Regulating Local Public Utilities by Profit-Sharing*

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Abstract

This paper concerns "profit-sharing" within an incomplete regulatory contract where a municipality delegates a risk-neutral firm to manage a local utility. Together with a price cap regulation (PCR) mechanism, the contract envisages the possibility of the municipality revoking the contract if the firm's profits are percieved "excessively" high. We show that when this threat is credible and the cost of exercising it is not too high, a long-term efficient equilibrium arises which guarantees the firm with an appropriate level of profits. The consequent regulation timing consists of an endogenous regulatory lag where the regulation has a PCR nature, followed by a period of ROR in which the firm is motivated to adjust its price downward to avoid contract recall. We also show that excessive revocation costs make the firm an unregulated monopolist with an infinite regulatory lag where ROR looks like a pure PCR.

Key words: Public utilities, Regulatory contracts, Profit-sharing, Stochastic games.
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1 Introduction

This paper investigates "profit-sharing" device in a regulatory contract signed between a municipality and a private firm for the supply of local public utility services. The type of administrative contract we study envisages for the municipality the temporary delegation of the utility provision to a private operator. While the ownership of the asset is maintained public, "the right to use it" becomes private. This form of delegation is justified by the presence of a residual segment of public utility industries which - notwithstanding technological change - is still a natural monopoly and is likely to remain so in the foreseeable future. The entire traditional business of a firm in a utility industry is no longer defined as a natural monopoly; however, in services such as sewage and fresh water, urban waste and provision of public transport, the problem of access facilities remains and - consequently - the problem of natural monopoly regulation with these residual segments. Here, therefore, delegation of the contract to a private firm becomes an alternative to direct public management or to full privatization of the asset.

Both the municipality and the private firm have potential gains from this delegation contract: on the one side, the private firm has returns guaranteed by the contract exclusivity and, on the other side, the municipality promotes efficiency and better and/or cheaper service injecting - through the private firm provision - technological, financial and managerial resources into the utility supply. The clean-cut allocation of functions between the municipality and the private firm - on the one hand planning, control and regulation of the utility and on the other hand management of the utility - is defined and ruled in the contract: in this perspective, the contract is itself a regulatory mechanism where the municipality has the position of residual decision maker with respect to the private firm as a consequence of property rights which it maintains[1].

In local public utility services, the contract is usually delegated under specific institutional features differently qualified at national level: in France, the country with the longest experience in this field, the *gestion déléguée* in public utility sectors allows for various forms of contracts[2] which are differ-

[1] The municipality's role of residual decision maker can also be related to the need to protect the customers' "right to be served". See about Goldberg (1976).

[2] Among these forms *concession, affermage, régie intéressée, gérence* are the most known and particularly used in the water sector (see about Carles and Dupuis, 1989; Lorraine, 1995).

ently characterized by the degree of delegation and by financial constraints on investments. Similarly, in Italy the *concessione* allows for two forms of contract[3] on the grounds of the relative weight given to planned new investment and to management of the utility itself. In Germany the industrial activities of local public utility (*Daseinsvorgsorgee*) are delegated through different forms of contracts which have to take account for the Lander's specific legislation in these sectors[4].

Notwithstanding their different designs, all these contracts have common features like the definition of a price regulation mechanism, an investment plan and quality objectives in the service provision. Moreover, at juridical level, all these contracts share a dual nature: the administrative (public) nature on the basis of which the municipality holds a favourable position within the contract, and the private nature via which the wishes of both parties are expressed and the agreement is determined. In other words, they are incomplete regulatory contracts where the municipality plays the role of residual claimer in the relationship with the firm whenever future contingencies unspecified in the contract occur.

Our analysis takes its cue from the evidence of these different positions of the two parts in the contract. In particular, we focus on the point that the municipality is able to exercise the role of residual claimer revoking the contract to the private firm: once the revocation applies, the management of the utility is back in the hands of the municipality which can choose from direct management, or privatization of the asset, or delegation of the contract to another private firm.

In the real world, revocation usually refers to the right of the municipality to remove the delegation of the utility provision *a)* in the event of breach of the contract by the private operator (i.e. it may occur when the firm does not respect the terms established in the contract) and *b)* in the case of redemption of the contract by the municipality itself (i.e. it may occur when political pressure - safeguarding collective welfare - induces the municipality to consider the firm's profits as "excessively" high). However, in these two

[3]These forms are called *concessione di costruzione e gestione* and *concessione di erogazione di servizio*: in the former new investment required by contract is primary with respect to the management of the utility, while in the latter management of the utility is the primary aim of the contract itself (see about Mameli, 1998).

[4]The most used forms of delegation contract in Germany are: *Verwaltungshelfer*, *Offentliche Eirichtung in privater Regie, Nutzungsubertragung, Betreiber* (see about Marcou, 1995).

dimensions the municipality's right to revoke has substantially different origins: while in the former it relates to conditions made clear in the contract, in the latter it belongs to the different positions of the two parties in the contract. In both the dimensions, the right of revocation is ruled in specific clauses of the contract where the timing, procedures and possible contractual indemnities belonging to the exercise of this municipality's right are defined[5].

Though in the regulation of firms managing public utility there is a well-developed literature concerning the use and effects of revocation when breach of contract occurs [6], no in-depth analysis has been performed when revocation occurs via exercise by the municipality of its right to redeem the contract. We move from this lack in literature, and investigates revocation when the right of redemption hold[7]: in particular, we consider how the threat of the municipality's revocation affects the private firm's decisions regarding development of its profits. We do this in a simple model where a risk-neutral firm has been delegated to manage an indivisible public project whose profits evolve stochastically over time. Moreover, by the above discussion, we assume that the municipality has the right, at any time, to revoke delegation and return to direct management if the project is a positive net present value investment. In this respect, therefore, revocation is analogous to a contractual claim that displays option-like characteristics where the municipality has the right - but not the obligation - to purchase an asset (the utility) of uncertain value for a present exercise price, and the value of such a claim is derived from the market value of the project. The exercise price refers to the sum necessary to overcome the obstacles to renewing direct management of

[5]It is interesting to stress here that while in Italy and in France these contracts are governed by administrative law, in Great Britain there is no law of public contract and the delegation of public utilities' provision to private operators (i.e. contracting out) is first and foremost a political choice that requires the issue of an *ad hoc* law for its implementation. Moreover, in Great Britain contracts between public authorities and private operators are subject to private law: this means that relations between public authorities and private firms, and between private firms and users of the service, are outlined only within the contract itself.

The model developed here - as will be seen in the following sections - can also be extended to contracting out where, within the contract of delegation to the private firm, there are explicit redemption clauses that reflect those considered in this analysis.

[6]Many contributions on this topic belong to the analogy between a regulatory contract and a law contract in determining appropriated damages to be paid in the event of breach by one of the parties (see about Lyon and Huang, 2000; Brennan and Boyd, 1997; Gregory and Spulber, 1997; Lyon, 1995; Miceli and Segerson, 1994).

[7]Then, in the remainder of the paper revocation and redemption are used as synonyms.

the service such as contractual indemnities on the value of the investment, technological costs, recruiting and training costs as well as litigation costs if the firm decides to sue the municipality for recalling the contract. The option will be exercised optimally when the value of the project exceeds a trigger value (i.e. an "allowed" level of profits) which is determined endogenously in the model although the optimal exercise time remains stochastic[8].

We offer an optimal regulatory mechanism where the commitment by the municipality to end the contract if the firm's "allowed" level of profits is exceeded ensures that the private operator will behave consistently with the contract itself: once the firm's costs or production conditions improve, it adjusts prices to keep its profits below the allowed level and therefore to prevent revocation. However, as the revocation threat is costly, a stochastic regulatory lag may follow during which prices are not revised and it is not optimal for the municipality to recall the contract.

We then look at the revocation from the perspective of collective welfare maximization and discuss the specific characteristics of the dynamic regulatory rule stemming from the continuous rate of hearing between the regulated firm and the municipality, as a tool for obtaining a long-term efficient equilibrium.

Our model is closest in spirit to the theory of monopoly regulation in a dynamic setting, in which mechanisms such as rate-of-return regulation (ROR) and price-cap-regulation (PCR) arise endogenously as a self-enforcing and mutually beneficial equilibrium[9]. However, in this literature both the "regulator" and the "regulated" firm share the same bargaining power (i.e. both players have the incentive to breach the contract) and they are not affected by regulatory lags. Although playing a crucial role in determining the incentive property of the regulation mechanism, these lags are of fixed time and exogenous whereas in our setup, the different bargaining positions of the two parties coupled with the municipality's option to revoke determine these lags *endogenously* as it is in the essence of ROR regulation (Laffont and Tirole,

[8]Brennan and Schwartz (1982) and Teisberg (1994) model the regulator's future options to cut high profits (with possibility of expropriation) as a perpetual call option which reduces the value of the regulated firm. Recently, referring to the French municipalities' negotiating disadvantage in the face of a cartelized water management, Clark and Mondello (2000) model the municipality's right to revoke delegation as a perpetual call option. However, these models do not investigate optimal regulatory policy within the regulation process.

[9]See for example Salant and Woroch (1991, 1992) and Gilbert and Newbery (1989).

1994, p.15). Price reviews are initiated by the municipality when revocation is worth exercising. This excludes that price renewals being perceived as the time in which the PCR takes some of the well-recognized inefficiencies of ROR. Only excessive revocation costs make the firm an unregulated monopolist, where ROR resembles a PCR with an infinite regulatory lag (Crew and Kleindorfer, 1996, p. 213). Furthermore, the result of an *endogenous* regulatory lag may also explain the empirical evidence indicating that, although contracts between local authorities and private operators are of limited duration, their renewals are often signed without any variations of contractual terms (Joskow and Schmalensee, 1986, p.7).

