Herd Behavior in Occupational Choice*

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Abstract

In this paper we examine why many professional labor markets are disturbed by cycles in the supply of new workers. We present a model where cycles in labor supply are a consequence of herd behavior in occupational choice. We also present evidence from nearly 150 West German labor markets which supports the herd behavior model of labor supply cycles.

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1 Introduction

In many professional labor markets, as for example that for engineers, the supply of new workers follows a cyclical time path: periods when the supply of labor is high in these markets alternate with periods when it is low (Freeman and Leonard 1978, Drost 2002). Cycles in labor supply are problematic because wages seldom are perfectly flexible; therefore labor supply cycles tend to translate into a cyclical sequence of surplus phases when many workers are unemployed and shortage phases when many jobs are vacant. Moreover, when this type of sequence occurs in a market that is essential for research and development, the repeated shortage of R&D workers leads to a slowdown in economic growth: the annual growth rate of the United States, for instance, is reduced by 0.2 to 0.6 percentage points in any year in which 500 thousand R&D jobs cannot be filled with qualified professionals (Romer 2000). Motivated by these problems, we will present a theoretical model in this paper that can explain labor supply cycles. We will also present empirical evidence which supports the model.

Our model differs from previous ones in the way it determines the supply of new workers available in a given professional labor market. In the model developed by Zarkin (1982, 1983, 1985) the supply of new workers depends on the wages workers expect to earn in this market, where wage expectations are assumed to be rational. Since there is evidence that even personnel executives in large firms tend to badly forecast future salaries (Leonard 1982), the assumption of rational wage expectations is quite doubtful, though. In the model developed by Freeman (1971, 1972, 1975a, 1975b, 1976a, 1976b) the supply of new workers depends on wage expectations as well, but this time expectations are assumed to be myopic and not rational. Even the assumption of myopic wage expectations could be too ambitious, however, because it seems that only a minority of students is able to accurately rank different occupations by starting salary (Betts 1996). In our model we therefore take the position that workers have only vague wage expectations, if any, and this lack of information induces them just to enter the labor markets they expect to be entered by many other workers (according to the maxim that it cannot be wrong what many others do). The supply of new workers thus is determined by some sort of herd behavior in our model, a type of behavior which is not unusual in low-information environments.

Before we go into the details, let us first give a short overview of our theoretical model and the empirical test on it. The model is based on two assumptions, the first of which is that the number of new workers entering a professional labor market in some year t is increasing with the number of new workers expected to enter this market in that year (assumption of herd behavior in occupational choice). The second assumption of the model is that the expectation about the number of new workers entering a market in t is derived by extrapolation from the number of new workers entering the market in t-1 and t-2 (assumption of extrapolative expectations about occupational choice). As these two assumptions

imply, the number of new workers entering a market in t depends on the number of new workers entering the market in t-1 and the number of new workers entering the market in t-2. The dynamics of the supply of new workers in a professional labor market thus is governed by a difference equation of order two, and as we will show in this paper, the time path solving this difference equation is determined by complex roots so that the path is characterized by cyclical ups and downs.

The empirical test of our model is based on three special properties of the difference equation: the coefficient of the lag-one term is larger than 0 and smaller than 2, the coefficient of the lag-two term is larger than -1 and smaller than 0, and the relation between the lag-one and the lag-two coefficient is equal to -2. To find out whether these properties can be validated empirically, we analyze time series data on the supply of new workers in nearly 150 West German professional labor markets. For each of these markets, we estimate an autoregressive process of order two, so obtaining estimates for the lag-one and the lag-two coefficient as well as for the relation between them. According to these estimates, there is indeed an overwhelming number of markets with cycles in the supply of new workers where the lag-one coefficient is larger than 0 and smaller than 2, the lag-two coefficient is larger than -1 and smaller than 0, and the relation between the two coefficients is in the neighborhood of -2. Some central predictions of the herd behavior model thus turn out to be correct, suggesting that we are on the right track when using the model to explain the cyclical behavior of worker supply observed in many professional labor markets. — In what follows, we will first present the theoretical model (section 2), then we will present the empirical evidence (section 3).

2 The Model

In our model we examine how the supply of new workers in a professional labor market $i \in \{1, 2, ..., N\}$ evolves over time $t \in \{1, 2, ...\}$, where any time period corresponds to one year. We denote the number of new workers who supply labor in market i at time t with $n_{i,t}$ and assume, for simplicity, that $n_{i,t}$ is a continuous variable. We think of this variable as being composed of a trend $n_{i,t}^*$ and a residual $n_{i,t}/n_{i,t}^*$: $n_{i,t} = n_{i,t}^* \cdot n_{i,t}/n_{i,t}^*$. The trend component $n_{i,t}^*$ is an exogenous variable and assumed to be a log-linear function of time. The residual component $n_{i,t}/n_{i,t}^*$ is endogenous and interpreted as the de-trended number of new workers entering market i at time t. The de-trended number of new workers is the variable we concentrate on in the rest of this paper. We want to explain why this number is cyclical over time and develop an explanation which is based on two assumptions.

The first assumption is that there is herd behavior in occupational choice. Because new workers have no or only very poor information about the pecuniary and nonpecuniary differences between occupations, they consider it best simply to imitate the occupational decisions of other new workers. Of course, when they make their choice of career, new workers do not know how the other ones will decide, and so all they can do is imitating the expected behavior of the others. The de-trended number of new workers entering market i at time t thus is an increasing function of the corresponding expected number $(n_{i,t}/n_{i,t}^*)^e$. Formally, the positive relation between these two numbers can be expressed as

$$\frac{n_{i,t}}{n_{i,t}^*} = h_i \left(\left(\frac{n_{i,t}}{n_{i,t}^*} \right)^e \right), \tag{1}$$

where h_i is a function of $(n_{i,t}/n_{i,t}^*)^e$ on $n_{i,t}/n_{i,t}^*$ with $h_i(1) = 1$ and $0 < h'_i < 1$. By imposing the condition $h_i(1) = 1$, we express that the de-trended number of new workers is one whenever corresponding expected number is; this condition is necessary to ensure that $n_{i,t}/n_{i,t}^*$ has a stationary state (see below). By requiring $h'_i > 0$, we indicate that the de-trended number of new workers is increasing with the expected number; this condition serves to introduce the herding effect into the model. By setting $h'_i < 1$, finally, we impose an upper bound to the herding effect; this upper bound is necessary to ensure that the stationary state of $n_{i,t}/n_{i,t}^*$ is stable (see below). Note that there is an index i being associated with the function h_i ; this index indicates that the relation between the actual and the expected detrended number of new workers need not be the same across professional labor markets — and it will not be the same, as the evidence presented in section 3 will show.

