Monetary Policy in Open Economies: Price Inertia and Inflation Targeting

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Abstract

In this paper we consider a two-country model. Each country is characterised by several different sources of nominal inertia. This distinguishes our model from others in the so called New Open Economy Macroeconomics and makes it a suitable framework within which analyse the stabilising properties of monetary policies. We show that the variance of inflation induced by domestic inflationary shocks is lower under CPI targeting than when we target a measure of output price inflation. In fact, market segmentation and staggered wage and price setting result in lower and more persistent foreign inflation responses to a domestic inflationary shocks. This inertia in foreign price adjustments is completely passed through into CPI inflation but not into output price inflation. These differences cannot be detected in traditional models that usually introduce sluggish adjustments of domestic output prices as the only source of inertia. Furthermore, we find a limited role for the exchange rate in affecting the stabilising properties of the rules.

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0. Introduction

Much of the contemporary analysis of optimal monetary policy forms part of the emerging macroeconomic consensus as embodied in the New Neo-Classical Synthesis (see Goodfriend and King (1997) for a discussion), which seeks to incorporate real and nominal rigidities into general equilibrium models based on optimising behaviour to provide a coherent framework for the analysis of policy issues[1]. However, the vast majority of this work is conducted within closed economy models and the optimal monetary policy rules that emerge from this analysis suggests that Central Banks should follow a Taylor rule (1993) which sets interest rate to minimise deviations of output and inflation from trend and target, respectively, with particular weight being given to the inflation target (see Woodford (2001), for example). It is only fairly recently that work has begun on assessing the robustness of these conclusions in the context of the open economy. In extending the analysis to an open economy, a key issue is what measure of inflation a central bank should target. In particular, a key point is whether consumer price inflation might be better target than output price inflation. Gali and Monacelli (1999) claim that a welfare optimising monetary policy for a small open economy should aim to stabilise completely the domestic price inflation. Similarly, Benigno (2001) shows that a policy pursuing domestic inflation stability is optimal even when financial markets are incomplete. Benigno and Benigno (2001a) considering two large economies find that the optimal combination of monetary policies is where both central banks stabilise producer price inflation. A corollary of their conclusions is that if both central banks respond optimally to the deviations of the producer price inflation from its planned path, there is no need for further international monetary policy coordination. Similarly, Obstfeld and Rogoff (2000) reject the necessity of an international monetary compact since it does not provide better outcome than inward-looking monetary policies set to pursue domestic inflation stability. Dealing with a small open economy, Clarida et al. (2001) claim that the optimal monetary policies in open economies are isomorphic to those in closed economies. Essentially all these papers argue that monetary policy should seek to minimise the distortions caused by nominal inertia (the only uncompensated distortion in the model) in an attempt to recreate the equilibrium that would emerge under flexible prices. However, these models are based on the assumption of complete exchange pass-through. In particular, they assume that prices are set in the producer's currency and that free-trade ensures that the law of one-price holds for individual goods, and purchasing power parity holds in terms of consumer price indices. Dealing with a two-country model and allowing for incomplete exchange rate pass-through in the setting of import prices, Corsetti and Pesenti (2001) and Corsetti and Dedola (2001), show that the exchange rate volatility can affect welfare. It follows that open economy variables such as the exchange rate, should be included in the optimal monetary rule. Similar results are obtained by Monacelli (1999) and Sutherland (2001) in the context of a small open economy. Aoki (2001) considering a two-sector model – one with sticky prices and one with flexible prices- shows that monetary policy should target inflation in the sticky price sector.

In our paper we consider a two-country model. In each country there are two sectors: the intermediate goods sector and the final goods sector. Intermediate goods are used in the production of final goods in both countries. Final goods enter the consumption basket of domestic and foreign consumers. Furthermore, we assume that workers are monopolistic suppliers of differentiated labour services and they set their wages according to Calvo contracts (1983). Producers in both sectors are also monopolistic suppliers of differentiated goods. More precisely we assume that market power allows them to price discriminate between domestic and foreign markets. Therefore, breaking down the law of one price (LOP) we allow for different price dynamics of the same good in different countries. Finally, since firms in both sectors set prices in buyers' currency[2] we introduce imperfect exchange rate pass-through. In fact, when a shock occurs and the exchange rate changes only a fraction of firms adjust prices to react to the exchange rate fluctuations. In this way we introduce many different sources of nominal inertia. In each country we have sluggish adjustment in nominal wages, in prices of domestic and imported intermediate goods and in prices of domestic and imported final goods. This makes our model a particularly suitable framework within which to address the question of what measure of inflation the central bank should target. In this paper we have chosen to restrict ourselves to consider two alternatives: output price inflation and consumer price index (CPI). The first choice is due the fact that, as already mentioned, a policy rule formulated in terms of a measure of output prices has been recognised as the optimal one even in open economy. However, most of these results have been derived in

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[1] The seminal publications in this area are Obstfeld and Rogoff (1995, 1996). For a survey of this literature see Lane (1999).

model where the sluggishness in output price adjustment is the only source of inertia. CPI inflation has been chosen since it is the measure of inflation targeted by central banks that implement an inflation targeting regime although other measures of inflation could be more appropriate[3]. The exchange rate enters directly in the definition of CPI inflation, it follows that under this regime central banks implicitly react to change in the real exchange rate. Since the exchange rate react immediately to changes in monetary policies while other nominal variables typically adjust with some inertia then CPI inflation targeting may behave differently from rules that target a measure of output price inflation. Notwithstanding, Ball (1998) asserts that in open economy central banks should target the real exchange rate and respond to its changes to avoid excessive fluctuations of inflation and output. For this reason we consider rules that in addition to either output price inflation or CPI inflation target also the level of the real exchange rate or its changes.

The plan of the paper is as follows. Section 1 outlines the model. In section 2 we explain how we calibrate our model. Section 3 details our results and shows that CPI inflation targeting implies lower volatility of inflation than output price inflation targeting. This results are due to a lower and more persistent response of foreign good prices to an inflationary shock in the home country. Therefore, we claim that the absence of this additional channel of distortion in previous studies can lead to some misleading conclusions. In our model, with sources of inertia realistically affecting pricing decisions in the home and foreign market at different stages of production, the movement in the exchange rate has only a limited role in affecting the stabilising properties of the rules, whether they target CPI or output price inflation. This is in contrast to models which make the simplifying assumption of complete exchange rate pass through, purchasing power parity (PPP) and a single source of inertia in the pricing of final goods. Section 4 concludes.

1. The model

This model consists of two symmetric countries denominated as Home (H) and Foreign (F). Each country is inhabited by several different types of agents: consumers, final

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[2] There is a fairly recent but already vast literature on the implications of local currency price (LCP) assumption. Examples include Devereux and Engel (1998, 1999) and Betts and Devereux (1996).

[3] Mankiw and Reis (2002) suggest that the central bank should stabilise a price index where the weight of each sector depends on the sector's characteristics, including size, cyclical sensitivity, sluggishness of price adjustment and magnitude of sectoral shocks.

good producers, intermediate good producers, the government and the central bank. We assume that in each period a new generation of consumer is born and that all consumers face a constant probability of death. The introduction of overlapping generation allows us to derive a well-defined steady state for consumption, the terms of trade and the net financial wealth around which the model can be linearised (for a discussion of this point, see Ghironi (2000))[4]. Intermediate goods are produced using only domestic labour inputs and sold to both domestic and foreign producers. The production function of the final good producer combines domestic labour and bundles of domestic and foreign intermediate goods. As with intermediate goods, final goods are sold in both markets. Therefore, the consumption bundles purchased by households and governments combine both domestic and foreign final goods. We assume that price to market (PTM) is possible and that prices are set in the buyer's currency. For simplicity and to allow consumers in both countries to hold positive financial assets in equilibrium we assume that given its public expenditure choices, the government raises taxes to keep the public debt constant[5]. Finally, the central bank is in charge of the monetary policy and sets nominal interest rates to respond to domestic and foreign shocks to the economy. We now proceed to outline the model, considering first the problem facing individual consumers, before aggregating across all consumers. We then turn to the pricing decisions of the representative firm in both intermediate and final good sector. The linearised version of the model required to render it suitable for numerical simulation is presented in an appendix.

1.1 The Consumer's problem

Consider a typical home consumer j in the cohort s who derives utility from a basket of consumption goods ($C(j,s)$) and real money balances ($\frac{M}{P}(j,s)$) and disutility from providing labour services ($l(j,s)$). We assume that the expected value of the consumer's utility ($E_t\left[U_t(j,s)\right]$) is obtained as follows:

[4] Overlapping generations are also considered in Smets and Wouters (2001).

[5] Since we are not interested in considering interactions between fiscal and monetary policies, we do not allow the government to deviate from the zero deficit constraint. Moreover, insulating the economy from the effects of fiscal policies we are able to enhance the consequences of different monetary policy rules without affecting the main conclusions.

$$E_t\left[U_t\left(j,s\right)\right]=\sum_{t=\tau}^{\infty}\left(\iota\rho\right)^{t-\tau}\left[\ln C_t\left(j,s\right)+\chi_t\ln\frac{M_t}{P_t}\left(j,s\right)+k_t\ln\left(1-l_t\left(j,s\right)\right)\right] \tag{1}$$

where ι is the probability of survival and ρ is the consumers' discount rate. We assume that the parameters χ_t and k_t are strictly positive and that both ι and ρ are between zero and one. The basket of consumption goods is defined by the following index:

$$C_t\left(j\right)=C_{H,t}\left(j\right)^{\gamma}C_{F,t}\left(j\right)^{(1-\gamma)}$$

where $\gamma\in\left(0,1\right)$. $C_{H,t}\left(j,s\right)$ and $C_{F,t}\left(j,s\right)$ are consumption CES sub-indexes of a continuum of differentiated final goods produced respectively in country H and F:

$$C_{H,t}\left(j,s\right)=\left[\int_{h=0}^{1}c_t\left(h,j,s\right)^{\frac{\theta-1}{\theta}}dh\right]^{\frac{\theta}{\theta-1}} \qquad C_{F,t}\left(j,s\right)=\left[\int_{f=0}^{1}c_t\left(f,j,s\right)^{\frac{\theta^*-1}{\theta^*}}df\right]^{\frac{\theta^*}{\theta^*-1}}$$

with θ (θ^*) > 0 the elasticity of substitution among home (foreign) final goods. Given the price of each differentiated final good, the price of any consumption bundle is obtained as the minimum expenditure required to buy one unit of this bundle. Consequently we have:

$$P_{H,t}=\left[\int_{h=0}^{1}p_t\left(h\right)^{1-\theta}dh\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t}=\left[\int_{f=0}^{1}p_t\left(f\right)^{1-\theta^*}df\right]^{\frac{1}{1-\theta^*}} \quad , \quad P_t=\frac{P_{H,t}{}^{\gamma}P_{F,t}{}^{(1-\gamma)}}{\psi}$$

where $\psi=\gamma^{\gamma}\left(1-\gamma\right)^{(1-\gamma)}$.

The consumer can hold her financial wealth in the form of home government bonds (B_H), foreign bonds (B_F) and money balances. Domestic bonds earn a nominal interest rate i, foreign bonds a return i^*. Consumers receive also shares in the profits of all domestic firms ($DIV(j,s)$). Furthermore, it is assumed that the consumers receive a premium from perfectly competitive insurance companies in return for their financial assets should they die; this effectively raises the rate of return from holding financial assets by $\frac{1}{\iota}$. Finally, the consumer receives a wage $W\left(i\right)$ for any unit of labour supplied[6], consumes both domestic and foreign final good bundles pays lump sum taxes $T\left(j,s\right)$ and receive lump sum transfers $TR\left(j,s\right)$. It follows that the consumer budget constraint in nominal terms is given by:

[6] As it will be explained below, workers provide different labour services. Those who provide a labour service of type i receive the nominal wage $W_t\left(i\right)$

$$M_t(j,s)+P_{H,t}C_{H,t}(j,s)+P_{F,t}C_{F,t}(j,s)+B_{H,t}(j,s)+B_{F,t}(j,s)\varepsilon_t=\frac{M_{t-1}(j,s)}{\iota}+$$
$$+B_{H,t-1}(j,s)\frac{(1+r_{t-1})}{\iota}+B_{F,t-1}(j,s)\frac{(1+r^*_{t-1})}{\iota}\varepsilon_t+W_t(i)l_t(j,s)+Div_t(j,s)-T_t(j,s)+TR_t(j,s)$$

where ε_t is the nominal exchange rate. Deflating by P_t we can rewrite it in real terms as follows:

$$m_t(j,s)+\frac{P_{H,t}}{P_t}C_{H,t}(j,s)+\frac{P_{F,t}}{P_t}C_{F,t}(j,s)+b_{H,t}(j,s)+b_{F,t}(j,s)\varepsilon_t=\frac{m_{t-1}(j,s)}{\iota}\frac{P_{t-1}}{P_t}+$$
$$+b_{H,t-1}(j,s)\frac{(1+r_{t-1})}{\iota}+b_{F,t-1}(j,s)\frac{(1+r^*_{t-1})}{\iota}\varepsilon^R_t+w_t(i)l_t(j,s)+div_t(j,s)-\tau_t(j,s)+\tau r_t(j,s) \tag{2}$$

where ε^R_t is the real exchange rate and r_{t-1} (r^*_{t-1}) is the ex ante real interest rate paid at time t on home (foreign) bonds issued one period earlier. Since the non-arbitrage condition implies that the usual uncovered interest parity (UIP) condition must hold, we have:

$$\frac{\varepsilon^R_t}{\varepsilon^R_{t+1}}=\frac{(1+r^*_t)}{(1+r_t)}$$

From the first order conditions derived from the maximisation of the expected utility (1) under the budget constraint (2) we obtain the usual consumption Euler equation:

$$E_tC_{t+1}(j)=\rho(1+r_t)C_t(j)$$

The optimisation also leads to the following money demand function:

$$m_t(j)=\chi_t\frac{1+r_t}{1+r_t-\frac{P_{t+1}}{P_t}}C_t(j)$$

Integrating the consumption Euler equation forward and substituting it into the intertemporal budget constraint we obtain the consumer's consumption function:

$$C_t(j)=\frac{1-\iota\beta}{1+\chi}\left\{\begin{array}{c}\frac{(1+r_{t-1})}{\iota}b_{H,t}(j)+\frac{(1+r^*_{t-1})}{\iota}b_{F,t}(j)+w_\tau(j)l_\tau+div_\tau(j)-\tau_\tau(j)+\tau r_\tau(j)\\ +\sum_{t=\tau}^{\infty}\frac{(\iota)^{t-\tau}\left(w_t(j)l_t(j)+div_t(j)-\tau_t(j)+\tau r_t(j)\right)}{\prod_{k=0}^{t-\tau-1}(1+r_{\tau+k})}+\frac{m_{t-1}(j)}{\iota}\frac{P_{t-1}}{P_t}\end{array}\right\}$$

From the first order conditions we can also easily derive the consumer's demand for bundles of home and foreign final goods:

$$C_{H,t}(j)=\gamma\left[\frac{P_{H,t}}{P_t}\right]^{-1}C_t(j) \quad \text{and} \quad C_{F,t}(j)=(1-\gamma)\left[\frac{P_{F,t}}{P_t}\right]^{-1}C_t(j).$$

The domestic demands of a typical home and foreign final good are given by:

$$C_t(h,j)=\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}C_{H,t}(j) \quad \text{and} \quad C_t(f,j)=\left[\frac{p_t(f)}{P_{F,t}}\right]^{-\theta}C_{F,t}(j).$$

1.1.1 Labour supply and wage setting

We assume that workers provide differentiated labour services indexed by $i\in(0,1)$. Moreover, for the sake of simplicity, we assume the same generation structure within each labour service type. It means that for any cohort s there are t^s workers for any type i. Workers of the same type are represented by a trade union which fixes the same nominal wage for all its members. There are different trade unions for any type of labour service. Therefore, we label the trade unions with the same index i. Given the nominal wage $W_t(i)$, workers flexibly supply the amount of labour to maximise their welfare:

$$l(i,s)=1-\frac{k_tC_t(i,s)P_t}{W_t(i)}$$

The trade unions are monopolistic supplier of a given type of labour service. We assume that in any period each trade union has a constant probability $(1-z_W)$ of signing a new wage-contract. Until a new wage-contract is signed the nominal wage is fixed. In the appendix we show that solving the optimisation problem of the representative trade union we have:

$$\tilde{W}_t(j)=\frac{\phi}{\phi-1}\frac{\sum_{t=\tau}^{\infty}(z_W\rho)^{t-\tau}k_t\left(1-l_t(i)\right)^{-1}W_t^{\phi}l_t(j)}{\sum_{t=\tau}^{\infty}(z_W\rho)^{t-\tau}(P_tC_t)^{-1}W_t^{\phi}l_t(j)}$$

where $\phi>0$ is the elasticity of substitution of labour, $l(i)$ is the per capita demand of labour service of type i and C_t is the per capita consumption derived in the following section. The market power of the trade unions allows them to set the nominal wages above the discounted sum of marginal costs. Once the nominal wage $\tilde{W}_\tau(i)$ is set, the trade union fixes $W_t(i)$ in order to satisfy in each period following constraint: $l_t(i) = (1-\iota)\sum_{s=0}^{\infty} \iota^s l_t(i,s)$. We assume that the difference between what the trade union raises and what it pays to its members through wages is then redistributed to all members with lump sum transfers: $TR_t = \left(\tilde{W}_\tau(i) - W_t(i)\right) l_t(i)$.

