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Business Cycles in Italy: The Facts and Some Models

Abstract

This paper applies several RBC models to the Italian economy to see whether they can explain the aggregate fluctuations observed in the data. The performance of these models is not too unsuccessful, but it depends crucially on the parametrization chosen and on the form of the utility function, in particular, the intertemporal elasticities of substitution of labour and consumption both affect the short-run dynamics and the volatilities generated by the model. The paper also analyses the introduction of more than one shock, considering Italy as a small open economy subject to exogenous interest rate shocks.
However, there are still some remaining difficulties, in particular in the labour market. Hours in Italy fluctuate more than employment, as opposed to the US, where the opposite happens. The Italian labour market is more regulated than the US one, and it is costlier for firms to adjust the number of employees during recessions. As a consequence, a lot of labour adjustment takes place along the intensive margin. Therefore, the original model is modified in two ways (i) by introducing adjustment costs of the labour force and by considering both hours and participation decisions; (ii) by introducing an underground sector. Both these modifications increase the elasticity of hours in the model, and thus they may provide an explanation for the observed cyclical behaviour of labour supply in the Italian economy.

JEL classification numbers: E32, J22
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Introduction

Since the early 1980s, one of the main challenges of modern economic research has been understanding, and possibly reproducing in a model, the aggregate fluctuations of the main macroeconomic variables. The so-called real business cycle theory (RBC) hinges on both neoclassical theory and dynamic general equilibrium theory to accomplish this task[2]. However, the set of stylized facts that RBC theory tries to mimic is not space-independent, since the cyclical behaviour of the main macroeconomic variables can be strikingly different across countries. This paper tries to ascertain whether a standard RBC model with separable preferences in consumption and leisure can explain the aggregate fluctuations in the Italian economy, whether in its closed economy form, or in the small open economy version.

It is recognisable that the remaining difficulties of RBC theory in reproducing Italian business cycles may be due to its institutional settings, especially in the labour market, where a departure from a purely Walrasian framework seems particularly promising. Therefore, the original model is modified by considering adjustment costs of the labour force as a source of labour rigidity. Agent heterogeneity is also introduced, as a necessary step to filter out the two sources of fluctuations in the labour input, hours and employment movements.

The paper is organised as follows. Section I reports the main facts of the Italian business cycles as a set of statistics extensively used in the RBC literature, and with the aim to have a definite benchmark for comparison. Section II presents and evaluates a standard RBC model, section III models Italy as a small open economy, and Section IV considers the same model with labour market imperfections and unemployment. Section V summarises and concludes.

I. The Main Facts of Business Cycles in Italy and the US

The RBC literature has focused on the United States most of the times, so it comes as no surprise that there are extensive accounts of the cyclical properties of the US economy[3]. Given the resurgent interest for the DSGE technique, such accounts can also be found for other countries, but statistics for the Italian economy are not as abundant as for the US. Moreover, while the main macroeconomic time series are available for the Italian economy, there is little information on hours worked in Italy. However, information on hours is really needed, otherwise it is meaningless to test almost all RBC models.

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[2] Since the early work of Kydland and Prescott (1982) the framework has been considerably expanded to account for monetary shocks as well as real shocks, in models with or without Keynesian features such as wage or price rigidities. As a result, the adjective “real” does not apply any more, and many people now prefer to refer to this approach as dynamig stochastic general equilibrium models (DSGE).

[3] A recent and very detailed one is provided by Stock and Watson (1999).

Apart from employment and hours worked, all the statistics for both Italy and the US are calculated using OECD quarterly data for the period 1970(I) to 1998(III), given that earlier data for Italy are not available from the OECD. The uniformity of the source for both countries ensures safe comparisons, which is the ultimate aspiration of this paper as it is certainly the case for the issuing body of those statistics. The OECD does not publish data on hours worked, therefore hours worked in the US come from the Bureau of Labor Statistics and are based on the Establishment Survey. Hours worked in Italy come from Istat[4] and refer to blue-collar workers in manufacturing firms with more than 5,000 employees. Given that those are the only data on hours worked in Italy, they are taken as a proxy for hours worked by all employed people. Wages and productivity calculations are therefore not entirely based on OECD data. In addition to this, employment data for Italy does not come from the OECD but from the Bank of Italy[5].

After the work of Kydland and Prescott (1990), a standard procedure for defining the empirical regularities of business cycle fluctuations that the model tries to replicate has taken ground in the literature. Kydland and Prescott emphasised that the choice of facts to report should be guided by neoclassical theory, and they advocated the provision of the following information for each variable: amplitude of fluctuations (standard deviation), degree of co-movement with GDP (contemporaneous cross correlation), and phase shift (cross correlation at different lags and leads). Another feature is the degree of persistence (first-order autocorrelation) in a series. In short, DSGE theory is interested in second-order moments only, leaving aside the issue of identifying the underlying trend along which the series fluctuate[6]. In addition to these statistics, international real business cycle (IRBC) literature provides information on the co-movements of macroeconomic variables across countries, as they are the focus of the attention in two-country models.

This paper does what Kydland and Prescott[7] did in their paper for the US, trying therefore to fill the informational gap for the Italian economy. It is also necessary an explicit comparison with the US, since models that proved successful in dealing with American

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[4] They are published first in the Indicatori Mensili series, then in Bollettino Mensile di Statistica. Data available and fit for use are from 1973(I) to 1995(IV) only.

[5] The variability of the employment data from the Bank of Italy is in line with what can be found on Italy in a few RBC studies, but employment data from the OECD displayed a disproportionately high volatility. The problem comes probably from the fact that the OECD uses standard units of labour for Italy only.

[6] The sort of decomposition used is not an irrelevant issue, but it can naturally affect the results obtained. The HP filter is the standard choice with many advantages, not least comparability with similar studies.

[7] Accounts of business cycle regularities for other countries can be found in Blackburn and Ravn (1992), and Christodoulakis, Dimelis and Kollintzas (1994), and especially in Ravn (1997) for business cycles at the international level.

fluctuations might not produce desirable results when the economy under scrutiny is another one. For the sake of neatness of the exposition, data on aggregate variables are grouped into two approximate categories: those referring to production and those referring to expenditure or demand components. Contemporaneous and non-contemporaneous co-movement of selected series for Italy ad the US are reported separately. In the light of Blackburn and Ravn's remark that properties of the data constitute empirical regularities only if they are invariant over time, some tests of stability of variance are also conducted.

All statistics refer to seasonally adjusted data, expressed in constant 1990 prices, detrended using the HP filter, with smoothing parameter equal to 1,600. Except for the current account and inventory investments, which are expressed as ratios to GDP, data used are in logarithms. A description of the definitions used and the source of the data is in Table 7.

1. *Production inputs*

Tables 1 and 2 report the main statistics that describe the cyclical properties of production inputs: standard deviations in percentage and relative to output, first-order autocorrelations, and cross-correlations with output at different lags and leads. A positive sign indicates that the series is procyclical, a negative sign indicates that the series is countercyclical, a number close to zero indicates that the series has no correlation with the cycle. If the maximum correlation is reached at time t + i (t - i), the cyclical component of the series tends to lag (lead) the cycle by i quarters. The correlation is "strong"

Table 1: Italy

	% st dev	*Rel st. dev.*	*1-st ord AC*	*Cross-correlation of output at time t with*										
				x(t-5)	*x(t-4)*	*x(t-3)*	*x(t-2)*	*x(t-1)*	*x(t)*	*x(t+1)*	*x(t+2)*	*x(t+3)*	*x(t+4)*	*x(t+5)*
GDP	1.44	1.00	0.84	-0.24	-0.04	0.23	0.56	0.84	1.00	0.84	0.56	0.23	-0.04	-0.24
Capital Stock	0.38	0.26	0.91	-0.56	-0.61	-0.57	-0.44	-0.19	0.12	0.39	0.56	0.61	0.56	0.45
Labour input:														
Employment	0.81	0.56	0.74	-0.53	-0.47	-0.29	-0.08	0.13	0.32	0.46	0.50	0.49	0.40	0.28
Per capita hours.	2.94	2.04	0.54	0.01	0.09	0.22	0.38	0.56	0.67	0.41	0.13	-0.07	-0.17	-0.32
Earnings:														
Wages per capita	1.82	1.26	0.33	-0.07	-0.01	0.12	0.24	0.32	0.33	0.22	0.09	-0.01	-0.05	-0.03
Wages per hour	2.95	2.05	0.50	-0.08	-0.11	-0.15	-0.21	-0.33	-0.44	-0.27	-0.08	0.08	0.16	0.33
Labour productivity:														
Output per capita	1.40	0.97	0.78	0.06	0.23	0.40	0.62	0.78	0.84	0.60	0.28	-0.05	-0.27	-0.40
Output per hour	2.51	1.74	0.39	0.03	0.02	-0.04	-0.12	-0.25	-0.35	-0.14	0.04	0.11	0.09	0.22

("weak") if the correlation coefficient is greater than or equal to 0.5 (between 0.2 and 0.5).

