Exchange Rate Volatility and Consumption Home-Bias*

Dudley Cooke†
Department of Economics
University of Warwick

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Abstract

This paper develops a stylised small open economy model with a closed form solution to study the behaviour of the exchange rate. Exchange rate volatility is a feature of the model when there is home-bias in consumption. In particular, when money demand is not responsive to changes in consumption the exchange rate overshoots in response to an increase in the money supply. Our results suggest that consumption home-bias is an important feature which should be incorporated into the modern approach to international finance.
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1 Introduction

This paper uses a general equilibrium framework to study the exchange rate in a small open economy. The idea that in a small open economy there is necessarily some degree of home-bias in consumption provides the motivation. Surprisingly, this is not a feature stressed in the international finance literature which, in general makes use of a consumption 'no home-bias' condition to enhance tractability.[1] An important result here is that home-bias does not involve any loss in tractability and leads to the same conclusions as Dornbusch (1976). This leads to a more general point that home-bias should be included in any small open

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†Address: Department of Economics, University of Warwick, Coventry CV4 7AL, United Kingdom. E-mail: d.k.cooke@warwick.ac.uk

[1] Clarida et al. (2001) allow for home-bias, but they do not focus the exchange rate. The exposition we present is therefore closest to Gali and Monacelli (2002) and Parrado and Valesco (2002).

economy analysis. As an illustration we consider the Mundell-Fleming model and contrast this with the new generation of sticky price models.[2]

The basic assumptions of the Mundell-Fleming model are: (i) the domestic economy produces one good; (ii) the foreign economy produces one good; (iii) the price of the foreign good in the domestic economy is taken as given and (iv) the rate of interest in the foreign economy is also taken as given by the domestic economy. Assumptions (i)-(iv) indicate that the domestic economy is small relative to the foreign economy. The final implicit assumption is (v) that domestic households prefer domestically produced goods; hence there is consumption home-bias. To see why consider what the small open economy assumption represents. If the assumption is taken literally then we can think of it as meaning that the share of the domestic good in the foreign economy's consumption basket is negligible. If the consumption basket is identical in both countries then the small open economy only consumes the foreign good and exports all of it's output. Therefore allowing for non-identical preferences is a necessary part of modelling a small open economy. As a real world example consider the US economy; vis-à-vis the majority of economies imported goods from individual countries form a negligible component of the US consumption basket. The converse is not true in the same countries as US imports do not form a majority of their consumption baskets. Hence the consumption home-bias assumption makes sense.[3]

The new generation of sticky price models are at odds with the Mundell-Fleming model for two reasons. First, they tend to focus on interdependent economies; that is, economies where the terms of trade and interest rates can be affected by either the domestic or foreign economy, similar to Mundell's (1968) two country model. This problem is easily remedied, and modelling a small open economy actually presents less of a technical challenge because it is possible to make use of a set of exogeneity assumptions. Therefore, assumptions (i)-(iv) do not present a substantial problem. The second important feature of these models is that consumers in the domestic and the foreign economy have identical preferences, i.e. there is no home-bias in consumption. If assumptions (i)-(iv) are taken as given then the omission of (v) has the effect described above. As yet, the modern approach to international finance does not allow for home-bias, but we have argued it is implicit in the Mundell-Fleming model and necessary in a small open economy model with microfoundations. If we want to realistically model a small open economy and specify preferences we have to take this into

[2]The structure of the new macro models is equally comparable with the Fleming (1962) and Mundell (1963, 1968) models despite the lack of rational expectations and dynamics which Dornbusch (1976) includes.

[3]Pushing this a little further we can also use a traded vs. non-traded goods argument, with the US having a large non-traded sector.

account.[4] This is especially critical given our example of the US. To further understand the significance of this we examine a well known implication of these assumptions.

Using a no home-bias condition the interest elasticity of money demand does not alter the volatility of the exchange rate as it would in Dornbusch (1976). The key point is that the short run nominal exchange rate does not overshoot its long run level in response to an unanticipated permanent money shock. Regardless of the interest elasticity of money demand the short-run change in the nominal exchange rate matches precisely the change in the nominal money supply. This finding arises because of the restriction of identical preferences across countries. As the law of one price (LOOP) is assumed to hold it is straightforward to demonstrate that consumption based purchasing power parity (PPP) also holds. Thus each country faces an identical real interest rate and consumption growth rate. If we assume an uncovered interest parity (UIP) condition holds then a direct implication is that exchange rate overshooting cannot occur after a money shock, and the change in the nominal exchange rate is just equal to the magnitude of the shock. From this simple example a natural question arises; does the no home-bias assumption rule out overshooting à la Dornbusch?[5]

Although home-bias is more relevant to small open economy analysis there have been attempts to relax the assumption of identical preferences and allow for consumption home-bias in a two country model. Warnock (1998) uses a modified consumption sub-index, that nests the Dixit-Stiglitz (1977) index used in the *redux* model of Obstfeld and Rogoff (1995). Except for this alteration the models are identical. Under Warnock's formulation consumption based PPP does not hold, and this has implications for the exchange rate. Warnock finds that if home-bias exists, and the consumption elasticity of money demand is less than one, then the short run exchange rate overshoots its long run level in response to a permanent unanticipated money shock. Thus it seems home-bias has significant effects. There are some clear problems with this approach. First, Warnock is forced to resort to some cumbersome log-linearisations, and hence the usual caveat applies that the solutions obtained are only approximate. Second, only the special case of mirror images is considered; where the degree

[4] Sutherland (2002) is a good example of a small open economy model where home-bias is not accounted for. Although the aims of this work are significantly different from those presented here.

[5] Other models have been developed that allow for overshooting. Two examples are Lane (2001) who developes a small open economy model with an explicit distinction betwen traded and non-traded goods. The degree of overshooting is shown to depend on the degree of non-traded goods in the domestic consumption basket. Betts and Devereux (2000) consider a two country model with pricing to market, and in this case the degree of pricing to market determines the degree of overshooting. Neither model assumes consumption home-bias, and neither is, strictly speaking, consistent with Dornbusch (1976).

of home-bias and the size of the economies are identical. Without this set of simplifying assumptions the model is not tractable and numerical simulations are required. Hence there is a need to develop a simple small open economy model with a closed form solution that can help determine whether such exchange rate dynamics are a necessary consequence of consumption home-bias.

To understand the model we develop consider some of the other implications of the Mundell-Fleming model. The Mundell-Fleming approach assumes that wage rigidities allow monetary policy to affect output in line with the standard IS-LM model, but with obvious differences in the open economy. To begin with, in an open economy the consumer price index (CPI) is a function of both domestic and foreign prices, and thus the CPI will change even when domestic prices do not. Therefore monetary policy has a direct link to prices via changes in the exchange rate (pass-through). Secondly, in a closed economy there is a one-for-one relationship between the real wage (nominal wage over CPI) and aggregate supply. This is not necessarily true in an open economy as the output of domestic firms is determined by the nominal wage deflated by the domestic price. Since the terms of trade change, different aggregate output levels are consistent with the same real wage. Finally, in the closed economy an increased money supply reduces the nominal interest rate; it also raises prices lowering the real wage, leading to an increase in output. In a small open economy, the foreign rate of interest is taken as given, thus increases in output come about through changes in the exchange rate and the mechanism by which output increases is different. In the model we present all three of these effects are clear because we assume a predetermined nominal wage and do not resort to log-linearisations common in much of the literature. We need not assume that the given foreign interest rate is matched by the domestic interest rate.

The important point of our approach therefore lies in the explicit modelling of non-identical preferences. To do this we assume that consumption of the domestic good forms a negligible fraction of the foreign economy's consumption basket. The converse is not true in the domestic economy. Thus, we refer to our model as a model with consumption home-bias as we incorporate assumptions (i)-(v) consistent with the Mundell-Fleming-Dornbusch approach. Due to the exogeneity restrictions we are able to obtain simple closed form solutions and show clearly the consequences of changes in monetary policy under floating exchange rates.