The second assumption our model is based on is that expectations about the de-trended number of new workers are derived by extrapolation from former detrended numbers. If new workers observe that the de-trended number of new workers entering market i was rising in the past, then they will expect that the rise goes on; if, however, they observe that it was falling, then they will expect the fall to be continued. Expectations about the de-trended number of new workers are therefore derived according to the equation

$$\left(\frac{n_{i,t}}{n_{i,t}^*}\right)^e = \frac{n_{i,t-1}}{n_{i,t-1}^*} \cdot \frac{n_{i,t-1}}{n_{i,t-1}^*} \div \frac{n_{i,t-2}}{n_{i,t-2}^*},$$
(2)

which states that the expected value of $n_{i,t}/n_{i,t}^*$ is equal to the past value of $n_{i,t}/n_{i,t}^*$ times the past growth factor of $n_{i,t}/n_{i,t}^*$. The reason why we model expectations this way is that other common ways to model expectations will not result in a cyclical pattern of $n_{i,t}/n_{i,t}^*$. Assume, for example, that expectations about the de-trended number of new workers are formed myopically so that $(n_{i,t}/n_{i,t}^*)^e = n_{i,t-1}/n_{i,t-1}^*$. Then combining this equation with equation (1) results in $n_{i,t}/n_{i,t}^* = h_i \left(n_{i,t-1}/n_{i,t-1}^*\right)$ with $h_i' > 0$, implying that the time path of $n_{i,t}/n_{i,t}^*$ is monotonic and not cyclical. Assume, as an alternative, that expectations about the de-trended number of new workers are formed rationally so that $(n_{i,t}/n_{i,t}^*)^e = n_{i,t}/n_{i,t}^*$. Then combining this equation with equation (1) yields

 $n_{i,t}/n_{i,t}^* = h_i \left(n_{i,t}/n_{i,t}^*\right)$, a static equation implying that the time path of $n_{i,t}/n_{i,t}^*$ does not exhibit any dynamics at all.

The assumptions of herd behavior expressed in equation (1) and of extrapolative expectations expressed in equation (2) are the only assumptions our model is based on. By taking them together, we obtain the reduced form of the model

$$\frac{n_{i,t}}{n_{i,t}^*} = h_i \left(\left(\frac{n_{i,t-1}}{n_{i,t-1}^*} \right)^2 \div \frac{n_{i,t-2}}{n_{i,t-2}^*} \right), \tag{3}$$

a nonlinear difference equation of order two, which determines the dynamic behavior of $n_{i,t}/n_{i,t}^*$. Before we start to examine the dynamics, however, let us first analyze the stationary state of $n_{i,t}/n_{i,t}^*$. The stationary-state value of $n_{i,t}/n_{i,t}^*$ follows from the fact that the number of new workers $n_{i,t}$ must be equal to its trend component $n_{i,t}^*$ in the stationary state; so the de-trended number of new workers must be equal to one in the stationary state, or $n_{i,t}/n_{i,t}^* = 1$. Note that the stationary state will exist if and only if $h_i(1^2 \div 1) = 1$; this is the reason why we have imposed the condition $h_i(1) = 1$ on the function of h_i .

We are now going to examine the dynamics of $n_{i,t}/n_{i,t}^*$, but because the difference equation in (3) is nonlinear, we restrict the analysis to the local dynamics in the neighborhood of the stationary state. To begin with, we log-linearize the difference equation in (3) around the stationary state, so obtaining

$$\log\left(\frac{n_{i,t}}{n_{i,t}^*}\right) = 2 \cdot h_i'(1) \cdot \log\left(\frac{n_{i,t-1}}{n_{i,t-1}^*}\right) - h_i'(1) \cdot \log\left(\frac{n_{i,t-2}}{n_{i,t-2}^*}\right),\tag{4}$$

where log denotes the natural logarithm. This equation is a linear difference equation of order two which determines the local dynamics of $\log(n_{i,t}/n_{i,t}^*)$, and because it is linear, we can analyze it by applying the usual methods. Before we will do this, we want to draw the reader's attention to three special features of this equation, however. Note first that, because we have assumed h'_i to satisfy the inequality $0 < h'_i < 1$ (otherwise there would be no herding or the model would be unstable, see below), the coefficient of the lag-one term of the equation, $2 \cdot h'_i(1)$, is larger than 0 and smaller than 2. Note second that, due to the same inequality, the coefficient of the lag-two term of the equation, $-h'_i(1)$, is larger than -1 and smaller than 0. Note finally that the relation between the lag-one coefficient $2 \cdot h'_i(1)$ and the lag-two coefficient $-h'_i(1)$ is equal to -2. If the model presented here is true, these three features of the difference equation should be observed in the majority of professional labor markets that exhibit cycles in the supply of new workers. We can therefore use the difference equation to evaluate the model empirically, which is exactly what we will do in the following section.