On a formal level, our paper builds upon two distinct streams of literature. The first one relates to the stochastic control techniques recently developed to identify optimal timing rules and optimal barrier regulations[10]. These techniques have been widely used in the literature of irreversible investments (Pindyck, 1991; Dixit, 1992; Dixit and Pindyck, 1994), and emphasize the role of the option value of delaying investment decision, i.e. the value of waiting for better (although never complete) information on the stochastic evolution of a basic asset. The second one considers the existence of efficient sub-game perfect equilibria for infinite-horizon-threat-games where, in the absence of a binding commitment for the threatener, it is an equilibrium for the victim to make a stream of payment over time (Klein and O'Flaherty, 1993; Shavell and Spier, 1996). The expectation of future payment keeps the threatener from exercising its threat. Indeed, we formulate a time-dependent game in continuous time, where optimal revocation for the municipality requires identification of the time at which to pay a sunk cost in return for a public project whose value is stochastic. The municipality does not revoke the contract until revenues that it expects to earn from managing the investment by itself is equal to the expected present value of the profits regulation that the firm adopts[11].

The plan of the paper is as follows: Section 2 describes the model focusing firstly on the contract and the timing, then on the firm's value and finally on the municipality's option to revoke. Section 3 examines the regulation that belongs to this scheme. Section 4 discusses results and the policy implications. Finally, the Appendix gives precise statements of the results derived

[10] We refer here to the works of Harrison and Taksar (1983) and Harrison (1985).

[11] See Moretto and Rossini (1997, 2001) for the formulation and application of these infinite-horizon-threat-games.

heuristically in Section 3 with all the proofs.

2 The Basic Framework

We begin with a description of the key features of the regulatory contract, then we turn to the performance of the regulatory mechanism and to the policy implications.

2.1 The regulatory contract and its timinig

We consider a simple model where a self-interested-risk-neutral municipality delegates a risk-neutral firm to manage a one-time sunk indivisible public project. Here, what is called "delegation" is the temporary (although long duration) supply of a local public service by a municipality to some private operator under contractual relationship[12]. For the simple contract we consider, the municipality maintains the ownership of the asset while the firm has the "right to use it" and we assume that no new investments are undertaken during the delegation[13]. At $t = 0$, the parties sign a contract specifying a price cap that consumers should pay for the service inclusive of an automatic adjustment clause such as $\hat{p}_t = \hat{p}e^{(RPI-x)t}$, where the price is allowed to increase by the difference between the expected inflation rate (the Retail Price Index, RPI) and an exogenously given expected increase in the productivity the firm should obtain over time (x). Moreover, the delegation contract also includes a revocation (redemption) clause by which the municipality always has the right to recall delegation if the firm's profits are perceived as "excessively" high, in favor of direct management of the utility[14]. However, to manage the utility the municipality has to pay a (sunk)

[12] In principle, our analysis could be applied to utilities of global range (national utilities), but given our assumption on revocation of the contract, the local dimension is more realistic. In fact, the management of a contract at national level can affect the delegated firm's bargaining power which, in turn, can affect the revocation decision (*regulatory capture*).

[13] For the analysis of a regulatory contract where new investments are negotiated between a municipality and a private firm see Dosi, Moretto and Valbonesi (2001).

[14] In our framework, the private operator will never refuse to operate because the utility is always a positive net present value project, as described below.

revocation cost I which, without any loss of generality, we assume does not include any contractual indemnities on the value of the asset.

The setting of the game is the following. At time zero, the municipality assigns the contract to the firm and negotiates the price celing $\hat{p}$ and the x factor. On the basis of the estimated revocation cost I and the expected evolution of the firm's profits, the municipality determines an upper trigger level of profits[15]. The firm is allowed to continue unaltered until this level is crossed. The first time the municipality ascertains that this trigger value has been crossed, it intervenes calling for a revocation of the contract. The firm reacts to the commitment of the municipality to end the contract by adjusting its price downward to keep its profits below the allowed level. Once reduced, the new price remains valid until profits cross the trigger level again, inducing a new price revision. The firm can choose to reduce its profits to guarantee the continuity of delegation or to deviate and keeps its profits, knowing that consequently the contract will end.

This simple setting captures the characterisitcs of a delegation contract where the PCR is negotiated under the threat of a more stringent renegotiation and where the renegotiation timing is determined endogenously by the dynamic of the contract, i.e. the PCR incorporates an endogenous "profit-sharing" mechanism.

2.2 The firm's value

Once set up, we assume that the single project allows some flexibility in its operation at each time $t \geq 0$, by varying certain inputs according to the following production function:

$$q_t = a_t l_t^{\varphi} \qquad \text{with } 0 < \varphi < 1 \tag{1}$$

where q_t denotes the production at time t, l_t is the operating input such as labor (or some intermediate input) and a_t is a technology-efficiency parameter whose value is determined stochastically. The operating input is a perfectly flexible factor which can be rented at the instantaneous price w_t whose value

[15] Asymmetric information at time zero between the firm and the municipality about the revocation cost does not preclude the timing of the game.

is also stochastic. The operating cash flow function is defined as:[16]

$$\pi(p_t, a_t, w_t) = \max_{l_t} p_t q_t - w_t l_t \tag{2}$$

subject to equation (1)and the price-cap $p_t \leq \hat{p}_t \equiv \hat{p}e^{(RPI-x)t}$. For sake of simplicity and without sacrificing in generality, we assume that in the above maximization the price constraint is always binding, which allows us to write the operating cash flow as:[17]

$$\pi(\hat{p}_t, \theta_t) = \Pi(\hat{p}_t)\theta_t \tag{3}$$

where:

$$\Pi(\hat{p}_t) = (1-\varphi)\varphi^{1-\xi}\hat{p}_t^{\xi}$$

and:

$$\theta_t = \theta(a_t, w_t) \equiv a_t^{\xi} w_t^{1-\xi} \qquad \text{with } \xi = \frac{1}{1-\varphi} > 1 \tag{4}$$

The new variable θ_t summarizes at every instant the business conditions for the project, and satisfies the conditions $\frac{\partial \theta_t}{\partial a_t} > 0$ and $\frac{\partial \theta_t}{\partial w_t} < 0$: it is higher the higher the productivity indicator a_t and the lower the flexible-factor rental cost w_t.

Uncertainty is introduced in the model by assuming that both a_t and w_t evolve over time according to geometric Brownian motions, with instantaneous rates of growth $\alpha_a \geq 0$, $\alpha_w \geq 0$ and instantaneous volatilities $\sigma_a > 0$, $\sigma_w > 0$. That is:

$$da_t = \alpha^a a_t dt + \sigma^a a_t dW_t^a, \quad a_0 = a$$

[16] In our framework the difference between PCR and fixed price regime is not relevant as the regulated firm does not face competition.

[17] For example, the operating profits function (3) can be obtained by fixing $p_t \leq \hat{p}_t$ and assuming that the firm faces a completely inelastic demand function. That is:

$$\pi(p_t; a_t, w_t) = \max_{p_t} p_t q_t - w_t l(q_t)$$

subject to: $D(p_t) \leq q_t$ and

$$D(p_t) = d_t p_t^{-\mu} \qquad \text{with } \mu \to 0$$

where the parameter d_t is an index of the position of the demand curve. This form of the demand function is in agreement with the findings of Joskow and Schmalensee (1986, p.3). These authors underline that the demand for utilities such as electricity, water and gas by most industrial customers and all residential customers is very inelastic especially in the short term.

$$dw_t = \alpha^w w_t dt + \sigma^w w_t dW_t^w, \quad w_0 = w$$

where dW_t^a and dW_t^w are the standard increments of two Wiener processes (possibly correlated), uncorrelated over time and satisfying the conditions that $E(dW_t^a) = E(dW_t^w) = 0$ and $E[(dW_t^a)^2] = E[(dW_t^w)^2] = dt$. In other words, we assume that the input's price and the factor's productivity are expected to grow at a constant mean rate, but the realized growth rates are stochastic, normally distributed and independent over time. These assumptions allow us to reduce the model to one dimension.

By expanding $d\pi(\hat{p}_t, \theta_t)$ and applying Itô's lemma for Brownian process it is easy to show that $\pi(\hat{p}_t, \theta_t)$ is driven by:

$$d\pi_t = \alpha\pi_t dt + \sigma\pi_t dW_t \quad \text{with } \pi_0 = \pi, \tag{5}$$

with:

$$\alpha \equiv [\alpha^\theta + \xi(RPI - x)],$$

where $\alpha^\theta \equiv \ \xi\alpha^a - (\xi - 1)\alpha^w + \xi(\xi - 1)(\frac{1}{2}(\sigma^a)^2 + \frac{1}{2}(\sigma^w)^2 - \gamma\sigma^a\sigma^w)]$, and:

$$\sigma \equiv \sqrt{(\sigma^a)^2\xi^2 + (\sigma^w)^2(\xi - 1)^2 - 2\gamma\sigma^a\sigma^w\xi(\xi - 1)}.$$

The drift and the standard deviation parameters of the process π_t are linear combinations of the corresponding parameters of the primitive processes a_t and w_t, with weights given by the exponents of (4) and $\gamma = E(dW_t^a dW_t^w)/dt$. Hence, making use of (3) and (5), and provided that $\rho - \alpha > 0$, the expected value at time t of discounted cash flows from an infinite-lived project can be expressed as $V_t = \frac{\Pi(\hat{p}_t)\theta_t}{\rho - \alpha}$, resulting in dV_t being given simply by:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t, \quad V_0 = V \tag{6}$$

In the remainder of the paper V_t, which evolves according to (6) with starting state V_0, is taken as the primitive exogenous variable for the municipality's delegation-revocation process. In the interest of simplicity, V_0 can be interpreted both as the project value and as the "reasonable" rate of return at the delegation time to induce the firm to manage the utility. However, as any "reasonable" rate of return on an investment could be imbedded directly through a contractual (fixed) price for the service, this formulation sacrifices no generality[18]. Finally, if revocation is carried out, the firm suffers a loss V, while the municipality derives a gain $V - I$. As $V > V - I$, a revocation implies a dead weight loss given that the firm's loss exceeds the local authority's gain.

