Standard Promotion Practices Versus Up-or-Out Contracts

Suman Ghosh
Florida Atlantic University

and

Michael Waldman
Johnson Graduate School of Management
Cornell University

December 2003.

Contents

Abstract

We observe different types of promotion mechanisms in different types of firms. In a wide variety of firms we observe that the jobs are structured as ladders, where workers are promoted to a higher level job or are relegated to a lower level job. In other variety of firms, such as in academic institutions, law firms or medical practitioners firms, the promotion structure is that of up-or-out contracts, where if a worker is not promoted to the higher level job he must be dismissed from the firm. The purpose of this paper is to study the reasons behind this differing phenomena. We find that there exists a both sided double moral hazard problem which leads to a commitment failure. The solution to that problem under different circumstances can give us the reasons as to why we observe each type of contracts in different firms.

Corresponding author: Suman Ghosh, Department of Economics, Florida Atlantic University, 777 Glades Road, Boca Raton, Florida 33431.

1. Introduction

We observe different types of promotion mechanisms in different types of firms. In a wide variety of firms we observe that the jobs are structured as ladders, where workers are promoted to a higher level job or are relegated to a lower level job. In other variety of firms, such as in academic institutions, law firms, accounting firms or medical practitioners firms, the promotion structure is that of up-or-out contracts, where if a worker is not promoted to the higher level job he must be dismissed from the firm. The purpose of this paper is to study the reasons behind this differing phenomena.[1]

The promotion process involves a double moral hazard problem. The firms want workers to invest in training which will increase their productivity, for which promotion serves as an incentive mechanism for them to do so. On the other hand, after workers have invested in training, the firms might not promote him. In other words there is a commitment problem for the firm. One way in which this commitment problem can be solved is, if it is in fact beneficial for the firm to promote the worker to the higher level job if he has invested in training. This can be possible if the different level jobs are sensitive to the workers ability. That is, if trained workers are assigned to the higher level job, the output is higher and thus it is in the employers benefit to promote the worker. For the workers, they are now assured that they will in fact be promoted once they train themselves (which presumably involves a cost) and can reap the higher wages associated with the higher level job. This explains the first type of promotion contracts where workers who train themselves and increase their ability are promoted, while the rest are kept at the lower level job.

Next, we have to address the question, then why do some firms offer up-or-out contracts? As Lazear (1991) notes, justifying up-or-out rules is problematic because one

---

[1] Throughout the paper, we refer to the first type of promotion mechanism as "promotion contracts" while the other as "up-or-out contracts".

must explain why a worker is considered productive one day but is terminated the next. The answer to this is related to the reasoning just given. Suppose the type of activity that is associated with a particular type of firm is such that output of jobs are not that much sensitive to ability. In that case, the reasoning outlined above does not hold. In this scenario, firms commit to up-or-out contracts since now the opportunity cost of not promoting the worker implies loosing the worker (remember that the worker has to be dismissed if he is not promoted) and hence getting a rent of zero from the workers' services. In the standard promotion case, the employer has an option to keep the worker in the lower level job but in the up-or-out case the employer has to commit to something more drastic since the logic for which promotions worked in the case when the jobs were sensitive enough to output does not hold here. This seems to be consistent with what we tend to observe, that is, the nature of the work in such occupations does not appear to vary significantly with the rank of the employee. For example, a university may have no incentive to promote a junior faculty member as the nature of the work he does will not change after promotion, so why incur the extra wage costs?

To address the above logic we build a two period model with two job levels, where the set-up is such that it is beneficial to assign workers of higher ability to the higher level job (see Sattinger (1975) for one of the first models in a similar framework). In that set-up we can vary the parameter determining the higher level job to account for the fact of "sensitiveness of output corresponding to the job level". We show that workers will invest in human capital if the higher level job is responsive enough to workers ability. Precisely, it means whether the output of the higher level job is significantly higher for a worker of high ability. In contrast, in the up-or-out case, because the firm looses the worker if not promoted, then they are willing to promote the worker for a lower value of the parameter which establishes the job sensitivity. Then we compare the life-time utility for workers and find that in the parameter range where workers invest in both cases (that is, up-or-out and promotion contracts), then they are better off in the promotion case, while when workers invest in the up-or-out case but not in the promotion case, workers are better off in the up-or-out case.

Next we consider a model with effort provision on the part of the workers, where effort serves as a signal to the employer about the ability of the worker. The result here is

reminiscent of Holmstrom's (1999) paper on managerial incentive problems. In that paper he found that as long as ability is unknown there are returns to supplying labor because output will influence perceptions about ability. By increasing its supply, the manager can potentially bias the process of inference in his favor. The manager is trapped in supplying the equilibrium level that is expected of him. We get a similar result in our model. The model of effort provision provides another rationale for promotion contracts versus up-or-out contracts.

The basic question as to why promotion seems more important than bonuses as a source of incentives in many firms was raised by Baker et. al (1988). The answers to this puzzle can be categorized into two fields. One is that promotion serves as an incentive for skill acquisition and secondly promotion serves as an incentive for effort. The issue of promotion serving as an incentive for effort has been addressed by Malcolmson (1984), Fairburn and Malcolmson (1997) and Gibbs (1995).

Kahn and Huberman (1988) address the question in a setting where the firm has only one job. The logic is similar: a worker is paid a high wage if promoted, hence a worker has an incentive to invest. Because the worker must be dismissed if he is not promoted, the firm has an incentive to promote high productivity workers. [2]

In a recent paper, Levine and Tadelis (2002) give an interesting rationale for the existence of up-or-out contracts in partnership firms. The up-or-out promotion scheme is an integral part of a partnership's commitment to guaranteeing the high quality of long term employees. Because current partners will promote only the best associates to a full partner share, those that are not of extremely high quality will be let go even if they might make a positive contribution to the firm's total profits. Hence none of the papers mentioned above give us the reason as to under what conditions we would tend to observe promotion contracts as against up-or-out contracts. They merely give a reason for each type of contract existing.

In this paper we explain the rationale from a different stand point but not restricted to partnership firms only. Our model generate results as to when exactly do we see

---

[2] Waldman (1990) extends Kahn and Huberman's analysis by considering what happens when human capital is general and there is asymmetric learning.

promotion contracts as against up-or-out contracts. In the second model we give another explanation of the same fact but from a different perspective.

Lastly, another important issue is that of asymmetric information. We assume that the firm in which the worker works in the first period observes the output of the worker perfectly but the outside firms just observes the job assignment decision of the original employer and bids wage offers accordingly. This is important since now allocation decisions of workers to jobs serves as a signal to outside firms about the workers ability. In order to conceal that information the employer might inefficiently not promote workers to higher level jobs. We will see that this issue will play a crucial role in our model.

The rest of the paper is organized as follows. In section 2, we construct our model. This section is divided into three sections. In (a) we analyze the standard promotions case where the workers are either promoted or kept in the lower level job. In (b) we analyze the up-or-out case where the workers are dismissed if they are not promoted. And finally, in (c) we compare and contrast the two types of contracts. In section 3, we perform the analysis with effort choice as the relevant choice variable for the worker, where effort serves as a signaling device. In Section 4 we discuss some empirical results and issues. In section 5 we conclude.

2. The Model with Specific Human Capital

We consider two cases. One, where there is a spot market contract, and second, where there is an up-or-out contract. In both the cases there are two jobs in which the worker may be allocated. A lower level job in which the workers are allocated when they join a particular firm and a higher level job in which they might be promoted in the next period. And we consider a worker's life span consisting of just two periods. A spot market contract simply specifies the wage the worker will receive while young. If a worker accepts a standard spot-market contract, then the worker's second period wage and firm are determined as follows. The employer observes the worker's productivity in period 1 and

5

then decides whether to promote him to the higher level job. The observed productivity is private information to the employer while the outside firms can observe the promotion decision of the employer. With this information the outside firms bid for the worker and thus his wage is determined in period 2. The workers observe the wage offered and decides on which firm to join. The first period wage is determined by the zero profit condition of the firm for the entire lifetime of the worker.

On the other hand, in the up-or-out contract, the employer decides on the first period wage as before but at the beginning of period 2 she has to decide whether to keep the worker or dismiss him. If she decides to keep the worker then the worker has to be promoted to the higher level job. The wage in the second period is determined by the simultaneous bidding of the employer and the outside firms.

Finally, another fact which necessitates mention at this stage is regarding the wage setting process. We assume that the employer and the outside firms give simultaneous wage offers to the worker and observing all the offers the worker decides on the firm to work for. This is an apparently restrictive assumption (as against modeling it where the employer can give counter-offers, as in Lazear (1986), Milgrom and Oster (1987) and Waldman (1990)) but it can be easily rectified by assuming that there is always an exogenous probability of the workers changing jobs irrespective of the wage offer (as in Greenwald (1986)). That will give us the same results as we have obtained here.