The series for capital is constructed using the perpetual inventory method described in

Barro and Sala-i-Martin (1995). Data on labour productivity and wages at quarterly frequencies are not available from the OECD, therefore they have been derived using, respectively, the series for GDP and compensation of employees, both in the national accounts. Compensation of employees is adjusted to take into account the earnings of self-employed workers, considered as entrepreneurial income[8] by the OECD. Only an approximate adjustment method can be adopted, and the explanation is in Table 7.

By looking at cross-correlations at different leads and lags, it can be inferred that cycles in the US have longer duration than in Italy. The volatility of the cyclical component of output is bigger in the US than in Italy, and the first-order

Table 2: USA

	% st dev	*Rel st. dev.*	*1-st ord AC*	*Cross-correlation of output at time t with*										
				x(t-5)	*x(t-4)*	*x(t-3)*	*x(t-2)*	*x(t-1)*	*x(t)*	*x(t+1)*	*x(t+2)*	*x(t+3)*	*x(t+4)*	*x(t+5)*
GDP	1.70	1.00	0.87	0.06	0.27	0.48	0.69	0.87	1.00	0.87	0.69	0.48	0.27	0.06
Capital Stock	0.43	0.25	0.94	-0.49	-0.40	-0.28	-0.10	0.11	0.36	0.56	0.71	0.80	0.83	0.79
Labour input:														
Employment	1.31	0.77	0.65	-0.09	0.07	0.24	0.40	0.56	0.71	0.76	0.69	0.56	0.43	0.30
Per capita hours.	0.39	0.23	0.74	0.10	0.24	0.39	0.57	0.70	0.75	0.54	0.26	0.05	-0.16	-0.30
Earnings:														
Wages per capita	1.25	0.74	0.87	0.53	0.62	0.65	0.65	0.58	0.45	0.25	0.06	-0.13	-0.31	-0.44
Wages per hour	1.10	0.65	0.82	0.57	0.62	0.60	0.53	0.41	0.24	0.10	-0.03	-0.17	-0.29	-0.39
Labour productivity:														
Output per capita	1.04	0.61	0.34	0.18	0.26	0.33	0.41	0.45	0.45	0.25	0.15	0.05	-0.05	-0.17
Output per hour	1.20	0.71	0.47	0.18	0.30	0.42	0.54	0.62	0.64	0.39	0.22	0.06	-0.10	-0.25

autocorrelation is not very different. As it is a stock variable, capital is characterised by the lowest volatility and the highest autocorrelation. The cross correlations of output and capital in the two countries have similar patterns but are not easy to interpret.

Employment is much more volatile, less persistent and more procyclical in the US than in Italy. Employment lags the cycle in both countries. The behaviour of hours also differs a lot

[8] There is of course a flavour of subjectivity in this interpretation, but without it the profit share in the US / Italy is too low / high for any explanation to be convincing. An explanation relies in fact on the imputed earnings of the self-employed.

between the two countries, however it has been already pointed out that employment and hours data do not come from a common source. Hours seem to be a lot more volatile, less persistent and less procyclical in Italy than in the US. Not surprisingly, since data on earnings and average labour productivity are derived from data on employment and hours, statistics on earnings and average labour productivity also differ a lot between the two countries. Wages per hour and output per hour are anticyclical in Italy, but not wages and output per employed. However, the different behaviour of hours in Italy from the US is so remarkable that some more information on the construction and definition of the data is certainly needed before making any conclusion.

2. *Expenditure components*

Private consumption is more volatile in the US and a little less persistent than in Italy. The cyclical component of consumption leads the cycle in the US, while in Italy the highest autocorrelation is the contemporaneous one. The behaviour of government consumption is remarkably different in the two countries. Deviations from trend are far more volatile and less

Table 3: Italy

	% st dev	*Rel st. dev.*	*1-st ord AC*	*Cross-correlation of output at time t with*										
				x(t-5)	*x(t-4)*	*x(t-3)*	*x(t-2)*	*x(t-1)*	*x(t)*	*x(t+1)*	*x(t+2)*	*x(t+3)*	*x(t+4)*	*x(t+5)*
GDP	1.44	1.00	0.84	-0.24	-0.04	0.23	0.56	0.84	1.00	0.84	0.56	0.23	-0.04	-0.24
Consumption:														
Private consumption	1.25	0.87	0.90	-0.19	0.04	0.28	0.52	0.71	0.80	0.77	0.62	0.41	0.22	0.07
Government cons.	0.54	0.37	0.75	0.32	0.28	0.23	0.10	0.01	-0.06	-0.09	-0.12	-0.12	-0.11	-0.04
Total consumption	0.97	0.68	0.91	-0.14	0.07	0.31	0.52	0.69	0.78	0.74	0.59	0.39	0.20	0.06
Investment:														
Total investment	6.92	4.81	0.76	-0.32	-0.16	0.10	0.43	0.73	0.89	0.76	0.52	0.20	-0.08	-0.27
Fixed investment	3.66	2.55	0.90	-0.20	-0.04	0.17	0.40	0.63	0.78	0.78	0.68	0.50	0.32	0.15
Inventory inv. / GDP	1.18	0.82	0.66	-0.36	-0.24	0.00	0.32	0.61	0.73	0.56	0.28	-0.06	-0.35	-0.50
Trade:														
Current account / GDP	1.20	0.83	0.77	0.44	0.27	0.05	-0.21	-0.45	-0.58	-0.60	-0.54	-0.37	-0.17	-0.04
Exports	3.94	2.74	0.53	0.15	0.17	0.23	0.28	0.33	0.32	0.17	-0.07	-0.24	-0.33	-0.37
Imports	4.80	3.34	0.67	-0.03	0.13	0.31	0.49	0.67	0.73	0.63	0.40	0.13	-0.11	-0.25

Table 4: USA

	% st dev	Rel st. dev.	1-st ord AC	Cross-correlation of output at time t with										
				x(t-5)	*x(t-4)*	*x(t-3)*	*x(t-2)*	*x(t-1)*	*x(t)*	*x(t+1)*	*x(t+2)*	*x(t+3)*	*x(t+4)*	*x(t+5)*
GDP	1.70	1.00	0.87	0.06	0.27	0.48	0.69	0.87	1.00	0.87	0.69	0.48	0.27	0.06
Consumption:														
Private consumption	1.37	0.81	0.88	0.34	0.53	0.68	0.81	0.89	0.87	0.69	0.51	0.30	0.07	-0.11
Government cons.	0.82	0.48	0.58	-0.01	-0.03	-0.09	-0.12	-0.14	-0.07	0.00	0.09	0.19	0.31	0.37
Total consumption	1.07	0.63	0.87	0.34	0.52	0.66	0.79	0.86	0.86	0.69	0.52	0.33	0.12	-0.04
Investment:														
Total investment	6.65	3.92	0.83	0.11	0.27	0.45	0.63	0.79	0.93	0.81	0.60	0.37	0.15	-0.09
Fixed investment	4.73	2.79	0.90	0.13	0.30	0.48	0.68	0.85	0.95	0.87	0.71	0.52	0.30	0.09
Inventory inv. / GDP	0.56	0.33	0.48	0.05	0.13	0.26	0.34	0.46	0.61	0.46	0.24	0.01	-0.15	-0.35
Trade:														
Current account / GDP	0.45	0.27	0.78	-0.53	-0.49	-0.48	-0.46	-0.46	-0.44	-0.38	-0.22	-0.06	0.07	0.22
Exports	4.39	2.59	0.78	-0.54	-0.44	-0.30	-0.12	0.10	0.33	0.45	0.52	0.53	0.54	0.47
Imports	5.34	3.15	0.81	0.35	0.43	0.54	0.63	0.74	0.80	0.72	0.52	0.30	0.08	-0.17

persistent in the US than in Italy. The cyclical component of government expenditures leads the cycle in Italy and exhibit negative correlations at t + i, while in the US there is almost no significant relation with the cycle except at the higher leads. Further interpretation of the cyclical behaviour of government expenditure in the two countries requires knowledge of the political-economic cycle, which could differ between the two.