For the time being we stick to the theoretical analysis, however, and determine the time path of $\log(n_{i,t}/n_{i,t}^*)$ that solves the difference equation in (4). This time path is $\log(n_{i,t}/n_{i,t}^*) = c_1 \cdot r_1^t + c_2 \cdot r_2^t$, where $c_{1,2}$ are constants which can

be specified by initial conditions and $r_{1,2}$ are characteristic roots which can be derived from the characteristic equation $r^2 - 2 \cdot h_i'(1) \cdot r + h_i'(1) = 0$. As the solution of the characteristic equation, $r_{1,2} = h_i'(1) \pm \sqrt{h_i'(1) \cdot (h_i'(1) - 1)}$, shows, the time path of $\log(n_{i,t}/n_{i,t}^*)$ will be unstable if $h_i'(1) > 1$, a case that cannot be observed in practise; to avoid this problem, we have chosen $h_i'(1) < 1$ as one of the properties of h_i . With $h_i'(1) < 1$, the characteristic roots are complex conjugates, and this implies that we can apply De Moivre's theorem and some elementary trigonometry to the time path $\log(n_{i,t}/n_{i,t}^*) = c_1 \cdot r_1^t + c_2 \cdot r_2^t$, so being able to express it as

$$\log\left(\frac{n_{i,t}}{n_{i,t}^*}\right) = k_1 \cdot \left(\sqrt{h_i'(1)}\right)^t \cdot \cos\left(k_2 + \arccos\left(\sqrt{h_i'(1)}\right) \cdot t\right),\tag{5}$$

where $k_{1,2}$ are functions of the constants $c_{1,2}$. As we can see by expressing the time path in this way, there are cyclical fluctuations in the motion of $\log(n_{i,t}/n_{i,t}^*)$ and, thus, in the motion of $n_{i,t}/n_{i,t}^*$. The herd behavior model can therefore explain why the de-trended number of new workers is cyclical over time.

By having shown that our model is able to explain cycles in the supply of new workers, we have made our main point. One additional point is worth mentioning, however, and this point refers to the question how the intensity of herd behavior affects the cyclicity of labor supply. As equation (5) shows, the damping factor $1/\sqrt{h_i'(1)}$ of labor supply cycles is decreasing and the period $2 \cdot \pi / \arccos(\sqrt{h_i'(1)})$ of labor supply cycles is increasing with the intensity $h_i'(1)$ of the herding effect. Labor markets where the herding effect is strong are therefore characterized by strong supply cycles — and, thus, by serious employment and growth problems —, while labor markets where the herding effect is weak do only exhibit weak supply cycles. Due to this linkage between the intensity of herd behavior on the one hand and the cyclicity of labor supply on the other hand, we should examine whether and, if yes, why herd behavior is more intense in some and less intense in other markets. This problem will be tackled in another paper, however; in this paper we will continue with an empirical evaluation of the model, instead.

3 The Evidence

As we have indicated above, we will evaluate the herd behavior model of labor supply cycles by empirically checking whether the lag-one coefficient, the lag-two coefficient, and the ratio of the two lag coefficients from the difference equation in (4) are indeed element of (0,2), (-1,0), and $\{-2\}$. In order to carry out this check, we will run an OLS regression and estimate the autoregressive process

$$\log\left(\frac{n_{i,t}}{n_{i,t}^*}\right) = a_i \cdot \log\left(\frac{n_{i,t-1}}{n_{i,t-1}^*}\right) + b_i \cdot \log\left(\frac{n_{i,t-2}}{n_{i,t-2}^*}\right) + u_{i,t}$$
 (6)

for a large number of professional labor markets i, so obtaining estimates for the lag-one coefficient a_i , the lag-two coefficient b_i , and their ratio a_i/b_i , with $u_{i,t}$ being a stochastic error term satisfying the conditions of the classical linear regression model. Having obtained these estimates, we will examine in how many markets where the characteristic roots $a_i/2 \pm \sqrt{a_i^2/4 + b_i}$ of the autoregressive process are complex so that the time path of $\log(n_{i,t}/n_{i,t}^*)$ is cyclical the lag-one coefficient a_i is situated in the interval (0,2), the lag-two coefficient b_i is element of the interval (-1,0), and the coefficient ratio a_i/b_i is found in a small interval around the number -2, say (-3,-1). If the number of markets characterized by these properties turns out to be large, we will take this result as evidence supporting the model presented in the last section.

To estimate the autoregressive process specified in equation (6) for a large number of professional labor markets, we need time-series cross-section data on the logarithmic de-trended number of new workers $\log(n_{i,t}/n_{i,t}^*)$ and, hence, on the number of new workers $n_{i,t}$ the logarithmic de-trended number is derived from. Before we can examine where to obtain these data, we have to answer the question which is the optimal way of measuring the number of new workers in the context of our model. Because the model focuses on occupational choice, which is more or less equivalent to educational choice in the case of professional occupations, it appears to be best to measure the number of workers newly entering a labor market by the number of freshmen newly enrolling in the field of study corresponding to the market. Time-series cross-section data on this number can be obtained from the Federal Statistical Office Germany: though this institution does not provide very detailed data on the enrollment of freshmen in its publications, it is ready to provide an exhaustive data set on freshmen enrollment on request. The data set that was provided to us consists of semiannual time series on the number of freshmen enrolling at German universities for 278 fields of study, with the longest series covering the period between summer term 1972 and winter term 2000/2001.

Due to various statistical problems, we cannot use the time series provided by the Federal Statistical Office directly in our empirical analysis. A first problem is that the time series refer to enrollment at West German universities until summer term 1992 and to enrollment at West as well as East German universities afterwards; as a consequence, we can only use the sub-series ending in summer term 1992 in the empirical analysis. A second problem is that winter enrollment is much larger than summer enrollment in most fields so that there are considerable seasonal fluctuations in the series provided by the Statistical Office; we therefore have to convert the sub-series ending in summer term 1992 from a semiannual to an annual frequency before they can be analyzed empirically. As a third problem, there are several among the converted sub-series that do not cover the whole period between 1972 and 1991; these series have to be omitted from the empirical analysis because they are too short for a reliable estimation of the lag-one coefficient a_i and the lag-two coefficient b_i . A fourth problem, finally, is

