

Commercial Activity as Insurance: the Investment Behavior of Non-Profit Firms*

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Abstract

We provide a new rationale for commercial activities by non-profit organizations whose primary concern is to supply mission output. We show that investment in commercial activity may be used to insure mission output against the uncertainty of donations, though possibly at the cost of lower expected mission output. In this case, the amount of commercial investment is positively related to the variance of donations and to the degree of risk aversion. These predictions are corroborated by empirical tests on data from non-profits operating in the state of New York.

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1 Introduction

Non-profit organizations play a significant role in many market economies and recently have been the subject of much economic analysis.¹ In the US, non-profits figure prominently in many sectors, including health, education, religion, social services and the arts; in 1999, their current operative expenditures comprised 8.5% of GDP (Independent Sector, 2002). Typically, a non-profit is set up with a specific mission, entitling it to apply for forms of tax exemption or other favorable treatment.² To pursue its mission the non-profit may supply the output at a loss (possibly setting a zero price), relying on donations for funding. In recent years, however, non-profits have become increasingly commercialized, for example undertaking additional activities unrelated to their mission. Weisbrod (1998) cites numerous cases, including the (commercial) health clubs launched by non-profit hospitals and retail shops opened by museums. He explains this development as being primarily a means for non-profit to raise revenue in order to cross-subsidize mission activity, though the relationship is more complicated if donors respond to commercialization by changing the amount they give.

The present paper considers another argument for commercialization, which rests on the potential volatility of donations, particularly at the level of the individual organization. If the profit on commercial activity is more certain than the income from donations, it may be advantageous for a non-profit to invest in creat-

¹See, for example, Weisbrod (1988, 1998) for overall treatments; Glaeser and Shleifer (2001) and Glaeser (2003a) for recent theorizing; and Malani, Philipson and David (2003) for a discussion of various theories and how they relate to the existing evidence.

²Generous tax-exemptions are granted to non-profits by IRC S 501(c) in the US and by Charity Law in the UK.

ing capacity for commercial activity, even at the cost of less capacity investment in mission activity. Investment in commercial activity may be viewed as the purchase of a type of insurance. If a good state of the world obtains (donations being high), the forgone investment in mission activity will result in a binding mission capacity constraint, holding back mission output. However, if a bad state obtains (donations being low), the ability to supply commercial output limits the extent by which, because of budgetary considerations, the non-profit will have to reduce its supply of mission output. We find the conditions under which such investment behavior is rational for a non-profit and then explore empirically whether this behavior obtains in practice.

To obtain sharp results, we formulate the model as simply as possible. To capture the idea that the non-profit is concerned primarily with supplying mission output, we assume that it has lexicographic preferences. Its priority is to maximize the expected value of a concave function of mission output. Its secondary objective is to maximize its expected terminal money holding (financial reserves) in the period considered. We begin with the benchmark case in which there is no uncertainty as to the level of donations and show that an appropriate balance of investment in mission activity and commercial activity maximizes mission output because of cross subsidization. We then introduce uncertainty over donations and characterize the conditions under which commercial activity will be undertaken as insurance and analyze how the level of commercial activity is related to various parameter values.

We test our results empirically using a sample of non-profits operating in the

State of New York. In the absence of appropriate time-series data, we proxy the volatility of donations in each National Taxonomy of Exempt Entities (NTEE) sector by the variance, across organizations in the same sector, of the ratio of donations to total revenue. We estimate a least-squares model in which commercial activity is explained in terms of the volatility of donations, losses made on mission activity, the size of the organization and other control variables. As a further check, we estimate a binary model based on the possibility of cross-subsidizing mission activity by commercial revenues rather than by donations. Our results confirm the positive relationship between commercial revenues and the variance of donations, and therefore support the hypothesis that commercial activity can be a form of insurance.

Section 2 gives the theoretical analysis, with the proofs relegated to an appendix. Section 3 provides the empirical analysis. Section 4 concludes.

2 The Model

Consider a non-profit whose mission is to supply a particular type of output. It begins period 0 with an endowment of money, one of the sources of which is past donations. To supply output in period 1, it must invest some or all of its endowment in creating capacity during period 0. For simplicity, we assume that the investment is in training workers, renting capital goods and buying raw materials, and so the firm will not be left at the end of the time horizon of the model with assets of market value. Consistent with its mission, once the investment has been sunk it will sell the output at a negative markup on variable

costs.

However, it must satisfy a break-even constraint and so subsidizes its mission activity in period 1. One potential source of subsidy is period-1 donations, but these have the critical characteristic that their amount is unknown in period 0, when the investment decision is made. Another way to subsidize mission activity is to invest some of the endowment in creating capacity to supply a *non-mission* ‘commercial’ output, on which a profit can be made. We assume that capacity is output-specific, i.e., commercial capacity cannot be used to make mission output, and vice versa. In contrast to donations, investment in commercial activity has a direct opportunity cost in that less capacity for mission activity is created. For simplicity, we assume that the return on investment is risk-free.³

The non-profit enters period-0 with a given endowment of funds I ; it allocates these funds between investment I_m in mission output and investment $I_c = I - I_m$ in commercial activity. The capacity constraints in period 1 are

$$q_m \leq \alpha_m I_m, \tag{1}$$

$$q_c \leq \alpha_c I_c, \tag{2}$$

where q_m and q_c are the respective mission and commercial outputs; and α_m and α_c are the respective output-investment ratios for mission and commercial activity. The investment costs are sunk in period 0, while in period 1 there will be variable costs of supplying the two types of good. The non-profit sells the mission and commercial outputs at given markups $\pi_m < 0$ and $\pi_c > 0$ per unit

³A further way of subsidizing mission activity would be by saving some of the endowment; but in our model we focus on commercial activity, implicitly assuming that commercial activity has a higher return than saving does. We return to this point below.

over the respective variable costs. The mission markup is set at a negative level because of altruism, while the positive commercial markup allows us to capture in a simple way the idea that a greater investment in commercial activity generates higher profits.⁴

At the start of period 1 the non-profit receives donations, represented by the random variable θ , which has two possible realizations, θ^H and θ^L , with respective probabilities p^H and p^L . $\theta^H \geq \theta^L \geq 0$; $p^H, p^L \in [0, 1]$ and $p^H + p^L = 1$. In period 0, $\{\theta^H, \theta^L, p^H, p^L\}$ are known, but the realization of θ is not observed until the beginning of period 1. The expected value of θ is denoted by $\theta^0 = p^H \theta^H + p^L \theta^L$.

In period 1 the non-profit decides on $\{q_m, q_c\}$, given its capacities in each sector, and given the other parameter values, including the realization of θ . We denote by M the amount of money with which it leaves period 1. The existence of a break-even constraint implies that M must be non-negative. The period-1 budget constraint is therefore,

$$\pi_m q_m + \pi_c q_c + \theta^J - M = 0, \quad M \geq 0, \quad J = H, L. \quad (3)$$

To capture the idea that the non-profit's primary concern is to supply mission output, we assume that its utility function U is lexicographic.⁵ Let q'_m and q''_m denote any non-negative values of q_m and let M' and M'' denote any non-negative values of M . Let \succsim denote 'at least as good as' for the non-profit. We assume

⁴A more general treatment would allow for endogeneity of the mission markup through variation of the sale price of mission activity. Note that a reason for a fixed markup might be that, for institutional reasons, this price is set at zero.