(a) The Case of Promotion Contracts

Within the economy there is only one good produced, and the price of this good is normalized to one. Workers live for 2 periods, and in each period labor supply is perfectly inelastic and fixed at 1 unit for each worker. There are 2 jobs: the output at each job is given by $d_1+c_1\eta_i$ and $d_2+c_2\eta_i$ respectively, where η denotes the intrinsic ability of worker i. We assume $c_2>c_1$ and $d_1>d_2$. Let η^* be the ability level at which a worker is equally productive at jobs 1 and 2. That is, η^* solves $d_1+c_1\eta_i = d_2+c_2\eta_i$. Thus given full information about workers abilities, the efficient assignment rule for period 2 is to assign worker i to job 1 if $\eta_i<\eta^*$, and to job 2 if $\eta_i>\eta^*$. The worker has two choices. If he invests

in firm specific human capital accumulation, then his output is augmented by a factor α, where $\alpha > 1$[3] . Thus allocating the worker in job 1 and job 2, will give outputs $\alpha(d_1 + c_1 \eta_i)$ and $\alpha(d_2 + c_2 \eta_i)$ respectively. The cost of investment is given by z. Firms and workers are risk neutral and are perfectly competitive. We assume the discount rate to be zero. Hence, when coming into the labor market, a young worker will attempt to maximize his expected lifetime income minus any cost incurred in the accumulation of human capital, if he decides to acquire any.

The ability of the workers varies uniformly from η_L to η_H. Workers do not know their ability but they know the distribution from which their ability is a draw. Employers can observe the ability of the worker after he has worked for a period in the firm. The outside firms cannot observe the ability. To make the case of promotion interesting we assume further that $d_1 + c_1 \eta_H < d_2 + c_2 \eta_H$, so that a positive fraction of the workers are always more efficient in job 2. Otherwise it is always the case that the workers are kept in job 1. We can write the same condition in terms of c_2,

$$c_2 > c_1 + \Delta, \text{ where } \Delta = (d_1 - d_2)/ \eta_H. \tag{1}$$

Finally, we use a restriction on z. We assume that the cost of investment, z, is not too high such that none of the workers ever invest and is not too low such that there is always investment so that the issue of type of contracts to provide incentives for workers to invest is irrelevant. We provide a more precise range of z later once we have defined the entire model.

Timing

In period 1, the workers decide whether to accumulate human capital or not. The current employer can observe the productivity of the worker in job 1 in period 1. Then at the beginning of period 2, the current employer decides whether to promote the worker. After observing the action of the current employer regarding the promotion decision, there are simultaneous wage offers given by the market and the employer. The workers joins the firm with the highest wage offer. Also we assume that if the offers are the same, then the worker stays with the current employer.

---

[3] See later for a more specific restriction on α.

<u>Employer</u>

Here we derive the cut-off ability worker who will be promoted by the firm. If a worker who has acquired training is employed in job 2 by the employer, the productivity of the worker is given by $\alpha(d_2+c_2\eta)$. While if he is employed in job 1 his productivity is given by $\alpha(d_1+c_1\eta)$. The outside firms' wage offers determines the workers wages. The outside firms can only observe the assignment of the workers. Thus a worker who is assigned to job 2 has an average ability level of $((\eta^*+\eta_H)/2)$, and who is assigned to job 1 has an average ability level of $((\eta^*+\eta_L)/2)$, where η^* is the cut-off ability worker who is promoted to job 2. Hence the corresponding wages that the worker has to be paid in job 2 and job 1 are $d_2+c_2((\eta^*+\eta_H)/2)$ and $d_1+c_1((\eta^*+\eta_L)/2)$ respectively.[4] Here we need a condition on α such that workers will not be fired from the firm. The condition that is needed is the following: $\alpha > \eta_H/\eta_L$. This is a sufficiency condition which basically comes from the fact that the specific human capital is high enough such that it is beneficial for the firm to keep the lowest ability worker in either job 1 or job 2 if he has invested.

Thus by equating the two profits for the employer (i.e keeping the worker in job 1 and promoting him to job 2), given workers invest we get:

$$\alpha(d_2+c_2\eta^*)-[\ d_2+c_2((\eta^*+\eta_H)/2)] = \alpha(d_1+c_1\eta^*) - [d_1+c_1((\eta^*+\eta_L)/2)] \qquad (2')$$

Therefore:

$$\eta^* = \frac{2(\alpha-1)(d_1-d_2)+(c_2\eta_H - c_1\eta_L)}{2(c_2-c_1)(\alpha-1/2)} \qquad (2)$$

For $\eta^* > \eta_H$ and $\eta^* < \eta_L$, it takes a value of zero. From the above equation we can see that given a fixed α we get a corresponding η^* which gives the marginal ability worker is promoted by the firm. Also, in a full information framework (as in the model of Gibbons and Waldman 1999) the efficient η, denoted by η', is given by $\eta' = (d_1-d_2)/(c_2-c_1)$. By comparing (2) with η', and by using the condition $d_1-d_2 < c_2\eta_H - c_1\eta_H$, we get that $\eta' < \eta^*$,

---

[4] More precisely the outside offer to workers when they are promoted is given by $\max\{\ d_2+c_2((\eta^*+\eta_H)/2), d_1+c_1((\eta^*+\eta_H)/2)\}$.

which shows the inefficiency due to the asymmetric information set up in our case. Lemma 1 establishes this result.

Lemma 1: *$\eta' < \eta^*$. (Waldman 1984)*

The above lemma basically gives the inefficiency associated with asymmetric information as was first pointed out in Waldman (1984). In a complete information model where the employer and the outside firms can observe the first period output of the worker perfectly, the proportion of workers promoted would have been ($\eta_H - \eta'$) while in our case it is ($\eta_H - \eta^*$). Given the asymmetric information set-up of our model we see that workers are inefficiently not promoted in order to conceal the true ability of the worker.

Also by substituting $\alpha = 1$ in equation (1) we get that $\eta^* > \eta_H$, which intuitively means that when the investment in specific human capital does not augment output then there is no promotion.

Remember that c_2 is the slope of the output function of job 2. Hence as c_2 increases (keeping c_1, d_1 and d_2 fixed), the productivity of workers assigned to job 2 increases. Lemma 2 below gives the effect of an increase in c_2 on the marginal worker.

Lemma 2: *d η^*/d c_2 ≤ 0 , where η^* is the marginal ability worker who is promoted to the higher level job.*

It is worth noting here that d η'/d c_2 ≤ 0 also, i.e, the cut-off level in the full information case as mentioned before also decreases with c_2. Intuitively what it means is that as c_2 increases, the fraction of workers who are promoted for an investment in human capital accumulation increases. Basically the employer, in order to compete with that higher wage might inefficiently decide not to promote the worker. If placing the worker in the lower job costs the firm high enough (since he might be able to produce a larger output in the higher level job), then they might decide to promote him even if that entails paying a higher wage. What Lemma 2 says is that as the parameter c_2 increases, the cost of inefficiently allocating the worker to job 1 increases and thus the employer will promote more of the workers.

<u>Workers</u>

Now let us consider the workers' decision. Remember that the cost of investment in human capital accumulation by the worker is z. If the worker is promoted then he gets a wage of $d_2+c_2((\eta*+\eta_H)/2)$, while if he stays in job 1 he gets a wage of $d_1+c_1((\eta*+\eta_L)/2)$. If the worker invests then the probability of getting promoted is given by $X^1 = Pr\ (\eta > \eta*) = (\eta_H-\eta*)/(\eta_H-\eta_L)$, and the probability of not getting promoted is $Pr\ (\eta < \eta*) = (\eta* - \eta_L)/(\eta_H-\eta_L)$.[5] This follows from the fact that the ability is a draw from a uniform distribution with range (η_L, η_H). Thus while making the decision on whether to acquire human capital, the worker will weigh his cost of acquiring with that of his expected gain in wages.

Thus the relevant inequality which has to be satisfied for the workers to take up the investment in specific human capital accumulation can be written as:

$[d_2+c_2((\eta*+\eta_H)/2) - \{d_1+c_1((\eta*+\eta_L)/2)\}]\ [\{(\eta_H-\eta*)/(\eta_H-\eta_L)\}] > z,$

where, the left hand side of the above inequality is the expected gain to the worker for investing in specific human capital accumulation, and the right hand side is the cost of acquiring human capital.

From above it can be shown that there exists a c_2* for which the above inequality is satisfied as an equality.