Table 1: Data I

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	Field	i	
	Shop/technology Mining	75 76	
	Mining Smelting and foundry engineering		Philosophy Religion
	Mine surveying		Archeology
	Chemical engineering		History
	Printing and reproduction technology		Library science
	Power engineering (excluding electrical engineering)		Media studies and communication science
	Light engineering		General literary studies
	Industrial and production engineering	83	General linguistics and Indogermanic languages and
	Medical technology	0.4	literature
	Glass and ceramics technology Plastics technology		Byzantine studies Greek
	Mechanical engineering		Latin
	Metallurgy		German language and literature
15	Engineering physics		Dutch language and literature
16	Textiles and clothing technology	89	Nordic/Scandinavian languages and literatures (Nordic
	Process engineering		philology, individual languages n.o.i.b.n.)
	Materials sciences		American studies/literature
	Electrical and electronic engineering		English language and literature French language and literature
	Vehicular engineering Aerospace engineering		Italian language and literature
	Navigation		Portuguese language and literature
	Naval architecture		Romance languages and literatures (Romance philology,
	Architecture		individual languages n.o.i.b.n.)
25	Interior decorating	96	Spanish language and literature
	Regional planning		Finno-Ugric languages and literatures
	Civil engineering		Russian language and literature
	Surveying (geodesy)		Slavic languages and literatures (Slavic philology)
	Mathematics Computer science		Egyptology African languages and literatures
	Astronomy and astrophysics		Arabic language and literature
	Physics		Non-European languages and cultures in Southeast
33	Biochemistry		Asia, Oceania, and the Americas
34	Chemistry	104	Hebrew language and literature
	Food chemistry		Indology
	Pharmacy		Islamic studies
	Anthropology (human biology)		Japanology
	Biology Geology/paleontology		Oriental and Old Oriental language and literature Chinese and Korean languages and literatures
	Geophysics		Central Asian languages and cultures
	Meteorology		Ethnology
42	Mineralogy	112	Folklore
	Oceanography		Psychology
	Geography		Education
	Economic and social geography		Education of the blind and visually handicapped
	Medicine (general medicine) Dental medicine		Education of pupils with learning disorders Education of the deaf and hard-of-hearing
	Veterinary medicine		Education of the dear and nard-of-nearing Education of the mentally handicapped and slow
	Landscape architecture	110	learners
	Agriculture	119	Education of the physically handicapped
51	Brewing and beverage technology	120	Education of pupils with learning disabilities
	Horticulture		Special education
	Food technology		Speech pathology
	Dairy farming Vitigulture and vinting		Education of the emotionally disturbed
	Viticulture and vinting Silviculture		Sports and physical education Art education
	Wood industry		Art and art history
	Domestic science and nutrition		Fine and graphic arts
	Elementary school social studies education		Sculpture
	Political science	129	Painting
	Social studies		Commercial art
62	Social sciences	131	Graphic design and communication design
63	Sociology	132	Textile design
64 65	Social work Social-welfare education		Handicrafts Performing and dramatic arts, directing
66	Law	134	Motion pictures and television
	Public administration		Dramaturgy
	Labor and business studies	137	92
69	Business administration	138	Instrumental music
	Economics	139	
	Economics education		Composition
72	Economic sciences		Music education
	Industrial engineering Protestant theology and religious studies		Music studies and history
74	rocestant theology and religious studies	143	Sound engineer

n.o.i.b.n. = not otherwise indexed by name (in the original list received by the Federal Statistical Office Germany)