Total investment is the expenditure component with the highest degree of volatility in the two countries. Much of the volatility comes from inventories, since total investments (that are given by fixed investments plus inventories, see data definitions in Table 7) are a lot more volatile than fixed investments[9]. Both total and fixed investments are strongly procyclical in the two countries, but in Italy the tendency of investments to affect positively future deviations from trend output lasts only for two-three periods.

The cyclical component of the current account on GDP is a lot more volatile in Italy than in the US, but about as persistent. It is countercyclical in both countries and also at different lags and leads, but in Italy it tends to affect positively future deviations of output from trend. By contrast, exports and imports are procyclical. Both have higher volatility and persistence in the US than in Italy. The movement of exports and imports with the cycle differs in the two countries. In the US deviations from trend exports lag positively the cycle, and the

[9] If this seems less apparent in the statistics of inventory investment in Tables 3 and 4, it is because those statistics are not for logarithms but for ratios on GDP, since the logarithm transformation is not feasible for data with negative values.

correlations with output of the cyclical component of imports stay high at different lags and leads. Also, in the US cyclical exports lead negatively the cycle, while in Italy they tend to affect positively future output, and this may be due to higher degree of openness of the Italian economy[10]. The different cycle length may explain why in Italy correlations of imports with output are quite short-lived, if compared with those in the US.

3. *Stability*

There is no guarantee that the cyclical properties of the main macroeconomic variables are really the stylized facts that one wants to reproduce in a model. Changes in institutions or policies may have a permanent impact on the cyclical variation as well as the trend component of each series.

In order to test the dynamic stability of cyclical fluctuations, the F-test for the equality of variance across subsamples has been performed, whose results are in Table 5. The data range has been divided into two subsamples, 1970(I) – 1984(II) and 1984(III) – 1998(III), purely on the grounds of a visual examination of graphed series, which suggested a change in volatility. In fact, for both countries and for most variables, volatility seemed to decrease since the mid-Eighties. The third quarter of 1984 is a clear turning point for Italy, but could well be adapted to the US too.

The F-test statistic computes the variance of the two subgroups in which the data are split. Calling L the subgroup with the larger variance, σ_L^2, and S the subgroup with the smaller variance, σ_S^2, then the F-statistic is given by:

$$F = \frac{\sigma_L^2}{\sigma_S^2}$$

Under the null hypothesis of equal variance and independent normal samples, this F-statistic has an F-distribution with $(n_L - 1)$ numerator degrees of freedom and $(n_S - 1)$ denominator degrees of freedom.

[10] The log transformation implies that all cyclical deviations from trend are in percentage terms and not in absolute terms.

Table 5: F-test for equality of variance in subsamples

	Italy		*USA*	
	Value	*Probability*	*Value*	*Probability*
GDP	3.76	0.000	5.95	0.000
Employment	1.00	0.999	2.67	0.000
Per capita hours.	2.02	0.020	3.25	0.000
Wages per capita	2.84	0.000	3.22	0.000
Wages per hour	1.07	0.820	2.65	0.000
Output per hour	2.58	0.001	2.30	0.002
Output per capita	2.43	0.004	3.24	0.000
Private consumption	1.78	0.031	4.40	0.000
Government cons.	1.20	0.504	1.00	0.997
Total consumption	1.70	0.048	3.54	0.000
Fixed investment	1.05	0.862	5.47	0.000
Inventory inv. / GDP	7.76	0.000	3.38	0.000
Total investment	2.83	0.000	4.94	0.000
Current account / GDP	3.70	0.000	1.31	0.306
Exports	1.56	0.096	3.14	0.000
Imports	2.01	0.010	6.88	0.000

By looking at Table 5, it can be concluded that the hypothesis of stability of the cyclical properties is rejected for almost every series. Even if the breaking point was chosen visually and it does not seem to be related to any change in policy or structural change, only one series in the US and a different one in Italy have ratios significantly close to 1 The other series become all significantly less volatile over time, all with lower cyclical variances. Note in particular that output is the variable whose standard deviation changes more over time: the F-statistic shows that in the US the standard deviation of output more than halved after the mid-Eighties.

These results are only approximate since they may depend on the method adopted to test dynamic stability, the F-test being only one possible choice. Nevertheless, they suggest that the set of statistics presented in the business cycle literature as empirical regularities may not point to any regularity at all.

4. *Comovements*

A lot of work in the IRBC area has been conducted to solve some empirical discrepancies between data and theory. One of those discrepancies is related to international comovements of output, consumption and productivity. The models reproduce them correctly, but with an unsatisfactory order of magnitudes. Since the models presented in this paper will not produce cross country correlation coefficients, the issue is touched here only briefly.

By looking at Table 6, one can observe a contemporaneous correlation of GDP in the two countries, and also that Italian GDP, private consumption and total investments lag the same US variables. The fact that the Italian business cycle lags the US one may explain the positive correlation of employment at forward lags. Negative signs are more difficult to interpret. There is no contemporaneous correlation of Italian exports with cyclical variations of US exports, and the pattern of cross correlations of deviations from trend imports is quite similar to that of deviations from trend GDP.

Table 6: Cross country correlations

	Cross-Correlation with same US variable at time t:										
Series for Italy:	*x(-5)*	*x(-4)*	*x(-3)*	*x(-2)*	*x(-1)*	*x*	*x(+1)*	*x(+2)*	*x(+3)*	*x(+4)*	*x(+5)*
GDP	-0.38	-0.32	-0.21	-0.03	0.20	0.39	0.54	0.61	0.61	0.54	0.45
Private consumption	-0.60	-0.56	-0.48	-0.38	-0.22	-0.03	0.17	0.33	0.44	0.50	0.53
Total investments	-0.39	-0.37	-0.25	-0.05	0.12	0.24	0.30	0.30	0.31	0.28	0.17
Exports	0.18	0.15	0.10	0.10	0.10	0.04	0.03	-0.07	-0.15	-0.28	-0.30
Imports	-0.52	-0.43	-0.28	-0.05	0.22	0.34	0.36	0.32	0.33	0.31	0.29
Employment	-0.36	-0.40	-0.41	-0.33	-0.23	-0.18	-0.10	0.02	0.16	0.23	0.24

Appendix

Table 7: Data definitions and sources

Series	*Definition*	*Source of data used in calculations*
GDP	Gross Domestic Product at current prices / GDP implicit price deflator	OECD Quarterly National Accounts
Capital Stock	Calculated by the following iteration: Net Stock (t) = Net Stock (t-1) + Total investment – (Capital Consumption / GDP implicit price deflator). Capital Consumption for Italy is annual data / 4.	Net Stock: OECD Flows and Stocks of Fixed Capital. Other: OECD Quarterly National Accounts
Employment	USA: Total Labour Force – Total Unemployment	USA: OECD Quarterly Labour Force Statistics Italy: Bank of Italy
Per capita hours.	Italy: monthly hours of blue-collar workers in manufacturing firms, de-indexed, adjusted for seasonality. Quarter averages of monthly data. USA: average weekly hours of prod workers. Quarter averages of monthly data.	Italy: ISTAT USA: Bureau of Labor Statistics
Wages per capita	Adjusted Compensation of Employees / Employment Adjusted Compensation of Employees = (Compensation of Employees / Consumer Price Index) * Employment / Employees	OECD Quarterly Labour Force Statistics and Quarterly National Accounts
Wages per hour	Adjusted Compensation of Employees / (Employment * Per capita hours)	
Output per hour	GDP / Employment	OECD Quarterly Labour Force Statistics and Quarterly National Accounts
Output per capita	GDP / (Employment * Per capita hours)	
Private consumption	Private Final Consumption Expenditure at current prices / Private Final Consumption implicit price deflator	OECD Quarterly National Accounts
Government cons.	Government Final Consumption Expenditure at current prices / Government Final Cons. implicit price deflator	OECD Quarterly National Accounts
Total consumption	Private cons. + Government cons.	
Fixed investment	Gross Fixed Capital Formation / GFCF implicit price deflator	OECD Quarterly National Accounts
Inventory inv. / GDP	(Increase in Stocks / GFCF implicit price deflator) / GDP	OECD Quarterly National Accounts
Total investment	(Gross Fixed Capital Formation + Increase in Stocks) / GFCF implicit price deflator	OECD Quarterly National Accounts
Current account / GDP	(Exports- Imports) / GDP	
Exports	Exports of Goods and Services at current prices / Exports implicit price deflator	OECD Quarterly National Accounts
Imports	Imports of Goods and Services at current prices / Imports implicit price deflator	OECD Quarterly National Accounts

II. A Dynamic Closed Economy Model

In this section a standard RBC model is presented and calibrated for Italy, in order to test its ability to capture the business cycle statistics presented in the previous section. The economy is populated by a large number of identical agents, households and firms. Utility is separable in consumption and leisure, therefore it is possible to check if the model can reproduce Italian business cycles under different values for the intertemporal elasticity of substitution of leisure. The latter parameter can generate very different values for the standard deviations of hours, and as a result output. All variables are in per capita terms and there is no growth.

