Market entry and roll-out with product differentiation*

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Abstract

This paper examines a general problem exemplified by post-auction (third generation—'3G') mobile telecommunications markets. When entering these (or any other) markets, firms must often decide on the degree of coverage ('roll-out') they wish to achieve. Prior investment must be sunk in order to achieve the desired (or mandated) coverage level. We study the private and social incentives of a would-be entrant into a market with horizontal product differentiation when choosing its level of roll-out. The endogenous extent of entry influences downstream retail prices; Bertrand or local monopoly pricing or a mixed strategy equilibrium may emerge. Importantly, entry may involve too much or too little roll-out from a social perspective, thus suggesting that regulatory intervention may be appropriate to achieve desired levels of competition in such settings.

KEYWORDS: COVERAGE, ROLL-OUT, ENTRY, REGULATION
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1 Introduction

This paper examines a general problem exemplified by post-auction (third generation—'3G') mobile telecommunications markets. A potential entrant to these (or other) markets must make a variety of decisions. Where to locate in product space and what price to charge are two examples. Issues of location and pricing in horizontally and vertically differentiated markets have received considerable attention (see e.g. Beath and Katsoulakos (1991)). This paper is about another decision faced by potential entrants: their level of market coverage. Entry decisions in models of product differentiation are typically modelled as involving an exogenously fixed cost, with resulting market shares arising as the result of post-entry competition. Yet they will also be determined by the extent of entry by the new firm—by the amount of the market it chooses to cover.

It is easy to think of situations where a firm might make a roll-out decision prior to entry. However, some interesting recent examples can be found in utilities regulation: in particular in telecommunications. Thus, in the UK, following the granting of a fifth licence for UMTS (3G) mobile operation following spectrum auctions in 2000, the winner (TIW UMTS (UK)) must now decide how fast to roll out its network. Although it faces externally-set targets here, the interim decisions on coverage are its own. Similar issues face post-auction entrants in other countries. Another example can be found in UK postal services, where the regulator (Postcomm) has recently licenced Hays plc to compete with the incumbent monopolist Consignia. Hays have agreed short-term coverage levels. Another company (Deya) is reportedly keen to enter this market with different coverage levels (100%). The fact that, in all these cases, the entrants have stressed the differentiated nature of their product, relative to incumbent facilities, makes them strong examples of the issues our paper aims to address.

Apart from the privately optimal level of coverage for an entrant into a differentiated market, these examples raise another question: what is the socially optimal level of coverage? Perhaps the entrant will opt for low coverage in order to relax downstream price competition, but this may not satisfy regulatory preferences. In the above examples, this question relates to the universal service obligations present in both telecommunications and postal services (see Cremer *et al.* (2001)). We therefore seek to compare the socially and privately optimal levels of coverage.

A variety of authors have looked at issues relating to our work. Thus, Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), look at existing duopolists' capacity decisions in advance of Bertrand pricing games. In each case, they deal with homogeneous products and established competitors. Similarly, Valletti *et al.* (2001) asks about the optimality of universal service obligations when output is homogeneous. Dixit (1980) looks at an incumbent's capacity decision in advance of a potential entrant's arrival and the prospect of Cournot competition with homogeneous outputs.

It is interesting that, in our paper, the entrant has the ability (through its roll-out decision) to influence the downstream retail price equilibrium (as Dixit's incumbent can). Other authors have looked at entry decisions with differentiated products, where the scale (and cost) of entry is fixed (see Beath and Katsoulakos (1991)). Prescott and Visscher (1977) and Mason and Weeds (2000) extend these models to look at the timing of entry (or of product introduction). Several authors have examined incentives for investment in telecommunications markets with exogenous entry costs (see Wildman (1997), Gans and Williams (1999), Carter and Wright (1999)). Laffont *et al.* (1998a) discuss the problem we consider below, but do not solve it. They indicate how market coverage may be modelled and describe the two-stage game we study, but they do not derive the variety of potential equilibria or compare this with a benevolent regulator's choice of coverage.[1]

The paper proceeds as follows. The main part is divided into two sections aimed at analysing markets with inelastic and elastic consumer demands. In Section 2, we examine a variant of a simple Hotelling model, in which consumers have unit demands. This limiting case of inelastic demands is convenient because it has a well-defined monopoly solution. In order to address the question of roll-out, we imagine the total market as a unit square and assume that price competition takes place on that portion of the square the entrant chooses to cover. In this way, we endogenise the entry cost and give entry a geographical interpretation. Here we identify three downstream price equilibria: two in pure strategies (Bertrand competition and 'local monopoly', where each firm behaves as a standard monopolist within its market area) and one in mixed strategies. Restricting ourselves to situations where a pure strategy equilibrium exists, we show that the entrant will never invest beyond that level which guarantees local monopoly, where both entrant and incumbent tacitly collude on market share. We show that this level of entry will never be socially optimal: the regulator will either wish for sufficient entry to stimulate price competition, or none at all (because entry costs are too great).

Section 3 relaxes the assumption of unit (inelastic) demands and examines the case of elastic demands. We cannot derive closed form solutions for this case, but simulations illustrate several important effects of these more elastic demands. In particular, the entrant may now invest sufficiently for price competition to take place because, depending on the relative elasticities for the two products, this may stimulate inroads into the incumbent's market. Again, regulatory preferences may differ from this result, with excessive or insufficient roll-out taking place. Section 4 concludes the paper, and considers what our results mean for the need to include coverage targets when

[1] In general, Armstrong (2001), p. 67, observes that modelling investment in network industries is an important next step for literature in this area.

regulators allow new entry into geographical markets.

2 The model with inelastic demands

2.1 Market coverage

The market consists of a unit square, corresponding to a geographcal area (see Figure 1). Consumers are uniformly distributed across the square and, in particular, on $[0, 1]$ along every horizontal axis in the square (each axis therefore forms a sub-market corresponding to a Hotelling model of horizontal product differentiation). Firm 1 (the incumbent) is situated (exogenously) at point 0 in each of these sub-markets and, by assumption, already covers the whole geographical market. Firm 2 (the entrant) must decide whether to enter (at point 1 in each of the sub-markets) and what proportion of the total market to cover (the value of μ in Figure 1). If entry takes place, the firms compete for market share within the common sub-markets with α (to be determined) of each market segment going to Firm 1; the incumbent retains a monopoly in the remaining $1 - \mu$ of the market. Thus, we envisage a two-stage process where Firm 2 chooses its level of coverage (μ), then price competition takes place (determining α). Clearly, $\mu = 1$ corresponds to universal competition.

