Investment Timing and Predatory Behavior in Duopoly with Debt

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Abstract

This paper examines the effect of debt and limited liability on investment timing in a duopoly with aggregate demand uncertainty and irreversible investment.
Two patterns of investment emerge, depending on the parameter values and debt-levels. If debt levels of the firms are sufficiently different, an increase in debt delays investment. As debt-levels become more similar, a race for market shares begins, as firms wish to commit to not quitting the market first in adverse states. Moreover, firms may engage in predatory investment, if the competitor has a comparably high debt burden. Therefore, debt has a strategic disadvantage.
So this paper can explain both the relative low debt-ratios observed in most industries and the strong investment activities in exceptionally highly leveraged industries.
Moreover, the model of this paper can explain predatory behavior of firms with neither relying on reputational, on network- or learning-effects, nor on defining predatory behavior as deviations from tacit collusion.
JEL classification: D81, G31, G33, L13

Keywords: Real Options, Duopoly, Predatory Behavior, Debt, Timing Game

1   Introduction

Interdependency between the financial and the real decisions of a firm is a phenomenon that has been studied in a number of empirical works and many theorists by now are convinced of the existence of this interdependency. However, the models used to explain this interdependency can neither replicate the observed debt-ratios in practice, which are rather modest compared to the substantial tax benefits of debt,[1] nor do they generate a non-monotone influence of leverage ratios on investment.[2] This paper attempts to explain both these empirical findings by the strategic situation of indebted firms in a duopolistic real options model of investment. Moreover, it models predatory behavior without defining predatory behavior as deviation from tacit collusion, without relying on learning or network effects, and without relying on Folk theorem alike reputational arguments. It rather builds upon a real options approach that understands predatory investment as the attempt of one firm driving a competitor out of the market if the early exercise of the investment option is more than compensated by the expected earlier exit of this competitor.

1.1   Imperfect competition and debt

Since the seminal paper of Brander and Lewis (1986), economists have drawn attention to the strategic effects of debt on competition.[3] Most of the empirical literature on the link between financial situation and investment decisions suggests that increasing debt reduces investment. In Brander and Lewis' contribution however, larger debt leads to fiercer competition, so that firms invest and produce more. In a monopolistic setting in which equity but not debt is the marginal source of finance one can easily build a model that replicates the general empirical findings (Jou, 2001). In a duopoly with equity being the marginal source of finance, however, endogenous bankruptcy decisions render predatory investment-strategies possible.

Although Brander and Lewis (1988) describe how predatory behavior may emerge in a two period setting with bankruptcy, a dynamic model of finance, investment timing, and predatory behavior has not been formulated yet. In this sense the present paper extends the Brander and Lewis contribution to cases in which investment is subject to non-convex costs of adjustment. In contrast to the Brander and Lewis (1988) model however, declaring bancruptcy is a truely endogenous decision, as the owners of the firm are allowed to pay for the firm's obligations with private

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[1] E.g. Schowalter (1999, p. 327) finds in a sample of 1641 manufacturing firms, that the average debt to asset ratio is about 0.25, whereas Fischer et al. (1989) obtain debt ratios somewhat between 0.3 and 0.7 as optimal debt ratios in their capital choice model.

[2] See e.g. figure 1, taken from Bayer (2002) or Busse (2001), who reports that price wars between airlines are predicted best as non-linear (threshold functions) in flow measures of liquidity.

Brander and Lewis (1988) provide an explanation for this finding in cases when there are bankruptcy costs which are proportional to the "magnitude" of bankruptcy. However, their model is a static one and cannot explain periods of predatory behavior triggered by changes in the environment.

[3] See e.g. Maksimovic (1990), Maksimovic and Zechner (1991) and Fries et al. (1997).
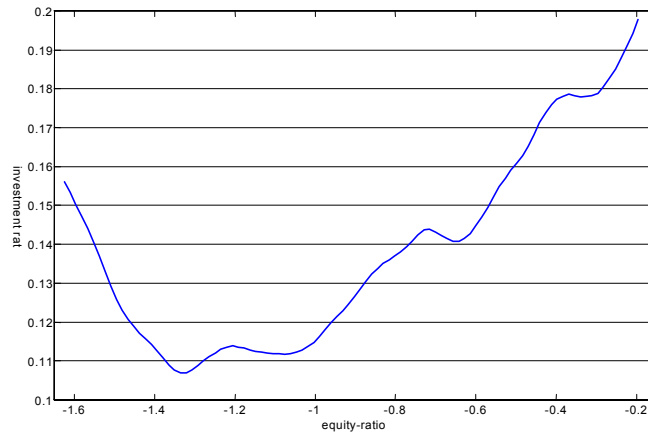
Figure 1: Investment rates conditional on the equity ratio, from Bayer (2002)

funds, if this is advantageous for them. Therefore, bancruptcy is declared only when
"optimal" and not neccessarily once the firm itself has insufficient funds.

In this (general) setting, changes in market conditions may trigger predatory
behavior in highly leveraged industries, but debt does not necessarily make output
market competition fiercer in general. This is also a central difference to Brander
and Lewis' contributions, where predatory behavior is simply defined as fiercer
competition in all states of nature and not a different policy regime triggered by
exogenous changes in the environment.

Moreover, the model of the present paper can also be read as a contribution
to the debt-in-industry-equilibrium literature. This literature typically has either
ruled out the possibility of predatory investment by assuming free market entry—
which makes predatory investment unprofitable—or assumed myopic behavior from
the outset.[4]

1.2 Imperfect competition, real options and predatory be-havior

Our model also extends the literature on real options in oligopoly to cases with
predatory investment. So far, most of this literature has ignored the strategic effects
of debt.[5] Nevertheless, debt can have important strategic effects and, especially in
the irreversible investment framework, influences investment decisions substantially:
If a firm has only limited liability, its owners have to decide to finance negative cash
flows from private funds or to default on the firms obligations in adverse states,
i.e. to declare bankruptcy. As declaring bankruptcy often is followed by an ir-

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[4]See for instance Fries et al. (1997) or Maksimovic and Zechner (1991).
[5]See for example Huisman and Kort (1999), Sparla (2001), or Weeds (2001).

reversible and complete exit of that firm from the market, a bankruptcy decision influences the payoff of the competitor and thus the value of the competitor´s investment. Therefore, the option to declare bankruptcy and the possibility of driving a competitor to bankruptcy (and thereby out of the market) may alter the prices of investment options significantly. The decision to declare bankruptcy obviously depends on the relative debt-burden a firm faces. Thus, investment not only can be used to drive the competitor into bankruptcy by lowering prices, but can also serve as a commitment device to not declare bankruptcy by altering the relative debt-obligation.

Therefore, allowing for predatory investment yields new insights and clearly distinguishes our model from existing work on exit decisions in duopoly in a real options framework: Sparla (2001) discusses partial but irreversible capacity reductions, only. Depending on the parameters of aggregate demand firms may end up in a war of attrition or a preemption game. However, as firms are assumed to be unable to increase capacity, predatory behavior does not emerge.

Joaquin and Khanna (2001) allow for both irreversible investment and exit decisions in their model of potential competition. Yet, predatory investment cannot occur due to their assumptions on revenues and costs: (rational) exit of the competitor imposes a *loss* on the remaining firm.

Our paper combines the real options approach with the strategic effects of debt literature. The basic setting is similar to the one of Jou (2001), who models a potential monopolist (or myopic investor) that enters the market and at the same time issues debt to finance investment.

In contrast to Jou, we model a duopoly. Both firms are assumed to operate with a given capacity and are indebted from the very beginning. Both firms can irreversibly increase capacity, but do not have any possibility of raising or lowering debt-levels. Therfore, debt is taken to be exogenous. The corporate tax-system is assumed to be classical, i.e. there is no shareholder's relief for corporate taxes, whereas interest payments are completely tax-deductible. This gives a tax-advantage of debt.

Although throughout this paper "debt" will be interpreted as a special financial obligation, by the way "debt" is to be modelled, all results equally apply to any fixed running cost (e.g. overhead costs) on which a firm may default. As a result, the model of this paper gives an explanation for predatory behavior itself in a wide class of situations, and is thus a contribution to the literature on predatory behavior, too. In contrast to existing models, that relate the financial situation of firms to predatory behavior (e.g. Bolton and Scharfstein, 1990, or Glazer, 1994), predatory outcomes in our model can be described as regimes that depend on the fluctuating state of demand. Insofar, predatory behavior emerges from time to time, triggered by changes in market conditions. As predatory investment emerges as a policy triggered by adverse market conditions, the present paper is especially related to and extends the results of Grenadier's (1996) real options-model on investment-

cascades to a non-collusive setting.

In contrast to contributions in this field which study price wars,[6] predatory behavior is not defined as a deviation from tacit collusion, but as investment at market conditions which yield negative net-present value for investment unless the endogenous nature of the exit decision of the competitor is taken into account. Our model also does not rely on the assumption that information is asymmetric among competitors, but only on the imperfectness of information.

As inFershtman and Pakes' (2000) model, heterogeneity of firms with respect to their fixed costs is one of the driving factors in the presented model. Other models of predatory behavior that do not rely on asymmetric information typically build upon network[7] or learning-curve effects (Cabral and Riordan, 1994 and 1997). Besides the work of Fershtman and Pakes, the empirical paper of Busse (2002) is most closely related. Busse finds for the airline industry that leverage is one of the main determinants for starting a price war in. Moreover, he finds that the probability of starting a price war reacts in a non-linear fashion to changes in the financial situation of a firm.

1.3 Main results

Our model will partly consist of equations that cannot be solved analytically, and so numerical simulations are presented where analytical solutions are not available. Nevertheless, it will be shown analytically as well as numerically that parameter constellations exist which lead to predatory behavior, i.e. firms invest not because investment itself has a positive present value, but to drive the competitor out of the market. However, the occurence of predatory behavior in equilibrium depends on the "competitiveness" of the market: When adjustment costs are high or demand is sufficiently elastic (low degree of competitiveness), predatory investment never occurs.

The numerical analysis yields furthermore that already a credible threat of predatory behavior lowers price triggers for investment substantially. However, predatory behavior only emerges in highly indebted industries. If predatory behavior does not occur in equilibrium, increasing debt increases the price trigger for investment of the firm increasing its debt. The price trigger for the other firm decreases. Therefore, only if the firm that changes its debt is the follower in equilibrium, increasing debt delays investment. Furthermore, competition and the possibility of predatory investment generally lowers the value of debt and therefore might explain the lower leverage ratios observed in practice, when compared with those predicted by the contingent claims literature.[8]

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[6] See Ordover and Saloner (1989) for a summary or for more recent contributions Fershtman and Pakes (2000) or Busse (2002).

[7] See Athey and Schmutzler (2001) for a fairly general model of investment and increasing dominance, that includes network-industries as a special case.

[8] See e.g. Fischer et al. (1989).

The paper will proceeds as follows: Section 2 outlines the model and presents the basic assumptions. Section 3 presents the expressions that determine the value of equity and price triggers for bankruptcy when both firms have already exercised their investment option. Section 4 derives the value of equity and the price-triggers for investment and bankruptcy when firms may invest. Section 5 shortly states the conditions which determine debt-value. Section 6 presents our numerical results and section 7 concludes. Detailed proofs are available in the appendix.

2   Model setup

We model a duopoly with quantity-competition and stochastic demand fluctuations in continuous time t, $t \in [0, \infty[$. Total production given, the price process $(P_t)_{t \geq 0}$ is assumed to be a geometric Brownian motion and shall be given by

$$P_t \quad = \quad D(Q_t)Y_t \quad , \tag{1}$$

$$dY_t \quad = \quad Y_t \left(\mu dt + \sigma dB_t \right) \quad . \tag{2}$$

B_t denotes a standard Brownian motion and Q_t denotes aggregate industry production, such that

$$Q_t = q_{1,t} + q_{2,t} \quad ,$$

with $q_{i,t}$ denoting production of firm i at time t. For the sake of simplicity, we assume that output is solely produced by a capital good which does not depreciate. Moreover, we assume that both firms already operate in the market with some initial production $\underline{q}_i$. However, both firms may irreversibly invest and increase production to $\overline{q}_i$ at cost C_i. This increase in production is assumed to be instantaneous. At time $t = 0$ each firm issues debt of unstated maturity with associated coupon payments b_i. Thereafter, a firm may not change its debt. Since we want to model heterogenous firms, we assume the "flow leverage" of the two firms to differ. "Flow leverage" is the ratio $l_i(q_i) := \frac{b_i}{q_i D(q_i)}$ of debt payments to monopoly earnings at $Y_t = 1$. Without loss of generality assume firm 2 is the firm with the higher leverage once both firms have invested.

Assumption 1: $l_1(\overline{q}_1) < l_2(\overline{q}_2)$.

For notational convenience, we define a function Δ for the relative price-change induced by a change in aggregate supply.

$$\Delta_{Q_1, Q_2} := \frac{P_t(Q_2)}{P_t(Q_1)} = \frac{D(Q_2)}{D(Q_1)} \tag{3}$$

The following assumption ensures, that price levels always exist, so that investment is profitable:

Assumption 2: Investment increases revenues of the investing firm, regardless if the other firm has invested or not, i.e.

$$D(\underline{q}_i + q_{-i})\underline{q}_i < D(\overline{q}_i + q_{-i})\overline{q}_i \tag{4}$$

As the tax-system is assumed to be classical, losses are fully offset. Therefore, there is a tax advantage of debt, and at tax-rate τ instantaneous net earnings of firm i are given by

$$(1 - \tau)(q_{i,t}P_t(Q_t, Y_t) - b_i) \quad . \tag{5}$$

However, firms may default and declare bankruptcy at any point in time, as they are assumed to have limited liability. If bankruptcy is declared, the firm stops coupon payments, leaves the market, and its assets are transferred to the creditors and sold at price λq_i. Although in a more general context λ may well be determined endogenously, here it will be treated as exogenous. Moreover, λ will only matter for determining the market value of debt and thus will not influence any decisions of the equityholders after debt has been issued.

Furthermore, we assume that the firm is unable to temporarily suspend production. Finally, the equityholders are assumed to have unlimited external resources. The risk-adjusted discount rate is ρ, which shall be larger than μ, i.e. $\rho > \mu$.

As Sparla (2001) argues, if the drift μ is strong compared to the variance σ^2, the probability that firms will not exit in finite time is strictly positive. However, this causes notational inconvenience as one root of the "fundamental quadratic equation" (see below) has to be "adjusted" to derive the correct value functions, see Sparla (2001) for details. To avoid this difficulty the drift is assumed to be not excessively large, i.e. $|\mu| < \frac{\sigma^2}{2}$.