1. *The model*

Each household seeks to maximise her expected utility over infinite sequences of consumption $\{c_t\}_{t=1}^{\infty}$ and leisure $\{l_t\}_{t=1}^{\infty}$ pairs:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] \tag{1.}$$

Households allocate their time between productive activities and leisure, and the total amount of time available to them is normalised to 1: $h_t + l_t = 1$.

Firms produce output y_t according to a Cobb-Douglas production technology, which exhibits constant returns to scale in capital k_t and hours h_t:

$$y_t = z_t k_t^{\alpha} h_t^{1-\alpha} \tag{2.}$$

Uncertainty comes from productivity disturbances, z_t. $\log z_t$ follows an AR(1) process:

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1} \qquad \varepsilon_t \quad \text{i.i.d. } N(0, \sigma^2) \tag{3.}$$

The law of motion for capital is:

$$k_{t+1} = (1-\delta) k_t + i_t \tag{4.}$$

where δ is the depreciation rate. Capital at time zero, k_0, is given. The resource constraint for the economy as a whole is:

$$c_t + i_t = z_t k_t^{\alpha} h_t^{1-\alpha} \tag{5.}$$

As there are neither taxes nor other distortions, the solution to the optimisation problem faced by a social planner is the same as in a decentralised economy where both firms and households are price takers.

Factor demand comes from the firms, which maximise profits. The rental rates of labour and capital are given by, respectively:

$$w_t = (1-\alpha)\, z_t k_t^{\alpha} h_t^{-\alpha} \quad (6.)$$

$$r_t^k = \alpha\, z_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta \quad (7.)$$

Households own all factors of production and all shares in firms, and choose optimally their supply of labour and capital. Optimality implies that these two efficiency conditions must hold:

$$\frac{w_t}{c_t^{\theta}} = A(1-h_t)^{-\gamma} \quad (8.)$$

$$\beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} \left(1 + r_{t+1}^k\right) \right] = 1 \quad (9.)$$

These two conditions, together with the laws of motion of capital and the shock, the price equations and the resource constraint of the household, determine the optimal path followed by the model economy. The model cannot be solved analytically. The behaviour of the model economy in response to the exogenous shocks is obtained using a standard solution technique, based on the log-linearisation of all the equations around a nonstochastic steady state.

2. *Calibration*

The choice of the parameters is made so as to choose the parameter values for which the steady state values of the model aggregates are equal to the long-run averages of the corresponding variables for the Italian economy. The same set of data used for the statistics of Part I is used for calibrating the parameters, using real per capita variables. Nevertheless, some parameters cannot be estimated in this way because they do not enter the steady state equations, and the procedure of Kydland and Prescott (1982) cannot be followed due to the lack of microeconometric studies for Italy. Therefore, the parameters are chosen according to the steady state equations whenever possible, otherwise they are set to a few plausible levels, in order to check how the model results change for different parametrizations.

The depreciation rate δ is calculated by the ratio Capital consumption / Net stock of capital, which gives a value of 0.0088 for Italy[10]. The coefficient of relative risk aversion θ is the most difficult to estimate, and it is very common to set it equal to 2. However, another possible choice is to set it equal to 1, so it is better to check the performance of the model, in terms of second-order moments, for both values.

The labour share of output (1-α) is calculated as the ratio Adjusted compensation of employees / GDP, which gives a value of α equal to 0.33.

γ is the inverse of the intertemporal elasticity of substitution of labour. Three cases are considered: γ equal to 1, 0 (infinite intertemporal elasticity of substitution, as in the indivisible labour model), and the intermediate case 0.5.

The discount rate β is calibrated from the steady state equation $\beta\left(1+r^k\right)=1$. The real interest rate can be calculated (once values for α and δ are given) from the output/capital ratio in the data, but this procedure is affected by measurement errors of the capital stock. However, this model does not have capital adjustment costs. In the “real” world, without capital adjustment costs, and abstracting from risk, the return on real activities must be equal to the return on financial activities. Because in the next model β must be calibrated using an exogenously given rate on an internationally traded bond, the same method is used here, using the same average short-term German real interest rate (3-month fibor minus the growth rate of the consumer price index), equal to 0.0078. In any case, a value of β equal to 0.99 is quite a common choice in the literature.

The parameter A is given by the steady state equation:

$$A=\frac{(1-h)^{\gamma}}{c^{\theta}}\,\frac{y}{h}\,(1-\alpha) \qquad (10.)$$

which can be rewritten as

$$A=(1-h)^{\gamma}\left(1-\delta\frac{k}{y}\right)^{-\theta}(1-\alpha)\,h^{-\theta}\left(\frac{y}{h}\right)^{1-\theta} \qquad (11.)$$

where y, and k are output and capital per employed. The capital-output ratio and the ratio of hours on output are already given by the steady state equation $\beta\left(1+r^k\right)=1$. γ and α are calibrated as above, and h is the fraction of time devoted to market activities, which is calibrated at 0.12, given data on hours worked[11].

Technology shocks are deviations of total factor productivity from its long-run average.

[10] Data definitions and sources as in Table 7.

Given the production function, a consistent estimate of total factor productivity is obtained from per-employed variables as follows:

$$\log z_t = \log y_t - \alpha \log k_t - (1-\alpha)\log h_t$$

and then the persistence and standard deviation of the technological shocks are obtained by fitting a first-order VAR. However, because the only data available on hours worked display a high standard deviation, it is evident that with this estimation method the standard deviation of output generated by the model will be too high, thus invalidating the whole exercise. However, it is quite difficult to believe that the standard deviation of hours in Italy is 7-8 times more than in the US. Some of it may be due to rigidities in the Italian labour market, which prevent adjustment along the extensive margin, thus leaving the burden of adjustment to hours only. But even accounting for this effect does not explain the huge standard deviation of hours in Italy, which may come from sampling, or statistical definitions. For this reason it is better to resort to the Kydland and Prescott's (1982) strategy of choosing the standard deviation of the shock so that the model reproduces the standard deviation of output in the data[12]. The autoregressive coefficient is set at 0.99, a value used in many studies.

Table 8: Parameter values

δ	α	h	β	ρ
0.0088	0.33	0.12	0.99	0.99

3. *Evaluation of the model for different parameter values*

All the statistics for the closed economy were computed on logarithms of HP-detrended data, generated by simulating the model for 225 periods, throwing away the first 25 observations, repeating 100 times, and then computing moments as averages over repetitions[13].

The calibration of the model based only on the steady state equations leaves some crucial parameters free to be set at different values. Those parameters are the intertemporal elasticities of substitution of consumption and hours, which greatly affect the model performance

[11] The average hours worked in a week (35.53), divided by total time available (16 hours × 7 days a week), and multiplied by the fraction of population that works (0.37).

[12] Maffezzoli (2000) calibrates a different RBC model for Italy using standard units of labour as an approximate measure of the labour input to measure Solow residuals. However standard units of labour are inferred from National Accounting data on GDP and employment data, thus they do not constitute a measure of hours of the same quality as survey data.

