# Endogenous Participation Risk in Speculative Markets

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**Abstract**

This paper analyses the dynamic implications of an asset pricing model with incomplete participation due to entry costs. It is shown that heterogeneity in entry costs may lead to the existence of multiple stochastic 'sunspot' equilibria whereby the number of agents in the market and asset prices endogeneously fluctuate over time absent fundamental uncertainty. Besides 'excess volatility', the equilibria under study generate *predictable* and *conditionally heteroskedastic* returns.

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*Keywords*: self-fulfilling expectations, asset price volatility

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# 1 Introduction

This paper analyses the impact of incomplete participation, due to the presence of entry costs, on the price dynamics of a speculative asset. It focuses on the possible destabilising nature of limited participation in asset markets, and shows that incomplete participation in the deterministic steady state may allows for the existence of stochastic equilibria where the arbitrary, self-fulfilling beliefs of investors cause fluctuations in both the proportions of agents present in the market and the associated asset prices.

From an empirical point of view, the fact that participation is incomplete in some speculative markets (like stock markets) is fairly uncontroversial. This point was forcefully made by Mankiw and Zeldes (1991) to explain the cross-sectional patterns of returns on different assets (i.e. the equity premium puzzle). Moreover, there is considerable evidence that stock market participation changes over time, as showed by the dramatic increase in stock market participation in Europe and the US during the 90's (Guiso *et al.*, 2003).

A natural explanation for incomplete market participation is that investors must pay an entry cost, reflecting fees, information gathering costs, processing costs and the like, before they can actually convey their buying or selling orders to the market. As a consequence, investors must weight the expected utility gains from joining against the utility loss from paying the entry cost. If investors differ in terms of certain characteristics such as preferences, wealth, or the level of the entry costs they have to pay, some agents may decide to participate while some others may not, thereby leading to situations of incomplete participation. If this explanation is correct, participation in speculative markets is driven by both entry costs and the risk/return trade-off face by investors. Since returns and risk typically change over time (see, for example, Poterba and Summers (1986, 1988) for early contributions), one should naturally expect participation to be time-varying as well, and these changing levels of participation to feed back onto equilibrium prices, returns, and risk. For example, Orosel (1998) shows that the price process implied by incomplete participation typically *magnifies* the effect of dividend innovations through the flow of agents in and out of the market, thereby creating a potential source of 'excess volatility' compatible with the assumption of full rationality.

The present paper demonstrates that the feedback loop between asset prices and participation levels may have more dramatic implications. More precisely, it is shown that the mere heterogeneity in entry costs may lead to the existence of multiple, self-fulfilling stochastic equilibria whereby the *ex ante* expectation that prices will fluctuate generates a pattern of changing participation which, through its effect on equilibrium prices, validates this expectation *ex post*. To illustrate this point clearly, the framework presented below assumes that the asset being traded bears no income risk, so that the only source of uncertainty lies in the volatility of investors' expectations.

The mechanism underlying such equilibria and their properties are, it is believed, reasonably intuitive. With linear mean-variance preferences, a high payoff volatility depresses the price of the risky asset so much that the resulting expected return is high enough to *increase* the expected utility from trading the asset (see, for example, Pagano, 1989a,b). Now suppose that investors expect market participation to switch randomly between 'high' and 'low' levels. For the high participation state to attract a broader class of investors, the (expected) return/risk trade-off must be more favorable there than in the low participation state. In our model, this is achieved when the high participation state has high payoff risk (and therefore low prices and high expected returns) relative to the low participation state. What is required for this mechanism to be operative is that the Markov-switching process driving the dynamics of participation be sufficiently asymmetric to induce substantial differences in the conditional variances of the asset payoff across states. When such is the case, expectations of changes in the level of participation become self-fulfilling and create a payoff risk arising independently from that of the asset's income, which we therefore label *endogenous participation risk*. This mechanism forms a possible cause of *excess volatility* in asset prices, since no fundamental uncertainty at all is needed for prices to fluctuate randomly. As will be made clear in the following, these equilibria also display some common properties of asset returns such as *predictability* and *changing conditional volatility*.

