

The possibility of a Pareto-Improving Pension Reform: more arguments.

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Abstract

This article provides simulation results of Pareto-improving transitions from pay-as-you-go to fully funded pension systems in an economy where agents are heterogeneous within generations. The possibility of such transitions for a wide range of parameters states that intergenerational heterogeneity should no longer be considered an obstacle when implementing Pareto-improving pension reforms. To maintain redistributive or insurance mechanisms supported by pay-as-you-go systems, I propose to replace social system with redistributive tax and transfer payments inside one generation. This would save the economy from the inefficiency related to the implicit taxes on pension contributions imposed by pay-as-you-go systems.

Keywords: *Pension reform, Pareto-improving transition, Heterogeneous population, Redistribution and insurance*

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1 Introduction

The privatisation of social security is one of the most important economic issues today. In the last thirty years, many countries have faced drastic demographic changes leading to a

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dramatic increase in dependency ratio and a reduction in payroll growth. Economists and policy makers world wide predict that this tendency will continue for at least 50 years¹, which will make pay-as-you-go systems even more inefficient. For a more detail discussion, see various World Bank publications, for example, World Bank (1994) and, more recently, Fox and Palmer (2001). A continuation of the current systems will force governments to either increase taxes or reduce the benefits for future generations, which would cause a decrease in welfare levels, as compared to those guaranteed by the present pension schemes. Thus, Feldstein and Samwick (2000) predict that the U.S. "current pure pay-as-you-go system can only maintain the benefits specified in current law by raising the payroll tax rate from 12.4 percent today to more than 17 percent by 2037 and nearly 19 percent by the end of the actuaries' 75 year forecast period."

Pay-as-you-go systems are inefficient even in the present demographic environment, since the rates of capital return exceeds the growth of wage bills². This conclusion was first drawn by Samuelson (1975). The inefficiency arises from the indirect taxation on pension contributions imposed by pay-as-you-go systems. In addition, it is fueled by the distortions to the labor market due to social security payroll taxes. According to Feldstein (1996), the relevant deadweight loss equals approximately one percent of GDP in the U.S. In closed economies, the damage from pay-as-you-go systems is even larger due to the reduction in savings. An increase in capital accumulation, caused by the pension reform will also increase pre-tax wages. The transition to a fully funded system might also cause larger economic growth in the presence of endogenous technical progress, as discussed in Wiedmer (1996).

The question considered in this paper is the one of how to introduce a better pension system without anybody suffering. On the one hand, the old generations must be compensated for their contributions to a pay-as-you-go system, and therefore, the young generations have to pay twice: taxed for pensions for the old, they also have to contribute to their own pension accumulation fund. On the other hand, a reduction in payroll taxes leads to higher wages

¹For demographic projections see, for example, the World Bank data base "World Development Indicators".

²According to Feldstein and Samwick (2000), the real rate of return will exceed the growth of the average real wage by at least 4.5% annually.

and larger capital supply, thereby providing some financing for a transition. This paper investigates whether and when those funds are sufficient for a Pareto-improving reform, with the purpose of convincing the readers that a Pareto-efficient transition can nearly always be implemented.

I start my paper by providing a systematic classification of earlier studies related to the possibility of a Pareto-improving pension reform in a representative agent economy (section 2). The existence of Pareto-improving transition depends on whether economy is open or close, how elastic labor supply is, the relationship between benefits and contributions and the policy instruments used during transition. The last section

Then, in section 3, I reject the intragenerational heterogeneity as an argument against the possibility of a Pareto-improving transition. In particular, I provide simulations of Pareto-improving transitions for economies with two types of agents, distinguished by productivity levels and preference parameterization. To maintain the pre-reform welfare level of less productive (or less lucky) households, I propose to make the redistribution within one generation. This mechanism endures the redistributive and insurance functions of a pay-as-you-go system, but saves the economy from the inefficiency related to the implicit taxes on pension contributions. As a result I disargued Brunner (1994, 1996) and Fenge and Schwager (1995) and show that intergenerational heterogeneity should no longer be considered an obstacle to implementing Pareto-improving pension reforms.

Section 4 concludes the paper.

2 A review of the literature on the existence of a Pareto-efficient transition in a representative agent's economy.

Studying previous literature on the existence of a Pareto-efficient pension reform, I realize that there exists a lot of articles reporting the possibility of Pareto-improving pension reform, as well as many of them state that such a reform is impossible. After closer consideration,

I find that the answer to this question depends on the type of reformed economy and the set of policy instruments which used during the reform³. This motivates me to write the present section, which concludes that a Pareto-improving pension reform is impossible for only two types of economy: an open economy with inelastic labor supply; and open economy with pension system where benefits are proportional to contributions (without lump-sum component). This allows me to conclude that Pareto-improving transitions from pay-as-you-go to fully funded pension systems nearly always exist.

2.1 Background

The early literature considers a pay-as-you-go system with a lump-sum form of pension benefits. Thus, Breyer (1989) proved the impossibility of an efficient debt-financed transition for both closed and open economies, using a simple two-period overlapping generations model (OLG2) and assuming the constancy of labor supply. Then, Homburg (1990) showed that a Pareto-improving transition is possible for an open economy with endogenous labor supply. Finally, Breyer and Straub (1995) proved the existence of a Pareto-efficient transition for a closed economy, where labor is endogenously chosen. Although the authors documented the possibility of an efficient transition, they did not provide a policy mechanism.

Later, researchers have investigated different policies to find out whether their implementations allow for intergenerational-efficient pension reforms. The first bulk of papers considers different debt-tax policies for reforming a lump-sum benefit pension system. Raffelhuschen (1993) provides an efficient policy, which uses both a public debt issue and lump-sum taxes and transfers for compensating transition generations. Kotlikoff (1995) also provides the simulations of Pareto-improved transitions using a lump-sum redistributive authority mechanism for the compensation of transition generations. Brunner (1994, 1996) rejected the use of lump-sum taxes for compensating losers as hardly being implemented in an economy with

³Orszag and Stiglitz (2001) (p. 24), for example, characterize the transition cost as unavoidable. "If the economy is dynamically efficient, one cannot improve the welfare of later generations without making intervening generations worse off." To support this point, the authors provide Breyer's (1989) argument for an open economy with exogenous labor supply, ignoring, however, the entire later literature.

heterogeneity inside one generation. However, Hirte and Weber (1997) simulates an efficient transition without using lump-sum taxes. They provide a transition implemented by debt issue to compensate for the first years of reform, and then consumption or income taxes for debt and interest repayments. Demmel and Keuschnigg (1999) proved the feasibility of a debt-financed Pareto-improving transition from a system with a lump-sum form of benefits, where the government gradually reduces payroll taxes.