1. 2 Aggregating across individual consumers

By assuming that each cohort is of size 1 when born, a cohort of age $t-\tau$ will have a size $\iota^{t-\tau}$. Therefore, the total size of the population is constant and equal to $\sum_{t=\tau}^{\infty} \iota^t = \frac{1}{1-\iota}$. The aggregate per capita money demand function is given by[7]:

$$m_t = \chi_t \frac{1+r_t}{1+r_t - \frac{P_t}{P_{t+1}}} C_t$$

The aggregate per capita consumption is:

$$C_t = \frac{1-\iota\beta}{1+\chi}\left[V_t + \left(1+r_{t-1}\right) b_{H,t-1} + \left(1+r_{t-1}^*\right) b_{F,t-1} + m_{t-1}\frac{P_{t-1}}{P_t} \right]$$

where V_t is the aggregate per capita human wealth after tax:

$$V_t = (1-\iota)\sum_{s=0}^{\infty} \iota^s \left\{ w_\tau(s) l_\tau(s) + div_\tau(s) - T_\tau(s) + \sum_{t=\tau}^{\infty} \frac{(\iota)^{t-\tau}\left(w_t(s) l_t(s) + div_t(s) - T_t(s)\right)}{\prod_{k=0}^{t-\tau-1}\left(1+r_{\tau+k}\right)} \right\}$$

1.3 The firms' problem

Both the intermediate good sector and the final good sector are populated by a continuum of firms producing differentiated goods indexed by $ih \in (0,1)$ and $h \in (0,1)$ respectively. Consequently, we model each producer as a monopolistic competitor that fixes its prices as a mark up over marginal costs. Moreover, we assume that firms can charge different prices in

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[7] We can express all variables in terms of aggregate per capita as follows: consider the generic variable $\chi(s)$, then $\chi_t = (1-\iota)\sum_{s=0}^{\infty} \iota^s \chi_t(j,s)$

the home and foreign markets. More precisely, we adopt the hypothesis of local currency price setting (LCP); in other words firms set the prices of their products in the buyer's currency. Since intermediate good producers and final good producers face a very similar problem, we describe in detail only the optimal choice of the representative intermediate good producer. The same procedure can be applied to solve the problem for the representative firm producing a final good.

1.3.1 Intermediate goods producers

We assume that a typical intermediate good (ih) is produced using only domestic labour and a linear technology:

$$I_t\left(ih\right) = A_t l_t\left(ih\right)$$

where A is the home labour productivity and $l_t\left(ih\right)$ is a composite labour factor defined by the following CES index:

$$l_t\left(ih\right) = \left(\int_{j=0}^{1} l\left(i, ih\right)^{\frac{\phi-1}{\phi}} di \right)^{\frac{\phi}{\phi-1}}$$

Consequently, the nominal marginal costs (MC) faced by the representative firm in the intermediate good sector are given by:

$$MC_t = \frac{W_t}{A_t}$$

where W_t is the wage index defined consistently with the bundle of labour services:

$$W_t = \left[\int_0^1 W_t\left(i\right)^{1-\phi} di \right]^{\frac{1}{1-\phi}}$$

Following Christiano et al. (2001), we assume that only a fraction $1-z_I$ of the firms can re-optimised in any given period. All the other firms index imperfectly their prices to the last period inflation rate in the market of their products. Define Γ_{I_i} as the degree of indexation to the past inflation in the domestic market of home intermediate good. If we assume that at time t a firm does not re-optimise, the prices it charges in the home and foreign markets are respectively:

$$\tilde{p}_t(ih)=\left(\frac{P_{I_H,t-1}}{P^P_{I_H,t-2}}\right)^{\Gamma_{I_H}}\tilde{p}_{t-1}(ih) \quad \text{and} \quad \tilde{p}^*_t(ih)=\left(\frac{P^*_{I_H,t-1}}{P^*_{I_H,t-2}}\right)^{\Gamma^*_{I_H}}\tilde{p}^*_{t-1}(ih)$$

where $P_{I_H,t}=\left[\int_0^1 P_t(ih)^{1-\theta_I}\,dih\right]^{\frac{1}{1-\theta_I}}$ and $P^*_{I_H,t}=\left[\int_0^1 P^*_t(ih)^{1-\theta_I}\,dih\right]^{\frac{1}{1-\theta_I}}$ are the price indexes of domestic intermediate goods in country H and F respectively.

Then the flow of present and future expected profits ($E_\tau\Pi_\tau$) in country H of the representative intermediate good producer is:

$$E_\tau\Pi_\tau=\sum_{t=\tau}^{\infty}z_I^{\,t-\tau}\prod_{k=0}^{t}\left(\frac{1}{1+r_{\tau+k}}\right)\left\{\frac{\Upsilon_t(ih)}{P_t}\left(\frac{\Upsilon_t(ih)}{P_{I_H,t}}\right)^{-\theta_I}I_{H,t}-\left(\frac{W_t}{P_tA_t}\right)\left(\frac{\Upsilon_t(ih)}{P_{I_H,t}}\right)^{-\theta_I}I_{H,t}\right\}$$

where $\Upsilon_t(ih)=\tilde{p}_t(ih)\left(\frac{P_{I_H,t-1}}{P_{I_H,\tau-1}}\right)^{\Gamma_{I_H}}$, I_H is the aggregate demand in country H for bundles of home intermediate goods, and $\theta_I>0$ is the elasticity of substitution across home intermediate goods. Profits maximisation leads to the following price choice for the home market:

$$\tilde{p}_\tau(ih)=\frac{\theta_I}{\theta_I-1}\frac{\left[\sum_{t=\tau}^{\infty}z_I^{\,t-\tau}\prod_{k=0}^{t}\left(\frac{1}{1+r_{\tau+k}}\right)\frac{W_t}{P_tA_t}\left(P_{I_H,t}\right)^{\theta_I}I_{H,t}\left(\frac{P_{I_H,t-1}}{P_{I_H,\tau-1}}\right)^{-\theta_I\Gamma_{I_H}}\right]}{\left[\sum_{t=\tau}^{\infty}z_I^{\,t-\tau}\prod_{k=0}^{t}\left(\frac{1}{1+r_{\tau+k}}\right)\left(P_{I_H,t}\right)^{\theta_I}\frac{I_{H,t}}{P_t}\left(\frac{P_{I_H,t-1}}{P_{I_H,\tau-1}}\right)^{\Gamma_{I_H}(1-\theta_I)}\right]}$$

Similarly, under the LCP assumption, the optimal choice for the foreign market is:

$$\tilde{p}^*_\tau(ih)=\frac{\theta_I}{\theta_I-1}\frac{\left[\sum_{t=\tau}^{\infty}z_I^{\,t-\tau}\prod_{k=0}^{t}\left(\frac{1}{1+r_{\tau+k}}\right)\frac{W_t}{P_tA_t}\left(P^*_{I_H,t}\right)^{\theta_I}I^*_{I_H,t}\left(\frac{P^*_{I_H,t-1}}{P^*_{I_H,\tau-1}}\right)^{-\theta_I\Gamma^*_{I_H}}\right]}{\left[\sum_{t=\tau}^{\infty}z_I^{\,t-\tau}\prod_{k=0}^{t}\left(\frac{1}{1+r_{\tau+k}}\right)\varepsilon_t\left(P^*_{I_H,t}\right)^{\theta_I}\frac{I^*_{I_H,t}}{P_t}\left(\frac{P^*_{I_H,t-1}}{P^*_{I_H,\tau-1}}\right)^{\Gamma^*_{I_H}(1-\theta_I)}\right]}$$

where I^*_H is the aggregate demand in country F for home bundles of intermediate goods. We can write the price level of home intermediate goods in country H as follows:

$$P^I_{H,t}=\left\{z_I\left[\left(\frac{P^I_{H,t-1}}{P^I_{H,t-2}}\right)^{\Gamma_{I_H}}P^I_{t-1}(h)\right]^{1-\theta_I}+(1-z_I)\,\tilde{p}^I_t(h)^{1-\theta_I}\right\}^{\frac{1}{1-\theta_I}}$$

Similarly, for the foreign market we have:

$$P^*_{I_H,t} = \left\{ z_I \left[\left(\frac{P^*_{I_H,t-1}}{P^*_{I_H,t-2}} \right)^{\Gamma_{I_H}} P^*_{t-1}(ih) \right]^{1-\theta_I} + (1-z_I)\, \tilde{p}^*_t (ih)^{1-\theta_I} \right\}^{\frac{1}{1-\theta_I}}$$

1.3.2 Final good producers

A firm producing final goods combines domestic and foreign intermediate goods and domestic labour services according to the following Cobb-Douglas production function:

$$Y_t(h) = \left[D_t (BH_t)^{\alpha} (BF_t)^{(1-\alpha)} \right]^{\beta} [A_t l_t]^{(1-\beta)}$$

where $\alpha, \beta \in (0,1)$ and D_t is the technology used in the production of final goods. BH_t and BF_t are bundles of home and foreign intermediate defined by the following CES aggregators:

$$BH_t = \left(\int_0^1 I(ih)^{\frac{\theta_I}{\theta_I - 1}} dih \right)^{\frac{\theta_I - 1}{\theta_I}} \quad , \quad BF_t = \left(\int_0^1 I(if)^{\frac{\theta_I^*}{\theta_I^* - 1}} dif \right)^{\frac{\theta_I^* - 1}{\theta_I^*}}$$

As in the previous section, we can derive the optimal prices that a domestic final good producer charges in country H and F respectively:

$$\tilde{p}_\tau(h) = \frac{\theta}{\theta - 1} \frac{\left[\sum_{t=\tau}^{\infty} z_P^{\,t-\tau} \prod_{k=0}^{t} \left(\frac{1}{1+r_{\tau+k}} \right) \frac{(P_{I_H,t})^{\alpha\beta} (P_{I_F,t})^{(1-\alpha)\beta} D_t^{\beta} (W_t / A_t)^{(1-\beta)}}{\vartheta P_t} (P_{H,t})^{\theta} \; Y_{H,t} \left(\frac{P_{H,t-1}}{P_{H,\tau-1}} \right)^{-\theta\Gamma_H} \right]}{\left[\sum_{t=\tau}^{\infty} z_P^{\,t-\tau} \prod_{k=0}^{t} \left(\frac{1}{1+r_{\tau+k}} \right) (P_{H,t})^{\theta} \frac{Y_{H,t}}{P_t} \left(\frac{P_{H,t-1}}{P_{H,\tau-1}} \right)^{\Gamma_H(1-\theta)} \right]}$$

and

$$\tilde{p}^*_\tau(h) = \frac{\theta}{\theta - 1} \frac{\left[\sum_{t=\tau}^{\infty} z_P^{\,t-\tau} \prod_{k=0}^{t} \left(\frac{1}{1+r_{\tau+k}} \right) \frac{(P_{I_H,t})^{\alpha\beta} (P_{I_F,t})^{(1-\alpha)\beta} D_t^{\beta} (W_t / A_t)^{(1-\beta)}}{\vartheta P_t} (P^*_{H,t})^{\theta} \, Y^*_{H,t} \left(\frac{P^*_{H,t-1}}{P^*_{H,\tau-1}} \right)^{-\theta\Gamma_H} \right]}{\left[\sum_{t=\tau}^{\infty} z_P^{\,t-\tau} \prod_{k=0}^{t} \left(\frac{1}{1+r_{\tau+k}} \right) \varepsilon_t (P^*_{H,t})^{\theta} \frac{Y^*_{H,t}}{P_t} \left(\frac{P^*_{H,t-1}}{P^*_{H,\tau-1}} \right)^{\Gamma_H(1-\theta)} \right]}$$

where $\vartheta = \left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\right]^{\beta}\beta^{\beta}(1-\beta)^{(1-\beta)}$. z_p is the probability of not re-optimising while Γ_H (Γ_H^*) is the degree of indexation to the past period inflation in the home (foreign) market. Y_H (Y_H^*) is the aggregate per capita domestic (foreign) demand of home bundles of final goods.

1.4 Government

We assume that government in the domestic economy faces the following real budget constraint

$$g_t - \tau_t + (1+r_{t-1})b_{t-1} = b_t + m_t - m_{t-1}\frac{P_{t-1}}{P_t}$$

where τ_t is the amount of taxes raises by the government, g_t is the government's expenditure and b_t is the amount of bonds issued by the government in period t. For the sake of simplicity we assume that the baskets of goods purchased by the government replicate exactly the private sector consumption indexes. As explained above, we assume that the government follows a rule to maintain its real debt constant (i.e. $b_{t-1} = b_t \;\; \forall\, t \in [\tau, \infty)$). It implies that in each period the amount of taxes raised have to satisfy the following constraint:

$$\tau_t = g_t + r_{t-1}\bar{b} - \left(m_t - m_{t-1}\frac{P_{t-1}}{P_t}\right)$$

1.5 Demand side of the economy

Before discussing the linearisation of the model, it is useful to specify the cost minimising demands for bundles of intermediate and final goods as well as the demand for labour.

