta_1 = 0$ in equation (3). This restriction stems from the fact that we do not include any information about real output and therefore cannot estimate extrinsic inflation persistence in response to business cycle shocks. Second, we exclude the possibility of shocks to π_t^P, i.e. $\eta_{2t} = 0 \; \forall t$. This restriction is motivated from the concern to keep, given the limited information set, the identification of π_t^P and π_t^T as simple as possible. Under this restriction, equation (4) can be rewritten, using equation (1), as:

$$\pi_{t+1}^P = (2 - \delta)\, \pi_t^P + (\delta - 1)\, \pi_{t-1}^P + \delta \eta_{1t} \tag{5}$$

This way of writing equation (4) shows that the univariate identification scheme boils down to the empirical restriction that (i) shocks to the central bank's inflation target, η_{1t}, have a unit long-run impact on observed inflation, (ii) inflation expectations can deviate from the central bank's inflation target over a long period of time and (iii) observed inflation is a stationary AR process around the perceived inflation target. Note that equation (5) is broadly consistent with the idea advocated by, among others, Young, Lane, Ng and Palmer (1991) that in order to introduce enough smoothness in estimates of unobserved trend components, they are best modelled as an integrated random walk process. Although strictly speaking the data generating process for π_t^P is not allowed to be an integrated random walk process, as $\delta > 0$, π_t^P will exhibit a similar smoothness in response to monetary policy shocks provided that δ is sufficiently close to 0. A similar specification of the data generating process of inflation expectations can be found in Doménech and Gomez (2003).

2.3 Multivariate identification

The univariate model exhibits two important drawbacks. First, identification of shocks to the central bank's inflation target stems from the purely statistical restriction that these shocks should have a unit long-run impact on inflation. Second, extrinsic inflation persistence cannot be estimated. Therefore, we add data on the central bank's key interest rate and real output. We use a variant of the widely used macroeconomic model of Rudebusch and Svensson (1999) to (i) identify the central bank's inflation target from information contained in the central bank's key interest rate and (ii) to measure extrinsic inflation persistence in response to shocks to the output gap from information contained in real output. Therefore, the baseline specification in equations (1)-(4) is extended with the following equations:

$$
\begin{aligned}
i_t &= \rho_2 i_{t-1} + (1-\rho_2)\left(r_t^* + \pi_t^P\right) + \rho_1(\pi_{t-1} - \pi_t^T) + \varepsilon_{2t} && (6)\\
y_t^r &= y_t^P + z_t && (7)\\
z_t &= \beta_2 z_{t-1} + \beta_3 z_{t-2} - \beta_4\left(i_{t-1} - \pi_{t-1}^P - r_{t-1}^*\right) + \varepsilon_{3t} && (8)\\
y_{t+1}^P &= \lambda_{t+1} + y_t^P + \eta_{3t} && (9)\\
\lambda_{t+1} &= \lambda_t + \eta_{4t} && (10)\\
r_{t+1}^* &= \gamma\lambda_{t+1} + \tau_{t+1} && (11)\\
\tau_{t+1} &= \theta\tau_t + \eta_{5t} && (12)
\end{aligned}
$$

where ε_{2t}, ε_{3t}, η_{3t}, η_{4t} and η_{5t} are mutually independent zero mean white noise processes.

The interest rate rule in equation (6) infers on the stance of monetary policy through comparing the central bank's key nominal interest rate, i_t, with a measure for the neutral stance of monetary policy. Following Laubach and Williams (2003), this measure is assumed to be the natural short-run nominal interest rate $\left(r_t^* + \pi_t^P\right)$, where r_t^* is the time-varying real short-term interest rate consistent with output equal to potential (cf. below). As the perceived inflation target π_t^P is the medium-run inflation anchor consistent with long-run inflation expectations, $r_t^* + \pi_t^P$ is the medium-run nominal interest rate anchor for monetary policy. The term $(\pi_{t-1} - \pi_t^T)$ captures the reaction of the central bank to deviations of inflation from its target, i.e. monetary authorities will increase the nominal interest rate i_t when observed inflation π_{t-1} lies above the inflation target π_t^T. The lagged interest rate i_{t-1} introduces a degree of nominal interest rate smoothing or policy inertia (Amato and Laubach 1999; English, Nelson and Sack 2003; Erceg and Levin 2003). We assume that the policy parameters ρ_1 and ρ_2 are time-invariant. Although Clarida, Gali and Gertler (1998) find that the policy parameters are unstable in a number of countries, this assumption is not in contradiction with their results. They estimate the parameters conditional on a constant inflation target, whereas we estimate the inflation target conditional on constant policy parameters. Both strategies are to a high degree observationally equivalent. The reason why we do so is that we are interested in the time-varying inflation target and less in the policy parameters. For examples of the same approach see e.g. Kozicki and Tinsley (2003) or Smets and Wouters (2005).

The interest rate rule enables us to extract information on shifts in the monetary policy regime contained in the key nominal interest rate i_t. Figures 1 and 2 present data for key nominal interest rates and inflation in the euro area and the United States since 1970. For a given fully credible central bank inflation target, inflation and the key nominal interest rate i_t should, over an entire business cycle, move around a fixed point on a 45 degree line with an intercept equal to the equilibrium real interest rate. This 45 degree line corresponds to the sum of the natural real interest rate and the perceived inflation target π_t^P, that equals the credible central bank inflation target π_t^T. However, the seven year moving average line of the data, which approximately filters out business cycle fluctuations, shows that from the 1970s until now inflation and interest rates did not move around a fixed point. This suggests that there have been substantial shifts in the central bank's inflation target.

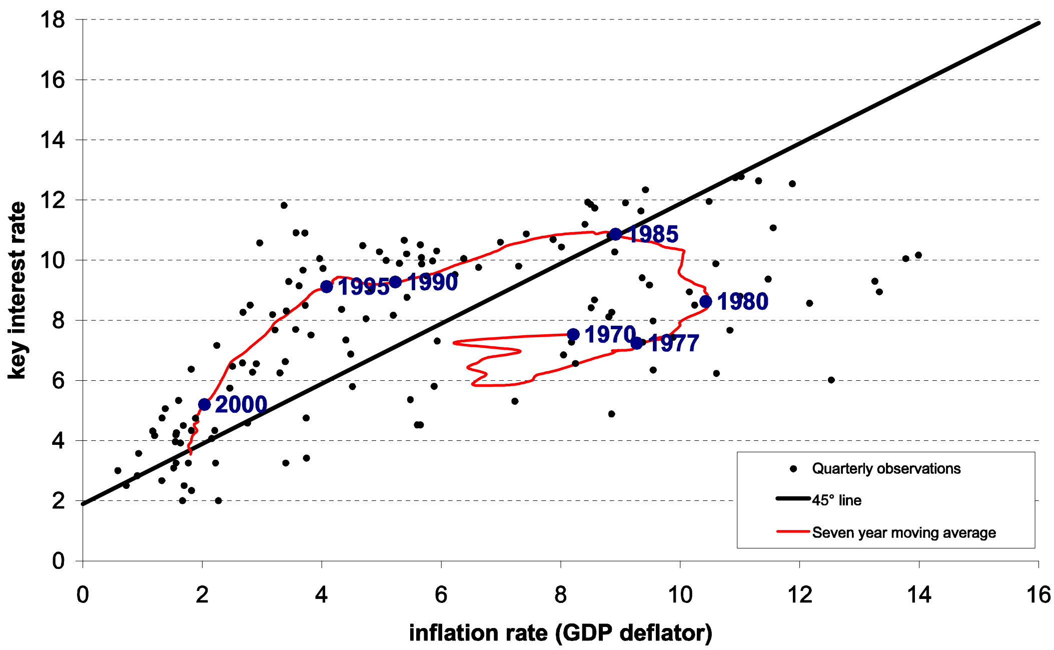

Figure 1: Shifts in the inflation target (euro area). The intercept is the mean of the real interest rate in the sample 1970Q2-2003Q4. As the sample begins in 1970Q2, the moving average will only start to contain seven years of data from 1977Q2. Therefore, the average is a slightly more volatile in the beginning of the sample.

