A Note on Corruption and Public Investment: The Political Instability Threshold

Frank Bohn*

Essex University

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Abstract

This paper studies the government's public investment decision problem. Above some critical value of political instability, no investment is optimal. The result can also be interpreted as an alternative explanation for some governments' reluctance in fighting corruption.

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* University of Essex, Department of Economics, Wivenhoe Park, Colchester CO4 3SQ, United Kingdom, email: fbohn@essex.ac.uk, phone: +44-1206-87-2394 (fax: -87-2724).

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All remaining errors are mine. – Comments most welcome.

1 Introduction

Political instability makes governments behave myopically, because beneficial effects of their policies may not accrue to them in the future. This may lead to underinvestment in infrastructure or reluctance in promoting structural change. Political instability may also explain the lack of determination in some governments' fight against corruption, even in cases when those governments do not benefit from corruption themselves.

This paper studies public investment under political instability. The main result is that there is a political instability threshold below which the government is so myopic that it does not want to invest at all. Going above the threshold leads to a strong increase in investment, at first, because additional political stability effectively increases the discount factor for the future. However, the additional investment increments (for more and more political stability) become smaller because marginal investment profitability goes down.

The result is obtained in a parsimonious model framework. The paper investigates a government's optimal choice between (i) public goods spending and (ii) public investment. To keep the revenue side as simple as possible a proportional income tax is modelled. Other sources of revenue could also be incorporated as done in other papers on political instability: seigniorage as in Cukierman, Edwards and Tabellini (1992), domestic debt as in Devereux and Wen (1998), or seigniorage and foreign debt as in Bohn (2000).[1] However, the idea is to show a fundamental mechanism. The results do not primarily depend on economic, but rather on political (stability) conditions.

Sections 2 and 3 present the intertemporal framework of the theoretical model. Sections 4 and 5 summarize and simplify the government maximization problem. Interior and corner solutions are discussed in sections 6 and 7. Section 8 concludes.

[1] As in this paper, Svensson (1998) also models public investment, but interprets it as property rights investment and studies its impact on *private* investment.

2 Government Preferences and Political Instability

The model captures the intertemporal decision problem of the government. It consists of two periods: period 1 (current period) and period 2 (next period). There are two sectors in the economy: (i) the government and (ii) the private sector. The model is specified in real terms.

Government preferences over periods 1 and 2 are given by the following utility function:

$$W = V_1(C_1) + H_1(G_1, F_1) + E\{\rho\,(V_2(C_2) + H_2(G_2, F_2))\}. \tag{1}$$

The $V.(\bullet)$ functions are concave and twice continuously differentiable utility functions of the government in private sector consumption C. The $H.(\bullet)$ functions are the utility functions in the government provision of public goods G and F. E is the expectation operator and $\rho < 1$ is the government's discount factor. Total government utility is additively separable in two senses: first, with respect to periods; and second, with respect to utility derived either from private consumption or from public goods provision.

Assuming two types of governments (i.e. policymakers) political instability comprises two features: (i) the probability of government change and (ii) political polarization. After the first period the incumbent government may loose office to the other set of policymakers with a fixed probability π; it stays in power with probability $(1 - \pi)$.[2] It is assumed that there are two ethnic or social groups. Each one benefits more from one of the two public goods. Each of the two types of government provides both types of public goods, but to differing degrees. Political polarization then depends on the differences of policymakers' preferences

[2] Technically, this random change of government at fixed intervals is referred to as Markov switching (or Markov chain). If several time periods were considered and their lengths were fixed, for instance, at six months, some governments would only be in power for half a year, fewer would last for a year, and fewer yet for any longer period of time. This is a simple way of describing government change, but it matches the situation in many developing or transitional countries. In Russia, for instance, there were 5 changes of government in 1998 and 1999 despite the fact that no Duma or presidential elections were held. President Yeltsin alternately replaced representatives of the nomenclature (Chernomyrdin, Primakov, Putin) with so-called reformist Prime Ministers (Chubais, Stepashin) in arbitrary and irregular intervals.

