Simple Cases of Multi-Level Models

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While much has been made of multi-level models, and specialized software for such models, in many cases standard methods can be used in estimating these models. Use of such standard methods is faster and easier, in many cases, than use of specialized software; further, the use of standard methods helps clarify what these models actually are estimating. I limit my discussion here to linear regression models and include a new, but incomplete, ado file that puts together the steps to match certain multi-level models.

Ways of Being Simple

Treating the nesting as a nuisance.

 Summary scores

ANOVA

 XT -type models

Rogosa s longitudinal random coefficient models via Regression

Dependence as a Nuisance

The above is the language used in Snijders, T. and Boskers, R, (1999), *Multilevel Analysis*, London: Sage Publications, to describe those cases in which the lack of independence caused by nesting within a higher level is nothing but a nuisance. *Real* multilevel models treat this dependence as something that is of analytical interest. Use of the -cluster- option in Stata treats this as a nuisance: that is, of no interest in itself. In other words, we only want to make inferences about the units at the lower level (e.g., students, employees, muscle fibers) and do not want to make inferences about the units at the higher level (e.g., classes, firms, whole muscles).

Summary Scores

When data are collected serially (longitudinally) on subjects, one simple but often effective method of analysis proceeds in two steps: (1) summarize the data for each subject in one number (mean, slope, AUC, etc.) and then, (2) analyze the summary scores. In particular, this is often more relevant to any real-world question than is repeated-measures ANOVA.

Matthews, JNS, Altman, DG, Campbell, MJ and Royston, P (1990), Analysis of Serial Measurements in Medical Research, *BMJ*, 300: 230-235 (also, Matthews, JNS (1993), A Refinement to the Analysis of Serial Data Using Summary Measures, *Statistics in Medicine*, 12: 27-37)

Frison, L and Pocock, SJ (1992), Repeated Measures in Clinical Trials, *Statistics in Medicine*, 11: 1685-1704 (see correction and comment: 13 (1994): 197-8)

Everitt, BS (1995), The Analysis of Repeated Measures, *The Statistician*, 44: 113-135

Senn, S, Stevens, L and Chaturvedi, N (2000), Repeated Measures in Clinical Trials: simple strategies for analysis using Summary Measures, 19: 861-77

Weinberg, JM and Lagakos, SW (2000), Asymptotic Behavior of Linear Permutation Tests Under General Alternatives, with Application to Test Selection and Study Design, *JASA*, 95: 596-607

ANOVA

In his article Hierarchical Linear Models and Experimental Design, (in *Applied Analysis of Variance in Behavioral Science*, 1993, ed. by L.K. Edwards, NY: Marcel Dekker, Inc., pp. 459-496), Stephen W. Raudenbush says that data analysis based on recently developed hierarchical linear models (1) duplicates the results of standard ANOVA models for an important class of experimental designs.... (P. 459) This is not the subject of my paper and I just want to urge StataCorp. to ask their in-house ANOVA expert to write an STB article pointing out the ways in which the -anova- command can be used to estimate multi-level models.

Stata s XT series

I know that the following is true for at least some of the XT models and I believe it is true for all of them: they can be used to analyze any two-level random intercept model. Thus, clearly this covers some of what is meant by multi-level models, but not a great deal. Unfortunately, there appears to be a fashion among users who don t read the manual carefully to treat these as general random coefficient models (which they are not). Note that though originally designed as models for longitudinal data, they can be used for cross-sectional two-level data also with no changes necessary.

Rogosa s Regression Model for Longitudinal Data

David Rogosa and Hilary Saner ((1995), "Longitudinal Data Analysis Examples with Random Coefficient Models," *Journal of Educational and Behavioral Statistics*, 20, 149-170) have presented a procedure for matching one type of multi-level model, that for longitudinal data, using just standard regression results. The procedure involves first estimating a model for each member of the lower level (e.g., each person) and then taking the slopes from those estimates and using them as the response (dependent) variable in a second-level model. By using some of the other results of the regressions, one can match other information, including the various variances normally presented by multi-level software.

In the following example, I use the first data set from the Rogosa/Saner article; this data is taken from the HLM manual. The command I use is:
. rogosa weight, id(ratid) time(time) w(time) b(z) detail

The Within varlist makes up the right-side variables for the within person regression; the Between varlist makes up the right-side variables for the between person regression. Each of within and between is required. The response variable for the between model is the slope or growth rate which is the coefficient of time from the within model. The first variable in the command is used as the response variable for the within model.

descriptive statistics for ind. slopes:

```
                                _Coef1
-------------------------------------------------------------------
        Percentiles      Smallest
  1%           19.7           19.7
  5%           19.7           23.4
 10%          21.55           23.6        Obs                   10
 25%           23.6           25.6        Sum of Wgt.           10

 50%           26.4                        Mean               26.76
                             Largest       Std. Dev.        4.44002
 75%           28.8           28.1
 90%           32.8           28.8        Variance        19.71377
 95%           36.3           29.3        Skewness         .6303867
 99%           36.3           36.3        Kurtosis         3.513578
```

approximate tests for within-person regression covariates other
than growth (a summary of the Wald statistics from the level-one
models):

```
Variable |       Obs        Mean    Std. Dev.       Min        Max
---------+-----------------------------------------------------------
 _JTest1 |        10    10.43698    3.467726    6.54905   16.99708
sample var. (between) of ind. growth =        19.71377
```

```
within person variance =                          10.53911
reliability of ind. slopes (w/b) =                .5346063

  Source |       SS       df       MS              Number of obs =      10
---------+------------------------------           F(  1,      8) =    4.08
   Model | 59.8772588       1  59.8772588          Prob > F      =  0.0782
Residual | 117.546709       8  14.6933386          R-squared     =  0.3375
---------+------------------------------           Adj R-squared =  0.2547
   Total | 177.423968       9  19.7137742          Root MSE      =  3.8332

-------------------------------------------------------------------------------
  _Coef1 |      Coef.   Std. Err.       t     P>|t|     [95% Conf. Interval]
---------+---------------------------------------------------------------------
       z |    .146866   .0727529      2.019   0.078     -.0209026     .3146346
   _cons |    2.96771   11.84815      0.250   0.809     -24.35416     30.28958
-------------------------------------------------------------------------------

Regression with Huber standard errors              Number of obs   =      10
                                                   R-squared       =  0.3375
                                                   Adj R-squared   =  0.2547
                                                   Root MSE        = 3.83319

-------------------------------------------------------------------------------
  _Coef1 |      Coef.   Std. Err.       t     P>|t|     [95% Conf. Interval]
---------+---------------------------------------------------------------------
       z |    .146866   .0737519      1.991   0.082     -.0232062     .3169381
   _cons |    2.96771   11.7294       0.253   0.807     -24.08033     30.01575
-------------------------------------------------------------------------------
```

Note that I have not (yet) implemented everything in the Rogosa/Saner article; in particular, I do not include bootstrapped standard errors and I do not include information on the correlation between change and initial status. However, the above results do match those presented in the article (and, according to Rogosa and Saner, in the HLM manual). A graph is also available and is particularly useful if the -by- option is used.

Note that the logic of the Rogosa/Saner procedure is described by them in their article as something that a Smart First Year Student (SFYS) would go through on understanding the substantive issue.

I don t know when I will have a chance to do more work on this program.

Conclusion

The existence of multi-level models has expanded our armory. However, many of the advantages of multi-level models (e.g., inferences about variables at each of the two levels and about their variances) are available via the use of standard techniques in (slightly) non-standard ways.