Smooth Transition Regression Models in UK Stock Returns

by

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ABSTRACT

This paper models UK stock market returns in a smooth transition regression (STR) framework. We employ a variety of financial and macroeconomic series that are assumed to influence UK stock returns, namely GDP, interest rates, inflation, money supply and US stock prices. We estimate STR models where the linearity hypothesis is strongly rejected for at least one transition variable. These non-linear models describe the in-sample movements of the stock returns series better than the corresponding linear model. Moreover, the US stock market appears to play an important role in determining the UK stock market returns regime.

Keywords: smooth transition models, forecasting, UK stock returns.

JEL Class: E44, E47

1. Introduction

A natural approach to modelling economic time series with non-linear models seems to be to define different states of the world or regimes, and to allow for the possibility that the dynamic behaviour of economic variables depends on the regime that occurs at any given point in time (Franses and van Dijk, 2000). Roughly speaking, two main classes of statistical models have been proposed which formalize the idea of existence of different regimes. The popular Markov-switching models (Hamilton, 1989) assume that changes in regime are governed by the outcome of an unobserved Markov chain. This implies that one can never be certain that a particular regime has occurred at a particular point in time, but can only assign probabilities to the occurrence of the different regimes. Hamilton applies a 2-regime model to the US GNP growth and concludes that contractions are sharper and shorter than expansions. Therefore, the US business cycles are found to be asymmetric. These models have been explored and extended in detail in a number of papers (see for example Engel and Hamilton, 1990, Hamilton and Susmel, 1994, Filardo, 1994).

A different approach is to allow the regime switch to be a function of a past value of the dependent variable. Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993) and Teräsvirta (1994) promote a family of univariate business cycle models called smooth transition autoregressive (STAR) models. These models can be viewed as a combination of the self-exciting threshold autoregressive (SETAR) and the exponential autoregressive (EAR) models. Markov-switching models imply a sharp regime switch, and therefore a small number (usually two) of regimes. This assumption is too restrictive compared to the STAR models. Two interpretations of a STAR model are possible. On the one hand, the STAR model can be seen as a regime-switching model that allows for two regimes where the transition from one regime to the other is smooth. On the other hand, the STAR model can be said to allow for a continuum of states between the two extremes (Teräsvirta, 1998).

The main advantage in favour of STAR models is that changes in economic aggregates are influenced by changes in the behaviour of many different agents and it is highly unlikely that all agents react simultaneously to a given economic signal. In financial markets, for example, with a large number of investors, each switching at different times (probably due to heterogeneous objectives), a smooth transition or a continuum of states between the extremes appears more realistic. According to Peters (1994) heterogeneity in investors' objectives arises from different investment horizons, geographical locations and various types of risk profiles. Further, investors may be prone to different degrees of institutional inertia (dependent, for example, on the efficiency of the stock markets in which they operate) and so adjust with different time lags. Thus, when considering aggregate economic series, the time path of any structural change is liable to be better captured by a model whose dynamics undergo gradual, rather than instantaneous adjustment between regimes. The STAR models allow for exactly this kind of gradual change whilst being flexible enough that the conventional change arises as a special case.

So far, the ST(A)R models have mainly been applied to macroeconomic time series. For example, Granger and Teräsvirta (1993) use then to analyse a non-linear relationship between US GNP growth and leading indicators. Öcal and Osborn (2000) employ STAR models to investigate non-linearities in UK consumption and industrial production. Skalin and Teräsvirta (1999) use this technique to examine Swedish business cycles. Applications in other areas such as finance is another challenging new area. To our knowledge, there are not very many applications in the finance literature. McMillan (2001) applies multivariate STAR models to the US stock market. Particularly, he examines the non-linear relationship between stock returns and business cycle variables. Lundbergh and Teräsvirta (1998) introduce the univariate STAR-STGARCH model, which is a generalization of the STAR and GARCH type specifications. This model allows plenty of scope for explaining asymmetries in both conditional moments of the underlying process. The authors suggest this model for applications to high frequency financial data. Other applications of STAR models in the finance literature include Sarantis (2001), Franses and van Dijk (2000) and Mills (1999).

The organization of this paper is as follows: In Section 2 we describe the STAR-type models and show that they possess some desirable features for modelling stock market returns. Section 3 discusses procedures used for specifying, estimating and evaluating such models. In Section 4 we report our results of fitting multivariate STAR models to quarterly UK stock market data, interpret the estimated models, discuss our findings and compare forecasts. Finally a few concluding remarks are stated in Section 5.

2. Definition of smooth transition (auto)regressive models

The 2-regime STAR model of order p is defined as,

$$y_t = \boldsymbol{\varphi}'_0 \mathbf{w}_t + (\boldsymbol{\varphi}'_1 \mathbf{w}_t) F(s_t;\gamma,c) + u_t, \qquad \{u_t\} \sim iid(0,\sigma^2) \quad (1)$$

where $F(s_t;\gamma,c)$ is the transition function bounded by zero and unity and s_t is the transition variable (determined in practice). The parameter c is the threshold and gives the location of the transition function, while γ defines the slope of the transition function. In (1), $\mathbf{w}_t = (1, y_{t-1}, ..., y_{t-p})'$ is the vector of explanatory variables consisting of an intercept and the first p lags of y_t, and $\boldsymbol{\varphi}_0 \equiv (\phi_{00},...,\phi_{0p})'$ and $\boldsymbol{\varphi}_1 \equiv (\phi_{10},,...,\phi_{1p})'$ are $(p+1)\times 1$ parameter vectors. In the empirical results in Section 4, $\boldsymbol{\varphi}'_0 \mathbf{w}_t$ and $\boldsymbol{\varphi}'_1 \mathbf{w}_t$ are labelled Part 1 and Part 2, respectively. It is straightforward to extend the model and allow for 'exogenous' variables as additional regressors. In this case, the model is called smooth transition regression (STR) model (Teräsvirta, 1998). In the STAR model as discussed in Teräsvirta (1994), the transition variable is assumed to be the lagged dependent variable. In our work, however, we allow the transition variable to be either a past value of the dependent variable or of an exogenous variable.

One form of transition function used in the literature is, the logistic function

$$F_L(s_t;\gamma,c) = \left(1+\exp(-\gamma(s_t - c))\right)^{-1}, \qquad \gamma > 0 \quad (2)$$

where (1) and (2) yield the logistic ST(A)R. The logistic function is monotonically increasing in s_t, with $F_L(s_t;\gamma,c) \to 0$ as $(s_t - c) \to -\infty$ and $F_L(s_t;\gamma,c) \to 1$ as $(s_t - c) \to +\infty$. In this work, we explore the idea that there are two distinct regimes in financial markets, namely bull markets and bear markets. In stock market terminology, bull (bear) market corresponds to periods of generally increasing (decreasing) market prices. Thus, bull (bear) markets are associated with periods when the returns are positive (negative). The LST(A)R specification can describe a situation where the bear markets (values of $F_L(s_t;\gamma,c)$ 'close' to zero) and the bull markets (values of $F_L(s_t;\gamma,c)$ 'close' to unity) phases of financial markets may have different dynamics. The slope parameter indicates how rapid the transition from 0 to 1 is as a function of s_t and c determines where the transition occurs. When $\gamma \to \infty$, $F_L(s_t;\gamma,c)$ becomes a step function and the transition between the regimes is abrupt. In that case, the model approaches a (SE)TAR model (Tong, 1990).