The same figures also reveal to what extent the perceived inflation target differed from

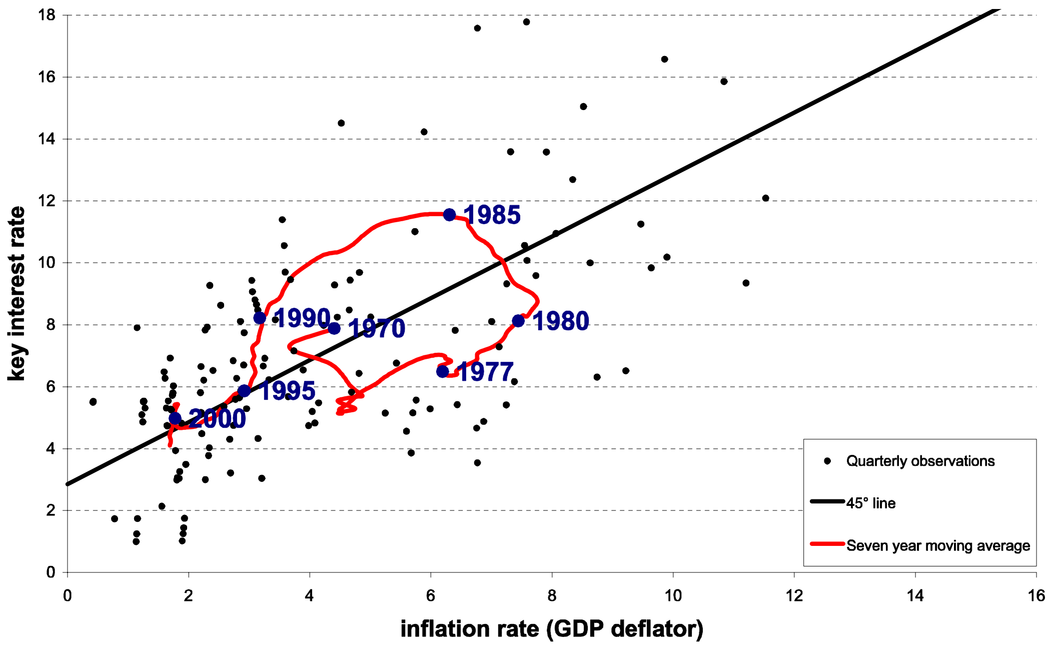

Figure 2: Shifts in the inflation target in the United States. See figure 1 for notes.

the central bank's inflation target at a certain point in time. Suppose we start from a point on the 45 degree line, e.g. a high inflation rate and a high key interest rate in the early 1980s. Now consider a central bank that wants to disinflate, i.e. the central bank reduces its target π_t^T. If the shift in π_t^T immediately feeds through into π_t^P, we would observe a contemporaneous decrease in the key interest rate. Graphically, this would correspond to a downward shift *along* the 45 degree line. As this is neither the case for the United States nor for the euro area in most of the sample, this shows that changes in the central bank's inflation target are usually only slowly reflected in the perceived inflation target. The only time this observation seems not to hold is for the period between 1994 and today in the United States. It suggests that during the last decade, the Federal Reserve was able to disinflate in a credible way by about 2 percentage points[6]. Note that, as Laubach and Williams (2003) point out, shifts in the natural real rate of interest could mislead our judgement of the stance of monetary policy if we would assume that the natural rate remains constant. Time variation in the natural rate implies that the intercepts in Figures 1 and 2 are also time-varying. Still it is hard to believe that the natural rate of interest was persistently lower in the seventies than in the eighties and nineties, which lets us conclude

[6]This seems to be confirmed by narrative evidence. Goodfriend (2002, p. 6) writes: "... in February 1994, the Fed started to announce its current intended federal funds rate target immediately after each FOMC meeting. This new practice made Fed policy more visible than ever. Every increase in the federal funds rate since then has attracted considerable attention."

that the interest rate rule indeed contains information about the timing and magnitude of shifts in the central bank's inflation target.

Equation (7) decomposes the log of real output y_t^r into potential output y_t^P and the output gap z_t. Equation (8) is an aggregate demand equation, relating the output gap z_t to its own lags and a term $\left(i_{t-1} - \pi_{t-1}^P - r_{t-1}^*\right)$ which captures monetary policy transmission. Following Harvey (1985), Stock and Watson (1998) and Laubach and Williams (2003), equations (9)-(10) model potential output as a random walk with drift, where the drift term λ_t varies over time according to a random walk process. The time-variation in λ_t allows for the possibility of permanent changes in the trend growth of real output, e.g. the productivity slowdown of the early 1970s.[7]

Laubach and Williams (2003) argue that the natural real rate of interest varies over time due to shifts in the trend growth of output and other factors such as households' rate of time preference. Therefore, equation (11) relates the real short-term interest rate r_t^* to the trend growth in potential output λ_t and a component τ_t that captures other determinants like time preferences. τ_t is assumed to be an AR process that, depending on the value for θ, can be either stationary or non-stationary.

Because we want to measure inflation persistence as the sum of the coefficients on the lagged inflation terms, the non-expectational autoregressive model presented above suits our purpose very well. In the case the economy is characterised by forward looking rational expectations, it can be considered as its reduced form representation. Rudebusch (2005), however, shows that in that case the reduced form representation of a simple forward looking monetary policy model would be subject to the Lucas critique. In this context Lansing and Trehan (2003) analytically show that the reduced form parameters depend on the policy parameters ρ_1 and ρ_2. As we model the economy in a reduced form around a time varying steady state inflation rate, this is not relevant for our extension. The policy parameters ρ_1 and ρ_2 remain constant and therefore the reduced form parameters are not affected by policy changes.

[7] Note that the random walk in equation (10) implies that y_t^P, and therefore also y_t, is an I(2) process. This seems inconsistent with the empirical evidence from Dickey-Fuller (DF) unit root tests that real output is I(1). Stock and Watson (1998) argue, though, that when the variance of η_{4t} is small relative to the variance of η_{3t}, Δy_t^P has a moving average (MA) root close to unity. Schwert (1989) and Pantula (1991) show that the size of the standard DF unit root test is severely upward biased in the presence of a large MA root. In this case, the standard DF unit root test is inappropriate to pick up a possible I(2) component in real output.

3 Estimation methodology[8]

3.1 State space representation

The structural time series models outlined in section 2 both include a number of unobserved components ($\pi_t^P, \pi_t^T, ...$). In order to estimate these models, it is necessary to write them into state space form[9]. In a state space model, the development over time of the system under study is determined by an unobserved series of vectors $\alpha_1, \ldots, \alpha_n$, which are associated with a series of observed vectors $y_1, \ldots, y_n$. A general linear Gaussian state space model can be written in the following form:

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H), \tag{13}$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t, \qquad \eta_t \sim N(0, Q), \qquad t = 1, \ldots, n, \tag{14}$$

where y_t is a $p \times 1$ vector of observed endogenous variables, modelled in the observation equation (13), x_t is a $k \times 1$ vector of observed exogenous variables and α_t is a $m \times 1$ vector of unobserved states, modelled in the state equation (14). The disturbances ε_t and η_t are assumed to be independent sequences of independent normal vectors. The matrices Z, A, T, R, H, and Q are parameter matrices.[10]

3.2 Kalman filter and smoother

Assuming that Z, A, T, R, H, and Q are known, the purpose of state space analysis is to infer the relevant properties of the α_t's from the observations $y_1, \ldots, y_n$ and $x_1, \ldots, x_n$. This can be done through the subsequent use of two recursions, i.e. the Kalman filter and the Kalman smoother. The objective of filtering is to obtain the distribution of α_t, for $t = 1, \ldots, n$, conditional on Y_t and X_t, where $Y_t = \{y_1, \ldots, y_t\}$ and $X_t = \{x_1, \ldots, x_t\}$. In a linear Gaussian state space model, the distribution of α_t is entirely determined by the filtered state vector $a_t = E(\alpha_t \mid Y_t, X_t)$ and the filtered state variance matrix $P_t = Var(\alpha_t \mid Y_t, X_t)$. The (contemporaneous) Kalman filter algorithm (see e.g. Hamilton, 1994, or Durbin and Koopman, 2001) estimates a_t and P_t by updating, at time t, a_{t-1} and P_{t-1} using the new information contained in y_t and x_t. The Kalman filter recursion can be initialised by the assumption that $\alpha_1 \sim N(a_1, P_1)$. In practice, a_1 and P_1 are generally not known though. Therefore, we assume that the distribution of the initial state vector α_1 is

$$\alpha_1 = V\Gamma + R_0\eta_0, \qquad \eta_0 \sim N(0, Q_0), \qquad \Gamma \sim N(0, \kappa I_r), \tag{15}$$

[8] The methodology outlined in this section was implemented using a set of GAUSS procedures. The code of these procedures is available from the authors on request.

[9] See e.g. Durbin and Koopman (2001) for an extensive overview of state space methods.

[10] The exact elements of the vectors y_t, x_t and α_t and the matrices Z, A, T, R, H, and Q for both the univariate and the multivariate model are specified in Appendix B.

where the $m \times r$ matrix V and the $m \times (m-r)$ matrix R_0 select the r elements of the state vector that are non-stationary and the $m-r$ elements that are stationary respectively. They are composed of columns of the identity matrix I_m and are defined so that, when taken together, their columns constitute all the columns of I_m and $V'R_0 = 0$. The unconditional variance matrix Q_0 of the stationary elements of the state vector is positive definite and can be computed from the model parameters. The $r \times 1$ vector Γ is a vector of unknown random quantities which, as we let $\kappa \to \infty$, is referred to as the diffuse vector. This leads to

$$\alpha_1 \sim N(0, P_1), \qquad P_1 = \kappa P_\infty + P_*, \tag{16}$$

where $P_\infty = VV'$ and $P_* = R_0 Q_0 R_0'$. The Kalman filter is modified to account for this diffuse initialisation implied by letting $\kappa \to \infty$ by using the exact initial Kalman filter introduced by Ansley and Kohn (1985) and further developed by Koopman (1997) and Koopman and Durbin (2003).