[13] Relative-volatility statistics may change in different simulations. Because of the certainty-equivalence property of the log-linear approximations, they do not depend on the variance of the innovations of the technology shock.

in terms of the second-order moments and the response of the variables to a technology shock. Table 9 describes the effects of different choices of the parameters on the most important moments. In this way sensitivity analysis is carried out, but instead of finely varying the parameters around a chosen value, a grid is chosen, which reflects some parameter choices made in other RBC studies.

The case $\gamma = 0$ corresponds to the indivisible labour model of Hansen (1985) and it implies an infinite intertemporal elasticity of substitution of the labour input. When $\gamma = 1$ the intertemporal elasticity of substitution of labour is equal to 1, and $\gamma = 0.5$ is the intermediate case. When γ increases the volatility of hours relatively to output decreases because households are less willing to substitute hours over time in response to the exogenous shock, and they become relatively more willing to substitute consumption instead of hours, so the volatility of consumption increases slightly. One explanation of the increase in the relative volatility of wages when γ grows could come from the fact that in this model wages are perfectly correlated with the capital-hours ratio. A growth in γ induces a reduction in consumption smoothing in favour of "leisure smoothing"[14]. If households care less about having a smooth pattern of consumption, they will not invest a lot after a shock. This could decrease the correlation between capital and hours, thus increasing the standard deviation of wages. Or alternatively, consumption smoothing implies a larger shift of the labour supply curve to the right after a shock, thereby causing larger volatility of hours but lower volatility of wages.

Table 9: Evaluation of the model for different parameter values

	Relative volatility with respect to output			***First-order autocorrelation***			***Contemporaneous correlation with output***		
	c_t	h_t	w_t	c_t	h_t	w_t	c_t	h_t	w_t
$\theta = 1$									
$\gamma = 0$	0.33	0.68	0.33	0.72	0.69	0.72	0.97	0.99	0.98
$\gamma = 0.5$	0.33	0.62	0.38	0.73	0.70	0.72	0.97	0.99	0.98
$\gamma = 1$	0.35	0.58	0.42	0.73	0.70	0.72	0.98	0.99	0.99
$\theta = 2$									
$\gamma = 0$	0.33	0.37	0.65	0.72	0.71	0.72	0.99	0.96	0.99
$\gamma = 0.5$	0.34	0.31	0.71	0.70	0.70	0.70	0.99	0.96	0.99
$\gamma = 1$	0.35	0.27	0.75	0.72	0.72	0.71	0.99	0.95	0.99

[14] Consumption smoothing and leisure smoothing depend on two separate parameters, but also on the relative importance of one parameter with respect to the other.

A higher level of θ should imply a decrease in the volatility of consumption, however, Table 9 reports volatilities with respect to output, and not in absolute terms. When θ is higher households are less eager to substitute consumption over time in response to a shock, and their labour supply becomes more rigid. That is, the income effect on labour supply becomes bigger. As a result, output follows closely the shock, with no amplification due to labour adjustment, and, by equation (10), consumption follows more closely wages and therefore output. The rigidity of labour supply when θ is high also explains the higher first-order autocorrelation of hours and the lower contemporaneous autocorrelation of hours with output when γ increases.

Figures 1 to 3 show the responses of output, hours, consumption and wages to a 1% positive technology shock. Responses are plotted for different values of γ, while θ stays fixed at 1. For $\theta = 2$ the responses of output, consumption and wages are simply dampened, and the impact of the shock on hours becomes very small.

As it is known, the indivisible labour model is the one that delivers the maximum contemporaneous impact of the shock on labour and output. However the above discussion about relative volatilities suggests that the impact on hours depends also on the relative importance of the intertemporal substitutability of consumption, with respect to the substitutability of hours.

Different values of γ do not affect or affect only slightly the responses of consumption and wages. In fact, giving separable preferences, the law of motion of consumption does not depend on hours, and wages are given by the capital-hours ratio, which can adjust only slowly.

4. *Results*

Values of θ between 1 and 2 are the most commonly used in the literature, and in practice there is some degree of subjectivity in the choice of the intertemporal elasticity of consumption. However, when γ is equal to zero all the fluctuation in the labour input can be attributed to employment instead of hours. This it has the advantage of abstracting from hours, since the only data available on hours worked in Italy led to not entirely reliable statistics. Therefore θ is chosen so as to generate an exact match of employment volatility in the data.

Table 10 reports the second order moments generated by the model with θ equal to 1.4 and γ equal to zero, and all the other parameters chosen as described in subsection II.2. It can be seen that the model is successful in matching the standard volatility of investment, and in generating a standard deviation for employment close to the one in the data. All the variables follow the cycle as expected, first-order autocorrelations are satisfactory for output, consumption, investment and employment[15]. Cross-correlations are satisfactory for output, consumption and investment. However, the model generates too low volatility for all the variables except employment and investment, and too high correlations with output for employment and wages.

One could exploit the fact that labour is indivisible and measure the Solow residuals only on employment instead of hours. In this case the model generates too much volatility: the standard deviation of output becomes 3.09 (but the relative standard deviations, the first-order standard deviations and the correlations with output stay the same).

The critical point of this RBC model, and many others, is that all the standard deviations and the other second-order moments are always drawn from the standard deviation of a single shock, and therefore the ability of the model to match real-world statistics is always limited. It is therefore interesting to see what happens if the stochastic structure of the model is enriched by introducing more than one shock. In order to try to do so, in the next section Italy is modelled as a small open economy, where uncertainty comes from both a technology shock and a shock in the world rate of return on bonds.

Table 10: Standard deviations and correlations with output $\theta = 1,\ \gamma = 0$

	St. dev. rel. to output.	*1-st ord AC*	*Cross-correlation of output at time t with*		
			x(t-1)	*x(t)*	*x(t+1)*
Output	1.00	0.70	0.70	1.00	0.70
Consumption	0.33	0.72	0.63	0.97	0.74
Investment	4.54	0.70	0.70	0.98	0.66
Employment	0.68	0.69	0.72	0.99	0.67
Wages	0.33	0.72	0.64	0.98	0.74
Interest rate	0.02	0.70	0.72	0.99	0.66

[15] Since the value chosen for ρ is quite high, the low autocorrelation of output must be regarded as an unavoidable feature of the model.

III. The Open Economy Model

The model presented in this section is the extension of the same closed economy model of the preceding section to the open world. There is a single asset, an internationally traded bond, the rate of return on which is exogenous. Models of this sort, in which the trade balance is not explicitly modelled, are used to model small open economies, because the return on the asset cannot be influenced endogenously. Italy is in fact a small, open country, where the ability to control the interest rate has been limited by the exchange rate mechanism first, and then by the single currency.

The same assumptions on the agents, the productive sector, and the institutional setting continue to hold, but we need two more assumptions: the first is that labour is internationally immobile, and the second is that there are adjustment costs for the stock of capital.

1. *The model*

As in the previous section, households maximise their expected utility over infinite sequences of consumption $\{c_t\}_{t=1}^{\infty}$ and leisure $\{l_t\}_{t=1}^{\infty}$ pairs:

$$U = E_0 \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] \tag{12.}$$

Households can consume more or less of what they earn in each period because they can sell or buy bonds in the international capital market. The amount of resources available to them is:

$$b_{t+1} + c_t + i_t = b_t(1 + r_t) + w_t h_t + \left(r_t^k + \delta\right) k_t \tag{13.}$$

b is the stock of traded bonds, r is the return on bonds. The total amount of time available to them is normalised to 1, that is $h_t + l_t = 1$.

As before, the production function is Cobb-Douglas with constant returns to scale, and firms and households act as price-takers. Only one good is produced, which can be consumed, transformed into capital or traded for foreign assets, thus the international capital market is in practice another channel to smooth consumption over time. The equilibrium rental prices for labour and capital are:

$$w_t = (1-\alpha)\, z_t k_t^{\alpha} h_t^{-\alpha} \tag{14.}$$

$$r_t^k = \alpha\, z_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta \tag{15.}$$

Arbitrage considerations lead to the equality between the rate of return on physical capital and the return on bonds in every period. To avoid this, adjustment costs for capital are introduced, therefore the law of motion for capital becomes:

$$k_{t+1} = (1-\delta)k_t + \Phi\left(\frac{i_t}{k_t}\right)k_t \qquad (16.)$$

where $\Phi(\cdot)$ is the adjustment cost function. $\Phi(\cdot)$ is concave and continuously differentiable, with $\Phi(\delta)=\delta$ and $\Phi'(\delta)=1$, so that in the steady state there are no adjustment costs. The ratio of the value of capital to its replacement cost gives the Tobin's q:

$$q_t = \left(\Phi'\left(\frac{i_t}{k_t}\right)\right)^{-1} \qquad (17.)$$

By the assumptions on the adjustment cost function, the Tobin's q is equal to 1 in the steady state. As in the Tobin's q theory of investment, capital is decreasing or increasing according to whether $q_t < \text{or} > 1$.