Monotonic transition might not always be successful in applications. The second function proposed by Teräsvirta and his co-authors is, the exponential function

$$F_E(s\ ;\gamma,c) = 1-\exp(-\gamma(s_t - c)^2)\,,\qquad \gamma > 0 \quad (3)$$

where (1) and (3) give rise to the exponential ST(A)R. The EST(A)R model may be interpreted as a generalization of the earlier EAR model of Haggan and Ozaki (1981), the more restrictive EAR case obtained under $c = \phi_{10} = 0$, that restriction making the EST(A)R model location invariant. The transition function is symmetric around c which makes the local dynamics the same for high and low values of s_t, whereas the mid-range behaviour is different. When $\gamma \to \infty$, then $F_E(s_t;\gamma,c) \to 1$ except a narrow range of values around the threshold. Thus for large values of γ it is difficult to distinguish an EST(A)R model from a linear one. It is not immediately obvious that the ESTAR model can capture stock market characteristics, since it would imply the same response to both bull and bear markets. It may be more appropriate for capturing distinctive responses to periods of 'extreme' (bull or bear) markets versus periods of more 'normal' markets.

3. Modelling procedure

Our modelling procedure is based on that proposed by Teräsvirta and his co-authors mentioned in the introduction but more systematically uses grid search procedures for the selection of the appropriate transition variable in the spirit of Öcal and Osborn (2000). Another difference is that we rely heavily on estimation of a linearised version of the ST(A)R model in which the transition function is fixed. The use of the linearised model, based on the work of Mejía-Reyes, Osborn and Sensier (2000), speeds model specification and we have found this procedure to work well in practice. Our modelling procedure consists of the following stages.

3.1 Specification of the linear model and linearity tests

Testing linearity against ST(A)R constitutes the first step of the modelling procedure. In order to test for linearity we first select a linear model with residuals, which are approximately white noise. We start with an order of 8 lags on all variables. A general-to-specific procedure is applied where the least significant (if non-significant) variable at any lag is dropped at each stage and the reduced linear model is re-estimated. Teräsvirta suggests that the lag order of the model could be determined by an order selection criterion such as the Akaike criterion (AIC). The selected linear model obtained by the general-to-specific procedure and based on the AIC is assumed to form the null hypothesis for testing linearity.

The problem of testing linearity against ST(A)R alternatives was addressed in Luukkonen, Saikkonen and Teräsvirta (1988). The test, to be referred as the particular LST(A)R linearity test in our work is carried out for different candidate transition variables. It can be obtained from the following auxiliary regression,

$$y_t = \boldsymbol{\beta}'_0 \mathbf{w}_t + \boldsymbol{\beta}'_1 \overline{\mathbf{w}}_t s_t + \boldsymbol{\beta}'_2 \overline{\mathbf{w}}_t s_t^2 + \boldsymbol{\beta}'_3 \overline{\mathbf{w}}_t s_t^3 + u_t^* \qquad (4)$$

Saikkonen and Luukkonen (1988) suggest testing linearity against EST(A)R alternative by using the auxiliary regression,

$$y_t = \boldsymbol{\beta}'_0 \mathbf{w}_t + \boldsymbol{\beta}'_1 \overline{\mathbf{w}}_t s_t + \boldsymbol{\beta}'_2 \overline{\mathbf{w}}_t s_t^2 + u_t^* \qquad (5)$$

where (5) is a restricted version of (4). This test is referred as the particular EST(A)R linearity test in our case.

Teräsvirta (1994) suggests that the above tests can also be used to select the appropriate transition variable. The statistic in (4) or (5) is computed for several candidate transition variables and the one for which the p-value of the test is smallest (strongest rejection of linearity) is selected as the true transition variable. Teräsvirta also provides a heuristic justification for using these tests (using a sequence of tests of nested hypotheses) to make the decision about the choice between LST(A)R and EST(A)R. Our decision, however, about the transition variable is based more systematically on the grid search procedure explained in next part. Further, we estimate both logistic and exponential versions of the model and choose between them at the evaluation stage.

We also assume that the transition variable is unknown and carry out the general linearity test as in Luukkonen et al (1988). However, our Luukkonen Saikkonen Teräsvirta (LST) test is a more parsimonious version of their economy version test and involves an auxiliary regression where the squared and cubed terms (not the cross products) of explanatory all variables are added and jointly tested for significance. This test referred as the LST test is reported together with Ramsey's RESET test based on squared and cubed fitted values.

3.2 Initial estimates and non-linear estimation

Once linearity is rejected against ST(A)R, the second stage in the modelling cycle is to select the appropriate transition variable and proceed to estimate the parameters of the model. For each candidate transition variable, a two-dimensional grid search is carried out using at least 250 values of γ (1 to 250 with the range extended if the minimizing value of γ is close to 250) and 40 equally spaced values of c within the observed range of the transition variable. Essentially, the transition variable series is ordered by value, extremes are ignored by omitting the most extreme 10 values at each end and 40 values are specified over the range of the remaining values. This procedure attempts guarantee to that the values of the transition function contain enough sample variation for each choice of γ and c. The model with the minimum RSS value from the grid search procedure is used to provide the γ, c and s_t for an initial estimate of the transition function. Note that the grid search procedure is carried out for both LST(A)R and EST(A)R specifications. Following Teräsvirta (1994) the exponent of the transition function is standardised by the sample standard deviation (LST(A)R model) or the sample variance (EST(A)R model) of the transition variable. This standardisation makes γ scale-free and helps in determining a useful set of grid values for this parameter.

Reducing the order of the model in the non-linear least squares (NLS) framework is obviously a computationally heavy procedure. However, there is another practical strategy one can follow. Note that giving fixed values to the parameters of the transition function makes the ST(A)R model linear in the remaining coefficients. The grid search mentioned above is used to obtain sensible initial values. Conditional on this transition function, the parameters of the ST(A)R model can be estimated by OLS and we call this model the linearised version of the ST(A)R model. To determine the order of the linear ST(A)R we follow a general-to-specific procedure and the selected model is based on the AIC criterion. The estimated coefficients from the linear ST(A)R along with the transition function parameters from the grid search are used as initial values in the non-linear estimation in the next stage. The preferred model is re-estimated (including the transition function parameters) by NLS in GAUSS using the Newton-Raphson algorithm and in RATS using the BHHH algorithm. However, we have found the BHHH algorithm to be preferable in practice[1]. After estimating the parameters of the ST(A)R, these are compared with those obtained from the linearised version since the latter is used for model specification.