Subsequently, the Kalman smoother algorithm is used to estimate the distribution of α_t, for $t = 1, \ldots, n$, conditional on Y_n and X_n, where $Y_n = \{y_1, \ldots, y_n\}$ and $X_n = \{x_1, \ldots, x_n\}$. Thus, the smoothed state vector $\widehat{a}_t = E(\alpha_t \mid Y_n, X_n)$ and the smoothed state variance matrix $\widehat{P}_t = Var(\alpha_t \mid Y_n, X_n)$ are estimated using all the observations for $t = 1, \ldots, n$. In order to account for the diffuse initialisation of α_1, we use the exact initial state smoothing algorithm suggested by Koopman and Durbin (2003).

Given the complexity of the multivariate model, we do not use the entire observational vector y_t in the filtering and smoothing algorithm. Following Koopman and Durbin (2000), the elements of y_t are introduced into the filtering and smoothing algorithms one at a time, i.e. the multivariate analysis is converted into a univariate analysis. As the data can then be analysed in univariate form, this approach offers significant computational gains, particularly for the treatment of initialisation by diffuse priors.

3.3 Bayesian analysis

The filtering and smoothing algorithms both require that Z, A, T, R, H, and Q are known. In practice, these matrices generally depend on elements of an unknown parameter vector ψ. One possible approach is to derive, from the exact Kalman filter, the diffuse loglikelihood function for the model under study (de Jong 1991; Koopman and Durbin 2000; Durbin and Koopman 2001) and replace the unknown parameter vector ψ by its maximum likelihood estimate. This is not the approach pursued in this paper. First, given the fairly large number of unknown parameters, especially in the multivariate model, the numerical optimisation of the sample loglikelihood function is quite tedious. Second, most of the unknown parameters in ψ have been estimated in the past for different countries and samples. Therefore, we analyse the state space models from a Bayesian point of view, i.e. we treat ψ as a random parameter vector with a known prior density $p(\psi)$ and estimate the posterior densities $p(\psi \mid y, x)$ for the parameter vector ψ and $p(\widehat{\alpha}_t \mid y, x)$ for the smoothed state vector $\widehat{\alpha}_t$, where y and x denote the stacked vectors $(y_1', \ldots, y_n')'$ and $(x_1', \ldots, x_n')'$ respectively, by combining information contained in $p(\psi)$ and the sample data. Essentially, this boils down to calculating the posterior mean $\overline{g}$:

$$\overline{g} = E[g(\psi) \mid y, x] = \int g(\psi) p(\psi \mid y, x) d\psi \tag{17}$$

where g is a function which expresses the moments of the posterior densities $p(\psi \mid y, x)$ and $p(\widehat{\alpha}_t \mid y, x)$ in terms of the parameter vector ψ.

In principle, the integral in equation (17) can be evaluated numerically by drawing a sample of n random draws of ψ, denoted $\psi^{(i)}$ with $i = 1, \ldots, n$, from $p(\psi \mid y, x)$ and then estimating $\overline{g}$ by the sample mean of $g(\psi)$. As $p(\psi \mid y, x)$ is not a density with known analytical properties, such a direct sampling method is not feasable, though. Therefore, we switch to importance sampling. The idea is to use an importance density $g(\psi \mid y, x)$ as a proxy for $p(\psi \mid y, x)$, where $g(\psi \mid y, x)$ should be chosen as a distribution that can be simulated directly and is as close to $p(\psi \mid y, x)$ as possible. By Bayes' theorem and after some manipulations, equation (17) can be rewritten as

$$\overline{g} = \frac{\int g(\psi) z^g(\psi, y, x) g(\psi \mid y, x) d\psi}{\int z^g(\psi, y, x) g(\psi \mid y, x) d\psi} \tag{18}$$

with

$$z^g(\psi, y, x) = \frac{p(\psi) p(y \mid \psi)}{g(\psi \mid y, x)} \tag{19}$$

Using a sample of n random draws $\psi^{(i)}$ from $g(\psi \mid y, x)$, an estimate $\overline{g}_n$ of $\overline{g}$ can then be obtained as

$$\overline{g}_n = \frac{\sum_{i=1}^{n} g\left(\psi^{(i)}\right) z^g\left(\psi^{(i)}, y, x\right)}{\sum_{i=1}^{n} z^g\left(\psi^{(i)}, y, x\right)} = \sum_{i=1}^{n} w_i g\left(\psi^{(i)}\right) \tag{20}$$

with w_i

$$w_i = z^g\left(\psi^{(i)}, y, x\right) / \sum_{i=1}^{n} z^g\left(\psi^{(i)}, y, x\right) \tag{21}$$

the weighting function reflecting the importance of the sampled value $\psi^{(i)}$ relative to other sampled values.

Geweke (1989) shows that if $g(\psi \mid y, x)$ is proportional to $p(\psi \mid y, x)$, and under a number of weak regularity conditions, $\overline{g}_n$ will be a consistent estimate of $\overline{g}$ for $n \to \infty$.

3.4 Computational aspects of importance sampling

As a first step importance density $g(\psi \mid y, x)$, we take a large sample normal approximation to $p(\psi \mid y, x)$, i.e.

$$g(\psi \mid y, x) = N\left(\widehat{\psi}, \widehat{\Omega}\right) \tag{22}$$

where $\widehat{\psi}$ is the mode of $p(\psi \mid y, x)$ obtained from maximising

$$\log p(\psi \mid y, x) = \log p(y \mid \psi) + \log p(\psi) - \log p(y) \tag{23}$$

with respect to $\widehat{\psi}$ and where $\widehat{\Omega}$ denotes the covariance matrix of $\widehat{\psi}$. Note that $p(y \mid \psi)$ is given by the likelihood function derived from the exact Kalman filter and we do not need to calculate $p(y)$ as it does not depend on ψ.

In drawing from $g(\psi \mid y, x)$, efficiency was improved by the use of antithetic variables, i.e. for each $\psi^{(i)}$ we take another value $\widetilde{\psi}^{(i)} = 2\widehat{\psi} - \psi^{(i)}$, which is equiprobable with $\psi^{(i)}$. This results in a simulation sample that is balanced for location (Durbin and Koopman 2001).

As any numerical integration method delivers only an approximation to the integrals in equation (18), we monitor the quality of the approximation by estimating the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ (Bauwens, Lubrano and Richard 1999, chap. 3, eq. 3.34). This error bound represents a 95% confidence interval for the percentage deviation of $\overline{g}_n$ from $\overline{g}$. It should not exceed 10%. In practice this can be achieved by increasing n, except when the coefficient of variation of the weights w_i is unstable as n increases. An unstable coefficient of variation of w_i signals poor quality of the importance density. This was exactly the problem encountered in the empirical analysis.

Note that the normal approximation in equation (22) selects $g(\psi \mid y, x)$ in order to match the location and covariance structure of $p(\psi \mid y, x)$ as good as possible. One problem is that the normality assumption might imply that $g(\psi \mid y, x)$ does not match the tail behaviour of $p(\psi \mid y, x)$. If $p(\psi \mid y, x)$ has thicker tails than $g(\psi \mid y, x)$, a draw $\psi^{(i)}$ from the tails of $g(\psi \mid y, x)$ can imply an explosion of $z^g\left(\psi^{(i)}, y, x\right)$. This is due to a very small value for $g(\psi \mid y, x)$ being associated with a relatively large vaule for $p(\psi)\, p(y \mid \psi)$, as the latter is proportional to $p(\psi \mid y, x)$. Importance sampling is inaccurate in this case as this would lead to a weight w_i close to one, i.e. $\overline{g}_n$ is determined by a single draw $\psi^{(i)}$. This is signaled by instability of the weights and a probabilistic error bound that does not decrease in n.

In order to help prevent explosion of the weights, we change the construction of the importance density in two respects (Bauwens *et al.* 1999, chap. 3). First, we inflate the approximate covariance matrix $\widehat{\Omega}$ a little. This reduces the probability that $p(\psi \mid y, x)$ has thicker tails than $g(\psi \mid y, x)$. Second, we use a sequential updating algorithm for the importance density. This algorithm starts from the importance density defined by (22), with inflation of $\widehat{\Omega}$, estimates posterior moments for $p(\psi \mid y, x)$ and then defines a new importance density from these estimated moments. This improves the estimates for $\widehat{\psi}$ and $\widehat{\Omega}$. We continue updating the importance density until the weights stabilise. The number of importance samples n was chosen to make sure that the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ does not exceed 10%.

4 Estimation results

We use quarterly data for the euro area and the United States from 1970Q1 to 2003Q4. The inflation series π_t is the annualised first difference of the log of the seasonally adjusted GDP deflator. For the interest rate, i_t, we use the annualised central bank key interest rate. This interest rate should be most appropriate to infer changes in the central bank's behaviour. Real output, y_t^r, is measured as the log of seasonally adjusted GDP at constant prices. See Appendix C for a more detailed data description. Given that we work with quarterly data, the number of AR terms in equation (3) is set equal to 4, i.e. $q = 4$.

4.1 Prior information

Prior information, presented in Table 1, about the unknown parameter vector ψ is included in the analysis through the prior density $p(\psi)$. Where possible prior information is taken from the literature. We use the same priors for the euro area and the United States. If no adequate information is available, we leave considerable uncertainty around the chosen priors. The prior distribution is assumed to be Gaussian for all elements in ψ, except for the variance parameters which are assumed to be gamma distributed.