Both the logarithms of the technology disturbance z_t and the return on bond r_t follow an AR(1) process:

$$\log z_{t+1} = \rho_z \log z_t + \varepsilon^z_{t+1} \qquad \varepsilon^z_t \quad \text{i.i.d. } N(0,\sigma^2_z) \qquad (18.)$$

$$\log r_{t+1} = c + \rho_r \log r_t + \varepsilon^r_{t+1} \qquad \varepsilon^r_t \quad \text{i.i.d. } N(0,\sigma^2_r) \qquad (19.)$$

The current account of this economy, which is savings minus investments, is given by:

$$CA = b_{t+1} - b_t(1+r_t) \qquad (20.)$$

which gives also the capital account of the economy. b_t is the amount of credit of the domestic economy at time t.

The optimality conditions of the household maximisation problem are:

$$c_t^{-\theta} w_t = A(1-h_t)^{-\gamma} \qquad (21.)$$

$$\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\theta}(1+r_{t+1})\right] = 1 \qquad (22.)$$

$$q_t c_t^{-\theta} = \beta E_t\left[c_{t+1}^{-\theta}\left(r^k_{t+1} + \delta + q_{t+1}\left(1-\delta-\Phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\frac{i_{t+1}}{k_{t=1}} + \Phi\left(\frac{i_{t+1}}{k_{t+1}}\right)\right)\right)\right] \qquad (23.)$$

The first condition is an intratemporal efficiency condition, relating hours of work with consumption choices, as in Section II. The other two conditions are arbitrage conditions, relating the return on physical capital and bond to the optimal intertemporal allocation of consumption. One consequence of the assumption of separability of preferences is that the logarithm of consumption follows a random walk, as equation (22) makes clear. This property is then inherited by all the other variables of the model, and it does not necessarily constitute a negative feature of the model[16]. However, the solution method based on linearly approximating the laws of motion and the equilibrium conditions of the model around the steady state becomes unreliable when the economy is hit by large shocks, and results of simulations must be taken with some care.

The optimal path followed by the model economy is again described by the laws of motion for capital and the shock, the price equations and the resource constraint of the household. The introduction of the international capital market requires two no-Ponzi games condition, on the 2 assets of the economy:

$$\lim_{t \to \infty} E_0 \prod_{s=1}^{t} \left(1 + r_s^k\right)^{-1} k_t = 0$$

$$\lim_{t \to \infty} E_0 \prod_{s=1}^{t} \left(1 + r_s\right)^{-1} b_t = 0$$

in order to prevent the economy to go forever into debt.

2. *Calibration*

Instead of taking arbitrary decisions on the intertemporal elasticities of substitution of consumption and leisure, the performance of the model for a few significant values of θ and γ is checked, as in Section II. The parameters δ and α are set at the same values used in the closed economy model. The introduction of the internationally traded bond requires also the calibration of its steady state level, which is not necessarily equal to zero: this model does not have a unique steady state, since any level of foreign asset holding is compatible with the steady state equations. However, Italy is a country that managed to balance its current account in the past, and the average of the current account on output for the period 1970(I) to 1998(III) is –0.005.

The calibration of A is done as in the closed economy, once the parameters θ and γ are decided, so as to ensure that in the steady state h is equal to 0.12. The equation is modified to take into account the steady-state share of the current account on output:

[16] Ways to ensure stationarity are time-non-separable preferences, as in Mendoza (1991), or finitely-lived agents, as in Cardia (19991).

$$A=(1-h)^{\gamma}\left(1-\delta\frac{k}{y}-\frac{b}{y}r\right)^{-\theta}(1-\alpha)\,h^{-\theta}\left(\frac{y}{h}\right)^{1-\theta} \quad (24.)$$

The elasticity of the marginal adjustment cost

$$\eta = -\frac{\Phi''}{\Phi'}\frac{i_t}{k_t}$$

is assumed to be always constant, so that the calibration procedure for the adjustment cost function $\Phi(\cdot)$ requires only one parameter. The volatility of investments depends on the elasticity η. If η is high investments display low volatility, if η is low investments are more volatile. The case $\eta=0$ corresponds to the linear case, with no adjustment costs. Here η is set at 1/15, the benchmark value used by Baxter and Crucini (1993).

The autoregressive process for the interest rate is estimated using data on short-term real rates in Germany, given that this is the country whose policy is more likely to affect Italian rates of return on bonds. The discount rate β is calibrated so that $\beta(1+r)=1$, given the steady state or averaged value of the rate of return on bonds, so it is set at the same value as in the closed economy. The variance of the innovations in the process for technology is again chosen so as to match the volatility of output in the data, while the autoregressive coefficient is set equal to 0.99.

Table 11: Parameter values

δ	α	h	β	$\frac{b}{y}$	η	ρ_z	ρ_r	σ_r
0.0088	0.33	0.12	0.99	-0.005	1/15	0.99	0.37	0.006

3. *Short-run dynamics*

Figures 4 to 7 present the response of the main economic variables to a 1% positive shock in technology and the interest rate. Because of equation (22) consumption is not stationary, and the other variables inherit the nonstationarity of consumption. The case of the indivisible labour model ($\gamma=0$) is omitted, because the responses of the variables after a technology shock and an interest rate shock were too peculiar for this model to be viable in the open economy. After both kinds of shocks, variables went too far away from the original steady state for the approximation method to be meaningful, and as a result standard deviations were excessive. This behaviour can be due to the assumption of separability of preferences, and it may suggest that (in this framework) preferences cannot be separable when the marginal utility of leisure is unitary, and the marginal utility of consumption is an integrated variable. However, this hypothesis must be checked only after the indivisible labour model is solved by means of a finer solution method than the log-linear approximation.

The technology shock increases output and wages, consumption and investment. Because equation (22) rules out consumption growth, the effect on consumption is a level effect. By equation (21) hours jump after the shock in order to ensure equality between the marginal utility of leisure and the marginal utility of consumption, as a result, output also jumps. After 40 quarters the wealth effect on assets causes hours, output and wages to decrease, and the economy settles at a new steady state where household can consume more, and work less, because they have positive foreign assets or less debt (they start from a negative b). When the intertemporal elasticity of leisure is lower the impact on hours is more moderate and therefore the in new steady state households will have less positive assets or moderate reduction of their debt. Responses are not plotted for a different intertemporal elasticity of consumption. $\theta = 2$ does not affect the shape and magnitude of the responses, but only the level effect on consumption, which becomes lower as one would expect.

From equation (21) it can also be seen why the indivisible labour model generated meaningless impulse responses. When γ is equal to zero wages have to jump with consumption and stay constant afterwards, but because capital and technology are increasing, hours must increase to keep the capital-hours ratio unchanged. As a result, the deviations of output and assets from the steady state become enormous.

The shock in the interest rate increases the cost of the debt, therefore households reduce it. If they had a positive b , an increase in assets would happen. Investment decreases because foreign assets are more rewarding than physical capital; consumption decreases initially so that it can grow afterwards, as long as the rate of return on foreign assets stays above the steady state level. Wages move in the same direction as consumption, to keep the marginal utility of leisure equal to the marginal utility of consumption. Output and hours jump in order to feed the growth in consumption, then they start decreasing because of the wealth effect on assets. However, in the new steady state households hold more foreign debt, and they pay the interest on it by working more hours. Different values of γ do not change the shape of the responses, but the initial increase in hours is higher when γ is lower. As with the technology shock, responses do not differ when θ is equal to 2, only consumption decreases less at the impact, and as a result in the new steady state households end up with a bigger debt.

The anomaly in the case of the indivisible labour model comes from the fact that the initial decrease in consumption is lower when $\gamma = 0$ then when $\gamma \neq 0$ because of equation (21). As a result, the economy goes too far way from the steady state and again variances become meaningless.