The next group of papers investigated the feasibility of a Pareto-improving pension reform, when benefits are proportional to contributions. Fenge (1995) opened the discussion. He proved the Pareto-efficiency of a pay-as-you-go system in an open economy with elastic labor supply, where benefits are proportional to contributions. In other words, he demonstrated that a Pareto-improving transition could not be implemented by changing payroll tax rates and the implicit return of contribution rates, combined with debt-financing. Wrede (1998) extended this result for a three-period model, doubting the simulation results provided by Hirte and Weber (1997), where the authors simulated a Pareto-efficient transition for both extreme cases of linkage between benefits and contributions. However, Hirte and Weber's results do not contradict Fenge's findings, since they investigate a closed economy. They also compared two transition paths starting from steady states, distinguished by the degree of benefits to compensations linkage, and found that transition from the lump-sum scheme can more easily be implemented. Kotlikoff (1995) confirms this result.

Valdes-Prieto (1997) provides a Pareto-improving transition, implemented by a gradual decrease in the rate on pension contributions and a simultaneous reduction in the use of payroll tax rate. This policy allows the government to carry out an efficient pension reform in an open economy with constant labor supply and stochastic income⁴, which is impossible for the same type of economy with deterministic incomes.

Belan, Michel and Pestieau (1998) introduced a saving subsidy form of pension benefits, in addition to generally considered lump-sum benefits and proportional to contribution forms. Assuming a constant labor supply and lump-sum benefits in an initial steady state, the

⁴This article considers deterministic income only. For more information about stochastic income and Social Security see De Nardi, Imrohoroglu and Sargent (1999) and Storesletten, Telmer and Yaron (1999).

authors showed that switching to saving subsidies improves the welfare of both current and future generations, which allows a pension reform to be implemented in one period for a closed economy. (See also Wigger (1998)).

Finally, Feldstein (1996) proposes the policy mechanism, which is the perfect opposite to debt-financed strategy. In particular, he suggested increasing the payroll tax rate and investing additional revenue, which would allow the government to accumulate the necessary for a transition fund. Although such a policy does not in itself allow the implementation of a Pareto-improving transition itself, it is an excellent addition to the policy schemes, which do not use debt but generate an additional welfare gain for the first generation in a transition, for example when a government increases the degree of linkage between benefits and contributions. I will refer to such a policy as "prefunding".

2.2 Classification.

Summarizing the results on the existence of a Pareto-improving pension reform, I classify the literature by four characteristics of reformed economies and transition policies. In other words, the existence of a Pareto-efficient pension reform depends on the answers to the following questions:

- 1) Is the economy closed or open?
- 2) Is labor supply elastic or constant?
- 3) What is the form of the pension benefits: lump-sum or proportional to the contribution?
- 4) Which policy mechanisms are implemented during the transition?

Table 1 summarizes the existing literature according to my classification.

Table 1.
Classification of the existing literature⁵

References and comments	Type of economy		Initial benefits		Labor supply		Transition policy				Result	
	Open	Closed	Lump sum	Proportional to contributions	inelastic	endogenous	Lump sum tax	Reduction of payroll tax	Switching to consumption tax	Debt issue		Saving subsidy
Breyer 1989												No
Homburg 1990												No
Breyer and Straub 1993												Yes
Raffelhuschen 1993												
Kotlikoff 1995												
Fenge 1995 OLG2												No
Wrede 1998 OLG3												
Hirte and Weber 1997												Yes
												Yes
Demmel and Keuschnigg 1999												Yes
Belan, Michel and Pestieau 1998												Yes

According to this classification, there are only two types of economy-pension system which can not be reformed on a Pareto-improving way. namely they are "open economy with constant labor supply" and "open economy when benefits are proportional to contributions". Both systems present the extreme cases, which can not be observed in practice.

2.2.1 Open vs. closed economy.

Pay-as-you-go systems are less destructive in open economies, but a closed economy can more easily be reformed.

The differences between open and closed economies are important for investigating a pension system for the following reasons. First, in the case of a closed economy, savings equal investments and, therefore, contribute as an important intergenerational pecuniary externality. In an open economy, a pay-as-you-go system⁵ can simply be considered as tax on

⁵The result is "yes" if the corresponding paper proves the existence of or provides a simulation of a Pareto-improving pension reform. The result is "No" if the paper proves the impossibility of implementing a Pareto-improving reform of the corresponding pay-as-you-go system in a corresponding economy by using the corresponding transition policy.

pension contributions, where the tax rate equals the difference between the interest rate and the wage fund growth. (See Valders-Prieto, 1997). Those taxes are thrown away without being used for financing government expenditures. In a closed economy, a pay-as-you-go system does not only create this wasteful tax on pension contributions, but also reduces the capital stock.

Closed economies can more easily be reformed. Pareto-improving transitions exist for closed economies regardless of the elasticity of labor or the form of pension benefits. Thus, if labor supply is independent of pay-roll tax rate, a Pareto-improving transition from a pay-as-you-go system with lump-sum benefits exists in a closed economy and can be implemented by saving subsidizing (Belan, Michel and Pestieau (1998)). This is impossible in an open economy, however (Breyer (1989)). Moreover, a debt-financing Pareto-improving transition from a pay-as-you-go system with proportional benefits is possible for a closed economy but impossible for an open one, as proved by Fenge (1995). The reform implementation is easy in a close economy because a capital provision, made by the old, has an impact on the wage rate earned by the young which, in turn, causes the changes in payroll tax revenue. Also, government borrowing has more complicated consequences in a closed economy. For example, it does not only increase government liabilities, but also enlargers the interest rate, thereby causing the growth of savings and increase in welfare of lending generations.

Summarizing, I would like to emphasize that the terms "closed" and "small open" represent two extreme theoretical structures. Everything in the world is located somewhere in between, containing some degree of "closeness". In other words, national savings are more or less important for national investment as well as for the wage and interest rates in any country. And if an economy is not perfectly "small open" it is possible to implement a Pareto-improving reform.

2.2.2 The elasticity of labor supply.

Once a pay-as-you-go system is financed by labor income taxes, the elasticity of labor supply is important for both the existence and the duration of a Pareto-improving reform. Although a pay as you go pension system creates less harm for the economy with constant labor supply,

a Pareto-improving transition can be implemented the easier the larger the response of labor supply to the value of pension benefits. Indeed, a pay-as-you-go system financed by payroll taxes in a deterministic open economy with exogenous labor supply is efficient, as shown by Breyer (1989).

But undersupply of labor is not the most important source of the inefficiency in a closed economy: the bad pecuniary intergenerational externalities created by insufficient savings seem to be more important. However, the degree of elasticity of labor supply plays an essential role in the choice of transition policy. Thus, a closed economy with constant labor should be reformed by substituting lump-sum pension benefits with saving subsidies (Belan, Michel and Pestieau, 1998). This is impossible for an economy with a large elasticity of labor supply and lump-sum benefits, and such an economy should be efficiently reformed by increasing the degree of linkage between benefits and contributions.