The priors for the AR coefficients φ_i are chosen from studies allowing for a break in the mean of the inflation rate. Levin and Piger (2004) for instance find a value of 0.36 for the sum of the AR coefficients of the United States GDP deflator. Gadzinski and Orlandi (2004) find a somewhat higher figure of 0.6 for the euro area. Finally we choose a prior for the sum of the AR coefficients of 0.4 for both the United States and the euro area. Our prior for δ is 0.15, which is the average of the parameter values determining signal extraction in Erceg and Levin (2003) and Kozicki and Tinsley (2003), or sticky information in Mankiw and Reis (2002). The prior for the variance of the inflation target shocks $\sigma_{\eta_1}^2$ corresponds, on average, to what Kozicki and Tinsley (2003) and Smets and Wouters (2005) find. The priors for the parameters that are only present in the multivariate model come from previous studies estimating variants of the model of Rudebusch and Svensson (1999). For the impact of the lagged output gap on inflation we choose a value of 0.2. The AR coefficients of the output gap equation are chosen in order to generate a hump-shaped response of output in reaction to a shock. This feature is often found in previous empirical studies (Gerlach and Smets 1999; Rudebusch and Svensson 1999; Laubach and Williams 2003; Rudebusch 2005). The parameter value for ρ_2 assumes considerable interest rate smoothing (Smets and Wouters, 2005). The parameter values for ρ_1 and ρ_2 are chosen so that the Taylor (1993) principle $\left(1+\frac{\rho_1}{1-\rho_2}=1.5>1\right)$ holds for deviations of π_t^P from π_t^T. The central bank reacts less vigorously $\left(\frac{\rho_1}{1-\rho_2}=0.5\right)$ in response to deviations of π_t from π_t^T. This is consistent with the view that an inflation-targeting central bank should only stabilise inflation in the medium run and pay less attention to short-term deviations.

Table 1: Prior information

	reference(s)	5 p.c.	Mean	95 p.c.
φ_1	-	0.04	0.20	0.36
φ_2	-	-0.06	0.10	0.26
φ_3	-	-0.11	0.05	0.21
φ_4	-	-0.11	0.05	0.21
$\sum_{i=1}^4 \varphi_i$	Gadzinski *et al.* (2004), Levin *et al.* (2004)	0.16	0.40	0.64
δ	Erceg *et al.* (2003), Kozicki *et al.* (2003) Mankiw *et al.* (2002)	-0.01	0.15	0.31
β_1	Gerlach *et al.* (1999), Rudebush (2005), Rudebush *et al.* (1999)	0.18	0.20	0.22
β_2		1.32	1.35	1.38
β_3		-0.50	-0.47	-0.44
β_4		-0.01	0.15	0.31
ρ_1	Taylor (1993)	0.02	0.05	0.08
ρ_2	Taylor (1993), Smets *et al.* (2005)	0.87	0.90	0.93
γ	Laubach *et al.* (2003)	3.67	4.00	4.33
θ		0.95	0.97	0.99
$\sigma^2_{\varepsilon_1}$	-	0.35	1.30	2.77
$\sigma^2_{\varepsilon_2}$	-	0.21	0.30	0.40
$\sigma^2_{\varepsilon_3}$	Laubach *et al.* (2003)	0.11	0.16	0.21
$\sigma^2_{\eta_1}$	Kozicki *et al.* (2003), Smets *et al.* (2005)	0.03	0.12	0.25
$\sigma^2_{\eta_2}$	-	$2.8e-5$	$1.0e-4$	$2.1e-4$
$\sigma^2_{\eta_3}$	Laubach *et al.* (2003)	0.26	0.37	0.49
$\sigma^2_{\eta_4}$		$4.5e-4$	$6.5e-4$	$8.8e-4$
$\sigma^2_{\eta_5}$		0.07	0.10	0.14

Note: All variances are expressed at annual rates except for $\sigma^2_{\varepsilon_3}$, $\sigma^2_{\eta_3}$ and $\sigma^2_{\eta_4}$ which are expressed as quarterly rates. The prior distribution is assumed to be Gaussian for all elements in ψ, except for the variance parameters which are assumed to be gamma distributed.

4.2 Posterior distributions

In this section we present estimates of the posterior mean $\overline{\psi}=E\left[\psi \mid y,x\right]$ of the parameter vector ψ and the posterior mean $\overline{\alpha_t}=E\left[\widehat{\alpha}_t \mid y,x\right]$ of the smoothed state vector $\widehat{\alpha}_t$. An estimate $\widetilde{\psi}$ of $\overline{\psi}$ is obtained by setting $g\left(\psi^{(i)}\right)=\psi^{(i)}$ in equation (20) and taking $\widetilde{\psi}=\overline{g}_n$. An estimate $\widetilde{\alpha}_t$ of $\overline{\alpha}_t$ is obtained by setting $g\left(\psi^{(i)}\right)=\widehat{\alpha}_t^{(i)}$ in equation (20) and taking

$\widetilde{\alpha}_t = \overline{g}_n$, where $\widehat{\alpha}_t^{(i)}$ is the smoothed state vector obtained from the Kalman smoother using the parameter vector $\psi^{(i)}$.

We also present the 5th and 95th percentiles of the posterior densities $p(\psi \mid y, x)$ and $p(\widehat{\alpha}_t \mid y, x)$. Let $F\left(\psi_j \mid y, x\right) = \Pr\left(\psi_j^{(i)} \leq \psi_j\right)$ with ψ_j denoting the j-th element in ψ. An estimate $\widetilde{F}\left(\psi_j \mid y, x\right)$ of $F\left(\psi_j \mid y, x\right)$ is obtained by setting $g\left(\psi^{(i)}\right) = I_j\left(\psi_j^{(i)}\right)$ in equation (20) and taking $\widetilde{F}\left(\psi_j \mid y, x\right) = \overline{g}_n$, where $I_j\left(\psi_j^{(i)}\right)$ is an indicator function which equals one if $\psi_j^{(i)} \leq \psi_j$ and zero otherwise. An estimate $\widetilde{\psi}_j^{5\%}$ of the 5th percentile of the posterior density $p(\psi \mid y, x)$ is chosen such that $\widetilde{F}\left(\psi_j^{5\%} \mid y, x\right) = 0.05$. An estimate $\widetilde{\alpha}_{j,t}^{5\%}$ of the 5th percentile of the jth element of the posterior density $p(\widehat{\alpha}_t \mid y, x)$ is obtained by setting $g\left(\psi^{(i)}\right) = \widehat{\alpha}_{j,t}^{(i)} - 1.645\sqrt{\widehat{P}_{j,t}^{(i)}}$ in equation (20) and taking $\widetilde{\alpha}_{j,t}^{5\%} = \overline{g}_n$, where $\widehat{\alpha}_{j,t}^{(i)}$ denotes the j-th element in $\widehat{\alpha}_t^{(i)}$ and $\widehat{P}_{j,t}^{(i)}$ is the (j, j)th element of the smoothed state variance matrix $\widehat{P}_t^{(i)}$ obtained using the parameter vector $\psi^{(i)}$. The 95th percentiles are constructed in a similar way.

4.2.1 Posterior distribution of the parameters

Tables 2 and 3 present the posterior mean and the 5th and 95th percentile of the posterior distribution of ψ for the euro area and the United States for both the univariate and multivariate model. Two important conclusions stand out. First, in the univariate model intrinsic inflation persistence, measured as $\sum_{i=1}^{q} \varphi_i$, amounts to 0.44 for the euro area and 0.80 for the United States. This is considerably lower than estimates from standard AR time series models. The multivariate intrinsic inflation persistence estimates amount to 0.45 and 0.75 for the euro area and the United States, and are in line with the results of the univariate specification. In the case of the United States, intrinsic inflation persistence is somewhat higher than in the euro area. Note that this result is consistent with Galí et al. (2001), who for the United States also find a relatively higher degree of backward-lookingness compared to the euro area. Second, expectations-based persistence, measured by $(1 - \delta)$, is at least as high or higher than intrinsic inflation persistence, i.e. higher than 0.74 for both economies across the different models. The persistence in the output gap, measured by the sum of β_2 and β_3, amounts to at least 0.9. This implies considerable extrinsic inflation persistence.

4.2.2 Posterior distribution of the states

Figures 3, 4, 5 and 6 show the dynamics of the inflation rate together with the central bank's inflation target and the perceived inflation target. These figures reveal considerable variation in the central bank's inflation target in both the euro area and the United States. The dynamics of the perceived inflation target show that inflation expectations adjust smoothly in response to shifts in the central bank's inflation target. The central bank's inflation target

Table 2: Posterior distribution univariate model (1971Q2:2003Q4)

	Euro area			United States		
	5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
φ_1	0.15	0.27	0.38	0.26	0.38	0.50
φ_2	0.01	0.11	0.22	0.07	0.19	0.31
φ_3	-0.18	-0.07	0.04	0.01	0.13	0.25
φ_4	0.02	0.13	0.23	-0.01	0.11	0.22
$\sum_{i=1}^{4} \varphi_i$	0.22	0.44	0.65	0.59	0.80	0.99
δ	0.16	0.26	0.37	0.10	0.24	0.37
$\sigma^2_{\varepsilon_1}$	1.34	1.66	2.06	1.15	1.42	1.76
$\sigma^2_{\eta_1}$	0.09	0.16	0.29	0.03	0.09	0.23

Note: The approximate covariance matrix $\widehat{\Omega}$ is inflated with a factor 1.2. For the US, the coefficient of variation of the weights stabilised after 3 updates of the importance function. For the Euro area updating was not necessary. With $n = 10000$, the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ is well below 10% for all coefficients.