⁵If the non-profit maximized a weighted sum of mission output and terminal money holding, our qualitative results would still hold, provided the weight attached to the mission output were sufficiently high. This is because higher mission output implies less terminal money holding.

that

$$\begin{aligned} \{E[(q'_m)^\gamma]; E(M')\} &\succsim \{E[(q''_m)^\gamma]; E(M'')\} \text{ whenever } E[(q'_m)^\gamma] > E[(q''_m)^\gamma] \\ \text{or } E[(q'_m)^\gamma] &= E[(q''_m)^\gamma] \text{ and } E(M') > E(M''); 0 < \gamma \leq 1. \end{aligned} \quad (4)$$

If $\gamma = 1$, the non-profit is risk neutral with respect to q_m ; as γ is reduced, the non-profit is increasingly risk-averse with respect to q_m . Its primary objective is to maximize $E[(q_m)^\gamma]$; but, for any given realization of $E[(q_m)^\gamma]$, it maximizes $E(M)$. Thus, its primary objective relates to mission activity and its secondary objective relates to terminal money holding. Because commercial output capacity cannot be used to produce mission output, an implication of (4) is that, since commercial activity is profitable, for any given investments $\{I_m, I_c\}$ the firm will use its commercial output capacity fully.⁶ The same is not true for mission output capacity, since the firm may lack funding to finance the supply of mission output.

2.1 The Benchmark Case: Certainty

We begin by finding the solution for the benchmark case in which $\theta^H = \theta^L = \theta^0$. The budget constraint (3) can then be rewritten, taking into account that $M \geq 0$, as

$$q_m \leq \frac{-1}{\pi_m} (\pi_c q_c + \theta^0). \quad (5)$$

This is depicted in $\{q_m, q_c\}$ -space in Figure 1. The budget constraint (5) is satisfied on or to the left of the line labelled BC . The line labelled II is the locus of

⁶Consider now the possibility that, instead of producing a commercial output, the firm saves the amount I_c , earning interest rate r . If $\pi_c \alpha_c$ is replaced by $1 + r$ in (3), the model can be reinterpreted as fitting this case. The firm would choose saving rather than commercial activity if $1 + r > \pi_c \alpha_c$, but it would prefer commercial activity if the inequality were reversed. For simplicity, we keep to the commercial-activity interpretation in developing the model.

combinations of the maximum possible outputs $I = q_m/\alpha_m + q_c/\alpha_c$. From (1), (2) and (5), this intersects BC at

$$I_c = \frac{-(\pi_m \alpha_m I + \theta^0)}{\pi_c \alpha_c - \pi_m \alpha_m} \equiv \bar{I}_c. \quad (6)$$

To avoid degenerate solutions, we assume that this intersection occurs in the interior of the positive quadrant:

$$\textit{Assumption A1: } I > I_{\min} \equiv -\frac{\theta^0}{\pi_m \alpha_m}.$$

A1 ensures that, if $I_m = I$, donations are not sufficient, given the negative markup π_m , to finance the supply of the maximum physically feasible mission output $\alpha_m q_m$. If A1 were violated, the non-profit would set $I_c = 0$.

[Figure 1 here]

The following lemma then holds.

Lemma 1 *Under certainty, the non-profit will invest such that the capacity constraints and the budget constraint bind simultaneously, i.e., $I_c = \bar{I}_c$ and $I_m = I - \bar{I}_c$. Then $q_m = \bar{q}_m$, where*

$$\bar{q}_m = \alpha_m (I - \bar{I}_c). \quad (7)$$

In Figure 1, this solution is found from the intersection of BC and II . The broken vertical and horizontal lines through this intersection give $\bar{q}_m = \alpha_m (I - \bar{I}_c)$ and $\bar{q}_c = \alpha_c \bar{I}_c$, respectively. The rationale for commercial activity is to subsidize mission activity. If all the endowment I were invested in mission activity, then,

because the markup on mission output is negative, the need to satisfy the budget constraint would result in some mission output capacity being unused. If, instead, a small amount of I were allocated to commercial activity, with the rest allocated to mission activity, the positive markup on commercial activity could be used to subsidize mission output. Mission output capacity would be reduced, but mission output would rise. Further increments to I_c similarly generate more mission output, up to the point represented in Lemma 1. When mission output capacity has fallen and mission output has risen so far that the mission output capacity constraint binds, all opportunities for cross-subsidization are exhausted.⁷

2.2 Uncertainty

Assume now that donations are higher in state of the world $J = H$ than in $J = L$: $\theta^H > \theta^L \geq 0$. To ease the exposition, we derive the general solution in two steps. First, we consider what would happen to outputs if the investment allocation were the same as in the benchmark case, but, for each of possible realization of θ , outputs were adjusted optimally, given capacities. Second, we allow for deviation of the investment allocation away from the benchmark solution, with outputs adjusted optimally for each possible deviation. Thus we consider the optimal

⁷From (6), there will be more investment in commercial activity if I or α_m is greater or if any of $\{\pi_m, \pi_c, \alpha_c, \theta^0\}$ is smaller. A greater endowment I results in greater investment in both activities, while a higher average product of investment in mission activity, α_m , by making mission output cheaper to produce, increases the need to invest in commercial activity to subsidize mission output. A lower value of π_m represents a more negative mission markup and so more commercial output capacity is required for cross-subsidization. A lower commercial markup π_c implies, *ceteris paribus*, less subsidization of mission activity. Commercial investment is therefore raised, partially to compensate for this change. A lower average product of investment in commercial activity, α_c , implies, *ceteris paribus*, that commercial output and cross-subsidization are less. Investment in commercial activity is therefore raised, partially to restore both commercial output and cross-subsidization. A lower expected level of donations θ^0 leads to a higher investment in commercial activity, partially to restore the amount of cross-subsidization.

deviation of the investment allocation due to uncertainty. When it is optimal to increase commercial investment above the amount in the benchmark case, a form of insurance is being purchased. In this case, compared to the benchmark solution, the non-profit reduces the mission output that will obtain if the good state of the world occurs, in order to raise the mission output that will obtain if the bad state occurs.

Consider first what happens to mission output if, despite uncertainty as to the budget constraint, investment allocations, and therefore output capacity constraints, are the same as in the solution depicted in Figure 1. If $\theta = \theta^L$, the budget constraint is to the left of BC and so the mission output capacity \bar{q}_m shown in the figure cannot be achieved. With commercial output held at its capacity level, mission output must be cut until the new budget constraint binds, with all available funds being used to subsidize mission activity, i.e., $M = 0$. When $\theta = \theta^H$, however, the budget constraint is to the right of BC and nothing prevents the mission capacity output \bar{q}_m from being achieved. The budget constraint is non-binding, i.e., $M > 0$. The effects on mission output are summarized in the following lemma.

Lemma 2 *If $I_c = \bar{I}_c$ and $I_m = I - \bar{I}_c$, mission output q_m is:*

$$\bar{q}_m^L = \bar{q}_m + \frac{\theta^0 - \theta^L}{\pi_m} < \bar{q}_m \quad \text{if } \theta = \theta^L; \quad (8)$$

$$\bar{q}_m^H = \bar{q}_m \quad \text{if } \theta = \theta^H. \quad (9)$$

Now consider the effect on mission output of small deviations of the investment allocation away from that in the benchmark solution. Specifically, consider the

allocation \hat{I}_c to commercial activity, where

$$\hat{I}_c = \bar{I}_c + \Delta, \quad \Delta \geq 0. \quad (10)$$

First, for numerically small Δ , we determine, for $\theta = \theta^J$, what mission output $q_m = \hat{q}_m^J$ ($J = H, L$) will be. Then we consider the optimal value of Δ .