2.2.3 The degree of linkage between benefits and contributions.

The third dimension of my classification is related to the form of the initial pension system. If benefits are proportional to contributions, the negative impact of pay-as-you-go systems on labor supply is much lower than in the case of lump-sum pensions. For example, labor supply does not depend on the contribution rate in an economy with logarithmic preferences when benefits are proportional to contributions. However, pension systems with a lower linkage between benefits and contributions can more easily be reformed. Thus, for the open economy, Fenge (1995) showed that an efficient transition is impossible if the benefits are proportional to the contributions, while its feasibility for the lump-sum form of benefits was proved by Homburg (1990). In the real world, almost all pension systems have a lump-sum component, which allows a Pareto-improving reform even in a perfectly open economy.

2.2.4 The choice of transition policy.

The existence and speed of a Pareto-improving pension reform do not only depend on the initial structure of the economy, but also on the instruments used by a government during a transition. The existing literature considers several instruments: switching to another tax

base, debt issue, an increase in the degree of linkage between benefits and contributions, the introduction of a saving subsidy. Below, I discuss all these measures in more detail .

The choice of tax base. According to Auerbach and Kotlikoff (1987), consumption tax is less destructive than income one, capital income or wage income taxes, and Kotlikoff (1996) repeats this result. The authors do not set Pareto-efficiency as a goal in either of these papers. To define the consumption tax rate, they use a revenue-neutral condition. However, for the welfare-neutral tax, the government should increase the tax rate up to the amount that allows compensating the old for having paid their consumption taxes. Such a large rate has too large a destructive effect on the economy and, therefore, switching to consumption tax in the first period does not help find additional financial sources for an efficient transition. Nevertheless, it can be used in later stages to reach a better steady state in an economy where a government must finance expenditures non-related to social securities. In other words, the implementation of a pension reform provides the financing of a global Pareto-improving tax reform, when a more distorting income tax can be replaced by a less distorting consumption tax. It also allows the repayment of part of the government debt and reduces interest expenditure which, in turn, leads to a further reduction in the tax burden.

Increasing linkage between benefits and contributions. Increasing the degree of linkage between benefits and contributions has been widely proposed as an improvement of pension systems. This measure causes a significant improvement even in the first period of reform for two reasons. First, the growth of labor supply will exceed capital growth, thereby shifting the interest rate upward, which would make the lending generation better off. Second, although the growth in labor supply reduces wages, the total payroll is growing, which allows the government to collect additional revenue without changing the tax rate. Maintaining pension on the promised level, the government can invest additional revenue in order to generate means for further reform. This investment will also create a favorable pecuniary externality, providing the economy with a larger capital stock. My simulations

show that an efficient transition can be implemented by a pure increasing linkage between benefits and contributions if a government has the option to reduce promised pensions while maintaining utility levels. Otherwise, if the government must commit on pension promises, the pure policy of increasing in linkage between benefits and contribution may not allow a Pareto-improving transition. However a combination with prefunding makes such a reform feasible.

Saving subsidy in a closed economy. The policy of a saving subsidy seems to be the most efficient in a closed economy, since the main source of inefficiency in the current pay-as-you-go systems is a capital underaccumulation due to low returns on pension contributions. The saving subsidy form of pension benefits has been proposed relatively recently by Belan, Michel and Pestieau (1998). In a closed economy, a saving subsidy increases capital accumulation and significantly improves the welfare of working generations. A larger capital stock increases the wages of the next generation, which allows a reduction in the payroll tax rate without loss in tax revenue. Moreover, pensions paid in the form of saving subsidies create lower distortions for the labor market than lump-sum pensions, which will be show later in the example of logarithmic preference⁶. Belan, Michel and Pestieau (1998) show that the government can make the efficient reform of an economy with constant labor supply in one period by using savings subsidies instead of lump-sum pensions. I find that savings subsidies are not sufficient in themselves for an economy with endogenous labor supply, if the government has obligations in term of the size of pensions. Nevertheless, a savings subsidy in combination with government prefunding makes it possible to complete a Pareto-improving reform when labor is elastic to the tax rate. This is also possible for savings subsidies if government obligations are expressed in terms of welfare. A savings subsidy can improve the debt financing strategy for the reform of pension systems characterized by a proportional relation between benefits and contributions.

⁶See formula (12) p. 27

Government prefunding There is another transition policy which I will call government prefunding⁷. This policy is the exact opposite of debt-financing and is realized via additional government savings, allowing the generation of a sufficient budget surplus to finance a reform. This policy can only be Pareto-improving in combination with another policy, which does not include the running of additional debt and allows the generation of additional revenue in the first period of transition. An example of such policies is the increase in the share of the pension in proportion to contributions and savings subsidies. Those transition methods can be significantly improved by prefunding. To complete the prefunding policy in isolation, the government must raise a payroll tax or reduce expenditures, thereby breaking the Pareto-improving constraint.⁸.

3 Heterogeneity is not an obstacle in implementing a Pareto-improving transition

The main goal of this section is to convince the readers that heterogeneity inside one generation is not an obstacle to completing an efficient pension reform. The first time heterogeneity was provided as an argument against a pension reform for an open economy in Brunner (1994) and for a closed one in Brunner (1996). In particular, the author introduced two types of householders with identical preferences but different productivity levels and concluded that if there is a great difference in earning abilities, the Pareto-improving transition is impossible. The main reason is that to maintain the same level of utility as in the initial steady state,

⁷Although the authors propose to place the means in the Individual Retirement Accounts, these savings are mandatory, and the householders have no access to their accounts before their retirement, when accumulated resources become withdrawable according to the design of the government pension plan. In the simple framework of my model, this operation is equivalent to a government investment.

⁸In earlier papers (1995, 1996, 1997), Feldstein and Samwick propose an increase in a payroll tax rate, leaving pension obligations unchanged, while later on (1999, 2000), they propose to use the exogenous gain generated by the US economy. (See also Martin Feldstein "Don't Waste the Budget Surplus", The Wall Street Journal, November 4, 1997. <http://www.nber.org/feldstein/wj110797.html>) This source is not generally available.

the government might have to use different tax-transfer rates for different individuals. In the case of imperfect information, it might be difficult to define what tax rate must be associated with a particular agent. An incorrect tax level, in turn, might make one householder worse off. I will refer to this argument as a "coordination problem". Subsection 3.1 disputes this by providing Pareto-improving transitions in the economy with two agents distinguished by productivity levels or preference parameters. In those transitions, the government only uses flat payroll taxes to compensate transition generations.