with respect to their public good provision. The government utility function H is specified for one type of government (for the other type, α must be replaced by $(1-\alpha)$):

$$H(G,F) = \frac{1}{\alpha(1-\alpha)} \min\{\alpha G, (1-\alpha)F\}. \tag{2}$$

For simplicity, their disagreement in public goods provision is parameterized symmetrically by α which is exogenous. The denominator in equation (2) is a normalization such that

$$H(G,F) \ = \ F + G \ =: X. \tag{3}$$

and the marginal public goods utility is unity. Without limiting the general validity of the analysis, it is assumed that $1 \leq \alpha \leq \frac{1}{2}$. When α equals half, the two types of government have identical preferences; the more distant α is from half, the more they disagree on how much to spend on each of the two public goods. If preferences of both policymaker types are very dissimilar, political polarization is large. Political polarization measured by α contributes to political instability because it accounts for the extend of preference changes given a change in government. For α equals half, the instability effect of a government change is eliminated.

3 Budget Constraints

The government budget constraints for both model periods (1 and 2) are:

$$I + G_1 + F_1 \ \leq \ \tau\bar{Y}. \tag{4}$$

$$G_2 + F_2 \ \leq \ \tau Y(I).$$

Government expenditure consists of two kinds: investment I; and consumptive spending F and G which is spent on the two types of public goods. Government revenue is modelled at a rudimentary level only. As, for instance, in Aghion and Bolton (1990) tax revenues are calculated from constant tax rate τ and income as tax base. The tax rate and first period income $\bar{Y}$ (an endowment) are exogenous, but second period income $Y(I)$ depends on public investment I in the previous period. We assume an increasing function, but decreasing marginal returns: $Y'(I) > 0$, $Y''(I) < 0$. Investment may be interpreted in terms of standard infrastructure investment or investment in any type of structural change leading to more efficiency and hence higher private sector production and income levels. It could, however, also be interpreted as anti-corruption measures with similar effects on production and income.[3]

The private sector budget constraints for both periods are simply:

$$C_1 \leq (1-\tau)\bar{Y}. \tag{5}$$

$$C_2 \leq (1-\tau)Y(I).$$

Each period real private consumption depends on real income net of non-distortionary taxes. The model could be interpreted in per capita terms, but the private sector is passive in the sense that it cannot take optimizing decisions on labor, savings or investment. Thus, the two private sector budget constraints are not directly linked intertemporally. In that regard the model is similar to the model in Cukierman, Edwards, and Tabellini (1992). Income growth is only generated by (anti-corruption) investment, not by private sector activity. These assumptions allow us to focus on the government and its decision problem. They may be justified in two ways: first, this is a short run model; and, second, growth in transition and developing countries is not so much determined by the private sector, but more by other factors like structural investment (as modelled here) or foreign direct investment (which is not captured in the model).

[3] The latter interpretation requires the assumption that corruption does always have a negative effect on output as confirmed in convincing empirical studies by Mauro (1995), Méon and Sekkat (2003) and others, but previously contested by Huntington (1968), Lui (1985), Beck and Maher (1986), Lien (1986) and others.

4 Government Maximization Problem

The government has two types of instruments to increase its utility: public investment in period 1 and public spending on each of the two public goods in both periods. Increasing this period's public spending raises contemporaneous public goods utility H. Higher investment this period increases future private sector income (and thereby private sector utility) as well as tax revenues (and thereby public goods spending and utility) in the following period.

The government decision problem is made tractable because of three assumptions: (i), public goods spending F and G does not appear in the private sector budget constraints (5); (ii), government objective function (1) is additively separable; (iii), the functional format of the polarization assumption embedded in equation (2) guarantees $H(G, F) = F + G$ (equation 3). Due to assumptions (i) and (ii) the government optimization problem can be decomposed into two problems: first, the optimal distribution of the total public goods spending between F and G (**distribution problem**); and second, the fundamental revenue and expenditure problem of the government (**fundamental problem**).