The paper is organised as follows. In Section 2 we describe the preferences and behaviour of firms, the representative agent, the government and the macroeconomic constraints. Section 3 considers the macroeconomic equilibrium and section 4 examines the reaction of the exchange rate and domestic output to exogenous money shocks. Section 5 concludes.

2 The Model

The model shares a number of features in common with the recent international finance literature that assumes optimising agents and nominal rigidities. There is a domestic country and a foreign country; the domestic country is negligible in size relative to the foreign country. Behaviour in the domestic country and foreign country is identical, except where stated. The domestic economy is characterised by a continuum of households $i \in (0, 1)$, which supply labour with union power. Labour is the only factor of production. Domestic households derive utility from a basket of goods consisting of those goods produced domestically and those imported, from holding money balances and from leisure time.[6] The domestic consumption basket need not be matched by the consumption basket in the foreign economy, allowing us to explicitly model home-bias. The timing of decisions is such that the money wage is negotiated in period $t-1$ and the levels of production, consumption, labour supply and nominal money are set in period t.

2.1 Firms

We adopt the Blanchard and Kiyotaki (1987) framework so that in the labour market union power reduces production below the competitive level. The production technology of firms is described by a homogeneous CES function,

$$y_{h,t}(i) = \left[\int_0^1 l_t(i)^{(\sigma-1)/\sigma} di \right]^{\sigma/\alpha(\sigma-1)}, \tag{1}$$

where $l_t(i)$ is the i^{th} individuals labour supply, $\alpha > 1$ implies decreasing returns to scale in production and $\sigma > 1$ measures the elasticity of input substitution, with higher substitution representing lower market power for workers and an increasing level of competition. Firms maximise nominal profits subject to the production constraint. From the firms cost minimisation process the conditional labour demand is, $l_t^d(i) = [w_t(i)/W_t]^{-\sigma} y_{h,t}^{\alpha}$, where $W_t = \left[\int_0^1 w_t(i)^{1-\sigma} di \right]^{1/(1-\sigma)}$ is the wage index and σ now measures the elasticity of demand

[6] Placing money in the utility function is a significant departure from Gali and Monacelli (2002) who specify a Taylor rule.

with respect to the relative wage. Substituting this into the expression for nominal profits, noting that symmetry implies a unitary relative wage, and maximising yields an expression for the supply of goods that depends on the nominal wage, the domestic price level and returns to scale in production,

$$y_{h,t} = [\alpha W_t / P_{h,t}]^{1/(1-\alpha)} . \tag{2}$$

A standard competitive labour demand condition is given by substituting (2) into the production function (1). For simplicity it is possible to set $\alpha = 1$, but with predetermined nominal wages, the price of the domestically produced good will also be predetermined such that the domestic component of the inflation rate will be independent of monetary surprises. This set of relations fully describes the behaviour of firms.

2.2 Households

The representative agent derives utility from the consumption of domestic and foreign produced goods, real money balances and leisure. The households utility function is $U_t = \sum_{t=0}^{\infty} \beta^t u\left(C_t, m_t, l_t\right)$, where $\beta \in (0,1)$ is the discount rate and all indices are suppressed for brevity. Utility depends positively on the first two arguments: C_t, consumption and m_t, real money balances, defined by the ratio of M_t, nominal money holdings in period t, and $P_t > 0$, the overall price level. Utility depends negatively on work effort, l_t, which is related positively to output.

Until this point we have not explicitly discussed the preferences for domestic and foreign goods. We assume the domestic households consumption preferences are Cobb-Douglas and hence the consumption index is given by $C_t \equiv {C_{h,t}}^n {C_{f,t}}^{1-n} / n^n (1-n)^{1-n}$, where $C_{h,t}$ and $C_{f,t}$ are consumption of the domestic and foreign good in the domestic economy, and $n \in (0,1)$ is a measure of the degree of openness. This formulation imposes a unitary intratemporal consumption elasticity between domestic and foreign goods, and is introduced to simplify the current account dynamics, allowing us to focus on the exchange rate. The corresponding consumption based price index (used as the nominal money deflator) is given by $P_t \equiv P_{h,t}^n P_{f,t}^{1-n}$. Cobb-Douglas preferences also exists in the foreign economy, but tastes are not identical, hence we suppose that n need not equal n^* (an asterisk denotes the foreign economy variable). We discuss this below.

The law of one price (LOOP) is assumed to hold for each good, where the price of the foreign good is given by $P_{f,t} = s_t P_{f,t}^*$, with s_t the nominal exchange rate and $P_{f,t}^*$ the exogenous foreign currency price. Because we allow for non-identical preferences PPP, which links CPI's need not hold, and furthermore the real exchange rate need not equal unity.[7] Introducing some further notation we can define $q_t = s_t P_t^*/P_t$ as the real exchange rate, $t_t = P_{f,t}/P_{h,t}$ as the inverse terms of trade, and the price ratio as $f_t = P_{h,t}/P_t$. As defined, an increase in q_t is a real depreciation in the home currency and a reduction in t_t represents an improvement in a country's terms of trade. We further make the assumption that the share of imports in the foreign economy's CPI is negligible (hence the foreign economy is effectively closed), or rather that $n^* \to 0$ such that $P_t^* \to P_{f,t}^*$. We can therefore write the real exchange rate as a function of the terms of trade and price ratio, $q_t = t_t f_t$, or a function of the terms of trade alone, $q_t = (P_{f,t}/P_{h,t})^n$.[8]

Domestic residents allocate wealth among two assets; real money balances and B_t, an internationally traded bond. The allocation of wealth is such that there is a nominal UIP condition which under perfect foresight relies on a simple arbitrage argument, thus $i_t/i_t^* = s_{t+1}/s_t$, where i_t is the gross nominal interest rate. Again an asterisk denotes the corresponding variable in the foreign economy. It is also possible to define a corresponding real uncovered interest parity (rUIP) condition using the real exchange rate and real interest rate, $r_t/r_t^* = q_{t+1}/q_t$[9]. Both UIP and rUIP hold in this model. As a simplification it is assumed that the domestic government issues no interest bearing debt and holds no interest bearing assets. This implies that the representative agent is confined to holding domestic money and interest bearing claims on foreigners.

Given all of the above the individual flow budget constraint for $t = 0.....\infty$ can be written as,

$$B_t + M_t + P_t C_t = i_{t-1} B_{t-1} + M_{t-1} + w_t l_t - \tau_t + \varphi_t, \tag{3}$$

where M_t is the level of nominal money holdings in period t, with $M_{t-1} > 0$ given. B_t are the holdings of one-period nominal bonds which pay interest i_t, where the initial stock of bonds is also given. τ_t is seigniorage revenue which is rebated lump-sum to the representative agent and φ_t are monopoly profits. Households maximise utility subject to the

[7] The differentials in PPP derive directly from preferences.

[8] The complete set of relations includes $t_t = f_t^{1/(1-n)}$ and $q_t = f_t^{n/(n-1)}$. The greater the degree of home-bias for the domestic economy (i.e. the larger n), then a given deterioration in the terms trade implies a greater depreciation in the real exchange rate. Benigno and Thoenissen (2002) refer to this as the home-bias channel.

