Extensions to `var` and `svar` Estimation

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Background

- Vector autoregressions (VAR) are commonly used in applied macro-economic and financial analyses
  - Dynamic models with minimal restrictions
    - Sims (1980); Lütkepohl (1993, 2005), Hamilton (1994), Enders (2004)
  - Useful for forecasting time series
  - Lack an economic interpretation

Background

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \cdots + A_p X_{t-p} + u_t$$

$$\left(I - A_1 L - A_2 L^2 - \cdots - A_p L^p\right) X_t = u_t$$

$$A(L) X_t = u_t$$

where $\mathbb{E}\left[u_t u_t'\right] = \Sigma$

Background

- Structural VARs use restrictions implied by theory to recover economic shocks
  - Short-run identification: Bernanke (1986), Sims (1986)
  - Long-run identification: Shapiro & Watson (1988), Blanchard & Quah (1989)
- Effectively a dynamic simultaneous equations framework

Background

- Structural VAR:

$$\Phi(L)\, X_t = \varepsilon_t \text{, where } \mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = I$$

- Structural vector moving average (VMA):

$$\begin{aligned} X_t &= \Phi(L)^{-1} \varepsilon_t = \Theta(L)\, \varepsilon_t \\ &= \Theta_0\, \varepsilon_t + \Theta_1\, \varepsilon_{t-1} + \Theta_2\, \varepsilon_{t-2} + \ldots \end{aligned}$$

Background

- A structural VMA allows computation of several objects of interest:
  - Impulse response functions
  - Forecast-error variance decompositions
  - Historical decompositions
- Stata® does not yet implement historical decompositions following `var` or `svar` estimation

Methodology

- If X_t has n series, then at most n structural shocks can be recovered
  - Given a series of shocks, the historical decomposition recovers the dynamics of X_{it} attributable to ε_{jt}, period by period
  - Because the VMA is infinite, one must choose a finite number of lags (K) of ε_{jt} to compute the historical decomposition

Methodology

- Formally,

$$X_{it} = \sum_{j=1}^{n} \sum_{k=0}^{K} \Theta_{ijk} \varepsilon_{jt-k} + \zeta_{it}$$

where K is the "truncation lag" and ζ_{it} is the "truncation error"

Application

- Model of real exchange rate determination for Chinese renminbi
  - Is the renminbi "overvalued" or "undervalued"?
  - Do the economic factors that account for G7 currency fluctuations also fit the renminbi?
  - Clarida & Galí (1994) use a structural VAR to explore real exchange rate determination for British pound, German mark, Canadian dollar, Japanese yen (all vis-à-vis the U.S. dollar)

Application

- Series: real output, real exchange rate, nominal aggregate, all relative to U.S.
- Data: annual, 1952 – 2004
  - Both official & black-market measures
- Identification: long-run restrictions
  - Nominal shocks have no LR real impact
  - Demand shocks have no LR impact on output

Application

- Specifically,

$$\begin{bmatrix} \Delta y_t \\ \Delta q_t \\ \Delta p_t \end{bmatrix} = \begin{bmatrix} \Theta_{11}(L) & \Theta_{12}(L) & \Theta_{13}(L) \\ \Theta_{21}(L) & \Theta_{22}(L) & \Theta_{23}(L) \\ \Theta_{31}(L) & \Theta_{32}(L) & \Theta_{33}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^S \\ \varepsilon_t^D \\ \varepsilon_t^M \end{bmatrix}$$

where the LR restrictions are imposed as:

$$\Theta_{13}(1) = \Theta_{23}(1) = 0$$

$$\Theta_{12}(1) = 0$$

Application

- Stata® implementation:
  - `matrix Theta = (.,0,0\.,.,0\.,.,.)`
  - `svar D.y D.q D.p, lreq(Theta)`
- Problem: want *cumulative* structural VMA representation
- Solution: create `csirf` object
  - Also create percentile bootstrapped CIs

Application

- Historical Decompositions
  - Extract structural VMA coefficients
  - Construct structural shocks
  - Construct historical contribution of each shock series based on K lags
- Stata® ado file: `hdecomp`
  - Still under construction; will switch to Mata