$[d_2+c*_2((\eta*+\eta_H)/2)-\{d_1+c_1((\eta*+\eta_L)/2)\}][\{(\eta_H-\eta*)/(\eta_H-\eta_L)\}] = z^6$ (3)

Proposition 1: *In the promotion contract case, there exists a c^*_2 such that in equilibrium, for $c_2 > c^*_2$, workers invest in human capital accumulation and the fraction of workers who are above the cut-off $\eta*$ are promoted. For $c_2 < c^*_2$, none of the workers invest and hence none of the workers are promoted.*

---

[5] There is a probability for workers to get promoted even if they don't invest, only worker's who are of extreme high quality are promoted in this case. However the assumption on α ($\alpha > \eta_H/\eta_L$) makes the probability of this occurrence zero such that workers who invest are at a significant enough advantage than even the highest ability worker who does not invest.

[6] $\eta*$ is a function of c_2, but for notational simplicity we are omitting writing the functional form here.

The left hand side of equation (3) consists of two parts. One is the wage effect, which is the increase in wages that workers receive when they are promoted. That is the first part of the left hand side. The second part gives the ex-ante probability of getting promoted if they invest. As c_2 increases, the wage effect becomes smaller (remember we had proved that $\eta^*(c_2)$ and $d\,\eta^*/d\,c_2 < 0$) since now the outside employers know that the average ability of the workers that are promoted is lower and thus bid a lower wage while the ex-ante probability of getting promoted becomes larger. We show that the latter effect dominates the wage effect for an increase in c_2. Hence there exists a c_2^* such that for values of c_2 lower than c_2^*, the left hand side is always smaller than the right hand side and thus it is not worth investing in firm specific human capital by the worker. Basically, for $c_2 > c_2^*$, both the employers' and the workers' incentives can be satisfied. For all of the propositions we assume a restriction on z, specifically we assume that $\phi(c_1 + \Delta) < z < \phi(c_2'')$, where $\phi(c_2) = [d_2 + c_2((\eta^* + \eta_H)/2) - \{d_1 + c_1((\eta^* + \eta_L)/2)\}] [\{(\eta_H - \eta^*)/(\eta_H - \eta_L)\}]$. c_2'' is the upper bound on the value of c_2 and $(c_1 + \Delta)$ is the lower bound on c_2 above which the issue of promotion makes sense, which comes from equation (1). This is basically made to guarantee existence.

Before we proceed to characterize the equilibrium in the spot market contract case we should mention one point. For values of $c_2 > c_2^*$, there are multiple equilibria, where all workers not investing is also an equilibrium. We assume that the workers can coordinate behavior such that the equilibrium realized is the one that is Pareto optimal for the workers in that period. Another way to put the assumption is that we restrict attention to Perfectly Coalition-Proof Nash equilibria.[7] This reasonable assumption helps us to have a neat characterization of the parameter space.

Let us relate our findings so far with the current literature. For $c_2 > c_2^*$, the equilibrium is such that the workers invest in human capital accumulation and the fraction of the workers that are above the cut-off are promoted. (Prendergast- 1993)

For $c_2 < c_2^*$, none of the workers invest in human capital and then we might need up-or-out contracts (Kahn and Huberman 1988, Waldman 1990) which we verify in the next subsection.

---

[7] See Bernheim, Peleg, and Whinston (1987) for a discussion of this refinement.

Thus, if the jobs are sufficiently different then we can employ promotion contracts, whereas if they are not, then it might be that an up-or-out contract maybe a better contractual device.

Since this is a two period model, in order to derive the expected life time utility of workers in the firm we need to calculate the wage that is offered to them in the first period, denoted by W^1. We break this up into two cases, namely, one in which workers do invest and the other case in which workers do not invest, as we derived above. Finally we introduce some notations where U_I^P denotes the lifetime utility of the workers who invest in the promotion contract case and U_N^P denotes the utility of the workers who do not invest in the spot market contract case, given that in equilibrium all other workers do invest.

(i) $c_2 > c_2^*$, i.e. the case where workers invest.

We find W^1 by imposing a net expected profit of zero condition for the employer since firms are perfectly competitive.

$$W^1 + X^1(d_2 + c_2((\eta_H + \eta^*)/2)) + (1-X^1)(d_1 + c_1((\eta_L + \eta^*)/2)) = d_1 + c_1((\eta_L + \eta_H)/2) + X^1\alpha(d_2 + c_2((\eta_H + \eta^*)/2)) + (1-X^1)\,\alpha\,(d_1 + c_1((\eta_L + \eta^*)/2)).$$

$$W^1 = d_1 + c_1((\eta_L + \eta_H)/2) + X^1(\alpha-1)\,[d_2 + c_2((\eta_H + \eta^*)/2)] + (1-X^1)\,(\alpha-1)\,[d_1 + c_1(\eta_L + \eta^*)/2)]$$

(4)

Here the left hand side of the above equation (the top equation) gives the expected wages that the worker will be paid in his career. Denoting the wage paid in the first period as W^1, the expected wages in period 2 consists of two components: (1) the wage paid if the worker is promoted to job 2 and (2) is the wage paid if the worker is not promoted. The respective probabilities are signified with (X^1) and $(1-X^1)$ respectively. The right hand side gives the expected productivity of the worker in the two periods. As in the wages paid, this takes into account the respective productivities if the worker is promoted and also the case where he is not promoted. The equality of the wages paid over the two periods with the productivity of the workers over the same time span is an artifact of the net expected zero profit condition of the employer.

Utility of the worker:

$U_I{}^P = W^1 + X^1(d_2 + c_2((\eta_H + \eta^*)/2)) + (1 - X^1)(d_1 + c_1((\eta_L + \eta^*)/2)) - z$

By substituting for W^1, in the expression for $U_I{}^P$ we get,

$U_I{}^P = d_1 + c_1((\eta_L + \eta_H)/2) + X^1\alpha [d_2 + c_2(\eta_H + \eta^*)/2)] + \alpha (1 - X^1) [d_1 + c_1((\eta_L + \eta^*)/2)] - z.$ (5)

Thus we get (5) by considering the wages that the worker is paid in his career (given he invests) and then subtracting the cost of investment, denoted by z.

    (ii) $c_2 < c_2^*$, i.e. the case where none of the workers invest.[8]

$W^1 + (d_1 + c_1((\eta_L + \eta_H)/2)) = d_1 + c_1((\eta_L + \eta_H)/2) + (d_1 + c_1((\eta_L + \eta_H)/2)).$

or, $W^1 = d_1 + c_1((\eta_L + \eta_H)/2)$

Note here that in this case none of the workers are promoted. And hence in each period the worker is paid the productivity corresponding to the ex-ante average productivity of a worker.

Utility of the worker:

$U_N{}^P = W^1 + (d_1 + c_1((\eta_L + \eta_H)/2))$

$U_N{}^P = 2(d_1 + c_1((\eta_L + \eta_H)/2))$ (6)

Thus the total utility of a worker in this case is just the summation of the wages that he is paid in each period, which is the ex-ante average productivity of a worker.

(b) Up-or-out Contracts

We showed in section (a), that under certain parametric conditions it is not efficient for the employer to use promotion to job 2 as an incentive device for workers to invest. In this section we consider the possibility where the employer can commit to an up-or-out contract. Basically, she commits to promote the worker if she continues with him,

---

[8] This is the case corresponding to Waldman (1984). Basically, without specific human capital accumulation none of the workers are promoted in equilibrium. In that paper the decision to acquire specific human capital was not endogenous though.

otherwise the worker must be dismissed. Our intention is to verify whether such up-or-out contracts can be a better contractual device under certain circumstances so that workers are better off compared to the spot contract regime. In other words, whether there exists a range of the parameter space of c_2, where workers did not invest in the promotions contract case but invest in the up-or-out case. Then we compare the lifetime utility of the workers in each of the cases.

Timing

The workers know that they will be dismissed if they are not promoted by the firm in period 2 and given this, at the beginning of period 1 they decide to invest (or not). At the beginning of period 2 the employer observes whether worker's have invested or not and also the individual ability. They decide either to keep a particular worker at the promoted level or else to dismiss the worker. The outside firms observes that decision and then offers wages to both types of workers. Workers who are given an option to stay with the employer decide on which firm to join. They join the firm offering the highest wage offer. As before, for a tie, worker's stay with the current employer. The other workers who are not kept by the firm take up one of the outside offers.