[18]In terms of cash flow, the local authority may set at time zero the price of service $\hat{p}$

2.3 What is the value of an option to revoke?

For the municipality, optimal revocation implies finding the time at which to pay the sunk cost I in return for a project whose value V evolves according to (6). If we denote the value of the municipality's revocation clause at $t = 0$ by $F_m(V)$, it is equivalent to valuing a perpetual call option, i.e.:

$$F_m(V) = \max_T E_0 \left[(V_T - I)e^{-\rho T} \mid V_0 = V \right] \tag{7}$$

where $T(V^*) = \inf (t \geq 0 \mid V_t - V^* = 0^+)$ is the unknown future time when the revocation is made and V^* is the value that triggers it. The maximization is subject to equation (6), ρ is the constant discount rate and V_0 is the value of the utility at time zero. To simplify discussion we assume, if not otherwise indicated, that $V_0 < V^*$ so that $T^* > 0$ (see Appendix for the general case). By an arbitrage argument and applying Ito's lemma, the value of the option to revoke held by the local authority is given by solution of the following Bellman equation (Dixit and Pindyck, 1994, p. 147-152):

$$\frac{1}{2}\sigma^2 V^2 F_m{}'' + \alpha V F_m' - \rho F_m = 0 \qquad \text{for } V \in (0, V^*], \tag{8}$$

where $F_m(V)$ must satisfy the following boundary conditions:

$$\lim_{x \to 0} F_m(V) = 0 \tag{9}$$

$$F_m(V^*) = V^* - I \tag{10}$$

$$F_m'(V^*) = 1 \tag{11}$$

If the value of the utility goes to zero, the value of the option should also go to zero. Efficient operation conditions (10) and (11) respectively imply that, at the trigger V^*, the value of the option is equal to its liabilities where I indicates the sunk cost for revoking the contract (*matching value condition*) and suboptimal exercise of the option is ruled out (*smooth pasting condition)*. By the linearity of (8) and using (9), the general solution is:

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so that the firm breaks even:

$$\Pi(\hat{p})\theta_0 \leq (\rho - \alpha)s_0$$

where s_0 is a "reasonable" rate of return (Joskow, 1973).

$$F_m(V) = AV^{\beta_1}, \tag{12}$$

A is a constant to be determined and $\beta_1 > 1$ is the positive root of the quadratic equation:

$$\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0 \tag{13}$$

Furthermore, as (12) represents the option value of optimally revoking, the constant A must be positive and the solution is valid over the range of V for which it is optimal for the municipality to keep the option alive $(0, V^*]$. By substituting (12) for (10) and (11) we get:

$$V^* = \frac{\beta_1}{\beta_1 - 1}I, \quad \text{with} \quad \frac{\beta_1}{\beta_1 - 1} > 1 \tag{14}$$

and:

$$A(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0,$$

Putting together (7), (10), (11) and (14), we can write the municipality's investment opportunity at time t as:

$$F_m(V_t) = \begin{cases} AV_t^{\beta_1} & \text{for all } V_t < V^* \\ \\ V_t - I & \text{for all } V_t \geq V^* \end{cases} \tag{15}$$

The optimal trigger value V^* indicates the firm's value for which the municipality will find it profitable to revoke or, in other words, the local authority will find it expedient to manage the public service by itself the first time V_t, randomly fluctuating, hits the upper threshold level V^*.

3 Firm performance under the threat of revocation

From the previous section, once the delegation is in place, the municipality does not have any incentive to revoke the contract as long as V_t is below the revocation level V^*. Indeed as, by (10) and (15), $V_t - I - AV_t^{\beta_1} < 0$ for all $V_t < V^*$, recalling the delegation implies a cost to the local authority which makes the (threat of) revocation not credible. On the contrary, for $V_t > V^*$ the local authority's gain from managing the utility is strictly positive, $V_t - I > 0$. Here, the threat is credible. This reveals the simple stationary nature that this extreme threat possesses: the first time V hits V^* revocation is carried out, the firm suffers the loss V^* and the municipality's gain is $V^* - I$. This extreme equilibrium represents the minimax point of the game[19].

To avoid revocation, the firm may be willing to reduce profits to keep V_t below V^* and then to guarantee the continuity of the contract. However, without a binding commitment a *one-time transfer,* based on the difference $V_t - V^*$, will be inefficient (Klein and O'Flaherty, 1993; Shavell and Spier, 1996). The firm knows that the municipality has an incentive to carry out the threat as soon as V^* is hit. In this respect, the municipality can set the length of the relationship whereas the firm cannot. If the firm makes a once-for-all reduction of its profits the first time V_t hits V^*, the local authority will revoke *immediately after* regardless of the level of the regulation. Furthermore, by backward induction, the same happens for any finite number of profit reductions. The firm does not have any incentive to regulate its profits to delay revocation. The municipality does not expect to see regulations and optimally carries out the threat as soon as V^* is hit. The unique sub-game perfect equilibrium is inefficient: the revocation is carried out regardless of the firm's gain by staying in the market[20]. To avoid this inefficiency the firm must *regulate in continuum its profits.* For $t \geq T^*$ the firm elects V^* as its ceiling and chooses to reduce expected profits via a downward adjustment of the PCR just enough to keep V_t from crossing the ceiling V^*, so that continuing the contract or revoking it makes no difference to the authority.

Our solution concept is subgame-perfect equilibrium in (non Markov) stationary strategies. In particular, we look for a regulatory function $r(.)$ mapping the past history of the observable variable V to the current firm's "profits regulation" chosen from $[0, \infty)$ such that $V_t < V^*$. A strategy rule for the municipality is a mapping $\phi(r(.))$ from the observation space of the municiaplity in [revoke, do not revoke].

The theory of the "regulated" Brownian motion can be used to character-

[19] We stress that the threat of revocation refers to V, and not to the current profits π.

[20] For $V > V^*$, "...the threatener's problem is that he will have an incentive to carry out his threat even if he is paid... Because this means that the victim will not prevent the threatened act by paying, he will not pay. The threatener cannot overcome this problem in a single (or finite) period setting, and his threat will therefore fail in this version of the model" (Shavell and Spier, 1996, p. 3-4).

ize the optimal stationary strategy[21]. Letting the firm start with the initial value V_0, the optimal stationary strategy from here on is a simple one: for $V_t < V^*$, it allows V_t to evolve according to the geometric Brownian motion (6); at V^* a costless "profits regulation" r_t is applied so that the "regulated" process $V_t^r \equiv V_t - r_t$ never goes above V^*[22]. Therefore, the overall process can be described as[23]:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t - dr_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \tag{16}$$

where the increment dr_t gives the sum the firm is willing to pay (i.e. the profits reduction that the firm is willing to bear) between t and $t+dt$ to keep the delegation contract alive. Moreover, the optimal profits regulation r_t, which represents the upside value of the project cut by the regulation, takes the form (see Appendix and figure 3)[24]:

$$r_t = [1 - \inf_{T^* \leq v \leq t} \left(\frac{V^*}{V_v}\right)] V_t \qquad \text{if } V_t \geq V^* \tag{17}$$

This profits control has several interesting features:

- Firstly, from (16) the sum the firm is willing to pay depends on the municipality's behaviour only through dt times units ago, which is interpreted as a reaction time. Specifically, if the firm does not wish to pay when $V_t \geq V^*$ it takes dt units of time for the municipality to analyze and react[25];

[21] See Harrison and Taksar (1983), and Harrison (1985) for a in-depth analysis of "regulated" Brownian motion.

[22] The assumption that the profits control is cost-free is not technically necessary for the results.

[23] By the characteristics of the profit regulation mechanism we maintain, without any confusion, the symbol V for the firm's regulated value (see Appendix).

[24] In technical terms, V^* is no longer an absorbing barrier but is a (reflecting) barrier control, while the optimal control r_t is a right-continuous, non-decreasing and non-negative adapted process.

[25] In continuous time repeated games there is no notion of *last time before t*. The real line is not well ordered and then induction cannot be applied. Continuous time can be seen as discrete-time with a length of reaction (or information lag) that becomes infinitely negligible to allow the threateners to respond immediately to the firm's actions. In Simon and Stinchcombe (1989), for example, a class of continuous strategies is defined so that any increasingly narrow sequence of discrete-time grids generates a convergent sequence of game outcomes whose limit is independent of the grid sequence. In Bergin and MacLeod

- Secondly, the optimal profits control r_t represents the cumulative amount of the project's value that the firm abandons up to time t. The firm must increase r_t fast enough to keep $V_t - r_t$ below V^* but wishes to exert as little control as possible subject to this constraint;

- Thirdly, r_t is parametrized by the initial condition V^* which, in turns, depends on the revocation cost I. An increase in I involves a reduction in r_t;

- Finally, as r_t depends only on the primitive exogenous process V_t, the "regulated" process $V_t - r_t$ is also a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81).

The first three properties make profits regulation related to past realizations of V_t and then to the history of the contract. Since V_t fluctuates stochastically over time, although the intervention is continuous, its rate of change is discontinuous. Furthermore, the last property is important as it effectively makes the "regulated" process (16) a function solely of the starting state. At the beginning of each period both the firm and the municipality can predict the evolution of V_t referring only to its current state which, in turn, makes any subgame beginning at a point at which revocation has not taken place equivalent to the whole game. After all, although the profits regulation is a non Markovian the "regulated" process yes.

The above strategies and the profits regulation mechanism (17) can improve upon non-cooperative outcomes. They imply an instantaneous response by the municipality when the firm departs from the profits regulation rule (17) with the minimax threat: **revocation**. Since the project is infinitely lived, the present value of foregone profits will ensure participation by the firm and the expectation of future profits regulations keeps the authority from exercising the threat.

Proposition Part I (Threat equilibria). For any $V^* > V_0 > 0$, if the firm regulates its profits with the non-decreasing proportional rule (17), then the following municipality strategy is a subgame-perfect equilib-

(1993) a class of *inertia strategies* represents a delay in response: an action at time t must also be chosen for a small period of time after t, with this small period of time tending to zero.

rium:

$$\phi(V_t, r_t) = \begin{cases} \textbf{Do not revoke} \\ \text{at } t \geq T^* \text{ if the firm has followed the rule } r_t \\ \text{to keep } V_t < V^* \text{ for } t' < t \\ \\ \textbf{Revoke} \\ \text{if the firm has deviated from } r_t \\ \text{at any } t' < t \end{cases}$$

Proof. see Appendix.