3.3 Evaluation of ST(A)R models

The validity of the assumptions underlying the estimation must be investigated once the parameters of the STR models have been estimated. We employ the Lagrange multiplier (LM) tests of Eitrheim and Teräsvirta (1996) derived for this purpose. As usual, the assumption of no error autocorrelation should be tested. Further, it is useful to find out whether or not there are non-linearities left in the process after fitting a STR model. That possibility is investigated by testing the hypothesis of no additive non-linearity against the alternative hypothesis that there is an additional STR component. Finally, the constancy of the parameters is tested against the hypothesis that the parameters change monotonically and smoothly over time. All the tests are carried out by auxiliary regressions. For details see Eitrheim and Teräsvirta (1996). Model evaluation also includes checking whether the estimates seem reasonable, and of course, checking the residuals for ARCH and normality.

3.4 Forecasting

Comparison of the forecasts from a ST(A)R model with those from a benchmark linear model might enable us to determine the added value of the non-linear features of the model. In this study, we consider one-step-ahead forecasts over the 1990:Q1-1999:Q3 and 1997:Q1-1999:Q3 periods, with the latter being a true out-of-sample comparison. The forecasts are generated as follows. After generating a first one-step

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[1] See for example the estimated standard errors of the coefficients of the models presented in Section 4.

ahead forecast for the first period, one observation is added, the estimates of the equation are updated and a second one-step ahead forecast for the second period is produced, and this is continued until the end of the sample.

The forecasts are evaluated according to three criteria, namely the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE) values and the Direction-of-Change criterion. Simply comparing the values of RMSE or MAE does not give us any idea of the significance of the difference. We therefore report the Diebold and Mariano (1995) (DM) predictive accuracy tests based on the squared prediction errors. In both cases, the null hypothesis is that there is no significant difference in the accuracy of the competing linear and ST(A)R models. We also report RMSE and MAE values for the competing models conditioning on being in a particular regime. That is, the forecasts are grouped depending on whether the transition function of the ST(A)R model is larger (bull markets) or smaller (bear markets) than 0.5. The direction of change results report the number of times where positive or negative stock returns are correctly indicated by the forecast (Total), along with the number of times positive and negative returns separately are correctly indicated by the models ($y>0$ and $y\leq 0$, respectively). The conditional forecasts and the directional change criterion are of particular interest since recent empirical studies have indicated that the forecast performance of regime-switching models depends on the regime in which the forecast is made (see for example, Pesaran and Potter, 1997 and Clements and Smith, 1999). Further, the directional change criterion can be particularly relevant for asset returns as investors may be more interested in accurate forecasts of the direction in which the stock market is moving than in the exact magnitude of the change. For this purpose, we also calculate the Pesaran and Timmermann (1992) (PT) nonparametric test for a comparison between the direction of change results, with the null hypothesis that each set of forecasts and the actual values are independently distributed[2].

4. Empirical results

This section presents three empirical applications of STR models and provides evidence on modelling UK stock returns non-linearities in a multivariate framework. Many recent studies conclude that stock returns can be predicted by time series data on economic variables; see Fama (1981, 1990), Schwert (1990), Black and Fraser (1995), Clare and Thomas (1992) and Pesaran and Timmermann (1995, 2000) among others. Further, in Chapter 2 of the Ph.D. thesis of the first author (Aslanidis) we also find financial and macroeconomic variables that characterize the evolution of UK stock market returns. For example, the US Standard & Poor's 500 (S&P-500) index appears to be the most significant among various financial and macroeconomic variables considered. We also find a role for the UK economic activity, interest rates, inflation and money supply in predicting UK stock prices.

To represent the UK stock market, we consider the Financial Times (FT) Actuaries All Share Index. For parsimony, we initially start with two explanatory variables. On the one hand, Gross Domestic Product is employed to represent UK economic activity, whereas on the other hand, S&P-500 is representative of the US stock market. Next we extend the logistic specification to include the UK short-term interest rates, inflation and broad money supply. In detail, the series we consider are the nominal Financial Times (FT) Actuaries All Share Index (10 April 1962=100), seasonally adjusted GDP in constant prices, nominal Treasury Bills 3-month yield

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[2] Because of the small forecast sample size in 1997-1999, the Diebold-Mariano, Pesaran-Timmermann

(TBY), Retail Price Index: All Items (1985=100) (RPI), seasonally adjusted Money Stock M4 (M4) and nominal US S&P-500 Composite Price Index (SP) at quarterly frequencies. The data is obtained from Office of National Statistics and Datastream. All variables employed are used in the form of first differences of the logarithms, except GDP and M4, which are transformed to D4DLnGDP (difference over 4 quarters of DLnGDP) and D6DLnM4 (difference over 6 quarters of DLnM4), respectively; these transformations are strongly supported by the data in a preliminary study[3]. The FT series is shown in Graph 1. In general, mid 1970s has been more turbulent than the remaining parts of the series. An exception to this rule is the stock market crash in October 1987. We use the series up to 1996:Q4 for estimation and testing and allow for a maximum of $p_{\max} = 8$ lagged first differences, such that effective estimation sample runs from 1967:Q2 until 1996:Q4 (119 observations).

4.1 Specification and estimation results

We start with a fully parameterized linear model allowing for a maximum of order of eight lags on DLnFT, D4DLnGDP and DLnSP variables. The selected AIC model obtained by the general-to-specific methodology is reported in the first column of Table 3. The diagnostics suggest the absence of ARCH components and serial correlation. It is not straightforward to interpret the estimated coefficients. In particular, the model implies negative endogenous effect after five and six quarters, positive after eight quarters, but negative overall. Next, the relationship of DLnFT with D4DLnGDP is of particular interest. It is seen that the nature of the effects of D4DLnGDP depends on the specific lag. Economic activity has positive effect after two and three quarters, but negative after a year. An expected result is that the

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tests as well as the conditional forecasts are computed only for the 1990-1999 period.

DLnSP_1 variable, which is the most significant, is positively associated with DLnFT. The model is estimated to have a positive intercept $\hat{\phi}_{00} = 0.022$ and is able to explain 37% of the variation in the dependent variable. In general, the linear specification seems an acceptable model.

The next stage is to test linearity against general and particular non-linearity. The linearity tests are displayed in Table 1, while Table 2 reports the grid search results. Note that the p-value (0.016) of the LST test indicates that linearity can be rejected. In particular linearity tests, the null is rejected in five out of eight cases. The strongest evidence of LSTR non-linearity occurs when D4DLnGDP_5 is used as the transition variable, while the lowest ESTR non-linearity p-values correspond to D4DLnGDP_5 or DLnFT_6. Admittedly, the statistical evidence of non-linearity is quite strong.

Based on the decision rule of the procedure of Teräsvirta (1994), the linearity tests suggest that D4DLnGDP_5 is the most appropriate of all potential transition variables in the case of LSTR. The grid search results, however, show that RSS is minimized when DLnSP_1 is considered as the switching variable. This finding contradicts the particular linearity test (p-value is 0.246). On the other hand, the inference about DLnFT_6 suggested by the particular ESTR non-linearity test is consistent with that implied by the grid search procedure.