Several authors have investigated the possible implications of incomplete participation for the multiplicity of equilibria in asset markets. In a pioneering contribution, Pagano (1989a) analysed the positive feedback between market volatility and the level of market participation. High participation lowers the impact of idosyncratic demand shocks on asset prices, thereby reducing the volatility of asset prices and increasing the attractiveness of participation. Multiple steady states can then appear, since the mere expectation that participation is high attracts more investors, whereas the expectation that it will be low deters investors from joining. In a related contribution, Allen and Gale (1994) have analysed the impact of aggregate liquidity shortages on the existence and properties of asset market equilibria. A common feature of these papers is to focus on the volatility and welfare implications of the (possibly multiple) levels of participation, i.e. the *steady states* of the corresponding models. In contrast, the present contribution focuses on self-fulfilling *stochastic* equilibria ('sunspot' equilibria), where the steady state is *unique* and the only source of uncertainty is *extraneous*.

The occurrence of sunspot equilibria in asset markets has long been recognised to be possible in dynamic macroeconomic models plagued with market imperfections. To take some of the most recent contributions, Azariadis and Chakraborty (1998) argue that sunspot equilibria offer a natural theory of excess volatility in asset prices, whereas Challe (2004) shows that they typically imply that asset returns are predictable. The model introduced below differs from these works by relating the self-fulfilling volatility in asset prices to the flows of agents in and out of the markets. Consequently, the present model is rooted in the so-called 'market microstructure' approach to market participation (e.g. Pagano (1989a,b)), as opposed to the dynamic general equilibrium framework favoured by macroeconomists.

Section 2 introduces the model and derives its deterministic equilibrium. Section 3 derives a sufficient condition for the existence of equilibria with endogenous participation risk. Section 4 discusses the volatility and welfare properties of these equilibria, and section 5 concludes.

## 2 The model

### 2.1 Assets and market structure

The framework of analysis consists of a (partial equilibrium) overlapping generations model where two period-lived agents maximise the utility of terminal consumption. Agents entering the market at date $t$ may trade two assets. One is a safe asset in perfectly elastic supply, which can be used to borrow or lend without limit at gross interest rate $R > 1$. The other asset (simply referred to as 'the asset' in the remaining of the paper) is in fixed supply, normalised to 1, and yields a constant dividend $d$ per share bought at date $t$. The price of the asset at date $t$ is denoted $p_t$.

There is a continuum of agents of mass 1 indexed by $i$ and uniformly distributed along the interval $[0, 1]$. Before entering the asset market and expressing selling or buying orders there, agent $i$ must pay the fixed cost $\epsilon(i) \geq 0$. From the budget constraints of agent $i$ at date $t$ and $t+1$, his/her terminal wealth, entirely consumed at date $t+1$, is:

$$C_{t+1}^{i} = R\left(e - z\epsilon\left(i\right)\right) + X_{t}^{i}\left(p_{t+1} + d - Rp_{t}\right), \tag{1}$$

where $X_t^i$ is the demand for fixed-supply assets by investor $i$ at date $t$, $e > 0$ his/her initial endowment (identical across agents), and $z$ is a dummy variable taking value 1 if the investor decides to enter the asset market and 0 otherwise.

The equilibria considered in this paper are in general stochastic and involve fluctuations in the price of the asset. The payoff risk thereby generated is measured by the conditional variance of asset prices, $\sigma_t^2 \equiv \mathrm{var}_t\left(p_{t+1}\right)$.

## 2.2 Preferences and entry decisions

Investors are endowed with constant absolute risk aversion utility functions and maximise the expected utility of terminal wealth. Denote $\gamma > 0$ the coefficient of absolute risk aversion of investors, and $\bar{C}^i$ the mean of $C_t^i$. Focusing on small fluctuations of $C_t^i$ around $\bar{C}^i$, we follow Levy and Markovitz (1979) in considering the quadratic approximation to the utility function around $\bar{C}^i$ and derive the following (local) linear mean-variance criterion[1]:

$$\max E_t \left(-\exp\left(-\gamma C_{t+1}^i\right)\right) \simeq a + b\left(E_t\left(C_{t+1}^i\right) - \frac{\gamma}{2}\operatorname{Var}_t\left(C_{t+1}^i\right)\right), \tag{2}$$

where $a \equiv -(1+\gamma)\exp\left(-\gamma\bar{C}^i\right)$ and $b \equiv \gamma\exp\left(-\gamma\bar{C}^i\right)$. Note that $\bar{C}^i$ may vary across agents, because entry costs and entry decisions may do. However, the utility function chosen ensures that optimal portfolios only depend on $\gamma$ and not on $\bar{C}^i$.