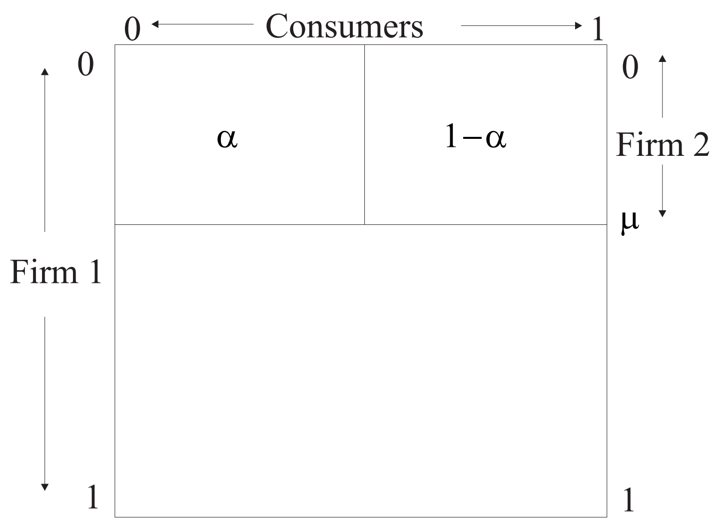

Figure 1: The market square

We derive the shares in each sub-market in the conventional way. In a representative sub-market, consumers derive a gross surplus of $\overline{s}$ when consuming from either firm and have (for now) unit demands (they buy 'one or none'). As mentioned earlier, this case can be thought of as the limit when elasticities tend to zero (and consumers have a maximum willingness to pay, $\overline{s}$). Its advantage over other inelastic cases is that it has a well-defined monopoly price so we can allow for a variety of possible market outcomes.

Consider a consumer at point $x \in [0, 1]$ on the horizontal axis. Then the net surplus when consuming from Firm 1 (located at $x = 0$) at price p_1 is $\overline{s} - p_1 - tx$, while that from Firm 2's product is $\overline{s} - p_2 - t(1 - x)$; t is the transport (or 'utility') cost of consuming away from the supplier. Market shares at $x = \alpha$ (if both firms can service the whole sub-market) are found from the indifference condition:

$$\overline{s} - p_1 - t\alpha = \overline{s} - p_2 - t(1 - \alpha)$$

$$\Rightarrow \alpha = \alpha(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t} \tag{1}$$

Thus, Firm 1's market share in the contested region is α and Firm 2's is $1-\alpha$ (assuming full coverage in this sub-market), as in Figure 1. Then, given Firm 2's initial coverage decision (μ), the respective shares for Firm 1 (the incumbent) and Firm 2 (the entrant) become

$$\alpha_1 = 1 - \mu(1 - \alpha), \qquad \alpha_2 = \mu(1 - \alpha)$$

We assume that investment is costly to the entrant, with the cost function being $d(\mu) = \gamma\mu^2/2,\ \gamma > 0$. Each unit produced has marginal cost of c.[2]

2.2 Retail prices

The firms play a two-stage game in which the entrant first sets μ, then price competition ensues in the retail market.[3] We therefore solve by backwards induction, beginning with prices conditional on μ. Profit functions for $\alpha \geq 0$ (post-investment) are

$$\pi_1 = (p_1 - c)\left[1 - \mu\left(1 - \alpha\right)\right], \qquad \pi_2 = (p_2 - c)\,\mu\left(1 - \alpha\right) \tag{2}$$

and it is straightforward to show that the reaction functions are

$$p_1(p_2) = \frac{1}{2}\left[\frac{t}{\mu}(2 - \mu) + c + p_2\right], \qquad p_2(p_1) = \frac{1}{2}(t + c + p_1) \tag{3}$$

[2]Thus, our assumption that Firm 1 will cover the whole market in the absence of competition is equivalent to $\overline{s} \geq c + 2t$. To see this, note that an incumbent monopolist would choose market share β to solve $\max_\beta \pi_1 = (\overline{s} - t\beta - c)\beta$. The solution $\beta = (\overline{s} - c)/2t \geq 1$ if the above inequality holds.

[3]For convenience, we restrict attention linear pricing, with no third-degree price discrimination (either along a given sub-market or across the incumbent's monopoly and non-monopoly markets). In this respect, our analysis departs from common practice in the telecommunications sector, although it is representative of many other settings. See Gabszewicz and Thisse (1992), Laffont *et al.* (1998a) and Laffont *et al.* (1998b) for examples relaxing these assumptions on a conventional 'Hotelling line'.

Solving these yields the Bertrand equilibrium prices

$$p_1^B = \frac{t}{3}\left(\frac{4-\mu}{\mu}\right) + c, \qquad p_2^B = \frac{t}{3}\left(\frac{2+\mu}{\mu}\right) + c \tag{4}$$

Notice that, when $\mu = 1$, we have $p_1^B = p_2^B = t + c$, i.e. marginal cost pricing with the firms also able to exploit the transport cost that provide an element of market power. Also $p_2^B < p_1^B \ \forall \mu < 1$; i.e. the entrant undercuts the incumbent.[4] Finally,

$$\frac{\partial p_1^B}{\partial \mu} = -\frac{4t}{3\mu^2} < 0, \qquad \frac{\partial p_2^B}{\partial \mu} = -\frac{2t}{3\mu^2} < 0$$

Thus, a larger entrant generates more price competition and pushes down retail prices.

Profit functions (2) hold for $\alpha \geq 0$. However, the retail prices may be such that this cannot be guaranteed so we must allow for the possibility of local monopoly as well. To consider this, recall (1) and the fact that $\alpha \geq 0$ when

$$\frac{p_2 - p_1}{2t} \geq -\frac{1}{2} \tag{5}$$

This condition may not hold because the entrant undercuts the incumbent in Bertrand equilibrium. Using (4) we have

$$\frac{p_2^B - p_1^B}{2t} = \frac{\mu - 1}{3\mu} \tag{6}$$

so that (5) holds when $\mu \geq 0.4$. From (1) α can be expressed as a function of μ

$$\alpha = \frac{1}{2} + \frac{\mu - 1}{3\mu} \tag{7}$$

and thus $\alpha \in (0,\ \frac{1}{2}]$ for $\mu \in (0.4,\ 1]$. Then, for investment levels $\mu \in (0,\ 0.4]$, the firms have the prospect of behaving as local monopolies, able to charge their monopoly prices (assuming full coverage of their own market segments):

$$p_1^M = \overline{s} - t, \qquad p_2^M = \overline{s} - t \tag{8}$$

In fact, the prospect of a local monopoly equilibrium needs closer examination. If Firm 2 chooses p_2^M, it may be profitable for Firm 1 to deviate from p_1^M in order to make inroads into 2's market share. In this case, Bertrand equilibrium should be the result but with $\mu < 0.4$ we know that a pure strategy Nash equilibrium on the reaction functions cannot exist. Accordingly, we need to establish two things. First, what is the

[4] This undercutting result can also be found in Laffont *et al.* (1998a).

retail equilibrium if a profitable deviation away from p_1^M is available to the incumbent? Second, what condition(s) characterise which of the possible equilibria will prevail?