For a sufficiently small numerical value of Δ , the procedure for deriving mission output is the same as in Lemma 2 (but with a deviation in the investment allocation). Thus, for realization θ^L , mission capacity cannot be used fully because the associated losses cannot all be covered. Substituting $\theta^J = \theta^L$, $q_c = \alpha_c(\bar{I}_c + \Delta)$ and $q_m = \hat{q}_m^L$ into (3), and then substituting from (6) and (7), yields

$$\hat{q}_m^L = \bar{q}_m^L - \frac{\pi_c \alpha_c}{\pi_m} \Delta \geq \bar{q}_m^L \text{ as } \Delta \geq 0. \quad (11)$$

For realization θ^L , if investment in commercial activity is set above (below) the benchmark level, mission output is greater (less) than when the investment is at the benchmark level.

For realization θ^H , however, the firm is unconstrained by its budget and so uses all of the capacity $\alpha_m(I - \bar{I}_c)$ to produce mission output, as given by (9). Investment in commercial activity above the benchmark level is associated with a cut in mission capacity and q_m is cut correspondingly. The opposite results obtain if investment in commercial activity is reduced below the benchmark level. Thus,

$$\hat{q}_m^H = \bar{q}_m^H - \alpha_m \Delta \leq \bar{q}_m^H \text{ as } \Delta \geq 0. \quad (12)$$

We can now examine the optimal value of Δ , beginning with the special case of risk-neutrality: $\gamma = 1$. Denote $p^L \bar{q}_m^L + p^H \bar{q}_m^H$ by $E(\bar{q}_m)$ and $p^L \hat{q}_m^L + p^H \hat{q}_m^H$ by

$E(\hat{q}_m)$. Then, from (8), (9), (11) and (12),

$$E(\hat{q}_m) \underset{\leq}{\overset{\geq}{\cong}} E(\bar{q}_m) \text{ as } \mu\Delta \underset{\leq}{\overset{\geq}{\cong}} 0 \quad \text{where } \mu \equiv p^H\alpha_m\pi_m + p^L\alpha_c\pi_c. \quad (13)$$

Hence, if $\mu > 0$, by setting a positive Δ the firm can raise expected mission output above the level associated with the allocation represented in Lemma 2. If $\mu < 0$ a negative value of Δ can raise expected mission output similarly. The effect of raising I_c by one unit is that, for realization θ^L , q_c can be increased by α_c , generating surplus $\pi_c\alpha_c$. Since a unit of mission output makes the negative profit π_m , the surplus may be used to subsidize $-\pi_c\alpha_c/\pi_m$ units of mission output. However, raising I_c by one unit also reduces I_m by one unit, which, for realization θ^H , reduces mission output by α_m . Weighting these effects by their respective probabilities, the expectation of q_m rises by $p^L(-\pi_c\alpha_c/\pi_m) - p^H\alpha_m$. Rearranging, this is positive if $p^H\alpha_m\pi_m + p^L\alpha_c\pi_c = \mu > 0$. By setting Δ either positive or negative, as appropriate, an increase in the numerical value of Δ raises expected mission output further, up to a certain point. This point is a corner solution - which we characterize below.

The optimal choice of Δ can be understood intuitively by reference to Figures 2 and 3, where BC and II are reproduced from Figure 1, and the lines BC^L and BC^H are the respective budget constraints (3), with $M = 0$, for the realizations θ^L and θ^H .

[Figure 2 here]

Consider Figure 2, which illustrates the case of $\mu > 0$. From (13), a positive Δ should be chosen, i.e., commercial capacity should be raised above $\alpha_c\bar{I}_c$ and

mission capacity lowered. In the figure, suppose first that $\Delta = 0$ is in fact chosen. Then output capacities are $\bar{q}_m = \alpha_m(I - \bar{I}_c)$ and $\bar{q}_c = \alpha_c\bar{I}_c$, as represented by point B (the benchmark). For realization θ^L , commercial output is set at the capacity level to gain as much revenue as possible, but mission output is held back by the budget constraint: $q_m = \bar{q}_m^L < \bar{q}_m$. This is shown by point A . For realization θ^H , the budget constraint BC^H is non-binding, i.e., both outputs are set at capacity levels, shown by point B ; $q_m = \bar{q}_m^H = \bar{q}_m$.

Now suppose that, instead, Δ is given a small positive value, so that commercial capacity is shown in the figure by the horizontal line $q_c = \alpha_c(\bar{I}_c + \Delta)$. At the intersection of this line with II (point C), the vertical line shows mission capacity. By setting a small positive value of Δ , rather than $\Delta = 0$, q_m is raised in the bad state of the world (point D is achieved, rather than point A) but reduced in the good state (point C is achieved, rather than point B). The configuration of parameter values, as explained above in interpreting a positive value of μ , is such that, by raising Δ slightly above zero, expected mission output increases.

The inequality $\mu > 0$ is more likely to hold if α_c , the productivity of commercial investment, or if π_c , the commercial markup rate, is high, because raising q_c in the bad state of the world then subsidizes a large amount of q_m . It is also more likely hold if π_m , the mission markup rate, is less negative, because the cross-subsidy in the bad state of the world then finances more units of q_m ; or if α_m , the productivity of mission investment, is low, because the amount of q_m forgone in the good state of the world, is then small. Finally, it is more likely to hold if p^L/p^H is high, i.e., if the bad state of the world is relatively more likely,

because it is in this state of the world that q_m is raised by having a positive Δ .

In Figure 2, a more positive Δ causes point D to slide further up BC^L and point C to slide further up II . Up to a limit, which we now discuss, each increment to Δ raises $E(q_m)$ further. Consider state of the world L . When D and C coincide with point E , where BC^L and II intersect, D cannot move further along BC^L , since the constraint II binds. An additional increment to Δ would then result in q_m being determined from the intersection of II and the (redrawn) binding capacity constraint $q_c = \alpha_c(\bar{I}_c + \Delta)$; i.e., further increments to Δ would cause q_m to fall in state of the world L as well as in state H . Hence, Δ should be set such that commercial capacity is given by a horizontal line through by point E . At E , points D and C have in effect shifted so far along BC^L and II , respectively, that they coincide. All mission-output uncertainty is eliminated.

Now suppose that $\mu < 0$. In this case the configuration of parameter values is such that expected mission output is raised by reducing Δ slightly below zero. The non-profit still undertakes commercial investment so that it can cross-subsidize mission activity, but, for the converse reasons to those already explained above for a positive μ , there is a positive payoff in terms of expected mission output from investing in less commercial capacity than in the benchmark case.

This is illustrated in Figure 3, where, as in Figure 2, if Δ were zero, outputs would be shown by points A and B in the bad and good states of the world, respectively. However, with a small negative value of Δ , commercial output capacity is shown by the horizontal line $q_c = \alpha_c(\bar{I}_c + \Delta)$. For each realization of θ , outputs are determined as we described above for a positive Δ . Hence, for the

realizations θ^L and θ^H outputs are represented by points F and G respectively. For similar reasons to those given for when $\Delta > 0$, now, Δ should be made more negative, up to the point where a corner solution obtains. Starting from $\Delta = 0$, as small negative increments are added to Δ , q_m falls in state of the world L and rises q_m in state H . The limit to adjustment of Δ occurs where, in state H , q_m no longer increases, that is, at K in the figure, where II and BC^H intersect. If Δ were made so negative that the q_c -capacity constraint passed below K , the binding budget constraint BC^H would cause q_m to be less than at K .