Employers

First let us find the cut-off ability worker who will be kept by the employer. For the workers that are kept in the firm, it signals higher ability. Let us denote the marginal ability worker who is kept in the firm (which we derive below) as η^{**}. The outside offer which those worker's get is $d_2+c_2((\eta_H+\eta^{**})/2)$.[9] The rest of the workers who leave the firm get an outside offer of $d_1+c_1((\eta_L+\eta^{**})/2)$. Now, if workers invest, then the output to the employer is $\alpha(d_2+c_2\eta)$ (remember that they have to be promoted if the firm keeps them). Now we can solve for η^{**} by equating the two profits.

$$\alpha(d_2+c_2\eta^{**})-d_2+c_2((\eta_H+\eta^{**})/2)=0 \qquad\qquad (7)$$

---

[9] Same as before where the outside offer is actually given by $\max\{d_2+c_2((\eta^*+\eta_H)/2), d_1+c_1((\eta^*+\eta_H)/2)$.

where the left hand side in equation (4) is the profit from the marginal worker who is kept in the firm while the right hand side is zero since the firm looses the worker. By solving for η^{**} from (7), we get:

$$\eta^{**} = \frac{c_2(\eta_H/2) - 2(\alpha - 1)d_2}{2c_2(\alpha - 1/2)}. \tag{8}$$

Hence, workers for whom the ability is greater than η^{**}, will be kept by the employer. Others leave the firm at a wage $d_1+c_1((\eta^{**}+\eta_L)/2$. One thing to note is that η^{**} is independent of c_1 and d_1. The reason behind this is that once the workers are not promoted then the firm loses the worker and hence do not have the option of keeping him in the lower level job as in the promotion contract case. Similar to Lemma 2 in the previous section we also get:

Lemma 2´: $d\,\eta^{**}/d\,c_2 < 0$.

Workers

The wage that is offered to workers who stay with the firm is $d_2+c_2((\eta_H+\eta^{**})/2$. Then for the workers, they will invest only if the prospect of staying in the firm is worthwhile. If the worker invests then the probability of staying in the firm is given by $Y^1=$ $\Pr(\eta > \eta^{**}) = (\eta_H - \eta^{**})/(\eta_H - \eta_L)$, and the probability of not getting promoted is $Y^2 = \Pr(\eta < \eta^{**}) = (\eta^{**} - \eta_L)/(\eta_H - \eta_L)$. Hence the relevant inequality for the worker is,

$$[d_2+c_2((\eta_H+\eta^{**})/2) - \{d_1+c_1((\eta_L+\eta^{**})/2)\}\,][(\eta_H - \eta^{**})/(\eta_H - \eta_L) > z. \tag{9}$$

In the above inequality, the left hand side gives the expected extra wages that the workers can earn if they invest, and the right hand side is the cost of the investment.

From (9) we can get the cut-off c_2, which we denote as c_2^{**}, above which up-or-out contracts will be effective. Basically, it is that c_2 for which (9) holds as an equality:

$$[d_2+c_2((\eta_H+\eta^{**})/2) - \{d_1+c_1((\eta_L+\eta^{**})/2)\}][(\eta_H - \eta^{**})/(\eta_H - \eta_L) = z. \tag{10}$$

Proposition 2: *In the up-or-out contract case, the equilibrium is such that for $c_2 > c^{**}_2$, all workers invest in human capital accumulation and the fraction of workers who are above the cut-off η^{**} are promoted. For $c_2 < c^{**}_2$, none of the workers invest.*

The intuition behind Proposition 2 is very similar to Proposition 1. Basically workers have an incentive to invest only if there benefit to investing is high enough to cover the cost of investment. For a very low value of c_2 (i.e., $c_2 < c^{**}_2$), the ex-ante probability of getting promoted and reaping the benefits of investment is not enough to cover the investment cost of z.

To derive the lifetime utility of the workers we do similar to what we did in the promotion contract case, we divide the parameter space into two regions: (i) $c_2 > c_2^{**}$, i.e. the case where workers invest, and (ii) $c_2 < c_2^{**}$, i.e. the case where workers do not invest.[10]

(i) $c_2 > c_2^{**}$.

Now let us calculate the wage that is offered by the employer in period 1, denoted by W^1. As in the previous case, we find that by imposing a net expected profits of zero condition for the employer.

$W^1 + Y^1(d_2 + c_2((\eta_H + \eta^{**})/2)) + (1-Y^1)(d_1 + c_1((\eta_L + \eta^{**})/2)) = d_1 + c_1((\eta_L + \eta_H)/2) + Y^1 \alpha (d_2 + c_2((\eta_H + \eta^{**})/2)).$

or, $W^1 = d_1 + c_1((\eta_L + \eta_H)/2) + Y^1(\alpha-1)(d_2 + c_2((\eta_H + \eta^{**})/2)) - (1-Y^1)(d_1 + c_1(\eta_L + \eta^{**})/2)).$

Here the left hand side of the above equation (the top equation) gives the expected wages that the worker will be paid in his career. Denoting the wage paid in the first period as W^1, the expected wages in period 2 consists of two components: (1) the wage paid if the worker is stays with the firm and (2) is the wage paid if the worker is fired. The respective probabilities are signified with (Y^1) and $(1-Y^1)$ respectively. We can interpret $(1-Y^1)$ as the turnover probability since these workers leaves their current employer and join an outside firm. The right hand side gives the expected productivity of the worker in the two periods. As in the wages paid, this takes into account the respective productivities if the worker stays and also the case where he is fired. We denote by U_I^U, the lifetime utility of the

---

[10] We use the Coalition Proof Nash Equilibrium as before to get rid of the issue of multiple equilibria.

workers who invest in the promotion contract case and by U_N^U, the lifetime utility of the workers when they do not invest in the promotion contract case.

Thus the total utility of the worker in two periods is:

$U_I^U = W^1 + Y^1(d_2+c_2((\eta_H+\eta^{**})/2)) + (1-Y^1)(d_1+c_1((\eta_L+\eta^{**})/2)) - z$

or, $U_I^U = d_1+c_1((\eta_L+\eta_H)/2) + \alpha Y^1 [d_2 + c_2(\eta_H+\eta^{**})/2)] - z.$ \hfill (11)

    (ii) $c_2<c_2^{**}$.

$W^1 + (d_1+c_1((\eta_L+\eta_H)/2)) = d_1+c_1((\eta_L+\eta_H)/2) + (d_1+c_1((\eta_L+\eta_H)/2)).$

or, $W^1 = d_1+c_1((\eta_L+\eta_H)/2)$

This is the same as in the spot market case, where the worker is paid just the ex-ante average productivity corresponding to job 1.

Utility of the worker:

$U_N^U = W^1 + (d_1+c_1((\eta_L+\eta_H)/2))$

$U_N^U = 2(d_1+c_1((\eta_L+\eta_H)/2))$ \hfill (12)

(c) Comparison between the Promotion Contract Case and the Up-or-out Contract Case.

    Let us compare the cut-off ability levels of workers in the two cases. We have derived that $\eta^* = \dfrac{2(\alpha-1)(d_1-d_2)+(c_2\eta_H-c_1\eta_L)}{2(c_2-c_1)(\alpha-1/2)}$ and $\eta^{**} = \dfrac{c_2(\eta_H/2)-2(\alpha-1)d_2}{2c_2(\alpha-1/2)}$. By comparing the two expressions we can derive that $\eta^{**}< \eta^*$. Basically the denominator is smaller and the numerator is greater in η^*.[11]

Lemma 3 : $\eta^{**}< \eta^*$.

    The above lemma says that the proportion of workers that are promoted if they invest is always higher in the case of up-or-out contracts. The intuition behind this result is the following. In the promotion contract case the employer has the option of keeping the worker in job 1 and get some of the rents from the output of the worker. Thus the incentive

---

[11] See proof of Lemma 3 in the appendix.

to promote is less in this case compared to the up-or-out contract case where the worker has to be dismissed if they are not promoted and thus the employer derives no rent from the worker.

Lastly, we can derive from the expression for η^{**} and η^*, that $d\,\eta^*/d\,c_2 = -\,\eta^*/(c_2 - c_1)$ and $d\,\eta^{**}/d\,c_2 = -\,\eta^{**}/c_2$.

Recap: For both the cases we have parameterizations in terms of c_2 such that for values of c_2 above c_2^*, workers invested in firm specific human capital in the spot contract case while for values above c_2^{**}, workers invested in the up-or-out contract case.

Before we proceed we should introduce another result which we use in deriving the Proposition 3 and which is given in Lemma 4.

Lemma 4 : $c_1 + \Delta < c_2^*$.

The above Lemma implies that the relevant range of c_2, above which the case of promotion is relevant as discussed before (below which all the workers will always be kept in job 1) is necessarily below the cut-off in the promotion contract case above which the workers invest in specific human capital accumulation.

An explicit comparison of the promotion and the up-or-out contract case is given in Proposition 3.