The 2-regime AIC STR models estimated in RATS are presented in the second and third columns of Table 3, while the corresponding GAUSS and linear STR specifications are shown in Table I in the Appendix. The estimated parameters of the models are close to those obtained from the grid search and the linear estimation of STR. The conspicuous detail, however, is the extremely small standard errors of the

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[3] Based on Chapter 4 of the Ph.D. thesis of the first author.

estimated parameters in GAUSS yielding enormous t-ratios. This result is in line with findings in Sensier, Osborn and Öcal (2001). On the other hand, it turns out that the BHHH algorithm produces reasonable standard errors and close to those obtained from the linear estimation of STR. It is also worth noticing that in the LSTR model, it turns out that the two programs produce different coefficients as well. Here, we have to state that the estimation of the slope parameter γ_L causes a lot of problems. In particular, joint estimation of all parameters does not work using the BHHH algorithm. To facilitate the estimation, we lower the initial value of γ_L down to 40, but convergence is still not reached. In such a case, we follow the recommendation of Eitrheim and Teräsvirta (1996) by fixing γ_L at a sufficient large value to get a step-shape transition function and estimate the remaining parameters of the model.

Both the LSTR and ESTR equations contain restrictions of the form $\phi_{0j} = -\phi_{1j}$, which are strongly suggested by the data. This means that the corresponding variables operate only when $F = 0$ or in the transition between the extremes. There are a few insignificant variables, but removing them has an adverse effect on the fit. According to the diagnostics the STR models form statistically adequate representations of the data since there is no sign of model inadequacy. In particular, there is no evidence of autocorrelation. The additive non-linearity test results imply that the models capture all non-linearities and therefore no two transition function models are examined. Further, the models pass the parameter constancy tests. Tests of no dynamic heteroskedasticity do not indicate any problem either. The LSTR model can account for the leptokurtosis more adequately than the ESTR one and is preferable according to the R^2 value. However, as measured by the σ, AIC and SBC values, the ESTR model represents better the dynamics of DLnFT.

Graphs 2-3 display the transition functions. The estimated slope parameter values for each model imply very different dynamics around the threshold parameters. According to the first model, the large value of the slope parameter implies almost instantaneous switch and consequently, the LSTR approaches very well a TAR model. This can also be seen in the second panel of Graph 2, where the transition function fluctuates only between zero and one; there are no intermediate values. The value of $\hat{c}_L = -0.003$ indicates approximately halfway point between the extremes. Thus, we can identify two UK stock market regimes associated with negative and positive values of DLnSP_1.

Different patterns are evident from the ESTR model where the small value of the switching parameter $\hat{\gamma}_E = 1.668$ implies that the "inner" regime and its associated coefficients apply over a relatively wide range of values. Actually, this can also be seen in the graph of the exponential function over time where a lot of the sample lies within the intermediate transition phase implying a smooth switch from one regime to other. Notice also that with few observations far beyond and to the left of the location parameter, we may have a situation where only the right side of F_E matters (Teräsvirta, 1994, Öcal and Osborn, 2000). Thus, in practice, this ESTR model behaves very similar to an LSTR one and a smooth transition from one regime to the other occurs for values of DLnFT_6 around zero. In such a case, F_E around zero can be associated with bear markets, and F_E close to one the bull market regime. In other words, F_E is effectively operating as one-sided exponential function.

From the information shown in Table 3 it can be seen that the STR specifications make a contribution in explaining FT returns over the linear model. The AIC and SBC values decrease while in terms of R^2 and σ the improvement is 14%

and 11 percentage points, respectively. These features, however, are partially reflected in the residuals of the linear and STR models (not shown). The non-linear models represent marginally better the period of 1974-75 (as seen from the size of the corresponding residuals). Interestingly, the STR (and linear) models are not able to foresee the large drop in 1987:Q4 associated with the market crash.

In the LSTR model, the implication of the estimated coefficients is that US bear markets are associated with an intercept $\hat{\phi}_{00} = 0.052$ while US bull markets imply an intercept $\hat{\phi}_{00} + \hat{\phi}_{10} = -0.004$ (effectively zero) for FT returns. At the extremes, the LSTR model implies, when $F_L = 1$:

DLnFT = - 0.009 - 0.095DLnFT_5 - 0.128DLnFT_6 + 0.198DLnFT_8+ 0.791DLnSP_1 - 1.644D4DLnGDP_5 + u

and when $F_L = 0$:

DLnFT = 0.053 - 0.095DLnFT_5 - 0.128DLnFT_6 + 0.791DLnSP_1 + 1.456D4DLnGDP_2 + 2.938D4DLnGDP_3 - 1.955D4DLnGDP_7 + u

It is seen that for increases in SP_1, the model implies negative D4DLnGDP effects at lag five whereas decreases in SP_1 lead to a different model with positive and richer overall D4DLnGDP effects. As anticipated, the invariant coefficient of DLnSP_1 is positive.

On the other hand, at the extremes the ESTR model implies, when $F_E = 1$:

DLnFT = 0.011 - 0.190DLnFT_5 + 0.139DLnFT_8 + 0.454DLnSP_1 + u

and when $F_L = 0$:

DLnFT = 0.049 - 0.190DLnFT_5 + 0.139DLnFT_8 + 0.454DLnSP_1 + 3.604D4DLnGDP_2 + 2.575D4DLnGDP_3 – 2.174D4DLnGDP_5 - 3.245D4DLnGDP_7 + u

In the ESTR case, both extremes are associated with positive intercepts, though the magnitude is larger when $F_E = 0$. Asymmetry is also implied by the ‘low’ phase, which is associated with richer explanatory dynamics than those of the ‘upper’ phase; the D4DLnGDP variables come into effect only when $F_E = 0$ and between the extremes. Notice also that the transition variable DLnFT_6 is not included as a regressor in the model. A comparison of the STR models with their corresponding linear model one reveals that the non-linear specifications imply more profound effects of DLnSP and D4DLnGDP on the DLnFT series.

Next we present an interesting LSTR model, which has richer explanatory dynamics than those considered previously. It can be seen as an extension of the LSTR model. Basically, the logistic specification is of particular interest since it is found that the US stock market drives FT regimes. In the ESTR model, on the other hand, the regimes are associated with endogenous dynamics. The starting LSTR equation includes the following variables in both parts of the model: DLnFT_2, DLnFT_3, D4DLnGDP_3, DLnTBY_1, DLnSP_1, DLnRPI_2 and D6DLnM4_1. These variables and their particular lags are chosen on the base of the accumulated evidence found in Chapter 4 of the Ph.D. thesis of the first author.

Table 4 reports the final model (estimated in RATS) selected by the minimized AIC value[4]. The GAUSS and OLS based models are reported in Table II in the Appendix. Although, not supported by the grid search results (D6DLnM4_1 appears as the most appropriate transition variable), we assume in advance that DLnSP_1 acts as the switching variable. This way is followed to connect the extended LSTR (labelled LSTR2) model with the previous LSTR specification. The estimated model appears quite representative of the data as suggested by the diagnostic tests. Nevertheless, its fit as measured by the R^2, σ or/and AIC/SBC values is worse than the fit of the LSTR specification presented in Table 3. There are also some hints of additional non-linearity associated with DLnFT_2, but it is not very strong and given the number of tests it does not cause much concern.