1.5.1 Intermediate goods

The representative final producer's demand for a domestic intermediate good ih is:

$$I_t(ih) = \left(\frac{P(ih)}{P_{I_H,t}}\right)^{-\theta_I} I_{H,t}(h)$$

where $I_{H,t}(h)$ is the demand of bundles of home intermediate goods of a typical final good producer in country H. Similarly, the demand for the foreign intermediate good if is:

$$I_t(if) = \left(\frac{P_t(if)}{P_{I_F,t}} \right)^{-\theta_I^*} I_{F,t}(h)$$

The demand of bundles of domestic intermediate goods of the representative home final good producer ($I_{H,t}(h)$**)** is obtained in two steps solving the cost minimisation problem of the final good. Firstly, we derive the firm's demand for bundles of intermediate goods ($I_t(h)$) (i.e. the demand for both home and foreign inputs):

$$I_t(h) = \left(\frac{\beta}{1-\beta} \right)^{(1-\beta)} \left(\frac{W_t}{P_{I,t}} \right)^{(1-\beta)} \frac{Y_{H,t}^w(h)}{A^{(1-\beta)} D^{\beta}}$$

where: $P_{I,t} = \frac{\left(P_{I_H,t}\right)^{\alpha} \left(P_{I_F,t}\right)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}$. $Y_{H,t}^w(h)$ is the aggregate per capita world demand of the a typical home final good h. Then we can write the demand for bundles of home intermediate goods as follows:

$$I_{H,t}(h) = \left(\frac{\alpha}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{P_{I_F,t}}{P_{I_H,t}} \right)^{(1-\alpha)} \frac{I_t(h)}{D_t}$$

Integrating across domestic final good producers we obtain:

$$I_{H,t} = \left(\frac{\alpha}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{\beta}{1-\beta} \right)^{(1-\beta)} \frac{\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} \right]^{(1-\beta)}}{A_t^{(1-\beta)} D_t^{(1+\beta)}} (w_t)^{(1-\beta)} \left(R_{H,t}\right)^{(\alpha\beta-1)} \left(R_{F,t}\right)^{\beta(1-\alpha)} Y_{H,t}^w$$

where $I_{H,t}$ is the demand for bundles of home intermediate goods in country H and $Y_{H,t}^w$ is the per capita world demand of home final goods bundles. We have also defined: $R_{H,t} = \frac{P_{I_H,t}}{P_t}$ and $R_{F,t} = \frac{P_{I_F,t}}{P_t}$. Similarly, the foreign demand for bundles of home intermediate goods is:

$$I_{H,t}^* = \left(\frac{1-\alpha}{\alpha} \right)^{\alpha} \left(\frac{\beta}{1-\beta} \right)^{(1-\beta)} \frac{\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} \right]^{(1-\beta)}}{A_t^{(1-\beta)} D_t^{(1+\beta)}} (w_t)^{(1-\beta)} \left(R_{H,t}^*\right)^{(2\alpha-2-\alpha\beta+\beta)} \left(R_{F,t}^*\right)^{(1-2\alpha+\alpha\beta)} Y_{F,t}^w$$

1.5.2 Final goods

The aggregate world demand for the final good h is described by the following equation:

$$Y_{H,t}^{w}\left(h\right)=\left[\frac{P_{t}\left(h\right)}{P_{H,t}}\right]^{-\theta}Y_{H,t}+\left[\frac{P_{t}^{*}\left(h\right)}{P_{H,t}^{*}}\right]^{-\theta}Y_{H,t}^{*}$$

where $Y_{H,t}$ $(Y_{H,t}^{*})$ is the aggregate per capita demand for bundles of home final good in country H and F respectively:

$$Y_{H,t}=C_{H,t}+G_{H,t}=\gamma\left(\frac{P_{H,t}}{P_{t}}\right)^{-1}C_{t}+\gamma\left(\frac{P_{H,t}}{P_{t}}\right)^{-1}G_{t}$$

$$Y_{H,t}^{*}=Y_{H,t}^{*}+G_{H,t}^{*}=\left(1-\gamma\right)\left(\frac{P_{H,t}^{*}}{P_{H,t}^{*}}\right)^{-1}C_{t}^{*}+\left(1-\gamma\right)\left(\frac{P_{H,t}^{*}}{P_{H,t}^{*}}\right)^{-1}G_{t}^{*}$$

Integrating across domestic firms leads to:

$$Y_{H,t}^{w}=Y_{H,t}+Y_{H,t}^{*}$$

1.5.3 Labour demand

As well as the demand for intermediate goods, the labour demand is obtained from the cost minimisation problem of the firms. Therefore, the demand for labour service of type j in the intermediate good ($l_{t}^{I}\left(j\right)$) is given by:

$$l_{t}^{I}\left(j\right)=\left(\frac{W_{t}\left(j\right)}{W_{t}}\right)^{-\phi}\left(\frac{I_{H,t}+I_{H,t}^{*}}{A_{t}}\right)$$

The demand for labour service of type k in the final good sector ($l_{t}^{I}\left(k\right)$) is as follows:

$$l_{t}^{F}\left(k\right)=\left(\frac{w_{t}\left(k\right)}{w_{t}}\right)^{-\phi}\left(\frac{1-\beta}{\beta}\right)^{\beta}\left(\frac{P_{I,t}}{W_{t}}\right)^{\beta}\frac{Y_{H,t}^{w}}{A_{t}^{(1-\beta)}D_{t}}$$

The aggregate labour demand for a typical worker j $(l\left(j\right))$ is simply the sum of the previous two expressions:

$$l_{t}\left(k\right)=\left(\frac{w_{t}\left(k\right)}{w_{t}}\right)^{-\phi}\left[\left(\frac{I_{H,t}+I_{H,t}^{*}}{A_{t}}\right)+\left(\frac{1-\beta}{\beta}\right)^{\beta}\left(\frac{P_{I,t}}{W_{t}}\right)^{\beta}\frac{Y_{H,t}^{w}}{A_{t}^{(1-\beta)}D_{t}}\right]$$

The above equation can be rewritten as follows:

$$l_t(k) = \left(\frac{w_t(k)}{w_t}\right)^{-\phi}\left\{\left(\frac{I_{H,t}+I^*_{H,t}}{A_t}\right)+\left[\frac{1-\beta}{\beta\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right]^{\beta}\frac{(R_{H,t})^{\alpha\beta}(R_{F,t})^{\beta(1-\alpha)}}{(w_t)^{\beta}A_t^{(1-\beta)}D_t}Y^{w}_{H,t}\right\}$$

2. Calibration

In parameterising the model, we assume a quarterly data period and the parameters we choose are given in table (3) along with the steady state values these imply. We do not distinguish between the elasticities of demand facing our imperfectly competitive firms in intermediate and final good sectors. The values we choose for θ and θ^* are taken from Gali et al. (2001) and they imply a price mark up of 1.1.

The quarterly discount rate ρ is slightly lower than that found in other studies (such as Kollman (1998) or Rotemberg and Woodford (1998), for example). The reason for this is that these studies assume infinitely lived consumers. Therefore, the usual higher discount rate is equivalent to an annual real interest rate of around 3%. Since the overlapping generation structure of our model raises the real interest rate, this slightly lower rate of time preference is consistent with the same equilibrium real interest rate found in literature.

The parameter ι is the probability of survival for our consumers and it implies an average working life of about 35 years. This seems to be a plausible measure of average time spent in employment, although it is admittedly a high probability of death if the model is taken literally. Nevertheless, such a parameter is necessary to generate a plausible steady state value of government debt relatively to GDP[8].

The parameters z_I and z_P are the probabilities that a firm in the intermediate good sector and in the final good sector respectively does not re-optimise prices in the next quarter. The value of 0.66 is taken from Leith and Malley (2002) and it implies that firms take on average nine months to reset their prices. They are also consistent with the estimates of Gali and Gertler (2001). Leith and Malley (op. cit.) provide also an estimate of probability z_w that a new wage contract is set in the next quarter. The value of 0.74 implies an average length of wage contracts of about one year.

[8] This point is also discussed in Leith and Wren-Lewis (2002). For a further exploration of this point see also Faruqee et al (1997)

Finally, the degree of indexation to the past period inflation is set equal to 0.3. The value we choose imply that in the hybrid New Keynesian Phillips curve the weights associated to the forward-looking components and to backward looking component are approximately 0.76 and 0.23. These values are consistent with the estimates of Leith and Malley (op. cit.) and Gali and Gertler (op. cit.).

The steady state these parameters imply are shown in table (2). The real interest rate has an annualised value of 3% and the steady state ratio of debt to GDP is around 70% which is consistent with the average level of debt in the euro area at the end of 2000 (ECB (2001)). The ratio of government spending to GDP is 23%. This value is also consistent with the data reported by Gali (1994) across OECD economies. Finally, the steady state values of the labour and intermediate good productivity are chosen in order to obtain a labour-income ratio of about 2/3 and a steady state value of aggregate per capita labour supply equal to 1/2 .

3. Interest rate policy rules and shocks

In this section we briefly discuss the policy rules we consider in our policy experiments. In closed economy, despite of its simplicity a Taylor rule seems to provide the best guidelines for a central bank if it aims to stabilise output gap and inflation around trend and target respectively. However, the same degree of consensus has not been reached in open economy. In particular, open economy models arise the issue of what measure of inflation should enter a Taylor rule. In this paper we do not aim to specify the best Taylor rule for our model. The relative weights that output gap and inflation should have in policy rules is still object of discussion even though recent research has moved towards the emphasis of the role of inflation volatility in the central bank's loss function. Therefore, the optimal solution of the inflation-output variability trade-off requires the derivation of a microfounded welfare function which is beyond the scope of this paper. Although a term in output gap has to be introduced in policy rules, any choice at this point would be arbitrary. Therefore, we have chosen to keep our policy rules as simpler as possible in order to emphasise the consequences of targeting different measures of inflation. In fact, the properties of the policy rules we implement do not depend on the choice of not include a term in output gap. The policy rules we choose are reported in the table below:

$$i_t = a_{\pi_H}\pi_{H,t} + a_{\pi}\pi_t + a_{\varepsilon}\varepsilon_t^R + a_{\varepsilon-1}\varepsilon_{t-1}^R$$

Rules	$\alpha_{\pi H}$	α_{π}	α_{ε}	$\alpha_{\varepsilon-1}$
1	1.5	0	0	0
2	0	1.5	0	0
3	1.5	0	1	0
4	1.5	0	0.5	0
5	0	1.5	0.5	0
6	1.5	0	0.5	-0.5
7	0	1.5	0.5	-0.5

Policy rule (1) simply targets the output price inflation while policy rule (2) targets CPI inflation. Comparing the dynamics of inflation and its variance under these rules allow us to asses some general properties of targeting different measure of inflation. Rule (3) is used to rule out the real exchange rate movement as a main source of excess inflation under output price inflation as it will be clear when we discuss the effects of a supply shock. In order to assess the properties of rules that react to movement in the exchange rate we modify the naive inflation targeting rules (1) and (2) adding a term for the level of the real exchange rate. Therefore, rule (4) targets output price inflation and the real exchange rate, while nominal interest rates in rule (5) respond to CPI inflation and the level of the real exchange rate. Rules (6) and (7) are similar to rules (4) and (5) but we replace the level of the real exchange rate with its first differences. In this way we can contrast the properties of these two different sets of rules. In fact, there is no consensus in literature on how the central bank should respond to movements in the exchange rate. In setting parameters for policy rules (4) and (5) we follow Taylor (2001) who claims that an exchange rate appreciation should induce the central bank to relax monetary policy. However, he suggests two different ways in which central bank can react to movements in the exchange rates: in the first case the central bank can adjust nominal interest rates whenever the real exchange rate is different from its steady state equilibrium, in the second case the central bank responds to changes in the real exchange rate. We consider the first case with policy rules (4) and (5) and the second one with rules (6) and (7).

Shocks

We consider two different types of domestic shock. The first is a negative shock to home labour productivity. We assume that labour productivity decreases of 1% for four quarters and then gradually returns to its steady state value. The second is a positive shock to home private sector's consumption. Similarly we assume a shock of size 1% that lasts for one year. Therefore we can evaluate our policy rules under both supply and demand shocks.

3.1 Domestic supply shock

Before discussing the simulation results of a shock to labour productivity under different policy rules, it is important to establish the general effects of this supply shock in our model. After a negative shock to domestic labour productivity, domestic firms in both sectors raise prices to protect their profits against the rise in real marginal costs; they also increase their demand of labour to offset the effects of the decline in labour productivity. The nominal interest rate rises in response to the increase in the inflation such that the real rates rise. Obviously, the exact magnitude of the rise in the nominal interest rate depends on the specific interest rate policy followed by the central bank. Consumption falls as a consequence of the rise in the real interest rate and this helps to offset the effects of the negative supply shock on inflation. However, in the home economy the inflation rises while output in both sectors falls. In contrast, in the foreign country the rise in inflation is accompanied by a rise in real GDP (deflated by the output prices) due to the improvement in the terms of trade. The real exchange rate appreciates because the response of the domestic monetary authorities is initially more aggressive than the foreign response. The initial appreciation of the exchange rate has a negative impact on the prices of imported goods and this further offsets the initial inflationary effects due to the labour productivity shock. In fact, the real exchange rate appreciation allows foreign exporters to charge relatively lower prices without reducing their profits. However, the subsequent depreciation of the real exchange rate represents an additional source of inflation for the home economy.

3.1.1 Targeting output price inflation vs CPI inflation

Under output price inflation targeting the initial appreciation of the real exchange rate (fig.5) is followed by a prompt depreciation, while under CPI targeting the initial appreciation is much smaller and the following convergence to the steady state is slower. The real exchange rate behaviour reflects the dynamics of the gap between home and foreign interest rates (fig.6). When the domestic shock occurs CPI inflation in the home country is mostly due to output price inflation. Similarly CPI inflation in the foreign country largely reflects the exportation of this inflation abroad. In other words, under output price inflation targeting the initial response of monetary authorities is relatively more aggressive in the home country and less aggressive in the foreign country. However, excluding the first quarter when the large appreciation of the real exchange rate reduces the size of imported inflation, CPI inflation in the county H is higher under output inflation targeting. In contrast, CPI inflation in country F is always higher under CPI inflation targeting, after the first quarter (fig.1). Furthermore, in country H, output price inflation is higher under output price inflation than under CPI inflation targeting. The opposite order applies to country F. The real exchange rate depreciation soon after its initial appreciation under output inflation targeting does not seem to significantly affect inflation dynamics, although it contributes to the inflationary consequences of the shock.

Interpreting the results

The explanation of these results lies in the dynamics of real marginal costs. The negative labour productivity shock dominates all other effects in the home economy and implies that the domestic firms' marginal costs increase; but then the real marginal costs gradually return to the steady state value as the effects of the shock vanish. In contrast, in the foreign country the initial jump of the real marginal costs can be either positive or negative. For intermediate good producers the real marginal costs decrease because of the fall in foreign real wages. For a final-good producer these effects are offset by rises in prices of imported intermediate goods. As a consequence, we observe that real marginal costs rises for the foreign producers who sell their good in country F. However, for the final good exporters who can benefit from the real exchange rate appreciation the real marginal costs initially fall. It is important to note that the initial gap between output price inflation and CPI inflation induces a rise in foreign firms' real marginal costs. Therefore, whatever is the direction of the initial jump foreign firms' real marginal costs rise. This pushes the foreign producers to raise prices to protect their profits against the rise in the marginal costs. The whole story can be better understood through an example Let's consider a firm in country F that produces an intermediate good and sell it in the its country. The firm's costs are given by wages paid to foreign workers. Firms deflate nominal wages by the price they charge in foreign market, while workers evaluate real wages in terms of CPI. The shock produces a fall in real wages and a rise in CPI inflation. Foreign workers increase nominal wages to protect their real wages against the CPI inflation. This turns out in a rise in firms' real marginal costs that after the initial fall are pushed above the steady state value. Firms react to the increase in real marginal costs raising prices to protect their profits. A similar explanation also applies to the final good producers. In other words, we can anticipate that in country F the domestic inflation is initially lower than the imported inflation but this order is reversed after few quarters (three or four quarters depending on the policy rule). Similarly, in the home country domestic output inflation is the main source of CPI inflation only at the beginning since imported inflation becomes relatively more significant within four quarters. When, in the home country, the monetary authorities target output price inflation, producers anticipate that after the initial aggressive response monetary policy in the home country will be less restrictive than under CPI inflation targeting. The opposite applies to the foreign country. The forward looking behaviour of producers then explains why domestic output inflation and CPI inflation are higher in the home country when output price inflation is targeted, and in the foreign country when CPI inflation is targeted.