Proposition 3 : *By comparing the cut-offs for the workers investment in both cases we get that $c^{**}_2 < c^*_2$. The different cases which arises depends on the following parametrizations.*

(i)   *If ($c_1 + \Delta < c^{**}_2 < c^*_2$) then for the region where $c_2 > c^*_2$, workers invest in both the regimes. Workers have a higher lifetime utility in the promotion contract regime in this case. When $c^{**}_2 < c_2 < c^*_2$, then workers invest only in the up-or-out case and have a higher lifetime utility than the promotion contract case. Lastly, when $c_2 < c^{**}_2$, then workers do not invest in either regimes and are indifferent amongst the two regimes.*

(ii)   *If ($c^{**}_2 < c_1 + \Delta < c^*_2$), then for the region where $c_2 > c^*_2$, workers invest in both the regimes. Workers are better off in the promotion contract regime in this*

case. When $c_2 < c^*_2$, then workers invest only in the up-or-out case and are better off than the promotion contract case.

Thus we indeed find that in certain circumstances an up-or-out contract is a better contractual device. The part (ii) of Proposition 3 is actually more interesting from an empirical stand point since that is what we generally observe in real circumstances, that is, jobs were the output is highly sensitive to the ability level ($c_2 > c^*_2$ in our model) we would observe promotion contracts while for jobs which are not as sensitive ($c_2 < c^*_2$ in our model) we would tend to observe up-or-out contracts.

Next we do the analysis with effort choice for the workers were effort might signal ability and we would analyze how that interacts with the promotion issue.

3. The Model with Effort Choice

In this section we consider the case where employees have a choice of whether to put in effort in the first period or not. For simplicity we assume that effort $e \in \{0, \bar{e}\}$ and $\bar{e} \in (0,1)$. The cost of effort is denoted by z. We also assume that effort has no effect on the output in the second period but influences output in the first period in the following way, that is, for a worker of ability η who exerts effort, $y = d_1 + c_1(\eta + e(\eta_H - \eta))$. Thus exerting effort augments the ability of the worker such that the effective ability ($\eta + e(\eta_H - \eta)$) remains within η_H. The motivation of modeling the effort process in this particular fashion is the following. It is reasonable to consider the number of mistakes a worker makes in the production process as a measure of ability. Thus a higher ability worker is less likely to make mistakes, which in turn affects his output, than a lower ability worker. For example, a worker of ability level η_H, will perform the job perfectly. So what the effort does is to reduce the number of mistakes that the worker is likely to perform. The above formulation captures this intuition quite succinctly. The employers can only observe output and from that infer the ability of the employees. Thus, by exerting high effort, the

employees try to bias the inference of the employer regarding the ability of the employee. Finally, in contrast to the previous section, the specific human capital accrues to the output of the employee in the second period, independent of the effort choice.

Summary of assumptions:

1) Employees put effort in the first period and $e \in \{0, \bar{e}\}$.

2) There is specific human capital accumulation if the employee is employed in the firm for the second period.

3) Employers can only observe output of the employee privately in the first period and then base their promotion decision on the basis of that.

4) The market observes the promotion decision and bids accordingly.

Before we proceed with the analysis, certain points needs to be mentioned upfront. Employers make the promotion decision by observing the output. The output is effected both by the ability of the worker and the effort exerted, in the manner as stated above. The significance of modeling the way effort affects the output is that the two ranges of the workers output corresponding to that with effort and without effort, overlaps. Thus the employer cannot decipher the true ability from the output. Hence effort serves a signaling device to the workers.

Another point to note is that in this case, there is a probability that workers can be promoted if they are of high enough ability even without effort provision. Specifically, this is true for workers above $\eta > (\eta^* + (\bar{e}(\eta_H - \eta^*)))$.[12] In this case the probability of getting promoted is given by $[(\eta_H - (\eta^* + (\bar{e}(\eta_H - \eta^*)))/(\eta_H - \eta_L)]$. So by investing in effort, the worker increases his chances of getting promoted by $[\{(\eta_H - \eta^*)/(\eta_H - \eta_L)\} - \{(\eta_H - (\eta^* + (\bar{e}(\eta_H - \eta^*)))/(\eta_H - \eta_L)\}]$. This simplifies to $[\{\bar{e}(\eta_H - \eta^*)/(\eta_H - \eta_L)\}]$.

As in the previous case, in the second period the market will bid $[d_2 + c_2((\eta^* + \eta_H)/2)]$ if the worker is promoted and $[d_1 + c_1((\eta^* + \eta_L)/2)]$ if the worker is kept in job 1. Now,

---

[12] This comes from the condition that a worker can be of high enough ability that the output produced is equal to the marginal worker who is promoted with effort provision. Specifically, $d_1 + c_1\eta = d_1 + c_1(\eta^* + \bar{e}(\eta_H - \eta^*))$, where η is the marginal ability worker who has the same output without effort provision as the marginal ability worker with effort provision.

20

knowing this, the workers decision whether to put effort or not comes from the following inequalities (as before):

If $[d_2+c_2((\eta^*+\eta_H)/2) -\{d_1+c_1((\eta^*+\eta_L)/2)\}] [\{\bar{e}(\eta_H-\eta^*)/(\eta_H-\eta_L)\}] > z$, then put effort, otherwise not.

Similarly for the Up-or-out case it is:

If $[d_2+c_2((\eta_H+\eta^{**})/2) -\{d_1+c_1((\eta_L+\eta^{**})/2)\}][\bar{e}(\eta_H-\eta^{**})/(\eta_H-\eta_L) > z$, the put effort, otherwise not. A formal proof of the above claims is given in Proposition 4.

Proposition 4: *Given employers form inferences about the workers ability from the output, and takes the promotion decision based on that, in equilibrium the workers behavior is characterized as follows:*

(i)   *If $[d_2+c_2((\eta^*+\eta_H)/2) -\{d_1+c_1((\eta^*+\eta_L)/2)\}] [\{\bar{e}(\eta_H-\eta^*)/(\eta_H-\eta_L)\}] > z$, then all the workers put effort.*

(ii)   *If $[d_2+c_2((\eta^*+\eta_H)/2) -\{d_1+c_1((\eta^*+\eta_L)/2)\}] [\{\bar{e}(\eta_H-\eta^*)/(\eta_H-\eta_L)\}] < z$, then none of the workers put effort.*

And η^* is given by (1). In the up-or-out case the corresponding inequalities are with η^{**}, where η^{**} is given by (8).

Analysis: Notice that even if the employer cannot observe the action of the worker directly, he can infer them from the inequalities and therefore, observing y, in equilibrium will be equivalent to observing η.

The above result is similar in spirit to Holmstrom (1999), since as long as ability is unknown there are returns to supplying labor, because output will influence perceptions about ability. By increasing its supply, the worker can potentially bias the process of inference in his favor. In equilibrium, this will not happen since the employer will know what effort level to expect and adjust the output measure accordingly. Thus the workers are trapped in supplying the equilibrium level that is expected of him if the cost of effort is not too high (see Proposition 4 above), because as in a rat race, a lower supply of labor will bias the evaluation procedure against him.

Before we proceed further, let us summarize our findings in the effort choice case, and contrast it with the investment in human capital case in the previous section. In

Section 1, since the human capital process was dependent on the investment by the workers, in the case where $c_2 < c_2^*$, none of the workers were promoted. Similarly for the up-or-out case it was for the parameter range $c_2 < c_2^{**}$ (Remember that with $\alpha = 1$, that is with no specific human capital accumulation, there was no promotion). But in Section 2, the human capital accrues irrespective of the effort choice decision of the workers. So there is always promotion. But in this case, for the parameter range $c_2 < c_2^*$, for the spot market case and $c_2 < c_2^{**}$, for the up-or-out case there is promotion with no effort provision in the first period. In the entire parameter range of c_2 the employer can correctly infer in equilibrium whether the agent exerted effort. Our next agenda, should be to do the welfare analysis for the workers. Proposition 5 summarizes the findings.

Proposition 5: *By comparing the cut-offs for the workers investment in both cases we get that $c^{**}_2 < c^*_2$. The different cases which arises depends on the following parametrizations.*

(i) *If $(c_1 + \Delta < c^{**}_2 < c^*_2)$ then for the region where $c_2 > c^*_2$, workers exert effort in the first period in both the regimes. Workers are better off in the spot contract regime in this case. When $c^{**}_2 < c_2 < c^*_2$, then workers exert effort only in the up-or-out case and are better off than the spot contract case. Lastly, when $c_2 < c^{**}_2$, then workers do not put effort in either regimes and are indifferent amongst the two regimes.*

(ii) *If $(c^{**}_2 < c_1 + \Delta < c^*_2)$, then for the region where $c_2 > c^*_2$, workers put effort in both the regimes. Workers are better off in the spot contract regime in this case. When $c_2 < c^*_2$, then workers exert effort only in the up-or-out case and are better off than the spot contract case.*

Analysis: In this case, the effort serves as a signal to bias the inference. And effort has a positive effect on output only in the first period. Hence, what makes the difference in the total lifetime utility of the workers is the wage in the first period. Because of the zero profit

condition of the firms, the case where workers provide effort, they get a higher wage in the first period.