The fundamental difference in the impulse responses of the closed and the open economy after a technology shock is related to the behaviour of hours and consumption. In the closed economy a positive technology shock makes working more attractive because of the increase in wages, and consumption grows as well. However, as long as consumption grows, the marginal utility of consumption decreases. For efficiency the marginal utility of leisure must decrease as well, and therefore hours go back quickly to the old steady state. In the open economy after a technology shock there is no sustained growth in consumption because this can happen only when the rate of return on bonds is above the steady state level.

3. *Results*

As stated earlier, the accuracy of simulations of models that do not have a unique steady state is questionable, and long simulations should be avoided. However, the moments presented here are not too big and remained quite stable when the simulation period was changed. Therefore, the simulation period is the same as in the previous section (225 quarters, less the initial 25), and statistics are averages over 100 repetitions. All data were transformed in logarithms (except the current account, which was divided by output instead) and then detrended.

Instead of assessing the model performance under different values for the intertemporal elasticities of substitution, it is perhaps more interesting to compare the results with the closed economy, for a given parametrization. We compare the results for the two artificial economies when $\theta = 1$ and $\gamma = 0.5$. The effect of different elasticities is quite predictable: when $\theta = 2$ the volatility of consumption is too low, and because households are less eager to substitute consumption over time labour supply is more rigid, and the volatility of hours is slightly diminished. When γ is equal to 1 labour supply is more rigid and the volatility of hours is lower, therefore is $\gamma = 0.5$ which gives a better performance in terms of the volatility of hours.

Tables 12 and 13 report the second moments for the open and the closed economy. Since the standard deviation of output was approximately matched using $\sigma_z = 0.006$, both the technology shock and the interest rate shock have the same standard deviation in the simulation. As it can be seen from the tables, the volatility of consumption is lower in the open economy: this happens because the foreign assets are an extra channel to smooth consumption over time, and any increase in productivity is spread over an infinite horizon. Households revise their pattern of consumption only when a change in the interest rate changes their permanent income, through a change in the present discounted value of assets. Because the technology shock has a level effect only, the correlation with output is lower than in the closed economy. Moreover, the interest rate shock has only a small effect on consumption, and it is not persistent. This fact contributes to explain, together with the level effect of the technology shock, the lower autocorrelation.

The volatility of investments is higher in the open economy. This happens because after a technology shock households want to spread over future periods the positive income effect and enjoy a constant level of consumption. The increase in consumption must be sustained initially by output, so it becomes optimal to invest initially a large sum, until the positive wealth effect on assets takes the burden of consumption. The interest rate shock has only a small, not persistent effect on investment, but it contributes to reduce the correlation with output. However, the volatility of investment depends a lot on the elasticity of the adjustment cost function.

The increased volatility of hours in the open economy can be also explained with the availability of an additional channel to smooth consumption over time. Households can enjoy more consumption in all future periods by means of reducing the foreign debt, but to do so they need a large initial increase in output, therefore hours (and investments) must jump after a technology shock. An analogous jump in output through a jump in hours is needed to feed consumption growth after an interest rate shock. Because in both shocks hours jump in order to increase output at impact, the correlation of hours with output stays high in the open economy. In the open economy the volatility of consumption is therefore decreased at the expenses of the volatilities of hours and investments. This happens because in the parametrization chosen the intertemporal elasticity of substitution of consumption is relatively higher than that of leisure.

Wages are competitive, equal to the marginal productivity of labour, so they are highly correlated with output. The low volatility of wages may be due to consumption smoothing, as it was explained for the closed economy.

In conclusion, the analysis shows that the open economy model is successful in generating an increased volatility of hours without the need to assume that the intertemporal elasticity of substitution of leisure is large, which may not be supported by microeconometric evidence. With respect to wages, the model performance is quite similar to the performance of the indivisible labour model in the closed economy, with respect to investment, the open economy model is not unsatisfactory if the definition of capital includes the inventories. Given that trade was not explicitly modelled, the model performance with respect to the current account is not unsatisfactory: the signs of the standard deviation and the negative correlations with current and future output are correct, and the autocorrelation is not distant from that in the data.

Table 12: Open economy: Standard deviations and correlations with output $\theta = 1$, $\gamma = 0.5$

			Cross-correlation of output at time t with		
	Rel st. dev.	***1-st ord AC***	*x(t-1)*	*x(t)*	*x(t+1)*
Output	1.00	0.72	0.72	1.00	0.72
Consumption	0.28	0.72	0.74	1.00	0.69
Investment	4.93	0.72	0.74	0.99	0.67
Hours	0.68	0.73	0.72	1.00	0.73
Wages	0.32	0.72	0.73	1.00	0.70
Current account / output	0.20	0.86	-0.37	-0.30	0.00

Table 13: Closed economy: Standard deviations and correlations with output $\theta = 1$, $\gamma = 0.5$

			Cross-correlation of output at time t with		
	Rel st. dev.	***1-st ord AC***	*x(t-1)*	*x(t)*	*x(t+1)*
Output	1.00	0.71	0.71	1.00	0.71
Consumption	0.33	0.73	0.64	0.97	0.74
Investment	4.42	0.70	0.71	0.99	0.67
Hours	0.62	0.70	0.72	0.99	0.67
Wages	0.38	0.72	0.66	0.98	0.74

As it was stressed before, these results depend on the parametrization chosen and on the form of the utility function. It is generally true, however, that the introduction of more than one shock can improve the performance of a RBC model along many dimensions.

In spite of these successes, there are still some drawbacks. First, the model was not able to replicate the very large volatility of hours observed in the data[17]. Lower values of γ did not increase significantly the relative standard deviation of hours: with $\gamma = 0.4$ it becomes equal to 0.70, with $\gamma = 0.1$ it becomes 0.76, and with $\gamma = 0$ the standard deviations degenerate. Second, the increased volatility of hours was obtained at the seemingly unavoidable cost of a

[17] Though, as it was already observed, data available on hours worked in Italy should be taken with some care.

reduction of the volatility of consumption, too low to match the data. This feature may be eliminated by introducing nonseparable preferences in consumption and leisure. Third, the low standard deviation of wages, too low to match the Italian data. As it was pointed out in Section II, the volatility of wages and hours seem to move in opposite direction when the degree of consumption smoothing changes. This feature may be overcome by making different assumptions on wage formation.

In both the open and the closed economy models the household decision to supply labour had only one dimension, hours, or in the case of the indivisible labour model, participation only. It is interesting to see whether another model, in which households make a decision on both their labour market participation and the hours of work, can generate a better match of the data. The high volatility of hours in Italy may be plausibly attributed to rigidities in the labour market, which prevent employers to fire after a bad shock. These rigidities may also be the cause of the lower standard deviation of employment in Italy, as compared to the US. To check if a RBC model can capture this feature of the data, the next Section introduces a model with both hiring and firing costs, as one of the many factors that can bring rigidity in the labour market.

IV. The Model with Firing Costs

The models of the previous sections did not make a distinction between the decision to participate or not in the labour market, and the decision on how many hours to supply. In reality, labour fluctuates along both the intensive margin and the extensive margin, a feature that can be captured only in a model where the two dimensions of variation of the labour input are explicitly modelled[18]. Moreover, it is quite plausible that hours in Italy fluctuate more than employment, as opposed to the US, where the opposite happens. The Italian labour market is more regulated than the US one, in particular there are some institutions that prevent firms to adjust the number of employees during recessions. As a consequence, a lot of labour adjustment takes place through the social security system. In fact, under some conditions, a troubled firm can have access to the so-called "Cassa Integrazione Guadagni", which allows not to employ some or all of her workers, without having to dismiss them, at least for a short period of time. Workers do not loose their job, and they receive a payment from the social security

[18] The modelling strategy used in this section follows quite closely the one proposed by Kydland and Prescott (1991).

system that covers almost entirely their wage. What is important is that they do not become unemployed, so this sort of labour adjustment is, as far as statistics are concerned, variation in hours and not “bodies”. It is also plausible that the same sort of restrictions which prevent the firms to fire work also in the opposite direction, making firms more reluctant to hire during expansions. If this is the case, a lot of labour adjustment will take place during expansions by resorting to overtime, instead of recruiting new workers. This causes again a change in hours worked, while employment is only mildly affected or not affected at all.