As we have no Bertrand equilibrium for $\mu < 0.4$, when Firm 1 deviates from p_1^M the entrant needs to find a lower price (than p_2^M) to maintain its local monopoly over μ. This is the 'limit price', p_2^L: the lowest price that makes the consumers who have the choice of buying from either of the two firms and who are located furthest from the entrant, indifferent between buying from the incumbent or the entrant; i.e.

$$\overline{s} - p_1(p_2^L) = \overline{s} - t - p_2^L \quad \Rightarrow \quad p_2^L = p_1(p_2^L) - t$$

Substituting this into (3), tells us that

$$p_2^L = \frac{2 - 3\mu}{\mu} t + c \tag{9}$$

Notice that this is unique. Firm 1's price in this case is

$$p_1(p_2^L) = \frac{2 - 2\mu}{\mu} t + c \tag{10}$$

We now ask whether this is the best price that Firm 2 can charge. Clearly it is not because, given a local monopoly, it would rather charge its full monopoly price p_2^M. However, it can only do this if Firm 1 is prepared to set p_1^M. When will this happen? A convenient way to approach this is to compare p_1^M with $p_1(p_2^M)$. If we have $p_1^M < p_1(p_2^M)$ then Firm 1 will set p_1^M by definition of its monopoly price (given μ). Alternatively, if we have $p_1^M > p_1(p_2^M)$ then by definition of Firm 1's best response function, it must gain by deviating from p_1^M (after all, p_1^M is one possible response to p_2^M). The lemma below compares the relevant prices.

Lemma 1 *$p_1^M < p_1(p_2^M)$ iff*

$$\mu < \frac{2t}{\overline{s} - c} \equiv \overline{\mu} \tag{11}$$

Proof Substitute p_2^M into (3) to give $p_1(p_2^M)$ and compare this with p_1^M. Next, note that if the inequality holds for $\mu = 0.4$ (the level of coverage at which local monopoly becomes a possibility) it also holds for lesser coverage levels. *Q.E.D.*

Thus, when $\mu < \overline{\mu}$, a local monopoly pure strategy equilibrium $\{p_1^M, p_2^M\}$ exists. Note that $\overline{\mu} > 0$ if $\overline{s} > c$, which is ensured by our assumption that $\overline{s} \geq c + 2t$. Further $\overline{\mu} < 0.4$ if $\overline{s} > c + 5t$, which is stronger than our assumption.

We can now state our first result:

Result 1 *For $\mu \in [0,\ 0.4]$, when (6) holds (i.e. $\mu < \overline{\mu}$) , the retail equilibrium (conditional on μ) is $\{p_1^M, p_2^M\}$. For $\mu \in (0.4, 1]$, the retail equilibrium is $\{p_1^B, p_2^B\}$.*

We now consider what happens when $\mu \in (\overline{\mu},\ 0.4)$, should this range exist. One possibility here might be a 'commitment equilibrium' where the entrant commits to charging p_2^L, so the incumbent sets p_1^M. However, in the absence of any credible commitment mechanism for the entrant, it would wish to respond to this by setting p_2^M. Yet Firm 1's best response to this is $p_1(p_2^M)$—given $\overline{\mu} < 0.4$—and Firm 2 then regrets setting p_2^M. Thus, the commitment equilibrium breaks down. We can, however, demonstrate the existence of a mixed strategy Nash equilibrium for this case, as the following result proves.

Result 2 *For $\mu \in (\overline{\mu},\ 0.4)$, should this range exist, there exists a mixed strategy equilibrium in retail prices, consisiting of strategies: Firm 1 plays p_1^M with probability $x^* \in (0,1)$ and $p_1(p_2^M)$ with probability $1-x^*$; Firm 2 plays p_2^M with probability $y^* \in (0,1)$ and p_2^L with probability $1-y^*$.*

Proof See Appendix.

Figures 2 and 3 illustrate these results. The relationships depicted are easily derived from (4) and (8).

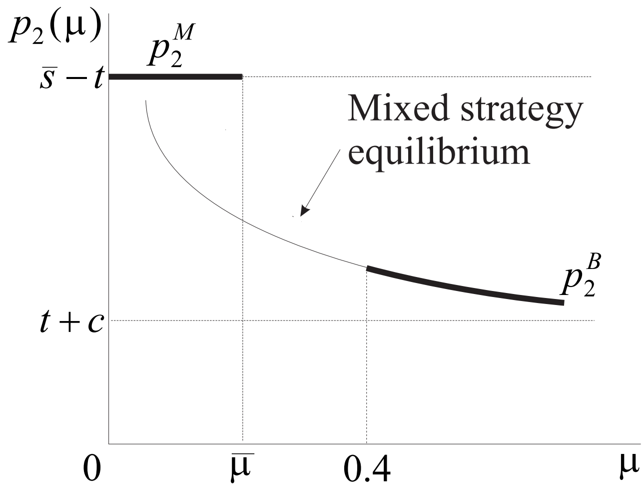

Figure 2: p_2 **when** $\overline{s} > c + 5t$

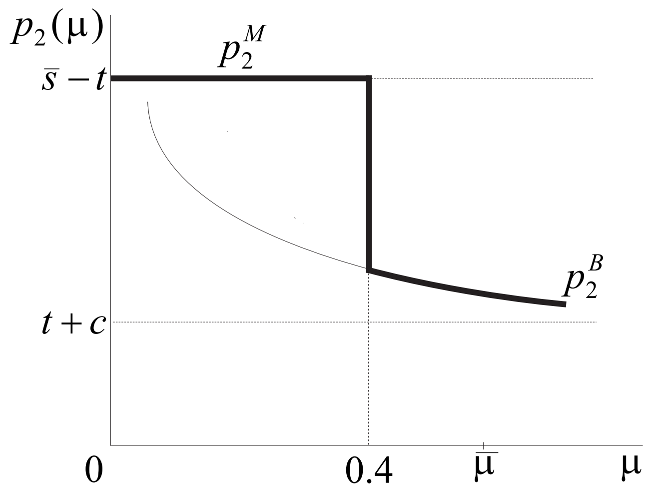

Figure 3: p_2 **when** $c + 2t < \overline{s} < c + 5t$

Hence, in the case where the two firms assume full coverage of their own market segments, they will either choose local monopoly prices or, if a profitable deviation from this is available to Firm 1, price competition will be 'too severe' to produce a pure strategy equilibrium. Lemma 1 tells us that the incumbent will be happy to charge its monopoly price provided the entrant is sufficiently small—there is not much market share to attract from the entrant. 'Smallness' here is determined by t and $\overline{s}-c$. When t is high, so the two firms' products are not very substitutable, there is little to be gained from price competition to attract custom and the incumbent is prepared to set p_1^M. Similarly, when the net surplus from attracting another customer ($\overline{s} - c$) is low, this also deters competitive behaviour by the incumbent.

2.3 Investment

For the remainder of this section, we restrict attention to $c + 2t < \overline{s} < c + 5t$ and thus rule out the complexities of a mixed strategy equilibrium.