Table 3: Posterior distribution multivariate model (1971Q2:2003Q4)

	Euro area			United States		
	5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
φ_1	0.15	0.25	0.36	0.20	0.31	0.41
φ_2	0.00	0.11	0.21	0.05	0.16	0.26
φ_3	-0.17	-0.07	0.04	0.03	0.14	0.24
φ_4	0.05	0.15	0.25	0.05	0.15	0.25
$\sum_{i=1}^{4} \varphi_i$	0.25	0.45	0.64	0.60	0.75	0.89
δ	0.14	0.24	0.35	0.06	0.20	0.34
β_1	0.18	0.20	0.22	0.18	0.20	0.22
β_2	1.34	1.37	1.40	1.34	1.36	1.39
β_3	-0.48	-0.45	-0.42	-0.48	-0.45	-0.42
β_4	0.10	0.16	0.22	0.09	0.13	0.17
ρ_1	0.02	0.04	0.06	0.02	0.05	0.07
ρ_2	0.87	0.90	0.92	0.86	0.89	0.91
γ	3.64	3.98	4.31	3.66	3.99	4.32
θ	0.95	0.97	0.98	0.95	0.97	0.98
$\sigma^2_{\varepsilon_1}$	1.27	1.57	1.94	0.98	1.21	1.49
$\sigma^2_{\varepsilon_2}$	0.25	0.30	0.36	0.63	0.72	0.82
$\sigma^2_{\varepsilon_3}$	0.08	0.11	0.15	0.11	0.15	0.20
$\sigma^2_{\eta_1}$	0.06	0.12	0.23	0.04	0.09	0.22
$\sigma^2_{\eta_2}$	$1.3e{-5}$	$5.2e{-5}$	$1.6e{-4}$	$1.3e{-5}$	$5.2e{-5}$	$1.6e{-4}$
$\sigma^2_{\eta_3}$	0.15	0.19	0.25	0.26	0.34	0.42
$\sigma^2_{\eta_4}$	$4.4e{-4}$	$6.2e{-4}$	$8.5e{-4}$	$4.3e{-4}$	$6.2e{-4}$	$8.6e{-4}$
$\sigma^2_{\eta_5}$	0.07	0.10	0.14	0.08	0.11	0.15

Note: The approximate covariance matrix $\widehat{\Omega}$ is inflated with a factor 1.2. The coefficient of variation of the weights stabilised after 1 update of the importance function for both the euro area and the United States. With $n = 10000$, the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ is well below 10% for all coefficients.

and the perceived inflation target identified in the univariate model are very similar to the ones identified in the multivariate model. This confirms that the permanent shifts in the perceived inflation target, which were identified in the univariate model using a statistical restriction, are indeed driven by shifts in the central bank's inflation target.

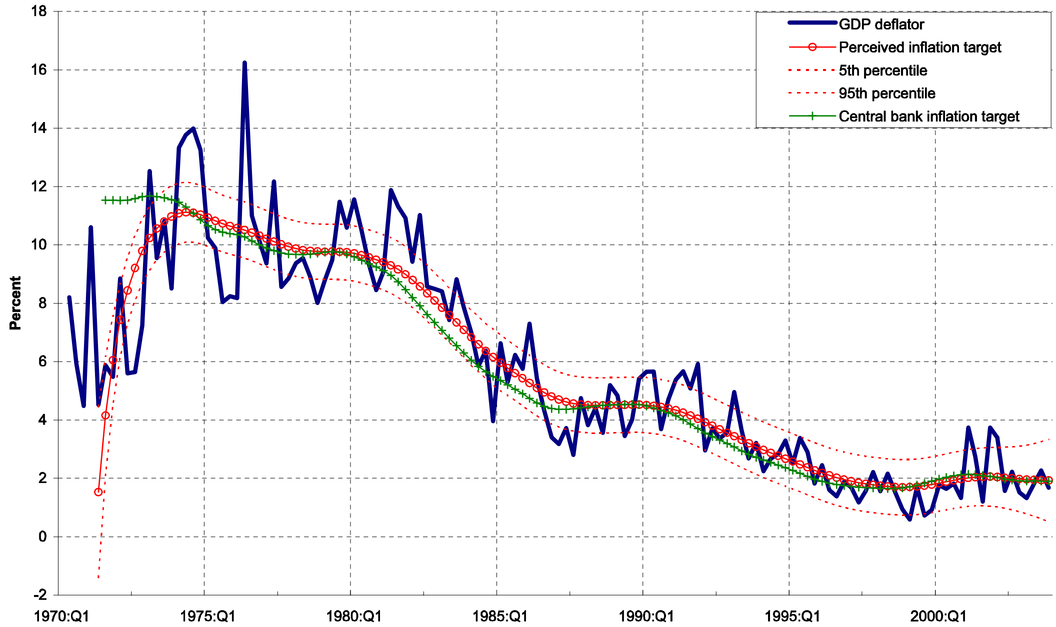

Figure 3: Smoothed univariate states for the euro area

The timing of the shifts in the central bank's inflation target seems to be in line with common knowledge about the historical conduct of monetary policy. A first disinflationary period is present in the early 1980s. In the United States, the univariately estimated inflation target decreased from 6% in the late 1970s to about 3% in the mid 1980s. This is matched by the disinflationary policy of Paul Volcker, who was appointed president of the Federal Reserve in 1979. A similar decrease, from about 10% to about 5%, is observed for the euro area. This decrease is more difficult to match with narrative evidence, as no unified monetary policy existed before 1999. Still, several future euro area member countries (e.g. Austria, Belgium, France, The Netherlands) were disinflating in the beginning of the eighties. For the euro area, a second disinflationary period is also present in the beginning of the nineties. Other future euro area member countries (e.g. Greece, Italy, Portugal, Spain) were then disinflating in order to comply with the Maastricht criteria. In the United States there seems to have been a somewhat less pronounced decrease in the central bank's inflation target over that period.

Finally, in an inflation targeting framework, where the short-term interest rate is the