The (optimal) distribution problem is not really interesting since its results hinge on specific (though quite sensible) assumptions for public goods utility H (assumption (iii)). Indeed, the mathematical solution of the distribution problem for public goods spending is only required for being able to solve the fundamental revenue and expenditure problem of the government. Due to assumption (iii) the fundamental problem of the government is independent of the actual government in power (see next paragraph). Nonetheless, the fact that there are two potential governments does have crucial implications for any government decision on the total amount of public goods spending as well as on public investment. In fact, the model is constructed that way to allow for the analysis of political instability by itself (as, for instance, in Devereux and Wen (1998) or Svensson (1998)) as opposed to analyzing the effect of different types of government with different objectives (as, for instance, in Aghion and Bolton (1990) or Tabellini and Alesina (1990)).

We proceed as follows. In the next paragraph, the solution for the optimal public goods **distribution problem** is used to simplify total government utility and, thereby, make the government maximization problem tractable. Then both the interior and the corner solution for the **fundamental problem** of government revenue and expenditure are discussed.

5 Simplifying Total Government Utility

Assumption (iii), which refers to the functional format of utility function H, has three specific implications. First, the optimal distribution of the total partial interest spending between F and G is crosswise symmetrical for both types, say i and k, of governments (when in power). Second, government utility H derived from type i's choice of F and G (when in power) is equal to government utility derived from type k's choice (when in power):

$$H^i(G^i, F^i) = G^i + F^i = X^i = X = X^k = G^k + F^k = H^k(G^k, F^k). \quad (6)$$

In either case, the marginal utility of public goods spending is unity. Third, the (real) total value of public goods spending H is normalized - for each government - by the sum of its arguments $(F + G)$, when chosen optimally by any incumbent government. For i and k representing different governments and $\alpha > \frac{1}{2}$ being assumed (without loss of generality), note, however, that government k's optimal choice for F and G is, of course, suboptimal for government i: $X^i = H^i(G^i, F^i) > H^i(G^k, F^k) = \frac{1-\alpha}{\alpha} X^i$.

On this basis, the government utility function (1), can be simplified. For each period separately, utility derived from private consumption and from partial interest spending is considered for the government in power in period 1 only. Superscripts are only used for the other government (marked by k). In period 1, this government's optimal choice for F and G results in $H(G_1, F_1) = X_1$. Thus first period utility is

$$V(C_1) + H(G_1, F_1) = V(C_1) + X_1 \quad (7)$$

If this government is still in power in period 2 (with probability $(1-\pi)$), it will choose F and G such that $H(G_2, F_2) = X_2$. If, however, this government looses power in period 2 (with probability π), it has to put up with the public goods spending chosen by the other government, i.e. $H(G_2^k, F_2^k) = \frac{1-\alpha}{\alpha} X_2$. Hence its second period total expected utility is:

$$
\begin{aligned}
E\ \{\ \rho\ \ (\,V(C_2)\ +\ H(G_2, F_2)\,)\} \qquad & (8)\\
=\ \rho\ \left(\,(1-\pi)\,(V(C_2) + X_2) \quad + \quad \pi\,(V(C_2) + \frac{1-\alpha}{\alpha} X_2)\,\right) & \\
=\ \rho\ \ (\,V(C_2)\ +\ \beta(\alpha, \pi) X_2\,) &
\end{aligned}
$$

Thus government utility in period 2 depends on three exogenous parameters: discount factor ρ, political polarization α and the probability of loosing power π. The latter two parameters are subsumed under quasi-exogenous parameter β, which is to represent political instability: $0 \leq \beta(\alpha, \pi) = (1-\pi) + \pi\frac{1-\alpha}{\alpha} \leq 1$. Note that political instability augments the effect of the discount factor: it lowers the valuation for the second period, i.e. it increases government myopia. Obviously, $\beta = 1$ if both governments have identical preferences ($\alpha = \frac{1}{2}$) or if the government stays in power with certainty ($\pi = 0$). For $\alpha = 1$ *and* $\pi = 1$, $\beta = 0$. In other words, β decreases with more political diversity (polarization $\alpha \uparrow$) and/or more political uncertainty (probability of government change $\pi \uparrow$).