We first ask the question of whether the entrant will want to compete with the incumbent firm.[5] To consider this, we begin by calculating the reduced-form profit function in the case where Bertrand competition prevails. This is given by

$$\pi_2^B = (p_2^B - c)\mu[1 - \alpha(p_1^B, p_2^B)] - d(\mu) = \frac{t}{18}\frac{(2+\mu)^2}{\mu} - \frac{\gamma\mu^2}{2} \tag{12}$$

Using this, we can derive our second result:

Result 3 *The entrant never invests beyond $\mu = 0.4$.*

Proof From (12) we have

$$\frac{\partial \pi_2^B}{\partial \mu} = \frac{t}{18}\frac{\mu^2 - 4}{\mu^2} - \gamma\mu = \frac{t}{18}\frac{(\mu-2)(2+\mu)}{\mu^2} - \gamma\mu < 0 \ \forall \mu \in [0,1]$$

Thus, the entrant prefers as little investment as possible in the presence of price competition. In particular, it will not want to invest past the local monopoly level. *Q.E.D.*

We now need to know whether the entrant chooses to invest as far as $\mu = 0.4$. Under monopoly prices the entrant's profit net-of-investment is

$$\pi_2^M = (p_2^M - c)\mu - d(\mu) = (\overline{s} - t - c)\mu - \frac{\gamma\mu^2}{2} \tag{13}$$

Maximizing π_2^M with respect to μ we have the first-order condition for optimal investment μ^*

$$\frac{\partial \pi_2^M}{\partial \mu} = \overline{s} - t - c - \gamma\mu^* \geq 0 \quad \Rightarrow \quad \mu^* \leq \frac{\overline{s} - c - t}{\gamma} \tag{14}$$

where the inequality holds at the boundary $\mu^* = 0.4$. We can now state a result characterizing the entrant's choice of coverage and the type of retail equilibrium that will prevail in the market[6]:

[5] The entrant behaves in analogous fashion to the incumbent in Dixit (1980): investment in the current period takes place in anticipation of its effect on the subsequent market game.

[6] To reiterate, we are assuming $c + 5t > \overline{s} \geq c + 2t > c + t$ throughout.

Result 4 *The entrant will choose*

$$\mu^* = \begin{cases} 0.4 \quad when \quad \frac{\overline{s}-c-t}{\gamma} \geq 0.4 \\ \in (0,\ 0.4] \quad when \quad \frac{\overline{s}-c-t}{\gamma} < 0.4 \end{cases}$$

The only retail equilibrium involves local monopoly prices.

Proof This follows first from (14). We must check that $\pi_2^M(\mu^*) > 0$ when $\mu^* < 0.4$ and that $\pi_2^M(0.4) > 0$ when $\mu^* > 0.4$. Substituting for μ^* from (14) in (13) gives $\pi_2^M(\mu^*) = \frac{\mu^*}{2\gamma} > 0$. Further, placing $\mu = 0.4$ in (13) gives $\pi_2^M(0.4) = \overline{s} - c - t - 0.08\gamma$. This is clearly positive when $\mu^* > 0.4$. Finally, we also need to check that $\pi_2^M(\mu^*) > \pi_2^B(0.4)$. Clearly, this is true since $\pi_2^M(\mu^*) \geq \pi_2^M(0.4) \geq \pi_2^B(0.4)$: the first inequality holds by definition of μ^* and the second holds by definition of monopoly prices. *Q.E.D.*

Result 4 tells us that the entrant may be willing to give up market share (i.e. restrict its local monopoly area) if the costs of investment are too high. Factors that increase μ^* are listed as follows:

Result 5 *Investment rises towards* $\mu^* = 0.4$ *as* $\overline{s}$ *rises and* c, t *and* γ *fall, ceteris paribus.*

Proof Differentiating (14) gives each of these. *Q.E.D.*

Clearly, factors that increase the monopoly price or margin, *ceteris paribus*, make extra investment (and local market share) worthwhile, while an increase in the cost of investment has the opposite effect. For this reason, products with high consumer value ($\overline{s}$) raise μ^*. Similarly, projects that are highly capital-intensive or geographically difficult to build (two interpretations of high γ) reduce investment in coverage, as does significant product differentiation (high t), where monopoly power is less needed.

2.4 Regulator's investment choice

The regulator seeks to maximize the sum of consumers' expected surplus plus industry profit net of investment costs.[7] In fact, the unit demands of the current framework mean that she is not worried about price (there are no deadweight losses), but does care about the expected transport costs faced by consumers—see Tirole (1988).

[7]We assume that the regulator does not directly choose prices, an assumption mirrored by the current plans for 3G mobile telecommunications. Also, note that we solve the regulator's problem without the firms' zero profit constraints that would usually accompany the regulator's problem; we check these are in fact satisfied for our subsequent simulations.

Once the regulator has chosen (and enforced) a level of investment, Result 1 tells us the resulting price equilibrium, given $\overline{\mu} > 0.4$. Thus, we must examine the regulator's welfare function across the local monopoly and Bertrand equilibria. In the first of these cases, we have

$$W^M = \overline{s} - c - \frac{t}{2} - \frac{\gamma(\mu_R^M)^2}{2}$$

Clearly, this means that the regulator will choose $\mu_R^M = 0$ whenever a monopoly equilibrium would arise from encouraging entry. The reasoning is straightforward: since (monopoly) prices are of no interest to the regulator, her chief concern is to economise on investment costs.

Now assume that the regulator chooses $\mu_R > 0.4$ (call this μ_R^B) so that a Bertrand equilibrium would result. Welfare is now

$$W^B = \overline{s} - c - \frac{t}{2} + \mu_R^B \frac{t}{2}\{1 - [\alpha^2 + (1-\alpha)^2]\} - \frac{\gamma(\mu_R^B)^2}{2}$$

Why might the regulator choose such an investment level? Although investment costs are incurred, there is now the prospect that investment will push down prices and lower α, thereby reducing expected transport costs. This channel was not available under monopoly, where $\alpha = 0$ by definition.

In order to examine the regulator's decision more closely, consider

$$\Delta W(\mu_R^B) \equiv W^B - W^M = \mu_R^B t\alpha(1-\alpha) - \frac{\gamma(\mu_R^B)^2}{2} \qquad (15)$$

(where we have used $\mu_R^M = 0$ and $1-\alpha^2-(1-\alpha)^2 = 2\alpha(1-\alpha)$). Notice that maximising ΔW is equivalent to maximising W^B; we work with the former. Straightaway we see that since $\mu_R^B = 0.4 \Rightarrow \alpha = 0$, we have $\Delta W(0.4) < 0$: the regulator will never set $\mu_R^B = 0.4$. Further, it is easy to show that any positive choice of μ_R^B will be unique since $\Delta W(\mu)$ is concave.[8]

Combining our observations so far with Result 4, we have

[8] We have the first order condition

$$\frac{\partial \Delta W}{\partial \mu} = t\alpha(1-\alpha) + \frac{t}{3\mu_R^B}(1-2\alpha) - \gamma\mu_R^B = 0 \qquad (16)$$

using $\frac{\partial\alpha}{\partial\mu} = \frac{1}{3(\mu_R^B)^2}$ (from (7)). Notice that both of the first terms are positive here, the second as a result of $\alpha \leq \frac{1}{2}$. The second order condition is

$$\frac{\partial^2 \Delta W}{\partial \mu^2} = t\alpha'(1-\alpha) - t\alpha\alpha' + \left[\frac{3\mu_R^B t(-2\alpha') - 3t(1-2\alpha)}{9(\mu_R^B)^2}\right] - \gamma = -\frac{2t\alpha'}{3\mu_R^B} - \gamma < 0$$

where we have again used $\frac{\partial\alpha}{\partial\mu} = \frac{1}{3(\mu_R^B)^2}$ to get to the last line. Thus, if (16) has an interior maximum, it is unique. A corner solution (at $\mu_R^B = 1$) will obviously be unique.