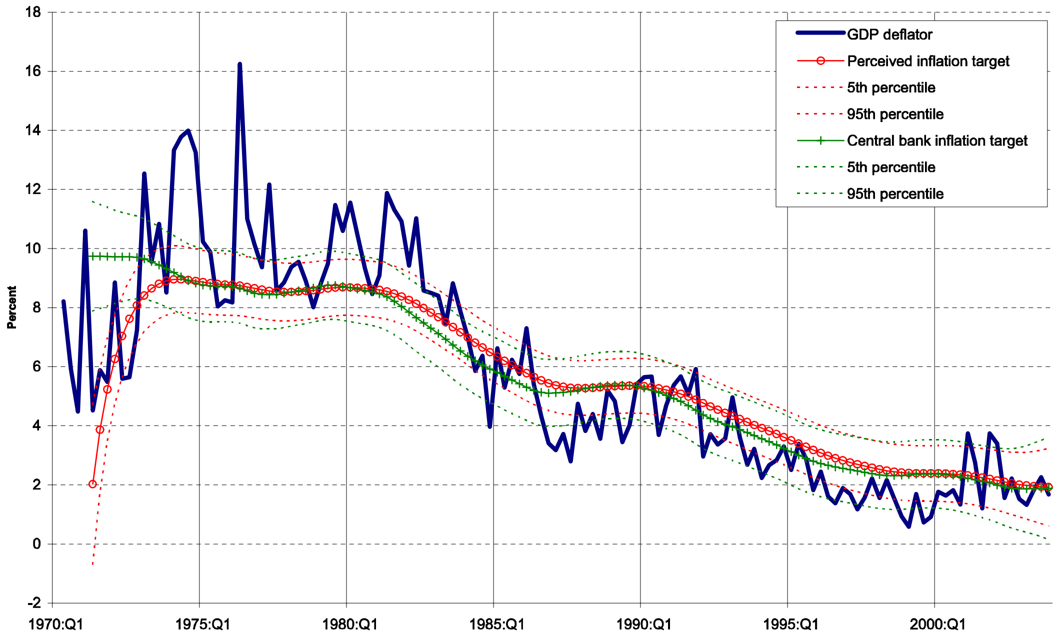

Figure 4: Smoothed multivariate states for the euro area

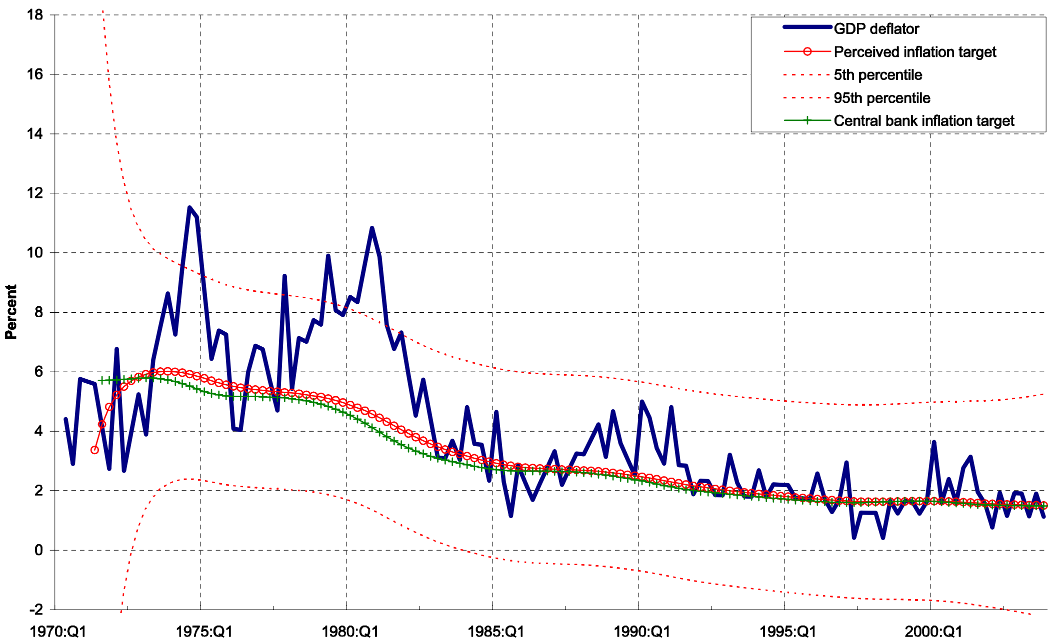

Figure 5: Smoothed univariate states for the United States

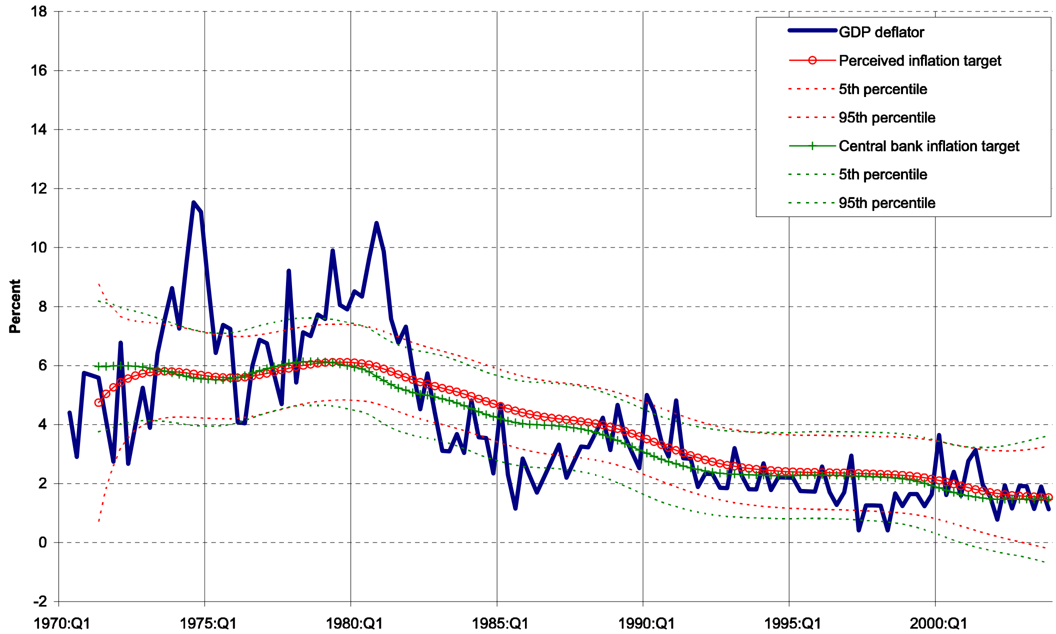

Figure 6: Smoothed multivariate states for the United States

primary policy instrument, the natural interest rate provides a metric for the stance of monetary policy. The natural rate of interest varies over time due to shifts in the trend growth of output and other factors such as households' rate of time preference. We took these variations explicitly into account in our model, so that when estimating shifts in the central banks' inflation target the results would not be misleading due to the shifts in the benchmark, namely the natural interest rate. Figures 7 and 8 show that especially variations in time preferences have driven the natural real interest rate over the last three decades in both the United States and the euro area. In addition, during the nineties a decrease in the trend growth rate has driven down the natural real interest rate in the euro area, whereas this does not seem to be the case for the United States.

4.2.3 Half life and impulse response analysis

An alternative way of analysing inflation persistence is to look at the half life and impulse response functions of different shocks to inflation. The former counts the number of periods for which the effect of a shock to inflation remains above half its initial impact. An important difference with the sum of estimated AR coefficients as a measure of persistence is that both the half life and impulse response analysis take all the roots of the AR equation into account while the sum of AR coefficients only measures the average speed of convergence. A second important difference with the point estimates of the AR coefficients is that different sources

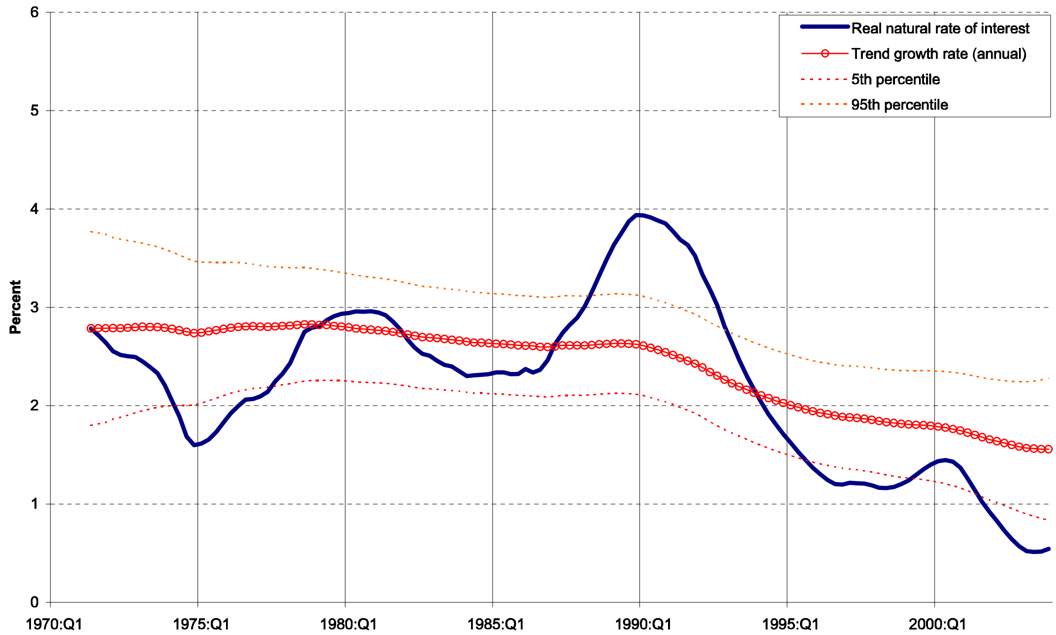

Figure 7: Smoothed multivariate states for the euro area

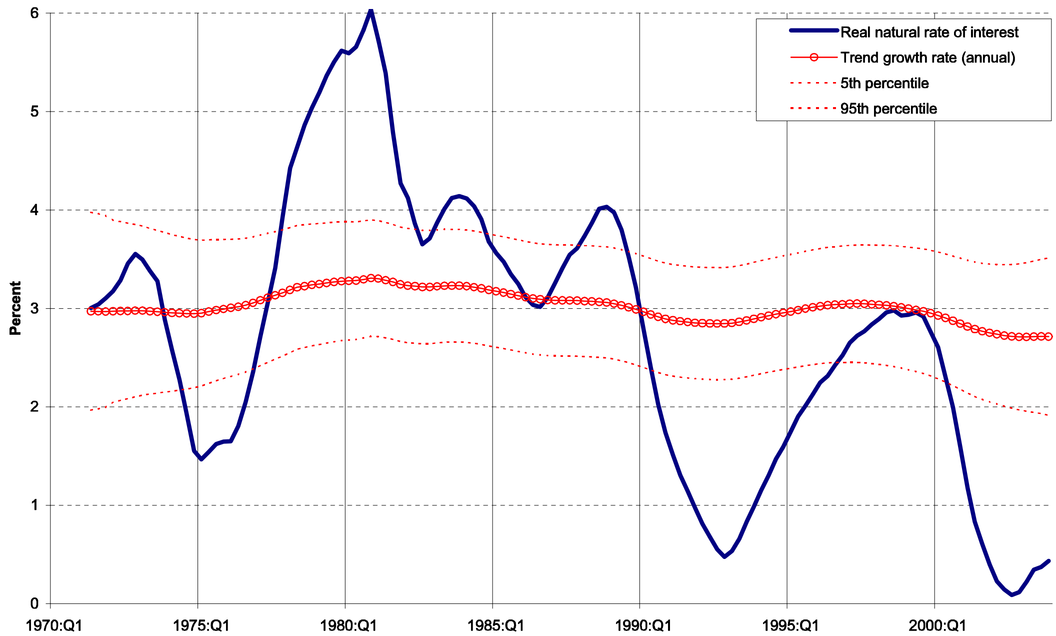

Figure 8: Smoothed multivariate states for the United States

Table 4: Half lives of inflation (quarters)

	Euro area	United States
Temporary inflation shock	1	1
Perceived inflation target shock	9	16
Output gap shock	13	19
Central bank target shock	∞	∞

of persistence in response to a shock can reinforce each other. The inflation dynamics in response to a shock will thus not only depend on the persistence in the variable that was shocked, but will also depend on the interaction with other variables. Therefore, also the persistence in the latter will play a role.