[9] By substituting the Fisher parity condition, $i_t/r_t = (P_{t+1}/P_t)$, into the UIP condition we find $r_t/r_t^* = (s_{t+1}/s_t)(P_{t+1}^*/P_t^*)(P_t/P_{t+1})$. With $q_t = s_t P_t^*/P_t$ defined as the real exchange the real UIP condition follows.

flow budget constraint (3), and the conditional labour demand constraint. The households' utility function is assumed to have the following semi-CRRA form,[10]

$$U_t = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + a m_t^{1-\epsilon} / \left(1 - \epsilon\right) - d l_t^{\kappa} / \kappa \right), \tag{4}$$

where $a > 0$, $d > 0$ is the marginal disutility of effort, $\kappa > 1$ and $1/\left(1 - \kappa\right)$ is the substitution elasticity of labour and $\epsilon > 0$ determines the magnitude of both the consumption and the interest elasticity of money demand. The intertemporal elasticity of consumption is one under logarithmic preferences. The first order conditions are:

$$P_{t+1} C_{t+1} = P_t C_t \beta i_t \tag{5}$$

$$m_t^{\epsilon} = a C_t i_t / \left(i_t - 1\right) \tag{6}$$

$$W_t = \frac{\sigma d}{\sigma - 1} P_t C_t l_t^{\kappa - 1}. \tag{7}$$

Equation (5) is the consumption Euler equation; the optimal consumption path such that an individual is indifferent between consuming an additional unit at time t or not consuming and using the interest made on the saving to consume goods in period $t+1$. Equation (6) is the money demand function giving the trade-off between not consuming an additional unit of output to acquire nominal money versus immediate consumption. Optimality requires the gain in utility from holding money be equal to the loss in interest from not holding foreign bonds. It is worth restating here that ϵ determines both the interest and consumption elasticity of money demand when utility is specified as (4), a point which is important in the later discussion. The wage equation (7) demonstrates that the optimal wage is a function of the monopolistic distortion σ, measured by the desired mark-up of households over marginal cost, and the marginal rate of substitution between consumption and leisure. This condition gives us a natural way of incorporating nominal rigidity into the model, as the money wage is negotiated one period in advance. These three conditions do not fully characterise the equilibrium as we need also to assume a no-Ponzi game condition. This describes the behaviour of households.

[10] This class of utility function is common in the 'new open economy macro' literature, see for example Obstfeld and Rogoff (1995).

2.3 Government

The government issues money by a lump-sum transfer to households. In aggregate we write the government budget constraint as,

$$\tau_t = M_{t-1} - M_t. \tag{8}$$

This behaviour leads to a money growth rate of $(1 + \mu_t) \equiv M_t/M_{t-1}$, and therefore the transfer is equal to $-\mu_t M_{t-1}$. As the counterpart of an increase (decrease) in the money stock is a lump sum transfer (tax) of equal size to all households there exists no marginal redistribution associated with government transfers. With the representative agent taking nominal prices as given when choosing a desired path of nominal money holdings and the government rebating revenues lump sum to the public inflation still discourages holding nominal balances as the money transfer is unrelated to the optimal money demand decision.

2.4 Macroeconomic Constraints

In equilibrium both the individual and government budget constraints need to be satisfied. The individual and government budget constraints, production function and labour demand condition give an expression for the short run current account [national budget constraint],

$$B_t = P_{h,t} y_{h,t} - P_t C_t + i_{t-1} B_{t-1}. \tag{9}$$

In words, the end of period bond level is equal to domestic output, minus the rate of absorption plus interest from claims on foreign bonds. In an open economy output and consumption are also related to the net export level. The market clearing condition is therefore characterised by a simple accounting identity[resource constraint],

$$y_{h,t} = C_{h,t} + C^*_{h,t}, \tag{10}$$

where $C^*_{h,t}$ is defined as aggregate consumption of the domestic good in the foreign economy. The final thing to consider is the short-run equilibrium demand for goods. Equation (10) is an accounting identity, and therefore to derive an equilibrium condition it is also necessary to take account of the demand for domestic and foreign goods at the micro level. Returning to the price and consumption indices it is possible to re-write consumption as,

$$P_tC_t = P_{h,t}C_{h,t} + \varepsilon_t P^*_{f,t}C_{f,t} = (1/n)P_{h,t}C_{h,t} = 1/\left(1-n\right)\varepsilon_t P^*_{f,t}C_{f,t}. \quad (11)$$

Equation (11) makes clearer the consequences of the assumptions made so far. First, we can express the relative demand of the domestic consumer for the domestic good as $C_{h,t}/C_{f,t} = t_t n/(1-n)$; second the intratemporal elasticity of substitution between the domestic and the foreign good is independent of the degree of monopolistic competition. The corresponding expression in the foreign economy is $C^*_{h,t}/C^*_{f,t} = 0$ as $n^* \to 0$. Taking the resource constraint and rearranging (11) allows us to describe equilibrium demand for goods in the domestic economy. Using, $C^*_{h,t} = g^*_t s_t/P_{h,t}$ and $C_{h,t} = n\Gamma_t/P_{h,t}$, where $\Gamma_t = P_tC_t$ and $g^*_t = n^*\Gamma^*_t$, we obtain an expression for the short-run equilibrium condition in the goods market,[11]

$$y_{h,t} = \left(n\Gamma_t + g^*_t s_t\right)/P_{h,t}. \quad (12)$$

Unlike a closed economy model aggregate output need not equal aggregate consumption as a trade deficit (surplus) is possible which satisfies excess demand (supply) in the domestic economy. This is captured by the second term in the brackets on the right hand side of equation (12), as $g^*_t s_t$ represents exports of the domestic good.[12] There is no direct analogy between equations (9) and (12) as (9) is a budget constraint and (12) is a clearing condition, but (12) allows us to solve for the real side of the economy as it connects the supply of goods and the real exchange rate, whereas (9) allows us to solve for the monetary sector. Equations (2), (5)-(9) and (12) are necessary conditions for an equilibrium as we need to take account of the optimal behaviour of the individual, firms, the government, and the market clearing conditions. We turn to this next.

3 Macroeconomic Equilibrium

We solve for the macroeconomic equilibrium in two stages. First, as a consequence of home-bias and the specification of the utility function we jointly account for the behaviour of the

[11] Notice that although n^* is effectively zero, it is also necessary that Ω^*_t is large. Therefore g^*_t represents demand in the foreign economy for the domestic good in terms of domestic currency. More explicitly we also assume that as $n^* \to 0$, $\Omega^*_t \to \infty$ in such a way that $g^*_t = n^*\Omega^*_t$ remains non-zero and finite.

[12] An equivalent expression for goods market clearing more common in the literature is $y_{h,t}f_t = n^*C^*_t + nC_t$, where f_t is the price ratio defined above. The same caveat applies for $n^*C^*_t$.

real and monetary sectors. Second, by using the national intertemporal budget constraint we describe the behaviour of the current account and are thus able to express the model in a familiar aggregate demand-aggregate supply form.

3.1 The Relationship Between the Real and Monetary Sectors

Before solving for the macroeconomic equilibrium it is important to return to the specification of preferences as this determines the solution method. Crucially we have assumed CRRA preferences for money balances in the utility function, whereas in a more restricted model with a unit interest elasticity of money demand ($\epsilon = 1$) preferences would be logarithmic. An important point is that by restricting the interest elasticity of money demand to equal unity it is possible to exploit a separability property between the real and monetary sectors.[13] Thus, we would only need to consider the monetary sector of the economy when solving the model. Alternatively, with the interest elasticity of money demand not equal to unity the domestic real interest rate influences the nominal interest rate and the separability property does not hold; hence with a non-unit interest elasticity we need to account for both real and monetary sectors simultaneously in the solution method. The separability property is discussed in more detail in section 3.2. The other reason we are forced to explicitly consider the real and monetary sectors of the economy together is because we allow for consumption home-bias. Under consumption home-bias the condition that the domestic and foreign real interest rates are equal does not hold.[14] This creates a further problem when looking at the monetary sector because we cannot simply appeal to the exogenous foreign interest rate, r_t^* to help solve the model. Even with a non-unit interest elasticity if there were no home-bias ($r_t = r_t^*$) the solution method would be greatly simplified as we could effectively ignore the real side of the domestic economy when solving the monetary side because the current nominal interest rate would only depend on the future nominal rate and exogenous parameters. Therefore to solve for the macroeconomic equilibrium we first describe the dynamic properties of the domestic real sector and demonstrate it's interaction with the monetary sector. What we require initially is an explanation of consumption, the real interest rate, and the real exchange rate in the domestic economy, independent of monetary factors. We then connect this to the current period accounting for the nominal rigidity. Although we cannot appeal to the exogenous foreign rate of interest in any solution to the monetary sector the analysis of the real sector is simplified by this exogeneity condition.

[13] This property is also noted in Ascari and Rankin (2002).