Under these assumptions, the roots of the so called "fundamental quadratic equation" (see e.g. Dixit and Pindyck, 1994) are given by

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left[\frac{1}{2} - \frac{\mu}{\sigma^2}\right]^2 + \frac{2\rho}{\sigma^2}} \quad , \tag{6}$$

which implies $\beta_1 > 1$ and $\beta_2 < 0$ and $\beta_1 + \beta_2 = 1 - 2\frac{\mu}{\sigma^2} > 0$.

Therefore, just as in Jou (2001, p. 72), the general solutions for the value of equity $E_i(P, b, q_i, q_{-i})$ and the value of debt $B_i(P, b, q_i, q_{-i})$ for firm i are given by the following equations:[9]

$$E_i(P, b_i, q_i, q_{-i}) = (1 - \tau)\left[q_i\frac{P}{\rho - \mu} - \frac{b_i}{\rho}\right] + a_{i1}(q_i, q_{-i})P^{\beta_1} + a_{i2}(q_i, q_{-i})P^{\beta_2}, \tag{7}$$

$$B_i(P, b_i, q_i, q_{-i}) = \frac{b_i}{\rho} + c_{i1}(q_i, q_{-i})P^{\beta_1} + c_{i2}(q_i, q_{-i})P^{\beta_2}. \tag{8}$$

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[9]See appendix for details. As usual, we denote by firm $-i$ the competitor of firm i.

Figure 2: Examples of possible sequences of events
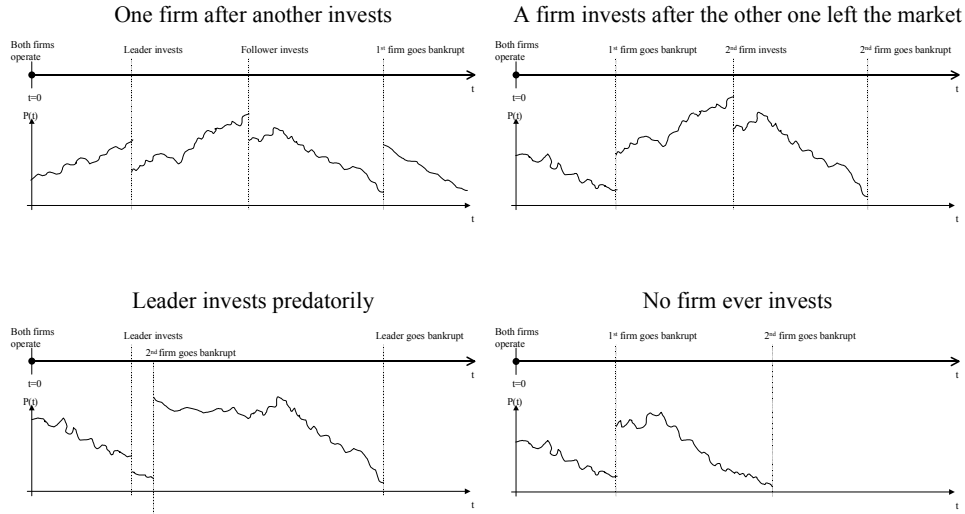
For notational convenience, b is dropped from the list of arguments of the value function, as it will remain constant throughout the subsequent analysis.

3   Value and bankruptcy without investment option

The derived formula for the value of equity still contains two unknowns a_{i1} and a_{i2}. These have to be solved for by deriving further conditions that reflect the optimality of investment plans. Therefore, strategic considerations have an important influence on these parameters. This means, the parameters a_{i1} and a_{i2} will in general vary with the state of production (q_i, q_{-i}).

We should hence discuss in a bit more detail the timing structure of the game. At each point in time an active firm may chose to

(1) invest and increase capacity to $\bar{q}_i$ if it has not invested yet,

(2) declare bancruptcy and become inactive from then on,

(3) or do nothing and wait.

As a result, the number of possible states of production (q_1, q_2) complicate the analysis a lot. The following two sections will derive and specify equity-value at the various states (q_i, q_{-i}), which also index subgames.

Figure 2 displays some of the possible sequences of action that may occur depending on the realization of the Brownian motion.

We do not assume which firm invests first, but let this be determined in equilibrium. Therefore, at the time the first firm invests the other firm must be at least indifferent between investing somewhat earlier or becoming the second firm that invests. We first discuss the behavior of both firms as second-mover. This allows us to obtain the valuation of the second-mover's position for both competitors. This

valuation is crucial to the competition for the position of the first-mover.

To some extent, this discussion cannot avoid some technical complexity. Most technical proofs are presented in the appendix.

3.1   The monopolist's case

Once one of the firms declares bankruptcy, the other firm becomes a monopolist. Therefore, we also have to consider the monopoly situation, although we want to model a duopoly. We start with determining the value functions of a monopolist who has already exercised its investment option. As our model is—for the case of a monopolist that has already invested—similar to Jou´s (2001) investment and financing model, we can primarily rely on his result to obtain the value functions.

Proposition 1 *Having invested, the monopolist´s value of equity and debt is given by*

$$
E(P,\overline{q},0) \;=\; (1-\tau)\left(\overline{q}\frac{P}{\rho-\mu}-\frac{b}{\rho}\right)+\frac{b\left(1-\tau\right)}{\rho(1-\beta_2)}\left(\frac{P}{P_{\overline{q},0}^{exit}}\right)^{\beta_2}, \tag{9}
$$

$$
B(P,\overline{q},0) \;=\; \frac{b}{\rho}+\left(\lambda\overline{q}-\frac{b}{\rho}\right)\left(\frac{P}{P_{\overline{q},0}^{exit}}\right)^{\beta_2}. \tag{10}
$$

Here, we denote by $P_{\overline{q},0}^{exit}=\frac{\beta_2(\rho-\mu)}{(\beta_2-1)\rho}\frac{b}{\overline{q}}$ the trigger price to declare bankruptcy.

Proof. *Denote the revenues process by $\widetilde{P}:=\overline{q}P$. This process has exactly the same properties as the price process in Jou (2001). The proposition then follows straightforward from Jou´s Proposition 1.* ∎

3.2   The duopolists' case

When both firms have invested, so that we are in state $(\bar{q}_1,\bar{q}_2)$, both firm still have to decide whether and when to declare bankruptcy. However, a priori it is not obvious which firm will declare bankruptcy first. But since we assumed the two firms to be differently leveraged, the only Markov-perfect equilibrium of the resulting exit game is the one in which the higher leveraged firm exits at its monopoly exit price. This is shown by the proposition below:

Proposition 2 *In all Markov-perfect equilibria in pure strategies of the $(\bar{q}_1,\bar{q}_2)$-subgame (exit after investment), the firm with the higher leverage (firm 2) chooses its monopoly exit price as the price trigger for bankruptcy $P_{\overline{q}_2,\overline{q}_1}^{exit,2}=P_{\overline{q}_2,0}^{exit,2}$, whereas firm 1 chooses as exit-price trigger some*
$P_{\overline{q}_1,\overline{q}_2}^{exit,1} \in \left]\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}{}^{-1}P_{\overline{q}_1,0}^{exit,1}, \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}{}^{-1}P_{\overline{q}_2,0}^{exit,2}\right[.$
Proof. *See appendix.* ∎

With the equilibrium exit strategies given by Proposition 2, which determine who stays in the market, we can now compute the equilibrium value functions

for equity and debt in a duopoly when both firms have exercised their investment option. As firm 2 behaves myopic, its value functions are the same as the monopolist's ones. However, the possible exit of firm 2 changes firm 1´s value. Therefore, the value functions of firm 1 need to be determined anew and the "option values" a_{11}, a_{12}, c_{11} and c_{12} of (7) and (8) have to be calculated.

Firstly, when prices tend to infinity, the bankruptcy option becomes worthless. This leads to $a_{11} = c_{11} = 0$.[10] Secondly, when the price tends towards the exit price of firm 2, the following value-matching conditions must hold:

$$E_1(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2) = E(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} P^{exit,2}_{\overline{q}_2,0}, b_1, \overline{q}_1, 0) \tag{11}$$

$$B_1(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2) = D(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} P^{exit,2}_{\overline{q}_2,0}, b_1, \overline{q}_1, 0) \tag{12}$$

This now yields for c_{12} and a_{12} after some simple algebraic calculations:

$$c_{12}(\overline{q}_1, \overline{q}_2) = \left(\lambda\overline{q}_1 - \frac{b_1}{\rho}\right)\left(\frac{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}}{P^{exit,1}_{\overline{q}_1,0}}\right)^{\beta_2}, \tag{13}$$

$$a_{12}(\overline{q}_1, \overline{q}_2) = g \cdot \frac{(1-\tau)}{1-\beta_2}\frac{b_1}{\rho}\left(\frac{1}{P^{exit,1}_{\overline{q}_1,0}}\right)^{\beta_2}, \tag{14}$$

with g being defined as

$$g := \underbrace{\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}}_{<1} - \underbrace{\beta_2\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} - 1\right)}_{>0}\left(\frac{\overline{q}_1 b_2}{\overline{q}_2 b_1}\right)^{1-\beta_2} > 1.$$

The stated inequalities are shown to hold in the appendix. With these terms at hand, we obtain for the value of equity and debt of firm 1 the following expressions

$$\frac{E_1(P, \overline{q}_1, \overline{q}_2)}{(1-\tau)} = \left(\overline{q}_1\frac{P}{\rho-\mu} - \frac{b_1}{\rho}\right) + g \cdot \frac{b_1}{(1-\beta_2)\rho}\left(\frac{P}{P^{exit}_{\overline{q}_1,0}}\right)^{\beta_2}, \tag{15}$$

$$B_1(P, \overline{q}_1, \overline{q}_2) = \frac{b_1}{\rho} + \left(\lambda\overline{q}_1 - \frac{b_1}{\rho}\right)\left(\frac{P}{P^{exit,1}_{\overline{q}_1,0}}\right)^{\beta_2}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}. \tag{16}$$

As one can now easily see, the presence of a competitor who leaves the market at a higher trigger price adds a factor g in (9) to the price of the exit-option. This factor is composed of the costs of postponed exit $\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}$ and the gain, when firm 2 exits, which is a "hedge" against bad states. This hedge outweighs the cost of waiting and in total increases equity-value. Moreover, value of firm 1´s equity now exhibits a kink where firm 2 exits and it does not necessarily increase monotonously increases in P anymore(see figures 3(a),(b) and appendix for details).

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[10] See e.g. Jou (2001) for details.

Figure 3: Equity-value of firm 1

(a) both $\frac{b^1}{\overline{q}^1}$ and $\Delta_{\overline{q}^1+\overline{q}^2,\overline{q}^1}$ large   (b) both $\frac{b^1}{\overline{q}^1}$ and $\Delta_{\overline{q}^1+\overline{q}^2,\overline{q}^1}$ small
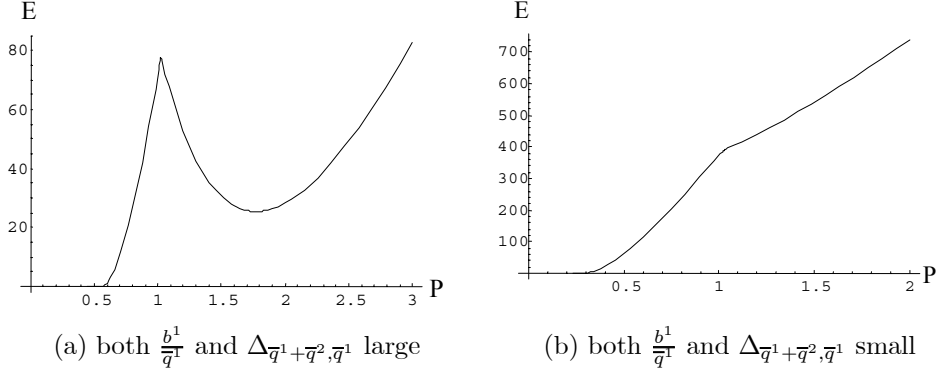
4   Value, optimal investment and bankruptcy

To describe the investment behavior in the model presented here, some assumptions are necessary to keep the model tractable. Some of them were mentioned above. However, further discussion of these assumptions is still open.

First of all, as debt payments are fixed, investment—financed by external equity— is the only way to decrease leverage. Although this seems too great a constraint at first glance, it is very much in line with the findings of Fries et al. (1997), who find that a small equityholder finds only extramarginal changes in leverage advantageous. Therefore, if only marginal changes are feasible by issuing or buying back marginal debt, investment—as an extramarginal change—is the only possibility to lower leverage.

Moreover, as leverage is changed by investment, the order of leverage ratios and therefore the order of exit prices may be reversed by investment. Furthermore, as we have made no assumptions about the sizes of the investment projects, the order of leverage ratios may differ before and after investment anyway. Therefore, we have to consider a number of sub-cases which depend on the parameters of the model.

A further issue concerns predatory behavior, i.e. firms might invest just to drive the price down to increase the probability of the other firm defaulting. Obviously, the possibility of profitable predatory investment again depends on the parameter-values and the order of leverage-ratios.

As no assumptions shall be made about which firm invests first, the two possible asymmetric orders of movement have to be considered. The order of movement will later be determined endogenously in equilibrium. However, as with ruling out equal leverage ratios, we assume simultaneous investment of both firms (or collusive strategies) to be ruled out by prohibitive costs of simultaneity.[11]

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[11] Instead we could as well assume firm sales to be decreasing if both firms invest simultaneously, i.e. $\Delta_{\underline{q}_1+\underline{q}_2,\overline{q}_1+\overline{q}_2\overline{q}_i} < \underline{q}_i$. However, such assumption would be stronger.

In order to obtain the sequence of price triggers for bankruptcy by the same reasoning used in Proposition 2 and to obtain a benchmark case, we start with the investment and bankruptcy decisions of a monopolist. We will leave the calculation of debt-values for a later section.