6 Interior Solution

The fundamental revenue and expenditure problem of the government can now be specified on the basis of government preferences as stated in (1) and equations (7) and (8). Government budget constraints (4) and private sector budget constraints (5) can be substituted into equations (7) and (8) for $F_t + G_t =: X_t$ and C_t, $t = 1, 2$, respectively. Then the government objective function is:

$$
\max_I \quad V((1-\tau)\bar{Y})\ +\ \tau\bar{Y} - I\ +\ \rho V((1-\tau)Y(I))\ +\ \rho\beta(\alpha, \pi)\tau Y(I) \qquad (9)
$$

The first order condition (FOC) with respect to the (remaining) policy variable I is as follows:

$$-1 \; + \; \rho V'((1-\tau)Y(I))\,((1-\tau)\,Y'(I)) \; + \; \rho\,\beta(\alpha,\pi)\,\tau\,Y'(I) \; = \; 0 \tag{10}$$

The FOC requires that the marginal utility of (giving up 1 unit of) public good provision in period 1 (which is unity due to assumption (2) on public goods utility H) must be equal to the (additional) utility derived (i) from (additional) second period consumption (due to the after-tax income effect of increased investment) *and* (ii) from the (additional) public goods provision in period 2 (due to the tax effect of increased investment). Note that the discounted marginal utilities for (i) and (ii) are different. As for (i), marginal utility V' is discounted by discount factor ρ. As for (ii), unity marginal utility of the public goods provision is discounted by $\rho\beta$.

The FOC is, of course, only a necessary condition. The sufficient condition for a maximum is that the second derivative of (9) must be negative. It can be that this is only true for β above a threshold value, $\beta > \beta^*$, where β^* is the value for which the second derivative equals 0.

We are interested in the effect of political instability β on public investment I. Remember that political instability parameter β introduced in equation (8) represents both the probability of government change π and political polarization α. Remember also that both π and α are negatively related to β, which takes values between 0 (complete instability) and 1 (perfect stability). Assuming now that (9) is a well-defined maximization problem (i.e. $\beta > \beta^*$), applying total differentials leads to

Proposition 1 (Interior Solution)

For $\beta > \beta^*$, the following perturbation results hold at the equilibrium:

(i) $\frac{dI}{d\beta} \; > \; 0$

(ii) $\frac{d\frac{dI}{d\beta}}{d\beta} < 0.$

Point (i) states that increasing β, i.e. less political instability, leads to more public investment (as long as $\beta > \beta^*$). As the government becomes less myopic, it is optimal to invest more into the future. Additional political stability effectively increases the discount factor for the future. This is intuitive and straightforward. However, point (ii) asserts that the (positive) marginal effect on investment of more political stability decreases. This is so because the marginal investment profitability goes down. Conversely, proposition 1 means that political instability leads to a depletion of public investment which accelerates for more and more instability (up to β^*).

7 Corner Solution

For two reasons, this is not the full story: first, β could be smaller than β^*; and second, public investment cannot be reduced at an ever increasing rate for increasing political instability (β decreased). The rest of the story is simple. For β below β^*, public investment must be zero. To see this consider the government's optimal choice problem for $\beta < \beta^*$. Formally, the government problem becomes a minimization problem as the second order condition turns positive. If, however, public investment is constrained to zero in this two period model, the optimal choice of the government is the corner solution.

Proposition 2 (Corner Solution)

For $\beta < \beta^*$, it is optimal for the government not to invest.

The proposition appears obvious from first inspection of the problem, but can be formally proved by using the Kuhn-Tucker conditions. The intuition is also simple. For small β, the government values the present much more than the future. Given such myopia, the government does not want to move resources from today to tomorrow. Hence there is no investment. In the real world, disinvestment (like the sale of infrastructure, e.g. train coaches) might actually result from large myopia.