Strictly speaking we should notify the cut-offs (represented by c_2) differently than the cut-offs in the case of the first section. This paragraph involves a discussion about that. The cut-offs in the effort case are higher than the human capital case. Mathematically it comes from the fact that the functions $\psi(c_2)$ and $\phi(c_2)$ from which we got the cut-offs in the first case are now $\bar{e}\,\psi(c_2)$ and $\bar{e}\,\phi(c_2)$ respectively. (One can observe this directly from the inequalities given in Prop 4) And since $\bar{e} \in (0,1)$, the values of c_2 in the spot market case and up-or-out case get respectively increased. Remember the equation from which we derived the cut-offs were $\psi(c_2) = z$ and $\phi(c_2) = z$ in each case. This has a very clear intuition. In the effort provision case there is a positive probability that the worker can be promoted even without providing effort in the first period. And thus, the workers needs to have more incentive (in the form a higher c_2) to invest in this case than the investment in human capital case.

4. Discussion

We have mentioned about the existence of up-or-out contracts in academic institutions, law firms, accounting firms and medical practitioner firms. The empirical work in this area is sparse and hence it is more of a stylized fact. Nonetheless it is worth mentioning about the relevant empirical work that has been done regarding up-or-out contracts. O'Flaherty and Siow (1995) in their study on law firms find that op-or-out rules operate as a screening device. Using data on New York law firms, they show that firm growth is a slow and uncertain process because performance as an associate is not an especially informative signal about whether a lawyer will make a good partner and because the costs of mistaken promotion are relatively high. The parameter estimates confirm to a number of restrictions that are needed for up-or-out contracts to be optimal.

Another interesting empirical study is that of Asch and Warner (2001) where they study the personnel policy of the United States military. The existence of up-or-out rules in the military is due to a separate reason. The military, for example, has a prespecified set of positions it must fill that is determined by technological considerations. Because it must promote someone to fill the upper-level vacancies, it has no incentive to incorrectly declare

good workers to be poor ones. No employer-side moral hazard problem exists. The authors argue that the existence of up-or-out rules in the military is due to lateral entry constraint. The lack of lateral entry forces the military to raise entry pay in an effort to attract a higher quality entry cohort that is capable of filling upper levels in the future. But heterogeneity in the entry cohort means that many individuals will be hired who are unsuitable in the upper-level positions even though they are perfectly suitable for the lower level tasks. As time goes by and promotion contests reveal such individuals to be unpromotable beyond some level, they may not separate voluntarily due to the fact that pay in the low ranks was set above within-rank productivity and contained a shadow value component. But the continued retention of unpromotable individuals imposes a shadow cost on the organization by reducing promotion opportunities of more junior, but more able, personnel. Without an up-or-out rule, the organization would have to raise pay in order to maintain the retention and effort of the more junior (but on the average more able) personnel. Because it is the lack of lateral entry that creates and enhances this shadow value, an up-or-out rule is more likely to be observed when lateral entry is not feasible.

5.Conclusion

In this paper we analyze the reasons as to why we observe different type of promotion contracts in firms. We first show under what circumstances a promotion might solve the hold-up problem associated with workers investment in training. We find that this is true when the higher level job is significantly more productive when a higher ability worker is assigned. This solves the hold-up problem in investing since workers now know that they will be promoted if they meet the cut-off level above which the employer promotes workers. On the other hand, it is now in the interests of the employer not to renege and promote workers efficiently since they will benefit from a correct assignment. Then we show why up-or-out contracts may be a better promotion mechanism than simple promotion contracts in jobs which are not that sensitive to worker productivity. Specifically in a simple promotion contract since workers have the option of keeping the worker to a lower level job they have an incentive to renege and not promote a worker after he undergoes training. Hence, he has to commit to something more drastic (such as dismiss the

worker if he is not promoted) in order to convince the worker that he will be promoted if eligible and make the worker invest.

We believe our framework has some very useful insights as to how we can capture some other contracting phenomena that we observe. For example, an extension of our basic model into a multi-period version (more than two periods) can help to explain the existence of tenure in academia). We believe the reason for tenure in academia may be because of the following reason. A person who works in an academic environment commits himself to some kind of specific investment. An academic certainly has lesser market value outside academia once he has worked in an academic institution as compared to say a corporate job. In a sense this reduces the person's general human capital. Hence the outside wage or the market wage will be less than the competitive wage offers in our model. Thus there is an added disincentive for a person in the academia to commit himself to such a situation. This can be averted if there is a tenure commitment by the employer (in this case the academic institution).[13] We believe that this might be a very fruitful area of future research.

---

[13] The standard explanation in the literature for the existence of tenure in academia is that of Carmichael (1983). His explanation is related to the hiring decision of the employer, where the people who hire new employees will face no threat of losing their job if they hire a very qualified person.

Appendix

Proof of Lemma 1: We need to prove that $\eta' < \eta^*$. Suppose not. That is, suppose $\eta' > \eta^*$. Our strategy in proving this lemma will be to show a contradiction in this case. We know that $\eta^* = \dfrac{2(\alpha-1)(d_1-d_2)+(c_2\eta_H - c_1\eta_L)}{2(c_2-c_1)(\alpha-1/2)}$ and $\eta' = (d_1-d_2)/(c_2-c_1)$. If $\eta' > \eta^*$, then we will get $(d_1-d_2)/(c_2-c_1) > \dfrac{2(\alpha-1)(d_1-d_2)+(c_2\eta_H - c_1\eta_L)}{2(c_2-c_1)(\alpha-1/2)}$. This simplifies to $(d_1-d_2) > c_2\,\eta_H - c_1\,\eta_L$. But from inequality (1) we know that $(d_1-d_2) < c_2\,\eta_H - c_1\,\eta_H$. And since $\eta_H > \eta_L$, $c_2\,\eta_H - c_1\,\eta_L > c_2\,\eta_H - c_1\,\eta_H$. Hence $(d_1-d_2) < c_2\,\eta_H - c_1\,\eta_L$, which leads to a contradiction.   #

Proof of Lemma 2: From $(1')$ we know that $\alpha(d_2+c_2\eta^*)-[\,d_2+c_2((\eta^*+\eta_H)/2)] = \alpha(d_1+c_1\eta^*) - [d_1+c_1((\eta^*+\eta_L)/2)]$. Just by applying the implicit function theorem we get $\partial\eta^*/\partial\,c_2 = -\eta^*/(c_2-c_1) < 0$. #

Proof of Proposition 1: From Lemma 2 we know that the cut-off ability workers who are promoted, η^*, varies with c_2. We get the cut-off c_2 (i.e c_2^*) from equation 3, which gives the marginal condition for the workers investing decision in the promotion contract case. Note that this is dependent on the costs of investment which is z.

$$[d_2+c_2((\eta^*+\eta_H)/2) - \{d_1+c_1((\eta^*+\eta_L)/2)\}]\,[\{(\eta_H-\eta^*)/(\eta_H-\eta_L)\} - \{(\eta_H-\alpha\eta^*)/(\eta_H-\eta_L)\}] = z$$

Because of the condition on α which ensures that workers are not fired, we know $\{(\eta_H-\alpha\eta^*)/(\eta_H-\eta_L)\}=0$. We can write the left hand side of the above equation with ϕ as:

$$\phi = [d_2+c_2((\eta^*+\eta_H)/2) -d_1+c_1((\eta^*+\eta_L)/2)\,]\,[\{(\eta_H-\eta^*)/(\eta_H-\eta_L)\}]$$

Using equation (1) we can simplify this further to:
$$\phi = \alpha\,[d_2+c_2\,\eta^* -\{d_1+c_1\,\eta^*\}]\,[\{(\eta_H-\eta^*)/(\eta_H-\eta_L)\}]$$
$$\text{or, } \phi = \alpha\,[d_2 -d_1+(c_2-c_1)\,\eta^*]\,[\{(\eta_H-\eta^*)/(\eta_H-\eta_L)\}]$$

Differentiating the above expression with respect to c_2 we get:

$$\partial\phi/\partial\, c_2 = [(c_2 - c_1)\, \alpha\;\; \partial\eta^*/\partial c_2 + \alpha\, \eta^*].\, X_1 + [\{(-1).\, \partial\eta^*/\partial c_2\}\, /\;\; \eta_H - \eta_L]\,.\, X_2$$

where $X_1 = [\{(\eta_H - \eta^*\,)/(\, \eta_H - \eta_L)\}]$

and $X_2 = \alpha\, [d_2 - d_1 + (c_2 - c_1)\, \eta^*]$

We have shown that $\partial\eta^*/\partial\, c_2 = -\, \eta^*/\, (c_2 - c_1)$

Therefore, $(c_2 - c_1)\, \partial\eta^*/\partial\, c_2 = -\, \eta^*$.