4.1 Investment- and bankruptcy-decisions of a monopolist

For a monopolist holding an investment and a bankruptcy option, we will label its bankruptcy trigger price as $P_{\underline{q},0}^{exit}$ and its investment price-trigger as $P_{\underline{q},0}^{inv}$. The following two value-matching conditions must hold for the equity-value function:

$$E(P_{\underline{q},0}^{exit}, \underline{q}, 0) = 0, \tag{17}$$

$$E(P_{\underline{q},0}^{inv}, \underline{q}, 0) = E(\Delta_{\underline{q},\overline{q}} P_{\underline{q},0}^{inv}, b, \overline{q}, 0) - C. \tag{18}$$

Together with the following smooth-pasting conditions these equations fully determine equity-value and price triggers. The two smooth-pasting conditions are:

$$\frac{\partial E(P_{\underline{q},0}^{exit}, \underline{q}, 0)}{\partial P} = 0, \tag{19}$$

$$\frac{\partial E(P_{\underline{q},0}^{inv}, \underline{q}, 0)}{\partial P} = \frac{\partial E(\Delta_{\underline{q},\overline{q}} P_{\underline{q},0}^{inv}, \overline{q}, 0)}{\partial P}. \tag{20}$$

Now, combining $(7),(17)$ and (19), an algebraic expression for the value of equity can be derived after some calculations:

$$
\begin{aligned}
E(P, \underline{q}, 0) = {} & (1-\tau)\left[\underline{q}\frac{P}{\rho-\mu} - \frac{b}{\rho}\right] \\
& + \frac{(1-\tau)}{\beta_2 - \beta_1}\left(\frac{\underline{q}P_{\underline{q},0}^{exit}}{\rho-\mu} - \frac{b}{\rho}\right)\left[\beta_1\left(\frac{P}{P_{\underline{q},0}^{exit}}\right)^{\beta_2} - \beta_2\left(\frac{P}{P_{\underline{q},0}^{exit}}\right)^{\beta_1}\right] \\
& + \frac{(1-\tau)}{\beta_2 - \beta_1}\frac{\underline{q}P_{\underline{q},0}^{exit}}{\rho-\mu}\left[\left(\frac{P}{P_{\underline{q},0}^{exit}}\right)^{\beta_2} - \left(\frac{P}{P_{\underline{q},0}^{exit}}\right)^{\beta_1}\right].
\end{aligned}
\tag{21}
$$

But the trigger prices $P_{\underline{q},0}^{inv}$ and $P_{\underline{q},0}^{exit}$, have to be calculated numerically from equations (18) and (20).

4.2 Investment- and bankruptcy-decisions in duopoly when one firm has invested

As mentioned above, both the case where firm 1 invests first and firm 2 invests second and the reversed one have to be considered before one is able to determine which firm invests first in equilibrium. The firm investing first will be called "leader" while the other firm will be called "follower". We begin with discussing the behavior of the follower. As the case where firm 2 is the follower is the simpler one, we start with discussing this case.

Firm 2 as follower

Proposition 3 *As a follower, firm 2 behaves myopically, i.e. just as a monopolist would behave that faces the same demand function as firm 2.*

   Proof. *As investment induces only costls at the point of time where the firm invests, equity value before investment must be smaller than equity value after investment. (The investment option must be worth less than the increase in revenues after investment* $(\Delta_{\bar{q}_1+\underline{q}_2,\bar{q}_1+\bar{q}_2}\cdot\overline{q}_2-\underline{q}_2)P)$. *Thus, we obtain* $P^{exit,2}_{\underline{q}_2,\overline{q}_2} > P^{exit,2}_{\overline{q}_2,0}$. *Moreover, proposition 2 yields that firm 2 leaves first after investment and behaves just as a monopolist would do. Thus, firm 2 can also not credibly threaten to exit second before investing, i.e. firm 2 leaves first and therefore cannot influence the (exit) behavior of firm 1.* ∎

So, as follower and just like in other games with preemption (e.g. Weeds, 2001), firm 2, the higher leveraged firm, does not need to take into account the strategic situation. Therefore, we obtain equations similar to the situation of a monopolist, determining the investment and bankruptcy price-triggers.

Firm 1 as follower

If firm 1 is the follower the strategic situation changes: In contrast to the simpler situation of firm 2, firm 1's actions affect the probability of firm 2 declaring bankruptcy. Hence, the strategic situation of firm 1 is much richer and firm 1 may invest not because it is "fundamentally" profitable but because this makes the exit of firm 2 more likely. As we have seen, firm 2 does not have this opportunity.

   Taking into account the potential exit of firm 2, a couple of different cases have to be considered. For the moment, take $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ as given. Of course $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ will be later endogenously determined in equilibrium. The two cases differ with respect to the number of price-triggers for investment. In the first case, there is a unique (high) price-trigger so that firm 1 invests if and only if the price gets larger than this trigger. In the other case there are two trigger prices: a high and a low one. Investment at the low price-trigger is predatory.

   However, in order to calculate the price-trigger(s) it is necessary to determine in a first step if predatory investment is profitable. To do so, we define two auxiliary "equity-value" functions. The first function to be defined is the equity value firm 1 would have if it could not invest. This is similar to the case in which both firms have already invested but $\overline{q}_1$ is replaced with $\underline{q}_1$ and the exit-price trigger of firm 2 is replaced with $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. $\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2)$ will denote this function. Given the exit price-trigger of firm 2 $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$, one can easily decide if $\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2)$ is similar to firm 1's or firm 2's value when both firms have invested. This function obviously defines a lower bound for $E_1(P,\underline{q}_1,\overline{q}_2)$.

   For predatory investment to be profitable it hence is *necessary* that

$$\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2) + C_1 = E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,\overline{q}_1,\overline{q}_2)$$

has more than one solution in P. The largest solution to this equation also defines a lower bound for the non-predatory investment price-trigger.[12]

Lemma 1 *(i) Other things equal*

$$\widetilde{E}_1(P, \underline{q}_1, \overline{q}_2) + C_1 = E_1(\Delta_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2} P, \overline{q}_1, \overline{q}_2) \tag{22}$$

has at most three solutions in $P > \max\{P^{exit,2}_{\overline{q}_2, \underline{q}_1}, P^{exit,1}_{\underline{q}_1, \overline{q}_2}\}$. We denote the solutions with $P^*(< P^{**})(< P^{***})$ respectively and the set of solutions by S. Moreover, if P is large the following inequality holds:

$$\widetilde{E}_1(P, \underline{q}_1, \overline{q}_2) + C_1 < E_1(\Delta_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2} P, \overline{q}_1, \overline{q}_2). \tag{23}$$

Proof. *See appendix.* ■

Although equilibrium exit-price-triggers will at first be derived in the next sub-section, individual optimality already puts a restriction to the exit-price-triggers, as the following Lemma shows. This Lemma proves useful in discussing the existence of predatory investment in our model.

Lemma 2 *(i) If firm 2 leaves the market first, $P^{exit,2}_{\overline{q}_2, 0} < P^{exit,2}_{\overline{q}_2, \underline{q}_1}$ holds.*
(ii) Moreover, in all cases $P^{exit,2}_{\overline{q}_2, \underline{q}_1} < \Delta^{-1}_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2} P^{exit,2}_{\overline{q}_2, 0}$.
Proof. *See appendix.* ■

The second auxiliary equity-value function to be defined, is the value that equity would have, if investment was not allowed for prices below $\max(S)$. Not allowing investment at prices below $\max(S)$ precludes predatory investment as long as $P^* > \Delta_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2, 0}$. We will denote this second auxiliary function by $\widehat{E}_1(P, \underline{q}_1, \overline{q}_2)$. Note that—given the exit strategy of firm 2—this function is well defined by the usual smooth-pasting and value-matching conditions.[13]

With these auxiliary functions at hand, three possible investment schemes can now be distinguished:

1. A situation may occur, where firm 2 exits at quite high prices in situation $(\overline{q}_1, \overline{q}_2)$, the demand is very inelastic,[14] and the costs of investing are low, so that the value-gain of equity of firm 1 in situation $(\overline{q}_1, \overline{q}_2)$ is always larger than the costs of investing. This however means that firm 1 would *invest at*

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[12] A remark to notation is necessary at this point: The solution to an equation of the form $lhs(P(Y)) = rhs(P(Y))$, like equation (22), is noted as a "price level" for which the equation is solved. However, this is slightly incorrect, as we rather obtain solutions in Y. E.g. the solution might be at a price-level, lower than the exit-price of one of the firms. Therefore, the exit-decision has to be taken into account, since it causes prices to rise. So we normalize for induced changes in price. Nevertheless we stick to the notion as "price level", as it is easier to interpret.
[13] See appendix for details.
[14] This means prices react strongly to changes in total output.

any price. This is the case if for all $P > P_{\overline{q},\underline{q}}^{exit,2}$

$$E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2) > C_1 + \widetilde{E}_1(P,\underline{q}_1,\overline{q}_2).$$

Therefore, in this case

$$E_1(P,\underline{q}_1,\overline{q}_2) \equiv E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2) - C_1. \tag{24}$$

2. If $\#S = 1$ or $\#S > 1$ but

$$\widehat{E}_1(P,\underline{q}_1,\overline{q}_2) = E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2) - C_1 \tag{25}$$

has one and only one solution in $\left[P_{\overline{q}_2,\underline{q}_1}^{exit,2},\infty\right[$, we have

$$E_1(P,\underline{q}_1,\overline{q}_2) = \widehat{E}_1(P,\underline{q}_1,\overline{q}_2) \quad .$$

This will usually be the non-predatory behavior case. However, if $P_{\overline{q}_2,\underline{q}_1}^{exit,2} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}^{-1}P_{\overline{q}_2,0}^{exit,2}$, we may obtain $P^* < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}^{-1}P_{\overline{q}_2,0}^{exit,2}$, which is predatory, though we will not refer to this case as predatory investment.

3. If $\#S > 1$ and (25) has more than one solution in $\left[P_{\overline{q}_2,\underline{q}_1}^{exit,2},\infty\right[$, firm 1 will predatory invest, i.e. firm 1 invests at low prices to crowd firm 2 out of the market. However, the exact structure of the value-function of firm 1 depends on the number of solutions to (25):

   (a) If (25) has two solutions, we will get an Investment/ No-Investment/ Investment scheme, i.e. a low price-trigger for which investment occurs and a high price-trigger for investment and a region of inactivity in between. See figure 4(a). We can then obtain both price-triggers and the equity value by applying standard boundary and smooth-pasting conditions for investment. However, for the predatory-investment price-trigger $P_{\underline{q}_1,\overline{q}_2}^{pred,1}$ (low price-trigger) the smooth-pasting condition does not need to hold and the border solution $P_{\underline{q}_1,\overline{q}_2}^{pred,1} = \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}^{-1}P_{\overline{q}_2,0}^{exit,2}$ may well be obtained.

   (b) If (25) has three solutions, the situation gets even more complex: If occasionally P is very low, firm 1 will not invest, but will invest as prices rise. However, smooth-pasting conditions do not need to hold in this situation. See figure 4(b). Starting between P^{**} and P^{***}, we obtain the same Investment/ No-Investment/ Investment scheme obtained under (a). Again note the possibility of a border-solution for the predatory-investment price-triggers.

To simplify the following analysis we shall rule out the latter case by assumption:

Figure 4: Solutions to (22)

(a) two solutions   (b) three solutions
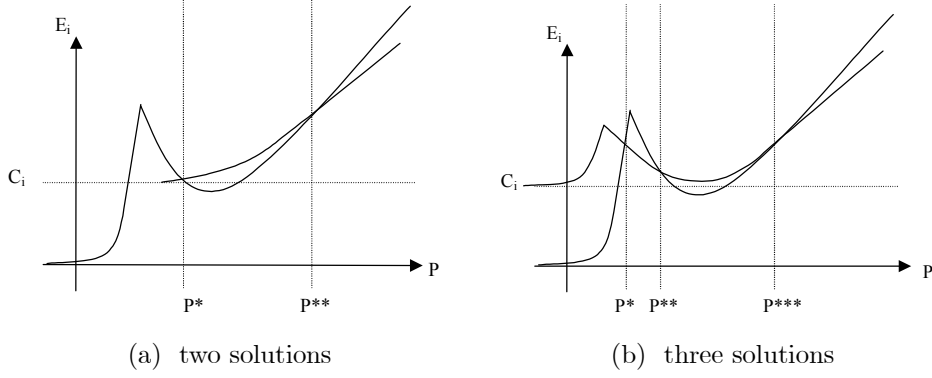
Assumption 3: The costs of investment shall be such that firm 2 will never invest at a price level that would lead to a type 3(b) predatory investment case with very low prices, where firm 1 would predatorily invest as soon as prices rise.

The following proposition discusses the general possibility of predatory investment. The exact equations describing equity value and determining price-triggers will be derived afterwards.

Proposition 4 *(i) If demand is sufficiently elastic, i.e. $\forall Q_1, Q_2 : \Delta_{Q_1, Q_2} \approx 1$, or if demand is not too inelastic and the costs of investment C_1 are sufficiently high, then predatory investment never occurs.*
(ii) If $g > \left(\frac{\overline{q}_1 b_1}{\overline{q}_2 b_2} \right)^{1 - \beta_2}$ and

(a) if firm 1 exits first when it would have no investment option and if

$$\widetilde{P}^{exit,1}_{\underline{q}_1, \overline{q}_2} < \Delta_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2, 0}, \tag{26}$$

then there exist investment-costs C_1, so that (22) has multiple solutions.

(b) if firm 2 exits first and if for the right-hand partial derivative $\frac{\partial E_1}{\partial P^+}$

$$\frac{\partial \widetilde{E}_1}{\partial P} \left(\Delta_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2, 0}, \underline{q}_1, \overline{q}_2 \right) > \frac{\partial E_1}{\partial P^+} \left(P^{exit,2}_{\overline{q}_2, 0}, \overline{q}_1, \overline{q}_2 \right) \Delta_{\underline{q}_1 + \overline{q}_2, \overline{q}_1 + \overline{q}_2}$$

holds, then there exist cost of investment C_1 so that predatory investment occurs.

Proof. *See appendix.* ∎

In case, predatory investment does occur, the exit value-matching condition for

firm 1 has to be modified to

$$E_1^{pred}(P_{\underline{q}^1,\overline{q}^2}^{pred,1},\underline{q}_1,\overline{q}_2) = E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1} P_{\underline{q}^1,\overline{q}^2}^{pred,1},\overline{q}_1,\overline{q}_2) - C_1 \quad . \tag{27}$$

Let $\frac{\partial}{\partial P^+}$ denote the right-hand partial-derivative. Due to Lemma 2 (and since we have to allow for a corner solution) we obtain for the necessary (smooth pasting) condition the following generalized expression:

$$\forall P \geq \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P_{\overline{q}_2,0}^{exit,2} :$$
$$\left(\frac{\partial E_1^{pred}}{\partial P}(P_{\underline{q}^1,\overline{q}^2}^{pred,1},\underline{q}_1,\overline{q}_2) - \frac{\partial E_1}{\partial P^+}(P_{\underline{q}^1,\overline{q}^2}^{pred,1},\overline{q}_1,\overline{q}_2)\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1} \right) \left(P_{\underline{q}^1,\overline{q}^2}^{pred,1} - P \right) \leq 0. \tag{28}$$

Corollary 1 *The predatory investment-price trigger $P_{\underline{q}^1,\overline{q}^2}^{pred,1}$ is always strictly larger than $P_{\overline{q}_2,\underline{q}_1}^{exit,2}$, $P_{\underline{q}^1,\overline{q}^2}^{pred,1} > P_{\overline{q}_2,\underline{q}_1}^{exit,2}$.*
    Proof. *Follows straightforward from Lemma 5.2 and the definition of the predatory investment-price trigger.* ∎

Equilibrium exit-strategies

    The exit strategy of firm 2 was taken to be given so far. However, exit strategies have to be determined as an equilibrium of both firms competing to monopolize the market.