The overall solution can be illustrated by two figures which plot optimal I and $\frac{dI}{d\beta}$ as functions of β. Figure 1 shows that there is no public investment for small β. However, from $\beta = \beta^*$ onwards, it is optimal for government to invest more and more for increasing β. That $I(\beta)$ is non-differentiable at β^*[4] and concave thereafter can be seen from figure 2. $\frac{dI}{d\beta}$ is zero for $\beta < \beta^*$. It becomes infinity at $\beta = \beta^*$ and decreases though still positive thereafter.

8 Conclusion

This paper captures the government decision problem between public investment and public consumption in a simple model of political instability. The chance of another government being in power and taking undesirable decisions in the future produces a negative spill-over onto today's government. This is the basis for the result of myopic government behavior in the literature.

In this paper, it is actually optimal for the current government to totally refrain from spending on public investment, if enough myopia is produced by political instability. As we increase political stability, we reach a threshold above which it is optimal to increase investment. At first, marginal investment is strong, because additional political stability effectively increases the discount factor for the future. Then, however, the additional investment increments become smaller, because marginal investment profitability goes down.

The paper offers an (alternative) explanation for appalling levels of public investment in some countries and/or governments' unwillingness to invest in the fight against corruption. For several reasons, the model is particularly relevant for certain developing or transition

[4] Note, however, that the level of investment is not necessarily zero at $\beta = \beta^*$. We know that it is positive, but there could be a jump to a higher level of investment depending on the precise format of the investment and utility functions.

countries. First, political instability in some of these countries is inherent to the political structure of the country rather than caused by electoral uncertainty as in Western democracies. Second, the disregard for private sector decisions on labor, consumption and investment would certainly not be suitable simplifications for industrialized countries, but may be seen as a first approximation in some developing or transition countries, where there is either no economic growth or it depends on external factors (like foreign direct investment).

However, relaxing these assumptions offers scope for future research. A possible extension would be to model the effect of public investment on growth, when the private sector optimizes its investment and consumption decisions. A further extension might be to capture the interaction between growth and political instability. It is, however, not obvious how to do this, because the typical median voter approach (as, for instance, in Tabellini and Alesina's political instability model) may not be suitable for less democratic countries. It may also be worth while exploring if there are any trade-off effects when other sources of government revenue are included.

Finally, this model could be tested empirically. Countries could be placed in different groups according to their level of investment. It could then be studied, if their investment behavior changes as their level as political instability changes over time. The model predicts that moderate changes of political instability would not affect the investment behavior of low-investment countries .

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A Optimal Public Goods Spending

The following exposition draws on Cukierman, Edwards, and Tabellini (1992). The same approach is also used in Svensson (1998). For convenience, polarisation assumption (2) which is embedded in the government utility function H for public goods spending is restated for the type i government:

$$H^i(G^i, F^i) = \frac{1}{\alpha(1-\alpha)} \min\{\alpha G^i, (1-\alpha)F^i\}. \tag{A.1}$$

Since (A-1) contains a minimum function, optimality can only be achieved for

$$(1-\alpha)F^i = \alpha G^i. \tag{A.2}$$

As the utility function H for the type k government is symmetrical according to its definition in section 2, so is the optimal distribution between F^k and G^k: $(1-\alpha)G^k = \alpha F^k$.

Government i's optimal total public goods spending X^i can be written as

$$X^i := F^i + G^i = \frac{G^i}{1-\alpha} = \frac{F^i}{\alpha}. \tag{A.3}$$

By reinserting into utility function (A-1) the optimal values for F and G in terms of X ($G^i = (1-\alpha)X^i$, $F^i = \alpha X^i$) a simple result for total public goods utility H is obtained:

$$\begin{aligned} H^i(G^i, F^i) &= \frac{1}{\alpha(1-\alpha)} \min\{\alpha(1-\alpha)X^i, (1-\alpha)\alpha X^i\} \\ &= X^i = F^i + G^i. \end{aligned} \tag{A.4}$$