[14] See Dornbusch (1983) for a related discussion.

3.1.1 Real Sector

An aggregate supply relation and a goods market relation characterise the real side of the model. We first consider these relationships under flexible wages. This is equivalent to considering periods $t \geq 1$ if we assume a one period nominal wage contract negotiated in period $t = -1$, and lasting until the end of period $t = 0$. We then turn to consider these relationships under fixed wages, i.e. those in period $t = 0$. This solution method will enable us to connect both periods so that we can see the effect of disturbances hitting the economy in the current period. In the following exposition it therefore helps to keep in mind the idea that exogenous disturbances hit the economy in the beginning of the current period. Solutions with both flexible and fixed wages follow a similar methodology.

In the following exposition we only present a limited number of expressions and focus on intuition leaving a more detailed algebraic derivation in Appendix A. The first step is to solve for the real exchange rate as a function of consumption, by equating the solutions for labour supply and labour demand. Under flexible wages we can assume that markets clear and thus the aggregate supply relation is given by combining the optimal wage condition for the individual (7) and the firms labour demand condition (2). The goods market clearing relation is just the short-run equilibrium condition for the home good given by equation (12). This can be expressed not only as a function of prices, but also as a function of the real exchange rate as the price ratio and the real exchange rate are uniquely related. The intersection of these two conditions gives the natural rate of output and real exchange rate, conditional on C_t. We can write this system compactly as,

$$q_t = q(C_t), \tag{13}$$

$$y_{h,t} = y_h(C_t). \tag{14}$$

The first step of the solution method demonstrates the similarities and differences of the microfounded and ad-hoc approaches. In many respects the supply side of the model we present is similar to the Sachs (1980) model of a small open economy, which in turn is based on the Mundell-Fleming assumptions. Solving the ad-hoc model is more straightforward because the aggregate supply and market clearing conditions can be solved simultaneously to determine the natural rate of output and the real exchange rate. Here we find that the real exchange rate is an implicit function of consumption, and therefore so is output. Clearly consumption appears in this system for the same reason it appears in money demand as agents are optimising. Although consumption plays an additional role compared to more traditional models there is still a tractable solution, and we can use (13) and (14) to consider the dynamic path of the real side of the economy in periods $t \geq 1$ onward. The second step is therefore to solve for r_t, q_t and C_t given the relations (13) and (14). From the rUIP condition we have $r_t/r^* = q_{t+1}/q_t$, where the foreign variable is assumed constant, such that $r_t^* = r^* \ \forall t$. Because $q_t = q(C_t)$ we can write the rUIP condition in terms of consumption,

$$r_t/r^* = q\left(C_t\right)/q\left(C_{t+1}\right). \tag{15}$$

Further suppose that the rest of the world is in a steady state, such that from the foreign consumption Euler equation we have $r^* = 1/\beta^*$. To allow for the possibility of a world steady state we also require $\beta = \beta^*$, and hence we can write $r^* = 1/\beta$. Using the domestic real consumption Euler equation and rearranging the resulting expression we obtain a first order difference equation in C_t. The expression for the difference equation in C_t can be written very simply as,

$$C_{t+1}/q\left(C_{t+1}\right) = C_t/q\left(C_t\right) \ \ \forall t \geq 1. \tag{16}$$

Equation (16) is self-contained, and therefore without a disturbance that alters the $q\left(C_t\right)$ function it follows that, $C_t = C_{t+1} \ \forall t \geq 1$. Therefore the consumption profile of the domestic economy is flat. From this we conclude that $q_t = q_{t+1} \ \forall t \geq 1$ since $q_t = q\left(C_t\right)$, that $y_{h,t} = y_{h,t+1} \ \forall t \geq 1$ since $y_{h,t} = y_h\left(C_t\right)$, and finally that $r_t = 1/\beta \ \forall t \geq 1$ since $r_t/r^* = q_{t+1}/q_t$. Thus the real side of the economy jumps immediately to the steady state for all periods $t \geq 1$. Importantly, this result does not impose the condition that the domestic economy is always in the steady state.[15] We now turn to the current period.

In the current period the money wage is fixed at $\overline{W}_0$, and with individuals off the labour supply curve we can only appeal to the labour demand condition (2) as a solution to this sector. This condition is not sufficient alone because it contains the domestic price level, which is endogenous. From the definitions of the real exchange rate and consumer price index we can express the domestic price as a combination of the real and nominal exchange rates which will be determined below, thus temporarily solving the endogeneity problem. The goods market clearing relation still holds as this is not subject to the wage rigidity and thus we have an equivalent system for the current period,[16]

[15] This argument ties down the rate of change of consumption, but not the initial value, which is dependent on the initial level of wealth.

[16] See Appendix A for more details.

$$
\begin{aligned}
q_0 &= \widehat{q}(C_0; s_0, \overline{W}_0), &(17)\\
y_{h,0} &= \widehat{y}_h\left(C_0; s_0, \overline{W}_0\right). &(18)
\end{aligned}
$$

We denote the current period functions with a circumflex. Notice the difference between the new system and the old system. The former is conditioned on consumption alone, the latter is conditioned on consumption, the fixed money wage and nominal exchange rate. The second stage is to solve for r_0, q_0 and C_0, and therefore we can repeat the previous steps. From the rUIP condition we have $r_0 = q_1/\beta q_0$ and from the first system $q_1 = q(C_1)$. Thus we can find an expression for the current real interest rate,

$$
r_0 = q\left(C_1\right)/\beta\widehat{q}\left(C_0; s_0, \overline{W}_0\right). \tag{19}
$$

In deriving (19) we have still only assumed the foreign economy is in a steady-state and that there is the possibility of a world steady state. Combining (19) with the domestic consumption Euler equation (5) and rearranging the resulting expression we are able to derive a second first order difference equation in consumption. Thus,

$$
C_1/q\left(C_1\right) = C_0/q\left(C_0; s_0, \overline{W}_0\right). \tag{20}
$$

Because of the assumption of wage contracts in the current period the real sector of the domestic economy will not be in a steady state, and we can only appeal to (19) to express the current period real interest rate. Because the current real interest rate is not invariant to the money shock we cannot say that in the current period the real sector is in a steady state, and thus we can only be certain we are in a steady state for periods $t \geq 1$. This completes the description of the real sector and we can now consider the monetary sector.

3.1.2 Monetary Sector

In the monetary sector we define the equilibrium for a given specification of monetary policy in terms of the money growth rate, $1 + \mu_t$. To describe the behaviour of the monetary sector we combine the money demand (6) and consumption Euler (5) equations to express the period t nominal interest rate as a function of the future nominal interest rate, the current domestic real interest rate, future money growth and the discount rate. Thus,

$$(i_t - 1)\, i_t^{\epsilon-1} = \left(1 + \mu_{t+1}\right)^{\epsilon} r_t^{\epsilon-1} \left(i_{t+1} - 1\right) / \beta i_{t+1}, \tag{21}$$

where $1 + \mu_{t+1} \equiv M_{t+1}/M_t$ is exogenous. The policy regime for money growth is such that $\mu_t = \mu \ \forall t \geq 1$ but μ_0 may differ from μ so the complete time path for μ_t is described by two parameters (μ_0, μ). What is immediately obvious from equation (21) is that if the interest elasticity of money demand is unity ($\epsilon = 1$), the current real interest rate does not influence the current nominal interest rate. This is a clear demonstration of the real-monetary separability property described above. Furthermore remember that without consumption home-bias $r_t = r_t^*$ and so even if $\epsilon \neq 1$, the analysis of section 3.1 would be unnecessary in order to solve (21) due to the exogeneity of r_t. Here however we have accounted for home-bias and the wage rigidity, as the dynamics of the nominal interest rate with a non-unit interest elasticity will depend on this rigidity. To solve (21) remember that although the real interest rate is endogenous we have expressions for the real side of the domestic economy in all periods. Therefore, since $r_t = 1/\beta \ \forall t \geq 1$, we can express the nominal interest rate for $t \geq 1$ as a function of the future nominal interest rate and exogenous variables alone. As the nominal interest rate is non-predetermined we need the system to be unstable in its forward dynamics, and therefore to satisfy the saddle path property. Thus, provided this holds for all periods after the current period we write the nominal interest rate as it's steady state value,

$$i_t = (1 + \mu) / \beta \ \forall t \geq 1. \tag{22}$$

Solving (21) recursively for the current period we then have simply,

$$(i_0 - 1) i_0^{\epsilon-1} = r_0^{\epsilon-1} (1 + \mu - \beta) / \beta \left(1 + \mu\right)^{\epsilon-1}. \tag{23}$$

Therefore to solve for the monetary sector we also need to use the current period real interest rate given by equation (19), and thus it becomes clear how rigid wages influence the effects of monetary policy in the current period. As with the real sector we cannot say that the monetary sector is in a steady state in the current period, but we are now in a position to consider tying down the current level of all the variables.