So the first term of $\partial\phi/\partial\, c_2$ is zero, and X_1 and X_2 are positive.

Thus $\partial\phi/\partial\, c_2 > 0$.

And since ϕ is a continuous function and $\phi(c_2{}')< z < \phi(c_2{}'')$, we know from the Intermediate Value Theorem (IVT) that there exists a $c^*{}_2$ for which $(\phi(.)-z)$ is equal to zero. #

Proof of Proposition 2: This proof is similar to the proof of proposition 1. The corresponding equation for the workers decision to invest is given by equation (10).

$$[d_2 + c_2((\eta_H + \eta^{**})/2 - \{d_1 + c_1((\eta_L + \eta^{**})/2)\}][(\eta_H - \eta^{**}\,)/(\, \eta_H - \eta_L)] = z.$$
$$\psi = [d_2 + c_2((\eta_H + \eta^{**})/2 - \{d_1 + c_1((\eta_L + \eta^{**})/2)\}][(\eta_H - \eta^{**}\,)/(\, \eta_H - \eta_L)].$$

Using (7) and the restriction on α, we can simplify the above equation as

$$\psi = [(d_2 - d_1) + \{(c_2 - c_1)\eta^{**}\}/2) + c_2\eta^{**}/2)][(\eta_H - \eta^{**}\,)/(\, \eta_H - \eta_L)].$$
$$\partial\psi/\partial\, c_2 = [(c_2 - c_1)\, /2)\, \partial\,\eta^{**}/\partial c_2 + (\eta^{**}\, /2) + c_2\, (\eta^{**}\, /2)].Y_1 + [\{(-1).\, \partial\,\eta^{**}/\partial c_2\}/\eta_H - \eta_L]\,.\, Y_2$$

We know that $\partial\,\eta^{**}/\partial c_2 = -\, \eta^{**}/\, c_2$. Hence $(c_2 - c_1)\, \partial\,\eta^{**}/\partial c_2 < \eta^{**}$. And since Y_1, Y_2 are positive we prove that $\partial\psi/\partial\, c_2 > 0$.

By applying the IVT as above we get a cut-off $c^{**}{}_2$ such that workers invest for values of c_2 above $c^{**}{}_2$. #

27

Proof of Lemma 3:

$\eta^* = \dfrac{2(\alpha-1)(d_1-d_2)+(c_2\eta_H-c_1\eta_L)}{2(c_2-c_1)(\alpha-1/2)}$ and $\eta^{**} = \dfrac{c_2(\eta_H/2)-2(\alpha-1)d_2}{2c_2(\alpha-1/2)}$. A direct

comparison of the denominators yields that $2(c_2-c_1)(\alpha-1/2)< 2(c_2)(\alpha-1/2)$. Now if

we prove that the numerator of η^* is larger than that of η^{**}, then we are done. Our strategy

in showing this would be to posit that it is otherwise and then find a contradiction. So

suppose, $2(\alpha-1)(d_1-d_2)+(c_2\eta_H-c_1\eta_L)< c_2(\eta_H/2)-2(\alpha-1)d_2$. After simplification

this comes to $c_2\dfrac{\eta_H}{2}+2(\alpha-1)d_1 <c_1\eta_L$. By using the assumption which we made before,

that is, $c_2 > c_1 + \Delta$, where $\Delta = (d_1 - d_2)/\eta_H$, we can reduce the above inequality further to

$c_1\dfrac{\eta_H}{2}+2(\alpha-1)d_1+\dfrac{d_1-d_2}{2}<c_1\eta_L$. Given $d_1>d_2$, and $\alpha>1$, we know that this is impossible.

Hence, the numerator is larger for η^*. This proves the claim that : $\eta^{**}< \eta^*$. #

Proof of Lemma 4:

We have to show that $(c_1 + \Delta) < c^*_2$, where $\Delta = (d_1 - d_2)/\eta_H$. Now suppose this is not

true. Then $(c_1 + \Delta) > c^*_2$. Consider values of c_2, where $c_2 \in (c^*_2, (c_1 + \Delta))$. Because in that

region $c_2<(c_1 + \Delta)$ all the workers will necessarily be kept in job 1 (remember the

significance of the term $(c_1 + \Delta)$, where for values of c_2 below this, even the highest ability

worker η_H is kept in job 1). In that case the left hand side of equation (3) in the text

becomes zero (none of the workers are promoted and so $\eta^* = \eta_H$). But we know that z is

positive. Hence there does not exist a c^*_2 in this case. Hence $(c_1 + \Delta) > c^*_2$, is not possible.

Thus it must be the case that $(c_1 + \Delta) < c^*_2$, which proves our result.   #

Proof of Proposition 3:

<u>Step 1</u>

First we have to show that $c^{**}_2 < c^*_2$. The defining equations from which we get c^{**}_2 and

c^*_2 are (10) and (3) respectively.

Rewriting the simplified forms of these two equations (see proofs of proposition 1 and 2

above)

$\psi = [d_2+c_2((\eta_H+\eta^{**})/2 -\{d_1+c_1((\eta_L+\eta^{**})/2)\}][(\eta_H- \eta^{**})/(\eta_H- \eta_L)].$

$\phi = [d_2+c_2((\eta^{*}+\eta_H)/2) -\{d_1+c_1((\eta^{*}+\eta_L)/2)\}] [\{(\eta_H- \eta^{*})/(\eta_H- \eta_L)\}]$

Step 2

Now, to show that $c^{**}_2 < c^{*}_2$, it is sufficient to show that for all values of c_2, $\phi(c_2) <$ $\psi(c_2)$. This is because from Proposition 1 and Proposition 2 we know that both $\partial\phi/\partial c_2$ and $\partial\psi/\partial c_2$ are positive. So starting from a very low c_2, where both ψ and ϕ are negative, if we increase c_2 then the above condition will give us the result about the value of c_2 (namely, c_2^{**} and c_2^{*}), where each of ψ and ϕ becomes zero.

First let us expand $\psi(c_2)$.

$\psi(c_2)= (d_2 -d_1)\{(\eta_H- \eta^{**})/(\eta_H- \eta_L)\} + 1/ 2(\eta_H- \eta_L)[c_2. (\eta_H)^2 - (c_2 - c_1) (\eta^{**})^2 - c_1 \eta_H \eta_L - c_1(\eta_H- \eta_L) \eta^{**}].$

$\phi(c_2) = (d_2 -d_1)\{(\eta_H- \eta^{*})/(\eta_H- \eta_L)\} + 1/ 2(\eta_H- \eta_L)[c_2. (\eta_H)^2 - (c_2 - c_1) (\eta^{*})^2 - c_1 \eta_H \eta_L - c_1(\eta_H- \eta_L) \eta^{*}].$

By using the fact that $\eta^{**}< \eta^{*}$, by comparing the above two expressions we get the result that $\phi(c_2) < \psi(c_2)$.

Thus it is proved that $c^{**}_2 < c^{*}_2.$

Step 3

Lastly we have to compare the welfare of the workers in each of the parameter spaces. In order to prove both parts (i) and (ii) we follow the following strategy. We first prove (i) entirely, i.e., for the entire parameter range for c_2. Part (ii) is just a part of (i) since here we neglect the region where $c_2< c_1 + \Delta$. In this region there is no issue of promotion as is shown before. Hence the parameter range of c_2, consists of just two zones.

Firstly, we show that in the parameter space where workers invest in both regimes then the workers are better off in the promotion contract regime. For this we compare

equation (5) with equation (11). Equation (5) is the life-time utility of the workers in the promotion contract case when they invest and equation (11) is the corresponding expression for the up-or-out contract case.

$U_I{}^P = d_1 + c_1((\eta_L + \eta_H)/2) + X^1 \alpha [d_2 + c_2(\eta_H + \eta^*)/2)] + \alpha (1 - X^1) [d_1 + c_1((\eta_L + \eta^*)/2)] - z$

$U_I{}^U = U^U = d_1 + c_1((\eta_L + \eta_H)/2) + \alpha Y^1 [d_2 + c_2(\eta_H + \eta^{**})/2)] - z.$

A comparison of the above two expressions shows that the lifetime utility of the workers in the promotion contract regime is higher in this parameter range.

Secondly for the parameter space where workers invest in the up-or-out contract ,but not in the spot contract case (where, $c_2{}^{**} < c_2 < c_2{}^*$), we compare $U^P{}_N$ and $U^U{}_I$ .It can be easily shown that $U^P{}_N < U^U{}_I$.