Table 2: Data II

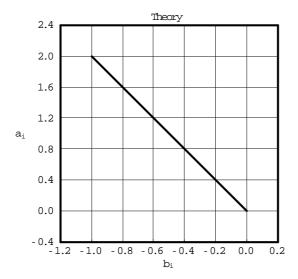
_	1050	1050	1054	1055	1050	1055	1050	1050	1000	1001	1000	1000	1004	1005	1000	1005	1000	1000	1000	1001
$\frac{i}{1}$	1972 51	1973 48	1974 59	1975 75	1976 46	1977 42	1978 35	1979 34	1980 25	1981	1982		1984	1985	1986 8	1987 5	1988	1989	1990	1991 74
2	66	50	125	236	278	370	321	345	362	395	405	506	385	287	244	182	120	91	176	125
3 4	99	119 1	159 3	239 21	304 14	234 26	$\frac{216}{28}$	198 25	$\frac{241}{20}$	319 10	320 20	454 15	375 11	288 11	316 7	261 8	305 8	315 2	279 3	$\frac{222}{4}$
5	1241	1277	911	1026	1153	1003	599	455	513	692	1105	1375	1062	1170	1204	1349	1391	1512	1643	1618
6 7	102 87	$\frac{117}{128}$	94 132	125 160	93 186	$\frac{94}{221}$	$\frac{132}{250}$	$\frac{192}{357}$	$\frac{152}{255}$	$\frac{132}{244}$	189 193	273 268	243 318	232 299	261 308	313 358	330 388	$\frac{292}{425}$	316 436	$\frac{314}{468}$
8	260	326	554	666	709	590	494	527	639	692	912		958		1019	1061	1086	1158	1084	1131
9 10	$\frac{58}{165}$	88	51 135	276 89	344 78	431 77	$\frac{423}{127}$	464	$\frac{865}{158}$	$\frac{1149}{230}$	$\frac{1418}{224}$		$\frac{1607}{248}$	1457	$\frac{1624}{289}$	1677	1696	$\frac{1865}{287}$	1907	$\frac{1906}{358}$
11	38	163 49	69	74	89	83	57	$\frac{144}{46}$	25	38	45	245 53	48	274 48	209 54	325 76	318 79	62	328 87	68
12	54	66	76	97	108	88	83	84	65	95	108		149	180	191	223	270	248	234	205
13 14	7278 82	7851 47	8310 55	9268 61	10571 55	9896 53	$9395 \\ 74$	8978 75	8738 69	43	13701 30	$14500 \\ 34$	$\frac{12715}{37}$	11787	12263 15	13788	14542 45	16407 50	17178 59	$\frac{15505}{74}$
15	244	401	378	413	414	410	396	361	354	384	489	587	529	517	488	616	808	867	879	857
16 17	$\frac{277}{687}$	279 739	$\frac{220}{645}$	325 903	$\frac{367}{1098}$	342 874	378 1096	346 969	373 849	477 1213	508 1313	677 1380	563 1310	647 1188	$\frac{582}{1203}$	616 1280	614 1406	$605 \\ 1727$	603 1754	$\frac{593}{1665}$
18	123	79	75	137	190	149	139	167	243	279	387	407	406	365	445	492	580	614	642	506
19 20	$8428 \\ 168$	9511 205	$9962 \\ 177$	$10140 \\ 221$	$\frac{11552}{223}$	$10147 \\ 237$	$9313 \\ 275$	8947 335	8440 344	$10859 \\ 375$	13394 463	15150	$13361 \\ 482$	$12379 \\ 487$	13063 509	$14525 \\ 548$	15980 661	16831 733	15494 685	$\frac{14064}{758}$
21	120	202	219	253	265	219	328	539	482	562	564	$625 \\ 639$	640	646	654	640	894	984	839	748
22	187	129	121	115	141	136	204	187	193	143	123	126	94	91	$\frac{114}{224}$	87	77	70	34	84
23 24	$\frac{135}{3197}$	$176 \\ 4171$	$\frac{150}{4069}$	193 3661	$\frac{249}{3808}$	$\frac{224}{3929}$	$\frac{217}{3960}$	$\frac{194}{4608}$	$\frac{211}{5187}$	$\frac{182}{5500}$	$179 \\ 5344$	$\frac{134}{5057}$	$\frac{196}{5289}$	$\frac{224}{5045}$	$\frac{224}{4759}$	$\frac{227}{4859}$	$\frac{242}{5228}$	$\frac{236}{5416}$	$\frac{229}{5519}$	$\frac{295}{5092}$
25	124	229	290	269	348	366	248	353	630	780	849	741	855	736	749	786	756	689	630	680
$\frac{26}{27}$	$\frac{21}{4738}$	75 5079	$101 \\ 4852$	$74 \\ 4797$	$\frac{115}{5439}$	$\frac{145}{4954}$	$\frac{236}{4379}$	$\frac{235}{4557}$	$\frac{216}{4654}$	$\frac{242}{5767}$	$\frac{226}{6119}$	$\frac{198}{7276}$	$\frac{243}{6624}$	$\frac{247}{4910}$	$\frac{249}{4095}$	$\frac{258}{4298}$	$\frac{276}{4759}$	$285 \\ 6122$	$272 \\ 8218$	$\frac{277}{9511}$
28	764	1030	1082	1232		1225	1050		1174	1171	1324	1391	1248	1022	857	715	744	786	802	930
29 30	$10255 \\ 741$	$9666 \\ 1051$	$9960 \\ 1147$	7928 1515		5730 1644	5078 2060			$4863 \\ 4051$		5213 6062	$4454 \\ 6264$		$4180 \\ 6288$	4821 6816	5847 8207	6501 8453	7814 8475	$7139 \\ 7571$
31	35	22	19	29	32	24	11	16	16	13	26	15	12	17	16	12	11	8	14	9
32 33	3556 65	3357 52	$3528 \\ 52$	3443 52	3633 66	3175 76	3146 101	3012 109	3432 98	4156 98	5094 92		5661 105	$\frac{5167}{125}$	5150 133	5623 137	5707 147	5739 150	6354 157	$\frac{5470}{179}$