[Figure 3 here]

The investment levels implicit in this discussion are summarized in the following proposition.

Proposition 1 *If the firm is risk neutral with respect to mission output, its optimal investment in commercial activity is $\hat{I}_c^+ > \bar{I}_c$ if $\mu > 0$, but $\hat{I}_c^- < \bar{I}_c$ if $\mu < 0$, where \bar{I}_c is commercial investment in the benchmark case and*

$$\hat{I}_c^+ = \bar{I}_c + \frac{p^H (\theta^H - \theta^L)}{\pi_c \alpha_c - \pi_m \alpha_m}, \quad (14)$$

$$\hat{I}_c^- = \bar{I}_c - \frac{p^L (\theta^H - \theta^L)}{\pi_c \alpha_c - \pi_m \alpha_m}. \quad (15)$$

Thus we have found that, depending on the sign of μ , if $\gamma = 1$, the outputs will be represented by point E in Figure 2 ($\mu > 0$) or by point K in Figure 3 ($\mu < 0$); i.e., the intersections of the two potential budget constraints with II give the firm's optimal behavior. Note that if $\mu > 0$ the non-profit chooses to reduce mission capacity below that in the benchmark case. We interpret the

investment in commercial capacity above the benchmark level as purchasing a form of insurance in the face of uncertainty over donations. If, however, $\mu < 0$, the revenue-generating power of commercial investment and/or the loss-generating effects of investment in mission activity are relatively low, and so the non-profit invests in less commercial capacity than in the benchmark case.

We can now examine the effects of risk aversion, i.e., $\gamma < 1$. If $\mu \geq 0$, the solution here is the same as in Proposition 1. When $\gamma = 1$ the solution is then shown by point E in Figure 2. Since, at E , the capacity constraint for q_m binds for each realization of θ , $\hat{q}_m^L = \hat{q}_m^H$, i.e., the firm experiences no risk. Since the expectation of q_m is maximized at E , this is also the optimal allocation for $\gamma < 1$. With risk neutrality, the firm invests in additional commercial capacity as insurance and so, *a fortiori*, it also does so when it is risk averse. This is summarized in the following.

Corollary 1 *If $\mu \geq 0$, risk aversion has no impact on the optimum allocation of investment to commercial activity.*

Now consider the case of $\mu < 0$. We focus on the maximization of the first argument $E[(q_m)^\gamma]$ in the firm's utility function (outputs being given by (11) and (12)). Thus, we are interested in the sign of

$$\frac{dE[(q_m)^\gamma]}{d\Delta} = \frac{-\gamma}{\pi_m} \left[p^H \pi_m \alpha_m (\bar{q}_m^H - \alpha_m \Delta)^{\gamma-1} + p^L \pi_c \alpha_c \left(\bar{q}_m^L - \frac{\pi_c \alpha_c \Delta}{\pi_m} \right)^{\gamma-1} \right]. \quad (16)$$

Setting this to zero, we have that, if an interior solution exists, $\Delta = \Delta^*(\gamma)$, where

$$\Delta^*(\gamma) = \frac{\pi_m (\Omega \bar{q}_m^H - \bar{q}_m^L)}{\pi_m \alpha_m \Omega - \pi_c \alpha_c}, \text{ where } \Omega \equiv \left[\frac{-p^H \pi_m \alpha_m}{p^L \pi_c \alpha_c} \right]^{1/(\gamma-1)}. \quad (17)$$

We have seen that, if $\mu < 0$ and $\gamma = 1$, optimum investment is represented by point K in Figure 3. If γ is reduced below unity by a small enough amount, this corner solution still obtains. However, below some value of γ , which we call $\hat{\gamma}$, $dE[(q_m)^\gamma]/d\Delta > 0$ at point K , and so a different solution, which is interior, may obtain.⁸ This is summarized in the next proposition.

Proposition 2 *Suppose $\mu < 0$. For I not too high, there exists a level of γ , denoted by $\hat{\gamma}$, where $\hat{\gamma}$ is given by $\Delta^*(\hat{\gamma}) = \hat{I}_c^- - \bar{I}_c$ with $\hat{\gamma} \in (0, 1)$, such that if $\gamma < \hat{\gamma}$, the optimum investment allocation to commercial activity is*

$$\hat{I}_c = \bar{I}_c + \Delta^*(\gamma) \equiv \bar{I}_c + \frac{\pi_m(\Omega\bar{q}_m^H - \bar{q}_m^L)}{\pi_m\alpha_m\Omega - \pi_c\alpha_c} > \hat{I}_c^-, \quad (18)$$

If $\gamma \geq \hat{\gamma}$ the optimal investment in commercial activity is $\hat{I}_c = \hat{I}_c^-$.

In Figure 3, if $\gamma \geq \hat{\gamma}$, investments at point K result in mission outputs \hat{q}_m^L and \hat{q}_m^H in states L and H , respectively. If $\gamma < \hat{\gamma}$, however, this prospect, with its relatively wide dispersion of possible mission output outcomes, is viewed as embodying too much risk. The firm now sets $I_c > \hat{I}_c^-$, so that in state L q_m increases, sliding to the right up BC^L , and in state H q_m decreases, sliding to the left up II ; i.e., the dispersion of possible outcomes is reduced. Thus for interior solutions, greater risk aversion is weakly associated with more investment in the commercial activity, at the cost of lower expected mission output. This is summarized in the following.

⁸The interior solution only obtains if I is not too high. As I becomes higher, point K slides up the budget constraint BC^H in Figure 3. Output in state of the world L increases as a proportion of output in state of the world H . Eventually, the reduction in the variance of mission output that obtains when commercial investment is raised above that obtaining at K is outweighed, in utility terms, by the associated loss on expected mission output.

Corollary 2 *If $\mu < 0$ and risk aversion is sufficiently great ($\gamma < \hat{\gamma}$), the optimal investment in commercial activity increases with risk aversion ($d\hat{I}_c/d\gamma < 0$).*

To complete our analysis, we consider the effect on the solution of greater risk. For this purpose, we let $p^H = 1/2$ and consider an increase in the variance of θ for constant mean θ^0 . Thus, we let $\theta^H = \theta^0 + \alpha$, and $\theta^L = \theta^0 - \alpha$. The variance of θ is then found to be given by

$$\alpha^2 = p^H p^L (\theta^H - \theta^L)^2. \quad (19)$$

For the case of $\mu > 0$, the effects of an increase in α can be seen from Figure 2. Since θ^L falls, point E moves up along the II boundary, i.e., commercial investment increases: the greater risk in this case is associated with more insurance. For the case of $\mu < 0$, it can be seen from Figure 3 that point K moves down along II . Hence, when $\gamma \geq \hat{\gamma}$, so that the solution is at point K , greater risk is associated with less commercial investment. If, instead, the non-profit is relatively risk averse ($\gamma < \hat{\gamma}$), the effect of greater risk is not apparent from the figure. However, we can show analytically that greater risk is then associated with more insurance.

Proposition 3 *(a) If $\mu > 0$, a higher variance α^2 is associated with a greater commercial investment. (b) If $\mu < 0$, a higher variance α^2 is associated with less commercial investment if $\gamma \geq \hat{\gamma}$, but with greater if $\gamma < \hat{\gamma}$.*

Finally, we note a related result concerning the effect on commercial investment of the non-profit having a greater endowment of funds I .