The estimated transition function implied for the model is plotted in Graph 4. Interestingly, the transition function has almost the same estimated location parameter ($\hat{c}_L = -0.006$) as the LSTR specification in Table 3. It is effectively centered at zero, hence implying that increases and decreases in SP_1 have asymmetric effects on FT returns. As to the estimated slope parameter value, this is greatly affected from the re-specification of the model. The switch from one FT regime to the other is less steep compared with the previous model (i.e. $\hat{\gamma}_L = 12.82$).

The LSTR2 specification implies that the US stock market initiates asymmetries in the FT process. In particular, the interaction term between the transition function and DLnSP_1 has a positive coefficient of 0.389, implying that increases in SP_1 have an effect of greater magnitude than decreases. Another implication is that DLnRPI_2 comes into effect only when $F_L = 1$. Further, the D4DLnGDP_3 and D6DLnM4_1 variables provide information only when $F_L = 0$ and in the transition period between the extremes. It is also interesting to notice that in general the sign of the coefficients of the explanatory variables is consistent with findings in the literature. Particularly, DLnTBY_1 and DLnRPI_2 enter with negative coefficients and this supports Fama (1981), Breen, Glosten and Jagannathan (1989) and Pesaran and Timmermann (1995, 2000). We also find a positive effect for D4DLnGDP_3. This result is consistent with the belief that changes in output, which affect expected future cash flows, have a positive effect on stock prices (Fama, 1990). As to the money supply, Pesaran and Timmermann (2000) find a negative association between UK money supply and stock prices. However, monetary growth may provide

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[4] To economise on space we do not report results for the corresponding linear model and linearity tests.

a stimulus to economic growth, which is likely to increase stock prices. Thus, the positive effect of D6DLnM4_1 is expected.

4.2 Forecasting comparisons

The forecasting results are reported in Table 5. The post-sample period forecasts suggest that the best model is the LSTR in terms of RMSE and MAE. In terms of directional changes the LSTR, LSTR2 and linear equations deliver similar accuracy forecasts, correctly predicting the sign of FT returns in 10 out of 11 quarters. It is, however, striking the bad performance of the LSTR2 model in terms of RMSE and MAE criteria (more than 30% larger than those obtained from the LSTR model). On the other hand, according to the 1990:Q1-1999:Q3 forecast period results, the linear model beats the corresponding non-linear versions. The linear model provides the smallest RMSE and MAE values, with approximately 21 percentage points gain over the ESTR specification (the worst model). However, the p-value of the DM test suggests that there is no significant difference between the forecasting ability of STR models and the linear model. In terms of directional changes, the best models are the linear and the LSTR2 specifications. As to the PS test the null hypothesis that the forecasts and actual values are independent is rejected, which implies good predictive performance for all models[5]. As to the conditional forecasts, Table 5 shows that the linear model (surprisingly perhaps) appears statistically more adequate than the STR models with the latter performing particularly bad during bear markets (for example, the RMSE and MAE values of ESTR are more than 40% higher than those obtained from the linear model). Overall, the forecasting results are not in agreement with the statistical adequacy of the non-linear models shown in the previous part. In the

[5] Note that for the linear and LSTR2 models the rejections are strong.

financial literature, this finding can be compared and contrasted with the mixed results in McMillan (2001) or the evidence of forecast gains from STAR models found in Sarantis (2001).

5. Conclusions

The empirical results of this paper can be summarized as follows: We have estimated acceptable STR models for UK stock market returns where the linearity hypothesis is strongly rejected. The STR models describe the in-sample movements of the FT series better than the linear model. Nevertheless, the STR cannot improve over the linear model in terms of forecasting. The estimates of the slope parameters indicate that the speed of the transition from one regime to the other is rather smooth, except the LSTR case. This is in contrast to the simple threshold models, which assume a sharp switch. The US stock market appears to play an important role in determining FT regimes, which reflects strong interdependence between UK and US stock markets. Overall this study has shown that there are financial and macroeconomic variables, which contain predictive information for stock returns in a non-linear framework. This complements the results in McMillan (2001) who provide evidence of STR predictability of stock returns by using mainly interest rates.