In order to decide whether to enter the market or not, investors must solve a two-stage decision problem (see, for example, Pagano (1989a)). First, they must compute their (notional) optimal asset demand conditional on participating, and the associated level of expected utility; and second, this level of utility must be compared to that associated with non participating $(a + beR)$.

*First stage.* Assume $\sigma_t^2 > 0$ (the case where $\sigma_t^2 = 0$ is the deterministic equilibrium analysed below). If investor $i$ decides to enter the market, then from (1) and (2) his/her asset demand is:

$$\hat{X}_t^i = \frac{E_t\left(p_{t+1}\right) + d - Rp_t}{\gamma\sigma_t^2} \tag{3}$$

*Second stage.* Investors participate in the market for risky assets if the expected utility from participating ($z = 1$, $X_t^i = \hat{X}_t^i$) is greater than that from non participating ($z = X_t^i = 0$). From eqs. (1) to (3), agent $i$ participates if, given the current asset price and the distribution of the asset payoff, the following inequality holds:

$$\frac{\left(E_t\left(p_{t+1}\right) + d - Rp_t\right)^2}{2R\gamma\sigma_t^2} \geq \epsilon(i) \tag{4}$$

Finally, let investors be ordered along $[0,1]$ in such a way that $\epsilon(i)$ is non decreasing. Given the entry condition (4), this means that if investor $i^*$ finds it worthwhile to enter the market for a given conditional payoff distribution $\left(E_t\left(p_{t+1} + d\right), \sigma_t^2\right)$, then all agents $i < i^*$ also find it worthwhile to do so.

[1] The body of the paper focuses on heterogeneity in entry cost. An extension of the basic model is presented in appendix B that generalises the results to the case of heterogeneity in *both* entry cost and risk aversion.

### 2.3 Deterministic equilibrium

It is useful to derive, as a benchmark case, the unique deterministic equilibrium of this economy. In this equilibrium, $\mathrm{Var}_t\left(C_{t+1}^i\right) = \sigma_t^2 = 0$ and preferences become linear in consumption (see eq. (2)). Prices are then given by a (risk-neutral) no-arbitrage condition according to which agents facing the *lowest* entry cost trade the asset until its gross return, net of entry cost, is equal to $R$. The resulting equilibrium price is:

$$\bar{p} = \frac{d - R\epsilon\left(0\right)}{R-1},$$

which we may call the 'market fundamental' of the asset, i.e. the price that would prevail if no extraneous uncertainty was affecting investors' beliefs. Note that, along the deterministic equilibrium, investors facing entry cost higher than $\epsilon\left(0\right)$ never enter the market, because for them the return on holding the asset, net of entry cost, is strictly less than $R$.

## 3 Existence of equilibria with endogenous participation risk

### 3.1 Conjectured stochastic equilibria

In the following we first conjecture, and then derive a sufficient condition for, the existence of equilibria where market participation and asset prices fluctuate according to the arbitrary (but rational, it turns out) expectations of investors. The equilibria considered here are stochastic and oscillate between two states, $l$ and $h$, the switches from one state to the other being governed by the following transition matrix:

$$\Pi = \begin{bmatrix} \pi^l & 1-\pi^l \\ 1-\pi^h & \pi^h \end{bmatrix}, \tag{5}$$

where $\pi^s \in (0,1)$ , $s = l, h$, denotes the probability of staying in state $s$ from the current to the next period. State $l$ denotes a state of low market participation, where all agents until $i_l \in (0,1)$ enter the market, whereas state $h$ denotes a state of high market participation where all agents until $i_h \in (i_l, 1)$ participate. Given the imposed ordering of agents along $[0,1]$, $i_l$ and $i_h$ denote both the *last entrants* and the *measures of agents* participating in state $l$ and $h$, respectively.