34	4343	4290	4430	5011	4972		4727	4077			7827		7565	6517	6199	6351		7134	7494	6655
35	413	$\frac{264}{1010}$	$\frac{143}{1038}$	126	107	126	208	159	211	240	240		252	244	229	251	$\frac{255}{1792}$	276	$\frac{264}{1724}$	238
36 37	$\frac{1179}{12}$	1010	1038	1190	$\frac{1210}{24}$	1458	1662 6	1734	48	62	61	1832 45	1895	1862	1889 69	1930 67	67	64	46	$\frac{1657}{56}$
38	3592	3556	3700	3242		4236	5002		5093				4916	4874	4879	5026	5117	5103	5385	5111
39 40	628 78	695 84	$842 \\ 122$	1625 323	$\frac{749}{213}$	$\frac{762}{183}$	$723 \\ 165$	779 139	1012 138	$\frac{1173}{179}$	$\frac{1070}{239}$	$\frac{967}{328}$	808 319	$\frac{757}{275}$	$751 \\ 235$	857 188	810 207	909 178	$\frac{1038}{250}$	$\frac{971}{221}$
41	167	216	301	497	179	171	158	105	125	167	178	254	242	187	150	150	175	219	269	228
42 43	236 61	290 55	293 56	517 145	336 58	285 48	290 61	$\frac{274}{34}$	$\frac{296}{41}$	$453 \\ 64$	$\frac{672}{52}$	624 46	696 55	455 60	366 56	297 73	301 72	331 78	373 59	342 66
44	1604	1605	1908	1440	1385	1580	1877	1979	2301	2322	2560		1973	1785	1813	2151	2486	2875	3368	3215
45 46	$\frac{14}{3721}$	$\frac{12}{3939}$	$\frac{16}{3867}$	$\frac{6}{4241}$	$\frac{11}{5252}$	$\frac{17}{6515}$	$\frac{23}{7407}$	$\frac{7}{8023}$	9 8919	6 8455	$\frac{23}{7651}$	38 7355	$\frac{29}{7624}$	35 7838	$71 \\ 8250$	81 9254	$\frac{79}{9422}$	$105 \\ 9254$	97 8179	$\frac{123}{7327}$
47	615	677	697	735	807	950	1140	1191		1234			1212		1345	1457	1568	1534	1513	1445
48	485	461	456	498	508	653	663	757	914	742	744		693	684	657	725	801	778	841	848
49 50	$\frac{375}{1030}$	$438 \\ 1386$	$\frac{407}{1651}$	$\frac{487}{2078}$	$\frac{416}{1919}$	$\frac{495}{2027}$	$\frac{594}{2275}$	$628 \\ 2341$	663 2430	$679 \\ 2414$	694 2366	$\frac{665}{2276}$	$661 \\ 2444$	$704 \\ 2347$	684 2292	$717 \\ 2105$	709 1904	$719 \\ 1924$	$812 \\ 1884$	$805 \\ 1798$
51	84	82	115	141	137	130	118	111	130	112	126	124	118	125	119	128	146	166	195	207
52 53	$\frac{163}{174}$	$\frac{194}{249}$	$\frac{300}{268}$	$\frac{300}{243}$	$\frac{306}{221}$	$\frac{353}{198}$	$\frac{341}{261}$	$\frac{392}{206}$	$\frac{423}{211}$	$\frac{431}{208}$	$\frac{437}{291}$	$\frac{451}{416}$	420 383	536 343	526 388	$\frac{515}{426}$	533 602	479 640	$\frac{451}{620}$	$\frac{446}{571}$
54	14	18	33	28	31	33	31	30	31	31	30	32	29	24	36	27	26	29	34	27
55 56	$\frac{23}{172}$	$\frac{17}{360}$	$\frac{32}{450}$	$\frac{45}{363}$	$\frac{56}{343}$	68 393	$\frac{56}{425}$	$\frac{52}{480}$	63 460	$\frac{65}{575}$	65 569	85 598	54 623	84 706	80 559	83 489	97 634	$\frac{129}{552}$	97 554	$\frac{108}{635}$
57	98	130	125	222	233	247	34	41	37	35	30	37	30	25	39	32	32	36	36	37
58 59	$\frac{1472}{47}$	1477 86	1502 80	1663 65	1612 60	1639 42	1653 43	$\frac{1592}{34}$	1476 33	$\frac{1583}{257}$	$1562 \\ 156$	$1654 \\ 135$	1524 156	1511 138	1408 151	$\frac{1382}{278}$	$\frac{1477}{409}$	1321 589	1282 920	$1332 \\ 604$
60	730	775	784	748	829	774	810	847	1138	$\frac{257}{1450}$	1585		1739		1731	2011	2062	2278	$\frac{920}{2647}$	2612
61	434	362	471	339	209	178	246	264	312	288	202		82	58	71	125	197	239	395	281
62 63	$819 \\ 1543$	$853 \\ 1581$	$1088 \\ 1494$		$\frac{1113}{1430}$							$1961 \\ 1232$								
64	6549	6927	7104	6691	6501	6945	6277	5652	3814	3749	3741	3347	2651	2496	2217	2124	2239	2319	2471	3111
65 66	$\frac{163}{7331}$	253 8756		339 10284	$403 \\ 9783$							2629 13263								
67	2	1614	971	1241	2797	3533	4118	5931	8090	4079	3659	3162	3224	3648	3855	4501	4613	4838	4179	4917
68 69	264 5935	193 5164			$\frac{213}{6533}$									127 15931					31 23445	64
	2308			1918	3021		2028	2148	3134	4060	4915	3789	3313	3330	3454	3622	3698	4458	3580	5125
	639	811								619				375		632				
72	5149	5889	5403	5577	6754	6157	6703	6092	7327	5140	9164	9994	9420	8799	9951	11708	12524	10229	13203	11690