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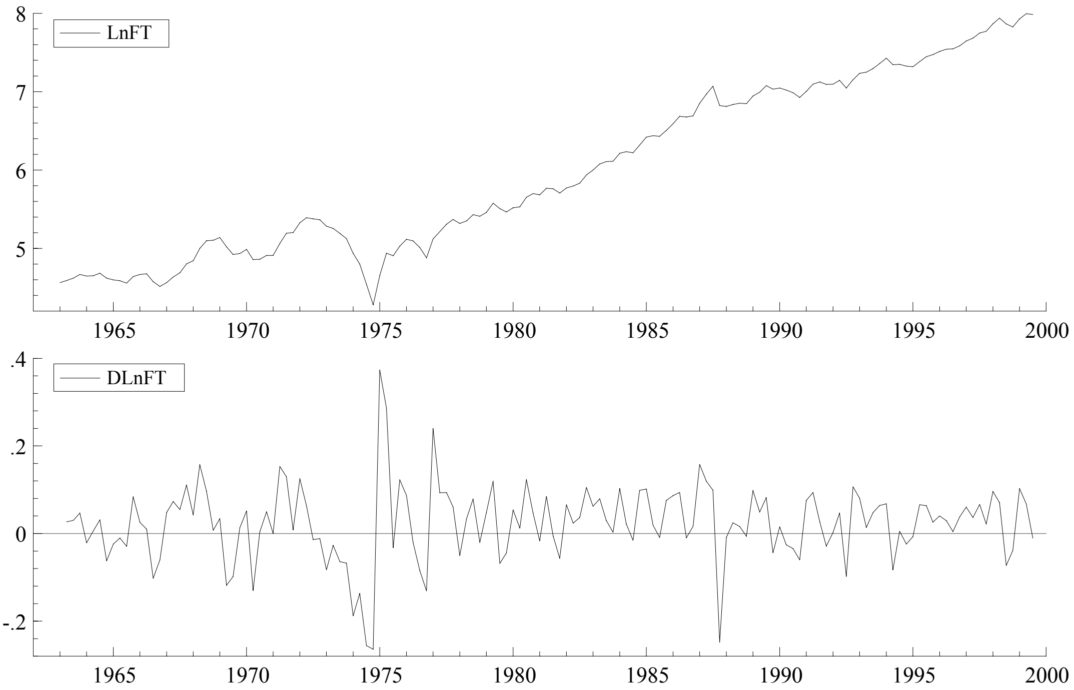
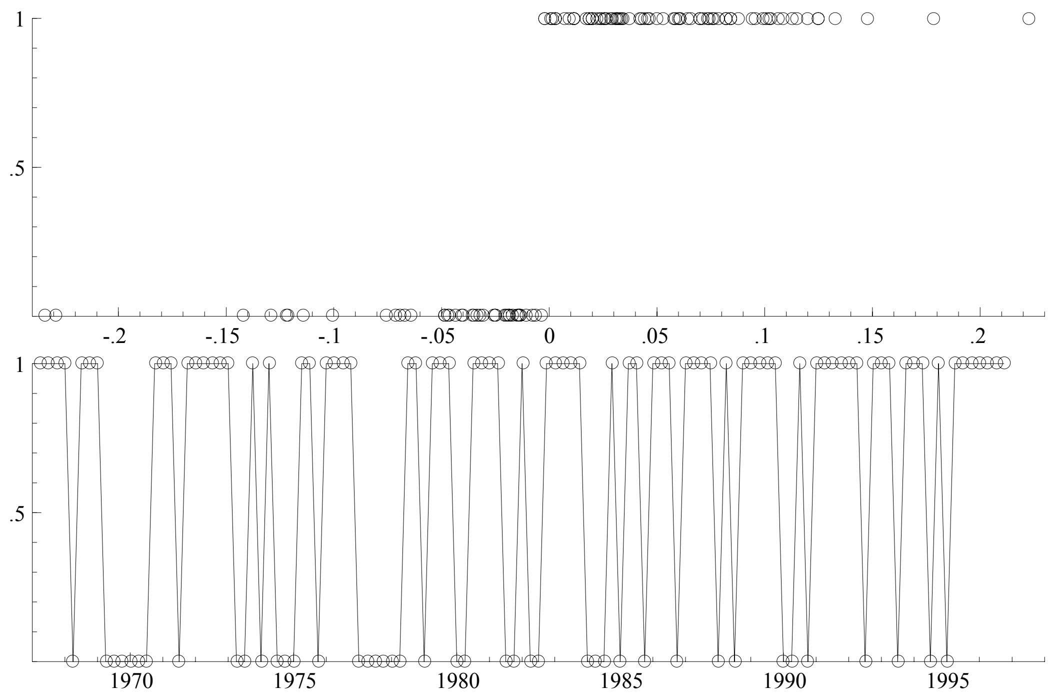
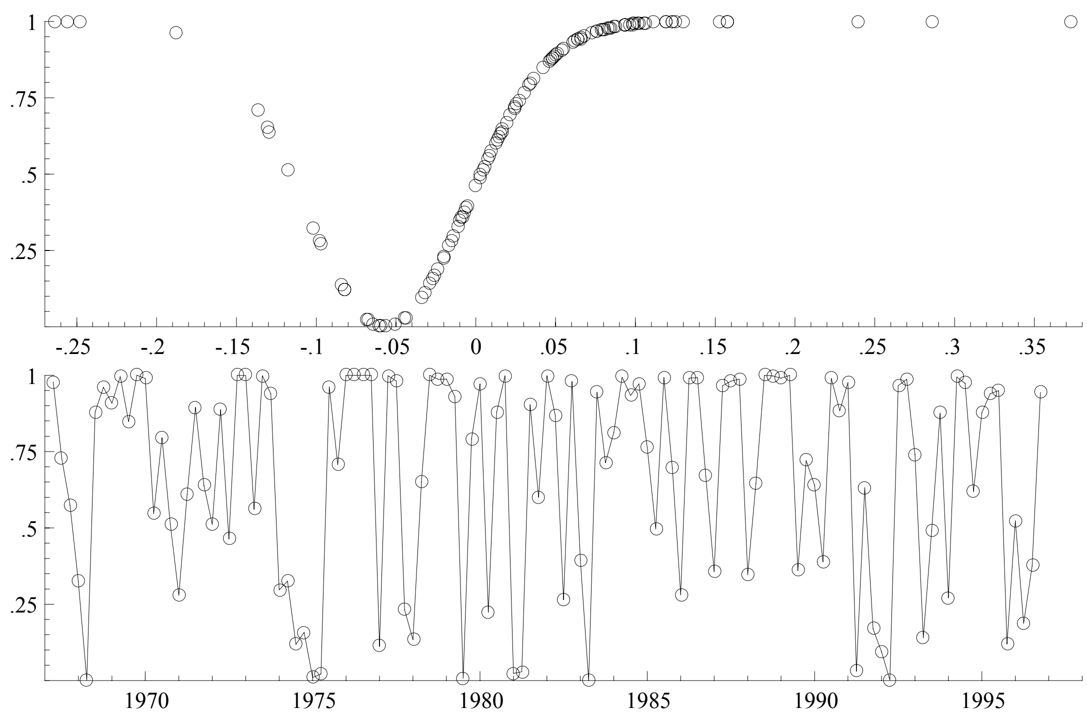


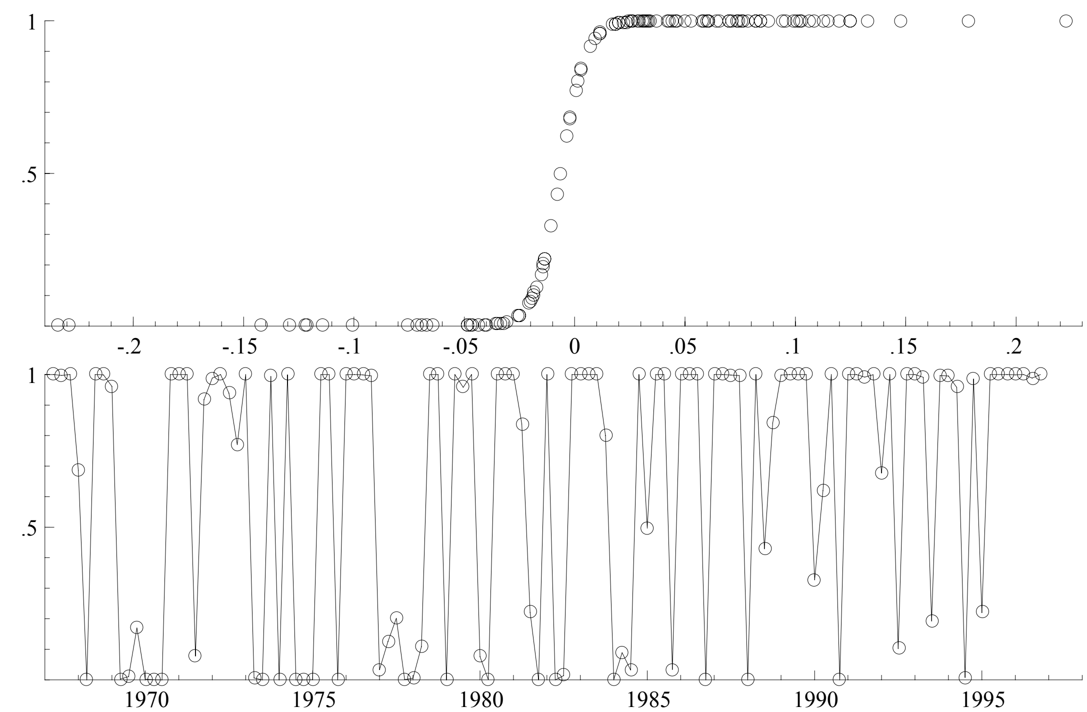
Graph 1: Quarterly observations on the log-level (upper panel) and returns (lower panel) of the UK FT series, 1963:Q2-1999:Q3.