According to the stationary strategy rule ϕ, the firm observes V_t, chooses an action (17) and the municipality stays ($\phi(V_t, r_t) =$ "Not Revoke" for all $t \geq T^*$) or, equivalently, at T^*, sets a continuous time control rule for each realization of V_t for any $t \geq T^*$[26]. The firm's value under profits regulation is obtained from V_t by imposition of an upper control barrier at V^*. Regulation increases to keep V_t lower than V^* and it is given by the cumulative amount of profits control exerted on the sample path of V_t up to t. Regulation is related to the history of the game and past value realizations, this makes $\phi(V_t, r_t)$ a time-dependent strategy. The local authority's "threat" strategy is adopted if the firm deviates from the regulation rule (17). The municipality believes that this mechanism, from initial date and state (T^*, V^*), is kept in use for the whole (stochastic) planning horizon. If the firm deviates, the local authority expects a fresh rule. The punishment for the firm deviating from the announced rule is revocation[27].

However, although the public project lives forever, profits regulation takes place within a finite (stochastic) time span. Owing to uncertainty, neither player can perfectly predict V_t each time. As V_t follows a random walk there is, for each time interval dt, a constant probability of moving up or down, i.e. of the game continuing one more period. The game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite[28].

[26]In our continuous time setting we can assume, without any loss of generality, that when the municipality is indifferent it may exercise the threat; (see footnote n.25).

[27]The firm cannot commit itself to changing the rule without losing its credibility. In this respect, a change in the profits regulation policy is perceived by the municipality as a stoppage of regulation.

[28]In a discrete-time and constant-payoffs game, Shavell and Spier (1996) propose a similar scheme, where the threatener uses a *threat strategy* with maximal punishments.

Proposition Part II (Regulation timing). As long as $V_t < V^*$ nothing is done. The first time V_t *crosses from below* V^*, at $T^* = \inf(t \geq 0 \mid V_t - V^* = 0^+)$, the firm regulates profits using (17) to keep the municipality indifferent to revoking. Regulation goes on up to the point where the unregulated firm's value V_t *crosses from above* the trigger V^* and the authority becomes (again) indifferent, i.e. $T^{*\prime} = \inf(t \geq T^* \mid V_t - V^* = 0^-)$.

Proof. see Appendix.

Since the authority's strategy is time-dependent, the firm cannot decide whether to continue or stop the regulation referring only to the current realization of V_t. If the regulated value $V_t - r_t$ goes below V^*, in the interval $[T^*, T^{*\prime})$ the firm may be willing to stop regulating profits to increase its value. However, for the sake of perfectness, earlier interruption is not allowed before $T^{*\prime}$. Earlier interruptions are not feasible as long as the threat of contract closure is credible. The credibility relies on the fact that the municipality's option to revoke if the firm deviates from r_t is always worth exercising at $V_t \geq V^*$, i.e. $F_m(V_t) \geq F_m(V^*)$. At $T^{*\prime}$, however, the firm is able to restore the process V_t and the game can start afresh. The timing of the game is shown in figure 1 below.

Figure 1 about here

4 Discussion and Policy Implications

Although our regulation mechanism is simple in nature, several novel implications follow from our analysis. We summarize the discussions of our results in the following items.

- **Profit-sharing and price adjustment**

As argued by Lyon (1996) and Crew and Kleindorfer (1996), most of the PCR plans implemented in recent years for monopoly regulation do not

Our continuous time framework calls for a refinement of the *threat strategy* as in footnote n.25.

simply cap prices. To prevent firms' profits increasing excessively, they also include limits, called deadbands, on how much firms can gain before triggering profit-sharing with customers[29]. In practice, these regulation plans require, in the event of the firm's profits going beyond a "pre-determined" level, the x factor to be automatically adjusted upward, making the price cap adjustment rate $RPI - x$ more stringent[30].

What is the profit deadband that should trigger revision of the price cap mechanism? And what should the revision level of the x factor be to optimise the expected welfare? The model presented above helps us to answer these questions[31].

First of all, it is worth stressing that the profit-sharing rule (17) is *endogenous:* it rises as optimal response from the continuous relationship between the firm and the municipality. Second, this rule is dynamic in nature: such a repetition of the relationship implicitly establishes the terms of a long-term contract which guarantees the firm with an "allowed" level of profits. Third, the optimal deadband is given by V^* (or V^* /I): the firm's value is allowed to evolve according to the geometric Brownian motion (6) until V^* is reached. At V^* the price adjustment rule $RPI - x$ is revised to stop the process V_t from going above V^*. From this moment onwards the Brownian motion describing the regulated profits is given by (16), i.e.:

$$dV_t = \left[\alpha^\theta + \xi(RPI - x')\right] V_t dt + \sigma V_t dW_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \quad (18)$$

[29] Among those favourable to proft-sharing see also Sappington and Sibley (1992); Sappington and Weisman (1996); Burns, Turvey and Weyman Jones (1998).

[30] Although some authors have called this variation of the PCR a "sliding-scale" regulation (Lyon, 1996; Sappington and Weisman, 1996), we prefer to call it PCR with a profit-sharing clause as variation of the x factor in the price cap mechanism serves to redistribute rents to customers, making the regulation more "fair". We maintain the term "sliding scale" regulation - as proposed by Joskow and Schmalensee (1986) - for a mechanism that encompasses ROR and PCR.

[31] Lyon (1996) in a static model explores the efficiency property of regulatory schemes that contemplate profit-sharing. He argues that total welfare can always be increased by switching from a scheme of pure PCR to one with sharing. Crew and Kleindorfer (1996) propose that the x factor be determined with a bargaining process between the firm and the regulator in the same way as the "allowed" rate of return is determined in the costs of service regulation.

where $x' = x - \frac{d \inf_{0 \leq v \leq t} (V^*/V_v)/dt}{\xi \inf_{0 \leq v \leq t} (V^*/V_v)} > x$ is the endogenous new price decrease factor[32]. Fourth, by the regulatory profit restriction x' the probability of an increase in the firm value decreases as the firm value rises (Teisberg, 1994).

Let's now discuss in detail the price adjustment behind the profit-sharing rule (17). Once the numerical value for V^* is known, by using (3) and (4), the optimal policy (14) can be written as $\Pi(p_t)\theta_t = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)I$, from which the boundary value for θ^* is given by:

$$\theta^*(p_t) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{I}{\Pi(p_t)} \tag{19}$$

For any given value of the price cap $\hat{p}_t$, random fluctuations of θ_t move the point (θ_t, p_t) horizontally to the left or right. If the point goes to the right of the boundary, then a price reduction is immediately undertaken, i.e. $p_t \leq \hat{p}_t$, so that the point shifts down to the boundary. If θ_t stays on the left of the boundary, no new price regulation is undertaken. Price reduction proceeds gradually to maintain (19) as an equality. For example, setting $RPI - x = 0$ so that $\hat{p}_t = \hat{p}$, by inverting (19) we can obtain the optimal boundary function $p(\theta_t)$ which determines the optimal price regulation as a function of the sole state variable θ_t and the parameter of the problem ξ :

$$p_t = \hat{p}\left(\frac{\theta^*}{\theta_t}\right)^{1/\xi} \qquad \text{with} \quad \frac{dp_t}{d\theta_t} < 0 \tag{20}$$

The boundary function for this case is shown in Figure 2.

Figure 2 about here

- **Sliding scale regulation**

[32] Panteghini and Scarpa (2001) consider a similar problem in a continuous time stochastic model of investment choices by a regulated firm. However, in their model the $RPI - x$ rule remains in place as long as profits are below an exogenously given level $\tilde{V}$, and, if $V_t > \tilde{V}$, the price decrease factor increases exogenously from x to x'.

As the municipality's goal is rent extraction, the profit-sharing rule (17) also establishes a connection between ROR and PCR. Simple algebra allows us to write (17) as a "one-side sliding scale" over a normalized "allowed" rate of return, similar to the formula proposed by Joskow and Schmalensee (1986, p. 29):

$$s_t^r = s_t + h_t\left(s^* - s_t\right), \quad \text{with } h_t = \begin{cases} 0 & , \text{ for } s_0 \leq s_t < s^* \\ \frac{1 - \inf\limits_{T^* \leq v \leq t} (V^*/V_v)}{1-(V^*/V_t)}, & \text{for } s_t \geq s^* \end{cases} \tag{21}$$

where $s_t^r = \frac{V_t^r}{I}$, $s_t = \frac{V_t}{I}$ and $s^* = \frac{V^*}{I}$. By (21), the actual rate of return under regulation s_t^r is given by the actual rate of return without regulation s_t, i.e. at prices that prevail in time t, plus the adjustment $s^* - s_t$, where the *revocation rate* s^* plays the role of the upper "allowed" rate of return. Thus, if at time t the earned rate of return goes above s^*, the output price is adjusted according to (19) to decrease the rate of return by the fraction $h_t \geq 1$ of the difference between the earned rate of return and the allowed rate of return. Contrasting with the formula of Joskow and Schmalensee in (21) h_t is time-dependent and not-decreasing[33]. That is, h_t is the optimal adjustment rate that keeps the municipality indifferent to revoking the contract or leaving the project to the firm. For the sake of perfectness, h_t cannot decrease when the difference between the earned rate of return and the allowed rate of return drops. In the period $0 \leq t < T^*$ where $s_t < s^*$, we will have $h_t = 0$ and $s_t^r = s_t$. During this regulatory lag the firm is allowed to earn the actual rate of return at the rates fixed at time $t = 0$ (i.e. $p_t = \hat{p}_t$, which represents a period of "pure" PCR). When $s_t \geq s^*$, in period $t \geq T^*$, the adjustment rate h_t jumps to 1 and it will remain at that value until $dV_t > 0$ so that $s_t^r = s^*$. The firm is allowed to earn a rate of return no greater than the upper rate $s^* = \frac{\beta_1}{\beta_1 - 1} > 1$ (i.e. we get a period of ROR regulation with $p_t < \hat{p}_t$,). However, in periods where $dV_t < 0$ we will have $h_t > 1$ in order to keep the difference $s_t^r - s_t$ constant at the highest level reached up to t.

[33]The formula proposed by Joskow and Schmalensee would adjust prices so that the actual rate of return s_t^r at new prices would be given by: $s_t^r = s_t + h\left(s^* - s_t\right)$, where s_t is the rate of return at the prices in the year t (old prices), h is a constant between zero and one and s^* is the ROR target. Hendricks (1975) and Brennan and Schwartz (1982) have also presented models of regulated firms in which regulatory policy is represented by predetermined bounds on the rate of return.