    Independently from the (q_i, q_{-i})-state, equilibrium exit strategies can be characterized as follows:

Definition 1 *A vector of (q_i, q_{-i})-state-contingent price-triggers $\bar{P}_i^{\#}$ is a **Markov-perfect best-response in pure strategies** of firm i given the vector of price-triggers $P_{-i}^{\#}$ of firm $-i$, if for all (q_i, q_{-i}) not exiting before the declared price-trigger is credible*

$$\forall P > \bar{P}_{q_i,q_{-i}}^{exit,i} : E_i\left(P, q_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) > 0. \qquad \text{(limited liability)}$$

Secondly, it also must be credible not to preempt on the own proposed price triggers for investment, i.e.

$$\forall P \leq \bar{P}_{\underline{q}_i,q_{-i}}^{inv,i} : E_i\left(P, \underline{q}_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) \geq E_i\left(P, \overline{q}_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) - C_i$$
$$\forall P \geq \bar{P}_{\underline{q}_i,q_{-i}}^{pred,i} : E_i\left(P, \underline{q}_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) \geq E_i\left(P, \overline{q}_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) - C_i. \qquad \text{(no preemption)}$$

For all price-triggers $P_i^{\#}$ (which may include predatory investment-triggers) that fulfill the above credibility constraints, the proposed price-triggers need to be optimal,

i.e.

$$E_i\left(\bar{P}^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}|\bar{P}^{\#}_i, P^{\#}_{-i}\right) \geq E_i\left(\bar{P}^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}|P^{\#}_i, P^{\#}_{-i}\right)$$
$$E_i\left(\bar{P}^{inv,i}_{q_i,q_{-i}}, q_i, q_{-i}|\bar{P}^{\#}_i, P^{\#}_{-i}\right) \geq E_i\left(\bar{P}^{inv,i}_{q_i,q_{-i}}, q_i, q_{-i}|P^{\#}_i, P^{\#}_{-i}\right). \quad \text{(optimality)}$$

Definition 2 *A **Markov-perfect equilibrium in pure strategies** is a pair of vectors $\left(P^{\#}_i, P^{\#}_{-i}\right)$, so that each is a best response to the other.*

Thus, a firm that defaults first, uses the value-matching and smooth pasting condition

$$E_i(P^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}) = \frac{\partial E_i}{\partial P}(P^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}) \overset{!}{=} 0 \qquad (29)$$

to determine its own exit price-trigger. If a firm expects to leave second, its value function flat in its own exit price-trigger. Thus, this firm is indifferent about the level of its own exit price-trigger on the margin. Therefore, there always is a multiplicity of equilibria which only differ with respect to the (virtual) exit price-trigger of the firm which exits second.

Now denote by $P^{ind,i}_{q_i,q_{-i}}$ the largest exit-price-trigger of firm i that lets the limited liability constraint of firm $-i$ hold with equality for some P'. If firm i chooses an exit price trigger below $P^{ind,i}_{q_i,q_{-i}}$, firm $-i$ cannot credibly threaten to exit later as the limited liability constraint would bind. Thus firm $(-i)$ exiting at the myopic price-trigger and the other firm choosing an exit price-trigger smaller than $P^{ind,i}_{q_i,q_{-i}}$ will be an equilibrium, if $P^{ind,i}_{q_i,q_{-i}}$ fulfills the other equilibrium conditions of firm i. Note that $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$ always holds since firm $-i$ would immediately exit in state $(q_{-i}, 0)$ at a price below $\Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$.

The following proposition describes all equilibria of the exit-game depending on the parameters of the actual environment.

Proposition 5 *(i) If for firm i the myopic exit price-trigger $P^{m\_exit,i}_{q_i,\bar{q}_{-i}}$, obtained from (29), is smaller than $P^{ind,i}_{q_i,q_{-i}}$, then firm $-i$ choosing $P^{m\_exit,-i}_{q_{-i},q_i}$ and firm i choosing a lower price-trigger is an equilibrium of the (q_i, q_{-i}) stage.*
(ii) If $\left[\Delta_{q_i+q_{-i},q_i}^{-1} P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] = \emptyset$, then firm i chooses $P^{m\_exit,i}_{q_i,\bar{q}_{-i}}$ as the exit-price-trigger in all equilibria of the (q_i, q_{-i}) stage.
(iii) If $P^{m\_exit,-i}_{q_{-i},q_i} > P^{ind,i}_{q_i,q_{-i}}$ and $\left[\Delta_{q_i+q_{-i},q_i}^{-1} P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] \neq \emptyset$, then firm i choosing some $P^{exit,i}_{q_i,q_{-i}} \in \left[\Delta_{q_i+q_{-i},q_i}^{-1} P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right]$ and firm $-i$ choosing $P^{m\_exit,-i}_{q_{-i},q_i}$ is an equilibrium of the (q_i, q_{-i}) stage, if this yields no incentive to predatorily invest for firm $-i$.
(iv) If both firms have an incentive to predatorily invest, instead of choosing their myopic exit price-trigger, then both firm preempt on predatory investment.
(v) If firm i predatorily invests, firm $-i$ cannot credibly threaten to deviate from choosing the myopic exit price-trigger to hinder i in investing predatorily.

Proof. *See appendix.* ∎

Remark 1 *Part (iii) of the last proposition gives rise to the problem of multiplicity of equilibria. For the numerical simulations the firm with the larger $P_{q_i,q_{-i}}^{ind,i}$ is selected as the one, who leaves second.*
This selection can be motivated by the following idea: Suppose firm $-i$ chooses an exit price-trigger marginally larger than $P_{q_i,q_{-i}}^{ind,i}$. Is the choice of $P_{q_i,q_{-i}}^{ind,i}$ still credible then? Of course not for the firm with the lower $P_{q_i,q_{-i}}^{ind,i}$.
Conversely, this rule can be interpreted as a notion of conservativeness in the following sense: Suppose the shareholders of firm i imagine the worst case, i.e. firm $-i$ defaults just one logical second before $i's$ (proposed) trigger price is reached. Then only if the proposed exit price trigger is larger than the own indifference price trigger, i has no incentive to exit before.
Another motivation for this rule would be that firms step by step and sequentially undercut each other's exit price-triggers before the actual game commences.

4.3   Investment- and bankruptcy-decisions in duopoly when no firm has invested yet

We shall now turn to the investment decision of the leader. Here however, the problem becomes more complex, as there may be a preemption game for both, predatory investment and "fundamental" investment decisions. However, if second mover advantages are not too strong, one can obtain a relatively simple rationale for the investment trigger prices.

To avoid further complication, we will make the following assumption according to the price-level (and investment costs) in $t = 0$:

Assumption 4: At the initial price-level P_0 at least one firm finds it unprofitable to invest and both firms find it unprofitable to declare bankruptcy. Moreover, P_0 lies between the preemption thresholds $\left(P_{pre}^{pred,i}, P_{pre}^{inv,i} \right)$, which will be defined below, i.e.

$$\min_{i=1,2}\{P_{pre}^{pred,i}\} < P_0 < \max_{i=1,2}\{P_{pre}^{inv,i}\} .$$

Non-predatory investment

To disentangle the interrelated decisions at the early stage of the game (when both firms still do not have invested) non-predatory investment is analyzed separately in a first-step. The exit and investment strategy of the respective other firm are assumed to be given for the moment. Just as we did when firm 1 was the follower, we construct a hypothetical value-function for the leading firm i. For this hypothetical value-function we again assume firm i would not have any investment option. Hence, this function exhibits a kink at the price-trigger where the follower, firm $-i$, invests.

Non-predatory investment of the leader (with the leader's role preassigned) is again defined as investment that occurs at prices larger than the largest price-level P^* for which this hypothetical value function intersects with the value at output $(\overline{q}_i, \underline{q}_{-i})$ less investment costs.[15] However, if this intersection is at a higher price than the investment price of firm $-i$, then firm i cannot profitably invest in a non-predatorily before firm $-i$ invests. Therefore, under the assumption that firm $-i$ does not predatorily invest, we obtain the following value-matching conditions for our second hypothetical value-function:

$$\widehat{E}_i(P^{exit,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, \underline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i} P^{exit,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, 0) \tag{30}$$

if firm i chooses an exit price-trigger larger than the one of firm $-i$ and

$$\widehat{E}_i(P^{exit,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) = 0 \tag{31}$$

otherwise. If firm i non-predatorily invests, we obtain

$$\widehat{E}_i(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} P^{inv,i}_{\underline{q}^i,\underline{q}^{-i}}, \overline{q}_i, \underline{q}_{-i}) - C^i \tag{32}$$

and if firm $-i$ invests first, we obtain

$$\widehat{E}_i(P^{inv,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, \underline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P^{inv,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, \overline{q}_{-i}). \tag{33}$$

The investment price-trigger $P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}$ is then determined as the solution $P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}} \in \Theta := \left[P^{pred,-i}_{\underline{q}_{-i},\underline{q}_i}, P^{inv,-i}_{\underline{q}_{-i},\underline{q}_i} \right]$ to the generalized smooth-pasting condition

$$\forall P \in \Theta : \left(\frac{\partial \widehat{E}_i}{\partial P}(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) - \frac{\partial E_i}{\partial P}(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \overline{q}_i, \underline{q}_{-i}) \cdot \Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} \right)$$
$$\times \left(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}} - P \right) \leq 0. \tag{34}$$

Proposition 5 can be used to find an exit-equilibrium, given the investment strategies of firm $-i$. Whether predatory investment at the early stage of the game is possible, can be checked by expressions analogous to those obtained in the case when firm 1 is the follower.

Equilibrium investment

The investment strategies of firm $-i$ obviously cannot be taken as given, i.e. we have to determine the exit- and investment-equilibrium at the early stage of the game simultaneously. Remember that we were merely interested in competitive outcomes. Thus, we assumed that collusive investment strategies are not feasible.

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[15]Again a case where investment is profitable at any price may occur. However, we will not further comment on this possible case.

We define as Huisman and Kort (1999) a function $\Phi_i(P)$ which represents the advantage of taking the role of the leader and investing at P instead of becoming the follower, when the other firm invests at price P.

$$\Phi_i(P) := \underbrace{E_i\left(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} P, \overline{q}_i, \underline{q}_{-i}\right) - C}_{\text{Value as leader}} - \underbrace{E_i\left(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P, \underline{q}_i, \overline{q}_{-i}\right)}_{\text{Value as follower}} \quad (35)$$

Note that this expression is independent of exit decisions in the early stage of the game and is thus well defined, although we have not identified the exit equilibrium, yet.

To describe the root-behavior of $\Phi_i(P)$ a bit more notation has to be introduced. We denote the state-$\left(\underline{q},\underline{q}\right)$ price at which firm i invests as follower by $\overline{P}_i$, i.e.

$$\overline{P}_i = \Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P^{inv,i}_{\underline{q}_i,\overline{q}_{-i}}$$

analogously we define $\underline{P}_i$ as the price at which firm i quits as follower, invests predatorily, or firm $-i$ quits, whichever happens first.

Proposition 6 *(i) On $M := [\max_{j=1,2}\{\underline{P}_j\}, \min_{j=1,2}\{\overline{P}_j\}]$*

$$\Phi_i(P) = 0 \quad (36)$$

has at most three solutions.
(ii) If $\overline{P}_i \leq \overline{P}_{-i}$, then $\Phi_i(P) = 0$ has at most two solution on M and only one additional one for $\overline{P}_i \leq P \leq \overline{P}_{-i}$, namely $\overline{P}_{-i}$ with $\Phi_i'(\overline{P}_{-i}) < 0$.
(iii) If $\overline{P}_i > \overline{P}_{-i}$, then $\Phi_i(\overline{P}_i) = 0$ and $\Phi_i(P) < 0$ for all $\overline{P}_i > P > \overline{P}_{-i}$.
(iv) If $\Phi_i(P) = 0$ has two solutions on M in case (ii) or three solutions in case (iii), then $\Phi_i(\max\{\underline{P}_i\}) > 0$. Moreover, there can only exist an additional solution on $\min_{j=1,2}\{\underline{P}_j\} < P < \max_{j=1,2}\{\underline{P}_j\}$ if $\underline{P}_i < \underline{P}_{-i}$.
    Proof. *See appendix.* ■

If there are two solutions to $\Phi_i(P) = 0$ in case (ii) (respectively three solutions in case (iii)) we will call the smaller one the preemption-threshold for predatory investment and the larger one preemption-threshold for non-predatory investment. They define a threshold at which firm i is just indifferent between being the leader and being the follower. We will denote these thresholds by $P^{pred,i}_{pre}$ and $P^{inv,i}_{pre}$ respectively. If there are less solutions, we only define a non-predatory preemption threshold.

If in case (ii) there is no solution to (36) on M then there is a solution for smaller P since the payoff of the leader's role is $-C_i < 0$ at its (effective) exit price, while it is zero for the same firm being follower. We then denote this solution by $P^{inv,i}_{pre}$. Moreover, we set $P^{pred,i}_{pre} = \overline{P}_i$. This is the only case in which $P^{inv,i}_{pre} > P^{pred,i}_{pre}$.

If in case (iii) $\Phi_i(P) < 0$ for all $P < \overline{P}_i$, we set $P^{inv,i}_{pre} = \overline{P}_{-i}$.

A typical problem that arises in timing games of (dis-)investment is the multiplicity of equilibria associated with Fudenberg and Tirole´s (1985) notion of perfect timing game equilibria. The non-uniqueness typically arises if none of the firms has an unilateral incentive to invest, while both firms have an incentive to preempt each other.[16] However, this non-uniqueness disappears when we look at the renegotiation-proof equilibria only and make the following additional assumption.

Assumption 5: Let i be the firm with the larger $\overline{P}_i$, then the largest solution to (36) for firm $-i$ shall be larger, than the optimal unconstrained[17] investment price trigger of firm i. Moreover, at least for one firm there exists an (interior) solution for $P_{pre}^{inv,i}$ on M.

This assumption assures that second-mover advantages not being "too strong".

