Flexibility in the Small Firm: the dynamics of market re-positioning and scale adjustment in the early stages of the life cycle

Gavin C Reid

Abstract

This paper examines flexibility in the small firm in two ways. First, it looks at the re-positioning of their main product markets that firms undertake in the early life cycle, in an attempt to best exploit their niche advantages. The market extent variables used are: local, regional, Scottish, national, and international. A transition probability approach is taken, estimating the probability of moving from one market are to another in a unit period. In this way, it is possible to compare the long run equilibrium of such a process, with the period by period adjustment. This examination of short run adjustment to a long period equilibrium provides insights into small firm flexibility as regards market area and niche exploitation. It is found that the speed of adjustment of small firms is relatively rapid, and they typically get close to the long period equilibrium in just a few periods of adjustment. This suggests high flexibility in the exploitation of market areas. Secondly, the paper estimates a model of the dynamics of small firm sales growth. This is a variant of a Gibrat's law type of model. It is shown that rapid sales growth is often achieved in the early life cycle. This process is log-linear in size, dynamically stable, and implies a plausible value for the long run equilibrium size of the small firm. Over short periods, of just a few years, however, most small firms were yet still below their equilibrium sizes, though a systematic tendency towards equilibrium was observed. Thus pervasive flexibility was evident in small firm behaviour, both in terms of niche exploitation and growth. Greater flexibility was observed in niche exploitation, as compared to overall scale.

Key words: Markov chains, Gibrat's Law, flexibility, Scottish small firms

JEL Classification Numbers: D21, L11, M13, R32

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Gavin C Reid[1]

1. Introduction

The objective of this paper is to show how two aspects of small firm flexibility can be modeled. The first involves flexibility in terms of moving to new markets, and the second involves flexibility in terms of change in scale, as measured by sales growth. Two types of models are used: a Markovian model of shifts, period by period, in the market extent for the main product; and a variant of a Gibrat Law model, in which the dynamics of small firm growth is estimated. Both models are estimated on a sample of Scottish small firms over the period 1994-1997. Evidence on 150 small business start ups, over a four year period, was obtained by field work methods, involving face to face interviews with entrepreneurs. It is on data from this field work that the models of the paper are estimated.

The paper demonstrates two key findings. (1) Shifts in main markets are often substantial for new business start ups, yet have a distinctive pattern over time. They show strong patterns of convergence over time, adapting towards the implied equilibrium position of the underlying dynamic process rather rapidly. Although small firms can be quite exploratory about their main markets in the periods shortly after launch, there is a strong tendency for them to retrench to local markets in the long run. (2) Patterns of sales growth of new business start ups conform to a well defined dynamic law, which is econometrically estimated. This law implies a convergent growth process. This has a stable long run equilibrium position. Further, this equilibrium is plausible, in terms of its implied ‘optimal’ small firm size being consonant with extant evidence. In comparing the dynamic processes (1) and (2) described above, it is found that flexibility is relatively larger for the market adjustment, as compared to the scale adjustment.

The paper first describes the sampling procedure, and the key features of the data set. It then looks at the Markovian model, followed by the model which is a variant of the Gibrat Law formulation. In each section, a dynamic model is estimated, and its adjustments to equilibrium is discussed. The general picture that emerges is of considerable flexibility of the small firm in its early life cycle. Unusually, this paper provides explicit trajectories of the adjustment processes by which this flexibility works itself out.

2. The Data

The evidence on which the models of this paper are estimated were all obtained by field work methods. This involved face to face interviews with owner managers of new business start ups in Scotland. As is usual with field work methods it is necessary to find "gate keepers" who provide "ports of entry" to the field. Here, the gate keepers were directors of enterprise incubators, known as Enterprise Trusts (ETs) in Scotland. They provide a range of business inception facilities including training, advice on sites, access to finance and more generally networking opportunities[2]. Directors of ETs were asked to provide random samples of new business start ups from their case loads. The only restriction set, was that the exact inception date of the enterprise needed to be known and that no more than three years should have elapsed since inception. A random sample of approximately half of the ETs in existence in 1993 was taken.

[Figure 1 near here]

By reference to Figure 1 it will be seen that the sampling area was from the main metropolitan concentrations on the West Coast of Scotland (including Glasgow) through the Central Belt to the metropolitan areas of the East including Edinburgh and then up North through the main population centres including Stirling, Perth, Dundee, Aberdeen, finally extending as far north as Inverurie. Thus the main population concentrations of Scotland were largely covered by a sampling area which has, roughly speaking, a thick, reversed-L shaped configuration. The sampling areas attaching to ETs were Gordon, Paisley, Strathkelvin, Clydesdale, Cumnock & Doone, Hamilton, Glasgow, Stirling, Perth, Alloa, Midlothian, Edinburgh, Grangemouth, Crossgates, Cupar, Dundee, Angus, Aberdeen, Inverurie. These sampling areas are indicated by the device of a circle in Figure 1. The radius of each circle has a magnitude proportional to the stratum size within that sampling area. The initial sample size was 150 small firms in the base year of 1994. The same firms were re-interviewed for 3 successive years. Extensive data were gathered on a wide range of attributes, including markets, finance, costs, business strategy, human capital, internal organization and technical change. Here only a small proportion of these data will be used, so a brief indication of general characteristics of the small firms sampled will be provided to set the scene for the empirical work. A more detailed analysis of the database is available in Reid (1999).