All the relevant variables (i.e. $\sigma^2$, $p$ and $X^i$) are now indexed by the state $s = l, h$. Given the assumed stochastic process, (conditional) price expectations are:

$$E\left(p^{s'}\middle| s = l\right) = \pi^l p_l + \left(1-\pi^l\right) p_h \tag{6}$$

$$E\left(p^{s'}\middle| s = h\right) = \pi^h p_h + \left(1-\pi^h\right) p_l, \tag{7}$$

where $s'$ denotes next period's state. Using (6) and (7), one finds the following simple expressions for the conditional payoff variances:

$$\begin{aligned}\sigma_s^2 &= E\left(\left(p^{s'} - E\left(p^{s'}\middle| s\right)\right)^2 \middle| s\right) \\ &= \pi^s \left(1-\pi^s\right)\left(p_l - p_h\right)^2,\ s = h, l. \end{aligned} \tag{8}$$

Eq. (8) shows that conditional variances depend on prices through $p_l - p_h$. In equilibrium, prices are in turn affected by these variances through their impact on individual demands $X_s^i$, $s = l, h$ (see (3)). Prices can be substituted out of (8) by considering the market-clearing conditions for the asset in each state.

### 3.2 Market clearing

Market clearing requires that market demand be equal to 1 in both states. Using eqs. (3) and (6)-(7), one can aggregate individual demands across investors and obtain:

$$\int_0^{i_l} X_l^i di = i_l \left(\frac{\left(1-\pi^l\right)p_h - \left(R-\pi^l\right)p_l + d}{\gamma\sigma_l^2}\right) = 1 \tag{9}$$

$$\int_0^{i_h} X_h^i di = i_h \left(\frac{\left(1-\pi^h\right)p_l - \left(R-\pi^h\right)p_h + d}{\gamma\sigma_h^2}\right) = 1 \tag{10}$$

Equations (9) and (10) can be solved for the equilibrium price of the asset in each state, $p_l, p_h$, as a function of the equilibrium measures of investors, $i_l, i_h$, the conditional variances, $\sigma_l^2, \sigma_h^2$, and the transition probabilities, $\pi^l, \pi^h$. One finds the following expression for the price difference appearing in (8):

$$p_l - p_h = \frac{\gamma\left(\sigma_h^2 i_h^{-1} - \sigma_l^2 i_l^{-1}\right)}{1 + R - \pi^l - \pi^h} \tag{11}$$

Substituting (11) into (8) and rearranging allows to solve for the conditional variances as functions of the measures of investors and the transition probabilities only. One finds:

$$\sigma_s^2 = \frac{\pi^s\left(1-\pi^s\right)\left(1+R-\pi^l-\pi^h\right)^2}{\gamma^2\left(\pi^h\left(1-\pi^h\right)i_h^{-1} - \pi^l\left(1-\pi^l\right)i_l^{-1}\right)^2},\ s = l, h. \tag{12}$$

So far the analysis has been concerned with the properties of the asset price distribution under two assumptions, namely (a) that the asset-price dynamics is governed by a 2-state markov chain; and (b) that the subsets of agents entering the asset market in each state have different measures. One now needs to analyse the conditions under which this assumed pattern of participation is consistent with the entry rule (eq. (4)).

### 3.3 Equilibrium measures of investors

The stochastic process postulated above can be a self-fulfilling one if the conditional payoff distribution assumed *ex ante* leads agents to sort themselves in and out of the market *ex post* in a way consistent with the original expectation. Using (9) and (10), one can substitute out the expression for the net payoff in eq. (4) and express the entry condition as a function of the conditional variances and the equilibrium measures of investors only. One finds that any stochastic equilibrium must satisfy the following inequalities:

$$\begin{aligned} \gamma\sigma_s^2/2Ri_s^2 &\geq \epsilon(i) \;\; for\ i \leq i_s \\ &< \epsilon(i) \;\; for\ i > i_s,\ s = l,h. \end{aligned} \tag{13}$$

The left hand side of (13) is the utility gain that an investor may expect from joining the asset market rather than staying out of it when the current state is $s$. For the decisions of investors as a whole to indeed generate state $s$, it must be the case that *only* investors facing entry cost less than $\epsilon(i_s)$ find it worthwhile to join (right hand side of (13)).