Table 4 reports half lives for four shocks to inflation considered in the multivariate model. The half life of a temporary shock (ε_{1t}) is only one quarter. For a shock to the perceived inflation target (η_{2t}), the half life is 9 and 16 quarters in the euro area and the United States respectively. For a shock to the output gap (ε_{3t}), the half life even amounts to 13 quarters in the euro area and to 19 quarters the United States. Finally, a shock to the inflation target (η_{1t}) is permanent and therefore its half life is equal to infinity. The latter result is obtained by construction because we assume a random walk process for the shifts in the central bank's inflation target. Still, it shows that ignoring a component with an infinite half life must create a considerable bias in the estimates of the other kinds of persistence.

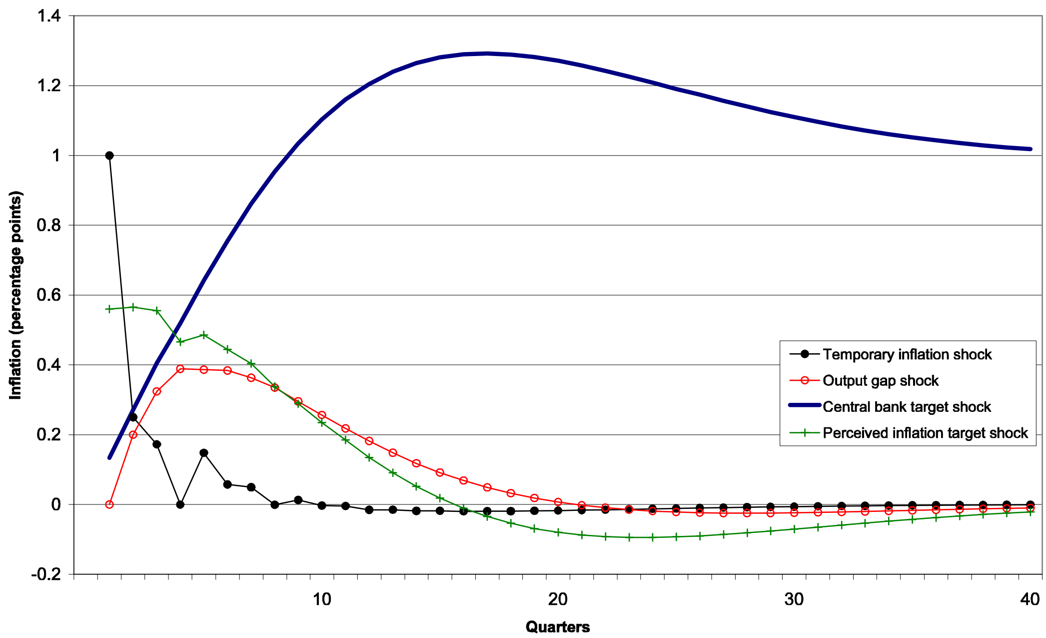

Figure 9: Impulse responses for the euro area

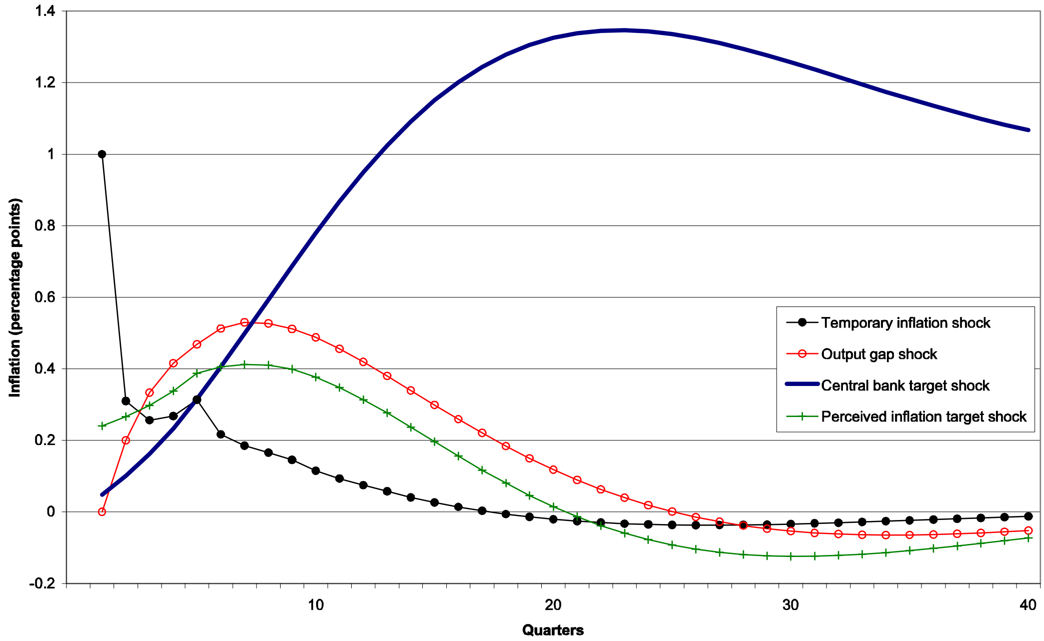

Figure 10: Impulse responses for the United States

A similar lesson can be learned from the impulse response functions in response to a unit shock in Figures 9-10. A shift in the central bank's inflation target (η_{1t}) has a permanent impact on inflation. Still, it takes various periods before the inflation rate stabilises at the new target, both in the euro area and in the United States. This is to a big extent due to considerable expectations-based persistence that creates persistent deviations of the perceived inflation target from the central bank's inflation target. In case of a shock to the output gap (ε_{3t}) or the perceived inflation target (η_{2t}), the response of inflation seems to be characterised by a similar degree of persistence. In case of a temporary shock to inflation (ε_{1t}), the convergence to the target goes much faster. According to the sum of the AR coefficients, intrinsic and expectations-based persistence are not statistically significantly different. Still, due to the persistence in the reaction of the central bank and the output gap, the number of quarters that inflation is affected by a difference between the perceived and the central bank's inflation target can be considerably higher.

5 Conclusions

This paper aims at measuring different sorts of inflation persistence, i.e. the sluggish response of inflation in response to different macroeconomic shocks. In the literature post war inflation persistence measures are often found to be close to that of a random walk. The main point stressed in this paper is that these unconditional estimates are hard to interpret as the data generating process of inflation can be decomposed in a number of distinct components, each of them exhibiting its own degree of persistence. First, shifts in the central bank's inflation target can induce permanent shifts in the mean inflation rate. Second, imperfect or sticky information implies that private agents have to learn about the true central bank's inflation target. As such, the inflation target perceived by private agents can persistently differ from the true central bank's inflation target. Third, persistence in the various determinants of inflation also introduces persistence in the observed inflation rate. As each of these components typically shows relatively high inertia, ignoring one of them might create an upward bias in estimates of intrinsic inflation persistence, which measures the sluggish response of inflation to various macroeconomic shocks related to rigidities in the wage- and price-setting mechanism.

Therefore, we measure inflation persistence in a structural time series model which explicitly models the various components driving inflation. We pursue both a univariate and a multivariate approach. Extracting information from the central bank's key interest rate we find confirmation that shifts in the central bank's inflation target induce a non-stationary component in the inflation rate. In addition, slow adjustment of inflation expectations in response to changes in the central bank's inflation target and persistence of shocks hitting inflation are important factors determining the observed inflation persistence. These components explain a large fraction of the high degree of persistence observed in the post-WW II inflation rate. Taking these components into account, intrinsic inflation persistence is found to be lower than the persistence of a random walk, i.e. the half life of a cost-push shock is only one quarter in both the euro area and the United States.

The implications for monetary policy are as follows. Our evidence indicates that in a stable inflation regime, where the central bank's inflation target does not change and where the public perception about this inflation target is well anchored, inflation persistence is relatively low. The results also imply that in the case monetary policy would again give rise to unstable inflation, it would afterwards be very hard to disinflate due to the slow adjustment of inflation expectations in response to changes in the inflation target. In the case of natural rate misperceptions (Orphanides and Williams 2004) this might however not be straightforward to avoid.

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Appendix A: Deriving an empirical specification for the perceived inflation target

Equation (4) can be derived using a variant of the sticky information model of Mankiw and Reis (2002) or the signal extraction problem of Erceg and Levin (2003) and Andolfatto *et al.* (2002). The difference between the two models is the way information about the central bank's inflation target π_t^T arrives to the firms. In the sticky-information model, exact information about π_t^T is available but not all firms update their information about π_t^T every period due to for instance information gathering costs. Therefore, aggregate prices do not respond immediately to changes in π_t^T. In the model of Erceg and Levin (2003) and Andolfatto *et al.* (2002) all firms update their information about π_t^T every period, but exact information about π_t^T is not available. This leads to a signal extraction problem. Aggregate prices will only respond to changes in π_t^T once firms have learned about the new central bank target. If learning is slow, aggregate prices will not respond immediately to changes in π_t^T.