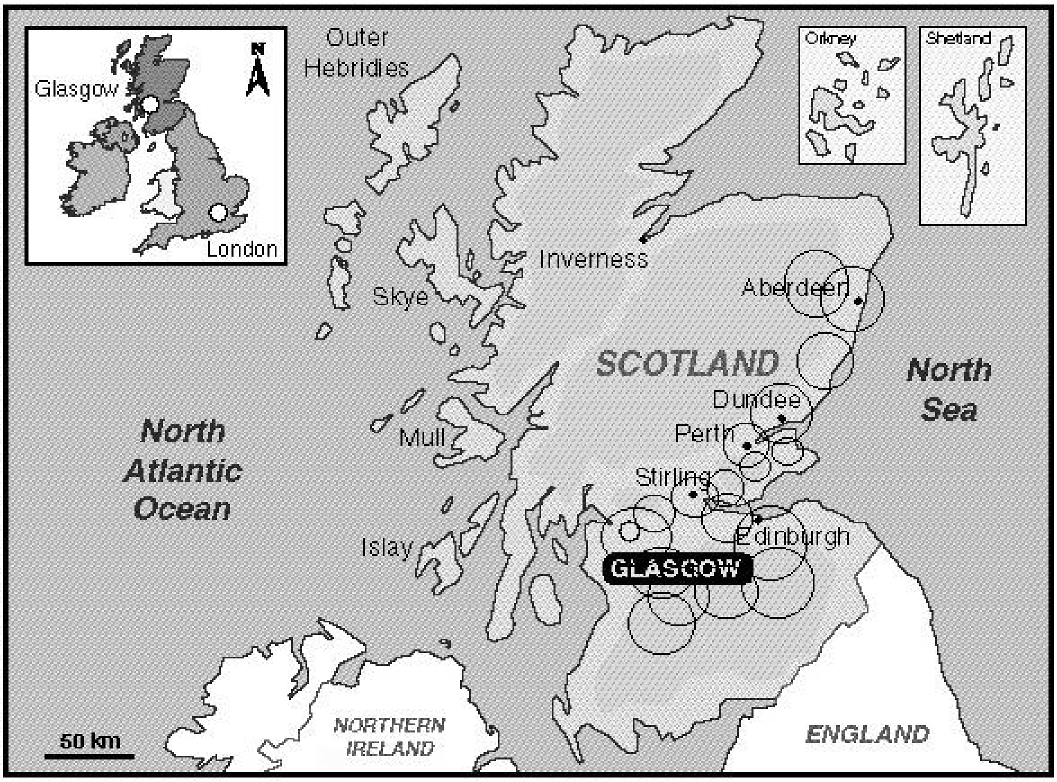

Glasgow
London
N
Outer
Hebridies
Orkney
Shetland
Inverness
Skye
Aberdeen
SCOTLAND
North
Sea
Dundee
Perth
North
Atlantic
Ocean
Mull
Stirling
Islay
Edinburgh
GLASGOW
NORTHERN
IRELAND
ENGLAND
50 km

Figure 1

The typical firm (in the sense of average or modal) of the sample is a micro-firm (i.e. having no more than 10 employees). This typical firm produced just under 50 products, which can be classified into four main product groups. Gross sales were around a quarter of a million pounds (for firms which survived) and about half that value for firms which ceased trading. Generally, survival rates were high, with 105 of the original 150 still being in business in the fourth year (1997). The number of firms exiting year by year was low at: 28 in year 2 (1995); another 5 in year 3 (1996); and a further 12 in year 4 (1997)[3].

[Table 1 near here]

In terms of how well the sample represents the population, Table 1 provides a reassuring picture. Essentially one has a reverse J-shaped distribution with a high proportion of the very smallest firm types, as measured by employees. The data presented provide the best comparison that could be made on available statistics. The Scottish data set for 1996 has rather more weight at the bottom end of the size scale (as does the UK in 1994), compared to the sample as initially selected for the project research in 1994. However, the effect is not marked. Based on this and other evidence, the sample is therefore thought to be a reasonable representation of the population of small firms in Scotland in the mid 1990s.

Business Size	**Sample 1994**	**UK 1994**	**Scotland 1996**
1-4	61.4	66.5	65.0
5-9	20.5	16.8	17.5
10-19	8.0	9.5	9.9
20-49	8.0	4.5	4.7
50-99	1.1	1.4	1.4
100+	1.1	1.3	1.5

Note: (a) Business size is measured by number of employees. (b) Figures are a percentage of the total number of businesses with 1 or more employees. (c) 1996 figures for Scotland are the earliest set available with are comparable to the sample.

Size Distributions of Sample and Population

Table 1

3. The Markovian Model

The first aspect of flexibility to be considered in this paper is that of ability of the small firm to change its principal market area. Firms in the sample were asked early in the interview (when dealing with Market Data) the following question:

"1.7 Do you consider your *main* market to be:

Local Regional Scottish British or International ?"

In the first year (1994), for example, firms replied: Local (34%), Regional (28%); Scottish (21%); British (11%); and International (6%). This same question was asked in follow up interviews for the next three years (1995-97). A frequency count was taken of these changes to estimate, by using the observed frequencies of moves, the probabilities of moving from one state to another. These probabilities were then used to achieve four things. First, they were used to forecast future patterns of main markets, under the assumption that these probabilities were stable over time. Second, they were used to investigate the pattern of change in main markets, period by period, in effect looking at adjustment to equilibrium. Third, they were used to compute the long run equilibrium pattern of main markets for the small firms. Fourth, they were use to compare short period adjustments to the ultimate long run equilibrium, to estimate the rate of convergence to equilibrium. To undertake these quantitative tasks, one requires recourse to the theory and techniques of Markov chain (also called 'process'), analysis. Though the use of Markov chains has been intermittent in industrial economics, a number of classical studies have used this technique, including Adelman (1958).

The principal theorems which are relevant to this type of modeling are as follows[4]. Consider a square (n×n) *matrix* defined by $(p_{ij}) = \mathbf{P}$. It has the properties that for its *elements* p_{ij} it is true that $p_{ij} \geq 0\ \forall ij$ and $\sum_j p_{ij} = 1\ \forall i$. As each element of a row is non-negative, and row sums are unity, **P** is said to be stochastic, because, in effect, each row defines a discrete stochastic distribution. A matrix of the form **P** is defined as a *transition probability matrix*. The word transition arises, because each row (or column) is said to refer to the *state* of a *Markov process* (or *chain*), and the process evolves by movements between states. Each element p_{ij} defines the probability of moving from the i'th to the j'th state of the process in one step, where this step is usually interpreted as one time period. This will be the interpretation used in this paper.

Higher powers of this matrix are defined by simple matrix multiplication, giving powers like $\mathbf{P}\,\mathbf{P} = \mathbf{P}^2$ and $\mathbf{P}\,\mathbf{P}^2 = \mathbf{P}^3$. Elements of the m'th power of this matrix are denoted p_{ij}^m. In a similar way to the above, p_{ij}^m refers to the probability of moving from the i'th to the j'th state of the Markov process in m time periods. Then if $\exists\ m'$ such that $\forall m > m'$ we have $p_{ij}^m > 0\ \forall ij$ then the Markov process is described as *ergodic*. This property is sometimes said to imply that the matrix **P** is *regular*, the implication being that, if it holds, every state is ultimately (in the sense of for a sufficiently large m) accessible from any other state of the Markov process. The word 'ultimately' has the connotation of 'in a finite number of time periods'. A property of a regular transition probability matrix is that $\mathbf{P}^n \rightarrow \mathbf{P}^*$ as $n \rightarrow \infty$. For such a $\mathbf{P}^* = (p_{ij}^*)$ the property will hold that $p_{ij}^* = p_{kj}^*$ for any i,k. That is, the rows of $\mathbf{P}^*$ are identical.

Thus regularity ensures that, powers of **P** tend to a limiting matrix with every row identical. This means, whatever the row, there is the same probability of getting to a specific column. Put another way, the Markov process 'has no memory' in the sense that the ultimate state is independent of the initial state. This notion is more clearly explained by considering an *initial state* vector $\mathbf{w}_0 = (w_i)$, where $\sum_i w_i = 1$ with $w_i \geq 0 \ \ \forall$ i. This state will become $\mathbf{w}_1 = \mathbf{w}_0 \mathbf{P}$ after one time period and $\mathbf{w}_2 = \mathbf{w}_1\mathbf{P} = \mathbf{w}_0\mathbf{P}^2$ after two time periods, and so on. Given the properties of w_i, **w** is often referred to as a *distribution*.