3.2 Appealing to the National Budget Constraint

Following the logic of the previous sections we need to consider the periods with and without the nominal wage rigidity. The key point now is that we connect the two periods by iterating the national budget constraint forward to obtain the national intertemporal budget constraint. Therefore the first step in solving the model involves looking at the current account and exchange rate. The second step involves appealing to the implicit systems developed above, and solving them in terms of exogenous variables so that, (13), (14), (17) and (18) can be expressed as a single, simpler system, with all endogenous variables tied down.

3.2.1 Current Account

The first thing to do is to consider the behaviour of the current account. The idea that in an open economy a trade surplus (deficit) is satisfied by an excess supply (demand) of goods is captured by equation (10) where $B_t = \Psi_t + i_{t-1}B_{t-1}$ is the current account expression and $\Psi_t = P_{h,t}y_{h,t} - \Gamma_t$ is the trade surplus. We can then express the national intertemporal budget constraint as,[17]

$$i_{-1}B_{-1} = -\sum_{t=0}^{\infty} (i_0 i_1 .....i_{t-1})^{-1} \Psi_t, \tag{24}$$

where B_{-1} is the initial bond stock and we define $i_0 i_1 .....i_{t-1} \equiv 1$ when $t = 0$. The first term on the right hand side of (24) is all future interest rates; the second term is the trade surplus which now takes the form $\Psi_t = \Gamma_t (n-1) + g_t^* s_t$ due to the assumption of Cobb-Douglas consumption preferences. The dynamics of the monetary sector describe the behaviour of the domestic nominal interest rate with starred variables exogenous. Splitting the intertemporal budget constraint into two elements; one where the nominal rigidity takes effect, and one where it does not, we can find the current period exchange rate. Using the fact that i_t is constant for periods $t \geq 1$ it is straightforward to demonstrate that $\Gamma_{t+1}(1+\mu) = \Gamma_t \ \forall t \geq 1$ and as a consequence $\Psi_{t+1}(1+\mu) = \Psi_t \ \forall t \geq 1$; or in words, that in all periods after the current period the trade surplus grows at a constant rate. From this we can re-write (24) as,

$$0 = \Psi_0 + i_0^{-1} \left[(1+\mu) / (1+\mu-\beta) \right] \Psi_1, \tag{25}$$

where we set $B_{-1} = 0$. Equation (25) follows from (24) as Ψ_1 represents all future levels of the trade surplus and the infinite sum, given the constant future interest rate, is just a

[17] This is also derived using a no-Ponzi game condition.

geometric series, where i_t $\forall t \geq 1$ is a function of exogenous parameters. By substituting out Γ_1 and s_1 we find Ψ_1 is a positive multiple of Ψ_0 and hence we can write,

$$0 = \left[1 - \beta \left(1+\mu\right) / \left(1+\mu-\beta\right)\right] \Psi_0. \tag{26}$$

From this it follows that $\Psi_0 = \Psi_1 = 0$ and therefore more generally $\Psi_t = 0$ $\forall t$. Thus in this model a zero current account condition always holds, even without a no home-bias assumption.[18] This is a key result which allows for an analytical solution. From this we can derive a condition for the nominal exchange rate,

$$s_t = \Gamma_t \left(1-n\right) / g^* \ \forall t. \tag{27}$$

As above, we also know $s_{t+1}(1+\mu) = s_t$ $\forall t \geq 1$ and as such we can relate the nominal exchange rate to the monetary policy variable, M_t by using the period $t \geq 1$ money demand function. Doing this we have simply,

$$s_t = x M_t \ \forall t \geq 1, \tag{28}$$

where x is given by,

$$x = \left(\left(1+\mu-\beta\right) / a(1+\mu)\right)^{1/\epsilon} y_{h,t}^{(\epsilon-1)/\epsilon} \left[\left(1-n\right) / g^*\right]^{(n(\epsilon-1)+1)/\epsilon} \ \forall t \geq 1.$$

The final thing to be determined is the level of the real exchange rate. From our assumptions over preferences we write $P_{f,t}^* = P_t^*$, which we normalise equal to one because $P_{f,t}^*$ is completely exogenous. Thus we have $q_t = s_t P_t^* / P_t = s_t / P_t$ and from the zero trade balance result $q_t = C_t \left(1-n\right) / g^*$. These conditions for the nominal and real exchange rates do not fully tie down the model and thus we need to go back and reconsider the implicit systems for the current period and for all future periods using the results above.

3.2.2 Tying Down Equilibrium

To tie down equilibrium in the current period we begin by considering equilibrium in periods $t \geq 1$. To offer a full solution to this model the key insight is the behaviour of the current

[18] Using the restriction of a unit intratemporal elasticity of substitution between domestic and foreign production to eliminate any current account dynamics was first demonstrated in Corsetti and Pesenti (2001).

account. Remembering that $\Psi_t = 0$ implies $C_t = y_{h,t} q_t^{(n-1)/n}$ and by using this in the implicit system (13)-(14) with flexible wages we arrive at an analogous system,

$$\begin{aligned} q_t &= q(y_{h,t} q_t^{(n-1)/n}), & (29) \\ y_{h,t} &= y_h(y_{h,t} q_t^{(n-1)/n}). & (30) \end{aligned}$$

Thus by the substitution of $\Psi_t = 0 \ \forall t$ into (13) and (14) we have tied down q_t, $y_{h,t}$ and C_t $\forall t \geq 1$. Therefore, although at the first stage we could not solve for domestic output and the real exchange rate simultaneously because each was implicitly dependent on consumption, in all periods after the current period it is possible to solve for domestic output, the real exchange rate and consumption simultaneously because we know the current account does not react to exogenous shocks. Above we suggested these equations describe the natural rate of output and the real exchange rate; therefore for periods $t \geq 1$ we write $q_t = \overline{q}$, $y_{h,t} = \overline{y}_h$ and $C_t = \overline{C}$, where upper-bar now denotes the variables natural rates. From (29)-(30) we can easily show that the natural rate of output depends only on preference parameters and technology,

$$\overline{y}_h = \left[(\sigma - 1) / \sigma \alpha d\right]^{1/\kappa\alpha}. \qquad (31)$$

As the monopoly distortion falls; that is as $\sigma \rightarrow \infty$, employment reaches the competitive level $\overline{l} = (1/d\alpha)^{1/\kappa}$. Equation (31) is independent of monetary policy. From (29)-(30) we can also show $\overline{q} = \overline{C}(1-n)/g^*$, and then using the zero current account condition, $\overline{\Psi} = 0$,we can relate $\overline{y}_h, \overline{q}$ and $\overline{C}$ to one another uniquely.