Proposition 7 *Under Assumption 5 the only Markov-perfect equilibrium for the preemption game for non-predatory investment is that the firm with the smaller $P_{pre}^{inv,i}$ takes the lead, and chooses an investment-price trigger, which is a solution to (34), where $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i} = P_{pre}^{inv,-i}, P_{\underline{q}_{-i},\underline{q}_i}^{pred,-i} = P_{pre}^{inv,i}$. We denote this investment price trigger by $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$.*

Proof. *First note that at $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ firm L indeed prefers to be the leader, moreover because of assumption 5 we have a preemption game for non-predatory investment: Suppose τ denotes the stopping-time associated with the optimal investment-price trigger of firm i. Then firm $-i$ has an incentive to invest at time $\tau - \epsilon$, as long as the price is above its preemption threshold. Therefore, when $P_t \in [P_{pre}^{pred,-L}, P_{pre}^{inv,-L}]$ firm $-L$ prefers to be the follower, while at $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ firm L profitable invests.* ∎

However, an existing preemption threshold for predatory investment does not necessary imply predatory investment to occur in a renegotiation-proof Markov-perfect equilibrium:

Proposition 8 *There can only occur predatory investment in a renegotiation-proof Markov-perfect equilibrium, if at least one firm can profitably invest predatorily, without assuming that the other firm invests predatorily, i.e. a $P' < P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ exists, such that*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} P', \overline{q}_i, \underline{q}_{-i}) - C \geq \widehat{E}_i(P', \underline{q}_i, \underline{q}_{-i}), \tag{37}$$

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[16] See e.g. Weeds (2001) or Sparla (2001).

Indeed, in some numerical simulations (not reported) we found non-renegotiation-proof predatory equilibria. Renegotiation-prooveness is but a strong assumption on rationality.

Therefore, we might in reality observe a circular situation of the following form: One agent takes pre-emptive, predatory action in threat of a predatory action of the other agent. This other agent however has no incentive to undertake that action as long as the first agent does not take action. The first agent however takes action as she is threatened. Triggered off by a sunspot, the situation escalates.

[17] This means $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i}$ is set to infinity in calculating the price-triggers.

where $\widehat{E}_i$ denotes the equity value without the possibility of predatory investment. If (37) has indeed such a solution, we will call predatory investment for firm i to be "fundamentally profitable".

Proof. (37) defines a kind of net-present value rule to predatory investment: The option to predatorily invest exists if and only if predatory investment can give a net-present-value gain. If therefore predatory investment is not profitable for both firms unless the other firm predatorily invests, then both firms can renegotiate not to invest predatorily. ∎

Proposition 9 *A solution to* (37) *implies that a predatory investment preemption threshold for firm i exists, i.e. there cannot be second-mover advantages for profitable predatory investment independent of how low P gets.*

Proof. *See appendix.* ∎

Therefore, if both firms have a predatory-investment preemption-threshold price, or if the non-predatory investment threshold of one firm is smaller than the predatory one of the other firm, we have a preemption game for predatory investment. However, both firms only enter this game, if at least one firm finds predatory investment fundamentally profitable. If only for one firm a predatory-investment preemption threshold is defined, and if this firm finds predatory investment fundamentally profitable, then it will predatorily invest, indeed. In this case it sets the predatory investment-price trigger according to a generalized smooth pasting condition analogous the one derived for firm 1 as follower.

Proposition 10 *(i) If there is a preemption game for predatory investment and $P_{pre}^{pred,i} < P_{pre}^{inv,-i}$ for both firms, the only Markov-perfect equilibrium (outcome) is that the firm with the higher $P_{pre}^{pred,i}$, takes the lead for predatory investment and invests at a price-trigger which is a solution to a version of (34) that is modified by defining $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i} := P_{pre}^{pred,i}$, $P_{\underline{q}_{-i},\underline{q}_i}^{pred,-i} := P_{pre}^{pred,-i}$ and by using the appropriate value matching conditions.*
(ii) If there is a preemption game for predatory investment, and $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ for firm i, and one of the firms has an unilateral incentive to invest in $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$,[18] then in all Markov-perfect equilibria firm $-i$ invests predatorily at $P_{pre}^{pred,i}$.
(iii) If there is a preemption game for predatory investment and $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ for one of the firms, and none of the firms have an unilateral incentive to invest on $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$, then in all renegotiation-proof Markov-perfect equilibria firm i predatorily invests at its unconstrained optimal predatory investment price-trigger or at $P_{pre}^{pred,-i}$, whichever is the higher price.

Proof. *See appendix.* ∎

Corollary 2 *If both firms wish to predatorily invest, then the equilibrium predatory*

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[18] *This means the unconstrained optimal investment price-trigger falls in this interval.*

investment price-trigger is strictly smaller than the equilibrium normal investment price-trigger.

The equilibrium exit strategies are determined analogously to the follower case. Therefore, we now have all equations that determine the equilibrium price-triggers of our model and so we can obtain numerical results and numerically check the importance of the strategic situation modelled in this paper.

5   Value of debt

For completeness, we briefly state the two value-matching conditions which generally characterized the value of debt:

$$B_i(P^{exit,i}_{q_i,q_{-i}}, b_i, q_i, q_{-i}) = \lambda q_i \ , \tag{38}$$

$$B_i(P^{Trig}, b_i, q_{i1}, q_{-i1}) = D_i(\Delta_{q_{i1}+q_{-i1},q_{i2}+q_{-i2}} P^{Trig}, b_i, q_{i2}, q_{-i2}). \tag{39}$$

P^{Trig} denotes an arbitrary decision price trigger that does not lead to an immediate exit of firm i and at which capacities are changed from (q_{i1}, q_{-i1}) to (q_{i2}, q_{-i2}). However, as we do not aim at deriving optimal leverage strategies, debt values are not reported in the numerical results.

6   Numerical results

Numerical solutions have been calculated for different parameter values. Table 1 contains the parameter-values for the non firm-specific parameters. In all calculations an isoelastic specification for the inverse demand-function has been used. Two base cases are considered. One to discuss the impact of changes in the leverage, and one to study the influences of the elasticity of demand and the tax rate.

Table 1: General Parameter Values, Base Cases

Tax Rate τ	0.3
Discount Rate ρ	0.05
Drift μ	0.03
Variance σ^2	0.1
Inverse-Demand	$Q^{-\xi}$

Tables 2, 3 and 4 report the results for those two base cases. First of all, investment price-triggers for the duopoly and the monopoly differ very significantly.[19]

For the follower, this is just the standard result of Cournot competition. For the preemption threshold of the leader, this is the usual effect of the strong first-mover advantages of the Stackelberg-leader. These first-mover advantages are additionally

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[19] This is also true for the trigger values of Y, which can be obtained by rescaling the price triggers by 1.86607 for the leader and 1.95912 for the follower, setting $D(\overline{q}) = 1$.

amplified in the presence of predatory behavior or if the order of exit is reversed by investment. These first-mover advantages drive the results of our base cases.[20] The (tax adjusted) net-present-value price triggers for investment in duopoly in the two base cases were (according to rule 1 below) 5.7 (8.2) and 7.6 (10.9) respectively.[21] If one assumes the other firm would invest, unless firm i invests (rule 2), then the net-present-value rule yields a tax-adjusted price trigger of 4.5 in base case I (a), which is only slightly below the equilibrium investment-price trigger of firm 1.

Moreover, in calculating the adjusted net-present value price-trigger of 4.5, the investment of firm 2 when the price-process reaches the follower's price-trigger is ignored. Therefore the "true" net-present-value rule investment-price trigger is larger than 4.5. In that sense, we can conclude that in some cases the threat of being forced to exit first outweighs the gains from waiting.

Comparing the results for the three sub-cases of Base Case I,

(a) Firm 1 has lower leverage before and after investment,

(b) Firm 1 has a higher leverage before, but lower leverage after investment and

(c) the size of the investment projects differs between Firm 1 and 2,

shows that the firm with the higher initial, the firm with the higher post-investment leverage, and the firm with the lower leverage in both states can prey in equilibrium. Therefore, our model includes not only the cases studied by Busse (2001) but also cases in which the financially healthier firm preys.

Table 5 now reports the effects of a change in leverage of firm 1 (relative to Base Case I(a)). In states, in which investment does neither change the ordering of exit price triggers, nor predatory investment would occur on the second stage, the effect of debt on investment-price triggers are rather minor.

Moreover, as firm 1 becomes leader in equilibrium, the probability of investment in duopoly shrinks with larger debt levels as long as debt levels stay intermediate.

However, if both firms become more similarly leveraged, debt starkly influences investment decisions. If firm 2 can expect that firm 1 leaves the market first (in case firm 2 becomes the leader) then—as we have seen—first-mover advantages become very strong. The first-mover advantages can be even as strong as firm 1 in equilibrium invests below the simple net-present-value price trigger.

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[20]Note, that although interior solutions for the non-predatory investment price-trigger were allowed for, in the cases reported, the leader always invested at the preemption threshold of the follower.

[21]

$$NPV_{Rule\_1} = \frac{\rho C_i}{(1-\tau)\left(\overline{q}_i \Delta_{\underline{q}_i + \underline{q}_{-i}, \overline{q}_i + \underline{q}_{-i}} - \underline{q}_i\right)} \qquad (40)$$

$$NPV_{Rule\_2} = \frac{\rho C_i}{(1-\tau)\left(\overline{q}_i \Delta_{\underline{q}_i + \underline{q}_{-i}, \overline{q}_i + \underline{q}_{-i}} - \underline{q}_i \Delta_{\underline{q}_i + \underline{q}_{-i}, \underline{q}_i + \overline{q}_{-i}}\right)} \qquad (41)$$

Table 2: Base Case I (a), (b): $\xi = 0.9$

	Firm 1 (a)	Firm 1 (b)	Firm 2
Production before Investment	18	17.9	18
Production after Investment	20	20	20
Investment-Costs	120	120	120
Coupon-Payment [Debt]	49.9	49.9	50
MONOPOLY			
Exit-Price Trigger after investment	0.4496	0.4496	0.4505
Exit-Price Trigger before investment	0.4980	0.4980	0.4990
Investment-Price Trigger	99.844	95.337	99.844
DUOPOLY			
Both Invested, Exit-Price Trigger	Firm 2 exits	Firm 2 exits	0.45049
Firm 1 Follower: Exit-Price Trigger	0.4867	0.4888	Firm 1 exits
Firm 1 Follower: Investment-Price Trigger	15.872	15.061	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	Firm 2 exits	0.4881
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	Firm 2 inv.	17.48
Preemption Threshold Non-Predatory Investment	4.57881	4.36741	4.895 (4.873)
Unilateral Incentive to predatorily Invest as Leader	No	No	Yes
Firm 1: Predatory Investment-Price Trigger	0.611624	0.6206	Firm 1 inv.

Table 3: Base Case I (c): $\xi = 0.8$

	Firm 1	Firm 2
Production before Investment	17	18
Production after Investment	21	20
Investment-Costs	160	120
Coupon-Payment [Debt]	70	70
MONOPOLY		
Exit-Price Trigger after investment	0.601	0.631
Exit-Price Trigger before investment	0.7296	0.695
Investment-Price Trigger	34.75	49.77
DUOPOLY		
Both Invested, Exit-Price Trigger	Firm 2 exits	0.631
Firm 1 Follower: Exit-Price Trigger	0.693	Firm 1 exits
Firm 1 Follower: Investment-Price Trigger	9.293	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	0.680
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	15.924
Preemption Threshold Non-Predatory Investment	3.579	3.109
Unilateral Incentive to predatorily Invest as Leader	No	Yes
Firm 1: Predatory Investment-Price Trigger	Firm 2 inv.	0.786

Table 4: Base Case II: $\xi = 0.7$

	Firm 1	Firm 2
Production before Investment	18	18
Production after Investment	20	20
Investment-Costs	160	160
Coupon-Payment [Debt]	40	50
MONOPOLY		
Exit-Price Trigger after investment	0.3604	0.4505
Exit-Price Trigger before investment	0.3963	0.4951
Investment-Price Trigger	43.9427	43.9679
DUOPOLY		
Both Invested, Exit-Price Trigger	Firm 2 exits	0.4505
Firm 1 Follower: Exit-Price Trigger	Firm 2 exits	0.4584
Firm 1 Follower: Investment-Price Trigger	19.59	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	0.4863
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	19.73
Preemption Threshold Non-Predatory Investment	9.055	9.4218
Unilateral Incentive to predatorily Invest as Leader	No	No
Firm 2: Exit-Price Trigger before Investment	Firm 2 exits	0.4974

Table 5: Effects of firm 1's debt on investment-price triggers

debt	$P^{inv,1}_{\underline{q}_1,0}$	$P^{inv,1}_{\underline{q}_1,\overline{q}_2}$	$P^{inv,1}_{pre}$	$P^{inv,2}_{pre}$
49.9*	99.8436	15.8715	4.57881	4.89525
49	99.8422	17.2948	6.0871	6.52665
48	99.8407	17.2942	6.08645	6.52679
47	99.8392	17.2935	6.08581	6.52694
40	99.8294	17.2894	6.08161	6.5279
35	99.8232	17.2868	6.07892	6.52851

* Firm 1 exits first as follower and also predatorily invests as leader.

Table 6: Effects of firm 2's debt on investment-price triggers

debt	$P^{inv,2}_{\underline{q}_2,0}$	$P^{inv,2}_{\underline{q}_2,\overline{q}_1}$	$P^{inv,2}_{pre}$	$P^{inv,1}_{pre}$
50	133.102	23.2316	8.57345	8.20269
55	133.108	23.2526	8.61058	8.1743
60	133.115	23.2749	8.64987	8.14432
70	133.13	23.3233	8.73448	8.07987
80	133.147	23.3762	8.8266	8.00988

Table 7: Effects of the elasticity of demand on investment-price triggers

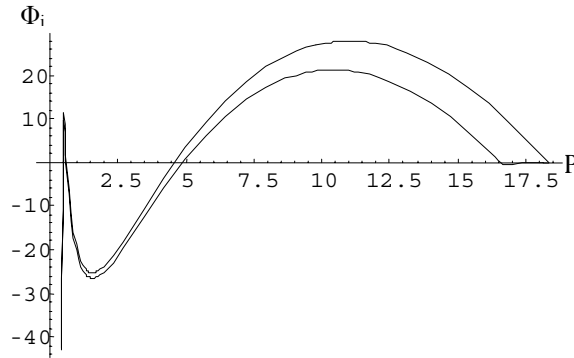| elasticity	$P^{inv,1}_{\underline{q}_1,0}$	$P^{inv,2}_{\underline{q}_2,0}$	$P^{inv,1}_{\underline{q}_1,\overline{q}_2}$	$P^{inv,2}_{\underline{q}_2,\overline{q}_1}$	$P^{inv,1}_{pre}$	$P^{inv,2}_{pre}$
-0.5	26.1318	26.1671	17.0162	17.1305	9.98429	10.3272
-0.6	32.8083	32.8389	18.2142	18.34	9.50913	9.86668
-0.7	43.9427	43.9679	19.5916	19.729	9.05488	9.42182
-0.8	66.2228	66.2419	21.1916	21.3402	8.61981	8.99124
-0.9	133.09	133.102	23.0725	23.2316	8.20269	8.57345

Figure 5: Gain, Φ_i, from becoming the leader, Base Case I (a)

Therefore, the model presented in this paper might indeed explain the U-shaped investment-debt relation found in figure 1 taken from Bayer (2002). Figure 5 shows the Φ_i functions for both firms corresponding to the first base case.