The final state of the process is then given as $\mathbf{w}^*$ where this is determined by the *fixed point* relationship $\mathbf{w}^* = \mathbf{w}^*\mathbf{P}$. That is the linear transformation **P** maps $\mathbf{w}^*$ into itself. Put another way, we have a way of finding that limiting $\mathbf{w}^*$, in the sense defined by $\mathbf{w}_n \to \mathbf{w}^*$ as $n \to \infty$, without having to compute higher powers of **w.** The vector $\mathbf{w}^*$ is the common row of $\mathbf{P}^*$. The upshot of this discussion is that the final distribution $\mathbf{w}^*$ is independent of the initial distribution $\mathbf{w}_0$. This is another way of expressing the idea noted above that a Markov process 'has no memory'. Finally, a point that has to be borne in mind is that $\mathbf{w}^* = \mathbf{w}^*\mathbf{P}$ cannot be solved directly, as **P** is singular (because its rows are linearly dependent), implying $|(\mathbf{I} - \mathbf{P})| = 0$. So the property $\sum_i w_i^* = 1$ with $w_i^* \geq 0 \ \ \forall$ i has to be used as well, as an auxiliary condition, to determine the *fixed point vector* $\mathbf{w}^*$.

Turning now to the data generated in the field work, and using the notation above, the initial vector of proportions in the classes local, regional, Scottish, national, international was given by

	LOC	REG	SCOT	UK	INT	
$\mathbf{w}_0$ =	[0.340	0.280	0.210	0.110	0.600E-01]	**(1)**

The states the Markov process has are five in number, and correspond to the market areas above. Using this same notation, the transition probability matrix **P** estimated[5], using raw relative frequencies, from four years of data (1994-97) on changes in main market areas was:

P =

	LOC	REG	SCOT	UK	INT
LOC	0.77	0.16	0.50E-01	0.90E-02	0.90E-02
REG	0.23	0.65	0.11	0.00	0.20E-01
SCOT	0.14	0.16	0.63	0.60E-01	0.20E-01
UK	0.00	0.00	0.12	0.82	0.60E-01
INT	0.00	0.00	0.40	0.20	0.40

(2)

Several points emerge from an inspection of the matrix in (2). (a) The first is that the principal diagonal of **P** (i.e. those elements p_{ii} $\forall i$) contains the largest elements. Thus, if a small firm starts (the row aspect of **P**) with a main market that is regional (REG) there is a 0.65 probability that it will still be regional the next period (the column aspect of **P**). The most 'absorbing' of the main market states is the UK market, with a 0.82 probability of a small firm which starts with that market remaining in that market to next time period. (b) However, there is still considerable flexibility in selection of main market between periods. This is clearly displayed in the matrix diagonals which are parallel to the principal diagonal. They tend to have the highest values next to those of the principal diagonal. Thus flexibility in main market is incremental, rather than radical. For example, there is a

0.23 probability that a small firm that starts with the region as its main market will have retrenched to a local market in the next period, and a 0.16 probability that a small firm which started locally will have become regional by the next period. Beyond these three diagonals, there is a little, but not much, action. For example, if the firm launched mainly in an international market, there is a zero probability that it will be in local or regional market in the next time period; and if a firm launched locally, it would have a tiny probability (0.009) of being mainly in an international market in the next period. (c) The international market generally has rather little activity. There is a less than evens chance (0.40) that if you launched internationally, you would remain international in the next period. If you did *not* launch internationally, there is only a slight probability that you will be international in the next period, irrespective of where you launched on the rest of the spectrum. For example, even if you launched with the UK as your principal market, there is only a 0.06 probability of this becoming international in the next period. This finding seems superficially to be consonant with recent policy views on small firm in Scotland, to the effect that unless they start with marketing intentions which are aimed at the international, they will never make this their main market. However, this overlooks the incremental approach which small firms can adopt to an international marketing standing. Briefly, the argument is that the diagonals of the estimated **P** which are adjacent to the principal diagonal (sometimes called the sub- and super-diagonals) may give small firms access to states (i.e. main markets) which may be denied to them on a one period basis. This argument will now be explored in more detail, as the evolution of the Markov process is considered.

Using the fixed point property above, the long run equilibrium distribution of the main market area for this Markov process, if it exists, is computed directly as:

$$\mathbf{w}^* = [0.402 \quad 0.272 \quad 0.194 \quad 0.108 \quad 0.216\text{E-}01] \qquad \mathbf{(3)}$$

Comparing (1) and (3), which is to say the *initial* distribution across market areas ($\mathbf{w}_0$), as compare to the long run or *final* equilibrium distribution ($\mathbf{w}^*$), it is observed that in long run equilibrium the 'weight' of the distribution has shifted down, towards the local main market state. The international main market has become an almost negligible state (down from 6% to 2%), and both Scotland and the UK as main markets have become less important. Above all, the local market has become the main market (up to 40% from 34%).

It is of interest to observe how this has come about, and to ask questions of the adjustment process to long run equilibrium, like how rapidly does it proceed, and is its effect monotonic for all states? Table 2 displays second, third, fourth and fifth powers of the transition probability matrix **P**.

[Table 2 near here]

It will be observe that the second power $\mathbf{P}^2$ produces a matrix with all elements positive. Thus **P** is a regular, stochastic matrix. This provides quantitative confirmation that we should expect powers of the matrix to converge[6].

$\mathbf{P}^2$

0.63	0.23	0.92E-01	0.19E-01	0.15E-01
0.34	0.47	0.16	0.12E-01	0.25E-01
0.23	0.22	0.43	0.92E-01	0.28E-01
0.16E-01	0.19E-01	0.19	0.69	0.75E-01
0.56E-01	0.64E-01	0.43	0.26	0.18

$\mathbf{P}^3$

0.55	0.26	0.12	0.29E-01	0.19E-01
0.39	0.39	0.18	0.28E-01	0.26E-01
0.26	0.25	0.30	0.10	0.32E-01
0.45E-01	0.46E-01	0.24	0.59	0.76E-01
0.11	0.12	0.38	0.28	0.98E-01

$\mathbf{P}^4$

0.50	0.28	0.14	0.40E-01	0.22E-01
0.41	0.34	0.19	0.42E-01	0.27E-01
0.33	0.26	0.27	0.11	0.33E-01
0.79E-01	0.76E-01	0.26	0.51	0.72E-01
0.17	0.15	0.33	0.27	0.67E-01

$\mathbf{P}^5$

0.47	0.28	0.16	0.51E-01	0.24E-01
0.42	0.32	0.19	0.55E-01	0.28E-01
0.35	0.27	0.24	0.12	0.34E-01
0.11	0.10	0.26	0.45	0.67E-01
0.21	0.18	0.29	0.26	0.55E-01

Second, Third, Fourth and Fifth Powers of Estimated Transition Probability Matrix P

Table 2

This process of convergence is most evident in Table 2. Very rapidly, previously inaccessible states become accessible (after just one period). Also, transitions which once had very low probabilities quickly assume quite large probabilities. To illustrate, it was impossible (i.e. probability zero) to go from an international main market to either a local or regional main market in just one period one. However, there is a finite but small probability of doing either in the second period period with probabilities 5.6% and 6.4% respectively. In the third, fourth and fifth periods, these probabilities have risen to (11%, 12%), (17%, 15%) and (21%, 18%). In fact the rise in these probabilities is rapid, given that for each period we are computing the power of **P** a year goes by in the life of the small firm. Put another way, these small firms display considerable flexibility, in terms of adaptation of their main market, in the early years of their existence.