Graph 2: Logistic function of LSTR model versus DLnSP_1 (upper panel) and over time (lower panel).



Graph 3: Exponential function of ESTR model versus DLnFT_6 (upper panel) and over time (lower panel).



Graph 4 Logistic function of extended LSTR2 model versus DLnSP_1 (upper panel) and over time (lower panel).

Table 1: Linearity tests: Linear model versus STR models.

General linearity <u>tests</u>		
RESET	0.393	
LST	0.016	
Particular linearity <u>tests</u>		
Transition variable	LSTR non-linearity	ESTR non-linearity
DLnFT_5	0.185	0.141
DLnFT_6	0.062	0.006
DLnFT_8	0.011	0.095
DLnSP_1	0.246	0.184
D4DLnGDP_2	0.028*	0.028
D4DLnGDP_3	0.013*	0.013
D4DLnGDP_5	0.006*	0.006
D4DLnGDP_7	0.039*	0.039

Notes: p-values of the F-variants of the LM-tests for STR type non-linearity using the preferred linear specification as a base model; the selection of the linear model is made using the AIC criterion; the transition variable in the particular non-linearity tests is assumed known; the asterisk (*) indicates that in these tests, the cubed terms are omitted from the regressors of the auxiliary regressions since they are very small and create near singularity of the moment matrix, omitting them does not affect the properties of the test statistics.

Table 2: Grid search results for the specification of the 2-regime STR models.

	LSTR			ESTR		
s_t	γ	c	RSS	γ	c	RSS
DLnFT_5	950	-0.065	0.4772	8	-0.098	0.4997
DLnFT_6	63	-0.005	0.5039	**2**	**-0.049**	**0.4813**
DLnFT_8	50	-0.070	0.5147	5	-0.081	0.4887
DLnSP_1	**950**	**-0.004**	**0.4742**	4	-0.069	0.4925
D4DLnGDP_2	1	-0.015	0.4977	1	0.019	0.5372
D4DLnGDP_3	950	0.019	0.4916	1	-0.015	0.5341
D4DLnGDP_5	8	-0.015	0.4749	1	-0.004	0.5433
D4DLnGDP_7	950	-0.015	0.5245	100	-0.007	0.5142

Table 3: Linear and 2-regime STR models.

Variable	Linear	LSTR Part 1	LSTR Part 2	ESTR Part 1	ESTR Part 2
Con	0.022	0.053	-0.062	0.049	-0.038
	(2.748)	[3.948]	[-2.826]	[3.079]	[-1.919]
DLnFT_5	-0.202	-0.095		-0.190	
	(-2.490)	[-1.285]		[-2.609]	
DLnFT_6	-0.178	-0.128			
	(-2.166)	[-1.734]			
DLnFT_8	0.169		0.198	0.139	
	(2.117)		[2.117]	[1.922]	
DLnSP_1	0.497	0.791		0.454	
	(5.030)	[4.522]		[4.969]	
D4DLnGDP_2	0.935	1.456	-1.456	3.604	-3.604
	(1.879)	[2.244]	[-2.244]	[3.606]	[-3.606]
D4DLnGDP_3	1.823	2.938	-2.938	2.575	-2.575
	(3.178)	[2.880]	[-2.880]	[2.527]	[-2.527]
D4DLnGDP_5	-1.026		-1.644	-2.174	2.174
	(-2.069)		[-2.892]	[-2.789]	[2.789]
D4DLnGDP_7	-0.958	-1.955	1.955	-3.245	3.245
	(-1.751)	[-2.432]	[2.432]	[-3.010]	[3.010]
s_t / γ / c		DLnSP_1 / 40 / -0.003 [-0.043]		DLnFT_6 / 1.668 / -0.057 [1.861] [-4.246]	
AIC / SBC	-5.085 / -4.875	-5.289 / -5.009		-5.305 / -5.048	
R-sq / σ	0.3688 / 0.0759	0.5107 / 0.0677		0.5099 / 0.0675	
Diagnostics					
Skewness	-0.220	-0.256		-0.812	
Ex kurtosis	1.584	1.181		2.008	
Normality	0.001	0.017		0.000	
ARCH(4)	0.220	0.762		0.919	
Autocorrelation(4)	0.779	0.881		0.985	
Non-linearity					
DLnFT_5		0.615		0.885	
DLnFT_6		0.286		0.713	
DLnFT_8		0.288		0.202	
DLnSP_1		0.725		0.812	
D4DLnGDP_2		0.092		0.119	
D4DLnGDP_3		0.079		0.186	
D4DLnGDP_5		0.547		0.570	
D4DLnGDP_7		0.780		0.756	
Constancy					
All	0.670	0.167		0.179	
Intercept	0.796	0.417		0.674	
Both intercepts		0.648		0.731	

Notes: Estimation period 1967:Q2-1996:Q4; the STR models are estimated by BHHH RATS algorithm; values in parentheses are t-ratios; diagnostic test results are presented as p-values; AIC and SBC are the Akaike and Schwarz Information Criteria values based on RSS; R-sq is the usual coefficient of determination; σ is the estimate of the residual standard deviation adjusted for degrees of freedom; skewness and ex. kurtosis are measured by conventional test statistics; normality refers to the test of Jarque and Bera (1980) for linear models, and to that of Lomnicki (1961) and Jarque and Bera (1980) for non-linear models; ARCH(4) is the LM test of Engle (1982) and considers ARCH effects of order 4; autocorrelation(4) is the LM test of residual autocorrelation of Godfrey (1978) and of Eitrheim and Teräsvirta (1996) for linear and non-linear models, respectively; non-linearity (not ignoring "holes") and constancy tests are the LM tests of Eitrheim and Teräsvirta (1996), the alternative to constancy is that the parameters change monotonically; the LSTR model has been estimated using a fixed value of $\hat{\gamma}$ =40 because the algorithm does not converge otherwise; see text for details; in this model the misspecification tests have been computed by omitting the partial derivatives with respect to the transition function parameters from the auxiliary regressions since they render the moment matrix near-singular; see Eitrheim and Teräsvirta (1996) for details.

Table 4: Extended 2-regime LSTR model.