A necessary condition for (13) to hold for $i_l \neq i_h$ is $\epsilon(i_h) > \epsilon(i_h)$. To derive a sufficient condition, assume that $\epsilon(.)$ is continuous and strictly increasing over $[0, i_1] \subset [0, 1]$, and choose $(i_l, i_h) \in (0, 1]$. (13) must then hold with equality for the last entrants, or *marginal investors* $i_l$ and $i_h$, who are exactly indifferent between entering and staying out of the market in states $l$ and $h$, respectively. This implies:

$$\sigma_s^2 = 2(R/\gamma)\, i_s^2 \epsilon(i_s),\ s = l,h. \tag{14}$$

The question whether stochastic equilibria exist amounts to asking whether, given $(i_l, i_h)$, $0 < i_l < i_h \leq i_1$, and the implied conditional variances in (14), there are transition probabilities $(\pi^l, \pi^h) \in (0,1)^2$ such that (12) holds, i.e. these variances are equilibrium ones. The following proposition provides a sufficient condition for it to be the case.

**Proposition 1 (Existence).** Assume that $\epsilon(.)$ is continuous and strictly increasing over $[0, i_1] \subset [0, 1]$. Then a sufficient condition for stochastic equilibria to exist is $2R < \gamma\epsilon(0)$.

The proof is in appendix. Note from (14) and the assumed continuity of $\epsilon(.)$ that equilibrium variances can be made as small as necessary for the approximation to the utility function to remain valid. Along stochastic equilibria different subsets of agents self-select themselves in and out of the market in $l$ and $h$, the equilibrium measures of investors $(i_l, i_h)$ and the transition probabilities $(\pi^l, \pi^h)$ being jointly determined (formally, $\pi^l$ and $\pi^h$ solve of a system of two equations parametrised by $i_l$ and $i_h$). Too high a safe return $R$ deters investors $i > 0$ from participating and eliminates the possibility of stochastic equilibria (the system has no solution).

# 4 Properties of equilibria with endogenous participation risk

Although the set of sunspot equilibria with varying participation levels is large, all such equilibria share some stable properties that allow for a clear interpretation of the mechanism at work. These are summarised in proposition 2 and 3 below.

**Proposition 2 (Prices, returns, and risk).** The price is lower, the expected payoff higher and the payoff risk higher in state $h$ than in state $l$.

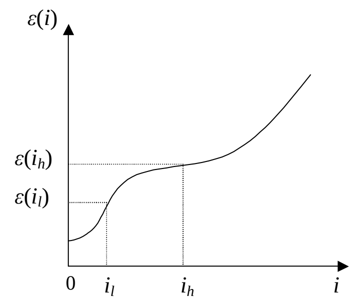
The fact that $\sigma_h^2 > \sigma_l^2$ follows immediately from (14) and the fact that $i_l < i_h$. In other words, stochastic equilibria are characterised by *conditional heteroskedasticity*, since along any stochastic equilibrium the conditional variance of the asset payoff changes over time. That $p_h < p_l$ can be seen by solving (14) for $\sigma_s^2 i_s^{-1}$ and substituting the corresponding expressions into (11). Varying participation levels are thus accompanied by fluctuations in asset prices absent income risk, i.e. prices feature *excess volatility*. Finally, that the expected asset payoff is higher in state $h$ than in state $l$ is a straightforward implication of $p_h < p_l$ and the fact that prices switch between states with strictly positive probability. Investors thus expect high returns in state $h$ and low ones in state $l$–returns are *predictable*.

Those three properties are closely connected. It is because the risk associated with holding the asset is higher in state $h$ than in state $l$ that the price is lower there, in spite of the larger number of market participants. At this point one may wonder why a state where the asset is high risk attracts more participants than a state where it is low risk. Wouldn't the lower risk associated with holding the asset in state $l$ also attract investors who are anyway keen on buying it in state $h$? The reason for this apparent paradox is that, for a certain level of risk, agents facing a high entry cost require a higher return to participate than agents facing a comparatively low one. In state $h$ the price is low, implying that *expected returns are high*, high enough to make it worthwhile for agents $i_l$ to $i_h$ to participate *in spite of the higher associated risk*. Conversely, in state $l$ the price of the asset is high, making expected returns low enough to deter agents $i_l$ to $i_h$ from participating, *in spite of the lower associated risk*.

The above analysis has emphasised that higher risk in state $h$ is compensated for by high expected returns. Conversely, lower risk in state $l$ state is 'paid for' by low expected returns. In fact, the overall effect of higher (lower) payoff volatility is in fact to *increase* (decrease) the expected utility gain from participating (see (13)), a well know property of optimal portfolio choice with mean-variance preferences. This mechanism, which explains why higher risk ultimately makes investment in the asset more attractive, also explains why endogenous participation risk *increases* rather than decreases welfare and participation levels.