Another point to observe about Table 2 is that rows of the higher powers of **P** become increasingly similar quite rapidly. By period five, the difference between the first and second rows of $\mathbf{P}^5$ is less than 5%, whereas in the first and second periods, the difference was marked. It will be observed that rows of **P** have also come some distance to approximating to the long run equilibrium, as represented by $\mathbf{P}^*$, the matrix limit. This process is even more evident if attention is focused on the initial distribution $\mathbf{w}_0$ and its successors in the sequence generated by the algorithm $\mathbf{w}_n = \mathbf{w}_0\mathbf{P}^n$. The first five iterations of this are given in Table 3. Again one sees the relatively rapid convergence to the long run equilibrium value. For example, the Local state, which accounts for most of the small firm flexibility, has adjusted to within 95% of its long run equilibrium value by five iterations (i.e.five years). The other probability weights in this vector are much closer, proportionally, to their long run values than even this, after five years. One also notes that adjustment, whilst *typically* monotonic, is not *necessarily* monotonic. For example, the adjustment to the UK weight initially rises from 0.110 (indeed, rises for all the iterations shown to 0.124) but must eventually fall, to reach the value of 0.108 in long run equilibrium.

[Table 3 near here]

Overall, the evidence from the transition probability matrix analysis, is that small firms exhibit considerable flexibility in switching between main markets. Further, the speed of adjustment towards long run equilibrium is quite rapid, with a large proportion of adjustment occurring within just a few years. This lends further credence to the mode of analysis and its conclusions, in the sense that these periods of almost full adjustment are sufficiently short that it is not unreasonable to assume that estimates of transition probabilities are approximately stable over the time period concerned. The next section turns to another form of dynamics and flexibility, relating to scale and its variation over time.

Initial distribution in 1994

LOC	REG	SCOT	UK	INT
0.340	0.280	0.210	0.110	0.600E-01

Distribution Projections for Years 1-5 Ahead

Year 1

0.355	0.270	0.217	0.117	0.434E-01

Year 2

0.366	0.267	0.215	0.121	0.374E-01

Year 3

0.373	0.266	0.213	0.123	0.352E-01

Year 4

0.379	0.267	0.211	0.124	0.344E-01

Year 5

0.382	0.268	0.210	0.124	0.342E-01

. . . .

Long Run Equilibrium Distribution

LOC	REG	SCOT	UK	INT
0.402	0.272	0.194	0.108	0.216E-01

Distribution Evolution of w_n Year by Year

Table 3

4. Variants of Gibrat's Law

The new business start ups that are typical of the sample examined often have owner-managers who are ambitious to see their firms grow rapidly. However, the growth process is fraught with uncertainty, and the possibility of re-trenching has also to be considered, as experiments in new niches may fail to be as successful as anticipated. In short, the entrepreneur must be flexible in adapting the scale of operation of her small firm to changed economic conditions. This type of flexibility, to grow rapidly, but at possibly variable rates, or even to contract, depending on evolving niche opportunities, is the focus of this second empirical section of the paper.

It is a more general form of flexibility than that considered in Section 3, because flexibility in the main market choice, plus much in addition, in terms of flexibility (e.g. in work force composition, like the ratio of full time to part time workers), is needed to accommodate to flexibility in terms of scale of operation. There are a number of measure of scale that could be used, but the results reported are not particularly sensitive to this choice. The one adopted here, for simplicity, is gross sales. The symbol used here for size, in a generic sense, is S_t , which denotes size in time period t. It is readily interpreted in terms of sale, and this will be done explicitly when estimates are discussed.

However, to start with, the general Gibrat's Law formulation, and its variants, will be briefly discussed, without any restriction on what is meant by size (S_t) - it could be sales revenue, output volume, capacity, headcount, assets, profit, whatever. Suppose markets expand at the rate γ and that all small firms share this common growth rate:

$$S_{t+1}/S_t = \gamma \qquad \textbf{(4)}$$

This is the Gibrat Law, or the law of Proportionate Effects, to the effect that growth is independent of size, see Sutton (1998, 242-243). If there is an endogenous effect of size on growth, one simple way of generalizing (4) is:

$$S_{t+1}/S_t = \gamma S_t^{(\beta - 1)} \qquad \textbf{(5)}$$

This is the most popular variant of Gibrat's Law, for which the Gibrat case falls out from (5) when $\beta = 1$. When $\beta > 1$ larger small firms have higher growth rates than smaller ones, and when $\beta < 1$ smaller small firms have higher growth rates than larger small firms. Finally, the variant (5) can be extended by multiplying it by an independently distributed, positive random variable $\mu_t > 0$, giving:

$$S_{t+1}/S_t = \gamma S_t^{(\beta - 1)} \mu_t \qquad \textbf{(6)}$$

Equation (6) can be expressed in a form suitable for econometric estimation by casting it in log-linear form:

$$\ln S_{t+1} = \ln \gamma + \beta \ln S_t + \ln \mu_t \qquad \textbf{(7)}$$

or

$$s_{t+1} = \alpha + \beta s_t + \varepsilon_t \qquad \textbf{(8)}$$

where, in obvious change of notation, $\ln S_{t+1} = s_{t+1}$, $\ln \gamma = \alpha$, $\ln S_t = s_t$ and $\ln \mu_t = \varepsilon_t$. It is equation (8) which is the focus of attention in this section of the paper. Once (8) is estimated as

$$s^e_{t+1} = a + b\, s_t \qquad \textbf{(9)}$$

Where e denotes expected value for the dependent variable, and (a,b) are regression estimates of (α,β). Equation (9) is an expression for a first order linear difference equation, for which the stability condition is $0 < b < 1$. If this condition holds, then the sequence $\{ s_t \}$ converges to an equilibrium value of s^*. Equilibrium is achieved when

$$s^e_{t+1}= s_t = s^* = a/(1-b) \qquad \textbf{(10)}$$

A useful way of representing the dynamics of (9) is by use of a phase diagram, with s_{t+1} on the vertical axis and s_t on the horizontal axis. The equilibrium set of points is then represented by those values of the size variable that are equal, period by period, that is for which $s_{t+1} = s_t \;\; \forall t$. This is of course the 45^0 line in the phase diagram.

Estimation of (9) by regression methods can proceed once variables have been expressed in constant prices. In this paper, where 1994, 1995, and 1997 magnitudes are used, they are expressed in 1994 prices[7]. Estimates for a regression of log size on log one-period-lagged size are reported in Table 4.