3.1.2 Rules that include the real exchange rate

In introducing the real exchange rate term in policy rules we aim to pursue two different results. Firstly we want to establish how important is the depreciation of the exchange rate after its initial appreciation to explain CPI inflation in home country. Then we are interested in evaluating whether policy rules that include the real exchange rate or changes in the real exchange rate perform better than rule that does not provide any explicit role for the real exchange rate.

Inflationary effects of the real exchange rate depreciation

The first experiment aims to evaluate the contribution of the real exchange rate depreciation in producing higher CPI inflation in home country. A reasonable objection to the different outcomes in terms of CPI inflation when the monetary authorities target output price inflation rather than CPI inflation is that most of the excess inflation comes from the real exchange rate depreciation after its initial appreciation. Under policy rule (3) monetary authorities target output price inflation and the real exchange rate. We assume that nominal interest rates respond quite strongly to deviations of the exchange rate from the steady state (we set this parameter equal to 1). We obtain a pattern for the real exchange rate that does not differ significantly from policy rule (2) in which nominal interest rates respond only to CPI inflation. However, CPI inflation under a policy rule (3) that targets both output price inflation and the real exchange rate is similar to CPI inflation under a rule that targets only output price inflation. This allows us to rule out the real exchange rate as the main source of CPI inflation even in the first quarters after the shock and to conclude that the different behaviour of inflation depends on which inflation rate monetary authorities choose to target.

Targeting the real exchange rate

Policy rule (4) targets both output price inflation and the real exchange rate. Nominal interest rates respond to domestic inflation as in policy rule (1) but we have added an extra term to target also the real exchange rate. Similarly, policy rule (5) differs from policy rule (2) since it targets both CPI inflation and the real exchange rate rather than CPI inflation only. Our findings reveal that policy rules that explicitly include the real exchange rate perform better in reducing inflation as long as the real exchange rate target contributes to the tightening of monetary policy. In fact, the variance of inflation decreases in foreign country where the exchange rate targeting implies a more aggressive response to the inflationary effects of the shock. In contrast, the variance of inflation increases in home economy where the exchange rate stabilisation requires a relatively less aggressive rise in nominal interest rate. The rise in variance of inflation in home country reveals that the easing in monetary policy dominates the benefits coming from the real exchange rate stabilisation

Nominal interest rate responding to changes in the real exchange rate

Under policy rule (6) nominal interest rates respond to output price inflation and changes in the real exchange rate. This rule takes into account more explicitly that after an easing in monetary policy due to the real exchange rate appreciation, nominal interest rates have to rise to offset the inflationary consequences of the real exchange rate depreciation. In other words, rather than target the real exchange rate monetary authorities offset the additional source of inflation due to the real exchange rate depreciation. The CPI inflation variance under this rule is lower than under rules in which nominal interest rates respond either to output price inflation only (1) or both to output price inflation and the real exchange rate levels (4). The reason of these findings lies in the rise of nominal interest rates, which is more aggressive than under the other two policy rules. As explained above the exchange rate channel does not seem to represent a main source of inflation. However, responding to the exchange rate changes monetary authorities respond more aggressively to an inflationary shock and this induces a fall in variance of inflation. Finally, with policy rule (7) nominal interest rates respond to CPI inflation and changes in the real exchange rate. Although the variance of inflation under this rule is slightly lower, the behaviour of CPI inflation does not differ significantly from policy rules that target CPI inflation alone (2). Our findings are consistent with results of other studies. For example Taylor (1999) finds that a policy rule reacting to the exchange rate does not yield a greater improvement in performance**.** The main explanation for this result lies on the fact that the initial appreciation of the real exchange rate is much smaller under CPI inflation targeting than under output price inflation targeting. It implies that the subsequent depreciation has negligible effects on nominal interest rate under this policy rule.

3.1.3 GDP volatility

After the labour productivity shock the GDP falls in country H (fig.3). Then a GDP stabilising policy requires a monetary expansion, while the inflationary consequences of the shock move the monetary authorities in the opposite direction. Therefore, after a supply shock monetary authorities in home country face a trade-off between inflation and output stabilisation. As already explained, in our paper we do not aim to calibrate an optimal monetary policy rule. However, even the very simply rules we are considering provide some useful insights. Policy rules that target output price inflation imply a relatively aggressive monetary response when the shock occurs. However, after this initial reaction monetary authorities are less aggressive against inflation than when they target CPI inflation. As a consequence GDP initially falls more heavily under policy rules that target output price inflation but it also converges faster to its steady state value. For example, policy rule (1) responding to output price inflation alone, produces the heaviest initial fall in home GDP which is, however, highest under this rule after five quarters. The properties of these different rules in stabilising GDP derive from the combination of these two effects.

3.2 Domestic demand shock

Before discussing the effects of a shock to domestic consumption under different policy rules, it is useful to outline the general implications of such a shock in the economy. A domestic real demand shock pushes up output; consequently labour demand as well as real wages increase[9]. The rise in real wages increases real marginal costs and induces firms to raise prices to protect their profits. Under the set of rules we consider, the monetary authorities, in order to offset the inflationary consequences of the shock, raise nominal interest rate such that real interest rates rise. Since, during the shock, consumers run down the holding of financial wealth, when the shock passes private sector consumption falls below the steady state level. Output falls below its steady state level while inflation is still positive because of the inertia in price adjustments. Since monetary policy in the home country is relatively more aggressive, the real exchange rate appreciates. The real exchange rate appreciation raises the relative price of home goods and this helps to offset the destabilising effects of the shock. In the foreign country, output increases because of the rise in exports while consumption initially falls because of the rise in real interest rates. In fact, the excess demand for foreign goods and the rise in imported inflation induces the foreign monetary authorities to raise interest rates in order to offset the inflationary consequences of the shock. The initial fall in foreign consumption helps to offset the inflationary effects of a rise in demand for foreign goods. However, because of the increase in private savings when the shock passes foreign consumption rises above its steady state level.

3.2.1 Targeting output price inflation vs CPI inflation

The main conclusions we have drawn for a supply shock are also valid for a shock to private sector demand. When targeting output price inflation (policy rule (1)), the monetary authorities initially react to the shock with an aggressive monetary policy (fig.7). In the first year, real interest rates are in fact higher under output price inflation targeting than under CPI inflation targeting (policy rule (2)). However the order is reversed after this period. Notwithstanding the initial aggressive monetary policy both CPI inflation and domestic

[9] The rise in real wages is easily explained in terms of labour-leisure choice. Since consumption rises consumers wish to increase their consumption of leisure as well as goods.

inflation are higher under output price inflation targeting (fig. 2). In the very first quarter CPI inflation is practically identical under these two rules because of the deflationary effects of the real exchange rate appreciation. As in the case of a supply shock, these results can be understood by considering the fact that domestic output price inflation is higher than imported inflation only in the first part of the shock. This order is reversed after about a year. Then, forward looking producers anticipate that targeting output price inflation monetary policy carry out a relatively less aggressive monetary policy even though at the beginning they seem to react more strongly.

Interpreting the results

Once again, the simplest explanation of these results lies in the dynamics of marginal costs. In the home country workers initially raise real wages as a consequence of their labour-leisure choice. When the shock passes and labour demand falls real wages and consequently firms' real marginal costs return gradually to their steady state values. Indeed, the return to the steady state is relatively fast since as explained above, consumption and labour demands fall below their steady state values. In the foreign country, after the initial jump, real wages decrease. This is due the fact that the consumers' labour-leisure choice works in the opposite direction such that the fall in consumption due to the rise in real interest rates induces workers to reduce real wages. However, when consumption increases workers demand for higher real wages. In other words, workers' behaviour initially reduces the rise in foreign firms' marginal costs, before contributing to the persistence of the inflationary effects of the shock.

3.2.2 Targeting the real exchange rate

Under policy rule (4) nominal interest rates respond to deviations of output price inflation and the real exchange rate from their targets. In the home economy the variance of inflation is lower than in case in which nominal interest rates respond only to output price inflation. However, the fall in variance of inflation does not come from the relatively less volatile exchange rate under policy rule (4). The real exchange rate, after an initial appreciation depreciates with respect to its steady state value[10]. Although less strong than the initial

[10] The depreciation of the real exchange rate reflects the financial conditions of the countries. Foreign consumers hold positive financial assets and their consumption is above the steady state value. Both variables converge

appreciation, the real exchange rate depreciation is more persistent. Therefore, the real exchange rate targeting implies after an initial easing in domestic monetary policy a slower return of the nominal interest rate to the steady state value. In other words, the more aggressive monetary policy reduces the variance of inflation in the home economy. It is interesting to notice that in the foreign country, despite of the more aggressive rise in nominal interest rates, CPI inflation is always higher with policy rules that target output price inflation and the real exchange rate (4) than with rules (1) that target output price inflation alone. This is explained considering the way in which optimal prices are set. In fact, forward-looking producers' anticipate that the real exchange rate targeting will implies a relatively less aggressive monetary policy. However, these results are reversed when monetary authorities target CPI inflation and the real exchange target (policy rule (5)). This result is simply due to the fact that the real exchange rate depreciation is insignificant and therefore monetary policy does not induce expectations of lower inflation.

Nominal interest rates responding to changes in the exchange rates

When nominal interest rates respond to output price inflation and to changes in the real exchange rate (policy rule (6)) the variance of inflation in the home economy is lower than when output price inflation alone or both output price inflation and the level of the real exchange rate are targeted. As in the case of a supply shock, responding to changes in the real exchange rate implies that the monetary authorities carry out a relatively more aggressive monetary policy. In fact, after the initial appreciation the real exchange rate rapidly depreciates. Responding to changes in the real exchange rate, nominal interest rates rise. In contrast, the subsequent appreciation of the real exchange rate is very slow and its impact on the nominal interest rate is negligible. For similar reasons the variance of inflation in foreign country increases. The same interpretation and the same conclusions apply also to policy rule (7) under which the monetary authorities target CPI inflation and respond to changes in the real exchange rate.

slowly to their steady state value because of the overlapping generation structure. The opposite happens to home

3.2.3 GDP volatility

The variances of GDP due to positive domestic demand shock (fig.4)under different policy rules can be interpreted as in the case of a supply shock. However, we should note that a demand shock does not raise an output inflation stabilisation trade-off. Both output and inflation increase after the shock, although the shock does not show any persistence on output while inflation return slowly to its steady state value. The variance of GDP in the home country is minimised when policy rule (6) is implemented. As explained, this rule implies the most aggressive response of monetary policy to an inflationary shock; since a demand shock move inflation and output move in the same direction this rule helps to stabilise GDP as well as inflation. It is also interesting to notice that the variance of GDP is slightly lower under policy rule (1) than policy rule (2). In other words, a pure output price inflation targeting produce a lower variance than when CPI inflation is targeted. The explanation for this results lies on the fact that the rise in real interest rate under output price inflation targeting is initially more aggressive than under CPI inflation targeting. Although as explained, this initial monetary policy response is not sufficient to generate expectation of lower future inflation, the rise in real interest rate helps to offset the effect of the shock on output. In fact, most of the variability in output is due to it initial jumping when the shock occurs. When the shock passes output immediately falls. Since under output price inflation monetary policy is aggressive in the very first part of the shock this helps to offset the initial rise in output and reduce its variance.

4. Conclusions

In this paper we have considered a two-country model. Each country is characterised by several different sources of nominal inertia: sluggish adjustments in nominal wages, in prices of domestic and foreign intermediate and final goods. Moreover, we have broken down the hypothesis of complete exchange rate pass-through introducing local currency price setting. Finally, market power allows firms in both sectors to price discriminate between domestic and foreign markets. In this framework we have analysed the stabilising properties of different inflation targeting rules. Our findings reveal that the variance of inflation in the home country

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consumers whose negative financial assets position forces them to save and pay back their debts.

is lower under CPI inflation targeting than output price inflation targeting. As shown in the paper, the explanation of these results lies in the interaction between all the different sources of inertia. Finally, we have found a limited role for the exchange rate in affecting the stabilising properties of our policy rules.

References

Aoki, Kosuke (2001). "Optimal Monetary Policy Responses to Relative-Price Changes*". Journal of Monetary Economics,* vol. 48, pp: 55-80.

Ball, Laurence, "Policy Rules for Open Economies" in John B. Taylor, ed. *Monetary Policy Rules*. University of Chicago Press, Chicago.

Benigno, Gianluca and Pierpaolo Benigno (2001b). "Price Stability in Open Economies". *Mimeo Bank of England and New York University.*

Benigno, Pierpaolo (2001). "Price Stability with Imperfect Financial Integration". *Mimeo*, New York University.

Betts, Caroline and Michael B. Devereux (1996). "Exchange Rate Dynamics in a Model of Pricing to Market". *European Economic Review*, vol 40: pp 1007-1021.

Calvo, Guillermo A. (1983). "Staggered Prices in a Utility Maximising Framework". *Journal of Monetary Economics*, vol. 12, pp: 383-398.

Clarida Richard, Jordi Galì and Mark Gertler (2001). "Optimal Monetary Policy in Open Economies: an Integrated Approach". *American Economic Review*, Paper and Proceedings: pp 248-252.

Corsetti, Giancarlo and Luca Dedola (2001), "Macroeconomic of International Price Discrimination". *Manuscript*.

Corsetti, Giancarlo and Paolo Pesenti (2001). "The International Dimension of Optimal Monetary Policy". *NBER Working Paper* No. 8230.

Devereux, Michael B. and Charles Engel (1998). "Fixed vs. Floating Exchange Rate: How Price Setting Affects the Optimal Choice of Exchange-Rate Regime". *Manuscript*.

Devereux, Michael B. and Charles Engel (1999). "The Optimal Choice of Exchange-Rate Regime: Price-Setting Rules and Internationalised Production". *Mimeo.* The University of British Columbia.

ECB (2001). "Euro Area Statistics". Monthly Bulletin, August 2001, pp: 1-83.

Faraqee, H., D. Laxton and S. Symansky (1997). "Government Debt, Life-Cycle Income and Liquidity Constraints: Beyond Approximate Ricardian Equivalence". IMF Staff Papers, vol. 44, n°3, pp: 374-382.

Galì Jordi and Tommaso Monacelli (2000). "Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy". *Mimeo, Pompeu Fabra*, November.

Galì, Jordi (1994). "Government Size and Macroeconomics Stability". European Economic Review, vol. 28, pp: 117-132.

Galì, Jordi and Mark Gertler (1999). "Inflation Dynamics: A Structural Econometric Analysis". Journal of Monetary Economics, vol. 44: pp 195-222.