Brunner (1994, 1996) correctly rejected lump-sum taxation as a policy instrument, but his main finding was based on incorrect assumptions. For example, Brunner (1996) assumed the impossibility of a Pareto-improving debt-financed transition for a closed economy. This assumption was theoretically refuted by Demmel and Keuschnigg (1999) by demonstrating that a transition to a funded system might be made welfare improving for all generations through a proper use of public debt and proportional payroll taxes to compensate any potential losers. Before that, Hirte and Weber (1997) simulated Pareto-improving debt-financed transition without using lump-sum tax. Despite the incorrectness of some assumptions, Brunner's argument is very important and must be carefully investigated. The importance of the argument of intra-cohort diversity is supported by recent research attention. Thus, Kotlikoff (1996) uses a Smetters and Walliser modification of the Auerbach-Kotlikoff Dynamic Life-Cycle Model, which incorporates twelve householder groups distinguished by productivity level. Discount rate heterogeneity is considered by Samwick (1998), where the author proposes a transition tax/benefit menu, investigating the dependence of a chosen option on a householder's time discount rate. Those papers do not pretend to achieve a Pareto-efficiency, however.

Brunner's argument is very often referred to as an obstacle in the implementation of a pension reform. This is not correct and must be rejected. Even if Brunner's argument is true for an economy with very big inequality, we still have to clarify, how big it should be. The existence is proven for homogeneous economy when income tax used during the transition. Therefore, it may happen, that a Pareto-improving pension reform exists for all reasonable calibration. I provide transition simulations for economies where earning abilities

or preference parameters are very different.

The second argument against pension reforms in a heterogeneous economy relates to the redistributive or the insurance function of a pay-as-you-go system. The simple abolishment of social security in this case can lead to the welfare loss of poor householders or general worthiness in case of very risky income and in the absence of another insurance mechanism. These arguments were provided by Fenge and Schwager (1995) and have been used by other researchers. Thus, Conesa and Krueger (1999) find "that the role of a pay as you go social security system as a partial insurance and redistributive device significantly reduces political support for a transition to an economy with a fully funded system". Storesletten, Telmer and Yaron (1998) attach importance to the risk-sharing aspect of social security and the fundamental trade-off of distortion versus risk sharing.

I find these arguments to be weak since redistributive or insurance mechanisms can be supported without running inefficient pay-as-you-go systems, which I propose to replace by taxes and transfers within one generation. This policy allows the government to maintain its redistributive or insurance function, while it helps us get rid of the inefficiencies related to indirect taxing of the pension contribution. Subsection 3.2 discusses this in more detail.

3.1 Differences in productivity or preferences still allow the Pareto-improving pension reform to be complete by using homogeneous tax rates.

In the present section, I will disprove the "coordination" argument by providing numerical simulations for economies with householders distinguished not only by productivity level but also by preferences. For this purpose, I will consider a closed economy with endogenous labor supply, where an initial pay-as-you-go system has a lump-sum pension benefit, but different groups of individuals receive different pensions proportional to the payroll tax payed by the total group. By using this construction, I avoid redistribution inside one generation, but capture all distortions related to low linkage between benefits and contributions. Such a pension system appears when the government pays a benefit that is not proportional to individual

contribution, but dependent on the contributions made by the part of society with the same occupation, professional qualification, regional location, union or other memberships. The elements of such pension systems were present in the Former Soviet Union where benefits were independent of individual efforts, but differed across regions and occupations. According to Edwards (1996), the pre-reform Chilean system had the same characteristics. Considering such a pension system allows me to separate the effects related to the coordination problem of a pension reform from the ones related to redistribution.

To simplify the presentation I provide technical framework in appendix 5.

3.1.1 The role of differences in productivity

In this subsection, I argue that at least for the logarithmic utility function, the difference in productivity levels is not important since the economy might be replaced by an equivalent representative consumer model, providing the same outcome of capital and labor supply as well as the welfare dynamics. Then, I discuss simulation results for the additive constant relative risk-averse (CRRA) preferences.

The solution to problems (24) and (25) is represented by formulas (1)-(4)

$$c_{yt}^i = \frac{\frac{Pen_{t+1}^i}{1+r_{t+1}} + w_t a^i (1 - tax_t)}{1 + \beta + \gamma}; \quad (1)$$

$$c_{ot+1}^i = c_{yt}^i \beta (1 + r_{t+1}); \quad (2)$$

$$l_t^i = 1 - \frac{c_{yt}^i \gamma}{w_t a^i (1 - tax_t)}; \quad (3)$$

$$s_t^i = w_t a^i (1 - tax_t) l_t^i - c_{yt}^i. \quad (4)$$

As already mentioned, I will consider an economy with a zero intergenerational redistributive effect. In my economy, householders pay a proportional labor income tax, and receive different lump-sum benefits, proportional to the contribution of the total group with the same productivity. Therefore, pensions are proportional to productivity levels and, as a result, both householders work the same number of hours and consume proportionally to the productivity level or to efficient labor supply. Formally, if (c, l) is the solution to the problem

for parameter $a^i = 1$, then $(a^i c, l)$ will be the solution to the general problem. Moreover, individuals' utilities are satisfied to relation (5):

$$U_t^2 = U_t^1 + (1 + \beta) \ln \left(\frac{a^1}{a^2} \right), \quad (5)$$

which is true for all periods, t . This means that the transition policy has the same impact on individuals' welfare dynamics. Since efficient labor supply and savings are proportional to productivity levels, the problem is equivalent to that of a representative agent with a productivity level equal to the weighted sum of the productivity of heterogeneous agents

$$a = \sum q^i a^i,$$

where q^i is a proportion of individuals with productivity a^i .

Therefore, if preferences are logarithmic, the introduction of different productivity levels does not principally change the model, even if the differences are very large.

The difference in productivity is important for other preferences, but nevertheless, a Pareto-efficient debt-financed transition exists. I consider an efficient transition for the CRRA utility function, given in formula (6)

$$U_t = \frac{c_{yt}^{1-1/\rho} - 1}{1 - 1/\rho} + \beta \frac{c_{ot+1}^{1-1/\rho} - 1}{1 - 1/\rho} + \gamma \frac{(1 - l_t)^{1-1/\rho} - 1}{1 - 1/\rho} \quad (6)$$

where $\rho > 0$ is a parameter determining how responsive individual labor supply is to changes in the wage rate. If $\rho > 1$, a householder works more when wages increase, while if $\rho < 1$, a growing wage causes increasing leisure. Logarithmic preference is a special case when $\rho = 1$, and the choice of labor supply does not depend on the wage.

Tables 2 and 3 provide efficient transition paths for $\rho = 1.67$ and 0.5, respectively.