[Table 4 near here]

```
102 OBSERVATIONS     DEPENDENT VARIABLE = LSALES2

  R-SQUARE =   0.8494     R-SQUARE ADJUSTED =   0.8479


                 ANALYSIS OF VARIANCE - FROM MEAN
                  SS         DF            MS           F
 REGRESSION    184.85         1.        184.85        563.944
 ERROR          32.779       100.       0.32779        P-VALUE
 TOTAL         217.63        101.        2.1548         0.000


 VARIABLE   ESTIMATED  STANDARD   T-RATIO
   NAME    COEFFICIENT   ERROR     100 DF   P-VALUE
 LSALES1   0.89754     0.3780E-01   23.75     0.000
 CONSTANT   1.4730     0.4162       3.539     0.001

 test b=1

 WALD CHI-SQUARE STATISTIC = 7.3490621 WITH 1 D.F.  P-VALUE= 0.00671

 test b=0.79796

 WALD CHI-SQUARE STATISTIC = 6.9418751 WITH 1 D.F.  P-VALUE= 0.00842
```

Regression of Log Real Sales 1995 on Log Sales 1994

Table 4

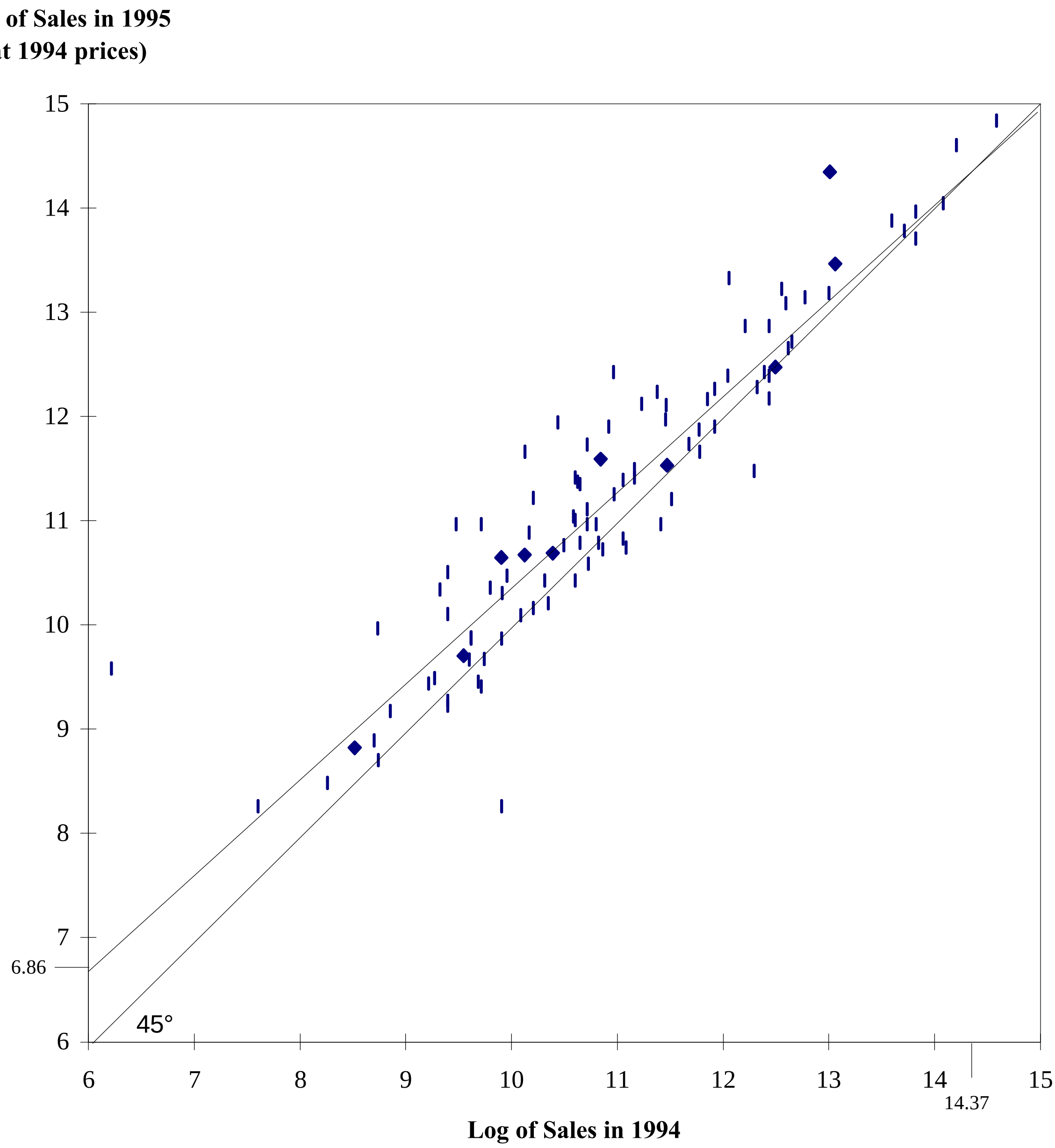
The regression reported on, in Table 4, is estimated as:

$$lsales2^{e} = 1.4730 + 0.8975\ lsales1 \qquad \textbf{(11)}$$

This is the estimated version of (9) above. The dependent variable is the natural log of real sales in 1995. The independent variable is the natural log of sales in the base period, 1994. The adjusted R^2 is high at 0.85, and the F value (563.9), as a test of goodness of fit of the overall regression, is highly statistically significant ($p = 0.000$). On a Wald test, the null hypothesis of H_0: $b = 1$ is clearly rejected ($p = 0.0067$). That is, the slope coefficient of equation (11) is highly significantly different from unity. This is an important finding, as it rejects Gibrat's Law, and further suggests a stable dynamic process of adjustment in small firm size. This rejection of Gibrat's Law ($b = 1$), in favour of the alternative $b < 1$ gives important status to small firms, as enjoying relatively greater growth prospects than large firms, and in this sense displaying greater flexibility[8].

I shall be presenting an alternative estimator to (11) in (12) below, which regresses the natural log of sales in 1997 (at 1994 prices) on the natural log of sales in 1994. There, in (12) the estimator has a smaller slope coefficient (b) of 0.79796 (see Table 5). On a Wald test of the hypothesis that the slope coefficient of the first estimated equation (11) (see Table 4) is equal to this value, as in (12), we find the hypothesis strongly rejected (p= 0.008). Thus the adjustment processes over a one year period and over a three year period are quite distinct.

[Figure 2 near here]



Note:

a) Fitted line: $\widehat{\text{lsales2}}$=1.4730+0.8975lsales1

b) Superimposed on 45° line for which lsales1=lsales2 (i.e. set of equilibrium values)

c) One period lag

d) Sales are gross sales at 1994 prices

Regression of Sales on Lagged Sales

Figure 2

The estimated regression equation (11) is graphed in the phase diagram of Figure 2. Also on this figure are the 45^0 line of equilibrium values, and the data points on which the estimated line was computed. Visual inspection indicates that the regression line is clearly a very good fit to the set of data points, as confirmed by the explicit statistical testing of Table 4. The slope of the regression line is low, at roughly 0.9. The equilibrium value for this process is, following the algebra of (10) above, $s^* = 1.4730 \div (1- 0.8975) \cong 14.37$. This is indicated in Figure 2. It is immediately apparent from Figure 2 that most small firms in 1994-95 were well short of the equilibrium position of the dynamic adjustment process of which they were a part. This picture of adjustment is different from the one reported upon by the author in an earlier study of small firm dynamics in Scotland, Reid (1993, Figure 11.3) where there was more dispersion about the equilibrium point. This is presumably because, in that study, the firms were older than in this study, allowing for greater adjustment about the equilibrium.