Galì, Jordi, Mark Gertler and J. David Lopez-Salido (2001). "European Inflation Dynamics". Forthcoming in *European Economic Review*.

Ghironi, Fabio (2000). "Towards New Open Economy Macroeconomics". *Mimeo*, Federal Reserve Bank of New York.

Goodfriend, M and R. King (1997). "The New NeoClassical Synthesis and the Role of Monetary Policy". *NBER Macroeconomics Annual*, pp 231-282.

Kollman, R (1998). "International Financial Markets and G& Business Cycles: A General Equilibrium Approach with Money and Nominal Rigidities". *Mimeo,* University Paris XII.

Lane, Philip (2001). "The New Open Economy Macroeconomics: A Survey”. *Journal of International Economics*, (August). (Also mimeo at Trinity College Dublin and CEPR Discussion Paper #2115, March 1999).

Leith, Campbell and Jim Malley (2002). “Estimated General Equilibrium Models for the Evaluation of Monetary Policy in the US and Europe”. Mimeo. University of Glasgow.

Leith, Campbell and Simon Wren-Lewis (2002). “Interaction Between Monetary and Fiscal Policy Under EMU”. *Mimeo*, University of Glasgow.

Mankiw, Gregory and Ricardo Reis (2002). “What Measure of Inflation Should a Central Bank Target?”. *ECB Working Paper* n° 170.

Monacelli, Tommaso (1999). “Open Economy Policy Rules under Imperfect Pass-Through”. *Mimeo New York University*.

Obstfeld Maurice and Kenneth Rogoff (1995). “Exchange Rate Dynamics Redux”. *Journal of Political Economy*, vol. 103, pp: 624-660.

Obstfeld Maurice and Kenneth Rogoff (1996). “Foundations of International Macroeconomics”, MIT Press, Cambridge.

Obstfeld Maurice and Kenneth Rogoff (2001). “Global Implications of Self-Oriented National Monetary Rules”. Forthcoming in *Quarterly Journal of Economics*.

Rotemberg,J.J. and M.Woodford (1998). “An Optimisazion-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version”. *NBER Technical Working Paper* n° 233.

Smets, Frank and Raf Wouters (2001). “Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy. *Mimeo*. European Central Bank.

Sutherland, Alan (2001), “Incomplete Pass-Through and the Welfare Effects of Exchange Rate Variability”. *Mimeo, St. Andrew's.*

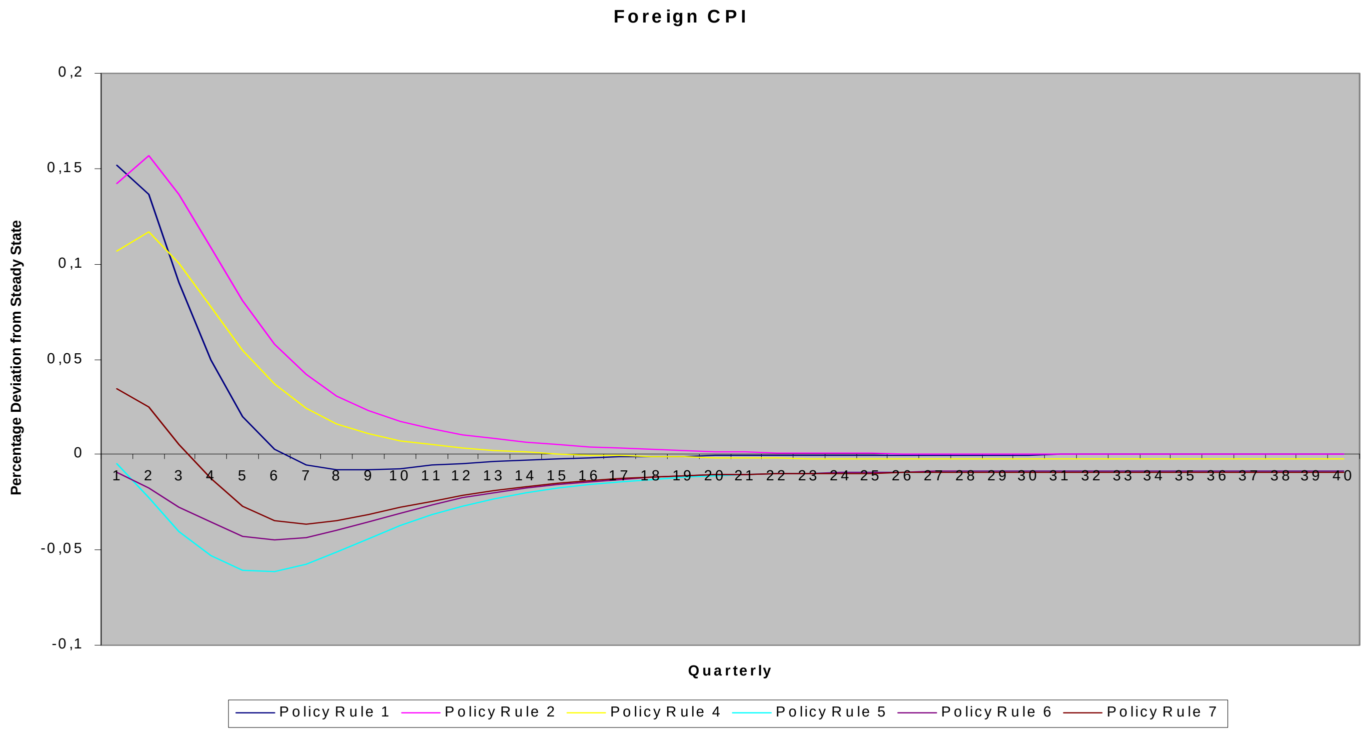
Taylor, John B. (1993). “Discretion Versus Policy Rules in Practise”. *Carnegie-Rochester Conference Series on Public Policy*, vol. 39, pp: 195-214.

Taylor, John B. (1999). “Robustness and Efficiency of Monetary Policy Rules as Guidelines for interest Rate Setting by the European Central Bank”. *Journal of Monetary Economics*, vol. 43, pp: 665-679.

Taylor, John B. (2001). “The role of the exchange rate in monetary policy rules”. *American Economic Review*, Paper and Proceeding vol. 91, pp: 263-267.

Woodford, M. (2001). “The Taylor Rule and Optimal Monetary Policy” *American Economic Review*, vol. 91(2), pp: 232-237.

Fig.1. CPI inflation. Supply Shock



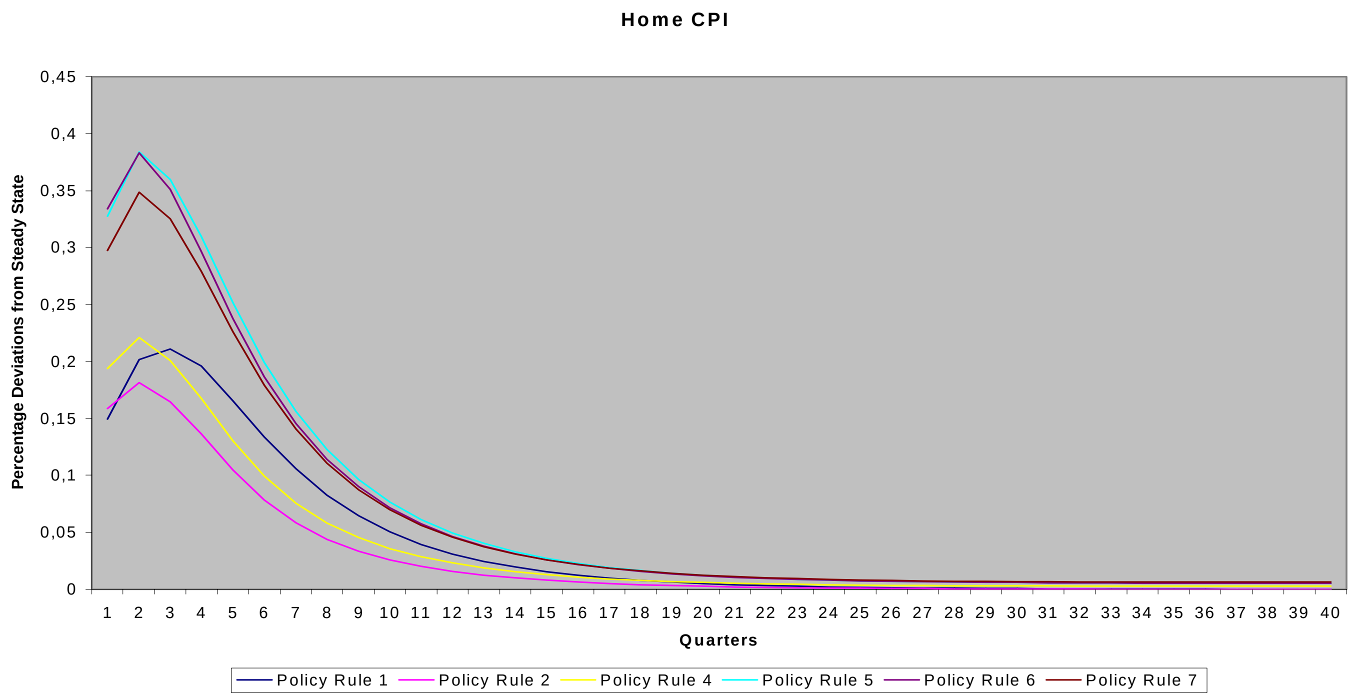
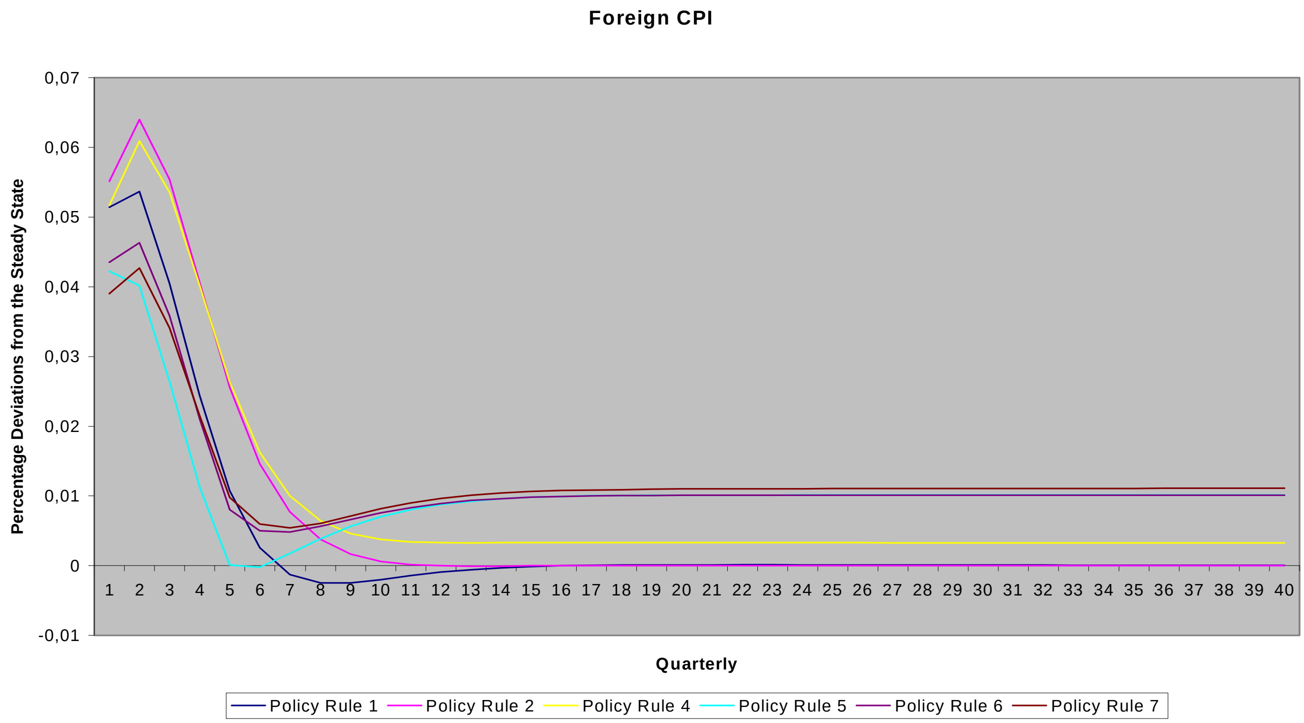