Corollary 3 (a) If $\mu > 0$, commercial investment is increasing in the endowment of funds I . (b) If $\mu < 0$, commercial investment is decreasing in I if $\gamma < \hat{\gamma}$, but is increasing in I if $\gamma \geq \hat{\gamma}$.

In general, it might be expected that a greater I would be associated with a greater scale of each activity. A non-profit with more resources will not only tend to supply more mission output, it will also need more commercial output for subsidization. However, if $\mu < 0$ and $\gamma < \hat{\gamma}$, commercial activity is decreasing in I . In this case the non-profit is relatively risk averse and cross-subsidization has a relatively low expected payoff (e.g., because the negative markup on mission activity is numerically small). At the margin, the higher endowment substitutes for some cross subsidization from commercial activity and allows an increase in mission capacity.

3 Empirical Analysis

Our theoretical analysis depicts cross-subsidization in the benchmark certainty case (Lemma 1) and then focuses on uncertainty over donations. We show that, if $\mu > 0$, uncertainty causes a risk-neutral non-profit to insure by undertaking additional commercial activity, but if $\mu < 0$ the uncertainty causes it to undertake less commercial activity (Proposition 1). However, when $\mu < 0$ and the non-profit is relatively risk averse, the reduction in commercial activity below the benchmark level is smaller than when the non-profit is risk neutral (Proposition 2). Greater uncertainty over donations leads to more commercial investment if either $\mu > 0$ or $\mu < 0$ and $\gamma < \hat{\gamma}$, though not if both $\mu < 0$ and $\gamma \geq \hat{\gamma}$ (Proposition 3). A

greater initial endowment of funds leads to more commercial investment if either $\mu > 0$ or $\mu < 0$ and $\gamma \geq \hat{\gamma}$, but if both $\mu < 0$ and $\gamma > \hat{\gamma}$ it leads to less commercial investment (Corollary 3).

We test our main results empirically by answering the following questions. The first is a preliminary question: is commercial activity undertaken to cross-subsidize losses in mission activity? Second, does greater uncertainty over donations increase commercial investment? The analysis of this question, both theoretically and empirically, is the main concern of the paper. The third question is a subsidiary one: is commercial activity increasing in the endowment of funds? We base our empirical analysis on US non-profits because of the extensive development of the non-profit sector in the US and the availability of micro-data collected from the non-profit tax returns.

3.1 Data Description

The sample used is of 17,396 non-profit organizations, operating in the State of New York, provided by IRS tax returns for the fiscal year 1999. The sample is limited to the organizations tax-exempt under §501(c)(3) of the IRC. We proxy mission revenues with program service revenues, defined by the IRS (2000) as those activities forming the basis of an organization's exemption from tax (mission activity, including user fees, third-party payments, government contracts, and some cross-subsidization). The measure of commercial revenues includes both commercial sales (from activities unrelated to the organization's mission) and returns on passive investments (e.g., interest and dividend receipts).

The volatility of donations in each NTEE sector is proxied by the variance,

across organizations in the sector, of the ratio of donations to total revenue. Thus, we assume implicitly that, in the absence of risk, donations would be evenly distributed within a sector. Consequently, the higher is the variance of donations in a sector, the greater is the risk associated with donations in that sector. All variables are in logarithms.

The following regression is estimated:

$$\begin{aligned}
CREV_i = & \beta_1 + \beta_2 VOL_i + \beta_3 AS_i + \beta_4 AS_i^2 + \\
& + \beta_5 DON_i + \beta_6 DON_i^2 + \beta_7 MREV_i^+ + \beta_8 MREV_i^- + \\
& + \beta_9 DCOM * MREV_i^+ + \beta_{10} DCOM * MREV_i^- + \beta_{10+n} \Psi_n + \varepsilon_i
\end{aligned} \tag{20}$$

where $CREV_i$ is revenue from commercial activities for the i -th non-profit in the sample⁹ and VOL_i is the cross-sectional variance of the ratio of donations to total revenues of the non-profits for the sector to which organization i belongs. AS_i measures the value of i 's assets at the beginning of the year, thereby proxying its size. Donations, DON_i , are measured by the contributions, gifts and grants received by i . $MREV_i^+$ and $MREV_i^-$ are proxies for the respective gain or loss by i in mission activity, which we measure by subtracting labor costs from gross mission revenues (thus we attribute all labor costs to mission activity).

We consider potential interactions among the dummy variables and possible non-linearities in the response of commercial activities with respect to AS_i and DON_i . By multiplying the regressors $MREV_i^+$ and $MREV_i^-$ by a dummy $DCOM$, which takes a unit value when commercial revenues exceed donations

⁹Revenue, rather than profit, from commercial activities is modeled because of the tendency of non-profits to allocate all joint costs to commercial activity, under-reporting commercial profits to avoid taxes.

and zero otherwise, we capture those non-profits whose main source of income is commercial sales. Ψ_n is a $[27 * 1]$ -vector of control dummy variables, including 24 (out of the 25) NTEE sector dummies, *DMFEES* (for those non-profits whose net mission revenues are higher than donations), *DCAP* (for capital-intensive organizations, proxied by nonlabor expenditure exceeding labor expenditure), and the commercial dummy *DCOM*.

An estimated $\beta_2 > 0$ would confirm the positive impact of volatility in determining commercial activity by non-profits. Cross-subsidization is tested for via a Wald test on the null $\beta_7 = \beta_8$ for organizations in which donations exceed commercial revenue and on $\beta_7 + \beta_9 = \beta_8 + \beta_{10}$ for organizations in which commercial revenue exceeds donations, while an estimated $\beta_4 > 0$ implies a positive impact of assets on commercial investment.

A problem with the dataset is that the IRS reports zero both for missing observations and for non-respondents. This can in principle bias the analysis, for we cannot eliminate all the zeros as possible non-respondents without creating sample-selection problems by arbitrarily eliminating the non-profits reporting zero revenues from the given cost/revenue source. This implies loss of information arising from organizations that, for instance, do not receive any donations, or do not cross subsidize. We try to cream off the possible nonrespondents by conditioning our sample on the following:

$$AS_i \neq 0, \quad EXP_i \neq 0, \quad LAB_i > 0, \quad TR_i > 0,$$

where EXP_i measures total expenditure, LAB_i is labor cost and TR_i is total gross revenue (from mission and commercial activity), each for i . We are consequently

eliminating as possible nonrespondents those non-profits with no assets, no labor expenditure - this may include completely voluntary organizations - and no revenues from any source. There is no particular reason to consider self-selection as a cause of non-responding.

The dimensions of our sample suggest log-linearization of the regression for an appropriate econometric treatment of the variables. To log-linearize without arbitrarily eliminating all the zero-filers as nonrespondents, we add to the zeros in the restricted sample a small positive quantity (\$1).¹⁰ The idea here is to retain in the conditioned sample the information supplied by organizations receiving, e.g., zero donations, without sacrificing the econometric reliability of our estimates.

A problem with the estimates might arise from the potential endogeneity of some right-hand side variables in (20). In particular, higher commercial revenues might either lower donations (when donors dislike commerce) or raise them (when donors reward self-help by non-profits).¹¹ Nevertheless, within the same fiscal year (the time horizon of our analysis), *DON* can be considered to be determined before *CREV*. This is because donors, unlike managers, cannot fully observe *CREV* until the following fiscal year, when balance sheets are published.

¹⁰Our results do not change qualitatively without the log-linearisation. However, our method improves the standard error of the regression and reduces normality failures considerably, improving the robustness of our results.