[Table 5 near here]

I turn now to the adjustment process over the longer period 1994-97. Table 5 reports on a regression which runs the natural log of sales in 1997 (in 1994 prices) against the natural log of sales in 1994. Here, the OLS method has been modified by using White's (1980) heteroskedastic consistent covariance matrix estimator, see Greene (1993, p391). As regards goodness of fit, the adjusted R^2 of 0.7073 is high, and the F-test for overall significance of the regression gives a highly significant test value of 162.94 ($p = 0.000$).

```
OLS ESTIMATION

      68 OBSERVATIONS     DEPENDENT VARIABLE = LSALES4


 USING HETEROSKEDASTICITY-CONSISTENT COVARIANCE MATRIX

  R-SQUARE =   0.7117     R-SQUARE ADJUSTED =   0.7073
```

ANALYSIS OF VARIANCE - FROM MEAN				
	SS	DF	MS	F
REGRESSION	108.06	1.	108.06	162.940
ERROR	43.772	66.	0.66322	P-VALUE
TOTAL	151.84	67.	2.2662	0.000

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 66 DF	P-VALUE
LSALES1	0.79796	0.9550E-01	8.356	0.000
CONSTANT	3.0251	1.108	2.731	0.008

```
test b=1

WALD CHI-SQUARE STATISTIC = 4.4758942 WITH  1 D.F. P-VALUE= 0.03438
```

Regression of Log Real Sales 1997 on Log Sales 1994

Table 5

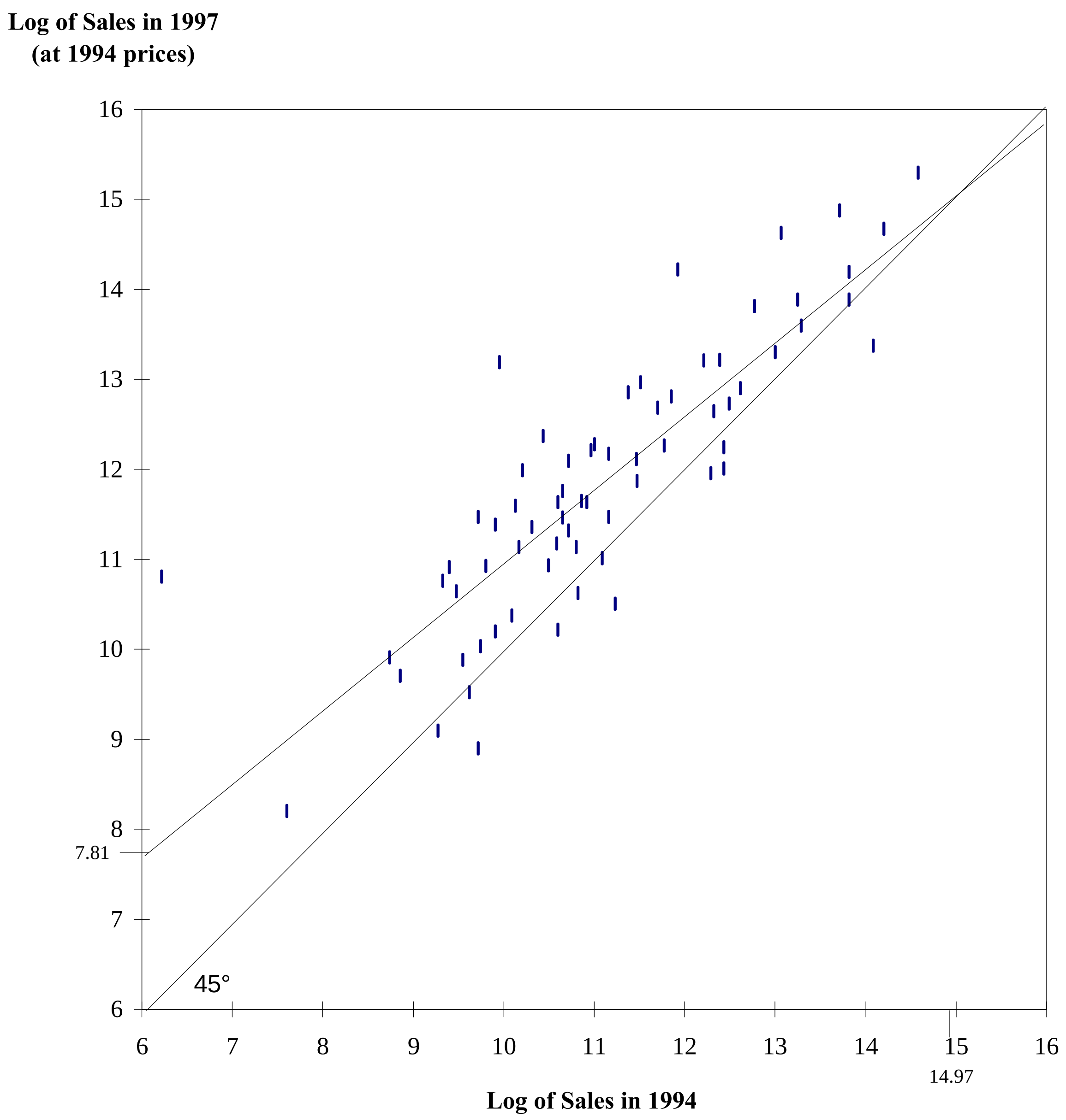
The estimated coefficients of the regression are also highly statistically significant ($p = 0.000$ for b; and $p = 0.008$ for a).

[Figure 3 near here]

The estimated regression is:

$$\text{lsales4}^{e} = 3.0251 + 0.79796 \text{ lsales1} \qquad \textbf{(12)}$$

This estimated equation is shown in the phase diagram of Figure 3. The main difference between (11) and (12) is in the slope coefficient. Further, one notes that the dispersion of data points is greater in Figure 3, reflecting the longer history of small firms for which data are displayed in that figure. For equation (12) a Wald test (of b=1) does indeed confirm that the slope coefficient is significantly different (at the 5% level) from unity ($p = 0.03438$), so again Gibrat's Law is refuted (see Table 5). Further, we note that the slope variable of (12) is, when tested against the slope variable of (11) above, definitely statistically significantly different. Approximately, one has a slope of 0.9 in the first case, and of 0.8 in the second case, and the difference is statistically significant ($p = 0.00842$), see bottom of Table 4.