Result 6 *Whenever the regulator chooses positive investment, this will involve* $\mu_R^B >$ 0.4 *and Firm 2's independent choice of coverage will be too low (since it is never above 0.4). Whenever the regulator chooses zero coverage, Firm 2's choice will be too high.*

From (15), note that $\Delta W(1) = \frac{1}{2}[\frac{t}{2} - \gamma]$ (since $\alpha = \frac{1}{2}$ when $\mu = 1$) . Hence, a *sufficient* condition to ensure that the regulator chooses $\mu > 0.4$ (so that the entrant's choice implies under-investment) is $\frac{t}{2} > \gamma$. Thus, the prospects of (some) coverage being socially optimal increase with transport costs (t) and decrease with investment costs (γ)—both of these are intuitive. Further, we can say that full coverage ($\mu = 1$, i.e. 'universal service') is socially optimal if $\Delta W'(\mu) > 0, \forall \mu \in (0.4, 1]$.

Next, suppose that μ_R^B is an interior solution to (16). Then we can use the implicit function theorem on (16) to perform comparative statics. In particular, because $\overline{s}$ and c do not feature in (16), we have

$$\frac{\partial \mu_R^B}{\partial \overline{s}} = \frac{\partial \mu_R^B}{\partial c} = 0$$

Further,

$$\frac{\partial \mu_R^B}{\partial t} = \frac{\alpha(1-\alpha) + \frac{1-2\alpha}{3\mu}}{\frac{2t\alpha'}{3\mu} + \gamma} > 0 \tag{17}$$

$$\frac{\partial \mu_R^B}{\partial \gamma} = \frac{-\mu}{\frac{2t\alpha'}{3\mu} + \gamma} < 0 \tag{18}$$

the first inequality being due to $\alpha \leq \frac{1}{2}$ (recall (7)). Thus, again, the regulator prefers greater coverage as 'transport costs' grow and as investment costs fall.

How does the entrant's investment choice compare with that of the regulator? Figures 4 and 5 indicate this by plotting both parties' optimal coverage as t and γ change respectively.[9] Figure 4 shows that, for our parameter values, Firm 2 enters the market but at too low a level for the regulator, who prefers a Bertrand equilibrium: there is under-investment. As t rises, the effect on expected transport costs leads the regulator to prefer increasingly extensive roll-out (see (17)). Accordingly, Firm 2's roll-out is increasingly sub-optimal.

Now consider Figure 5. For very low values of the investment cost parameter (γ) the regulator chooses over 50% market coverage for Firm 2 (and Bertrand equilibrium), but the firm prefers its maximum 40% coverage. As investment costs rise, however, the regulator prefers no entry (see (18)), and Firm 2's entry decision becomes one with over-investment. As γ increases past 8, the entrant gradually lowers its coverage level

[9]Our baseline parameters are $\overline{s} = 5,\ c = 0.5,\ t = 1,\ \gamma = 1$; hence, $\overline{s} > c + 2t$ and $\overline{\mu} > 0.4$. For convenience, the figures denote the regulator's choice of investment by μ_R regardless of whether a Bertrand or local monopoly equilibrium occurs.

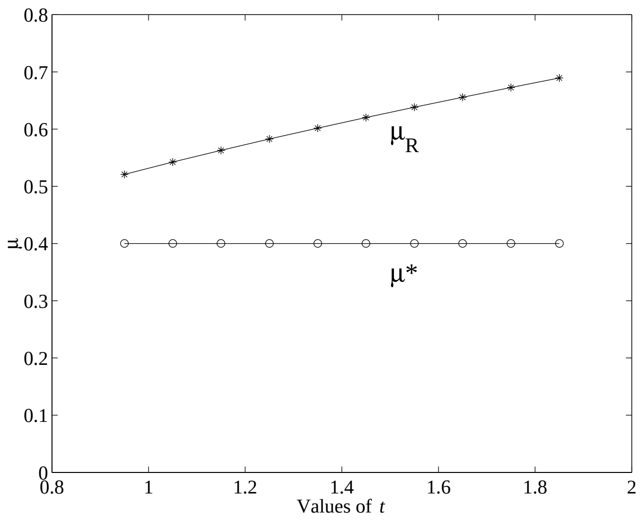

Figure 4: Changes in transport costs (t) with unit demands

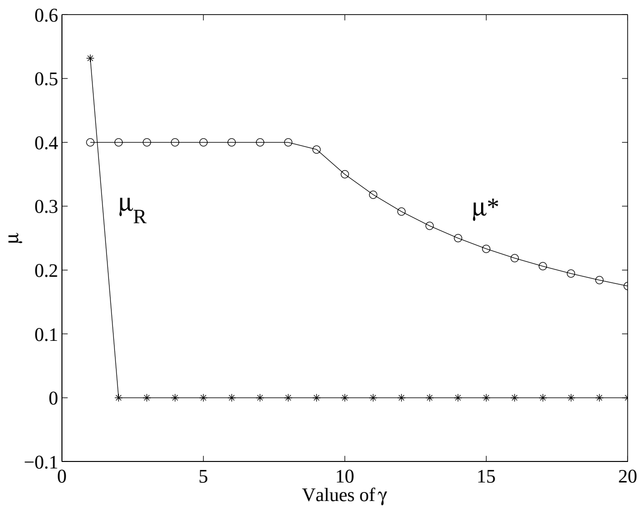

Figure 5: Changes in investment cost (γ) with unit demands

($\frac{\bar{s}-c-t}{\gamma} = 0.38 < 0.4$ when $\gamma = 9$). As the investment cost parameter continues to rise, μ^* falls and the level of over-investment decreases (in the limit, $\mu^* \to \mu_R^B = 0$ as $\gamma \to \infty$).