**Proposition 3 (Participation and welfare).** Average participation and ex ante welfare are higher along stochastic equilibria than in the deterministic one.

Proposition 3 follows straightforwardly from the fact that investors' 'reservation' utility from non participating is $a + beR$, which they *all* get in the deterministic equilibrium, whereas total market participation is *chosen* by agents to be higher along any stochastic equilibrium. Moreover, only investor $i = 0$ participates in the deterministic equilibrium, whereas participation is strictly higher in *both* states of any stochastic equilibrium.




# 5 Conclusion

This paper has investigated the connection between the presence of entry costs in the market for a speculative asset and the existence of stochastic equilibria with varying participation levels. In these equilibria the *ex ante* expectation that prices fluctuate over time generates a time-pattern of participation in the market that validates this original expectation *ex post*, in spite of the asset bearing no income risk. Such equilibria therefore display 'endogenous participation risk' since the only source of uncertainty affecting the asset payoff is generated by self-fulfilling changes in participation levels that have no connection with changing market fundamentals.

The key mechanism making such self-fulfilling fluctuations possible is the self-selection of some subset of investors (those facing a relatively high entry cost) out of the market in one of the sunspot states. For such agents, the decision to exit periodically is the outcome of a rational decision weighting the expected return against the risk associated with holding the asset, given the conjectured payoff distribution. Because these sunspot equilibria are associated with fluctuations in asset prices that are disconnected from fundamental uncertainty, they offer a simple explanation for *excess volatility* in asset prices that does not rely on irrational expectations or other behavioural considerations. They are also consistent with other stylised stock market facts such as the *predictability* and the *conditional heteroskedasticity* of asset returns, another couple of widely documented features of actual stock markets.

# 6 Appendix

## 6.1 Proof of Proposition 1

Using (12) and (14) and rearranging, we find that any stochastic equilibrium must satisfy:

$$\begin{aligned} 2R\gamma\left(\frac{i_h\epsilon(i_h)}{i_l\epsilon(i_l)}-1\right)^2\epsilon(i_l) &\equiv \alpha=\frac{\left(1+R-\pi^l-\pi^h\right)^2}{\pi^l\left(1-\pi^l\right)} \\ 2R\gamma\left(1-\frac{i_l\epsilon(i_l)}{i_h\epsilon(i_h)}\right)^2\epsilon(i_h) &\equiv \beta=\frac{\left(1+R-\pi^l-\pi^h\right)^2}{\pi^h\left(1-\pi^h\right)}, \end{aligned}$$

or

$$\alpha\pi^l\left(1-\pi^l\right) = \left(2+r-\pi^l-\pi^h\right)^2 \tag{A1}$$

$$\beta\pi^h\left(1-\pi^h\right) = \left(2+r-\pi^l-\pi^h\right)^2 \tag{A2}$$

Since $\alpha,\beta>0$, solutions to (A1)-(A2) satisfy $\left(\pi^l,\pi^h\right)\in(0,1)^2$. Moreover,

$$\alpha/\beta=\pi^h\left(1-\pi^h\right)/\pi^l\left(1-\pi^l\right)=i_h^2\epsilon(i_h)/i_l^2\epsilon(i_l)>1 \tag{A3}$$

Let us focus on solutions where $\pi^l,\pi^h\geq 0.5$. From (A3), solutions to (A1)-(A2) solve:

$$\pi^h = 0.5+0.5\sqrt{1-4(\alpha/\beta)\pi^l\left(1-\pi^l\right)} \tag{A4}$$

$$\pi^h = 1+R-\pi^l-\sqrt{\alpha\pi^l\left(1-\pi^l\right)} \tag{A5}$$

Define $F\left(\pi^l\right)$, continuous over $\left[0.5+0.5\sqrt{1-\beta/\alpha},1\right]$, as follows:

$$F\left(\pi^l\right)=0.5+R-\pi^l-\sqrt{\alpha\pi^l\left(1-\pi^l\right)}-0.5\sqrt{1-4(\alpha/\beta)\pi^l\left(1-\pi^l\right)}.$$

Solutions to (A4)-(A5) are roots of $F\left(\pi^l\right)$. Since $F(1)=R-1>0$, a sufficient condition for at least one $\pi^l$ to satisfy $F\left(\pi^l\right)=0$ is

$$F\left(0.5\left(1+\sqrt{1-\beta/\alpha}\right)\right)=R-0.5\left(\sqrt{\beta}+\sqrt{1-\beta/\alpha}\right)\leq 0.$$