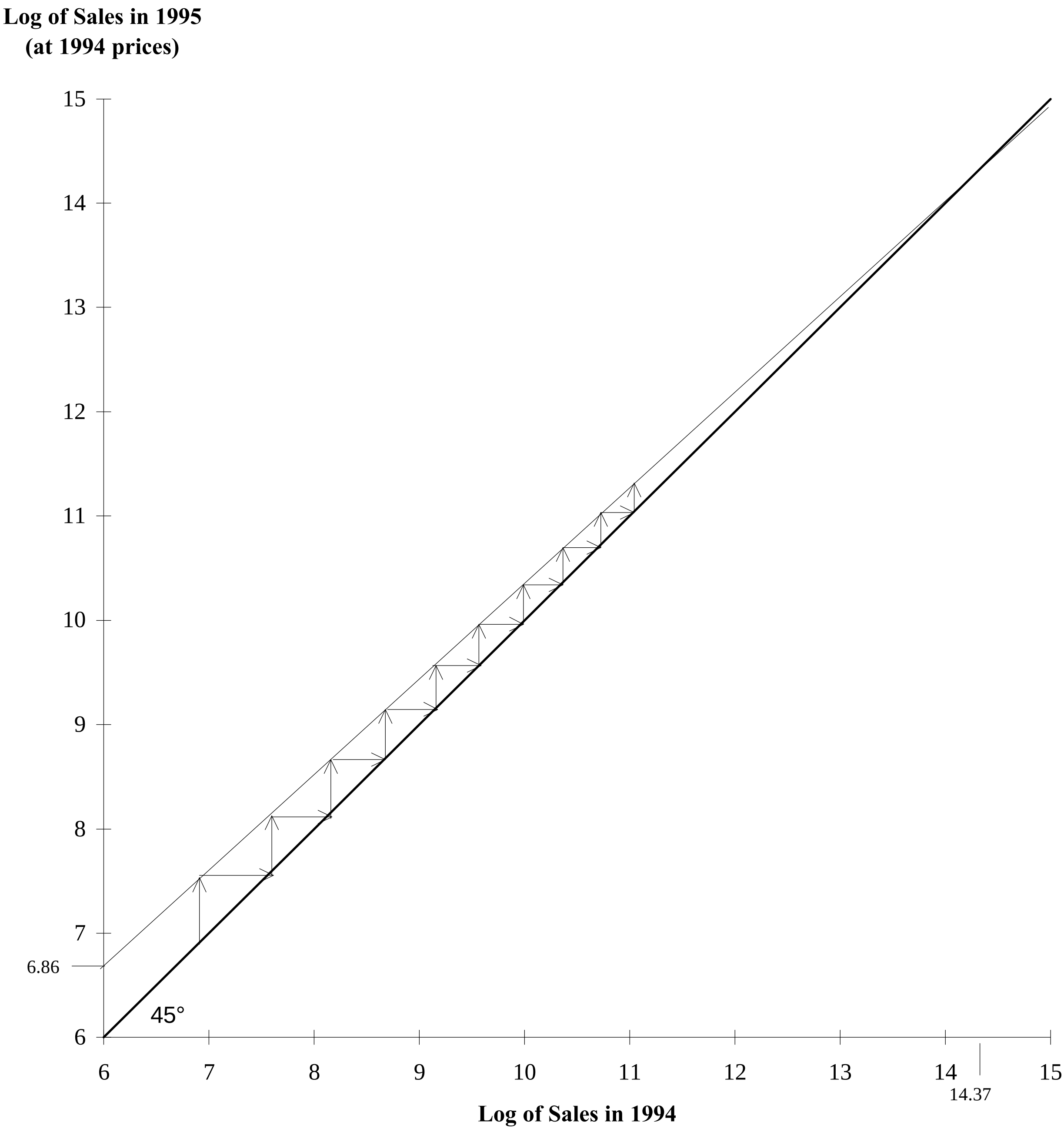
Note:
a) Fitted line: $\widehat{lsales4}$=3.025+0.7979lsales1
b) Superimposed on 45° line for which lsales1=lsales4 (i.e. set of equilibrium values)
c) Three period lag
d) Sales are gross sales at 1994 prices

Regression of Sales on Lagged Sales
Figure 3

In terms of the adjustment process, the equilibrium value implied by the dynamic equation (12) is s* = 3.0251 ÷ (1- 0.79796) ≅ 14.97. In terms of gross sales this equilibrium value is exp (14.97) = £ 3,181,227 for equation (12). This is to be compared with the equilibrium value of exp (14.37) = £ 1,741,051 in the case of the estimated equation (11). In real terms this difference is considerable (approx. 87%), although the use of logs of variables previously masked this feature of the results. Put briefly, the equilibrium position has risen considerably.

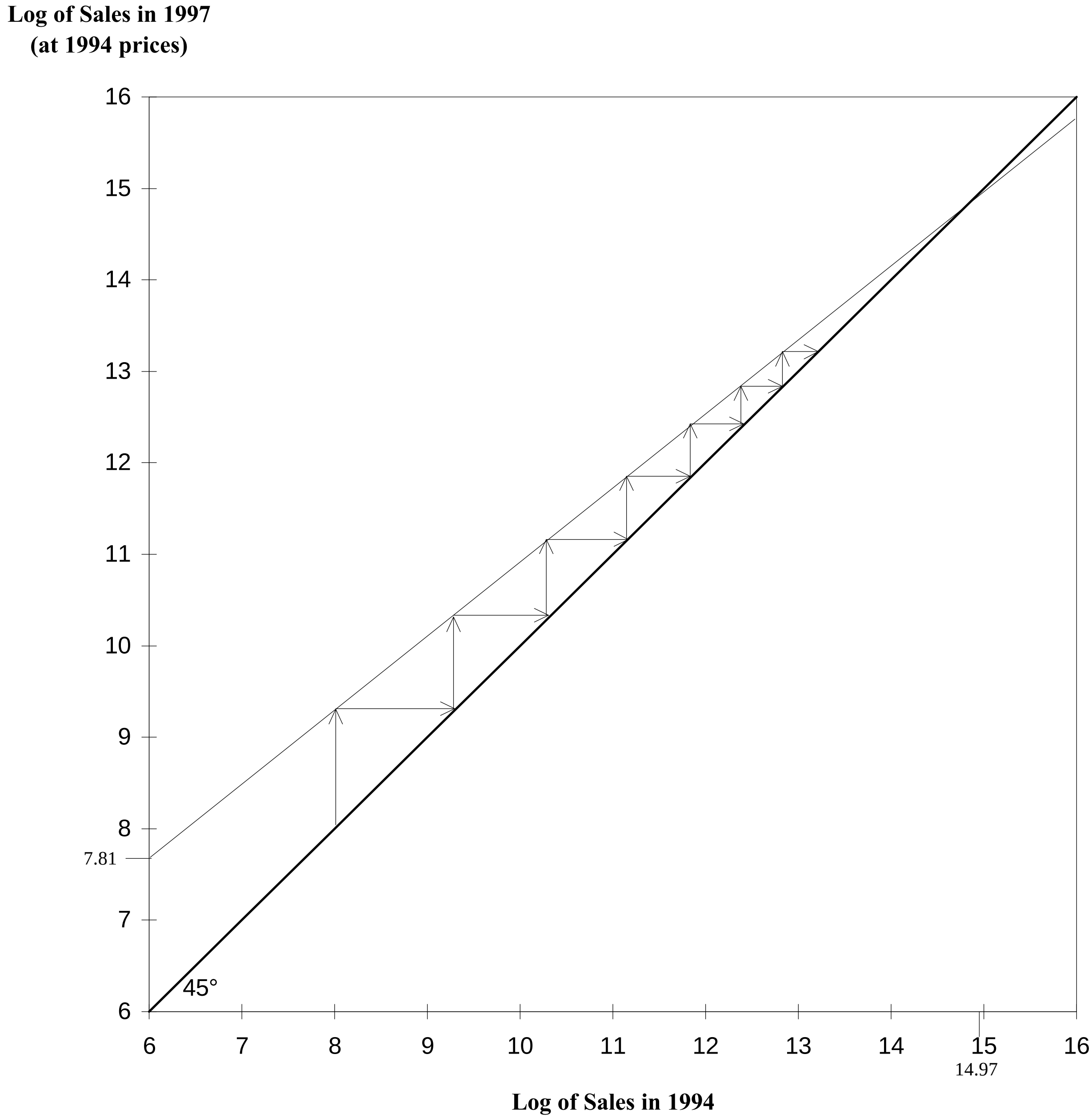
These equilibrium values are generally considerably greater than the average size of the small firms in the sample. In 1995, the average gross sales for small firms in the sample were £ 226, 000, and in 1997 they were £ 336, 000 (both in 1994 prices). For equation (11) small firms were approximately one eighth (on average) of their long period equilibrium values, and for (12) they were approximately one ninth (on average) of their long run equilibrium values. In ratio terms, these difference between equations (11) and (12) are not great, but both imply a lot of adjustment has yet to occur[9]. This evidence further illustrates the flexibility of adjustment that occurs in these small firms.