Table 2.
Different Productivity Levels and CRRA Preference ($\rho > 1$)

Parameters															
alpha	beta	theta	prod1	prod2	rho										
0.3	0.7	0.3	1	0.01	1.67										
t	debt	tax rate	welf 1	welf 2	s1	s2	L1	L2	r	w	Cy 1	Cy 2	Co 1	Co 2	
ss	0	30.00%	0.0%	0.0%	0.042	0.0000	0.403	0.030	1.472	0.354	0.058	4.4E-05	0.146	0.0001	
-1	0	30.00%	0.0%	0.0%	0.042	0.0000	0.403	0.030	1.472	0.354	0.058	4.4E-05	0.146	0.0001	
0	0	29.99%	0.0%	0.0%	0.042	0.0000	0.403	0.030	1.472	0.354	0.058	4.4E-05	0.146	0.0001	
1	0.000091	29.77%	0.1%	0.1%	0.042	0.0000	0.405	0.031	1.475	0.354	0.058	4.4E-05	0.146	0.0001	
2	0.00061	28.90%	0.2%	0.3%	0.044	0.0000	0.409	0.031	1.477	0.354	0.059	4.5E-05	0.148	0.0001	
3	0.00138	26.94%	0.5%	0.8%	0.047	0.0000	0.419	0.032	1.474	0.354	0.061	4.7E-05	0.150	0.0001	
4	0.002075	23.00%	1.0%	1.6%	0.054	0.0000	0.436	0.035	1.453	0.356	0.065	5.2E-05	0.153	0.0001	
5	0.00057	16.56%	2.2%	3.1%	0.067	0.0001	0.463	0.038	1.380	0.364	0.074	6.1E-05	0.154	0.0001	
6	0	3.76%	18.1%	20.0%	0.086	0.0001	0.493	0.043	1.223	0.383	0.096	8.4E-05	0.175	0.0002	
7	0	0.00%	29.9%	32.7%	0.097	0.0001	0.507	0.046	1.043	0.410	0.112	1.0E-04	0.190	0.0002	
8	0	0.00%	33.9%	37.1%	0.100	0.0001	0.512	0.046	0.964	0.424	0.117	1.1E-04	0.195	0.0002	
9	0	0.00%	35.2%	38.5%	0.102	0.0001	0.514	0.047	0.940	0.429	0.119	1.1E-04	0.196	0.0002	
10	0	0.00%	35.7%	39.0%	0.102	0.0001	0.514	0.047	0.933	0.430	0.119	1.1E-04	0.197	0.0002	
new ss	0	0.00%	35.9%	39.2%	0.102	0.0001	0.514	0.047	0.930	0.431	0.119	1.1E-04	0.197	0.0002	
Increasing in a New Steady State					146%	197%	28%	54%	-37%	22%	105%	147%	36%	64%	

Table 2 is consistent with Kotlikoff's (1995) paper, where the author states that the welfare gains for the poorer income group exceed those of the richer group. Moreover, a low-productive class starts to benefit earlier than high-productive ones. This occurs as soon as the net tax wage reaches an initial level, which must be reduced in the first periods of debt-financed transition, following the reduction in capital. A decreasing wage is compensated by payroll tax reductions; these two factors together create a positive impact on the growth of labor supply. These results held for all $\rho > 1$.

Table 3 presents an efficient transition for a small $\rho = 0.5$. As expected, employment is lower in a new steady state but households benefit from a higher capital stock and wages. The gain from a transition is larger for the more productive group.

Table 3.
Different Productivity Levels and CRRA Preference ($\rho = 0.5$)

Parameters															
alpha	beta	theta	prod1	prod2	rho										
0.3	0.7	0.3	1	0.01	0.5										
t	debt	tax rate	welfare1	welfare2	s1	s2	L1	L2	r	w	Cy 1	Cy 2	Co 1	Co 2	
ss	0	30.00%	0.00%	0.00%	0.031	0.000	0.747	0.967	2.760	0.270	0.110	1.4E-03	0.179	2.3E-03	
-1	0	30.00%	0.00%	0.00%	0.031	0.000	0.747	0.967	2.760	0.270	0.110	1.4E-03	0.179	2.3E-03	
0	0.00017	29.93%	0.00%	0.00%	0.031	0.000	0.747	0.967	2.760	0.270	0.110	1.4E-03	0.178	2.3E-03	
1	0.00096	29.39%	0.01%	0.00%	0.032	0.000	0.748	0.967	2.774	0.270	0.110	1.4E-03	0.179	2.3E-03	
2	0.00253	28.73%	0.02%	0.00%	0.034	0.000	0.749	0.968	2.763	0.270	0.110	1.4E-03	0.179	2.3E-03	
3	0.00435	27.80%	0.03%	0.00%	0.036	0.000	0.750	0.968	2.763	0.270	0.110	1.4E-03	0.179	2.3E-03	
4	0.00675	26.47%	0.06%	0.02%	0.039	0.000	0.752	0.968	2.758	0.271	0.111	1.4E-03	0.179	2.3E-03	
5	0.00836	25.23%	0.10%	0.05%	0.041	0.001	0.753	0.968	2.736	0.271	0.111	1.4E-03	0.178	2.3E-03	
6	0.00788	23.99%	0.83%	0.78%	0.044	0.001	0.753	0.968	2.668	0.274	0.113	1.5E-03	0.177	2.3E-03	
7	0.00395	22.11%	0.47%	0.42%	0.050	0.001	0.754	0.968	2.510	0.282	0.115	1.5E-03	0.169	2.2E-03	
8	0	11.84%	12.52%	12.52%	0.069	0.001	0.746	0.967	2.092	0.305	0.132	1.7E-03	0.176	2.3E-03	
9	0	0.00%	39.68%	39.25%	0.089	0.001	0.725	0.964	1.561	0.345	0.161	2.1E-03	0.204	2.7E-03	
10	0	0.00%	50.05%	49.23%	0.097	0.001	0.719	0.962	1.293	0.374	0.172	2.3E-03	0.215	2.9E-03	
new ss	0	0.00%	55.14%	54.08%	0.099	0.001	0.715	0.962	1.186	0.388	0.177	2.4E-03	0.219	3.0E-03	
Growth					216%	228%	-4.3%	-0.6%	-57%	44%	61%	67%	23%	28%	

3.1.2 Difference in preferences should not be considered a barrier to a pension reform

Motivated by Brunner's idea on heterogeneity in each generation, I investigate an economy with two types of individuals distinguished by preferences. Simulating numerically an economy where two householders have different time discount and leisure to consumption preferences, I find that Pareto-improving debt-financed transitions exist. In my experiment, I set different β ($\beta_1 = 0.7$; $\beta_2 = 0.5$), different γ ($\gamma_1 = 2.5$; $\gamma_2 = 5$) and different ρ ($\rho_1 = 0.7$, $\rho_2 = 1.11$). The results are presented in tables 4, 5 and 6, respectively.