The non-decreasing property of h_t makes the one-side sliding scale (21) similar to an "insurance premium" based on the rate of return s_t, paid in continuous time and in advance by the firm to avoid revocation. The firm starts paying the first time s_t goes above s^* (the first occurrence time) and cannot stop or reduce it since this would cancel its coverage. It continues paying even when "things get better" (profits decrease as well as the municipality's option value of revoking the contract) in order to have the option of being active next time the value goes above s^*. When the firm's current rate of return goes again above s^* (the second occurrence time), the firm will be asked to increase its premium to maintain the coverage. It follows that the new regulation is higher, since the firm pays the premium due after the "second occurrence" (see figure 3 in the Appendix).

- **Revocation as consistent regulatory policy**

We can highlight the municipality's optimal revocation timing by comparing the opportunity costs of currently revoking the contract and the corresponding benefits of optimally postponing the decision. This can be done by evaluating the difference $F_m(V_t) - V_m^0(V_t)$ where, by (15), $V_m^0(V_t) = V_t - I$ is the net value of the public project when it is acquired at time t, and $F_m(V_t) = AV_t^{\beta_1}$. If we assume $V_t < V^*$ so that the municipality finds it optimal to wait before revoking, we get:

$$F_m(V_t) - V_m^0(V_t) = I + AV_t^{\beta_1} - V_t \qquad (22)$$

The first term on the r.h.s. of (22) is the direct cost of revocation. The second term is the value of the option, and since revocation implies "killing" this option, in (22) it appears as an opportunity cost of current revocation. The third term is the current value of the project and is thus an opportunity benefit. Since $V_t < V^*$ and $F_m(V_t) - V_m^0(V_t) > 0$, the direct cost plus the opportunity cost are greater than the opportunity benefit, and the revocation decision should be delayed.

Finally, it is important to stress that we get the same result if, reverting the point of view, (22) is written as $I - (V_t - AV_t^{\beta_1})$ where the term in brackets represents the value of the regulated firm reduced by the municipality's future options to be revocated (Brennan and Schwartz, 1982; Teisberg, 1994).

- **Revocation and welfare.**

Our option to revoke is similar in spirit to the option to own studied by Nőldeke and Schmidt (1998). In a hold up problem in which two parties have to make relationship-specific investments, Nőldeke and Schmidt show that an option to own contract where one party owns the firm initially while the other has the option to buy it at a price specified (in the contract) at a later date, induces both parties to invest efficiently. They also show that this result is robust to renegotiation and uncertainty, and that it permits specification of side payments for the joint surplus between the parties.

In our specific instance, the higher the cost of revoking the contract the higher the option to revoke. However, a higher value of the option to revoke increases the value of waiting for better information on the evolution of the public project before the local authority commits itself to recall delegation. In particular, the expected value of cumulative future profit reductions (equations (27) and (40) in Appendix) can be expressed, at time t, as :

$$\begin{aligned} R(V_t; V^*) &= E_t \left\{ \int_t^\infty e^{-\rho(s-t)} dr(V_s) \mid V_t^r = V_t \right\} \\ &= (\rho - \alpha) E_t \left\{ \int_t^\infty e^{-\rho(s-t)} r(V_s) ds \mid V_t^r = V_t \right\} \\ &= B(V^*) V_t^{\beta_1}, \end{aligned} \tag{23}$$

with $B(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0$ and V_t is the firm's regulated value as in (16). Equation (23) is the firm's expected cumulative controls in terms of profit reductions. The adoption of the policy rule (17) means that it makes no difference to a "local community" whether it receives benefits from the firm's profit regulations or from the local authority's maximization of the discounted customers' surplus, i.e.[34]:

$$A(V^*) V_t^{\beta_1} - B(V^*) V_t^{\beta_1} = 0, \qquad \text{for } t \geq T^*.$$

- **Regulatory information and regulatory commitment**

Although it is universally recognised that the PCR offers considerable advantages compared to ROR in improving firms' efficiency, Crew and Kleindorfer (1996) argue that: "Price cap renewal, in theory and in practice, is

[34] Formal proof (see Appendix) shows that the municipality in revoking the contract does so in rational expectation of subsequent (marginal) profit regulations by the firm. It turns out that for the municipality this makes no difference to the trade-off between revoking now and waiting another instant, i.e. the municipality's option value is identically zero. See Leahy (1993) for the same result in the context of a competitive industry.

recognized as the most likely time for PCR to adopt some of the inefficiences of ROR...(p.212)". In this regard, it is important to underline the endogenous nature of the regulatory lag resulting from our model. In this specific instance, the price adjustment behind the profit-sharing rule (17) is parametrized by the deadband V^* (or *revocation rate* s^* if we refer to (21)). Hence, in addition to the parameters of the model, the key variable for valuing the option to revoke and thus the municipality's position during the delegation period is the direct cost I which - in turn - depends, excluding indemnities, on training and hiring costs as well as on litigation costs. Thus, information on production and demand/cost data that the municipality uses to write the regulatory contract are fundamental in determining the length of the regulatory lag. This effect could be weighted with respect to the well-known tradeoff in ROR literature between a short regulatory lag that promotes allocative efficiency but is bad for productive efficiency, and a long regulatory lag that produces the opposite effect on allocative and productive efficiency.

In the same work, Crew and Kleindorfer (1996) also argue that a major issue in incentive regulation is commitment: "If a company is concerned that the regulator will penalize it at the end of or even during the price-cap period if it is successful, it may not pursue efficiency as strongly as implied by the apparent incentives of PCR. Thus, the notion that the regulator will not renege on the terms of PCR is very important for efficiency to be achieved....(p.218)". However, they subsequently admit that as the regulators' goal is rent extraction it is not difficult to recognise that they have limited incentives to commit, and that this difficulty is at the base of the recent growth of regulatory contracts which incorporate sharing rules: "Such devices provide sharing of gains to ratepayers and therefore might be seen to be less vulnerable to reneging by the regulator if the company does well. In addition, such devices, in limiting how well the company can do, make the regulator less likely to renege....(p.218)".

However, in the process we described in this paper, in addition to the trade-off between commitment and reneging raised by Crew and Kleindorfer, it also becomes crucial to highlight the credibility of the municipality to pursue these sharing rules, that is to revoke the contract when the *revocation trigger* V^* is reached. This credibility is relevant for the renegotiation process itself since it determines the municipality's bargaining power with the delegated firm and - in turn - the timing of contract renewal. Indeed, if the revocation costs, on the one hand, measure the "inefficiencies" the local authority incurs by direct management and are, therefore, used to positively evaluate the decision to delegate the public service to a private operator, on the other hand they raise the problem of the irreversibility of the delegation once it is made. In the case of local provision of the utilities we refer to, after the delegation has taken place the municipal authority plays the role of a regulator with respect to the private firm: the inexperience of the municipal authority in this role can negatively affect its credibility and thus determine a negotiating disadvantage (Clark and Mondello, 2000).

- **Market expectations**

As long as public projects are, in general, not traded assets, their growth rate α may actually fall below the equilibrium total expected rate of return $\hat{\alpha}$ required in the market by investors from an equivalent-risk traded financial security, i.e. $\delta \equiv \hat{\alpha} - \alpha > 0$ (McDonald and Siegel, 1986). Relying on the asset price equilibrium relationship $\hat{\alpha} - r = \lambda\sigma$, we are able to evaluate the municipality's value of the option to revoke, replacing α with the risk-adjusted rate of growth $\alpha - \lambda\sigma = r - \delta$ and behaving as if the world were risk neutral: where r is the risk-free rate of interest, δ is the below-equilibrium return shortfall and λ is the utility's market price of risk (Brennan and Schwartz, 1982). The allowed rate of return becomes:

$$s^* = s^*(r, \lambda, \sigma)$$

Although it seems reasonable to assume that utilities with higher "capital costs" will be allowed to earn higher rates of return, i.e. $\frac{\partial s^*}{\partial r} > 0$, the empirical evidence that a higher systematic risk, as measured through the market price of risk λ, results in a higher allowed rate-of-return, i.e. $\frac{\partial s^*}{\partial \lambda} > 0$ (Fan and Cowing, 1994) is also confirmed. Finally, a higher volatility also increases the allowed rate of return, i.e. $\frac{\partial s^*}{\partial \sigma} > 0$, but for reasons other than those related to interest rates and systematic risk. From section 3 we know that an increase in the instantaneous variance, σ^2, of the revenue process reduces β_1 and then increases the *option multiply* $\frac{\beta_1}{\beta_1 - 1}$. As a result, when the economic environment becomes more volatile, the market value of the public project can go up, but it also increases the municipality's value of keeping the revocation opportunity alive. Thus, the allowed rate of return s^* is higher since the authority optimal policy is to lag behind in revoking the contract with the firm.

- **Final remarks**

The paper has modelled the regulation of a local public utility as a long-term relationship between a firm and a municipality. The repetition of the relationship may substitute long-term contracts and guarantee utilities with an appropriate level of profits. Furthermore, since the price and its adjustment mechanism is contractually fixed when the contract is signed and the firm is the residual claimant for its profits, a stochastic regulatory lag exists where the regulation has a price cap nature. Excessive revocation cost makes the firm an unregulated monopolist with an infinite regulatory lag. This PCR is followed by a period of ROR in which the firm is induced to adjust its price downward to keep its profits below the allowed level set by the authority and avoid revocation.

A Appendix: The threat game

We prove that the municipality scheme proposed is a perfect equilibrium belonging to the class of efficient perfect equilibria (which may be very large) for the continuous time threat-game described in the text.

1) Regulation mechanism

We define the regulation as the negative increment dV_t to let V_t stay at V^*,that is, a policy control is a process $Z = \{Z_t, t \geq 0\}$ and a regulated process $V^r = \{V_t^r, t \geq 0\}$ such that

$$V_t^r \equiv V_t Z_t, \quad \text{for} \quad V_t^r \in (0, V^*], \tag{24}$$

where:

- *i)* V_t is a geometric Brownian motion, with stochastic differential as in (6);
- *ii)* Z_t is a decreasing and continuous process with respect to V_t ;
- *iii)* $Z_0 = 1$ if $V_0 \leq V^*$, and $Z_0 = V^*/V_0$ if $V_0 > V^*$ so that $V_0^r = V^*$;
- *iv)* Z_t decreases only when $V_t^r = V^*$.