Fig.2 CPI Inflation. Demand Shock



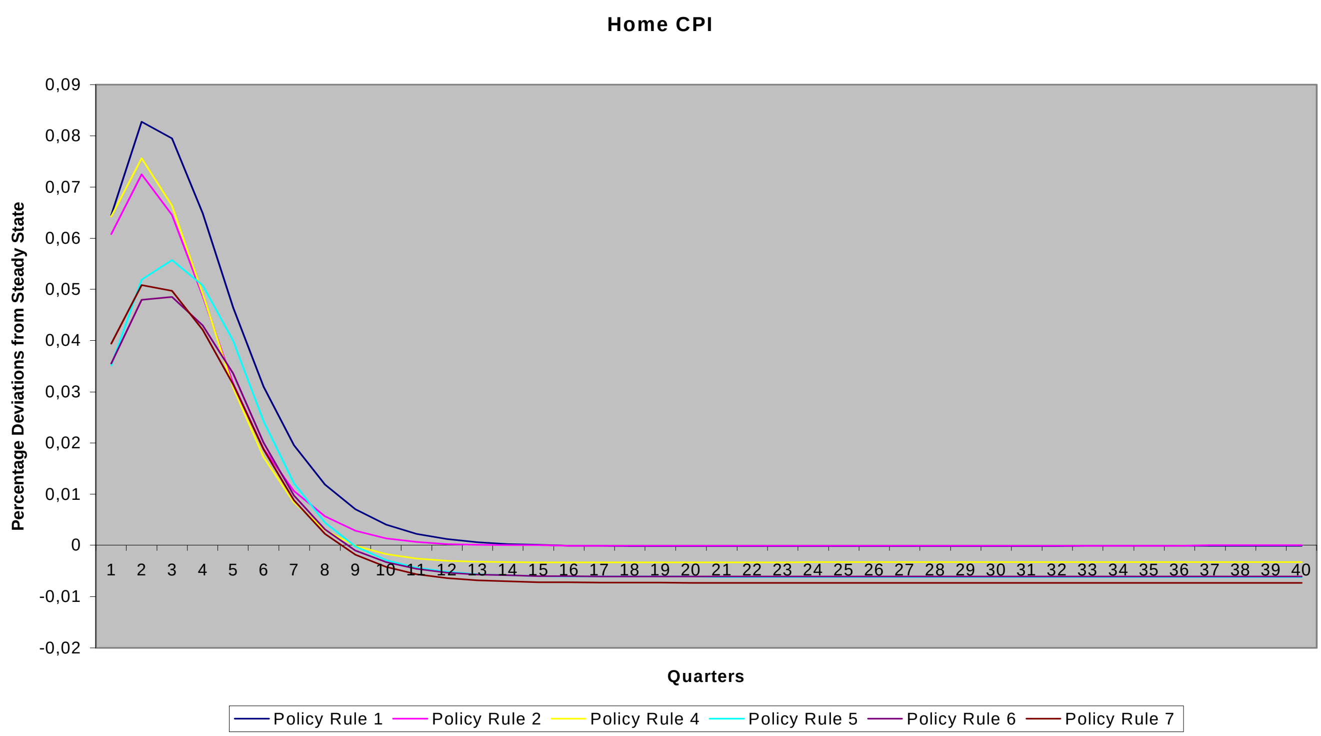
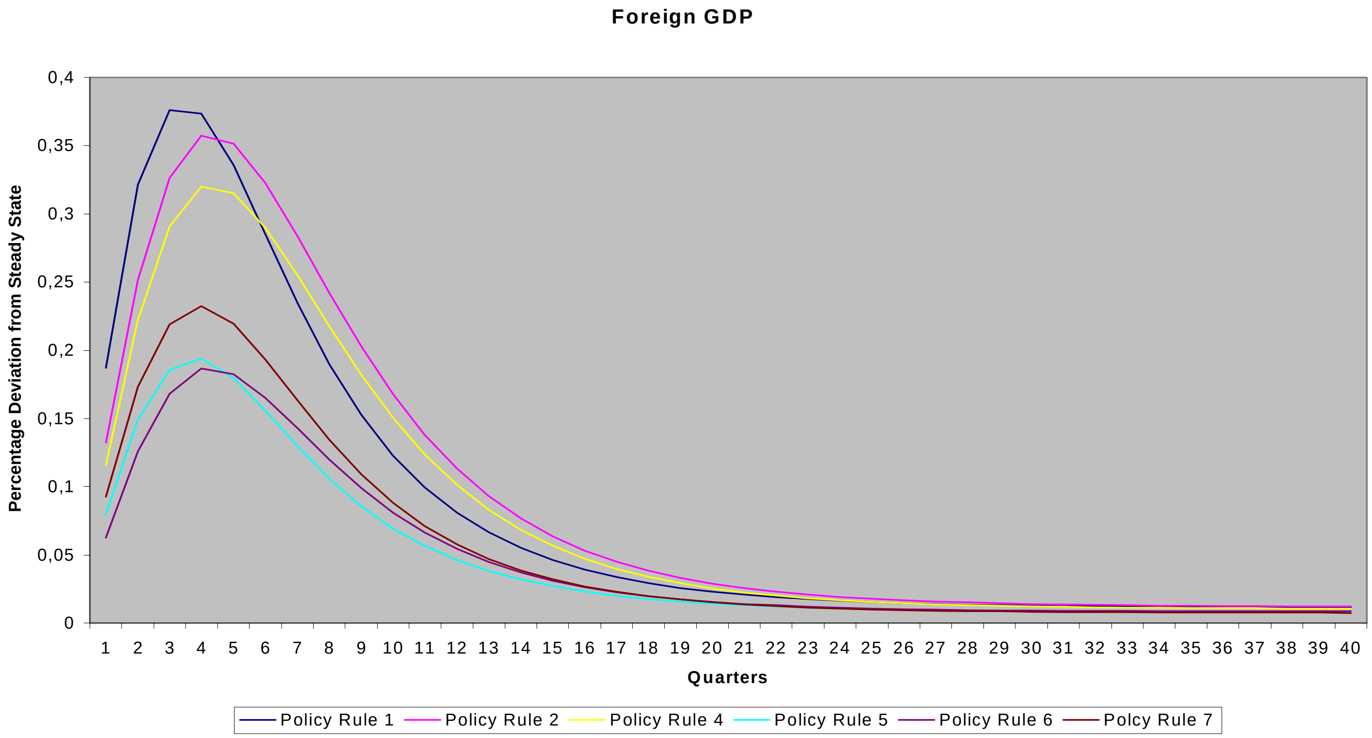
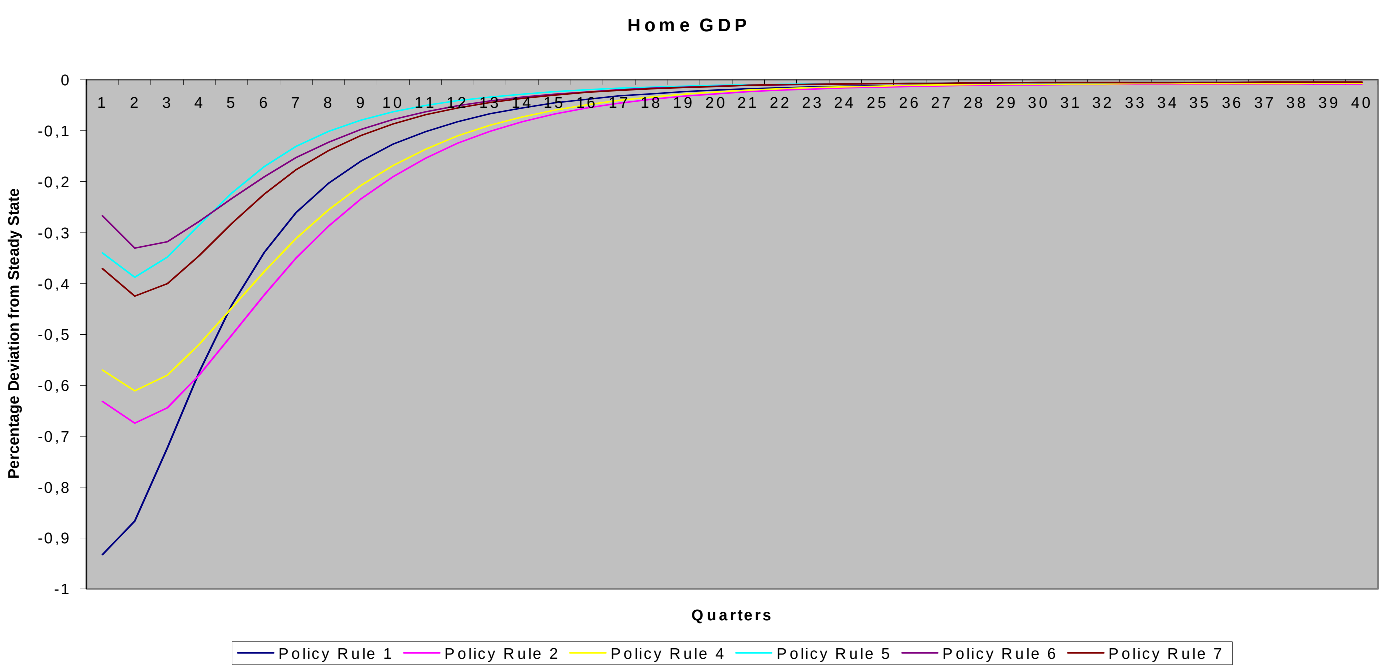


Fig.3 GDP. Supply Shock





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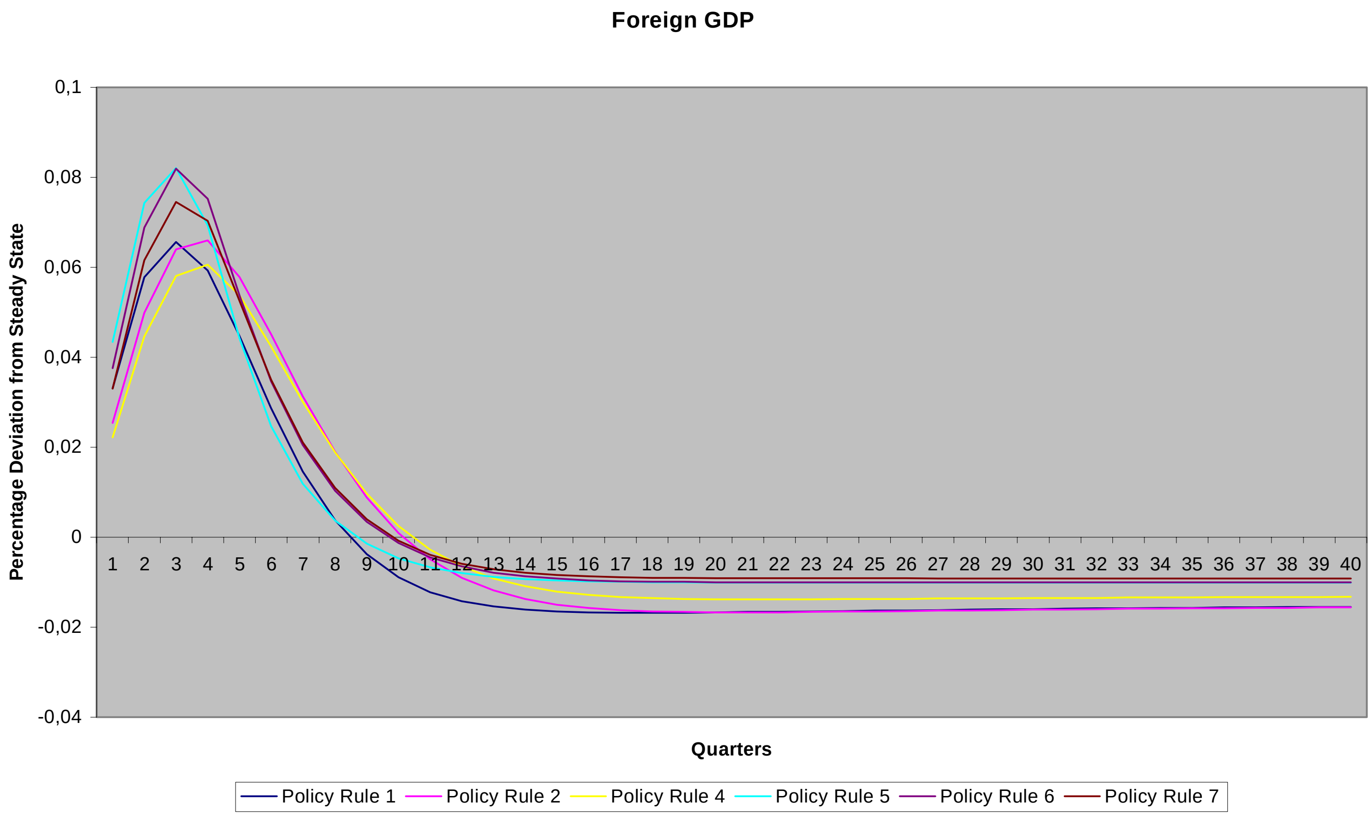


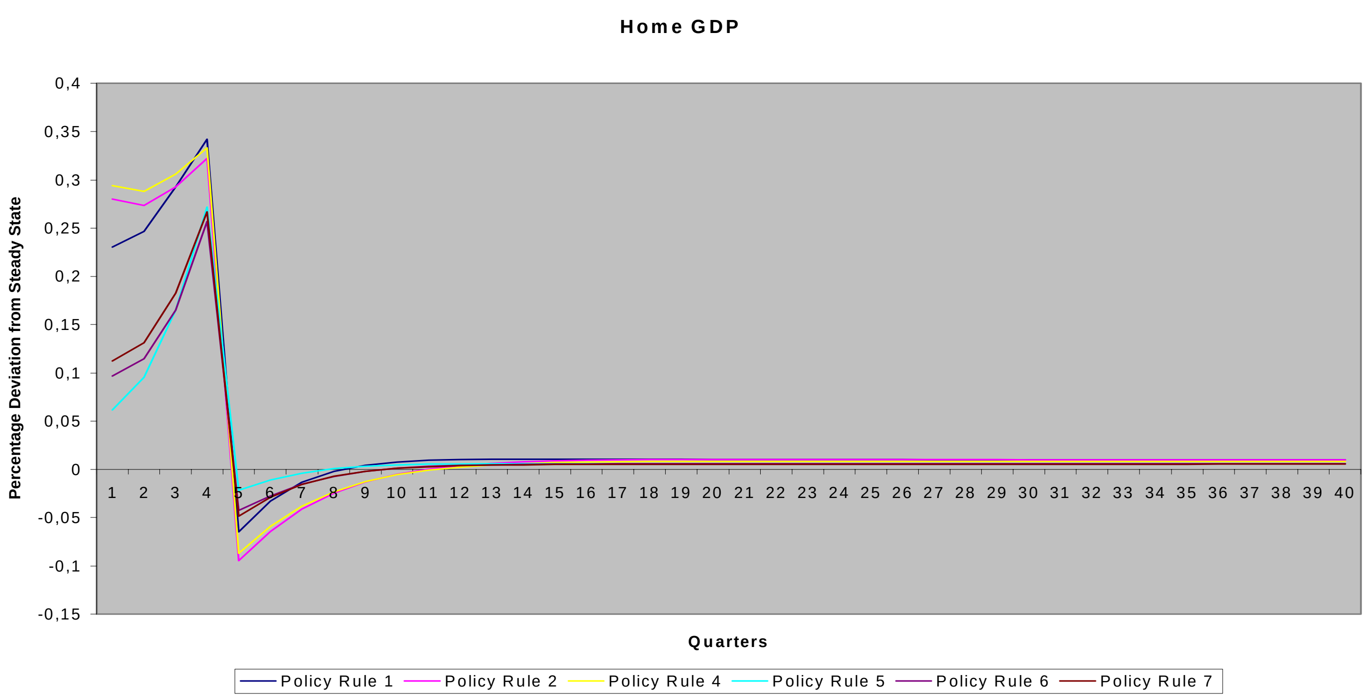
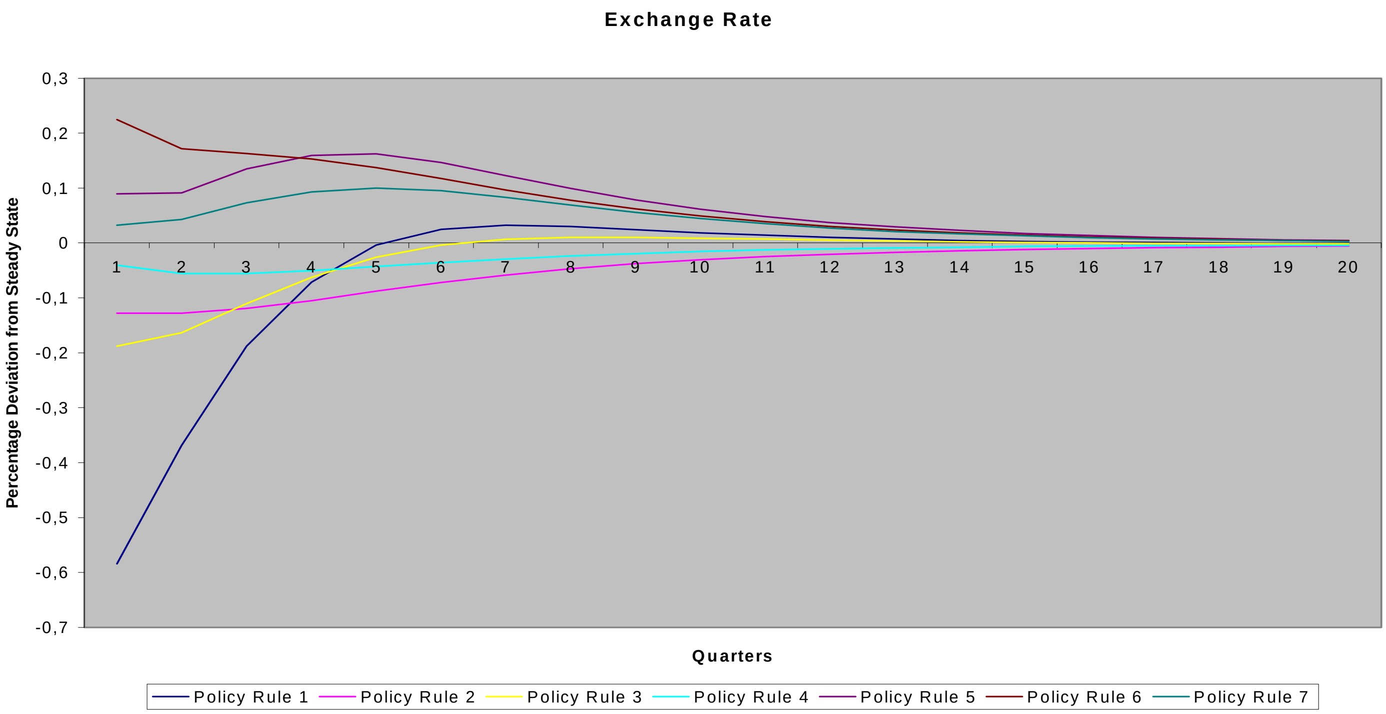