Lastly for the parameter space where workers invest in none of the regimes we know that both $U^P{}_N$ and $U^U{}_N$ are given by $2(d_1 + c_1((\eta_L + \eta_H)/2))$. Hence workers are indifferent for both the spot contract and up-or-out contract case. #

Proof of Proposition 4: The equilibrium that we are supposed to verify is the following: all workers exert effort and the firms equilibrium strategy is to promote workers whose realized output is greater than that corresponding to output for an ability level η^*. Given the firms' strategy the worker's best response will be such that he will invest given the condition $[d_2 + c_2((\eta^* + \eta_H)/2) - \{d_1 + c_1((\eta^* + \eta_L)/2)\}] [\{\bar{e} (\eta_H - \eta^*)/(\eta_H - \eta_L)\}] > z$ holds. What this says is that the expected increased benefit of exerting effort is greater than the disutility of effort. Remember in this case there is always a positive probability that the worker will get promoted even if he does not exert effort. The condition given above is precisely the one given in the proposition. Now, from the firms' side, we have to check whether promoting workers above ability level η^* is in fact their best strategy. Notice that even though the employer is not able to observe the workers ability directly, it is able to infer them by solving the above equation. Therefore, observing output, will in equilibrium be equivalent to observing ability. Then firms decision rule is exactly the same as that of the previous model where they could observe ability, and we know that the solution to that is they promote workers above ability η^*. The corresponding equation in the up-or-out case is given by (8). #

Proof of Proposition 5: In order to prove both parts (i) and (ii) we follow the following strategy. We first prove (i) entirely, i.e., for the entire parameter range for c_2. Part (ii) is just a part of (i) since here we neglect the region where $c_2 < c_1 + \Delta$. In this region there is no issue of promotion as is shown before. Hence the parameter range of c_2, consists of just two zones.

(i) Firstly, we show that in the parameter space where workers exert effort in the first period in both regimes then the workers are better off in the spot contract regime. For this we have to find the wage that is paid in the first period to the workers. As before we use the next expected profits of zero for the firms to derive that.

Specifically,

$W^1 + X^1(d_2 + c_2((\eta_H + \eta^*)/2)) + (1-X^1)(d_1 + c_1((\eta_L + \eta^*)/2)) = d_1 + c_1\{((\eta_L + \eta_H)/2) + \bar{e}((\eta_H - \eta_L)/2)\} + X^1\alpha(d_2 + c_2((\eta_H + \eta^*)/2)) + (1-X^1)\alpha(d_1 + c_1((\eta_L + \eta^*)/2))$.

$W^1 = d_1 + c_1((\eta_L + \eta_H)/2) + X^1(\alpha-1)[d_2 + c_2((\eta_H + \eta^*)/2)] + (1 - X^1)(\alpha-1)[d_1 + c_1(\eta_L + \eta^*)/2)] + c_1\{\bar{e}((\eta_H - \eta_L)/2)\}$.

Note that the wage is the same as in section 1 but only the effort augmenting factor $(c_1\{\bar{e}((\eta_H - \eta_L)/2)\})$ is added to the wage since effort has a positive effect on output in period 1. Similarly in the up-or-out case:

$W^1 = d_1 + c_1((\eta_L + \eta_H)/2) + Y^1(\alpha-1)(d_2 + c_2((\eta_H + \eta^{**})/2)) - (1-Y^1)(d_1 + c_1(\eta_L + \eta^{**})/2)) + c_1\{\bar{e}((\eta_H - \eta_L)/2)\}$.

By comparing the life time utility in both cases:

$U_I^P = d_1 + c_1((\eta_L + \eta_H)/2) + X^1\alpha[d_2 + c_2(\eta_H + \eta^*)/2)] + \alpha(1 - X^1)[d_1 + c_1((\eta_L + \eta^*)/2)] + c_1\{\bar{e}((\eta_H - \eta_L)/2)\} - z$

$U_I^U = U^U = d_1 + c_1((\eta_L + \eta_H)/2) + \alpha Y^1[d_2 + c_2(\eta_H + \eta^{**})/2)] + c_1\{\bar{e}((\eta_H - \eta_L)/2)\} - z$.

Thus it is basically the same as in Proposition 3 except the term $c_1\{\bar{e}((\eta_H - \eta_L)/2)\}$ has been added. A comparison of the above two expressions shows that the lifetime utility of the workers in the spot contract regime is higher in this parameter range.

Secondly for the parameter space where workers exert effort in the up-or-out contract but not in the spot contract case (where, $c_2^{**} < c_2 < c_2^{*}$), we compare U^s_N and U^u_I.

$U^P_N = d_1 + c_1((\eta_L + \eta_H)/2) + X^1\alpha [d_2 + c_2(\eta_H + \eta^*)/2)] + \alpha(1 - X^1)[d_1 + c_1((\eta_L + \eta^*)/2)]$.

$U^u_I = d_1 + c_1((\eta_L + \eta_H)/2) + \alpha Y^1 [d_2 + c_2(\eta_H + \eta^{**})/2)] + c_1\{\bar{e}((\eta_H - \eta_L)/2)\} - z$.

From the above two expressions it is evident that $U^P_N < U^U_I$, if $\bar{e} > (2z)/\{c_1(\eta_H - \eta_L)\}$.

Lastly for the parameter space where workers invest in none of the regimes we know that both U^P_N and U^U_N are given by $2(d_1 + c_1((\eta_L + \eta_H)/2))$. Hence workers are indifferent for both the promotion contract and up-or-out contract case.

References

1) Asch, B.J., and J. Warner. 2001, "A Theory of Compensation and Personnel Policy in Hierarchical Organizations with Application to the United States Military", *Journal of Labor Economics,* vol 19, no.3: 523-562.

2) Baker, G.P, M.C.Jensen, and K.C. Murphy. 1988, "Compensation and incentives: practice vs. theory", *Journal of Finance,* 43: 593-616.

3) Bernheim, B.D., B. Peleg and M. Whinston. 1987, "Coalition- Proof Nash Equilibria I. Concepts", *Journal of Economic Theory,* XLII: 1-12.

4) Carmichael, H.L., 1983, "Incentives in Academics: Why is there tenure?" *Journal of Political Economy,* 96: 453-472.

5) Fairburn, J.A. and J.M. Malcolmson. 1997, "Performance, Promotion and the Pater Principle", *mimeo* (University of Sussex), 43: 593-616.

6) Gibbs, M. 1995, "Incentive compensation in a corporate hierarchy", *Journal of Accounting and Economics,* 19: 247-277.

7) Gibbons, R and M. Waldman. 1999b,"A Theory of Wage and Promotion Dynamics Inside Firms", *Quarterly Journal of Economics,* 114: 1321-1358.

8) Greenwald, B. 1986, "Adverse Selection in the Labor Market", *Review of Economic Studies* 53: 325-347.

9) Holmstrom, B., 1999, "Managerial Incentive Problems: A Dynamic Perspective", *Review of Economic Studies* 66: 169-182.

10) Kahn, C and G. Huberman, 1988, "Two-sided uncertainty and 'up-or-out' contracts", *Journal of Labor Economics* 6: 423-444.

11) Lazear, E. 1986, "Raids and Offer Matching" in *Research in Labor Economics*, vol.8, ed. Ronald Ehrenberg, 141-165, JAI Press, Greenwich, CT.

12) Lazear, E. 1991, "Pay Equality and the Psychology of Organizations", *Journal of Economic Perspectives* 5: 561-580.

13) Levin, J and S. Tadelis, 2002, "A Theory of Partnerships", *mimeo* (Stanford University).

14) Malcomlson, J.M., 1984, "Work incentives, hierarchy, and internal labor markets", *Journal of Political Economy*, 92: 486-507.

15) Milgrom, P. and S. Oster, 1987, "Job discrimination, market forces, and the invisibility hypothesis", *Quarterly Journal of Economics,* 102: 453-476.

16) O' Flaherty, B. and A. Siow, 1995, "Up-or-Out rules in the Market for Lawyers", *Journal of Labor Economics,* vol 13, no. 4: 709-735.

17) Ricart I Costa, J., 1988, "Managerial task assignments and promotion", *Econometrica* 56: 449-466.

18) Sattinger, M., 1975, "Comparative Advantage and the distribution of earnings and abilities", *Econometrica* 43: 455-468.

19) Waldman, M. 1984, "Job Assignments, Signaling and Efficiency", *Rand Journal of Economics,* 15:255-267.

20) Waldman, M. 1990. "Up-Or-Out Contracts: A Signaling Perspective", *Journal of Labor Economics*, 8: 230-250.