Table 6 reports the investment price triggers for different debt-levels of firm 2 (with the base case II specifications). For firm 2, investment becomes more likely for lower debt levels. This is in line with Jou´s (2001) findings for monopolists and empirical evidence. Interestingly, different debt levels influence firm 2´s investment decision much stronger in a duopoly than in monopoly.

Unlike in a static model of Cournot-competition, here, increasing the elasticity of demand make investment of the leader more likely, whereas the investment of the follower becomes less likely, as shown in table 7. Here again, the strong first-mover advantages drive the result.

Although only results for firms that are relatively symmetric were reported, solutions have been calculated for cases more asymmetric in production or investment costs. Yet, in the asymmetric cases the results do not change much, except for predatory outcomes to be more likely, i.e. we obtain predatory equilibria also for cases with a greater difference in leverage ratios or more elastic demands.

Generally, predatory outcomes are likely if firm 1 is small compared to firm 2 and both are heavily leveraged.

7   Conclusions

In this paper the real options in duopoly literature was extended to allow for simultaneous irreversible investment and exit decisions. The duopoly was modelled in continuous time and firms were assumed to hold debt because of tax advantages. The firms were allowed to default on their obligations at no costs for the equityholders.

It was shown that allowing for endogenous bankruptcy decisions alters the strate-

gic situation significantly. Firms may invest, not because investment is fundamentally profitable, but because this makes the exit of the competitor more likely. This however, affects the option value of the investment options themselves. Therefore, in the model presented debt not only induces an agency problem, but also has a negative strategic effect, which reduces the value of debt. This might explain the low debt-ratios found in practice compared to the substantial tax benefits of debt.

Moreover, a discontinuous effect on debt on investment incentives was found: For moderate debt-levels investment tends to decrease with increasing debt. However, if debt levels get large enough, investment-incentives become very strong, as both firms seek to become leader and monopolize the market when revenues decrease in the presence of adverse shocks. This strategic effect of debt might explain the non-monotone influence of debt on investment as shown in figure 1. Moreover, this gives an explanation for predatory behavior in a dynamic setting neither relying on the asymmetry of information among competitors, nor on learning-curve or network-effects. Our numerical examples show that in equilibrium both the firm with the higher and the firm with the lower debt may invest predatory.

As a further result and in stark contrast to Sparla (2001), who considers partly reduction of capacity, it was shown that even small heterogeneity with respect to th induce an asymmetric exit equilibrium, if only bankruptcy decisions are considered.

For further research several extensions can be made: First of all the choice of debt may be endogenised. Moreover, collusive behavior and symmetry of the firms could be allowed for as in Weeds (2001). Glazer (1994) finds in a two period version of the Brander Lewis (1986) model, that debt can make collusive behavior more likely in the first period. As a special case of the present model, market entry as in Jou (2001) could be studied. Moreover, welfare issues and issues of competition policy are open for research.

Finally one might allow firms to have multiple options to adjust leverage and capacity.

8   Appendix

In the following appendix, we first derive the functional form of the value function involved in our model. Thereafter, the proofs which were omitted in the main text are presented.

8.1   Deriving the value functions

Treating $E_i(P, b_i, \overline{q}_i, \overline{q}_{-i})$ as an asset value and using (2) yields according to Itô´s Lemma for its expected capital gain:

$$\mathbb{E}\left[\frac{dE_i(P, b_i, q_i, q_{-i})}{dt}\right] = \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 E_i(P, b_i, q_i, q_{-i})}{\partial P^2} + \mu P \frac{\partial E_i(P, b_i, q_i, q_{-i})}{\partial P} \quad (42)$$

This expected capital gain plus the dividend $(1 - \tau)[q_i P - b_i]$ should be equal to the normal return $\rho E_i(P, b_i, q_i, q_{-i})$ to prevent any arbitrage profits from arising. This yields the differential equation

$$\rho E_i(P, b_i, q_i, q_{-i}) =$$
$$\frac{\sigma^2}{2}P^2 \frac{\partial^2 E_i(P, b_i, q_i, q_{-i})}{\partial P^2} + \mu P \frac{\partial E_i(P, b_i, q_i, q_{-i})}{\partial P} + (1 - \tau)[q_i P - b_i] \quad (43)$$

A particular solution to this equation is

$$E_i(P, b_i, q_i, q_{-i}) = (1 - \tau)\left[q_i \frac{P}{\rho - \mu} - \frac{b_i}{\rho}\right] \quad (44)$$

The complementary solution involves terms in the form P^β, for each solution β to the fundamental quadratic equation

$$\beta^2 \frac{\sigma^2}{2} + \beta\left(\mu - \frac{\sigma^2}{2}\right) - \rho = 0 \quad (45)$$

as given in (6). (See Dixit and Pindyck (1994) for details.)

8.2   Proofs of the propositions of the main text

8.2.1   Proof of Proposition 2

Lemma 3 *Under the assumptions of our model*

$$g \geq h := \left[\left(\Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_1}\right)^{\beta_2} - \beta_2\left(\Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_1} - 1\right)\left(\frac{\overline{q}_1 b_2}{\overline{q}_2 b_1}\right)^{1 - \beta_2}\Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_2}^{-(1 - \beta_2)}\right] \geq 1 \quad (46)$$

holds for all $\Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_1} \geq 1 \geq \Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_2}^{-(1 - \beta_2)}$.

Proof. *For notational convenience we denote* $a := \Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_1} - 1$; $b := -\beta_2 > 0$ *and* $r := \frac{\overline{q}^1 b^2}{\overline{q}^2 b^1}$. *By definition of flow leverage, we obtain* $r = \frac{l_2}{l_1}\frac{\Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_2}}{\Delta_{\overline{q}_1 + \overline{q}_2, \overline{q}_1}}$. *Therefore,*

we have $\frac{r}{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}}(a+1) = \frac{l_2}{l_1} > 1$ *by assumption, so* $\frac{r}{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}} > \frac{1}{a+1}$ *follows. Rewriting h yields:*

$$h(a) = \left(\frac{1}{a+1}\right)^b + bar^{1+b}\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-(1+b)} \leq \left(\frac{1}{a+1}\right)^b + bar^{1+b} = g(a) \qquad (47)$$

and

$$\frac{\partial h(a)}{\partial a} = b\left[\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-(1+b)} r^{1+b} - \left(\frac{1}{a+1}\right)^{1+b}\right] = b\left(\frac{1}{a+1}\right)^{1+b}\left[\left(\frac{l_2}{l_1}\right)^{1+b} - 1\right] > 0 \tag{48}$$

Now, note that $h(0) = 1$ *which completes the proof.* ∎

Lemma 4 *For any* $P_{\overline{q}_2,\overline{q}_1}^{exit,2}$ *and* $P \geq P_{\overline{q}_2,\overline{q}_1}^{exit,2} \geq \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-1}P_{\overline{q}_2,0}^{exit,2} > 0$ *equity value of firm 1 is positive.*

    Proof. *To obtain negative equity values dependent on choosing* $P_{\overline{q}_2,\overline{q}_1}^{exit,2}$, *a necessary condition would be that* $\exists \hat{P}_{\overline{q}_2,\overline{q}_1}^{exit,2} > 0 : \min_{P>0}\{E_1(P,b_1,\overline{q}_1,\overline{q}_2)\} \leq 0.$
Using (11) *we obtain as a general solution for the equity-value of firm 1:*

$$\frac{1}{1-\tau}E_1(P,b,\overline{q},\overline{q}_2) = \frac{\overline{q}P}{\rho-\mu} - \frac{b}{\rho} + OpV P^{\beta_2} \tag{49}$$

$$OpV := \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} - 1\right)\frac{\overline{q}}{\rho-\mu}\left(\hat{P}_{\overline{q}_2,\overline{q}_1}^{exit,2}\right)^{1-\beta_2} + \frac{1}{1-\beta_2}\frac{b}{\rho}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}^{-1}P_{\overline{q}_1,0}^{exit,1}\right)^{-\beta_2}. \tag{50}$$

Therefore, we obtain for the price $P_{\min}$ *which minimizes value:*

$$P_{\min} = OpV^{\frac{1}{1-\beta_2}}\left[-\frac{1}{\beta_2}\frac{\overline{q}}{\rho-\mu}\right]^{-\frac{1}{1-\beta_2}}. \tag{51}$$

Note that this price may be smaller than $\hat{P}_{\overline{q}_2,\overline{q}_1}^{exit,2}$. *Using* $P_{\min}$ *to calculate the minimal equity-value* $E_{\min}$ *of firm 1 yields.*

$$\begin{aligned}
E_{\min} \quad &:= \frac{1}{1-\tau}E_1(P_{\min},b_1,\overline{q}_1,\overline{q}_2) \\
&= \frac{\overline{q}}{\rho-\mu}OpV^{\frac{1}{1-\beta_2}}\left[-\frac{1}{\beta_2}\frac{\overline{q}}{\rho-\mu}\right]^{-\frac{1}{1-\beta_2}} - \frac{b}{\rho} + OpV^{1+\frac{\beta_2}{1-\beta_2}}\left[-\frac{1}{\beta_2}\frac{\overline{q}}{\rho-\mu}\right]^{-\frac{\beta_2}{1-\beta_2}} \\
&= \underbrace{\left[(-\beta_2)^{\frac{1}{1-\beta_2}} + (-\beta_2)^{\frac{\beta_2}{1-\beta_2}}\right]\left[\frac{\overline{q}}{\rho-\mu}\right]^{-\frac{\beta_2}{1-\beta_2}}}_{>0}OpV^{\frac{1}{1-\beta_2}} - \frac{b}{\rho}. \tag{52}
\end{aligned}$$

Since OpV *is increasing in* $\hat{P}_{\overline{q}_2,\overline{q}_1}^{exit,2}$ *so is* $E_{\min}$. *Therefore, the smallest minimal equity value is obtained at* $\hat{P}_{\overline{q}_2,\overline{q}_1}^{exit,2} = \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-1}P_{\overline{q}_2,0}^{exit,2} = \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-1}rP_{\overline{q}_1,0}^{exit,1}$ *(with* r *defined as in the Lemma 3). Substituting this back in* (50) *we obtain (making further use*

of the definition of $P_{\overline{q}_1,0}^{exit,1}$)

$$
\begin{aligned}
OpV_{\min}\left(P_{\overline{q}_1,0}^{exit,1}\right)^{\beta_2} &= \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}-1\right)\frac{\overline{q}^1 P_{\overline{q}^1,0}^{exit,1}}{\rho-\mu}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-1}r\right)^{1-\beta_2}+\frac{1}{1-\beta_2}\frac{b}{\rho}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2} \\
&= \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}-1\right)\frac{\overline{q}^{\frac{-\beta_2(\rho-\mu)}{(1-\beta_2)\rho}}\frac{b}{\overline{q}}}{\rho-\mu}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-1}r\right)^{1-\beta_2}+\frac{1}{1-\beta_2}\frac{b}{\rho}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2} \\
&= \left[-\beta_2\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}-1\right)\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-1}r\right)^{1-\beta_2}+\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}\right]\frac{1}{1-\beta_2}\frac{b}{\rho} \\
&= h\frac{1}{1-\beta_2}\frac{b}{\rho}.
\end{aligned}
\tag{53}
$$

This now yields for $P_{\min}^{\min}$:

$$
\begin{aligned}
P_{\min}^{\min} &= h^{\frac{1}{1-\beta_2}}\left[\frac{\frac{1}{1-\beta_2}\frac{b}{\rho}}{-\frac{1}{\beta_2}\frac{\overline{q}}{\rho-\mu}}\right]^{\frac{1}{1-\beta_2}}\left(P_{\overline{q}_1,0}^{exit,1}\right)^{-\frac{\beta_2}{1-\beta_2}} \\
&= h^{\frac{1}{1-\beta_2}}\left[P_{\overline{q}_1,0}^{exit,1}\right]^{\frac{1}{1-\beta_2}}\left(P_{\overline{q}_1,0}^{exit,1}\right)^{-\frac{\beta_2}{1-\beta_2}}=h^{\frac{1}{1-\beta_2}}P_{\overline{q}_1,0}^{exit,1}.
\end{aligned}
\tag{54}
$$

Thus, the global minimum of the equity value is

$$
\begin{aligned}
\frac{1}{1-\tau}E_1(P_{\min}^{\min},b_1,\overline{q}_1,\overline{q}_2) &= h^{\frac{1}{1-\beta_2}}\frac{\overline{q}_1 P_{\overline{q}_1,0}^{exit,1}}{\rho-\mu}-\frac{b_1}{\rho}+h^{\frac{1}{1-\beta_2}}\frac{b_1}{\rho}\left[P_{\overline{q}_1,0}^{exit,1}\right]^{-\beta_2}\left[h^{\frac{1}{1-\beta_2}}P_{\overline{q}_1,0}^{exit,1}\right]^{\beta_2} \\
&= h^{\frac{1}{1-\beta_2}}\left[\frac{\overline{q}_1 P_{\overline{q}_1,0}^{exit,1}}{\rho-\mu}-\frac{b_1}{\rho}+\frac{1}{1-\beta_2}\frac{b_1}{\rho}\right]+\left(h^{\frac{1}{1-\beta_2}}-1\right)\frac{b_1}{\rho} \\
&= \left(h^{\frac{1}{1-\beta_2}}-1\right)\frac{b_1}{\rho}>0.
\end{aligned}
\tag{55}
$$

The last equality follows from the definition of $P_{\overline{q}_1,0}^{exit,1}$, whereas the inequality is result of Lemma 3. This completes the proof. ∎

Proposition 11 (Proposition 2 main text) *In all Markov-perfect equilibria in pure strategies of the $(\overline{q}_1,\overline{q}_2)$-subgame (exit after investment), the firm with the higher leverage (firm 2) chooses its monopoly exit price as the price trigger for bankruptcy $P_{\overline{q}_2,\overline{q}_1}^{exit,2}=P_{\overline{q}_2,0}^{exit,2}$, whereas firm 1 chooses any price $P_{\overline{q}_1,\overline{q}_2}^{exit,1}$, such that $P_{\overline{q}_1,\overline{q}_2}^{exit,1}\in\widetilde{P}:=]\Delta(\overline{q}_1+\overline{q}_2,\overline{q}_1)^{-1}P_{\overline{q}_1,0}^{exit,1},\Delta(\overline{q}_1+\overline{q}_2,\overline{q}_2)^{-1}P_{\overline{q}_2,0}^{exit,2}[.$*

    Proof. *First note that under the proposed equilibrium strategy firm 2 never becomes a monopolist. Therefore, only the actual price and not the quantity of the competitor matter for firm 2. Thus firm 2 behaves myopic. Therefore, the value of firm 2 under the proposed strategy is zero at $P_{\overline{q}_1,\overline{q}_2}^{exit,2}$, which then is indeed the optimal trigger price.*
Secondly, we have to show that firm 2 cannot profitably choose an exit price trigger smaller than $P_{\overline{q}_1,\overline{q}_2}^{exit,1}$. To see this, suppose firm 2 chooses a lower price trigger. Then firm 2 becomes a monopolist after firm 1 exits. However, as the price after firm 1 has left the market is still below firm 2´s monopoly exit price-trigger, the value

associated with this strategy must be negative.