Application

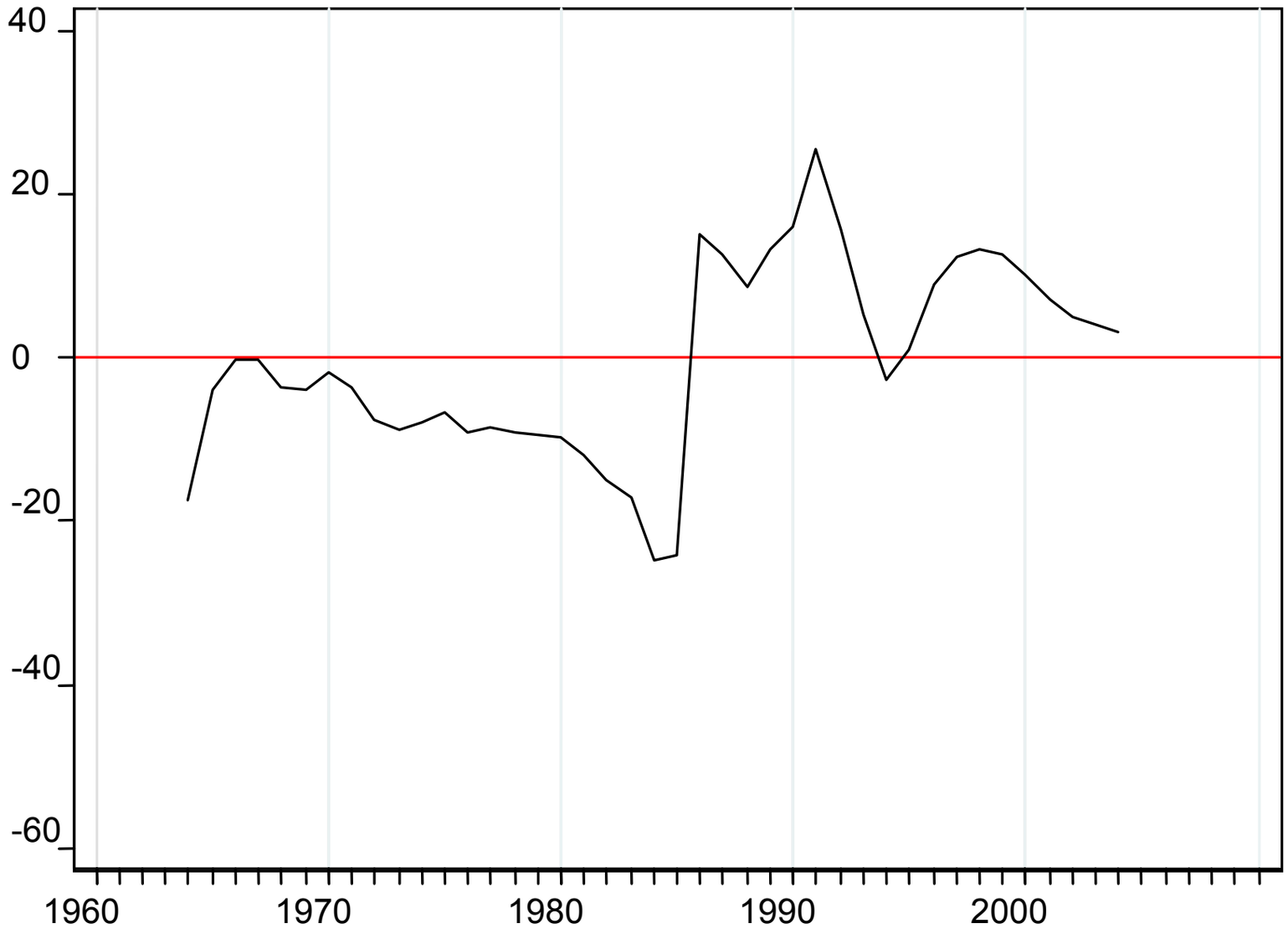
- Is renminbi over- or undervalued?
  - Purchasing power parity (PPP) based models suggest significantly *overvalued*
  - Panel PPP models suggest barely to significantly *undervalued*
  - Behavioral equilibrium models are mixed
- Question: what constitutes the "equilibrium" real exchange rate?