Tying down consumption for future periods turns out to be the important step in tying down all remaining endogenous variables in the model. We now turn to the current period. By substituting $\Psi_0 = 0$ into (17)-(18) it is again possible to solve for $y_{h,0}, q_0$ and C_0, but this time conditional on s_0. Therefore we can relate output and the nominal exchange rate via real sector relations alone and obtain a closed-form solution,

$$y_{h,0}^{\alpha} = \left[s_0 g^* / (1-n) \alpha \overline{W}_0\right], \qquad (32)$$

which describes the aggregate supply relation. The next step is to think about using this relationship in the monetary sector. Using the monetary relations first involves rearranging the current period difference equation (23), then using the rUIP condition, $r_0 = q_1/q_0\beta$, to substitute out the real interest rate and noting that output and the real exchange rate are uniquely related as above to determine a relation between output and the nominal interest rate. Substituting in the UIP condition, $i_0 = s_1/s_0\beta$, finally yields a relation between output, the nominal exchange rate and the monetary policy variable, M_1, thus,

$$y_{h,0}^{n(1-\epsilon)} = (1/\gamma)\left(\frac{xM_1}{s_0\beta} - 1\right)\left(\frac{xM_1}{s_0\beta}\right)^{\epsilon-1}, \tag{33}$$

where $xM_1 = s_1$, and γ is given by,

$$\gamma = \left((1+\mu-\beta)/\beta^{\epsilon}(1+\mu)^{1-\epsilon}\right)\overline{y}_h^{n(\epsilon-1)},$$

where $\overline{y}_h$ is the natural rate of output so that both x (above) and γ contain only exogenous parameters. Equation (33) describes the aggregate demand relation. This method therefore transforms the current period implicit system for $y_{h,0}$, q_0 and C_0 into a simple pair of implicit equations for $y_{h,0}$ and s_0. It also ties down the current period, and in all future periods variables are at their natural rate levels so that we are now in a position to consider the effects that exogenous money shocks have on output and the exchange rate.

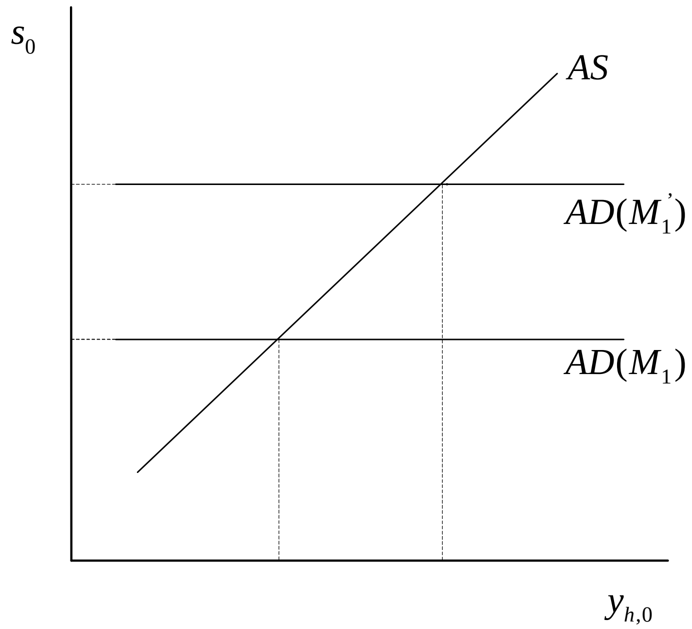
4 Results

In the analysis we limit ourselves to considering permanent unanticipated changes in the money supply. We are interested in explaining two things; the short-run change in the nominal exchange rate and the behaviour of domestic output. We discuss the results in two parts. First, since we have an aggregate demand and supply system we can show diagrammatically the effects of an exogenous increase in the money supply. Second, we show how the reaction of the exchange rate to a money shock changes as the interest elasticity of money demand changes.

Before doing this we review some of the background to the model presented in section two in terms of previous literature, noting significant results. The most obvious starting point is the simple overshooting model proposed by Dornbusch (1976). The key findings are that the degree of exchange rate overshooting is altered by the interest elasticity of money demand, and overshooting itself rests on the differential speed of adjustment of prices in the goods and asset markets. Our model maintains the Keynesian features, although the rigid price is the price of labour. The second natural benchmark is the Sachs (1980) model of a small open economy. Like the overshooting model the Sachs model suggests that there will be some volatility in the short-run nominal exchange rate in response to unanticipated money shocks. Like the overshooting model the Sachs model also implies that following an exogenous shock the domestic economy may run a current account imbalance, although it is important to realise the overshooting model does not incorporate the wealth effects which would result from the changing stocks of net foreign assets over time. The significant difference between the two models is that Sachs specifies a supply side to study the effects of nominal wage rigidities and wage indexation and Dornbusch does not. In our model one thing is already clear; there will be no current account effect. The natural microfounded extension of the Sachs (1980) model is the Redux model discussed in the introduction. It keeps the Dornbusch assumption of sticky prices and includes a supply side, but specifies all relationships consistent with maximising behaviour. The key result of interest is that after a permanent unanticipated money shock the exchange rate does not overshoot its long run value. This is explained by the UIP condition and consumption based PPP which tie down consumption differentials. In this case the microfoundations appear to be restrictive. Therefore by relaxing the no home-bias assumption what happens to the exchange rate after a money shock?

4.1 Money Shocks when $\epsilon = 1$

Because we have a two variable system it is reasonably simple to plot the loci in $(s_0, y_{h,0})$ space. The most important feature we need to take into account is the interest elasticity of money demand as this determines the slope of the monetary relation, (33). To simple things we begin by setting $\epsilon = 1$, which would be equivalent to a utility function where the logarithm of real money balances enters. The first thing to note is that the slope of (32) is unambiguously positive in $(s_0, y_{h,0})$ space, as the aggregate supply relation does not depend on ϵ, and $\alpha > 1$ and $n \in (0, 1)$. The second point is that if we set $\epsilon = 1$ then (33), the aggregate demand relation is horizontal. Drawing this out we have.



***Effects of a Money Shock when* $\epsilon = 1$.**

To understand the effects of shocks on the system as a whole it is straightforward to see that an exogenous increase in the money supply, i.e. M_1' to M_1, shifts the monetary relation up and this leads to an increase in output and a depreciation of the nominal exchange rate. Intuitively, output increases because increases in the money supply cause prices to rise, which leads to a lower real wage. As labour is demand determined in the short-run there is an increase in the labour supply, and ultimately an increase in output.

To understand more precisely how the exchange rate behaves we can appeal to equations (32) and (33) and implicitly differentiate them to determine the sign of $\partial y_{h,0}/\partial M_1$ and $\partial s_0/\partial M_1$. The calculations are presented in the Appendix B. Here we work through the intuition. Differentiating and applying Cramer's rule to the appropriate derivatives we find that when there is a permanent unanticipated increase in the money supply output always increases and the exchange rate always depreciates (this is always true when $\epsilon > 0$). The most striking thing is that when the interest elasticity of money demand equals unity a change in the money stock generates a proportional change in the exchange rate . Therefore the reaction of the economy to a money shock when $\epsilon = 1$ can be summarised by two equations:

$$\partial y_{h,0}/\partial M_1 = \frac{g^* \overline{W}_0 s_0 / M_1}{y_{h,0}^{\alpha-1} (1-n)}, \tag{34}$$

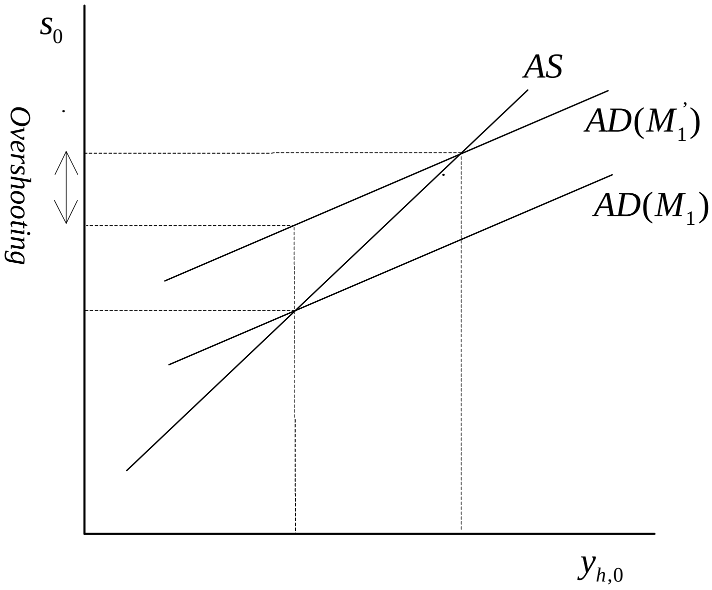
$$\partial s_0/\partial M_1 = s_0/M_1. \tag{35}$$