Figures 4 and 5 confirm Result 6 and make an important point that is worth summarising as follows:

Result 7 *It is possible for the entrant to engage in excessive roll-out (over-investment) or insufficient roll-out (under-investment) from a social perspective. Either case implies a role for regulation in entrants' roll-out decisions.*

3 Elastic demands

The assumption of unit demands is a convenient way to model inelastic demands when monopoly equilibria may be of interest, as we have said. However, it will not always be realistic. Further, there are reasons to believe that increasingly elastic demands may influence our results. For example, unit demands may be responsible for the entrant's incentive to avoid Bertrand competition: unit demands limit the gains available from heavy investment which pushes down prices. Similarly, the regulator's ambivalence towards the direct price effects of investment (as opposed to the indirect effects on transport costs through α) hinges on the unit demands assumption. Thus, it seems important to investigate the effects of more elastic demands.[10]

[10] Laffont *et al.* (1998a) and Armstrong (1998) both suggest that an existence problem may arise in the pricing equilibrium game when elastic demands are present. The reason involves the interplay

We retain our earlier framework but, instead of $\overline{s} - p_i$, we assume a representative consumer's net surplus when consuming from Firm i at price p_i is

$$v(p_i) = \frac{p_i^{-(\eta_i - 1)}}{\eta_i - 1}$$

so that

$$\frac{\partial v(p_i)}{\partial p_i} = -q_i, \qquad q_i = p_i^{-\eta_i}, \qquad i = 1, 2$$

In order to allow for the possibility of monopoly prices, it is necessary to assume that $\eta_i > 1,\ i = 1, 2$. Using 'hats' to denote values for the elastic case, we have an immediate analogy with (1):

$$\hat{\alpha} = \frac{1}{2} + \frac{v(\hat{p}_1) - v(\hat{p}_2)}{2t} \tag{19}$$

so that market shares are $\hat{\alpha}_1 = 1 - \hat{\mu}(1 - \hat{\alpha})$ and $\hat{\alpha}_2 = \hat{\mu}(1 - \hat{\alpha})$. The new profit functions are

$$\hat{\pi}_1 = (\hat{p}_1 - c)[1 - \hat{\mu}(1 - \hat{\alpha})]\hat{q}_1, \qquad \hat{\pi}_2 = (\hat{p}_2 - c)\hat{\mu}(1 - \hat{\alpha})\hat{q}_2$$

while the Bertrand reaction functions are given implicitly by

$$\frac{\hat{p}_1^B - c}{\hat{p}_1^B} = \frac{\hat{\alpha}_1}{\frac{\hat{\mu}}{2t}\hat{p}_1^B\hat{q}_1^B + \hat{\alpha}_1\eta_1}, \qquad \frac{\hat{p}_2^B - c}{\hat{p}_2^B} = \frac{\hat{\alpha}_2}{\frac{\hat{\mu}}{2t}\hat{p}_2^B\hat{q}_2^B + \hat{\alpha}_2\eta_2} \tag{20}$$

It is readily apparent that, in general, these are highly non-linear with no straightforward closed-form solution. For our purposes, this presents no great problem because we seek to illustrate significant changes brought about by moving to more elastic demands. We do this by numerical examples.

Again it may not always be the case that $\hat{\alpha} \geq 0$ or, in terms of prices

$$\frac{(\hat{p}_1^B)^{-(\eta_1 - 1)}}{\eta_1 - 1} - \frac{(\hat{p}_2^B)^{-(\eta_2 - 1)}}{\eta_2 - 1} \geq -t$$

In fact monopoly situations will arise when the above is reversed. Clearly, we can no longer say that this inequality will be satisfied by $\hat{\mu} \geq 0.4$. In such cases, we should in principle allow for local monopoly and mixed strategy Nash equilibria. The former involves

$$\frac{\hat{p}_1^M - c}{\hat{p}_1^M} = \frac{1}{\eta_1}, \qquad \frac{\hat{p}_2^M - c}{\hat{p}_2^M} = \frac{1}{\eta_2}$$

We again restrict attention to parameter values for which $\hat{p}_1^M < \hat{p}_1(\hat{p}_1^M)$, where $\hat{p}_1^M$

between retail prices and interconnection charges (both papers model two-way access in telecommunications, something not in our model).

is implicitly defined using (20), reducing the equilibria to either local monopoly or Bertrand. Investment decisions can be characterised as before. Analysis of the various profit functions makes clear that we can no longer rule out a Bertrand equilibrium since the entrant's Bertrand profit function need not be monotonically decreasing in $\hat{\mu}$. This may confirm our earlier intuition: it may be possible to find examples where the entrant chooses to compete under more elastic demands.

Now consider the regulator's welfare function. For monopoly and Bertrand equilibria respectively, we have

$$\hat{W}^M = (1-\hat{\mu}_R^M)v(\hat{p}_1^M) + \hat{\mu}_R^M v(\hat{p}_2^B) + (1-\hat{\mu}_R^M)(\hat{p}_1^M - c)\hat{q}_1^M + \hat{\mu}_R^M(\hat{p}_2^M - c)\hat{q}_2^M - \frac{t}{2} - \frac{\gamma(\hat{\mu}_R^M)^2}{2}$$

$$\begin{aligned}\hat{W}^B = {} & \hat{\alpha}_1(\hat{p}_1^B,\hat{p}_2^B)v(\hat{p}_1^B) + \hat{\alpha}_1(\hat{p}_1^B,\hat{p}_2^B)(\hat{p}_1^B - c)\hat{q}_1^B \\ & + \hat{\alpha}_2(\hat{p}_1^B,\hat{p}_2^B)v(\hat{p}_2^B) + \hat{\alpha}_2(\hat{p}_1^B,\hat{p}_2^B)(\hat{p}_2^B - c)\hat{q}_2^B \\ & - \frac{t}{2}\{(1-\hat{\mu}_R^B) + \hat{\mu}_R^B[(\hat{\alpha}(\hat{p}_1^B,\hat{p}_2^B))^2 + (1-\hat{\alpha}(\hat{p}_1^B,\hat{p}_2^B))^2]\} - \frac{\gamma(\hat{\mu}_R^B)^2}{2}\end{aligned}$$

Figures 6–9 illustrate several features of the equilibria that may now emerge, maintaining the same baseline parameters as before, with the addition that $\eta_1 = \eta_2 = 1.5$. The first thing to note about all the figures is that, now, $\hat{\alpha} > 0$ in all cases: as suggested earlier, more elastic demands make price competition more attractive to the entrant because of the extra responsiveness to demand that they imply.[11] This observation is illustrated (for changes in t) in Figure 6. An interesting feature of Figure 6 is that higher values of t cause the entrant and the regulator to lower $\hat{\alpha}$—i.e. to raise the entrant's market share in those areas where it rolls out its service. This is achieved by lower retail prices (unlike in (4), where less substitutable products raise Bertrand prices). The reason is that elastic demands can cause higher transport costs to push down firms' sales even if they do not lose custom (with unit demands, no custom is lost when t changes—see (7)—and each consumer's purchases remain the same, by definition). Thus, the firms now have an incentive to lower price even though they appear to have less need to compete with each other.