Table 4.
Different Preferences for Leisure Relative to Consumption

Parameters																				
alpha	beta	gamma1	gamma2	theta																
0.3	0.7	2.5	5	0.3	t	debt	tax rate	welfare1	welfare2	s1	s2	L1	L2	r	w	Cy 1	Cy 2	Co 1	Co 2	
ss	0	0.00%	0.0%	0.0%	0.03	0.02	0.372	0.228	1.887	0.3183	0.056	0.034	0.113	0.069						
-1	0	30.00%	0.0%	0.0%	0.03	0.02	0.372	0.228	1.887	0.3183	0.056	0.034	0.113	0.070						
0	0.00006	29.94%	0.0%	0.0%	0.03	0.02	0.372	0.229	1.888	0.3182	0.056	0.034	0.113	0.070						
1	0.00035	29.67%	0.0%	0.0%	0.03	0.02	0.373	0.229	1.888	0.3182	0.056	0.034	0.113	0.070						
2	0.00045	29.30%	0.1%	0.1%	0.03	0.02	0.374	0.230	1.884	0.3185	0.056	0.035	0.113	0.070						
3	0.00025	28.37%	0.1%	0.1%	0.03	0.02	0.377	0.232	1.873	0.3193	0.057	0.035	0.113	0.070						
4	0	25.08%	0.1%	0.2%	0.03	0.02	0.385	0.239	1.839	0.3218	0.059	0.037	0.113	0.070						
5	0	15.99%	0.8%	0.9%	0.05	0.03	0.405	0.254	1.719	0.3313	0.066	0.042	0.110	0.069						
6	0	0.00%	26.7%	26.9%	0.06	0.04	0.405	0.254	1.367	0.3655	0.087	0.055	0.130	0.081						
7	0	0.00%	35.8%	36.0%	0.07	0.04	0.405	0.254	1.129	0.397	0.094	0.059	0.137	0.086						
8	0	0.00%	38.7%	38.9%	0.07	0.04	0.405	0.254	1.067	0.406	0.097	0.061	0.139	0.087						
9	0	0.00%	39.6%	39.8%	0.07	0.04	0.405	0.254	1.049	0.409	0.097	0.061	0.139	0.087						
10	0	0.00%	39.8%	40.0%	0.07	0.04	0.405	0.254	1.043	0.410	0.098	0.061	0.140	0.088						
new ss	0	0.00%	39.9%	40.1%	0.07	0.04	0.405	0.254	1.041	0.411	0.098	0.061	0.140	0.088						
Increasing in a New Steady State					155%	160%	9%	11%	-45%	29%	75%	78%	23%	26%						

Table 4 presents an efficient transition path for an economy with two individuals distinguished by the parameter of the preference for leisure to consumption. A welfare analysis shows that the householder with a larger preference for leisure starts to enjoy a pension reform earlier and his gain in a new steady state is larger.

Table 5.
Different Time Discount Rates

Parameters														
alpha	beta1	beta2	gamma	theta										
0.3	0.8	0.5	2.5	0.3										
t	debt	tax rate	welfare1	welfare2	s1	s2	L1	L2	r	w	Cy1	Cy2	Co1	Co2
ss	0	0.00%	0.00%	0.00%	0.0306	0.0178	0.387	0.344	2.005	0.3101	0.053	0.057	0.128	0.086
-1	0	30.00%	0.01%	0.01%	0.0306	0.0178	0.387	0.344	2.005	0.3101	0.053	0.057	0.128	0.086
0	0	29.89%	0.05%	0.06%	0.0307	0.0179	0.387	0.344	2.005	0.3101	0.053	0.057	0.128	0.086
1	0.00019	29.74%	0.18%	0.20%	0.0309	0.0180	0.387	0.345	2.002	0.3103	0.053	0.057	0.128	0.086
2	0.000384	29.64%	0.22%	0.25%	0.0311	0.0182	0.387	0.345	1.999	0.3105	0.054	0.057	0.128	0.086
3	0.000526	29.51%	0.20%	0.24%	0.0313	0.0183	0.388	0.346	1.998	0.3106	0.054	0.057	0.128	0.086
4	0.000725	29.06%	0.18%	0.27%	0.0319	0.0189	0.389	0.347	1.996	0.3107	0.054	0.058	0.129	0.086
5	0.001085	27.66%	0.21%	0.50%	0.0338	0.0204	0.393	0.360	1.983	0.3116	0.055	0.059	0.128	0.086
6	0.001805	23.67%	0.82%	1.71%	0.0391	0.0247	0.402	0.369	1.933	0.3150	0.058	0.062	0.128	0.086
7	0	16.14%	0.79%	3.21%	0.0508	0.0341	0.419	0.375	1.788	0.3257	0.064	0.068	0.124	0.083
8	0	0.00%	25.97%	30.56%	0.0666	0.0448	0.419	0.375	1.433	0.3681	0.083	0.090	0.146	0.099
9	0	0.00%	34.79%	40.16%	0.0723	0.0485	0.419	0.375	1.186	0.3884	0.090	0.097	0.153	0.103
10	0	0.00%	37.56%	43.19%	0.0740	0.0497	0.419	0.375	1.120	0.3980	0.093	0.099	0.156	0.105
newss	0	0.00%	38.77%	44.51%	0.0748	0.0503	0.419	0.375	1.093	0.4021	0.094	0.101	0.157	0.105
Increasing in a New Steady State					144%	182%	8%	9%	-45%	30%	76%	77%	22%	23%

Table 5 provides a simulation result for an economy where householders have different time discount rates. At the beginning of a transition, when interest rates should necessarily increase due to government borrowing, an individual with a smaller time discount rate will have a somewhat larger welfare gain, while later, when the interest rate falls to a lower level as compared to that in an initial steady state, a householder representing the other type will obtain more in terms of the welfare measure presented by formula (22).