Fig 5 Exchange Rate. Supply and Demand Shock



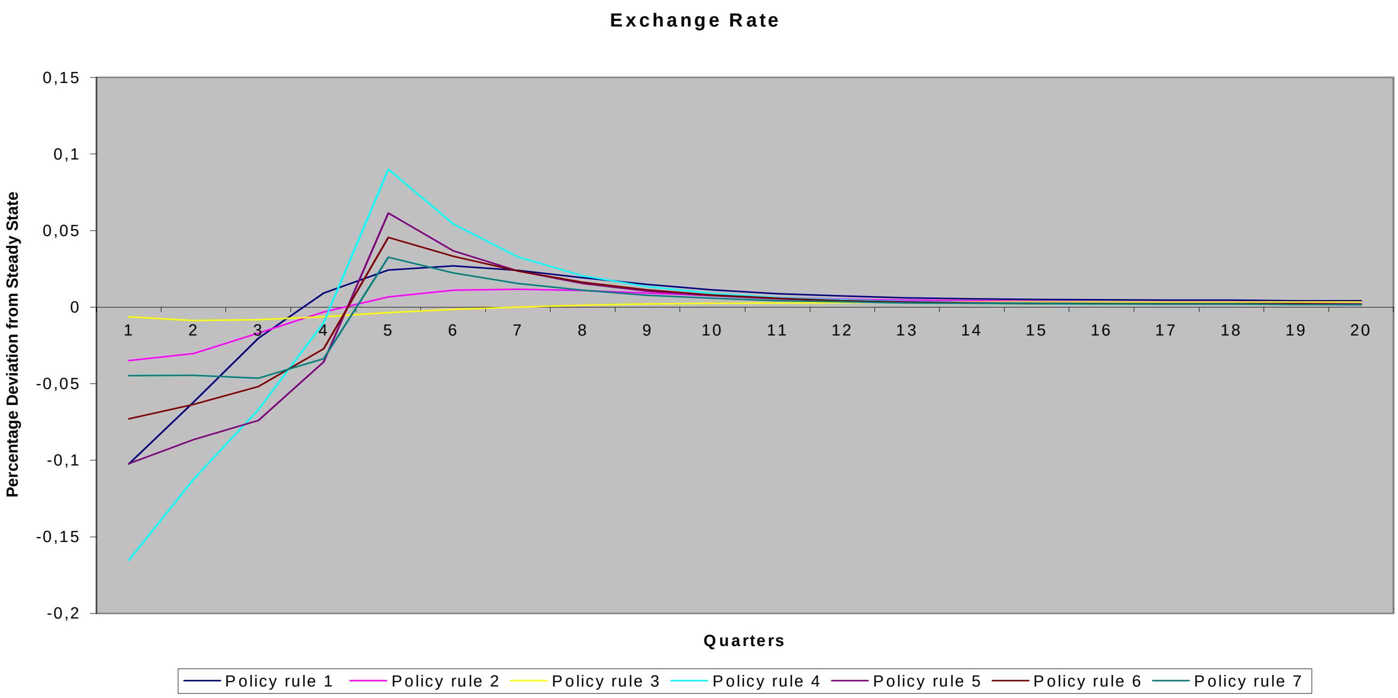
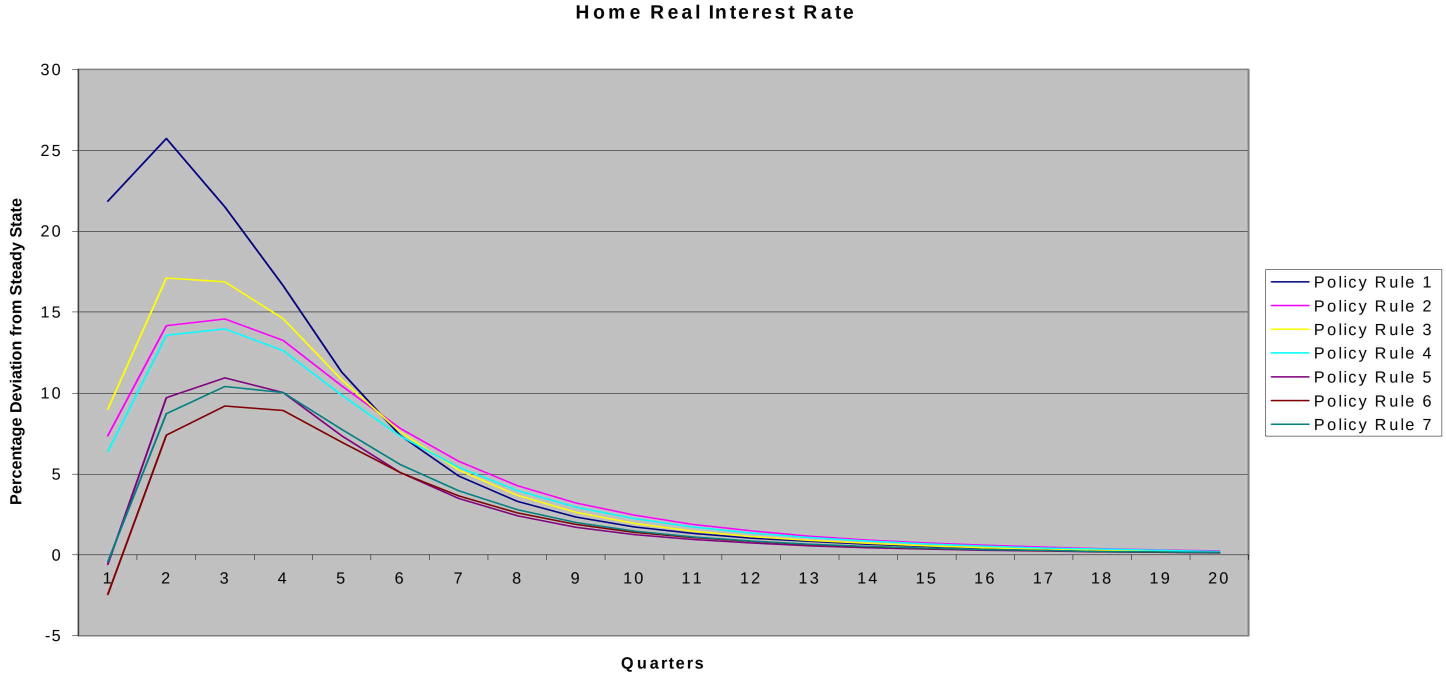


Fig.6 Real Interest Rate. Supply Shock



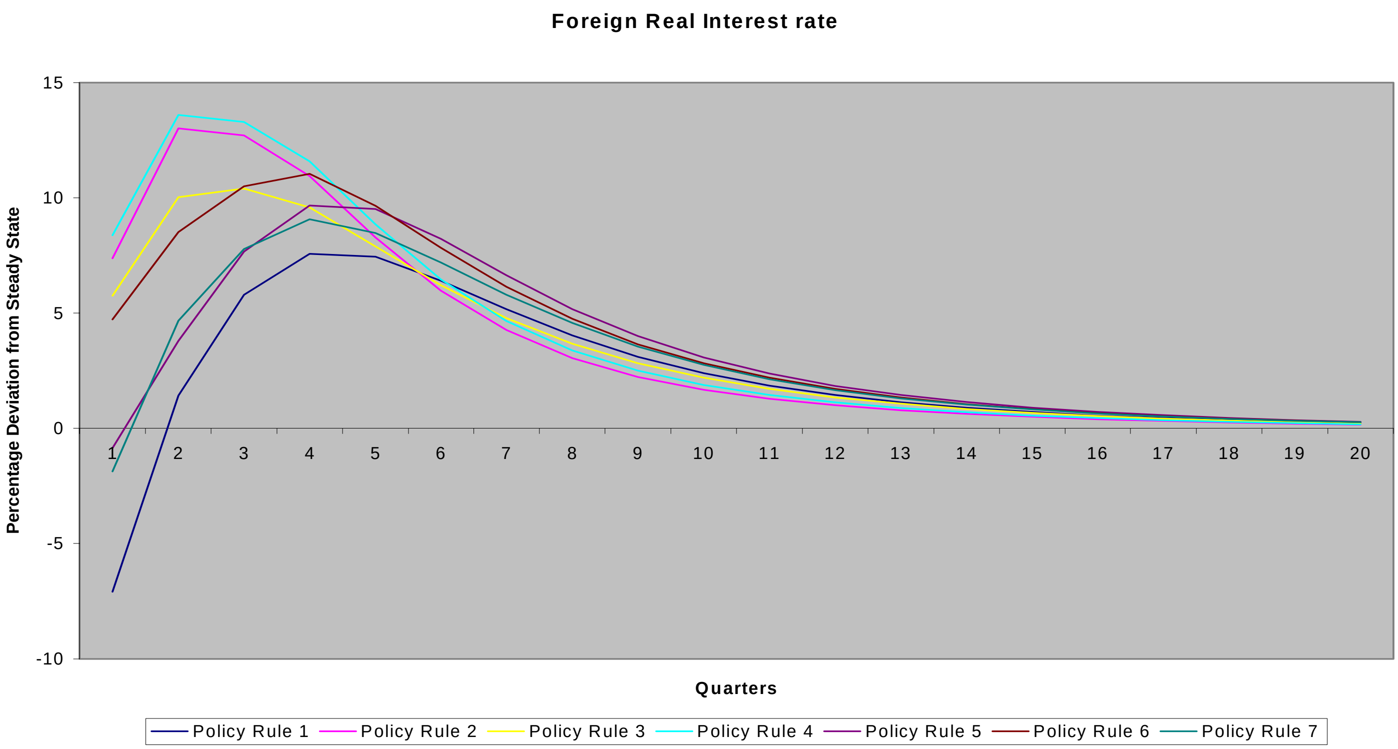
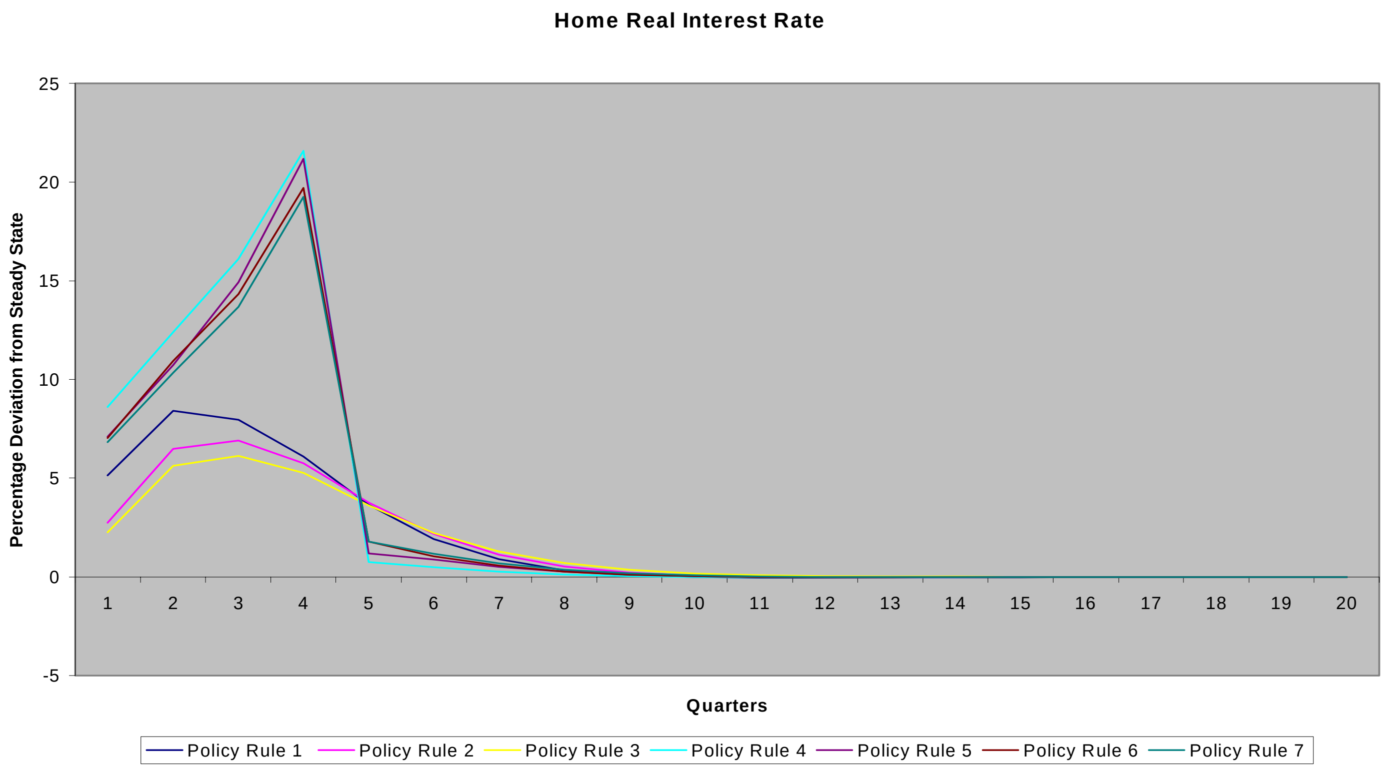