Figure 7 demonstrates the effects of changes in t on the entrant's roll-out decision and that of the regulator: both involve lower roll-out as t rises. Both are influenced here by the above argument that higher transport costs can (and in the current case do) push down retail prices and increase the entrant's market share. Thus, both have

[11] As a result of this, our figures now relate to a different market structure than Figures 4 and 5. Hence, the two sets of figures are not directly comparable.

an incentive to economise on entry costs by lowering $\hat{\mu}$. The regulator is also worried about the direct effect of transport costs, however, and this accounts for her lower curve in Figure 7. In order to reduce expected transport costs, she needs to lower $\hat{\alpha}$. Under certain conditions, this can be achieved by lowering $\hat{\mu}_R$. From (19) we have

$$\frac{\partial \hat{\alpha}}{\partial \hat{\mu}_R} = \frac{1}{2t}\left(\hat{q}_2 \frac{\partial \hat{p}_2}{\partial \hat{\mu}_R} - \hat{q}_1 \frac{\partial \hat{p}_1}{\partial \hat{\mu}_R}\right)$$

If we now assume that $\partial \hat{p}_1/\partial \hat{\mu}_R \approx \partial \hat{p}_2/\partial \hat{\mu}_R \equiv z$, $z > 0$ and $\hat{p}_1 > \hat{p}_2$, then we have

$$\frac{\partial \hat{\alpha}}{\partial \hat{\mu}_R} \approx \frac{z}{2t}(\hat{q}_2 - \hat{q}_1) > 0$$

Although we cannot confirm that the above assumptions hold generally, numerical solutions confirm that they hold for the current range of parameter values. Thus, the regulator economises on transport costs by reducing entrant roll-out and increasing its share in the contested market segment. The entrant does not internalise this externality.[12]

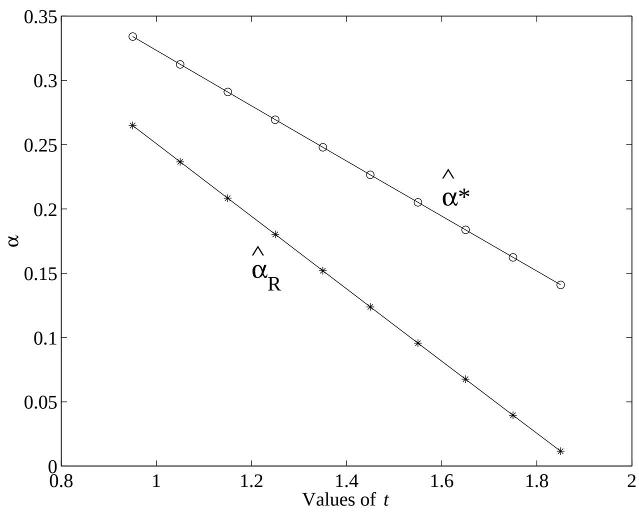

Figure 6: Contested market shares with elastic demands

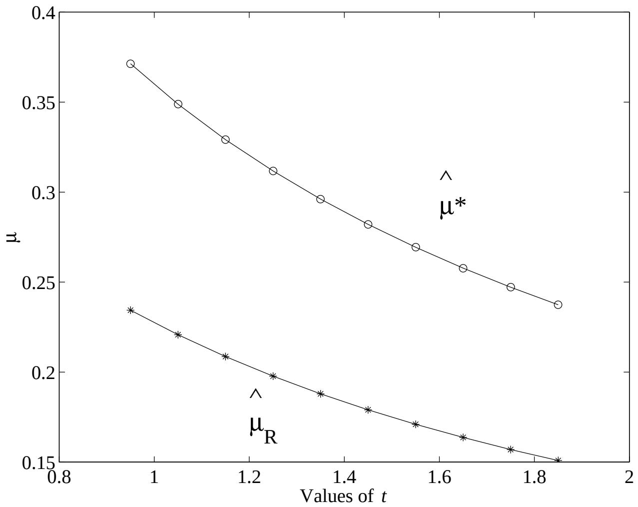

Figure 7: Changes in transport costs (t) with elastic demands

Figure 8 shows that desired coverage falls for both regulator and firm as investment costs (γ) rise, with both over-investment (for low γ) then under-investment (for high γ) taking place. Figure 9 considers the effects of one of the elasticity parameters introduced in this section, η_2. It shows that increases in the entrant's elasticity of demand reduce coverage for both itself and the regulator; with excessive roll-out oc-

[12]These arguments seem unlikely to hold in general. For instance, at some point, one would expect the regulator's concern for transport costs in the $1 - \hat{\mu}$ segment to dominate as $\hat{\mu}$ falls.

curring over the range we select. The negative slopes reflect that fact that more elastic demands sharpen price competition and, therefore, permit a reduction in investment costs while still allowing reasonable market share (for the entrant) and low prices (for the regulator).

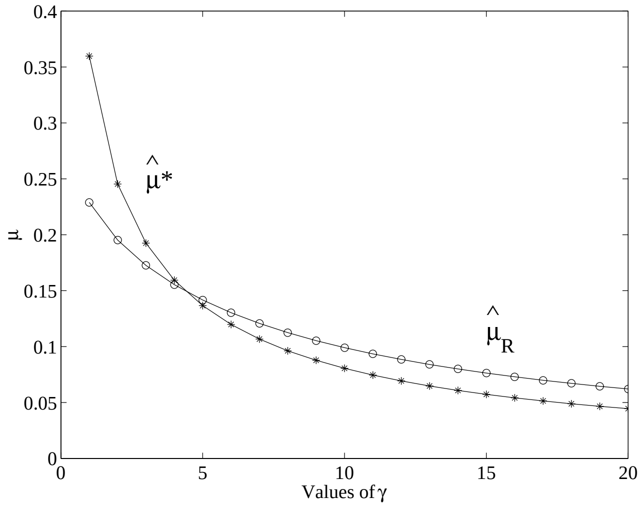

Figure 8: Changes in investment cost (γ) with elastic demands

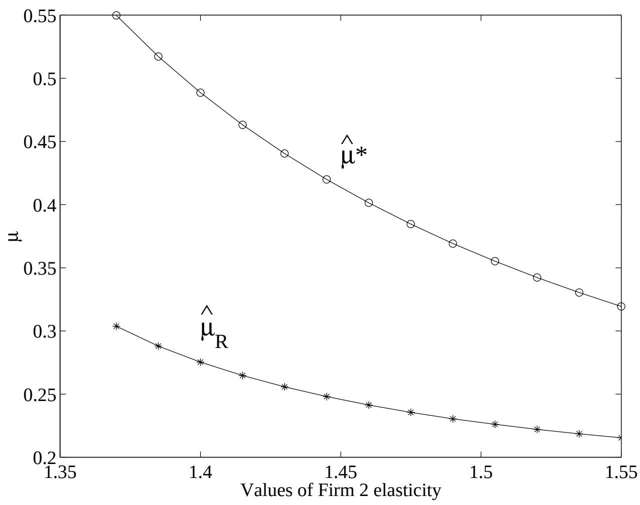

Figure 9: Changes in the entrant's elasticity of demand (η_2)

We finish this section by stating its main finding:

Result 8 *When demands are elastic, the entrant may be prepared to invest in sufficient roll-out to generate Bertrand price competition. Relative to the social optimum, both under-investment and over-investment can occur.*

4 Conclusions and further work

We have amended the standard Hotelling framework to allow an interpretation of geographical coverage and to endogenise a firm's costs of entry into this market, dependent on it chosen level of coverage. We find entry can result in several possible pricing equilibria. Which in fact arises is sensitive to the structure of consumer demands. With unit demands (or, perhaps more generally, relatively inelastic demands) entry will only take place when the entrant and incumbent tacitly collude to produce a local monopoly, with both firms charging normal monopoly prices, or (possibly) with a mixed strategy equilibrium involving local monopoly and limit pricing. Relatively elastic demands, however, make Bertrand competition more likely, because they encourage the entrant to enter and price away the incumbent's business. In both cases, the entrant's scale of entry may be too large or too small from a social perspective.