Table 2: Data II (continued)

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115 14 16 46 16 15 18 32 40 29 38 26 14 19 17 13 21 28 22 30 39 116 72 147 263 70 21 23 22 48 52 71 75 77 82 67 42 25 64 82 98 75 113 118 56 139 221 125 91 140 330 317 354 362 425 349 301 176 186 201 238 257 309 308 119 80 134 224 103 33 74 357 314 343 170 176 185 161 162 178 180 121 141 107 217 176 137 112 160 167 21 328 152 129 193 238																					
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Table 3: Results I

		b_i	a_i/b_i	$a_i^2/4 + b_i$	i		b_i	a_i/b_i	$a_i^2/4 + b_i$
$\frac{\iota}{1}$	$\frac{a_i}{0.9320}$	-0.1418	$\frac{a_i/b_i}{-6.5747}$	$\frac{a_i/4 + b_i}{0.0754}$	73	$\frac{a_i}{1.1001}$	-0.4061	$\frac{a_i/b_i}{-2.7090}$	$\frac{a_i/4 + b_i}{-0.1035}$
2	0.9141	-0.2412	-3.7903	-0.0323	74	1.1487	-0.4180	-2.7480	-0.0881
3	0.9050	-0.3825	-2.3661	-0.1777	75	1.0057	-0.3699	-2.7188	-0.1171
4	0.6683	-0.1471	-4.5449	-0.0354	76	0.6564	-0.4007	-1.6380	-0.2930
5	1.1075	-0.5338	-2.0748	-0.2271	77	0.3718	0.1515	2.4539	0.1860
6	0.2722	-0.4546	-0.5987	-0.4361	78	0.9617	-0.3488	-2.7576	-0.1175
7	0.5559	-0.0534	-10.4157	0.0239	79	0.3149	0.0832	3.7862	0.1080
8	0.9403	-0.6512	-1.4441	-0.4301	80	0.6198	-0.0529	-11.7222	0.0432
9 10	0.5357 1.0031	$0.1622 \\ -0.4716$	$3.3020 \\ -2.1267$	$0.2340 \\ -0.2201$	81 82	$0.4571 \\ 0.6923$	$-0.3098 \\ -0.2907$	$-1.4754 \\ -2.3816$	$-0.2576 \\ -0.1709$
11	0.8140	-0.4716 -0.2600	-2.1267 -3.1303	-0.2201 -0.0944	83	0.0925 0.9259	-0.2907 -0.3857	-2.3810 -2.4007	-0.1709 -0.1714
12	0.9284	-0.3225	-2.8789	-0.1070	84	0.1471	0.1958	0.7511	0.2012
13	0.9300	-0.8178	-1.1371	-0.6016	85	0.7250	-0.1961	-3.6970	-0.0647
14	1.0657	-0.4464	-2.3872	-0.1625	86	0.9665	-0.3347	-2.8874	-0.1012
15	0.8168	-0.3000	-2.7227	-0.1332	87	1.0163	-0.3933	-2.5841	-0.1351
16	0.3995	0.0955	4.1831	0.1354	88	0.6097	-0.1911	-3.1910	-0.0981
17	0.1922	-0.2720	-0.7068	-0.2627	89	0.4182	0.1773	2.3591	0.2210
18 19	0.6625	-0.5602	-1.1826	-0.4505	90	1.0625	-0.3254	-3.2654	-0.0432
20	0.9839 0.3872	-0.6832 -0.0913	-1.4403 -4.2410	-0.4411 -0.0538	91 92	1.0829 1.2904	$-0.2370 \\ -0.5075$	$-4.5701 \\ -2.5429$	$0.0562 \\ -0.0912$
21	0.6575	-0.0913 -0.1834	-3.5845	-0.0338 -0.0754	93	1.1816	-0.3239	-3.6480	0.0251
22	0.2662	0.2240	1.1883	0.2417	94	0.5297	-0.0217	-24.3997	0.0484
23	0.4417	0.0051	86.0516	0.0539	95	1.3417	-0.4746	-2.8271	-0.0246
24	0.8583	-0.2466	-3.4802	-0.0625	96	0.9287	-0.1717	-5.4090	0.0439
25	0.9397	-0.2744	-3.4251	-0.0536	97	0.2445	-0.1564	-1.5631	-0.1415
26	0.8621	-0.0944	-9.1312	0.0914	98	0.8552	-0.3989	-2.1437	-0.2161
27	1.2667	-0.8495	-1.4911	-0.4484	99	1.0006	-0.4995	-2.0030	-0.2493
28 29	1.1900	-0.4753 -0.5248	$-2.5035 \\ -2.4955$	-0.1213	100 101	0.2555	0.1109	2.3041	0.1272
30	1.3097 1.2706	-0.3248 -0.3625	-2.4955 -3.5051	$-0.0960 \\ 0.0411$	101	$0.8856 \\ 0.3132$	$-0.3612 \\ 0.0100$	-2.4517 31.3040	$-0.1652 \\ 0.0345$
31	-0.0595	-0.3742	0.1590	-0.3733	103	0.7115	-0.1562	-4.5548	-0.0296
32	0.9969	-0.4278	-2.3302	-0.1794	104	0.0643	-0.1021	-0.6302	-0.1010
33	0.6777	-0.2286	-2.9642	-0.1138	105	0.4719	-0.0951	-4.9644	-0.0394
34	1.0368	-0.5784	-1.7926	-0.3096	106	0.4973	-0.2059	-2.4147	-0.1441
35	0.7675	-0.3449	-2.2252	-0.1977	107	0.6755	0.0846	7.9839	0.1987
36	1.1889	-0.3783	-3.1426	-0.0249	108	0.6115	-0.3674	-1.6645	-0.2739
37	0.1980	-0.3227	-0.6138	-0.3128	109	1.3271	-0.6388	-2.0776	-0.1985
38 39	$0.9082 \\ 0.2332$	-0.3991 -0.2347	-2.2753 -0.9940	$-0.1929 \\ -0.2211$	$\frac{110}{111}$	$0.9640 \\ 1.2711$	$-0.5045 \\ -0.4968$	$-1.9105 \\ -2.5586$	$-0.2722 \\ -0.0929$
40	0.6685	-0.3560	-1.8776	-0.2443	112	0.8454	-0.1589	-5.3205	0.0198
41	0.6618	-0.3121	-2.1208	-0.2026	113	1.3292	-0.5646	-2.3543	-0.1229
42	0.8237	-0.2662	-3.0945	-0.0966	114	1.0573	-0.6047	-1.7486	-0.3252
43	0.1437	-0.0717	-2.0050	-0.0665	115	0.3067	-0.1817	-1.6880	-0.1582
44	0.9529	-0.4895	-1.9467	-0.2625	116	0.6625	-0.4602	-1.4397	-0.3504
45	0.4308	-0.0385	-11.1854	0.0079	117	0.4815	-0.2442	-1.9721	-0.1862
46	1.6086	-0.8281	-1.9425	-0.1812	118	0.9013	-0.3626	-2.4856	-0.1595
47	1.4733	-0.6813	-2.1624	-0.1387	119	0.7876	-0.5031	-1.5655	-0.3480
48 49	0.7072 0.3966	-0.0415 0.2939	-17.0338 1.3497	0.0835 0.3332	$\frac{120}{121}$	$0.6440 \\ 0.4609$	$-0.4462 \\ -0.5663$	$-1.4432 \\ -0.8139$	$-0.3425 \\ -0.5132$
50	1.2067	-0.3578	-3.3726	0.3332 0.0062	121	0.4609 0.6578	-0.3663 -0.3741	-0.8139 -1.7586	-0.3132 -0.2659
51	0.8591	-0.3650	-2.3539	-0.1805	123	0.3886	-0.3685	-1.0546	-0.3307
52	0.6872	-0.1926	-3.5690	-0.0745	124	1.3149	-0.6295	-2.0887	-0.1973
53	0.7694	-0.3221	-2.3884	-0.1741	125	0.4065	-0.0500	-8.1303	-0.0087
54	0.0865	-0.0829	-1.0441	-0.0810	126	0.5678	-0.2438	-2.3288	-0.1632
55	0.2615	-0.0454	-5.7582	-0.0283	127	0.9439	-0.5881	-1.6049	-0.3654
56	0.5985	-0.2024	-2.9575	-0.1128	128	0.5932	-0.3795	-1.5632	-0.2915
57 58	0.6093 0.4313	$-0.0791 \\ 0.0113$	-7.7060 38.1864	$0.0137 \\ 0.0578$	129 130	$0.1527 \\ 0.9427$	$-0.1230 \\ -0.1675$	$-1.2415 \\ -5.6285$	$-0.1172 \\ 0.0547$
58 59	0.4313 0.5385	-0.0113 -0.1523	-3.5367	-0.0578	131	0.9427 0.6557	-0.1675 -0.1657	-3.6285 -3.9557	-0.0547 -0.0583
60	0.9281	-0.1323 -0.4294	-3.3367 -2.1614	-0.0798 -0.2141	132	0.4023	-0.1637 -0.0634	-6.3459	-0.0229
61	1.3071	-0.6911	-1.8913	-0.2640	133	1.0817	-0.3743	-2.8902	-0.0818
62	0.5215	0.0259	20.1202	0.0939	134	0.0649	-0.1817	-0.3571	-0.1806
63	1.3430	-0.7524	-1.7850	-0.3015	135	0.7306	-0.4554	-1.6045	-0.3219
64	1.2269	-0.4939	-2.4842	-0.1176	136	0.3443	0.1009	3.4129	0.1305
65	0.9493	-0.2674	-3.5498	-0.0421	137	0.1911	-0.0832	-2.2967	-0.0741
66	1.1808	-0.6173	-1.9128	-0.2687	138	0.1449	0.0302	4.7948	0.0355
67	0.8596	0.0799	10.7651	$0.2646 \\ -0.0284$	139	0.4444	0.1551	2.8644	0.2045
68 69	$-0.0949 \\ 0.5286$	$-0.0307 \\ 0.0250$	3.0906 21.1191	-0.0284 0.0949	$\frac{140}{141}$	$0.2546 \\ 1.0664$	$0.1243 \\ -0.4082$	$2.0480 \\ -2.6121$	$0.1405 \\ -0.1240$
70	0.3280 0.4675	-0.3332	-1.4032	-0.2785	142	0.9282	-0.4082 -0.3067	-3.0263	-0.0913
71	0.9171	-0.2619	-3.5023	-0.0516	143	-0.2902	-0.0992	2.9243	-0.0782
72	-0.1596	-0.0553	2.8863	-0.0489					