Since $\sqrt{1-\beta/\alpha}\geq 0$, the latter expression is true whenever

$$4R^2\leq\beta=2R\gamma\left(1-i_l\epsilon(i_l)/i_h\epsilon(i_h)\right)^2\epsilon(i_h)$$

Since $\epsilon(i_h)\geq\epsilon(0)$, the previous inequality is in turn satisfied if

$$2R/\gamma\epsilon(0)\leq\left(1-i_l\epsilon(i_l)/i_h\epsilon(i_h)\right)^2 \tag{A6}$$

If $2R/\gamma\epsilon(0)<1$, and given the assumed continuity of $\epsilon(.)$, it is always possible to choose $i_l,i_h\in(0,i_1]$ such that $i_l\epsilon(i_l)/i_h\epsilon(i_h)$ is small enough for (A6) to hold.

## 6.2 Double heterogeneity

This appendix generalises the analysis to the case where both entry costs and risk aversion are heterogenous. Call $\gamma(i)$ the degree of risk aversion of investor $i$, and assume that $\gamma(i)$ and $\epsilon(i)$ are continuous. Rank investors in increasing order of $\gamma(i)\epsilon(i)$ and assume that $\gamma(i)\epsilon(i)$ is strictly increasing over $[0, i_1] \subset [0,1]$. The market-clearing conditions are now:

$$\begin{aligned}\int_0^{i_l} X_l^i di &= \left(\frac{(1-\pi^l)p_h - (R-\pi^l)p_l + d}{\sigma_l^2}\right)\int_0^{i_l}\gamma(i)^{-1} di = 1 \\ \int_0^{i_h} X_h^i di &= \left(\frac{(1-\pi^h)p_l - (R-\pi^h)p_h + d}{\sigma_h^2}\right)\int_0^{i_h}\gamma(i)^{-1} di = 1,\end{aligned}$$

where $\int_0^{i^*}\gamma(i)^{-1} di$ is continuous and strictly increasing in $i^*$. The resulting price difference and equilibrium variances, which generalise eqs. (11) and (12), are:

$$p_l - p_h = \frac{1}{1+R-\pi^l-\pi^h}\left(\frac{\sigma_h^2}{\int_0^{i_h}\gamma(i)^{-1} di} - \frac{\sigma_l^2}{\int_0^{i_l}\gamma(i)^{-1} di}\right)$$

$$\sigma_s^2 = \frac{\pi^s(1-\pi^s)\left(1+R-\pi^l-\pi^h\right)^2}{\left(\pi^h(1-\pi^h)\left(\int_0^{i_h}\gamma(i)^{-1} di\right)^{-1} - \pi^l(1-\pi^l)\left(\int_0^{i_l}\gamma(i)^{-1} di\right)^{-1}\right)^2}, \tag{A7}$$

for $s = l, h$. The consistency conditions about the decisions of investors and the corresponding state of participation (analogues of eqs. (13) and (14)) are now:

$$\begin{aligned}\left(\sigma_l^2/2R\gamma(i)\right)\left(\int_0^{i_s}\gamma(i)^{-1} di\right)^{-2} &\geq \epsilon(i) \;\; for\; i \leq i_s \\ &< \epsilon(i) \;\; for\; i > i_s,\;\; s = l, h,\end{aligned}$$

and, for the marginal investors in each state:

$$\sigma_s^2 = 2R\epsilon(i_s)\gamma(i_s)\left(\int_0^{i_s}\gamma(i)^{-1} di\right)^2,\;\; s = l, h. \tag{15}$$

Equating the variances in (A7) and (A8) gives a system of two equations and two unknowns similar to (A1)-(A2). The sufficient condition for stochastic equilibria to exist is now $2R < \gamma(0)\epsilon(0)$. Note that heterogeneity in entry cost is not *necessary* for stochastic equilibria to exist, provided that some heterogeneity in risk aversion is present and the sufficient condition is satisfied (in this case $\epsilon(.)$ is flat and $\gamma(.)$ is strictly increasing). The properties of equilibria with heterogenous risk aversion are similar to those stated in proposition 2 and 3 for the case of single heterogeneity.

## References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