Applying Ito's lemma to (24), we get:

$$d\,V_t^r = \alpha\,V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}, \qquad V_0^r \in (0, V^*]$$

where $V_t^r \frac{dZ_t}{Z_t} \equiv V_t dZ_t = -dr_t$ is the infinitesimally small level of value given up by the firm. In terms of the regulated process V_t^r, we can write:

$$r_t \equiv r(V_t) = V_t - V_t^r \equiv (1 - Z_t)V_t, \tag{25}$$

Although the process Z_t may have a jump at time $t = 0$ it is continuous and maintains V_t below the barrier using the minimum amount of control, in that control takes places only when V_t crosses V^* from below with probability one in the absence of regulation. Therefore, in the case of $V_0 < V^*$, we get $V_t^r \equiv V_t$, with initial condition $V_0^r \equiv V_0 = V$, and $Z_t = 1$. At $T^* \equiv T(V^*) = \inf(t \geq 0 \mid V_t - V^* = 0^+)$ the regulation starts so as to maintain $V_t^r = V^*$.

The firm regulates the project's value by the amount $r_t = V_t - V_t^r \geq 0$ every time V^* is hit.
Finally, the same conditions $(i) - (iv)$ uniquely determine Z_t with the representation form (Harrison,1985; proposition 3, p. 19-20):[35]

$$Z_t \equiv \begin{cases} \min(1, V^*/V_0) & \text{for } t = 0 \\ \inf\limits_{0 \leq v \leq t} (V^*/V_v) & \text{for } t \geq 0 \end{cases} \tag{26}$$

Figure 3 about here

2) Cost of regulation

Let's now indicate with $R(V^r; V^*)$ the expected value of future cumulative losses in terms of the firm's value due to the regulation. The rational player evaluates R considering an infinite life project:

$$\begin{aligned} R(V_0^r; V^*) &= E_0 \left\{ \int_0^\infty e^{-\rho t} dr(V_t) \mid V_0^r \in (0, V^*] \right\} \\ &= -E_0 \left\{ \int_0^\infty e^{-\rho t} V_t dZ_t] \mid V_0^r \in (0, V^*] \right\} \end{aligned} \tag{27}$$

Since V_t^r is a Markov process in levels (Harrison, 1985, proposition 7, p.80-81), we know that the above conditional expectation is in fact a function solely of the starting state.[36] Keeping the dependence of R on V_t^r active

[35] This is an application of a well-known result by Levy (1948), for which the process:

$$\ln V_t^r \equiv \ln V_t + \ln Z_t \equiv \ln V_t - \inf_{0 \leq v \leq t} (\ln V_v - \ln V^*)$$

has the same distribution as the "reflected Brownian process" $\mid \ln V_t - \ln V^* \mid$.

[36] For $V_0 = V > V^*$ optimal control would require Z to have a jump at zero so as to ensure $V_0^r = V^*$. In this case the integral on the right of (27) is defined to include the control cost r_0 incurred at $t = 0$, that is (see Harrison 1985, p.102-103):

$$\int_0^\infty e^{-\rho t} dr_t \equiv r_0 + \int_{(0,\infty)} e^{-\rho t} dr_t$$

where $r_0 = V - V_0^r$.

and assuming that it is twice continuously differentiable, by Ito's lemma we get:

$$\begin{aligned} dR &= R' dV_t^r + \frac{1}{2} R''(dV_t^r)^2 \\ &= R'(Z_t dV_t + V_t dZ_t) + \frac{1}{2} R'' Z_t^2 (dV_t)^2 \\ &= R'(\alpha V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}) + \frac{1}{2} R'' Z_t^2 \sigma^2 dt \\ &= \frac{1}{2} R'' \sigma^2 V_t^{r2} dt + R' \alpha V_t^r dt + R' \sigma V_t^r dW_t + R' V_t^r \frac{dZ_t}{Z_t} \end{aligned} \tag{28}$$

where it has been taken into account that for a finite-variation process like Z_t,$(dZ_t)^2 = 0$. As $dZ_t = 0$ except when $V_t^r = V^*$ we are able to rewrite (28) as:

$$dR(V_t^r; V^*) = [\frac{1}{2}\sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha V_t^r R'(V_t^r; V^*)]dt \tag{29}$$

$$+\sigma V_t^r R'(V_t^r; V^*)\, dW_t - R'(V^*; V^*) dr(V_t)$$

This is a stochastic differential equation in R. Integrating by part the process Re^{-rt} we get (Harrison, 1985, p.73):

$$e^{-\rho t} R(V_t^r; V^*) = R(V_0^r; V^*) + \tag{30}$$

$$+ \int_0^t e^{-\rho s} \left[\frac{1}{2}\sigma^2 V_s^{r2} R''(V_s^r; V^*) + \alpha V_s^r R'(V_s^r; V^*) - \rho R(V_s^r; V^*) \right] ds$$

$$+\sigma \int_0^t e^{-\rho s} V_s^r R'(V_s^r; V^*)\, dW_s - R'(V^*; V^*) \int_0^t e^{-\rho s} dr(V_s)$$

Taking the expectation of (30) and letting $t \to \infty$, if the following conditions apply:

(a) $\lim_{l \to 0} \Pr[T(l) < T(V^*) \mid V_0^r \in (0, V^*]] = 0$ for $l \leq V_t^r < V^* < \infty$, where $T(l) = \inf(t \geq 0 \mid V_t^r = l)$ and $T(V^*) = \inf(t \geq 0 \mid V_t^r = V^*)$;

(b) $R(V_t^r; V^*))$ is bounded within $(0, V^*]$;

(c) $e^{-\rho t} V_t^r R'(V_t^r; V^*)$ is bounded within $(0, V^*]$;

(d) $R'(V^*; V^*) = 1$;

(e) $\frac{1}{2}\sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha V_t^r R'(V_t^r; V^*) - \rho R(V_t^r; V^*) = 0$,

we obtain $R(V^r; V^*)$ as indicated in (27). Condition (a) says that the probability that the regulated process V_t^r reaches zero before reaching another point within the set $(0, V^*]$ is zero. As V_t^r is a geometric type of process this condition is, in general, always satisfied (Karlin and Taylor, 1981, p. 228-230). Furthermore, if condition (a) holds and $R(V^r; V^*)$ is bounded then conditions (b) and (c) also hold. According to the linearity of (e) and using (d), the general solution has the form:

$$R(V_0^r; V^*) = B(V^*)(V_0^r)^{\beta_1}, \tag{31}$$

with:

$$B(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0. \tag{32}$$

As for $V_0 \leq V^*$, $Z_0 = 1$ and $V_0^r = V_0 = V$, then $R(V_0^r; V^*) = R(V; V^*)$. On the other hand, if $V_0 > V^*$, we get $Z_0 = V^*/V_0$, so that $V_0^r = V^*$ and $R(V_0^r; V^*) = R(V^*; V^*)$.

3) The value of revocation

Although the firm prefers to regulate rather than close (i.e. the loss from closure is larger than the (expected) cost of regulation), it always prefers to stop regulation if the threat of revocation is not carried out, i.e. $r_t = V_t - V_t^r \geq 0$, for all $t \geq T^*$. To simplify discussion we assume that $V_0 < V^*$ so that $T^* > 0$. While regulation reduces the project's value but keeps the firm's contract alive, the municipality is not in the same condition. Indicating with $F_m^r(V; V^*)$ the municipality's option value when the firm pretends to control its profits, it can be expressed, at time zero, by:

$$F_m^r(V; V^*) = \max E_0 \left\{ (V_T^r - I)e^{-\rho T} \mid V_0 = V \right\} \tag{33}$$

or using $r_t = V_t - V_t^r = (1 - Z_t)V_t$:

$$F_m^r(V; V^*) = \max E_0[(V_T - I)e^{-\rho T} - (V_T - V_T^r)e^{-\rho T} \mid V_0 = V] \tag{34}$$

In (34) the municipality's option value, with a barrier control on V_t, takes account of two terms depending upon the joint evolution of V_t and V_t^r. The first $(V_T - I)$ is the net project's value without the barrier, while $(V_T - V_T^r)$ is the reduction in value due to the regulation. Again, keeping the dependence of F_m^r on V_t^r active and assuming it is twice continuously differentiable, by Ito's lemma we obtain:

$$dF_m^r = \frac{1}{2} F_m^{r''} V_t^{r2} \sigma^2 dt + F_m^{r'} \alpha V_t^r dt + F_m^{r'} \sigma V_t^r dW_t + F_m^{r'} V_t^r \frac{dZ_t}{Z_t} \tag{35}$$

As $dZ_t = 0$ except when $V_t^r = V^*$ the above differential equation becomes:

$$dF_m^r(V_t^r; V^*) = [\frac{1}{2}\sigma^2 V_t^{r2} F_m^r{}''(V_t^r; V^*) + \alpha V_t^r F_m^r{}'(V_t^r; V^*)]dt \tag{36}$$

$$+\sigma V_t^r F_m^r{}'(V_t^r; V^*)\, dW_t - F_m^r{}'(V^*; V^*) dr(V_t)$$

Integrating by part the process $F_m^r e^{-\rho T^*}$ gives:

$$e^{-\rho T^*} F_m^r(V_T^r; V^*) = F_m^r(V; V^*) + \tag{37}$$

$$+ \int_0^{T^*} e^{-\rho s} \left[\frac{1}{2}\sigma^2 V_s^{r2} F_m^r{}''(V_s^r; V^*) + \alpha V_s^r F_m^r{}'(V_s^r; V^*) - \rho F_m^r(V_s^r; V^*) \right] ds$$

$$+\sigma \int_0^{T^*} e^{-\rho s} V_s^r F_m^r{}'(V_s^r; V^*)\, dW_s - F_m^r{}'(V^*; V^*) \int_0^{T^*} e^{-\rho s} dr(V_s)$$