Fig.7 Real Interest Rate. Demand Shock



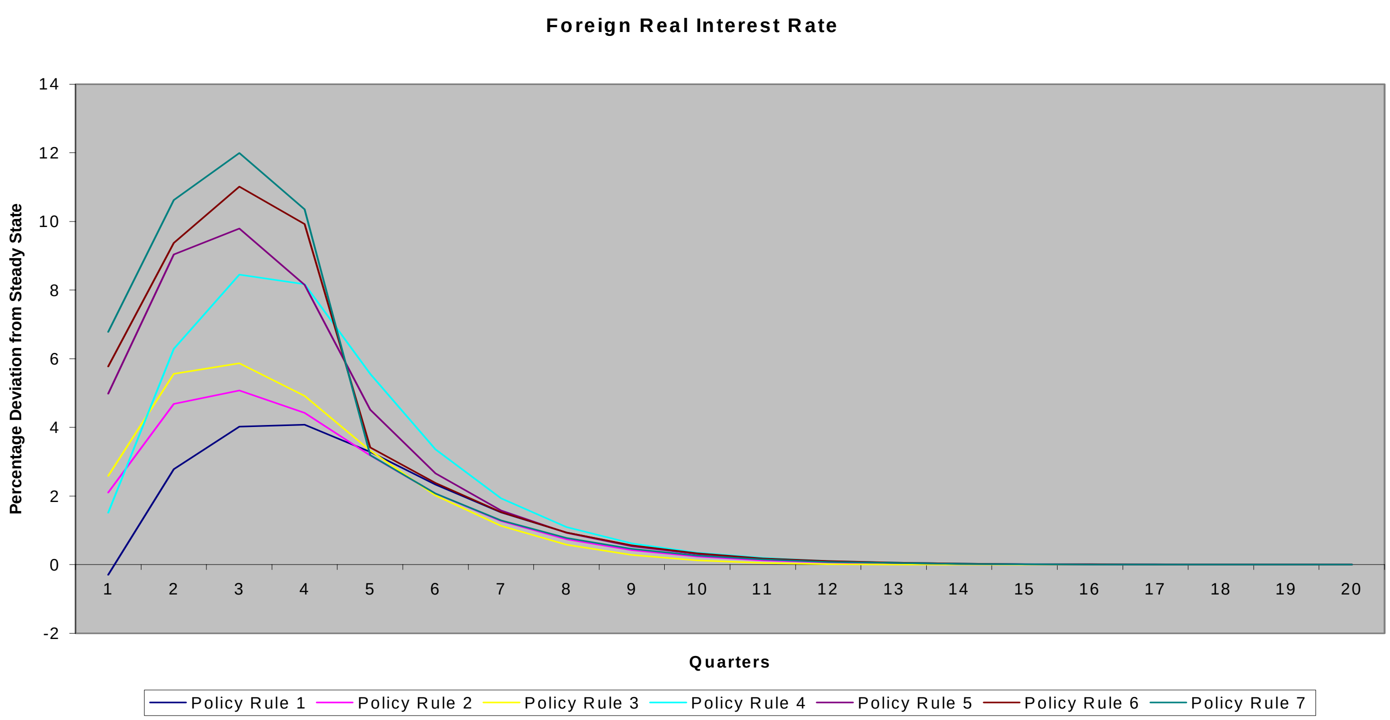


Table 1. CPI inflation and GDP variance. Home Demand Shock

	Home Cpi* Inflation variance	Foreign Cpi* Inflation variance	Home GDP variance	Foreign GDP variance
Policy Rule 1	0.40403	0.12652	0.32543	0.02303
Policy Rule 2	0.27183	0.20493	0.36134	0.02558
Policy Rule 3	0.33282	0.16743	0.36151	0.02068
Policy Rule 4	0.29267	0.19739	0.38936	0.020536
Policy Rule 5	0.20892	0.11694	0.11581	0.024687
Policy Rule 6	0.16785	0.14391	0.11934	0.02594
Policy Rule 7	0.1821	0.14145	0.1387	0.02229

∗ Annualised Inflation

Table 2. CPI inflation and GDP variance. Home Supply Shock

	Home Cpi* Inflation variance	Foreign Cpi* Inflation variance	Home GDP variance	Foreign GDP variance
Policy Rule 1	3.52037	0.8461	2.96468	0.77721
Policy Rule 2	2.06595	1.42765	2.40115	0.81448
Policy Rule 3	3.73472	0.59864	2.15821	0.65452
Policy Rule 4	3.15394	0.74879	1.93531	0.65176
Policy Rule 5	10.40888	0.4334	0.59207	0.20935
Policy Rule 6	9.90713	0.26446	0.51578	0.21047
Policy Rule 7	8.51468	0.20316	0.81151	0.30724

∗ Annualised Inflation

Table 3. Parameters and Steady State values

Parameters	Values	Steady State	Values
ρ	0.993	y	1
θ	11	c	0.77
ϕ	11	g	0.23
ι	0.9929	t	0.25
z_I	0.66	b	2.8
z_P	0.66	m	1.03
z_w	0.75	l	0.5
χ	0.01	w	1.33
γ	0.6	I_H	1.548
α	2/3	I_H^*	0.774
β	2/3	r	0.075
A	6.9	e	1
B	1.49	V	51.68

Appendix A

Log-linear version of the model for the home country

Money demand equation:

$$\hat{m}_t = \hat{c}_t - \frac{\hat{r}_t}{1+\delta} - \frac{\pi^e_{t+1}}{\delta} \tag{1A}$$

where δ is the steady state value of the interest rate.

Domestic demand for home final goods:

$$\hat{y}_{H,t} = \gamma\left(\frac{\overline{c}}{\overline{y}_{H,t}}\hat{c}_t + \frac{\overline{g}}{\overline{y}_{H,t}}\hat{g}_t + (1-\gamma)\hat{s}_t\right) \tag{2A}$$

Foreign demand for home final goods

$$\hat{y}^*_{H,t} = (1-\gamma)\left(\frac{\overline{c}^*}{\overline{y}^*_{H,t}}\hat{c}^*_t + \frac{\overline{g}^*}{\overline{y}^*_{H,t}}\hat{g}^*_t + \gamma\hat{s}^*_t\right) \tag{3A}$$

World demand for home final goods:

$$\hat{y}^W_{H,t} = \hat{y}_{H,t} + \hat{y}^*_{H,t} \tag{4A}$$

Domestic demand for home intermediate goods:

$$\hat{I}_{H,t} = (1-\beta)\hat{w}_t + (\alpha\beta-1)\hat{R}_{H,t} + \beta(1-\alpha)\hat{R}_{F,t} - (1-\beta)\hat{A}_t - (1+\beta)\hat{D}_t + \hat{y}^w_{H,t} \tag{5A}$$

Foreign demand for home intermediate goods:

$$\hat{I}^{*}_{H,t} = (1-\beta)\hat{w}_t + (2\alpha - 2 - \alpha\beta + \beta)\hat{R}^{*}_{H,t} + (1-2\alpha+\alpha\beta)\hat{R}^{*}_{F,t} - (1-\beta)\hat{A}_t - (1+\beta)\hat{D}_t + \hat{y}^{w}_{F,t} \quad (6A)$$

Consumption equation:

$$\hat{c}_t = \frac{1-\iota\rho}{1+\chi}\left[\frac{\overline{m}}{\overline{C}}(\hat{m}_{t-1} - \hat{\pi}_t) + \frac{\overline{V}}{\overline{C}}\hat{v}_t + (1+\delta)\frac{\overline{h}}{\overline{C}}\hat{h}_{t-1} + \delta\frac{\overline{h}}{\overline{C}}\hat{r}_{t-1}\right] \quad (7A)$$

Human wealth equation:

$$\begin{aligned}\hat{v}_t = {} & \frac{\iota}{1+\delta}\hat{v}_{t+1} - \frac{\iota\delta}{(1+\delta)^2}\hat{r}_t + \gamma\frac{\overline{C}}{\overline{V}}\hat{c}_t + \gamma\frac{\overline{G}}{\overline{V}}\hat{g}_t + \gamma\frac{\overline{C}^{*}\overline{\varepsilon}^{R}}{\overline{V}}(\hat{c}^{*}_t + \hat{\varepsilon}_t) + \gamma\frac{\overline{G}^{*}\overline{\varepsilon}^{R}}{\overline{V}}(\hat{g}^{*}_t + \hat{\varepsilon}_t) + \\ & + \frac{\overline{\varepsilon}^{R}\overline{R}^{*}_{H}\overline{I}^{*}_{H}}{\overline{V}}(\hat{\varepsilon}^{R}_t + \hat{R}^{*}_{H,t} + \hat{I}^{*}_{H,t}) - \frac{\overline{R}_F\overline{I}_F}{\overline{V}}(\overline{R}_{F,t} + \overline{I}_{F,t}) - \frac{\overline{\tau}}{\overline{V}}\hat{\tau}_t\end{aligned} \quad (8A)$$

Labour demand equation:

$$\begin{aligned}\hat{l}_t = {} & \frac{1}{\overline{l}}\left(\frac{1-\beta}{\beta}\right)^{\beta}\frac{(\overline{R}_H)^{\alpha\beta}(\overline{R}_F)^{\beta(1-\alpha)}}{(\overline{W}^{R})^{\beta}A^{(1-\beta)}}\overline{Y}^{w}_{H}\left(\alpha\beta\hat{R}_{H,t} + \beta(1-\alpha)\hat{R}_{F,t} - \beta\hat{w}_t - (1-\beta)\hat{A}_t - \beta\hat{D}_t + \hat{y}^{w}_{H,t}\right) \\ & + \frac{1}{\overline{l}}\frac{\overline{I}_{H,t}}{\overline{A}}(\hat{I}_{H,t} - \hat{A}_t) + \frac{1}{\overline{l}}\frac{\overline{I}^{*}_{H,t}}{\overline{A}}(\hat{I}^{*}_{H,t} - \hat{A}_t) +\end{aligned} \quad (9)$$

Tax equation :

$$\hat{\tau}_t = \frac{\delta\overline{b}}{\overline{\tau}}\hat{r}_{t-1} - \frac{\overline{m}}{\overline{\tau}}(\hat{m}_t - \hat{m}_{t-1} + \pi_t) + \frac{\overline{g}}{\overline{\tau}}\hat{g}_t \quad (10A)$$

The terms of trade:

$$\hat{s}_t = \hat{s}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad (11A)$$

$$\hat{s}_t^* = \hat{s}_{t-1}^* + \pi_{H,t}^* - \pi_{F,t}^* \tag{12A}$$

Real wage equation:

$$\hat{w}_t = \frac{(1-\iota z_w)(1-\rho\iota z_w)}{1+\rho(\iota z_w)^2}\left(\hat{k}_t + \mu\hat{l}_t + \hat{c}_t\right) + \frac{\rho\iota z_W}{1+\rho(\iota z_w)^2}\left(\hat{w}_{t+1}^e + \pi_{t+1}^e\right) + \frac{\iota z_W}{1+\rho(\iota z_w)^2}\left(\hat{w}_{t-1} - \pi_t\right) \tag{13A}$$

Price inflation equation of domestic intermediate goods in the home country:

$$\pi_{H,t}^I = \frac{(1+\overline{r}-z_I)(1-z_I)}{\left(1+\overline{r}+\Gamma_{I_H}\right)z_I}\hat{mc}_{I_H,t} + \frac{1}{1+r+\Gamma_{I_H}}\pi_{H,t+1}^{I^e} + \frac{(1+\overline{r})\Gamma_{I_H}}{1+\overline{r}+\Gamma_{I_H}}\pi_{H,t-1}^I \tag{14A}$$

Intermediate goods producer’s real marginal costs in the home country:

$$\hat{mc}_{I_H,t} = \hat{w}_t - \hat{R}_{H,t} - \hat{A}_t \tag{15A}$$

Price inflation equation of domestic intermediate goods in the foreign country:

$$\pi_{H,t}^{I^*} = \frac{(1+\overline{r}-z_I)(1-z_I)}{\left(1+\overline{r}+\Gamma_{I_H}^*\right)z_I}\hat{mc}_{I_H,t}^* + \frac{1}{1+\overline{r}+\Gamma_{I_H}^*}\pi_{H,t+1}^{I^{*e}} + \frac{(1+\overline{r})\Gamma_{I_H}^*}{1+\overline{r}+\Gamma_{I_H}^*}\pi_{H,t-1}^{I^*} \tag{16A}$$

Intermediate goods producer’s real marginal costs in the home country:

$$\hat{mc}_{I_H,t}^* = \hat{w}_t - R_{H,t}^* - \hat{A}_t - \hat{\varepsilon}_t \tag{17A}$$

Ratio between domestic intermediate goods price and consumer price index in the home country:

$$\hat{R}_{H,t} = \hat{R}_{H,t-1} + \pi_{H,t}^I - \pi_t \tag{18A}$$

Ratio between foreign intermediate good price and consumer price index in the home country:

$$\hat{R}_{F,t} = \hat{R}_{F,t-1} + \pi^{I}_{F,t} - \pi_t \tag{19A}$$

Price inflation equation of domestic final goods in the home country:

$$\pi_{H,t} = \frac{(1+\overline{r}-z_P)(1-z_P)}{(1+\overline{r}+\Gamma_H)z_P}\hat{mc}_{H,t} + \frac{1}{1+\overline{r}+\Gamma_H}\pi^{e}_{H,t+1} + \frac{(1+\overline{r})\Gamma_H}{1+\overline{r}+\Gamma_H}\pi_{H,t-1} \tag{20A}$$

Final goods producer's real marginal costs in the home country:

$$\hat{mc}_{H,t} = (1-\beta)\left(\hat{w}^{R}_t - \hat{A}_t\right) + \alpha\beta\hat{R}_{H,t} + (1-\alpha)\beta\hat{R}_{F,t} + (1-\gamma)\hat{s}_t - \beta\hat{D}_t \tag{21A}$$

Final goods producer's real marginal costs in the foreign country:

$$\pi^{*}_{H,t} = \frac{(1+\overline{r}-z_P)(1-z_P)}{(1+\overline{r}+\Gamma^{*}_H)z_P}\hat{mc}^{*}_{H,t} + \frac{1}{1+\overline{r}+\Gamma^{*}_H}\pi^{P^{*e}}_{H,t+1} + \frac{(1+\overline{r})\Gamma^{*}_H}{1+\overline{r}+\Gamma^{*}_H}\pi^{*}_{H,t-1} \tag{22A}$$

Final goods producer's real marginal costs in the home country:

$$\hat{mc}^{*}_{H,t} = (1-\beta)\left(\hat{w}_t - \hat{A}_t\right) - (1-\gamma)\hat{s}^{*}_t + \alpha\beta\hat{R}_{H,t} + (1-\alpha)\beta\hat{R}_{F,t} - \hat{\varepsilon}^{R}_t - \beta\hat{D}_t \tag{23A}$$

Domestic CPI inflation:

$$\pi_t = \gamma\pi_{H,t} + (1-\gamma)\pi_{F,t} \tag{24A}$$

The real exchange rate equation:

$$\hat{\varepsilon}^{R}_t = \hat{\varepsilon}^{R}_{t+1} + \frac{\delta}{1+\delta}\left(\hat{r}^{*}_t - \hat{r}_t\right) \tag{25A}$$

Home country aggregate budget constraint:

$$\hat{h}_t = (1+\delta)\hat{h}_{t-1} + \delta\hat{r}_{t-1} - \frac{\overline{b}\delta}{\overline{h}}\hat{r}_{t-1} - (1-\gamma)\left(\frac{\overline{c}}{\overline{h}}\hat{c}_t + \frac{\overline{g}}{\overline{h}}\hat{g}_t\right) + \gamma\left(\frac{\overline{\varepsilon}^R\overline{c}^*}{\overline{h}}\left(\hat{c}_t^* + \hat{\varepsilon}_t\right) + \frac{\overline{\varepsilon}^R\overline{g}^*}{\overline{h}}\left(\hat{g}_t^* + \hat{\varepsilon}_t\right)\right) +$$
$$+\frac{\overline{\varepsilon}^R\overline{R}_H^*\overline{I}_H^*}{\overline{h}}\left(\hat{\varepsilon}_t^R + \hat{R}_{H,t}^* + \hat{I}_{H,t}^*\right) - \frac{\overline{R}_F\overline{I}_F}{\overline{h}}\left(\hat{R}_{F,t} + \hat{I}_{F,t}\right)$$

The model is closed once we introduce a monetary policy rule for both countries. Variables representing technology or preference shocks are assumed exogenous. We model their deviations from the steady state as a first order autoregressive processes:

Labour productivity:

$$\hat{A}_t = \xi_a\hat{A}_{t-1} + \xi_{a,t} \tag{27A}$$

Intermediate goods productivity:

$$\hat{D}_t = \xi_D\hat{D}_{t-1} + \xi_{D,t} \tag{28A}$$

Consumers' preferences:

$$\hat{k}_t = \xi_k\hat{k}_{t-1} + \xi_{k,t} \tag{29A}$$