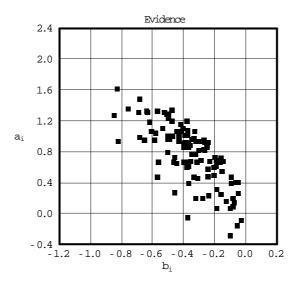


Figure 1: Results II

that some of the remaining series include zero enrollment numbers in one year or other; because computing $\log(n_{i,t}/n_{i,t}^*)$ from $n_{i,t}$ is only possible when data are nonzero, we are forced to exclude these series from the empirical analysis as well. After processing the data in this way, we are left with a sample which is documented in tables 1 and 2: it consists of time series for 143 fields of study, with each series being composed of annual data ranging from 1972 to 1991.

When we transform the data for $n_{i,t}$ included in our sample into logarithms $\log(n_{i,t})$, remove a linear least-squares trend $\log(n_{i,t}^*)$, and use the resulting data for $\log(n_{i,t}/n_{i,t}^*) = \log(n_{i,t}) - \log(n_{i,t}^*)$ to estimate the autoregressive process specifield in equation (6) for each field, we obtain the results documented in table 3. As the table shows, there are 108 out of 143 fields where $a_i^2/4 + b_i$ is negative so that $a_i/2 \pm \sqrt{a_i^2/4 + b_i}$ is complex and the time path of $\log(n_{i,t}/n_{i,t}^*)$ is cyclical. In 104 and, thus, in 96 percent of these fields the lag-one coefficient a_i is situated in the theoretically predicted interval (0,2); this is an empirical result which strongly supports the herd behavior model of labor supply cycles. In 108 and, thus, in 100 percent of the fields the lag-two coefficient b_i is element of the theoretically predicted interval (-1,0), a result that provides even stronger support for the herd behavior model. In 74 and, thus, in 69 percent of the fields the coefficient ratio a_i/b_i is found in the interval (-3,-1) and, hence, in some neighborhood of the theoretically predicted value -2, another result that supports the model to be evaluated. All in all, we find that the empirical results documented in table 3 confirm the theoretical predictions of the herd behavior model, a finding that becomes even more evident when we regard the two diagrams which are depicted in figure 1.

The upper diagram depicted in the figure shows those combinations of a_i and b_i which conform to the theoretical predictions of the herd behavior model: because $a_i \in (0,2)$, $b_i \in (-1,0)$, and $a_i/b_i = -2$, they all lie on a straight line joining the points (0,0) and (-1,2). The lower diagram shows the combinations of a_i and b_i which conform to the empirical results listed in table 3: since the herd behavior model can only be applied to fields where the motion of $\log(n_{i,t}/n_{i,t}^*)$ is cyclical, we have omitted all combinations that refer to non-cyclical fields from the diagram. As we can see by comparing the diagrams, most of the combinations observed empirically are in the close neighborhood of the combinations predicted theoretically; while the theoretical combinations are located on the line $a_i = -2 \cdot b_i$ (for $-1 < b_i < 0$), the empirical combinations are located near the OLS regression line $a_i = -1.9931 \cdot b_i$ (with $R^2 = 0.4794$), which is almost identical. Given this coincidence of the empirical and theoretical results, we are confident to be on the right track when using the herd behavior model to explain the cyclical behavior of worker supply observed in many professional labor markets.

We have now reached the end of this section, but before finishing it, we will briefly return to a point already discussed at the end of section 2: the relationship between the intensity of herd behavior on the one hand and the cyclicity of labor supply on the other hand. Returning to this point is worthwhile because we

have numbers for the intensity of herding effects now; since $h'_i(1) = a_i/2$ —this equation follows from equations (4) and (6)—, the results in table 3 imply that herding effects range between 0.0322 in Hebrew language and literature and 0.8043 in general medicine. When we insert these numbers into the formulas for the damping factor and the period of labor supply cycles, $1/\sqrt{h'_i(1)}$ and $2 \cdot \pi/\arccos(\sqrt{h'_i(1)})$, we find that the cycles arising in Hebrew have a damping factor of 5.5762 and a period of 4.5187 years, whereas the damping factor and the period of the cycles arising in medicine are 1.1151 and 13.7106 years. In the field with the weakest herding effect, Hebrew, cycles are therefore extremely weak and should not give rise to any remarkable problems; in the field with the strongest herding effect, medicine, cycles are extraordinarily strong, however, and give rise to serious economic problems whenever wages are not perfectly flexible in the medical labor market. It is therefore quite important to find out why herding effects differ across fields or markets, a question which we will examine in our next paper.

4 Conclusion

In this paper we have examined the question why the supply of new workers exhibits periodic ups and downs in a large number of professional labor markets, a phenomenon which may have serious consequences for employment and economic growth. We have proceeded from the assumption that new workers have only poor information about occupational alternatives when making their choice of career, and this assumption distinguishes our approach from other ones where new workers are assumed to be well informed about present or even future wages. Drawing upon the observation that people who have to make a decision without having enough information tend to imitate each other, we have presented a model with herd behavior in occupational choice where herding, when combined with one other assumption, leads to a cyclical behavior of the supply of new workers. Apart from this model, we have also presented empirical evidence, which strongly supports the herd behavior explanation of labor supply cycles.

At the end of this paper let us briefly describe our next project, the explanation of differences in the intensity of herd behavior across professional labor markets, and sketch our preliminary theoretical and empirical results. As it stands by now, our theoretical model attributes differences in the intensity of herd behavior to differences in the size of labor markets: while in large markets a change in the number of new workers easily becomes public so that a herding effect can start, in small markets a change in the number of new workers may stay unnoticed and a herding effect cannot arise. The empirical evidence seems to confirm this relationship: using the same data we have used in this paper, we find that there is indeed a positive relation between the intensity of the herding effect on the one hand and the size of a labor market on the other hand. Of course,

these results are purely preliminary; whether they will bear closer examination will only be known when our coming project is finished.

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