Taking the expected value of (37), if the following conditions apply:

(a) $e^{-\rho t} V_t^r F_m^{r\prime}(V_t^r; V^*)$ is bounded within $(0, V^*]$

(b) $F_m^r(V_{T^*}^r; V^*) = V_{T^*}^r - I$

(c) $F_m^r{}'(V^*; V^*) = 0$;

(d) $\frac{1}{2}\sigma^2 V_t^{r2} F_m^r{}''(V_t^r; V^*) + \alpha V_t^r F_m^r{}'(V_t^r; V^*) - \rho F_m^r(V_t^r; V^*) = 0$

we obtain the expression for $F_m^r(V;V^*)$ as in (33). Now the two conditions (b) and (c) together with the fact that at T^* the regulation starts so as to keep $V_t^r = V^*$ (i.e. compare condition (c) with condition (11)), give $F_m^r(V;V^*) = 0$. If the municipality rationally anticipates the firm's future profits regulation its option value is always null.
From (34) and (31), a heuristic but direct way of looking at the same result is to see $F_m^r(V;V^*)$ as the difference between the municipality's option value to manage the utility, $F_m(V) = A(V^*)V^{\beta_1}$, and the firm's expected value of future cumulative controls due to the regulation, $R(V) = B(V^*)V^{\beta_1}$, that is:

$$F_m^r(V_t;V^*) = A(V^*)V_t^{\beta_1} - B(V^*)V_t^{\beta_1} = 0$$

In other words, it should make no difference whether the "community" receives the benefits in terms of the firm's regulation (lower profits) or by direct transfers from the municipality.

4) Optimal threat strategy and perfect equilibrium

Since V_t follows a random walk there is, for each time interval of small length dt, a constant probability that the game will continue one more period. The game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite. Neither player is able to perfectly predict V_t at each date and the regulation scheme described by (25) with the form (26) is viewed by both contenders as a stationary strategy for evaluating all future value reductions.[37] In the strategy space of the agency it appears as:

[37] It is well known that infinitely repeated games may be equivalent to repeated games that terminate in finite time. At each period there is a probability that the game continues one more period. The key is that the conditional probability of continuing must be positive (Fudenberg and Tirole, 1991, p.148). Integrating the differential form (6), the geometric Brownian motion can be expressed as:

$$V_{t+dt} = V_t e^{dY_t}$$

where $dY_t = \mu dt + \sigma dW_t$ and $\mu = \alpha - \frac{1}{2}\sigma^2$. The differential dY_t is derived as the continuous limit of a discrete-time random walk, where in each small time interval of length Δt the variable y either moves up or down by Δh with probabilities (Cox and Miller, 1965, p. 205-206):

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\sqrt{\Delta t}}{\sigma}\right), \quad \Pr(\Delta Y = -\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\sqrt{\Delta t}}{\sigma}\right)$$

$$\phi(V_t, r_t) = \begin{cases} \text{Do not revoke at } t \geq T^* \text{ if the firm} \\ \text{plays the rule } r_t = (1 - Z_t)V_t \text{ for } t' < t \\ \\ \text{Revoke if the firm deviated from} \\ r_t = (1 - Z_t)V_t \text{ at any } t' < t \end{cases}$$

where $\phi(V_t, r_t)$ is the strategy at t with history (V_t, Z_t). The municipality's "threat" strategy is chosen if the firm deviates by regulating V_t less than r_t or by abandoning $r_t = (1 - Z_t)V_t$ as a rule to evaluate future regulations. The authority must believe that the regulation, from the initial date and state (T^*, V^*), will be kept in use for the whole (stochastic) planning horizon. If the firm deviates, the local authority believes that the firm will switch to a different rule in the future and knows for sure that the municipality will revoke immediately after. The municipality does not revoke in t if $r_{t'} \geq V_{t'} - V_{t'}^r$ for all $t' \leq t$, because value controls are expected to continue with the same rule and $F_m^r(V) = 0$ for all $t \geq T^*$. If $r_{t'} < V_{t'} - V_{t'}^r$ for some $t' < t$ the municipality expects a different rule and carries out the threat, switching from $F_m^r(V_t) = 0$ to $F_m(V_t) \geq V^* - I$. The game is over.

To prove this, let's first consider R as in (27). For each $t' > T^*$, integration by parts gives:

$$\int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = \tag{38}$$

$$e^{-\rho(t-t')} V_t Z_t - V_{t'} Z_{t'} + \rho \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds - \int_{t'}^{t} e^{-\rho(s-t')} Z_s dV_s$$

---

or defining $\Delta h = \sigma\sqrt{\Delta t}$:

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu \Delta h}{\sigma^2}\right), \quad \Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 - \frac{\mu \Delta h}{\sigma^2}\right)$$

That is, for small Δt, Δh is of order of magnitude $O(\sqrt{\Delta t})$ and both probabilities become $\frac{1}{2} + O(\sqrt{\Delta t})$, i.e. not very different from $\frac{1}{2}$. Furthermore, considering again the discrete-time approximation of the process Y_t, starting at $V^* e^{+\Delta h}$, the conditional probability of reaching V^* is given by (Cox and Miller, 1965, ch.2):

$$\Pr(Y_t = 0 \mid Y_t = 0 + \Delta h) = \begin{cases} 1 & \text{if } \mu \leq 0 \\ e^{-2\mu \Delta h/\sigma^2} & \text{if } \mu > 0 \end{cases}$$

which converges to one as Δh tends to zero.

Taking expectation of both sides and using the zero expectation property of the Brownian motion (Harrison, 1985, p.62-63), we have:

$$E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}V_s dZ_s = E_{t'}[V_t Z_t e^{-\rho(t-t')}] - V_{t'}Z_{t'} + (\rho-\alpha)E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}V_s Z_s ds \tag{39}$$

By the Strong Markov property of V_t^r[38], it follows that $E_{t'}[V_t Z_t e^{-\rho(t-t')}] = E_{t'}[V_t Z_t]E_{t'}[e^{-\rho(t-t')}] = V^* E_{t'}[e^{-\rho(t-t')}] \to 0$ almost surely as $t \to \infty$, so that:

$$E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}V_s dZ_s = -V_{t'}Z_{t'} + (\rho-\alpha)E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}(V_s - r_s)ds$$

Since $-V_{t'}Z_{t'} + (\rho-\alpha)E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}V_s ds = 0$, substituting in (27) and rearranging we get:

$$R(V_{t'}; V^*) = (\rho-\alpha)E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}r_s ds \tag{40}$$

Secondly, let's assume (t', t) is an interval in which r_s is flat so that $V_s^r \leq V^*$, and t is the first time in which $dZ_t > 0$. Considering the decomposition (39) we can write (40) as:

$$\begin{aligned} R(V_{t'}; V^*) &= (\rho-\alpha)\left\{E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{\int_{t}^{\infty} e^{-\rho(s-t')}r_s ds\right\}\right\} \\ &= (\rho-\alpha)\left\{E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{e^{-\rho(t-t')}\int_{t'}^{\infty} e^{-\rho(s-t')}r_s^* ds\right\}\right\} \end{aligned}$$

where we have defined $V_s^{r*} = V_{t+s}^r$ and $r_s^* = r_{t+s} - r_t$ for $t' \leq t$. Applying, again, the Strong Markov Property of V_t^r we get:

$$\begin{aligned} R(V_{t'}; V^*) &= E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{e^{-\rho(t-t')}E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')\infty}r_s^* ds\right\} \\ &= (\rho-\alpha)E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{e^{-\rho(t-t')}R(V_{t'}; V^*)\right\} \\ &= (\rho-\alpha)E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + R(V_{t'}; V^*)E_{t'}\left\{e^{-\rho(t-t')}\right\} \end{aligned}$$

Since $r_s = r_{t'} \equiv V_{t'} - V_{t'}^r$ for all $s \in (t', t)$ we can simplify the above expression as:

[38] The Strong Markov Property of regulated Brownian motion processes stresses the fact that the stochastic first passage time t and the stochastic process V_t^r are independent (Harrison, 1985, proposition 7, p.80-81).

$$R(V_{t'};V^*) = \frac{(\rho-\alpha)}{\rho} r_{t'} = \frac{(\rho-\alpha)}{\rho}(V_{t'} - V_{t'}^r) \tag{41}$$

>From (41), any application of controls $r_{t'} < V_{t'} - V_{t'}^r$, leads to a reduction of (40) for all $t \geq t'$ and then to $F_m^r(V_t; V^*) > 0$. Furthermore, the firm does not regulate more than r_t since, by doing so, it does not increase the probability of a delayed closure. It does not pay less, since $r_t < V_t - V_t^r$ induces closure making it worse off, i.e. $0 < V_t$. Finally, as V_t^r is a Markov process in levels, it is immediate by (40) that any sub-game beginning at a point at which revocation has not taken place is equivalent to the whole game. The strategy ϕ is efficient for any sub-game starting at an intermediate date and state (t, V_t). We have sub-game perfection.

6) Non-decreasing path of r_t within $[T^*, T'^*)$.

So far we have implicitly assumed that, once started at T^*, the regulation goes on forever. Earlier interruptions are not feasible as long as the threat of closure by the municipality is credible. Credibility relies on the fact that the agency's option-to-revoke the contract if the firm deviates from r_t is always worth exercising at $V_t > V^*$, i.e. $F_m(V_t) \geq F_m(V^*)$. As the decision rule strategy depends on the history of the game, the authority expects regulation to continue according to the rule r_t and any premature stop could make it no longer subgame-perfect.