Application

- Our approach:
  - Long-run restrictions imply a time-varying "equilibrium" real exchange rate as a function of identified supply and real demand shocks
  - Historical decomposition yields estimate of "equilibrium" real exchange rate
  - Compute percentage gap between current and "equilibrium" real rate

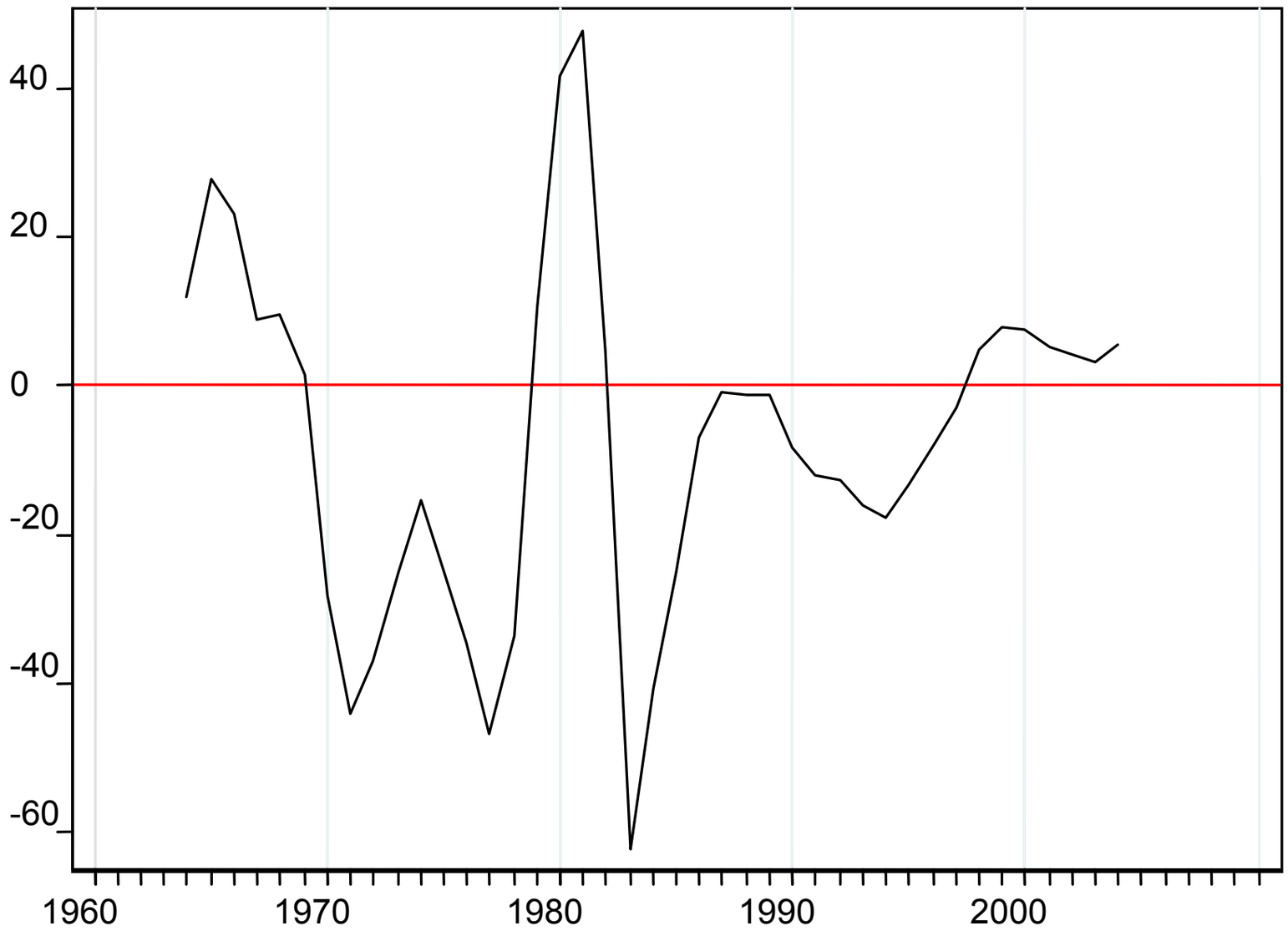
Application

Model 1: Output, Official XR, Prices



Application

Model 2: Output, Black-Market XR, Money



Conclusion

- Constructed cumulative structural IRFs
  - Also constructed percentile bootstrapped CIs
- Constructed historical decompositions
- Computed a theoretically-motivated "equilibrium" real exchange rate for Chinese renminbi
  - Renminbi appears slightly overvalued since mid-1990s

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