¹¹Also, if managers dislike commerce, higher donations might cause them to reduce commercial activity. However, we would expect this effect to be relatively small, given that the non-profit has already invested in, e.g., training workers and renting capital goods to undertake a higher level of commercial activity, and that any reduction in commercial revenue would reduce the non-profit's ability to invest further in mission activity

3.2 Regression Results

Initially, we estimate the model by ordinary least squares (OLS), using White's robust standard errors because of heteroskedasticity. All estimations are undertaken in E-views 4.1. The empirical results for the parsimonious model are shown in Table 1, where p -values are shown in brackets. Volatility of donations has a positive impact on commercial activities, confirming our main proposition. Cross-subsidization is tested by allowing for possible asymmetries in the response of commercial activities to gains and losses in mission activity. Our results show that the asymmetric response holds only for non-profits for which commercial revenue exceeds donations. This is formally tested by a Wald test on the restriction $\beta_7 + \beta_9 = \beta_8 + \beta_{10}$. The null of equal response is rejected at conventional statistical levels (F -statistic=108, p -value=.00). This means that, for this group of non-profits, commercial activities are undertaken mainly to cross-subsidize the mission. Some caution is nevertheless needed in interpreting this result, since the separation between positive and negative markups in our data is not perfect, given the impossibility of allocating fixed costs appropriately between the activities.

Commercial revenues are also positively correlated to donations, though less than proportionally. This complements our finding of a positive association between commercial revenues and donation risk: non-profit managers reduce their commercial activities only when donations are exceptionally high. Furthermore, capital-intensive non-profits and those with greater assets on average make higher commercial revenues. In terms of our model, a greater scale of an organization

implies that more commercial revenue is needed for cross-subsidization. A similar result is obtained by Cordes and Weisbrod (1998), although they relate the size of assets to mission revenues. Their explanation is that bigger dimensions of the non-profit allow it better to exploit possible cost-complementarities, favoring commercial activity.¹²

These explanatory variables also determine whether a non-profit relies primarily on commercial revenue, as shown by our estimation of binary models, with *DCOM* as the dummy dependent variable. The Probit and Logit regression results are presented in columns (2) and (3) of Table 1. The results show that the volatility of donations has a positive marginal effect on the probability of being primarily commercially financed. Also, as in the OLS model, we report an asymmetric effect of gains and losses in mission activity on commercial financing.¹³ Thus, our empirical results confirm the theoretical predictions that commercial financing is determined by the risk associated with donations, and it is aimed at cross-financing mission losses.

¹²The sector of activity also matters in this relationship: membership of *NTEE5* (the health sector) on average lowers commercial revenue, while membership of *NTEE20* (charities) is associated with more commercial revenue (though the statistical significance is low). This result is only apparently in contrast with the well-known increasing commercialization of hospitals, given our definition of commercial activities. We consider as commercial activities the ones undertaken for pure cross-subsidization and therefore unrelated to the mission activity. Commercialization of non-profit hospitals generally refers to the increasing adoption of user fees, and therefore of mission revenues in our model. This is why hospitals, receiving relatively stable revenue from user fees, on average make less revenue from commercial activity, while charities, not relying consistently on user fees, need to obtain more revenue from commercial activities.

¹³The marginal effect of the size of the organisation is estimated (at the sample mean) at $-.28 + 0.1 * AS$, which is positive for most of our sample, supporting the cost-complementarity hypothesis. Only small non-profits do not benefit from the cost-complementarities, and are consequently less inclined to commercial financing. The marginal effect of donations on the probability of commercial financing is less than proportional.

4 Conclusions

A well-known motivation for the supply of commercial, i.e., non-mission, output by non-profits, is that mission activity can thereby be cross-subsidized. In this paper, by formulating the investment decision of non-profit for which donations are stochastic, we have extended the ‘cross-subsidization’ argument to what is, in effect, the purchase of insurance. The supply of non-commercial output can enable a non-profit to improve its provision of mission output in the state of the world in which donations are low, at the cost of a reduction in mission activity in the state of the world in which donations are high. We show that the non-profit will gain from this purchase of insurance if a simple inequality regarding parameter values is satisfied.

This argument is supported by our empirical evidence, which, as well as confirming the existence of cross-subsidization, shows that commercial activity is positively related to the uncertainty over donations. It is also supported by our finding that commercial activity is significantly greater when financial losses on mission activity are larger, whereas the relationship for gains is not significant.

Appendix: Proofs

Lemma 1. Solving (1), (2) and (5), all with equality, and $I_m + I_c = I$, yields (7). If $I_c > \bar{I}_c$, then $I_m < I - \bar{I}_c$, and so, from (1), $q_m < \bar{q}_m$. If $I_c < \bar{I}_c$, then, from (5), $q_m < \bar{q}_m$. QED

Lemma 2. (i) Realization θ^L . When $\theta^H = \theta^L = \theta^0$, $I_c = \bar{I}_c$, $q_m = \bar{q}_m$, $q_c = \alpha_c I_c$ and (5) binds, i.e., (3) holds with $M = 0$. Hence, when $\theta^L < \theta^0$, (3) cannot be

satisfied unless q_m is cut. Given U , q_m should be reduced such that (3) holds with $M = 0$. Using (3), (6) and (7), (8) is obtained.

(ii) Realization θ^H . There are more donations than in the benchmark case, so benchmark outputs can be at least achieved. But as q_c and q_m are each at full capacity, neither can be increased, while a cut in q_m would reduce U . Hence, (9) holds. QED

Proposition 1. Given the reasoning in the text, \hat{I}_c^+ and \hat{I}_c^- are obtained from the intersections of BC^L and II and of BC^H and II , respectively. For each realization θ^H and θ^L , we use $I_c = I - I_m$, (1) and (2) each with equality, and (3) with $M = 0$. (14) and (15) are then obtained. QED

Proposition 2. (18) is obtained by substituting (17) into (10) and then comparing with (15). Let $\hat{I}_c^- - \bar{I}_c = \Delta^s$. We show that (i) $d\Delta^*(\gamma)/d\gamma < 0$ and (ii) $\Delta^*(\hat{\gamma}) = \Delta^s$, where $\hat{\gamma} \in (0, 1)$. Together, (i) and (ii) imply that, if $\gamma > \hat{\gamma}$, then $\Delta^*(\gamma) < \Delta^s$ (a corner solution), while, if $\gamma < \hat{\gamma}$, then $\Delta^*(\gamma) > \Delta^s$ (an interior solution).

(i) By assumption, $\mu < 0$ and so, from (13), $-p^H\pi_m\alpha_m/p^L\pi_c\alpha_c > 1$. Therefore, from (17), $d\Delta^*/d\gamma = (\partial\Delta^*/\partial\Omega).(d\Omega/d\gamma) < 0$.