However, in an optimal Brownian path there is a positive probability of the primitive process V_t crossing V^* again starting at an interior point of the range (V^*, ∞). In this case, the firm may be willing to stop regulation. That is, the firm regulates its value until $V_t \geq V^*$, letting the agency expect the regulation to continue in the future according to the same rule $r_t = (1-Z_t)V_t$, but when V_t reaches, for the first time after T^*, a predetermined level, say $V' \leq V^*$, it stops the regulation. The authority will face a jump from zero to $F_m(V') \leq F_m(V^*)$ making the threat of revocation no longer credible. To see this, consider the possibility of the firm's regulation terminating at time T' with $T^* < T' < \infty$, where $T' = \inf(t \geq T^* \mid V_t \geq V')$ is the first hitting time of $V' \leq V^*$ when regulation is on. The municipality's option value starting at any $t \in [T^*, \infty)$can be expressed as:

$$\tilde{F}_m^r(V_t; V') = P(V'; V_t) E_t[F_m^r(V_{T'}) e^{-r(t-T')}] + \tag{42}$$

$$(1 - P(V'; V_t)) \max E_t[(V_T^r - I)e^{-r(t-T)}]$$

where $P(V'; V_t)$ is the probability of the unregulated process V_t reaching $V' \leq V^*$ starting at an interior point of the range (V^*, ∞), which is equal to (Cox and Miller, 1965, p. 232-234):

$$\Pr(T' < \infty \mid V_t) \equiv P(V'; V_t) = \left(\frac{V_t}{V'}\right)^{-2\mu/\sigma^2}$$

with $\mu = (\alpha - \frac{1}{2}\sigma^2)$ [39]. As the starting point is now any $t \in (T^*, \infty)$, we can immediately see in (42) the dependence on both V_t^r and V_t. Recalling that the option value in the case of regulation is zero and that at time T' when the contract is revoked it is simply $F_m^r(V_{T'}) = F_m^r(V')$, we get:

$$\tilde{F}_m^r(V_t; V') = P(V'; V_t) E_t[F_m^r(V')e^{-r(T'-t)}]$$

According to the Strong Markov Property of V_t^r equation (42) becomes:

$$\tilde{F}_m^r(V_t; V') = P(V'; V_t) F_m^r(V') \left(\frac{V_t}{V'}\right)^{\beta_2} \tag{43}$$

where $\beta_2 < 0$ is the negative root of (13). Since at t the unregulated process V_t is greater than V' and $P(V'; V_t) \left(\frac{V_t}{V'}\right)^{\beta_2} = \left(\frac{V_t}{V'}\right)^{\beta_2 - 2\mu/\sigma^2} \leq 1$, we obtain $\tilde{F}_m^r(V_t; V') \leq F_m^r(V')$ for all $t \in [T^*, T')$, which implies that:

$$\tilde{F}_m^r(V_t; V') = F_m^r(V^*) \left(\frac{V'}{V^*}\right)^{\beta_1} \left(\frac{V_t}{V'}\right)^{\beta_2 - 2\mu/\sigma^2} \leq F_m^r(V^*) \tag{44}$$

Therefore, to avoid revocation the regulation continues until time $T'^* \equiv T'(V^*) = \inf(t \geq T^* \mid V_t - V^* = 0^-)$ when the trigger V^* is hit again (for the first time) after T^*. The game ends and can then be restarted afresh.

[39] This probability is $P(V'; V_t) = 1$ for $\mu \leq 0$, see footnote n. 37.

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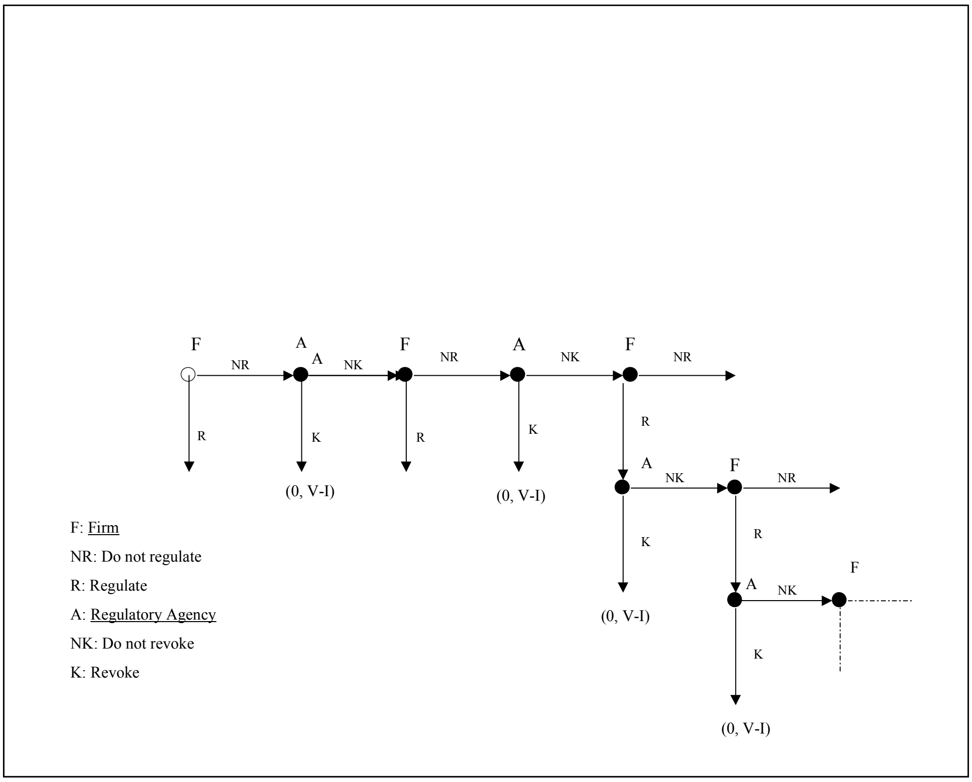

Figure 1: Discrete time representation of the game (dominant strategies)

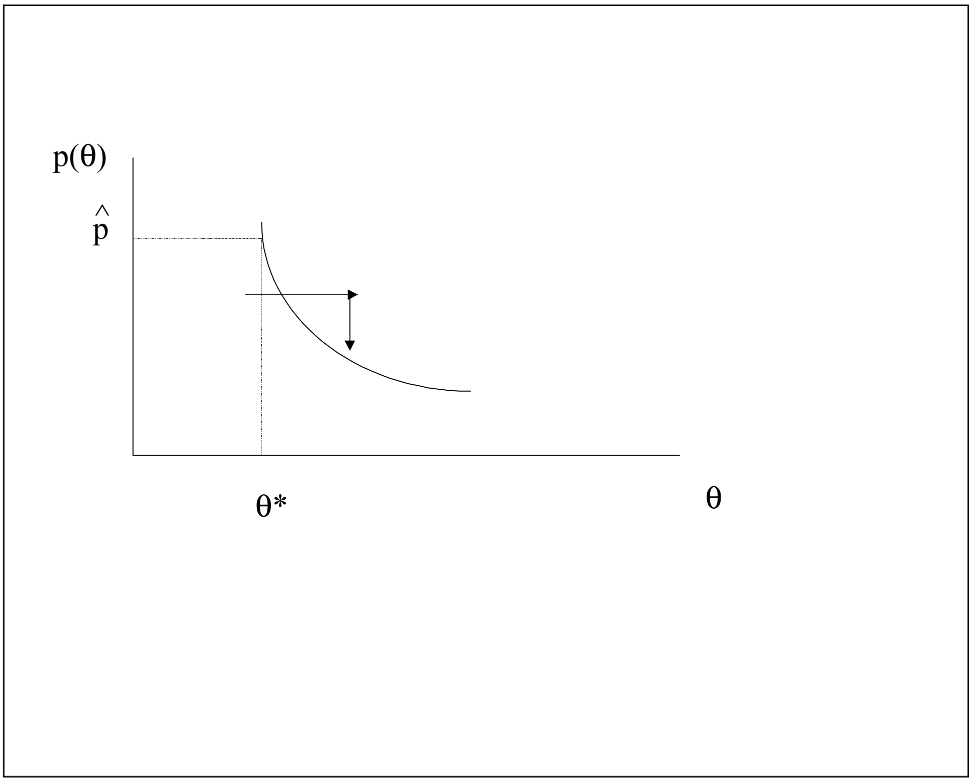

Figure 2: Price regulation under threat of revocation

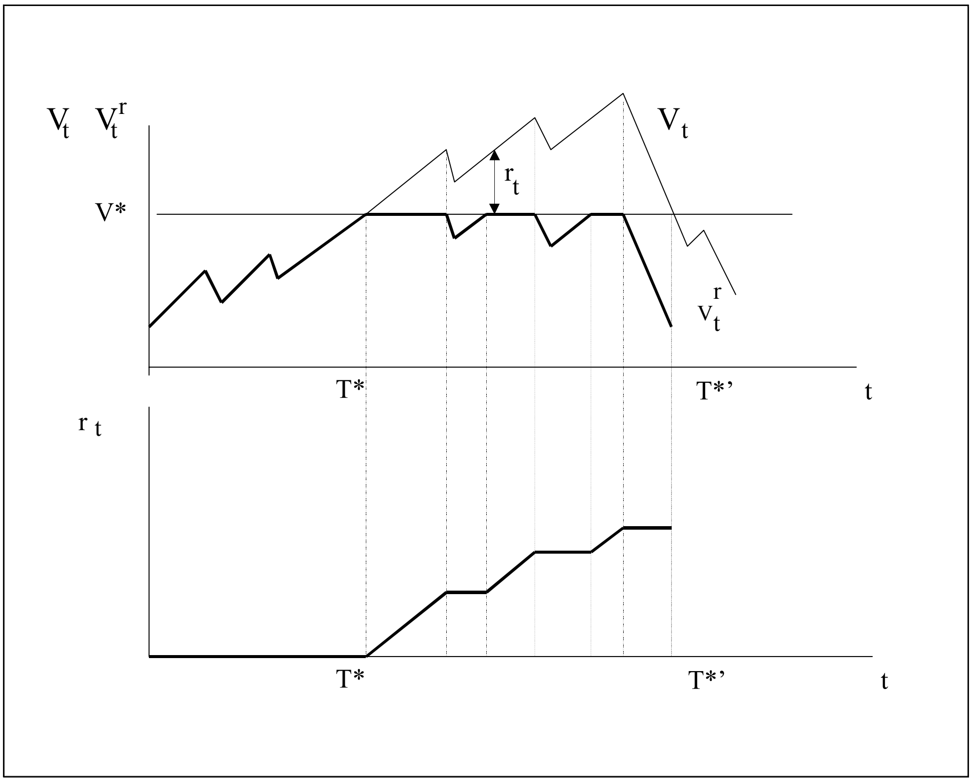

Figure 3: Threat game timing