Table 6.
Different CRRA parameters

Parameters															
alpha	beta	theta	rho 1	rho 2											
0.3	0.7	0.3	0.70	1.11											
t	debt	tax rate	welfare1	welfare2	s1	s2	L1	L2	r	w	Cy1	Cy2	Co 1	Co 2	
ss	0	30.00%	0.0%	0.0%	0.0397	0.0431	0.6830	0.5652	2.0044	0.3102	0.109	0.080	0.183	0.182	
-1	0	30.00%	0.00%	0.0%	0.0397	0.0431	0.6830	0.5652	2.0044	0.3102	0.109	0.080	0.183	0.182	
0	0.000086	29.96%	0.0%	0.0%	0.0397	0.0431	0.6830	0.5653	2.0059	0.3101	0.109	0.080	0.183	0.182	
1	0.000959	29.69%	0.0%	0.1%	0.0403	0.0436	0.6836	0.5663	2.0074	0.3100	0.109	0.080	0.183	0.182	
2	0.002120	29.32%	0.0%	0.2%	0.0412	0.0444	0.6844	0.5677	2.0052	0.3101	0.109	0.080	0.183	0.183	
3	0.003190	28.74%	0.0%	0.4%	0.0425	0.0455	0.6855	0.5697	2.0003	0.3104	0.109	0.080	0.183	0.182	
4	0.003520	27.70%	0.1%	0.9%	0.0448	0.0475	0.6872	0.5727	1.9828	0.3116	0.110	0.082	0.182	0.181	
5	0.001820	25.37%	0.6%	2.2%	0.0501	0.0518	0.6902	0.5787	1.9306	0.3152	0.112	0.084	0.180	0.178	
6	0.000000	18.63%	1.1%	4.5%	0.0669	0.0657	0.6999	0.5967	1.8031	0.3246	0.118	0.092	0.174	0.169	
7	0.000000	3.00%	23.1%	28.5%	0.0900	0.0879	0.6898	0.6063	1.4792	0.3533	0.146	0.120	0.198	0.194	
8	0	0.00%	36.4%	42.5%	0.1013	0.0993	0.6832	0.6089	1.2023	0.3861	0.162	0.136	0.213	0.209	
9	0	0.00%	40.7%	47.1%	0.1050	0.1030	0.6812	0.6097	1.1044	0.4004	0.168	0.141	0.218	0.214	
10	0	0.00%	42.0%	48.5%	0.1062	0.1042	0.6806	0.6100	1.0764	0.4049	0.169	0.143	0.220	0.215	
newss	0	0.00%	42.60%	49.09%	0.1065	0.1045	0.6803	0.6101	1.0646	0.4068	0.170	0.144	0.220	0.216	
Increasing in a New Steady State					168%	143%	-0.4%	7.9%	-47%	31%	57%	80%	20%	19%	

Table 6 provides an efficient debt financed transition for an economy with two types of householders, distinguished by the concavity of the preference function. At the beginning of the transition, when the after tax wage is lower as compared to an initial steady state, a government should take more care about a householder whose labor supply is more elastic with respect to wage, which means that ρ is larger. At this stage, representatives for the other type are better off, while later on, when the net of tax wage grows, the individual with larger ρ gains more.

3.2 Redistributive and insurance role of a pay-as-you-go system

When linkages between contributions and benefits are low, a pay-as-you-go system receives additional functions of redistributive and insurance mechanisms. Therefore, the simple abolishment of social security in this case can lead to a welfare loss for poor householders. This argument against pension reform is provided by Fenge and Schwager (1995), where the authors pay attention to the redistributive effect of a pay-as-you-go system across one generation. Similar arguments were used by other researchers until recently. Nevertheless, a Pareto-improvement is possible if a pay-as-you-go system is replaced by other mechanisms

maintaining a sufficient level of redistribution. Such replacement would maintain the utility welfare level of the poor and insure agents against an idiosyncratic productivity shock. However, an inefficient tax on pension contributions related to interest rate dominance over population growth would be eliminated. The best policy would be to tax the oldest and pay transfers to the youngest. In this way, the government would exploit the difference between the population growth rate and the rate on capital returns. If such a policy is not feasible for political reasons, the government can still achieve a welfare progress for everybody by intragenerational redistribution; i.e. a transition to an economy with a progressive wage income tax or a proportional payroll tax and lump-sum transfers within one generation.

4 Concluding summery

The purpose of this paper is to demonstrate that Pareto-improving transitions from pay-as-you-go to fully funded pension systems nearly always exist. Based on the review of existent literature as well as my own simulations, I find that such a transition is possible for all reasonable assumptions on reformed economies. I also discuss some details related to the execution of such transitions. In particular, the policy used in transition should be chosen depending on the economic characteristics of the initial pension system. Thus, for the closed economy saving subsidizing is more efficient when labor supply is exogenous, while a strengthening of the linkage between benefits and contributions is preferable when the coefficient of elasticity between leisure and consumption is large. The answer to the existence question depends drastically on the policy instruments chosen by the government.

My second finding is that intragenerational heterogeneity in productivity or preferences does not prevent the execution of a Pareto-improving transition to a fully funded system with redistribution within one generation.

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5 Appendix : Technical framework

I conduct my research by experimenting with listed transition methods and applying them to different types of economies. For this purpose, I use a two period overlapping generations model.

Formula (7) describes a householder's saving decision made under budget constraint (8).

$$\max_{c_y, c_o, l} U_t = \ln(c_{yt}) + \beta \ln(c_{ot+1}) + \gamma \ln(1 - l_t); \quad (7)$$

$$\begin{aligned} st. \quad &: c_{yt}(1 + tc_t) + \frac{c_{ot+1}(1 + tc_{t+1})}{(1 + r_{t+1} + \Phi_{t+1})} = \\ &= w_t l_t (1 - tax_t) + \frac{\Omega_{t+1}(1 - tax_t)w_t l_t + Pen_{t+1}}{1 + r_{t+1} + \Phi_{t+1}}. \end{aligned} \quad (8)$$

Here, c_{yt} , c_{ot} constitute the consumption of young and old households in period t , respectively, and l_t is labor. Terms r and w represent the interest rate and the wage. β and γ are test parameters; in particular, β is a time discount factor, while γ indicates the intensity of household preferences for leisure relative to consumption. I consider a combination of three types of pension benefit forms: lump-sum, proportional to contributions, and proportional to savings; using the following notation:

Pen_{t+1} – a lump-sum pension benefit;

Ω_{t+1} — the coefficient of return on after tax labor income;⁹

Φ_{t+1} — a savings subsidy rate.

Notations tax_t and tc_t represent payroll and consumption tax rates, which the government can set up to finance its expenditures.

The solution to problems (7), (8) is given by equations (9)-(13).

$$\frac{1}{\lambda} = \frac{w_t(1 - tax_t) [1 + r_{t+1} + \Phi_{t+1} + \Omega_{t+1}] + Pen_{t+1}}{(1 + \beta + \gamma) (1 + r_{t+1} + \Phi_{t+1})}, \quad (9)$$

$$c_{yt} = \frac{1}{\lambda(1 + tc_t)}, \quad (10)$$

$$c_{ot+1} = \frac{\beta(1 + r_{t+1} + \Phi_{t+1})}{\lambda(1 + tc_{t+1})}, \quad (11)$$

$$l_t = \frac{1 + \beta}{1 + \beta + \gamma} - \frac{\gamma Pen_{t+1}}{(1 + \beta + \gamma) [1 + r_{t+1} + \Phi_{t+1} + \Omega_{t+1}] w_t(1 - tax_t)}, \quad (12)$$

$$s_t = w_t(1 - tax_t)l_t - c_{yt}(1 + tc_t),$$

$$s_t = \frac{w_t(1 - tax_t)}{1 + \beta + \gamma} \left[\beta - \frac{\Omega_{t+1}}{(1 + r_{t+1} + \Phi_{t+1})} \right] - \frac{Pen_{t+1}}{(1 + \beta + \gamma)} \left[\frac{\gamma}{[1 + r_{t+1} + \Phi_{t+1} + \Omega_{t+1}]} - \frac{1}{(1 + r_{t+1} + \Phi_{t+1})} \right]. \quad (13)$$