A.1 A sticky-information model

As in Mankiw and Reis (2002) we assume that firms reset their prices every period, but infrequently gather information about the central bank inflation target π_t^T, which is readily available in every period. Following Mankiw and Reis (2002) the log of a firm's optimal price p_t^*, which can be derived from a firm's profit maximisation problem, is given by $(A.1)$:

$$p_t^* = p_t + \alpha z_t \tag{A.1}$$

$$p_t^* = p_t \qquad \text{assuming } z_t = 0 \ \ \forall t \tag{A.2}$$

$$p_t^* = p_{t-1}^P + \pi_t^T \tag{A.3}$$

where p_t is the log of the aggregate price level, z_t is the output gap and α is a positive coefficient. This equation tells us that a firm's desired relative price rises in booms and falls in recessions. If we abstract from price adjustment to output gap fluctuations we can set the output gap equal to zero, so that the firms' optimal price p_t^* will be equal to the aggregate price level p_t. The current aggregate price level p_t is identical to last period's aggregate price level p_{t-1}^P, which is consistent with last period's perceived inflation target, and the current period's central bank inflation target π_t^T. In the absence of short-run macroeconomic fluctuations and given that in the long run inflation is a monetary phenomenon, current period's inflation must be equal to the central bank's inflation target.

In this model, however, only a fraction λ of the firms updates its information about π_t^T to calculate a new optimal price. The probability of updating information is the same for each firm, i.e. independent of the timing of the last update. The other firms continue to set their prices based on old information about π_t^T.

A firm that last updated its beliefs about the inflation target j periods ago sets its price x_t^j:

$$x_t^j = E_{t-j} p_t^* \tag{A.4}$$

$$= p_{t-1-j}^P + (j+1)\pi_{t-j}^T \tag{A.5}$$

The aggregate price level is the average of the prices of all firms, given by:

$$p_t^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_t^j \tag{A.6}$$

The perceived inflation target can be calculated from (A.6) as:

$$\pi_t^P = p_t^P - p_{t-1}^P \tag{A.7}$$

$$= \lambda \pi_t^T + (\lambda - 1)\, p_{t-1}^P + \lambda \sum_{j=0}^{\infty} (1-\lambda)^{j+1} (p_{t-2-j}^P + (j+2)\pi_{t-j-1}^T) \tag{A.8}$$

Substituting out p_{t-j}^P using (A.6) and rearranging yields:

$$\pi_t^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \pi_{t-j}^T, \tag{A.9}$$

which is equivalent to:

$$\pi_t^P = (1-\lambda)\pi_{t-1}^P + \lambda \pi_t^T \tag{A.10}$$

A.2 A signal extraction problem

Both Erceg and Levin (2003) and Andolfatto *et al.* (2002) assume that monetary authorities set nominal interest rates in line with their inflation target, π_t^T, using an interest rate rule. Observing the central bank's interest rate, private agents can therefore infer on the central bank's inflation target from their knowledge of the central bank's interest rate rule. An information problem arrises from the assumption that the interest rate set by the central bank can shift due to both transitory and permanent monetary policy actions. Transitory policy actions can be interpreted as (i) deviations from the interest rate rule in response to various transitory shocks hitting inflation and/or (ii) imperfect control of monetary authorities over the interest rate. Permanent policy actions are shifts in the central bank's inflation target π_t^T. Consequently, private agents must solve a signal-extraction problem to disentangle transitory and permanent policy actions using shifts in the nominal interest rate. This can be done using the Kalman filter. This optimal filtering solution gives rise to a learning rule that resembles adaptive expectations processes.

In particular, we assume that the central bank's inflation target evolves according to equation (1) while monetary policy is described by the interest rate rule in equation (6).

More information on this interest rate rule can be found in subsection 2.3. Permanent monetary policy actions stem from η_{1t} in equation (1). Transitory policy actions stem from ε_{2t} in equation (6). An optimal estimate $E_t\pi_t^T$ of π_t^T based on the information contained in i_t can be obtained recursively using the Kalman filter as:

$$E_t\pi_t^T = E_{t-1}\pi_{t-1}^T - k_g\nu_t \tag{A.11}$$

where ν_t captures the new information contained in i_t, i.e. $\nu_t = i_t - E_{t-1}i_t = \rho_1\left(E_{t-1}\pi_t^T - \pi_t^T\right) + \varepsilon_{2t}$ and where for simplicity r_t^* is assumed to be a constant $\overline{r}$. k_g is the Kalman gain parameter that measures the speed at which private agents update their beliefs about the monetary policy target π_t^T in response to the new information contained in ν_t. It is given by

$$k_g = \frac{1}{2}\frac{\sigma_{\eta_1}^2}{\sigma_{\varepsilon_2}^2}\left(-\rho_1 + \sqrt{\rho_1^2 + 4\frac{\sigma_{\varepsilon_2}^2}{\sigma_{\eta_1}^2}}\right) \tag{A.12}$$

Equation (A.12) shows that k_g is increasing in the signal-to-noise ratio $\sigma_{\eta_1}^2/\sigma_{\varepsilon_2}^2$ and decreasing in the reaction ρ_1 of the central bank to deviations of inflation from its target.

As from equation (1) we have that $E_{t-1}\pi_t^T = E_{t-1}\pi_{t-1}^T$ and setting $\pi_t^P = E_t\pi_t^T$ using equation (2), equation (A.11) can be rewritten as:

$$\pi_t^P = (1-\rho_1 k_g)\,\pi_{t-1}^P + \rho_1 k_g\pi_t^T - k_g\varepsilon_{2t} \tag{A.13}$$

Appendix B: State Space representations

B.1 Univariate model

$$y_t = \begin{bmatrix} \pi_t \end{bmatrix}; \; \alpha_t = \begin{bmatrix} \pi_t^P & \pi_{t-1}^P \end{bmatrix}'; \; x_t = \begin{bmatrix} \pi_{t-1} & \dots & \pi_{t-q} \end{bmatrix}';$$

$$Z = \begin{bmatrix} (1-\sum_{i=1}^{q}\varphi_i) & 0 \end{bmatrix}; \;\; A = \begin{bmatrix} \varphi_1 & \dots & \varphi_q \end{bmatrix}; \;\; T = \begin{bmatrix} 2-\delta & \delta-1 \\ 1 & 0 \end{bmatrix};$$

$$R = \begin{bmatrix} \delta & 0 \end{bmatrix}'; \;\; \varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \end{bmatrix}; \;\; \eta_t = \begin{bmatrix} \eta_{1t} \end{bmatrix}; \;\; H = \begin{bmatrix} \sigma_{\varepsilon_1}^2 \end{bmatrix}; \;\; Q = \begin{bmatrix} \sigma_{\eta_1}^2 \end{bmatrix}$$

B.2 Multivariate model

$$y_t = \begin{bmatrix} \pi_t & i_t & y_t^r \end{bmatrix}'; \; x_t = \begin{bmatrix} \pi_{t-1} & \pi_{t-2} & \dots & \pi_{t-q} & y_{t-1} & y_{t-2} & i_{t-1} \end{bmatrix}';$$

$$\alpha_t = \begin{bmatrix} \pi_t^T & \pi_t^P & \pi_{t-1}^P & y_t^P & y_{t-1}^P & y_{t-2}^P & \lambda_t & \lambda_{t-1} & \tau_t & \tau_{t-1} \end{bmatrix}';$$

$$A = \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_q & \beta_1 & 0 & 0 \\ \rho_1 & 0 & \dots & 0 & 0 & 0 & \rho_2 \\ 0 & 0 & \dots & 0 & \beta_2 & \beta_3 & -\beta_4 \end{bmatrix};$$

$$Z = \begin{bmatrix} 0 & (1-\sum_{i=1}^{q}\varphi_i) & 0 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ -\rho_1 & (1-\rho_2) & 0 & 0 & 0 & 0 & (1-\rho_2)\,\gamma & 0 & (1-\rho_2) & 0 \\ 0 & 0 & \beta_4 & 1 & -\beta_2 & -\beta_3 & 0 & \beta_4\gamma & 0 & \beta_4 \end{bmatrix};$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & (1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \;\; R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \delta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} & \varepsilon_{2t} & \varepsilon_{3t} \end{bmatrix}'; \;\; \eta_t = \begin{bmatrix} \eta_{1t} & \eta_{2t} & \eta_{3t} & \eta_{4t} & \eta_{5t} \end{bmatrix}';$$

$$H_t = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon_3}^2 \end{bmatrix}; \;\; Q_t = \begin{bmatrix} \sigma_{\eta_1}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta_2}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\eta_3}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta_4}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\eta_5}^2 \end{bmatrix}$$

Appendix C: Data

- **Inflation**: quarterly inflation rate, defined as $400(\ln P_t - \ln P_{t-1})$, with P_t the seasonally adjusted quarterly GDP deflator. Sources: AWM (Fagan et al, 2005) and BIS;

- **Real output**: quarterly $\ln(GDP_t)$, with GDP_t the seasonally adjusted quarterly GDP in constant prices. Sources: AWM (Fagan et al, 2005) and BIS. The estimated output gap is expressed in percent deviation of current output from potential output, namely $100 * \left(y_t^r - y_t^P\right)$;

- **Key interest rate**: quarterly central bank key interest rate. Sources: NCB and ECB calculations and BIS.