(ii) At $\gamma = 0$, $\Omega = -p^L\pi_c\alpha_c/p^H\pi_m\alpha_m$, and so

$$\Delta^*(0) = \frac{\pi_m}{\pi_m\alpha_m\Omega - \pi_c\alpha_c} \left\{ \frac{\mu}{-p^H\pi_m\alpha_m} \left[\alpha_m I + \frac{\alpha_m(\pi_m\alpha_m I + \theta^0)}{\pi_c\alpha_c - \pi_m\alpha_m} \right] - \frac{\theta^0 - \theta^L}{\pi_m} \right\}$$

Since this is decreasing in I , Δ_0 reaches its maximum value at the minimum admissible level of I , which (from A1) is $I_{\min} = -\theta^0/\pi_m\alpha_m$. Now, note that

$$\Delta_0(0, I_{\min}) = \frac{1}{\pi_m\alpha_m\Omega - \pi_c\alpha_c} \left[\frac{\mu\theta^0}{-p^H\pi_m\alpha_m} - (\theta^0 - \theta^L) \right] > 0,$$

so that $\Delta_0(0) > 0$ for I not too high (the critical value of I obtains when $\Delta^*(0, I) = 0$). Since, when $\gamma = 1$ we have a corner solution (Proposition 1), we also know that $\Delta_0(1) < \Delta^s$. Given (i), this proves that $\hat{\gamma} \in (0, 1)$, for I not too high. QED

Corollary 2. From Proposition 2, when $\gamma \leq \hat{\gamma}$ the optimal investment is $\hat{I}_c^- = \bar{I}_c + \Delta^*(\gamma)$. Since $d\Delta^*(\gamma)/d\gamma < 0$, the result follows. QED

Proposition 3. This follows from substituting for $p^H = p^L = 1/2$ and $(\theta^H - \theta^L) = 2\alpha$ into (14) and (15) and (18), and then differentiating with respect to α . QED

Corollary 3. As for Proposition 3, we need consider only part (b) for $\gamma < \hat{\gamma}$. Substituting from (8) and (9) into (17) to eliminate \bar{q}_m^L and \bar{q}_m^H (17), and since $\hat{\gamma} \in (0, 1)$ (see Proof of Proposition 2), we find that $d\Delta^*(\gamma)/dI < 0$ and $\partial\hat{\gamma}/\partial I < 0$. Since, for $\gamma \leq \hat{\gamma}$, $\hat{I}_c^- = \bar{I}_c + \Delta^*(\gamma)$, it follows that $d\hat{I}_c^- / dI < 0$. QED

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Table 1. Regression Results.

	OLS <i>CREV</i>	PROBIT <i>CREV > DON</i>	LOGIT <i>CREV > DON</i>
β_1	-1.22 [.23]	-.55 [.32]	-19 [.38]
VOL_i	.04 [.00]	.04 [.00]	.08 [.00]
AS_i	.24 [.05]	-.40 [.00]	-.77 [.00]
AS_i^2	.03 [.00]	.02 [.00]	.05 [.00]
DON_i	.25 [.00]	.13 [.00]	.26 [.00]
DON_i^2	-.01 [.00]	-.02 [.00]	-.04 [.00]
<i>DMFEES</i>	-.24 [.00]	.61 [.00]	1.16 [.00]
<i>DCAP</i>	.63 [.00]	.36 [.00]	-.66 [.00]
<i>DCOM * MREV</i> ⁺	-.01 [.24]	-.01 [.25]	-.02 [.36]
<i>DCOM * MREV</i> ⁻	.16 [.00]	.146 [.00]	.26 [.00]
<i>NTEE5</i>	-.18 [.06]	-	-
<i>NTEE20</i>	.18 [.10]	-	-
Adjusted R²	44	54	54
S.E.	2.84	.27	.27
H₀ : $\beta_7 = \beta_8$	108.26 [.00]		
F	738 [.00]		
LR	-	6513 [.00]	6548 [.00]
H-L	-	21 [.01]	19 [.01]
Andrews Statistic	-	23 [.01]	24 [.00]
% correct predictions		89%	90%