Considering a closed economy, I assume that the production function has a Cobb-Douglas form (14), relating output to capital k_t , and labor l_t .

$$Y_t = k_t^\alpha l_t^{1-\alpha}. \quad (14)$$

Profit maximization by representative firms in the economy and competitiveness imply the following expressions for wage and capital price:

$$w_t = (1 - \alpha)k_t^\alpha l_t^{-\alpha}, \quad (15)$$

$$r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha}, \quad (16)$$

⁹ $\Omega_{t+1} = \Omega'_{t+1} \frac{tax}{1-tax}$, where Ω'_{t+1} is the coefficient of returns to contribution, which should be equal to GDP growth in steady state in a zero debt economy, or to 1 according to my simple specification. I choose this notation for simplicity.

where k_{t+1} is capital per worker in the next period.

For simplicity, I assume that the only government function is to support a pension system. Then capital per worker is defined as current workers' savings s_t minus government debt in formula (17) in the assumption of zero population growth. The government debt should be excluded from aggregate investment because it is used to finance the government expenditures which do not include investment in my simple framework.

$$k_{t+1} = s_t - debt_t, \quad (17)$$

where $debt_{t+1}$ equals government debt.

In an open economy, the wage and the interest rate are constant.

The government has the next budget constraint:

$$debt_t + T_t = P_t + (1 + r_t) debt_{t-1}, \quad (18)$$

$$T_t = tax_t w_t l_t + tc_t (c_{ot} + c_{yt}), \quad (19)$$

$$P_t = Pen_t + l_{t-1} w_{t-1} (1 - tax_{t-1}) \Omega_t + \Phi_t s_t, \quad (20)$$

where T_t and P_t represent total tax revenues and total pension payments to the old, respectively. Equations (7)- (20) provide a complete description of the simulated model.

In this framework, the government sets policy parameters ($Pen_t, \Omega_t, \Phi_t, tax_t, tc_t, debt_t$) in any time period t satisfying budget constraint (18). The Pareto-improving condition implies that for any time period t ,

$$U_t \geq U_{ss}, \quad (21)$$

where U_t is a lifetime utility of the individual born at period t , while U_{ss} is the utility level of the individual living in an initial steady state. This puts additional restrictions on government policy.

I use a variation of the solution method described in Auerbach and Kotlikoff (1987). Following these authors, I make the calculation in three stages: (1) solving for the initial steady state, (2) solving for the final steady state, (3) choosing the parameterization and solving for the transition. In stage (3), I choose policy parameters so that restriction (21)

is satisfied. I made my simulations assuming that in period 100, the economy has already converged to new steady states. This is more than enough; for comparison, the Auerbach and Kotlikoff (1987) simulation model provides the economy with 150 years to reach a new steady state in a fifty-five period OLG model.

For the welfare analysis, I use a wealth equivalent defined as the proportion by which a householder living in an initial steady state needs to increase his consumption in every period, in order to reach the life-time utility value of currently young households. Expression (22) formalizes the definition

$$U(c_{yt}, c_{ot+1}, l_t) = U(c_{yss} * (1 + wel_t), c_{oss} * (1 + wel_t), l_{ss}). \quad (22)$$

For logarithmic preferences, a welfare measure wel_t is calculated by formula (23)

$$wel_t = \exp\left(\frac{U_t - U_{ss}}{1 + \beta}\right) - 1, \quad (23)$$

where U_{ss} is equal to the householder's utility in the initial steady state.

5.0.1 Technical modifications for the simulation of a heterogeneous model with different pension funds but lump-sum benefits.

Here, I list the additional formulas needed to simulate a two-agent model.

The average efficient labor supply equals weighted average of efficient labor provided by all households

$$l_t = \sum q^i a^i l_t^i.$$

Here, a^i is the productivity level of an individual from group $i = 1, 2$; q^i is a proportion of the corresponding group; while l_t^i is the working time of the representative of group i . In other words, l_t^i is a solution of problems (7) and (8) where the wage is substituted by the efficient wage

$$\max_{c_{yt}^i, c_{ot+1}^i, l_t^i} U_t^i = \ln(c_{yt}^i) + \beta^i \ln(c_{ot+1}^i) + \gamma^i \ln(1 - l_t^i); \quad (24)$$

$$st. \quad : \quad c_{yt}^i + \frac{c_{ot+1}^i}{1 + r_{t+1}} = w_t a^i l_t^i (1 - tax_t) + \frac{Pen_{t+1}^i}{1 + r_{t+1}}. \quad (25)$$

The following equation defines savings

$$s_t^i = w_t a^i (1 - tax_t) l_t^i - c_{yt}^i.$$

The savings per efficient worker equal the weighted average savings

$$s_t = \sum q^i s_t^i.$$

The formula defined output, the wage per efficient unit, the interest rate and the capital supply (14)-(17) remain unchanged. Equations (19)-(20) should be slightly modified

$$T_t = \sum q^i tax_t w_t a^i l_t^i \quad (26)$$

$$P_t = \sum q^i Pen_t^i. \quad (27)$$

Further, I assume that society splits equally between two groups of individuals, or that $q^i = 1/2$.

A complete sensitive analysis has been done for all simulations provided in this section.

5.1 Transition from pay-as-you-go to "redistribution among young"

To consider the described policy, I must make some modifications. Thus, equation (25) is modified to (28)

$$st. : c_{yt}^i + \frac{c_{ot+1}^i}{1 + r_{t+1}} = w_t a^i l_t^i (1 - (tpen_t + tt_t)) + transfer_t^i + \frac{Pen_{t+1}^i}{1 + r_{t+1}}. \quad (28)$$

In a simulated economy, payroll taxes are used for three purposes. The government pays pensions to the old, transfers to the young and execute debt service. I assume that the government sets payroll taxes at the rate $tpen_t$ and issues new debt for expenditures related to the pension system and debt running, while tt_t is the share of payroll only used for transfers to the young.

Under these assumptions, equations (18)-(20) should be replaced by (29)-(31)

$$Pen_t^i = \frac{1}{n}(debt_t + Pen_Taxes_t - (1 + r_t) debt_{t-1}), \quad (29)$$

$$Pen_Taxes_t = \sum tpen_t w_t a^i l_t^i \quad (30)$$

$$transfer_t^i = \frac{1}{n} \sum tt_t w_t a^i l_t^i. \quad (31)$$

where n is a number of different income groups.

The equations for saving should also be modified

$$s_t^i = w_t a (1 - (tpen_t + tt_t)) l_t^i + transfer_t^i - c_{yt}^i.$$