Two general, opposing, effects govern this result. Investment involves duplication of facilities which suggests that over-investment may occur. However, it also increases competition and can lower prices (and expected transport costs), which is beneficial for consumers/regulators but not for firms; this may induce under-investment. The total impact of these effects on actual private investment depends, as we have shown, on the nature of demands and the costs of investment.

Our results suggest that it may be inappropriate for regulators to leave the market to determine investment levels by a new entrant (as is the case in, say, the 3G mobile phone context). If contractual clauses regarding roll-out are to be enforced, the regulator needs credible sanctions to encourage the entrant to abide by these. Further, to the extent that aspects of roll-out are non-verifiable/non-contractible, our results suggest that unregulated roll-out may not achieve first-best levels of coverage. Depending on the properties of consumer demands, our results also suggest that downstream pricing may be less competitive than the entry of a second firm might suggest. Thus, the idea (implicit in the plans for regulating 3G mobile operators) that downstream competition may push retail prices down needs careful scrutiny.

It is interesting that none of our numerical examples lead to universal service coverage by the entrant being socially optimal (or event close to being so). This has been true for a large variety of parameter settings and suggests that the model may need modification to make gains from such provision worthwhile. Thus, for example, within a model of homogeneous consumer demands, the introduction of network externalities may help encourage this.

There are a variety of ways in which the paper can be developed. Within the current, we might consider endogenous location, price discrimination (say, between the incumbent's monopolised customers $(1-\mu)$ and those for which it competes $(\mu\alpha)$ and allowing two-part tariffs (see Laffont *et al.* (1998a)) and more general non-linear tariff schemes (see Wilson (1993)), all of which can be shown to influence the results from Hotelling's original analysis. Other interesting developments could be introduced to model particular markets (and issues) more closely. Thus, thinking about telecommunications networks, the introduction of access charges (to be bargained over before, say, investment in coverage) would allow us to see how the incumbent can use its negotiating strategy to influence a potential opponent's scale of entry. On the same theme, one could examine a dynamic version of this set-up, where the timing as well as the scale of investment became the focus of attention. This would allow us to compare privately optimal roll-out speeds with socially optimal ones.

Each of these would shed light on current policy toward the 3G mobile market in the UK. However, as noted in the Introduction, there are also close analogies with these issues in the postal sector as it is gradually deregulated. Because it allows for an endogenous scale of entry, and a geographical interpretation of coverage, these are be natural application for our model. Indeed, as governments/regulators continue to seek ways to encourage competition and entry into formerly monopolised industries, it is likely that others will emerge. Further, being extensions towards issues in network economics, they confirm that our model may be fruitful in addressing Armstrong (2001)'s call for work on investment in such industries; a key issue for future research.

Appendix

Proof of Result 2 Consider the following mixed strategy: Firm 1 plays p_1^M with probability x and $\tilde{p}_1 \equiv p_1(p_2^M)$ with probability $1-x$; Firm 2 plays p_2^M with probability y and p_2^L with probability $1-y$. Some algebra yields

$$
\begin{aligned}
\{\pi_1(p_1^M, p_2^M), \pi_2(p_1^M, p_2^M)\} &= \{(1-\mu)(\overline{s}-t-c), \mu(\overline{s}-t-c)\} \\
\{\pi_1(p_1^M, p_2^L), \pi_2(p_1^M, p_2^L)\} &= \{(1-\mu)(\overline{s}-t-c), (2-3\mu)t\} \\
\{\pi_1(\tilde{p}_1, p_2^L), \pi_2(\tilde{p}_1, p_2)\} &= \{(1-\mu)\left[t\frac{1-\mu}{\mu} + \frac{\overline{s}-c}{2}\right], (2-3\mu)t\} \\
\{\pi_1(\tilde{p}_1, p_2^M), \pi_2(\tilde{p}_1, p_2^M)\} &= \{\frac{\mu}{2t}\left[(1-\mu)\frac{t}{\mu} + \frac{\overline{s}-c}{2}\right]^2, \left[\frac{1+\mu}{2} - \frac{\mu(\overline{s}-c)}{4t}\right](\overline{s}-t-c)\}
\end{aligned}
$$

It is straightforward to show that

$$\pi_1(\tilde{p}_1, p_2^M) > \pi_1(p_1^M, p_2^M) = \pi_1(p_1^M, p_2^L) > \pi_1(\tilde{p}_1, p_2^L) \tag{A.1}$$

$$\pi_2(p_1^M, p_2^M) > \pi_2(p_1^M, p_2^L) = \pi_2(\tilde{p}_1, p_2^L) > \pi_2(\tilde{p}_1, p_2^M) \tag{A.2}$$

In a mixed strategy Nash equilibrium Firms 1 and 2 maximize the following respectively:

$$
\begin{aligned}
E\pi_1 &= x\{y\pi_1(p_1^M, p_2^M) + (1-y)\pi_1(p_1^M, p_2^L)\} \\
&\quad +(1-x)\{y\pi_1(\tilde{p}_1, p_2^M) + (1-y)\pi_1(\tilde{p}_1, p_2^L)\} \\
&= x\pi_1(p_1^M, p_2^M) + (1-x)\{y\pi_1(\tilde{p}_1, p_2^M) + (1-y)\pi_1(\tilde{p}_1, p_2^L)\} \\
E\pi_2 &= y\{x\pi_2(p_1^M, p_2^M) + (1-x)\pi_2(\tilde{p}_1, p_2^M)\} \\
&\quad +(1-y)\{x\pi_2(p_1^M, p_2^L) + (1-x)\pi_2 t(\tilde{p}_1, p_2^L)\} \\
&= y\{x\pi_2(p_1^M, p_2^M) + (1-x)\pi_2(\tilde{p}_1, p_2^M)\} + (1-y)\pi_2(p_1^M, p_2^L)
\end{aligned}
$$

Hence Firm 1 chooses $x = 0$ or $x = 1$ according to $y\pi_1(\tilde{p}_1, p_2^M) + (1-y)\pi_1(\tilde{p}_1, p_2^L) \gtreqless$

$\pi_1(p_1^M, p_2^M)$, i.e. $x = 0$ or $x = 1$ according to

$$y \gtreqless \frac{\pi_1(p_1^M, p_2^L) - \pi_1(\tilde{p}_1, p_2^L)}{\pi_1(\tilde{p}_1, p_2^M) - \pi_1(\tilde{p}_1, p_2^M)} = y^*$$

Similarly for Firm 2, $y = 0$ or $y = 1$ according to

$$x \gtreqless \frac{\pi_2(\tilde{p}_1, p_2^L) - \pi_2(\tilde{p}_1, p_2^M)}{\pi_2(p_1^M, p_2^M) - \pi_2(\tilde{p}_1, p_2^L)} = x^*$$

From (A.1) and (A.2) x^* and $y^* \in (0, 1)$: the Nash equilibrium is at (x^*, y^*). *QED*

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