We can now see that the denominator in equation (A-1) was chosen as a normalisation such that the marginal public goods utility is unity. Furthermore, given that utility function (A-1) is symmetrical for both types of government, the optimal values for F and G are crosswise identical ($F^i = G^k$ and $G^i = F^k$) and

$$H^i(G^i, F^i) = X^i = X = X^k = H^k(G^k, F^k). \tag{A.5}$$

B Second Order Condition

For (9) to be a well-specified maximisation problem, the second derivative with respect to I must be smaller or equal to 0:

$$\rho * V''((1-\tau)Y(I)) \; * \; ((1-\tau) * Y'(I))^2 \tag{B.1}$$

$$+ \quad \rho * V'((1-\tau)Y(I)) \; * \; ((1-\tau) * Y''(I)) \tag{B.2}$$

$$+ \quad \rho * \beta(\alpha,\pi) * \tau * Y''(I) \quad \leq \quad 0$$

$$\Longleftrightarrow$$

$$\underbrace{\Big(\rho * V''((1-\tau)Y(I))\Big)}_{-} \; * \; \underbrace{\Big(((1-\tau) * Y'(I))^2\Big)}_{+} \tag{B.3}$$

$$+ \quad \underbrace{\Big(\rho * Y''(I)\Big)}_{-} \; * \; \underbrace{\Big(\tau * \beta(\alpha,\pi) - (1-\tau) * V'((1-\tau)Y(I))\Big)}_{?} \; \leq \; 0$$

A sufficient condition for this to hold is:

$$\tau * \beta(\alpha,\pi) \; \geq \; (1-\tau) * V'((1-\tau)Y(I)). \tag{B.4}$$

Given that the marginal utility of H is normalised at unity, the condition could be rewritten as:

$$\tau * \beta(\alpha,\pi) * H'(G_2, F_2) \geq (1-\tau) * V'((1-\tau)Y(I)). \tag{B.5}$$

If we ignore the tax rate for a moment (set $\tau = .5$), condition (B.4) requires the effective marginal public goods utility in period 2, $\beta * H_2'$, to be greater or equal to second period marginal private sector utility V_2'. Given some political instability ($\beta < 1$) this means that, at the margin, the policy maker must attribute less importance to private consumption than to total public goods provision.

Equation (B.2) does, however, hold for weaker conditions as well. The marginal private sector utility V_2' could also be greater than the effective marginal public goods utility $\beta * H_2'$, as long as $\beta * H_2'$ is sufficiently close to V_2'. For V_2' above (but close to) unity, β must be relatively close to 1, where 1 signifies perfect political stability (no polarisation and/or no chance of government change). For V_2' below unity, the following holds: the farther V_2' from unity (i.e. the less important private consumption is relative to public consumption), the more political instability is permitted.

In fact, equality in (B.2) defines β^*, a threshold level for β. For $\beta > \beta^*$ the government choice problem (9) is a well-defined maximisation problem producing an interior solution for the optimal level of public investment I.

C Perturbation Results

Proposition 1 (i) – given $\beta > \beta^*$:

$$\frac{dI}{d\beta} = -\frac{\frac{\partial W_I}{\partial \beta}}{\frac{\partial W_I}{\partial I}} = -\frac{\overbrace{\rho * \tau * Y'(I)}^{+}}{\underbrace{\frac{\partial W_I}{\partial I}}_{-}} > 0 \tag{C.1}$$

Proposition 1 (ii) – given $\beta > \beta^*$:

$$\frac{d\frac{dI}{d\beta}}{d\beta} = \frac{\overbrace{-\rho * \tau * Y''(I) * \frac{\partial W_I}{\partial I}}^{-} - \overbrace{-\rho * \tau * Y'(I) * \rho * \tau * Y''(I)}^{+}}{\underbrace{\frac{\partial W_I}{\partial I}^2}_{+}} < 0 \tag{C.2}$$