To see that $P^{exit,1}_{\overline{q}_1,\overline{q}_2}$ is indeed an equilibrium trigger price, first note that, if firm 2 chooses $P^{exit,2}_{\overline{q}_2,\overline{q}_1}$, all trigger prices below $P^{exit,2}_{\overline{q}_2,\overline{q}_1}$ yield the same payoff given P. Because of the above Lemmata this payoff must be positive. Moreover, it cannot be rational to exit earlier than firm 2 as firm 1 would then forego monopoly profits, i.e. the positive payoffs.

Choosing any price-trigger outside $\widetilde{P}$ for firm 1, however, cannot be part of an equilibrium strategy as firm 2 could profitably deviate and set a price slightly smaller and obtain monopoly profits.

Last, we have to show, that $\widetilde{P}$ is non-empty. This now follows straightforward from assumption 1, our assumption regarding the leverage ratios. ∎

8.2.2   Proof of Lemma 5.1

Lemma 5 (Lemma 5.1 main text) *(i) Other things equal*

$$\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) + C_1 = E_1(\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P, b_1, \overline{q}_1, \overline{q}_2) \tag{56}$$

has at most three solutions in $P > \max\{P^{exit,2}_{\overline{q}_2, \underline{q}_1}, P^{exit,1}_{\underline{q}_1, \overline{q}_2}\}$. We denote the solutions with $P^*(< P^{**})(< P^{***})$ respectively and the set of solutions by S.

(ii) The sign of $\frac{\partial\left[\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) - E_1(\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P, b_1, \overline{q}_1, \overline{q}_2)\right]}{\partial P}$ changes from one solution to the next.

(iii) If the number of solutions is odd, we have

$$\frac{\partial\left[\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) - E_1(\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P, b_1, \overline{q}_1, \overline{q}_2)\right]}{\partial P}\Bigg|_{P=P^*} < 0;$$

(iv) if there are two solutions, we have

$$\frac{\partial\left[\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) - E_1(\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P, b_1, \overline{q}_1, \overline{q}_2)\right]}{\partial P}\Bigg|_{P=P^*} > 0.$$

Proof. *(i) Due to Lemma 3, if firm 2 exits first, $\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2)$ exhibits a kink, and this kink must be at a smaller P than the kink in $E_1(\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P, b_1, \overline{q}_1, \overline{q}_2)$. Define the continuous function*

$$f(P) := \widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) + C_1 - E_1(\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P, b_1, \overline{q}_1, \overline{q}_2). \tag{57}$$

In case $\Delta_{\underline{q}^1 + \overline{q}^2, \overline{q}^1 + \overline{q}^2} P^{exit,1}_{\underline{q}_1, \overline{q}_2} < P^{exit,2}_{\overline{q}_2, 0}$ it has the following functional form (for $P >$

$\max\{P^{exit,2}_{\overline{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\overline{q}_2}\})$:

$$f(P) = \begin{cases} x_{11}P + x_{12}P^{\beta_2} + C & \text{if } \Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P < P^{exit,2}_{\overline{q}_2,0} \\ x_{21}P + x_{22}P^{\beta_2} + C & \text{if } \Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P \geq P^{exit,2}_{\overline{q}_2,0} \end{cases}. \tag{58}$$

Otherwise

$$f(P) = x_{31}P + x_{32}P^{\beta_2}. \tag{59}$$

Hence, f must be either concave or convex on each subset. Consequently $f(P) = 0$ can have at most four solutions.

Now note that as sales are increasing with investment $\lim_{P\to+\infty} f(P) = -\infty$. Moreover, consider that f keeps its functional form until firm 1 exits in monopoly. As $\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) \geq 0$, we can conclude $f(P) > 0$ at the normalized price where firm 1 exits in monopoly. Therefore, the number of solutions to $f(P) = 0$ must be odd on the set of price-levels larger than the monopoly exit-price. However, $\left\{P \,\middle|\, P > \max\{P^{exit,2}_{\overline{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\overline{q}_2}\}\right\}$ is subset of this set. This completes the proof. (ii)-(iv) follow trivially. ∎

8.2.3 Proof of Lemma 2

Lemma 6 (Lemma 2 main text) *(i) If firm 2 leaves the market first $P^{exit,2}_{\overline{q}_2,0} < P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ holds.*
(ii) Moreover, in all cases $P^{exit,2}_{\overline{q}_2,\underline{q}_1} < \Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P^{exit,2}_{\overline{q}_2,0}$.
    Proof. *(i) The first inequality follows from the fact that the possible investment of firm 1 lowers equity-value of firm 2. Thus, $P^{exit,2}_{\overline{q}_2,0} < P^{exit,2}_{\overline{q}_2,\underline{q}_1}$.*
(ii) We firstly show that

$$\forall P < P^{inv,1}_{\underline{q}_1,\overline{q}_2} : E_2(\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P, \overline{q}_2, \underline{q}_1) > E_2(P, \overline{q}_2, \overline{q}_1).$$

$\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P$ maps situation $(\overline{q}_2, \overline{q}_1)$ to $(\overline{q}_2, \underline{q}_1)$ prices and situation $(\overline{q}_2, \underline{q}_1)$ differs from $(\overline{q}_2, \overline{q}_1)$ for firm 2 only in the different prices which correspond to the same Y_t. Therefore, the decrease in equity value of firm 2 that is caused by the existence of an investment option of firm 1 is always smaller than the decrease caused by investment itself. Thus, the stated inequality follows. This inequality itself implies that the equity-value of firm 2 at price $\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P^{exit,2}_{\overline{q}_2,0}$ must be positive, because

$$E_2(\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P^{exit,2}_{\overline{q}_2,0}, \overline{q}_2, \underline{q}_1) > E_2(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_2, \overline{q}_1) = 0.$$

Thus, $P^{exit,2}_{\overline{q}_2,\underline{q}_1} < \Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P^{exit,2}_{\overline{q}_2,0}$, which concludes the proof. ∎

8.2.4   Proof of Proposition 4

Proposition 12 (Proposition 4 main text) *(i) If demand is sufficiently elastic, i.e. $\forall Q_1, Q_2 : \Delta_{Q_1,Q_2} \to 1$, respectively if demand is not too inelastic and the costs of investment C_1 are sufficiently high, then predatory investment never occurs.*
(ii) If $g > \left(\frac{\overline{q}_1 b_1}{\overline{q}_2 b_2}\right)^{1-\beta_2}$ and

(a) if firm 1 exits first when it had no investment option and if

$$\widetilde{P}^{exit,1}_{\underline{q}_1,\overline{q}_2} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \tag{*}$$

then there exists an investment-cost C_1, so that (22) has multiple solutions.

(b) if firm 2 exits first and if for the right-hand partial derivative $\frac{\partial E_1}{\partial P+}$

$$\frac{\partial \widetilde{E}_1}{\partial P}\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \underline{q}_1, \overline{q}_2\right) > \frac{\partial E_1}{\partial P+}\left(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2\right) \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}$$

holds, then there exists the cost of investing C_1, so that predatory investment occurs.

Proof. *(i) As $\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} \to 1$ the value of a monopolist and of the value a duopolist (after investment) converge and firm 1 cannot gain anything by firm 2´s exit. Therefore, predatory investment must become unprofitable.*
(ii) First note from the proof of Lemma 4 that, if $g > \left(\frac{\overline{q}_1 b_1}{\overline{q}_2 b_2}\right)^{1-\beta_2}$ and if $P' \in \left(P^{exit,2}_{\overline{q}_2,0}, P_{\min}\right)$, then $\frac{\partial E_1}{\partial P}(P', \overline{q}_1, \overline{q}_2) < 0$. Moreover, $P^{exit,2}_{\overline{q}_2,0}$ corresponds to the same Y as $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$ does in situation $\left(\underline{q}_1, \overline{q}_2\right)$.
(a) If firm 1 exits first if not predatorily investing in situation $\left(\underline{q}_1, \overline{q}_2\right)$, equity value is upward sloping and $\widetilde{P}^{exit,1}_{\underline{q}_1,\overline{q}_2} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$ by assumption (*). Thus, shifting $\widetilde{E}_1 + C_1$ by altering C_1 yields a solution of (22) on

$$\left]\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P_{\min}\right[$$

and another one on $\left]\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P_{\min}, +\infty\right[$.
(b) If firm 2 would exit first, due to Lemma 2, the peak in $E_1(P, \overline{q}_1, \overline{q}_2)$ lies at $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0} > P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. Consider the costs C' that give

$$\widetilde{E}_1\left(P, \underline{q}_1, \overline{q}_2\right) + C' < E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2\right) \tag{60}$$

for all $P \geq \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$ except for one point P'. At this point either both functions are tangentially or $P' = \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$. The assumption on the derivative rules out the latter case.

Therefore, at costs C' firm 1 invests for all prices $P \geq \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}^{-1} P_{\overline{q}_2,0}^{exit,2}$ and

$$\hat{E}_1\left(P,\underline{q}_1,\overline{q}_2\right) \equiv E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2\right) - C'.$$

Now take costs to be equal to $C' + \varepsilon$, $\varepsilon > 0$. Assuming that there is only one price trigger for investment P^{inv}, will lead to a contradiction: For this trigger $P' \leq P^{inv}$ holds. However, defining the stopping-time $\tau(p) := \inf\{t \in \mathbb{R} | P_t = p\}$ the difference in value for $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}^{-1} P_{\overline{q}_2,0}^{exit,2} < P < P'$ evaluates as

$$\hat{E}_1\left(P,\underline{q}_1,\overline{q}_2|C'\right) - \hat{E}_1\left(P,\underline{q}_1,\overline{q}_2|C'+\varepsilon\right)$$

$$= \mathbb{E}\left[\int_0^{\tau\left(P^{inv}\right)}\left(E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P_t,\overline{q}_1,\overline{q}_2\right) - C' - \widetilde{E_1}\left(P_t,\overline{q}_1,\overline{q}_2\right)\right)e^{-\rho t}dt \,\middle|\, P_0 = P\right]$$

$$\geq \mathbb{E}\left[\int_0^{\tau\left(P'\right)}\left(E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P_t,\overline{q}_1,\overline{q}_2\right) - C' - \widetilde{E_1}\left(P_t,\overline{q}_1,\overline{q}_2\right)\right)e^{-\rho t}dt \,\middle|\, P_0 = P\right] > 0.$$

Both inequalities follow from (60) (and $\tau(P^{inv}) \leq \tau(P')$). Thus, a marginal change in costs would lead to a non-marginal drop in value (the last integral does not depent on ε), whereas this would not be true for a system of two price-triggers of investment depending on ε. Thus for ε small enough two price-triggers are optimal. ∎

8.2.5   Proof of Proposition 5

Proposition 13 (Proposition 5 main text) *(i) If for firm i the myopic exit price-trigger $P_{q_i,\overline{q}_{-i}}^{m\_exit,i}$ obtained from (29) is smaller than $P_{q_i,q_{-i}}^{ind,i}$, then firm $-i$ choosing $P_{q_{-i},q_i}^{m\_exit,-i}$ and firm i choosing a lower price-trigger is an equilibrium of the (q_i,q_{-i}) stage.*
(ii) If $\left[\Delta_{q_i+q_{-i},q_i}^{-1}P_{q_{-i},0}^{exit,i}, P_{q_i,q_{-i}}^{ind,i}\right] = \emptyset$, then firm i chooses $P_{q_i,\overline{q}_{-i}}^{m\_exit,i}$ as the exit-price-trigger in all equilibria of the (q_i,q_{-i}) stage.
(iii) If $P_{q_{-i},q_i}^{m\_exit,-i} > P_{q_i,q_{-i}}^{ind,i}$ and $\left[\Delta_{q_i+q_{-i},q_i}^{-1}P_{q_{-i},0}^{exit,i}, P_{q_i,q_{-i}}^{ind,i}\right] \neq \emptyset$, then firm i choosing some $P_{q_i,q_{-i}}^{exit,i} \in \left[\Delta_{q_i+q_{-i},q_i}^{-1}P_{q_{-i},0}^{exit,i}, P_{q_i,q_{-i}}^{ind,i}\right]$ and firm $-i$ choosing $P_{q_{-i},q_i}^{m\_exit,-i}$ is an equilibrium of the (q_i,q_{-i}) stage, if this yields no incentive to predatorily invest for firm $-i$.
(iv) If both firms have an incentive to predatorily invest, instead of choosing their myopic exit price-trigger, then both firm preempt on predatory investment.
(v) If firm i predatorily invests, firm $-i$ cannot credibly threaten to deviate from choosing the myopic exit price-trigger to hinder i in investing predatorily.

   Proof. *(i) Firm value is increasing in the exit price of the competitor. If undercutting the price-trigger of firm i is not credible even when i chooses the myopic price-trigger, then $-i$ cannot threaten to exit second.*
(ii) $\left[\Delta_{q_i+q_{-i},q_i}^{-1}P_{q_{-i},0}^{exit,i}, P_{q_i,q_{-i}}^{ind,i}\right] = \emptyset$ implies that leaving second always yields positive equity value for all credible exit price triggers of the competitor i. Therefore,

in this case firm $-i$ will leave second, as this increases value at the myopic exit price-trigger. Note that since $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$, the interval can never be empty for both firms.

(iii) This was largely discussed in the main text. It remains to be mentioned that if firm $-i$ invests predatorily, this decreases equity value below the value obtained by behaving myopically, since the competitor will only invest predatorily (if not pre-empting) if she expects to leave second after investment.

(iv) See main text.