	LSTR2	
Variable	Part 1	Part 2
Con	0.027	
	[2.230]	
DLnFT_2	-0.179	
	[-2.325]	
DLnFT_3		
D4DLnGDP_3	3.926	-3.926
	[5.725]	[-5.725]
DLnTBY_1	-0.105	
	[-2.138]	
DLnSP_1	0.293	0.389
	[1.558]	[1.358]
DLnRPI_2		-1.408
		[-2.281]
D6DLnM4_1	2.214	-2.214
	[2.245]	[-2.245]
s_t / γ / c	DLnSP_1 / 12.82 / -0.006	
	[0.892] [-0.661]	
AIC / SBC	-5.220 / -4.986	
R-sq / σ	0.4574 / 0.0707	
Diagnostics		
Skewness	-0.173	
Ex kurtosis	1.277	
Normality	0.013	
ARCH(4)	0.430	
Autocorelation(4)	0.890	
Non-linearity		
DLnFT_2	0.019	
DLnFT_3	0.062	
D4DLnGDP_3	0.462	
DLnTBY_1	0.972	
DLnSP_1	0.447	
DLnRPI_2	0.674	
D6DLnM4_1	0.658	
Constancy		
All	0.306	
Intercept	0.240	

Notes: Estimation period 1967:Q2-1996:Q4; the model is estimated by BHHH RATS algorithm; see notes of Table 3 for information about the statistics reported in table.

Table 5: Forecast performance.

Measurements	Linear	LSTR	LSTR2	ESTR
Forecast period: 1997:Q1-1999:Q3				
RMSE	0.0340	0.0309	0.0442	0.0355
MAE	0.0266	0.0245	0.0358	0.0278
Direction-of-Change				
Total	10/11	10/11	10/11	9/11
$y > 0$	8/8	8/8	8/8	8/8
$y \leq 0$	2/3	2/3	2/3	1/3
Forecast period: 1990:Q1-1999:Q3				
Unconditional				
RMSE	0.0405	0.0487	0.0454	0.0511
MAE	0.0322	0.0365	0.0343	0.0385
DM		0.412	0.413	0.395
Direction-of-Change				
Total	31/39	30/39	31/39	29/39
$y > 0$	27/28	26/28	27/28	26/28
$y \leq 0$	4/11	4/11	4/11	3/11
PT	0.003	0.011	0.003	0.043
Forecast period: 1990:Q1-1999:Q3				
Conditional 1: LSTR vs Linear	LSTR	Linear		
Bull markets F>0.5 (29 obs)				
RMSE	0.0417	0.0379		
MAE	0.0320	0.0304		
Bear markets F<0.5 (10 obs)				
RMSE	0.0649	0.0471		
MAE	0.0493	0.0372		
Forecast period: 1990:Q1-1999:Q3				
Conditional 2: LSTR2 vs Linear	LSTR2	Linear		
Bull markets F>0.5 (32 obs)				
RMSE	0.0419	0.0395		
MAE	0.0326	0.0322		
Bear markets F<0.5 (7 obs)				
RMSE	0.0588	0.0445		
MAE	0.0421	0.0322		
Forecast period: 1990:Q1-1999:Q3				
Conditional 3: ESTR vs Linear	ESTR	Linear		
Bull markets F>0.5 (27 obs)				
RMSE	0.0417	0.0414		
MAE	0.0308	0.0310		
Bear markets F<0.5 (12 obs)				
RMSE	0.0676	0.0384		
MAE	0.0558	0.0347		

Notes: Forecast evaluation of linear, LSTR, LSTR2 and ESTR models; one-step ahead forecasts; RMSE = Root mean square error; MAE = mean absolute error; the row headed DM contains the p-value of the statistic of Diebold and Mariano (1995) test; this test is based on the squared prediction errors of STR vs linear model; the p-value of the DM statistic comes from the standard normal distribution; the row headed PT contains the p-value of the statistic of Pesaran and Timmermann (1992) test; this statistic is asymptotically normal; in the directional forecasts the first value gives the number of correct forecasts whereas the second value gives the number of observations; bull (bear) markets relate to forecasts for the which the value of the transition function in each STR is larger (smaller) than 0.5 at the forecast origin.

APPENDIX

Variable	Linear LSTR Part 1	Linear LSTR Part 2	Linear ESTR Part 1	Linear ESTR Part 2	LSTR Part 1	LSTR Part 2	ESTR Part 1	ESTR Part 2
Con	0.053 (4.030)	-0.057 (-2.802)	0.049 (3.111)	-0.036 (-1.823)	0.052 (2.139)	-0.056 (-1.287)	0.049 (1.939)	-0.038 (-0.866)
DLnFT_5	-0.120 (-1.618)		-0.186 (-2.556)		-0.111 (-1.375)		-0.190 (-2.330)	
DLnFT_6	-0.136 (-1.779)				-0.131 (-1.614)			
DLnFT_8		0.202 (2.075)	0.131 (1.770)			0.195 (2.092)	0.139 (1.735)	
DLnSP_1	0.722 (5.294)		0.453 (4.832)		0.717 (7.891)		0.454 (4.968)	
D4DLnGDP_2	1.538 (2.535)	-1.294 (-1.501)	4.064 (4.100)	-4.571 (-3.666)	1.520 (6.370)	-1.520 (-6.370)	3.604 (11.83)	-3.604 (-11.83)
D4DLnGDP_3	2.272 (2.824)	-2.116 (-2.090)	2.640 (2.563)	-2.686 (-1.976)	2.274 (9.574)	-2.274 (-9.574)	2.575 (9.708)	-2.575 (-9.708)
D4DLnGDP_5		-1.669 (-2.958)	-2.260 (-2.858)	2.259 (2.069)		-1.664 (-7.203)	-2.174 (-7.838)	2.174 (7.838)
D4DLnGDP_7	-2.050 (-2.635)	1.998 (1.972)	-3.172 (-2.933)	2.890 (2.137)	-2.058 (-7.584)	2.058 (7.584)	-3.245 (-10.00)	3.245 (10.00)
s_t / γ / c	DLnSP_1 / 950 / -0.004		DLnFT_6 / 2 / -0.049		DLnSP_1 / 993.8 / -0.003 (13.73) (-0.050)		DLnFT_6 / 1.668 / -0.057 (6.824) (-1.718)	

Variable	Linear LSTR2 Part 1	Linear LSTR2 Part 2	LSTR2 Part 1	LSTR2 Part 2
Con	0.026 (2.271)		0.027 (1.032)	
DLnFT_2	-0.191 (-2.475)		-0.179 (-2.118)	
DLnFT_3				
D4DLnGDP_3	3.938 (5.973)	-4.577 (-4.846)	3.926 (15.06)	-3.926 (-15.06)
DLnTBY_1	-0.101 (-2.055)		-0.105 (-1.523)	
DLnSP_1	0.284 (1.527)	0.423 (1.487)	0.293 (3.062)	0.389 (2.302)
DLnRPI_2		-1.507 (-2.410)		-1.408 (-5.571)
D6DLnM4_1	2.233 (2.334)	-1.817 (-1.598)	2.214 (7.109)	-2.214 (-7.109)
s_t / γ / c	DLnSP_1 / 10 / -0.004		DLnSP_1 / 12.82 / -0.006 (13.27) (-0.206)	

Table I & II: 2-regime linear STR and non-linear STR models; estimation period 1967:Q2-1996:Q4; the linear STR models are estimated by OLS; the non-linear STR models are estimated by Newton-Raphson GAUSS algorithm; values in parentheses are t-ratios.