[Figures 4 and 5 near here]



Note:
a) Fitted line: $\widehat{lsales2}$=1.4730+0.8975lsales1
b) Superimposed on 45° line for which lsales1=lsales2 (i.e. set of equilibrium values)
c) One period lag
d) Sales are gross sales at 1994 prices

Adjustment of Sales to Equilibrium Value
Figure 4



Note:

a) Fitted line: $\widehat{lsales4}$=3.025+0.7979lsales1

b) Superimposed on 45° line for which lsales1=lsales4 (i.e. set of equilibrium values)

c) Three period lag

d) Sales are gross sales at 1994 prices

Adjustment of Sales to Equilibrium Value
Figure 5

Figures 4 and 5 provide an illustration of the different adjustment processes to equilibrium implied by the estimated equations (11) and (12). For clarity, data points on which the estimates were constructed have been removed. In Figure 4 equation (11) alone is displayed, superimposed on the 45^0. Similarly, in Figure 5, estimated equation (12) is displayed. Also superimposed is a possible adjustment path towards equilibrium, in each case. The step lengths are shorter in Figure 4, given any initial starting size, compared to Figure 5. This is essentially because the same processes is being discussed in each case, but from the perspective of different time intervals. In this sense, the adjustments of Figure 4 are embedded in the adjustments of Figure 5. Thus, in Figure 4, year by year adjustments are being illustrated, whereas in Figure 5, adjustments are taking place over three year intervals, in each step. Over the longer time interval, these small firms have to be more flexible in their scale change responses. These difference in step length and paths to equilibria are of course a direct reflection of the different speeds of adjustment of the dynamic processes implied by the different slope estimators reported in estimated equations (11) and (12).

5. Conclusion

The aim of this paper has been to explore two types of dynamics of small firm adjustment. The first concerns 'market re-positioning': that is, the moving to new niches for principal products, as a type of flexible response to changed market opportunities. The second is changes in scale, according to some size measure, in this case, sales. If growth rates are high, as indeed they typically are for small firms close to inception, considerable demands of flexibility are imposed on the firm.

The dynamics in question have been examined in terms of two models, which are estimated on primary source data from 150 new business start ups. The first is a Markov chain model, which was estimated on data about changes in a small firm's main market. The second is a variant of a Gibrat's Law type of model, which examines the dependence (or otherwise) of growth and scale of the firm. Both models enable dynamic paths to be traced for small firms, towards a well-defined long term equilibrium. Both models also allow statements to be made about the stability of the adjustment process. It was found that adjustment was relatively rapid in the case of main product markets, with a high proportion of adjustment occurring in just a few periods. In the case of sales growth, Gibrat's Law was refuted. However, a stable adjustment process was discovered, but one which required rather many periods, before getting close to equilibrium, and in the process required considerable scale adjustment, and in that sense, flexibility.

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Footnotes

[1] Professor of Economics, and Director, Centre for Research into Industry, Enterprise, Finance and the Firm (CRIEFF), University of St Andrews, Scotland, KY16 9AL. This paper was first presented to a meeting of the Network of Industrial Economists (NIE), on the theme of 'Industrial Dynamics', at the University of Reading, 19th December, 2000. I should like to thank the following participants for their useful comments: Marc Casson, Brian Loasby, Peter Hart, Ken Simons, and John Sutton. Any faults that this paper yet may contain, remain my own responsibility. The work behind this paper was supported by a research grant from the British Academy on the theme of 'Flexibility in the Small Firm'. I am most grateful for this support. I should also like to thank Kirsty Hopkins of CRIEFF who acted as Research Assistant on this project.

[2] For further details on the Enterprise Trust as a business incubator see Reid and Jacobsen (1988, Ch. 5).

[3] These relatively high survival rates are partly attributable to the fact that all small firms came through business incubator units (ETs), but no doubt an additional feature was the relatively successful state of the Scottish macro-economy over this period of time.

[4] See, for example, the classical test book by Emanuel Parzen (1960, Ch. 3) *Modern Probability Theory and its Applications*.

[5] To save on notation, I have not put a hat over this **P** to denote 'estimate'. However, the **P** of (2) is indeed an estimate. It is estimated from all data over the period 1994-97 pertaining to all reported 'state to state' shifts of nominated main product markets over a one year period. The estimates for each cell are the normalized raw frequencies.

[6] It also allows one to use the direct fixed-point method for computing the long run equilibrium given in $\mathbf{w}^*$ of (3) above.

[7] The deflator for 1995 was 1.035, and the deflator for 1997 was 1.093. The retail price index was used for deflation.

[8] It is also saying that smaller small firms grow faster than larger small firms. The original evidence in favour of Gibrat's Law, suggesting there is no size effect at all, is nicely summarised in Sutton (1998, Ch 10). Contrary evidence for UK quoted companies (1948-60) is adduced in Singh and Whittington (1968) who find a positive relationship between size and growth ($b>1$). However, this is prima facie implausible, implying as it does an unstable adjustment process. Kumar (1985), using a similar set of company data, relating to the next sixteen years of evidence, found a negative relationship between size and growth. Since the 1970s, evidence has confirmed the result that $b<1$, as summarised in the paper by Hart (2000, Table 1).

[9] In this sense, the log scale of Figures 2 and 3 overstates the extent to which adjustment has been completed. However, as compared with the adjustment of market position alone

(Section 3 above), the scale adjustment considered here involves a great deal more modification of the small firm's operations e.g. workforce, capacity utilisation, debt etc.