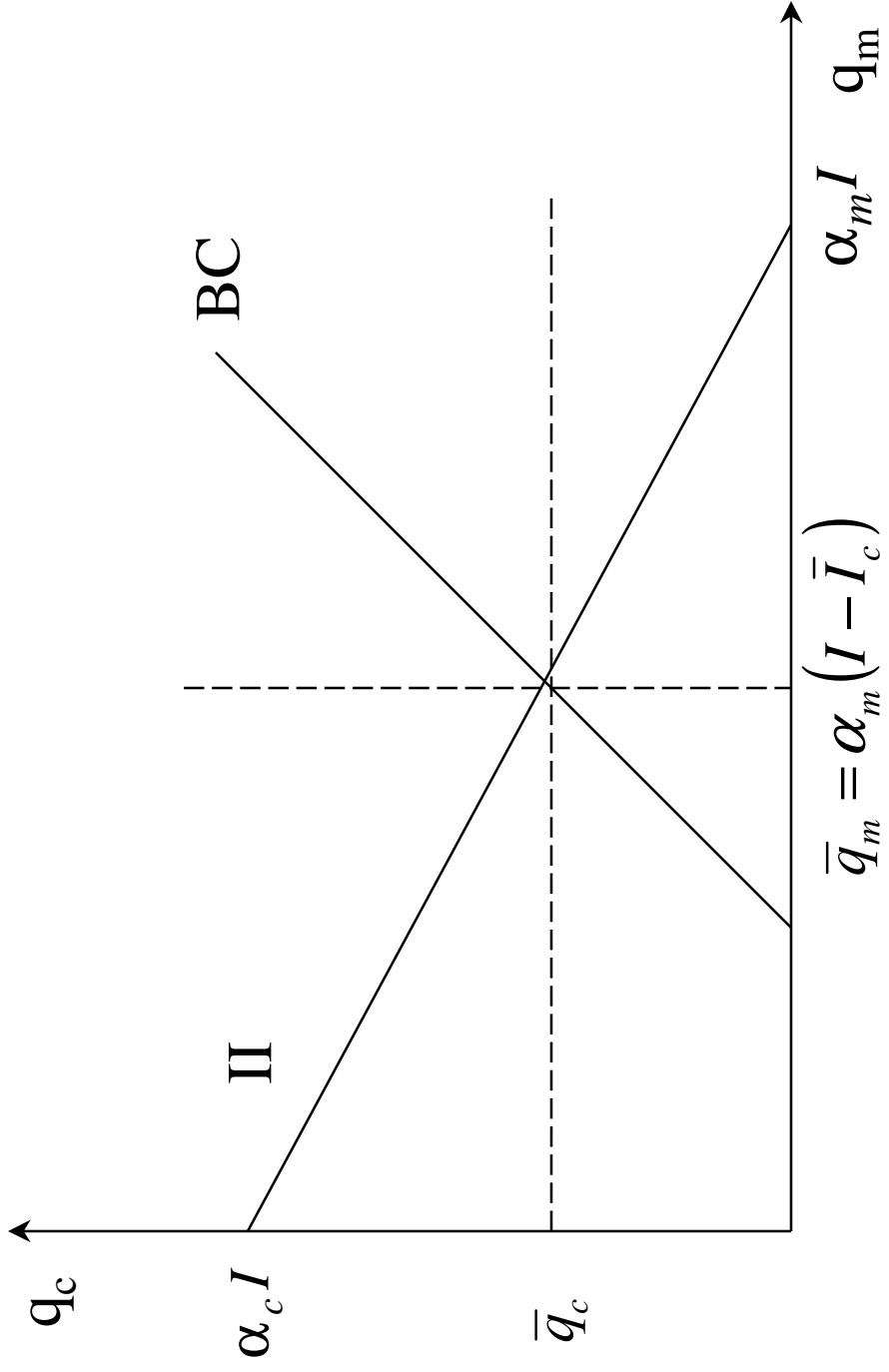


Fig.1 The Benchmark Case

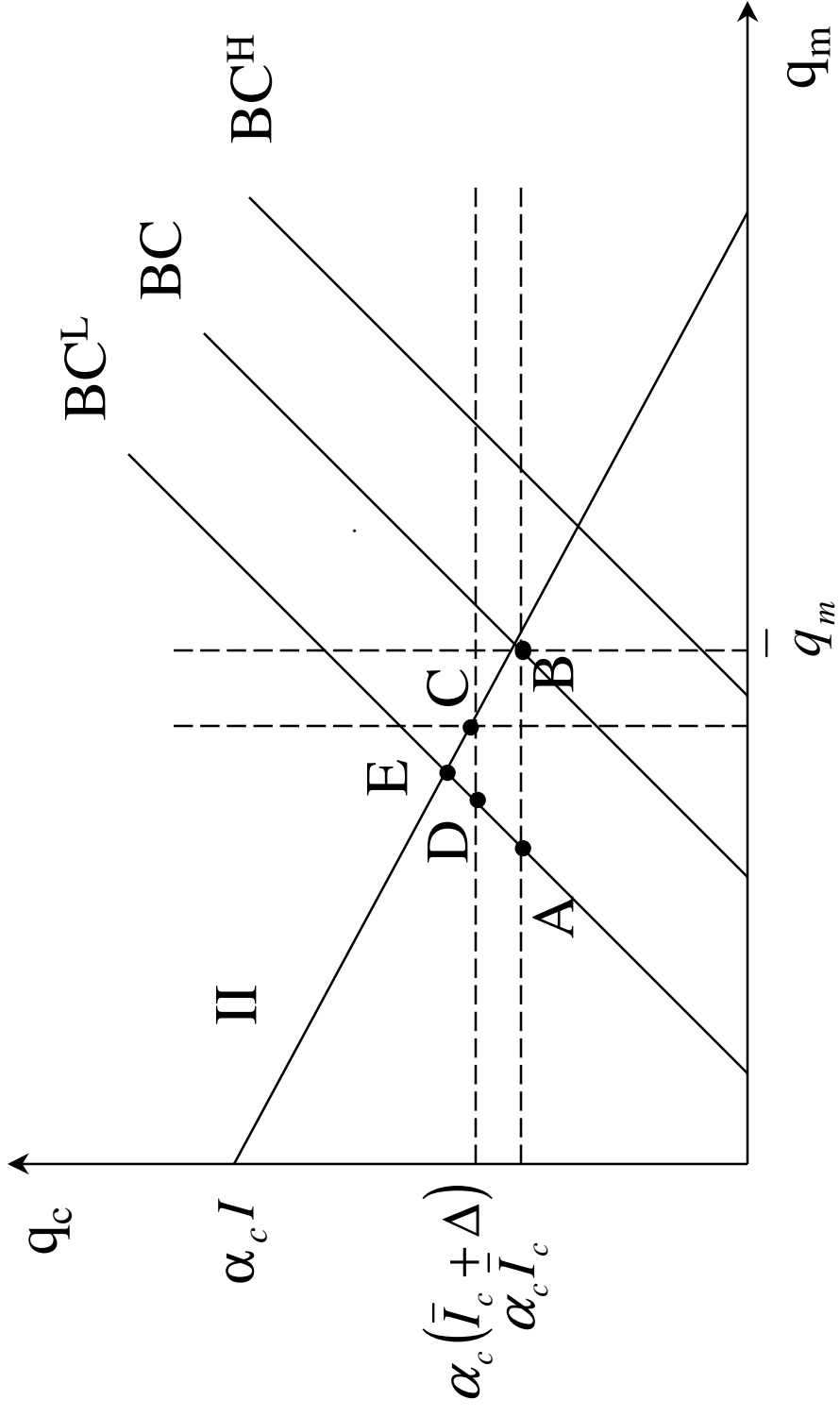


Fig.2 $\mu > 0$

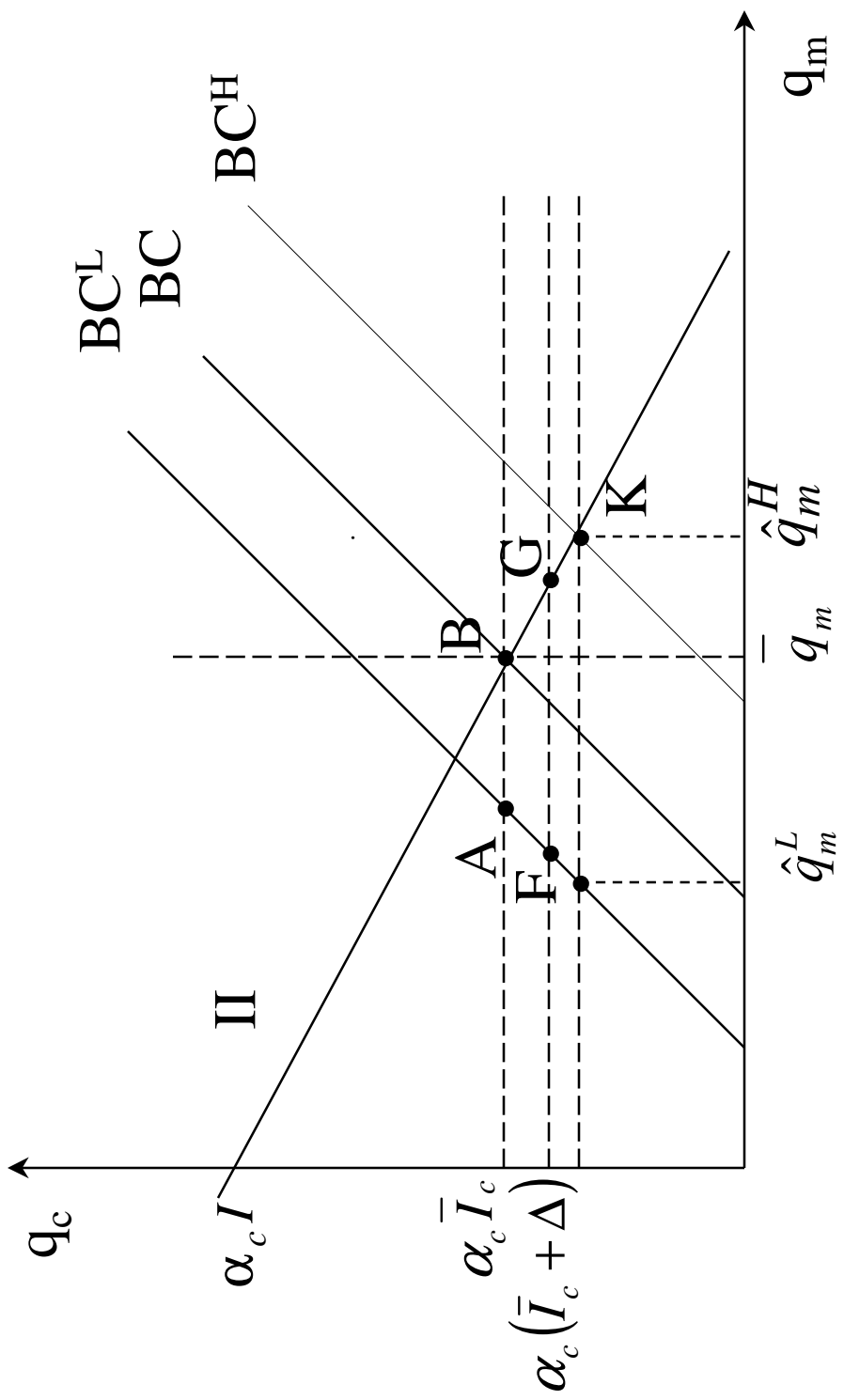


Fig.3 $\mu < 0$