(v) If firm -i would exit earlier it would forgo profits. Therefore, this is not credible. Exiting later is also not credible. If firm -i exits later, payoff would become negative. Thus, firm -i cannot credible threaten to set an exit price different to the one determined by the smooth-pasting conditions if firm i predatorily invests. ∎

8.2.6 Proof of Proposition 6

Lemma 7 *Let $f(P) = x_0 + x_1 P + x_2 P^{\beta_1} + x_3 P^{\beta_2}$; $P > 0$ and $x_1 > 0$, $x_2 < 0$, then f has at most three roots. Moreover if at P' $f(P') > 0$, there can only be two roots of f for $P < P'$.*

*   **Proof.** We have to consider two cases:*

Case 1: $x_3 \leq 0$, then f is concave and therefore has at most two roots.

Case 2: $x_3 > 0$. We firstly show that the second derivative changes its sign at most once:

Suppose $f''(P^*) = P^{*-2} \left[\beta_1 (\beta_1 - 1) x_2 P^{*\beta_1} + \beta_2 (\beta_2 - 1) x_3 P^{*\beta_2} \right] = 0$. Then

$$f'''(P^*) = -2P^{*-3} \left[\beta_1 (\beta_1 - 1) x_2 P^{*\beta_1} + \beta_2 (\beta_2 - 1) x_3 P^{*\beta_2} \right]$$
$$+ P^{*-2} \left[\beta_1 (\beta_1 - 1) \beta_1 x_2 P^{*\beta_1} + \beta_2 (\beta_2 - 1) \beta_2 x_3 P^{*\beta_2} \right] \quad . \quad (61)$$

However, the first term is zero and therefore:

$$f'''(P^*) = P^{*-2}[\underbrace{\beta_1 (\beta_1 - 1) \beta_1}_{>0} \underbrace{x_2 P^{*\beta_1}}_{<0} + \underbrace{\beta_2 (\beta_2 - 1) \beta_2}_{<0} \underbrace{x_3 P^{*\beta_2}}_{>0}] < 0 \quad . \quad (62)$$

This implies that the second derivative changes its sign at most once, dividing the function in a convex and a concave part. Now suppose $f(P^*) < 0$, then there may be two roots larger than P^*. However, as $f(P^*) < 0$, at the next smallest root $f'(P)$ must be negative, but as f is convex, there are no more roots. The case $f(P^*) \geq 0$ follows analogously. ∎

Lemma 8 *$\Phi_i(P)$ can be represented by $\Phi_i(P) = x_{0i} + x_{1i}P + x_{2i}P^{\beta_1} + x_{3i}P^{\beta_2}$ for $P \in M := [\max_{j=1,2}\{\underline{P}_j\}, \min_{j=1,2}\{\overline{P}_j\}]$; with $x_{1i} > 0$, $x_{2i} < 0$. Moreover, $\Phi_i(P)$ is also continuous on $[\min_{j=1,2}\{\underline{P}_j\}, \max_{j=1,2}\{\overline{P}_j\}]$*

Proof. *First note that $\Phi_i(P)$ has the stated functional form since it is a difference of functions of the type given in (6) (which are analytic on M). It is clear, that the followers equity value must be a convex function. Moreover, the leader´s value decreases by the potential entry of the follower, therefore $x_2 < 0$. That sales are increased by investment implies $x_1 > 0$. Continuity follows from the value-matching conditions* ∎

Proposition 14 (Proposition 6 main text) *(i) On $M := [\max\limits_{j=1,2}\{\underline{P}_j\}, \min\limits_{j=1,2}\{\overline{P}_j\}]$*

$$\Phi_i(P) = 0 \tag{63}$$

has at most three solutions.

(ii) If $\overline{P}_i \leq \overline{P}_{-i}$, then $\Phi_i(P) = 0$ has at most two solution on M and only one additional one for $\overline{P}_i \leq P \leq \overline{P}_{-i}$, namely $\overline{P}_{-i}$ with $\Phi_i'(\overline{P}_{-i}) < 0$.

(iii) If $\overline{P}_i > \overline{P}_{-i}$, $\Phi_i(\overline{P}_i) = 0$ and $\Phi_i(P) < 0$ for all $\overline{P}_i > P > \overline{P}_{-i}$.

(iv) If $\Phi_i(P) = 0$ has two solutions in case (ii) or three solutions in case (iii) on M then $\Phi_i(\max\{\underline{P}_i\}) > 0$. There may only exist an additional solution for $\min\limits_{j=1,2}\{\underline{P}_j\} < P < \max\limits_{j=1,2}\{\underline{P}_j\}$ if $\underline{P}_i < \underline{P}_{-i}$.

Proof. *(i) follows straightforward from the last two Lemmata.*

(ii) At $\overline{P}_i$ firm i invests as follower, therefore

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \underline{q}_i+\overline{q}_{-i}} P, b_i, \underline{q}_i, \overline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) - C \quad \forall P \geq \overline{P}_i. \tag{64}$$

This implies $\Phi_i(P) > 0$ if $\overline{P}_i \leq P < \overline{P}_{-i}$ and $\Phi_i(\overline{P}_{-i}) = 0$, since sales of the leader are larger before the follower has invested and

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} \overline{P}_{-i}, b_i, \overline{q}_i, \underline{q}_{-i}) - C = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\overline{q}_{-i}} \overline{P}_{-i}, b_i, \overline{q}_i, \overline{q}_{-i}) - C. \tag{65}$$

(iii) At $\overline{P}_{-i}$ firm $-i$ invests as follower, therefore

$$\forall P \geq \overline{P}_{-i} : E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} P, b_i, \overline{q}_i, \underline{q}_{-i}) - C$$
$$= E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) - C. \tag{66}$$

Therefore, as long as $P < \overline{P}_i$ firm i finds it unprofitable to invest as follower and obtain $E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) - C$. Hence, the right-hand term must be smaller than the equity value of i being the follower when $P < \overline{P}_i$.

(iv) $\Phi_i(\max\limits_{j=1,2}\{\underline{P}_j\}) > 0$ follows from the continuity of Φ_i and in case (ii) from $\Phi_i(\bar{P}_i) > 0$, respectively $\Phi_i(\bar{P}_{-i}) < 0$ in case (iii). Define $f(P)$ as stated in Lemma 7. Now suppose that firm i exits at $\underline{P}_i \geq \underline{P}_{-i}$, then $\Phi_i(P) > f(P)$ for $P < \underline{P}_i$, since the value of firm i as follower is a convex function with derivative

zero at $\underline{P}_i$. Therefore, for $\Phi_i(P)$ to have an additional root, $f(P)$ must have an additional root, too. However, due to Lemma 7, $f(P)$ cannot have that additional root.

If firm i predatorily invests at $\underline{P}_i$. Then for all $\min\limits_{j=1,2}\{\underline{P}_j\} < P < \max\limits_{j=1,2}\{\underline{P}_j\}$

$$\Phi_i(P) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} P, b_i, \overline{q}_i, \underline{q}_{-i}) - E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) > 0 \tag{67}$$

because the sales of the follower are lower, and firm $-i$ exits later. ∎

8.2.7   Proof of Proposition 9

Lemma 9 *If firm i invests predatorily as follower, then $\Phi(\underline{P}_i) > 0$.*

   Proof. *At the price at which firm $-i$ exits after firm i has predatorily invested as follower $\Phi = 0$ must hold. However, in state $(\overline{q}_i, \overline{q}_{-i})$ firm $-i$ will exit later than in state $\left(\overline{q}_i, \underline{q}_{-i}\right)$. Moreover, prices are lower before $-i$ exits if i is the follower. Therefore, firm i's value as leader must be larger than firm i's value as follower at the price at which firm i predatorily invests as follower, i.e.*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} \underline{P}_i, b_i, \overline{q}_i, \underline{q}_{-i}) - C \geq E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\overline{q}_{-i}} \underline{P}_i, b_i, \overline{q}_i, \overline{q}_{-i}) - C \quad. \tag{68}$$

∎

Lemma 10 *Only a firm i that effectively stays in the market longer than its competitor in state $\left(\underline{q}_i, \overline{q}_{-i}\right)$ could have second mover advantages of profitable predatory investment.*

   Proof. *Suppose that firm i leaves the market first in state $\left(\underline{q}_i, \overline{q}_{-i}\right)$. Furthermore, suppose firm $-i$ invests at P' and at P' predatory investment would be profitable for firm i, too, i.e.*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} P', b_i, \overline{q}_i, \underline{q}_{-i}) - C \geq \widehat{E}_i(P', b_i, \underline{q}_i, \underline{q}_{-i}) \quad. \tag{69}$$

Investment of firm $-i$ will lower prices and therefore decrease the revenues of firm i compared to the situation $\left(\underline{q}_i, \underline{q}_{-i}\right)$. Then, as firm i leaves first, it cannot gain of any investment-induced change in the probability of firm $-i$ exiting. Therefore, its value as follower must be less than $\widehat{E}_i(P', b_i, \underline{q}_i, \underline{q}_{-i})$, so that there cannot be any second mover advantages, i.e. $\widehat{E}_i(P', b_i, \underline{q}_i, \underline{q}_{-i}) > E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \underline{q}_i+\overline{q}_{-i}} P', b_i, \underline{q}_i, \overline{q}_{-i})$. ∎

Lemma 11 *If firm i does not predatorily invest as follower, then*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i}, \underline{q}_i+\overline{q}_{-i}} P, b_i, \underline{q}_i, \overline{q}_{-i}) \leq \widehat{E}_i(P, b_i, \underline{q}_i, \underline{q}_{-i}) \quad.$$

   Proof. *The option of firm $-i$ investing decreases firm i's value to a lesser extent than the drop in revenues caused by investment itself does. Because of that,*

$\widehat{E}_i(P, b_i, \underline{q}_i, \underline{q}_{-i}) < E_i(\Delta_{\underline{q}_i + \underline{q}_{-i}, \underline{q}_i + \overline{q}_{-i}} P, b_i, \underline{q}_i, \overline{q}_{-i})$ can only hold, if firm i would have to expect some costs not included in $\widehat{E}_i$ for low P or if there would be any expected gain in revenues associated with investment of firm $-i$. However, as firm i as follower was assumed to not predatorily invest, there cannot be any expected gain, since firm $-i$'s investment decreases the probability of firm $-i$ leaving the market. Moreover, there are no costs ignored in deriving $\widehat{E}_i$ (except for the cost of possible predatory investment, which - if profitable - increases value). Therefore, the stated inequality follows. ∎

Proposition 15 (Proposition 9 main text) *A solution to* (37) *implies that a predatory investment preemption threshold for firm i exists, i.e. there cannot be second-mover advantages for profitable predatory investment independent of how low P gets.*

    Proof. *In case firm i exits first or exists last, but does not predatorily invest, the proposition follows straightforward from the last two lemmata. Hence, we only need to discuss the case where firm i would predatorily invest as follower. However, due to Lemma 10 $\Phi(\underline{P}_i) > 0$. Therefore, and because of Proposition 14(iv) we either have first-mover advantages of investment for all P, or we have an preemption threshold price for predatory investment which is larger than $\underline{P}_i$.* ∎

8.2.8   Proof of Proposition 10

Proposition 16 (Proposition 10 main text) *(i) If there is a preemption game for predatory investment and $P^{pred,i}_{pre} < P^{inv,-i}_{pre}$ for both firms, the only Markov-perfect equilibrium (outcome) is that the firm with the higher $P^{pred,i}_{pre}$ takes the lead for predatory investment and invests at a price-trigger which is a solution to a version of* (34) *that is modified by defining $P^{inv,-i}_{\underline{q}_{-i}, \underline{q}_i} := P^{pred,i}_{pre}$, $P^{pred,-i}_{\underline{q}_{-i}, \underline{q}_i} := P^{pred,-i}_{pre}$ and by using the appropriate value matching conditions.*
(ii) If there is a preemption game for predatory investment, and $P^{pred,i}_{pre} > P^{inv,-i}_{pre}$ for firm i, and one of the firms has an unilateral incentive to invest on $\left[P^{inv,-i}_{pre}, P^{pred,i}_{pre} \right]$, then in all Markov-perfect equilibria firm $-i$ invests predatorily at $P^{pred,i}_{pre}$.
(iii) If there is a preemption game for predatory investment and $P^{pred,i}_{pre} > P^{inv,-i}_{pre}$ for one of the firms, and none of the firms have an unilateral incentive to invest on $\left[P^{inv,-i}_{pre}, P^{pred,i}_{pre} \right]$, then in all renegotiation-proof Markov-perfect equilibria firm i predatorily invests at its unconstrained optimal predatory investment price-trigger or at $P^{pred,-i}_{pre}$, whichever is the higher price.

    Proof. *(i) Suppose firm i wishes to invest at a price $P' < P^{pred,-i}_{pre}$. Define τ to be the corresponding stopping time. Then firm $-i$ would have an incentive to preempt and invest at a smaller price at time $\tau - \varepsilon$. Therefore, predatorily investing below $P^{pred,-i}_{pre}$ cannot be part of an equilibrium. However, at prices between $P^{pred,-i}_{pre}$ and $P^{inv,-i}_{pre}$ firm $-i$ wishes to become follower and will therefore not preempt. Moreover at prices below $P^{pred,i}_{pre}$ firm i wishes to become leader, so that the*

described solution P^* to the generalized smooth pasting condition (34) (allowing for border and non-border solutions) is indeed optimal for firm i, given firm $-i$ would invest as soon as prices hit the preemption thresholds. Hence, this is indeed an optimal strategy for firm given that firm i would invest at all prices lower than P^*.

(ii) If one of the unconstrained investment price triggers—say for firm j—lies between $P_{pre}^{pred,i}$ and $P_{pre}^{inv,j}$, then not only threatening to invest is for firm j credible, but also threatening to not invest is not credible. Thus, the firms wish to preempt until $P_{pre}^{pred,i}$ is reached, where firm i is indifferent between becoming leader or follower. As investment-price trigger, firm $-i$ will choose its unconstrained-optimal predatory-investment price trigger (if this is possible). The unconstrained price-trigger is then determined by a smooth-pasting condition. If the constraint binds, $P_{pre}^{pred,i}$ is chosen as investment price-trigger.

(iii) We only need to argue that investing on $[P_{pre}^{pred,i}, P_{pre}^{inv,-i}]$ cannot be renegotiation-proof. Suppose one firm would invest at $P^* \in [P_{pre}^{pred,i}, P_{pre}^{inv,-i}]$. Then, since neither firm has an unilateral incentive to invest at some price $P \in [P_{pre}^{pred,i}, P_{pre}^{inv,-i}]$, both firms would find it profitable to renegotiate and sign an incentive compatible contract that investment should be carried out at the proposed price-triggers for predatory and non-predatory investment. ∎

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