Some obvious conclusions follow from this. First, $(\partial s_0/\partial M_1)(M_1/s_0) = 1$; the short run change in the exchange rate is proportional to the increase in the money supply. To some extent this result is to be expected. If we set the interest elasticity to unity in the difference equation (23) then it is straightforward to demonstrate $i_0 = (1+\mu)/\beta$ so that the current real interest rate does not affect the current nominal interest rate, and therefore the interest rate jumps directly to it's steady state value. Thus a change in the money stock has no effect on the nominal interest rate and therefore the nominal exchange rate increases each period by the money growth rate; or rather, $\overline{s}(1+\mu) = s_0$. The crucial feature of a unit interest elasticity is that the home-bias assumption plays no role in the monetary sector because of it's separation from the real sector, as we discuss above. The mechanism underlying the result is that money demand responds one-for-one with the change in consumption when $\epsilon = 1$, and this is why the interest rate does not change. This corresponds to the type of effect we see in the *redux* model by virtue of the elimination of home-bias from the monetary sector. A second result is that when the economy is more open, i.e. when n is lower, monetary policy is less effective at increasing output.

4.2 Money Shocks when $\epsilon \neq 1$

The natural extension is to ask what happens when the interest elasticity of money demand does not equal unity. With a non-unit interest elasticity we are forced to explicitly consider the effect of the rigidity in the wage level on the monetary sector. Again the best way to understand how the economy behaves is to draw the loci in $(s_0, y_{h,0})$ space. The slope of (32) is unambiguously positive, but the slope of (33) is either positive or negative depending on whether $\epsilon > 1$ or $\epsilon > 1$, respectively. We draw the diagram for the $\epsilon > 1$ case as this is empirically the more plausible.[19]

[19] Empirical estimates suggest a value of ϵ between 9 and 20. See Mankiw and Summers (1986) and Keonig (1990).



Exchange Rate Overshooting when $\epsilon > 1$.

With a money shock (again $M_1' > M_1$) we can see the effect that a reduction in the interest elasticity has on the behaviour of the exchange rate. An upward shift in the monetary locus leads to an increase in output for the same reasons as before but the exchange rate now increases more than proportionally to the shock. The degree of overshooting can be clearly seen in figure two. Thus we find that the short run exchange rate overshoots in response to the money shock, given $\epsilon > 1$.[20] The reasons are very similar to Dornbusch (1976); first, there is a differential speed of adjustment in the product and asset markets; second, money demand is not highly sensitive to changes in consumption. Overshooting is greater the lower the sensitivity (i.e. higher ϵ) of money demand to changes in the interest rate. Thus the asset market compensates more and more for the distortion produced by the rigid money wage.

The Dornbusch model is also capable of generating an undershooting result, of which a necessary condition is that the money demand is sensitive to changes in output. As Rogoff (2002) notes this is quite unrealistic, but the overriding difference between the model we present (and this class of models in general) and the original overshooting model is that the interest elasticity determines the relationship between money demand and consumption, not money demand and output. Thus a quicker response may be less unrealistic. To examine

[20] As an aside to our description the interest elasticity of money demand is also the key parameter generating overshooting in Betts and Devereux (2000) and Lane (2001), although the former also requires a PTM assumption, and the latter a traded, non-traded goods distinction to generate the overshooting effect. Our analysis rests on the simple, but appealing description of home-bias.

this question in Appendix B we derive a general expression for the degree of exchange rate volatility, but to better understand what happens we can appeal to equation (33) directly. To explain the key features of the exchange rate more intuitively we therefore write down the monetary equation in terms of the current nominal interest rate and current level of output. From equation (33) we have,

$$y_{h,0}^{n(1-\epsilon)} = (1/\gamma)(i_0 - 1)\, i_0^{\epsilon-1}, \tag{36}$$

where γ is fixed. What we want to determine is the effect on the nominal interest rate from an increase in the money supply as we can then determine the effect on the nominal exchange rate. If the interest rate changes when there is a money shock, as it must do unless $\epsilon = 1$, the exchange rate will either under or overshoot its long run level; we have already demonstrated that when there is an increase in the money supply the nominal exchange rate will depreciate. Therefore to determine the change in the exchange rate all we have to do is determine how the interest rate changes, and this depends on the output level which we know always increases in response to an increase in M_1. We would expect the magnitude of the change in the interest rate to depend on the interest elasticity. Differentiating both sides of (36) with respect to $y_{h,0}$ and rearranging we find,

$$\partial i_0/\partial y_{h,0} = \gamma n (1-\epsilon)\, i_0^{1-\epsilon} y_{h,0}^{n(1-\epsilon)-1} / \left[\epsilon - (\epsilon - 1)/i_0\right]. \tag{37}$$

We know that the final term on left hand side of (37) is positive as $i_0 > (\epsilon - 1)/\epsilon \; \forall \epsilon > 0$, and the sign of the derivative therefore only depends on the magnitude of the interest elasticity of money demand. As a check if this is unity we should have the no overshooting result in the previous section as there is no effect on the nominal interest rate, i.e. $\partial i_0/\partial y_{h,0} = 0$. We find, if $\epsilon > 1$, then $\partial i_0/\partial y_{h,0} < 0$, so that the nominal interest rate falls and the current level of the exchange rate overshoots it's long run level. We also find the inverse is true; when $\epsilon < 1$ the exchange rate undershoots. Notice also that the change in the exchange rate will depend on all of the underlying parameters.

From this we can offer a tentative conclusion as the results appear to suggest that consumption home-bias is an assumption inherently linked to the ad-hoc models of Mundell, Fleming and Dornbusch, although they need never specify it. This is important because it is rarely assumed in the type of analysis popularised by Obstfeld and Rogoff (1995), a model which has become the workhorse of the modern approach to international finance.

To reiterate, when there is an increase in the money supply, the current real interest rate falls. With $\epsilon > 1$ this change has a negative effect on the nominal interest rate. If $\epsilon < 1$ the result is reversed and we find that a lower real interest rate induces a higher nominal interest rate. Returning to the current period UIP condition $i_0 = \overline{s}/s_0\beta$, when the nominal interest rate rises it requires that the current period nominal exchange rate rise by less than the long run nominal exchange rate, or rather it undershoots. The degree of undershooting increases as the interest elasticity rises. Therefore we find that when there is a permanent unanticipated increase in the money supply an interest elasticity greater than one induces undershooting, an interest elasticity less than one induces undershooting, and a unit interest elasticity induces no additional effect.

5 Conclusions

We have developed a stylised small open economy model with consumption home-bias. The model is simple enough to be solved analytically, but shows some of the salient features we would expect from any model of an open economy. In particular, with a low (or high) elasticity of money demand, the nominal exchange rate is increasingly volatile when exogenous money shocks occur. If money demand is sensitive to changes in the interest rate then the exchange rate undershoots its long run level in response to an increase in the money stock; if money demand is not very sensitive to changes in the interest rate the short run exchange rate overshoots. This is in line with many of the ad-hoc models since Dornbusch (1976), but runs contrary to the newer generation of models with fully specified preferences. The reason becomes apparent when we realise that vast majority of microfounded models do not incorporate consumption home-bias. We argue this is an implicit assumption in the overshooting model. The one feature this model does share with many other microfounded models is that the current account is zero; when an exogenous shock occurs there is no change in the net asset position of the domestic economy. It is possible to generalise a number of features of this model, for example to explicitly allow for the production of non-traded goods or more complex wage setting behaviour; this would not alter the basic result.

A Real Sector

Here we offer a more detailed derivation of the schematic analysis presented in the text. To determine the implicit system for the real sector when wages are flexible first equate the solutions for labour supply and labour demand. Combining the optimal wage condition for the individual (7) and the firms demand condition (2) yields an aggregate supply relation,

$$y_{h,t} = \left[\frac{\alpha d\sigma}{\sigma - 1} C_t q_t^{(1-n)/n} \right]^{1/(1-\kappa\alpha)} \quad \forall t \geq 1. \tag{A1}$$

As $n \in (0, 1)$ and $\alpha > 1$, equation (A1) is positively sloped in $(1/q_t, y_{h,t})$ space. To derive the goods market equilibrium condition take the short-run equilibrium condition for the home good given by equation (12). This can be expressed not only as a function of prices, for example $n\Gamma_t / P_{h,t} = nC_t / f_t$, but also as a function of real exchange rate as the price ratio f_t and the real exchange rate q_t are uniquely related; thus $nC_t/f_t = nq_t^{(1-n)/n} C_t$. As such we write,

$$y_{h,t} = nq_t^{(1-n)/n} C_t + g^* q_t^{1/n} \ \forall t. \tag{A2}$$

We assume foreign consumption levels do not vary over time, such that $C_t^* = C^* \ \forall t$ and hence $g_t^* = g^* \ \forall t$. Given the same conditions, in $(1/q_t, y_{h,t})$ space, (A2) is negatively sloped. The intersection of (A1) and (A2) give the natural rate of output and real exchange rate. This system cannot be solved explicitly for q_t, but can be described by two implicit functions given in the text as (13) and (14). The fact that the system cannot be solved explicitly is a direct result of the functional form of utility and the assumption of consumption home-bias.

Now consider the current period. Intuitively, in the labour market if there is some disturbance the individual is off the labour supply curve and thus the market clearing condition implicit in (A1) does not hold. Therefore in the current period we can only appeal to the labour demand condition (4) as a solution to the real side of the economy. This condition is not sufficient alone because it contains $P_{h,0}$, which is endogenous. From the definitions of the real exchange rate and the CPI we can express the domestic price as a function of the real and nominal exchange rates, thus $P_{h,0} = s_0 / q_0^{1/n}$ which solves the endogeneity problem. The counterpart to (A1) under fixed wages is therefore,

$$y_{h,0} = \left[\alpha \overline{W}_0 q_0^{1/n} / s_0 \right]^{1/(1-\alpha)}. \tag{A3}$$

Although we are assuming fixed wages, equation (A2) is not dependent on this assumption as it derives from the micro-demands and the resource constraint. As the goods market condition always holds using (A2) and (A3) we have an equivalent implicit system for the current period, given by (17) and (18) in the text.

B Comparative statics

To determine the effect of a permanent unanticipated money shock we appeal to the implicit function theorem and differentiate (32) and (33). To make the exposition clearer we denote the system as

$$
\begin{aligned}
0 &= F_r(y_{h,0}, s_0), && \text{(B1)} \\
0 &= F_m(y_{h,0}, s_0; M_1), && \text{(B2)}
\end{aligned}
$$

where F_r describes the equilibrium real side relations, and F_m the equilibrium monetary side relations. Thus the reaction of the economy to a money shock, i.e. $\partial y_{h,0}/\partial M_1$ and $\partial s_0/\partial M_1$ is captured by:

$$
\begin{bmatrix} \partial F_r/\partial y_{h,0} & \partial F_r/\partial s_0 \\ \partial F_m/\partial y_{h,0} & \partial F_m/\partial s_0 \end{bmatrix} \begin{bmatrix} \partial y_{h,0}/\partial M_1 \\ \partial s_0/\partial M_1 \end{bmatrix} = - \begin{bmatrix} 0 \\ \partial F_m/\partial M_1 \end{bmatrix}. \tag{B3}
$$

The derivatives are:

$$
\partial F_r/\partial y_{h,0} = \alpha y_{h,0}^{\alpha-1} \tag{B4}
$$

$$
\partial F_r/\partial s_0 = g^*/\left(n-1\right)\alpha \overline{W}_0 \tag{B5}
$$

$$
\partial F_r/\partial M_1 = 0 \tag{B6}
$$

$$
\partial F_m/\partial y_{h,0} = \gamma n(1-\epsilon) y_{h,0}^{n(1-\epsilon)-1} \tag{B7}
$$

$$
\partial F_m/\partial s_0 = \left(xM_1\right)^{\epsilon-1}\left(s_0\beta\right)^{-\epsilon} s_0^{-1}\left\{\left(xM_1\right)\epsilon + s_0\beta\left(1-\varepsilon\right)\right\} \tag{B8}
$$

$$
\partial F_m/\partial M_1 = \left(xM_1\right)^{\epsilon-2}\left(s_0\beta\right)^{-\epsilon} x \left\{s_0\beta\left(\varepsilon-1\right) - \left(xM_1\right)\epsilon\right\}. \tag{B9}
$$

We can sign all of these derivatives; (B4) is positive and (B5) is negative. As the gross nominal interest rate $i_0 = (xM_1/s_0\beta) > (\epsilon - 1)/\epsilon \ \forall \epsilon > 0$ we also find (B8) is positive and (B9) is negative. Equation (B7) is positive if $\epsilon < 1$, and negative if $\epsilon > 1$. Applying Cramer's Rule we can express the derivatives of interest as,

$$\partial y_{h,0}/\partial M_1 = \frac{(\partial F_r/\partial s_0)(\partial F_m/\partial M_1)}{(\partial F_r/\partial y_{h,0})(\partial F_m/\partial s_0) - (\partial F_m/\partial y_{h,0})(\partial F_r/\partial s_0)}, \tag{B10}$$

$$\partial s_0/\partial M_1 = -\frac{(\partial F_r/\partial y_{h,0})(\partial F_m/\partial M_1)}{(\partial F_r/\partial y_{h,0})(\partial F_m/\partial s_0) - (\partial F_m/\partial y_{h,0})(\partial F_r/\partial s_0)}. \tag{B11}$$

Although these expressions are a little complicated we can also sign them. They imply that in response to a positive money shock output always increases and the exchange rate always depreciates for $\epsilon > 0$, as we would expect. To demonstrate this it is necessary to realise that the numerators in (B10) and (B11) are positive and negative respectively. The denominators are common and therefore an increase (decrease) in output is always consistent with a depreciation (appreciation) in the nominal exchange rate. To demonstrate the impact of the shock assume,

$$(\partial F_r/\partial y_{h,0})(\partial F_m/\partial s_0) > (\partial F_m/\partial y_{h,0})(\partial F_r/\partial s_0), \tag{B12}$$

noting (32) and (33); or rather, $0 = F_r(y_{h,0}, s_0)$ and $0 = F_m(y_{h,0}, s_0; M_1)$, we can rewrite condition (B12) in terms of exogenous parameters and the nominal interest rate. For (B12) to hold we therefore require,

$$i_0 > \frac{1 - n/\alpha}{\epsilon/(\epsilon - 1) - n/\alpha}, \tag{B13}$$

which is trivially satisfied as $n/\alpha < 1$ and $\epsilon/(\epsilon - 1) > 1$ for $\varepsilon > 0$. The right hand side of (B13) is always less that one, whereas the left hand side, which is the gross nominal interest rate, is always greater than one. Hence, both $\partial y_{h,0}/\partial M_1$ and $\partial s_0/\partial M_1$ are positive for $\epsilon > 0$.

To demonstrate the main result of the paper a little more formally differentiate (B11) with respect to ϵ and evaluate the derivative at $\epsilon = 1$. This gives:

$$\frac{\partial}{\partial \epsilon}\left(\frac{\partial s_0}{\partial M_1}\right)_{\epsilon=1} = \left(\frac{s_0}{M_1}\right)^2 \left[\frac{g^* na(1+\mu)s_0\overline{W}_0}{(1-n)y_{h,0}^{\alpha}}\right] > 0. \tag{B12}$$

Thus, when ϵ increases (decreases) slightly the exchange rates' reaction to a money shock increases (decreases) and thus there is overshooting (undershooting) in the short run.

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