In this section both firing and hiring costs are introduced, in order to see if the model of the previous sections can generate fluctuations in hours worked and employment that match those in the Italian data. Firing and hiring costs are not the only source of labour rigidity, but they can be thought as a modelling tool. Different scenarios are possible. Firing and hiring costs can be due to legislation only, or they may be generated by market inefficiency: for example, the hiring cost can be a search cost.

1. *The model*

There is a continuum of agents on the interval [0,1]. The utility function of each agent is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right]$$

where c_t is consumption and l_t is leisure. The time endowment of each individual is one, but if the individual decides to work he loses for commuting an amount of time equal to $1-\psi$, where $\psi \in (0,1]$ is a fixed parameter. Consequently, the amount of leisure that the individual enjoys is equal to one if he does not work, and if he works it is equal to $\psi - h_t$, where h_t are hours worked. ψ models the cost of participation to the labour market.

Firms produce a composite good, which is given by the number of hours times a constant returns to scale production technology:

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t$$

where y_t is output, z_t is the technology shock, k_t is capital and n_t is employment, so unemployment is $1-n_t$. Contrary to by Kydland and Prescott (1991), the adjustment costs of the labour force are quadratic, and they are paid by the firms whenever they lay off workers or recruit new ones:

$$AC_t = \frac{\phi}{2}(n_t - n_{t-1})^2$$

The law of motion for capital is:

$$k_{t+1} = (1-\delta)k_t + i_t$$

where δ is the depreciation rate. Capital at time zero, k_0, is given, as well as employment at time –1, n_{-1}. The logarithm of the technology shock follows an AR(1) process.

Unemployment is the only source of heterogeneity, since agents are the same in this economy. As a consequence, all the unemployed have the same level of consumption, and all employed work the same number of hours and consume the same amount of the composite good. Individuals choose employment lotteries and either there is a market where individuals can insure themselves, or full employment insurance is provided by firms. Because markets are complete, the welfare theorems hold. Hence the conditions that define the optimal path of this model economy are the solution to the social planner's problem, which maximises the sum of agents' utilities:

$$\max \quad E_0 \quad \sum_{t=0}^{\infty} \beta^t \left\{ (1-n_t)\left[\frac{(c_t^u)^{1-\theta}-1}{1-\theta} \right] + n_t \left[\frac{(c_t^e)^{1-\theta}-1}{1-\theta} + A\frac{(\psi - h_t)^{1-\gamma}-1}{1-\gamma} \right] \right\}$$

$$s.t. \quad c_t + i_t = z_t k_t^{\alpha} n_t^{1-\alpha} h_t - \frac{\phi}{2}(n_t - n_{t-1})^2$$

$$c_t = (1-n_t)c_t^u + n_t c_t^e$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1} \qquad \varepsilon_t \quad \text{i.i.d. } \mathrm{N}(0,\sigma^2)$$

$$k_0 \ , n_{-1} \quad \text{given}$$

where c_t^u is consumption of the unemployed and c_t^e is consumption of the employed. It follows that optimality requires:

$$A(\psi - h_t)^{-\gamma} = c_t^{-\theta} z_t k_t^{\alpha} n_t^{-\alpha} \tag{25.}$$

$$c_t^{-\theta} = \beta E_t \left[c_{t+1}^{-\theta}\left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{t+1} - \delta\right)\right] \tag{26.}$$

$$A\frac{(\psi - h_t)^{1-\gamma}-1}{1-\gamma} + c_t^{-\theta}(1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} h_t = c_t^{-\theta}\phi(n_t - n_{t-1}) - \beta E_t \left[c_{t+1}^{-\theta}\phi\,(n_{t+1} - n_t)\right] \tag{27.}$$

Because preferences are separable in leisure and consumption, $c_t^u = c_t^e = c_t$. Equations (25) and (26) are the familiar intratemporal and intertemporal efficiency conditions. Equation (27) describes the optimal allocation of agents between employment and unemployment. The left-hand side is the marginal benefit of adding an additional worker to production, and the right-hand side is the present discounted value of the marginal cost.

2. *Calibration*

This economy has a unique steady state in which the shock is equal to its mean value and the variance of innovations is zero. Dropping time subscripts, in the steady state the following must be true:

$$\frac{(\psi - h)^{1-\gamma} - 1}{1-\gamma} = -(1-\alpha)(\psi - h)^{-\gamma} h$$

The above equation can be solved for ψ, once α and h have been calibrated and a choice of γ has been made.

Equation (26) gives the capital-output ratio in the steady state:

$$\frac{y}{k} = \frac{1}{\alpha}\left(\frac{1}{\beta} + \delta - 1\right)$$

from which the output-employment ratio $\frac{y}{n}$ can be derived. Then a consistent measure of the parameter A is given by the following:

$$A = (\psi - h)^{\gamma}\left(1 - \delta\frac{k}{y}\right)^{-\theta}\left(\frac{y}{n}\right)^{1-\theta} n^{-\theta} h^{-1}$$

which shows that n, the fraction of population that works, must also be calibrated at some sensible level. n is therefore set equal to the average employment -population ratio.

The fraction of time h devoted to market activities by working people is given by the average hours worked in a week (35.53), divided by total time available (see note 11).

The parameters θ, γ and ϕ are set to a benchmark parametrization, and then some sensitivity analysis is performed.

Table 14: Values employed in the calibration

δ	α	β	h	n	ρ
0.0088	0.33	0.99	0.32	0.37	0.99

The parameters δ, α, β, ρ and are set at the same values used in the other two sections. As before the variance of innovations of the process for technology is set at the level that reproduces the standard deviation of output in the data. The values of the parameters are reported in Table 14.

3. *Results*

Table 15 reports the standard deviations of employment, hours, consumption and investment, as a fraction of the standard deviation of output, obtained by simulating the model for different values of the adjustment cost. Since the feature of the data to be reproduced is the higher standard deviation of hours than the one of employment, only the value $\gamma = 0$, or infinite intertemporal elasticity of substitution of hours, is considered. When $\phi = 0$ there are no labour adjustment costs, and, by concavity of the utility function, hours do not fluctuate. The standard deviation of consumption is exactly the same as in the model of Section II, and the standard deviation of employment is equal to the one of hours in the same model. Since hours do not move and the power of *n* is the same as *h* in the production function, this implies that when $\phi = 0$ the policy functions are the same as in the model of Section II, except that *n* is substituted for *h*.

Table 15: Model results

	Relative standard deviation			
	n	h	c	i
$\theta = 2,\ \gamma = 0$				
$\phi = 0$	0.37	0.00	0.33	4.19
$\phi = 1$	0.43	0.54	0.34	4.34
$\phi = 1.5$	0.40	0.82	0.29	4.82
$\phi = 2$	0.33	1.00	0.22	5.29
$\phi = 2.5$	0.25	1.07	0.15	5.56
$\phi = 3$	0.17	1.08	0.09	5.67
$\theta = 1,\ \gamma = 0$				
$\phi = 0$	0.68	0.00	0.33	4.20
$\phi = 1$	0.68	0.85	0.28	4.87
$\phi = 1.5$	0.51	1.05	0.19	5.39
$\phi = 2$	0.36	1.10	0.12	5.60
$\phi = 2.5$	0.25	1.10	0.07	5.67
$\phi = 3$	0.17	1.09	0.04	5.68

As Table 15 shows, an increase in the adjustment cost parameter ϕ is successful in increasing the relative volatility of hours with respect to employment. ϕ takes values between 1 and 3, which imply that the marginal cost of a 1-percent change in the employment level is between 0.002 and 0.007 percent of a quarter's output. These values are well below the level employed by Cogley and Nason (1995), who proposed an estimate for the marginal cost equal to 0.36 percent, however, higher values introduced complex roots in the approximate solution of the model. When ϕ increases the supply of hours is more elastic and there is more consumption smoothing, so the relative volatility decreases for consumption and increases for investment.

An increase in θ tends to induce a decrease in the relative volatility of consumption when there are adjustment costs. As in the model of Section II, this happens because the increase in θ makes the labour supply curve more rigid, because households are less willing to substitute consumption over time. In fact, employment and hours tend to fluctuate less when θ is equal to 2. When the intertemporal elasticity of substitution of leisure γ is equal to zero, the left-hand side of equation (25) becomes a constant, and this explains, together with equation (27), why adjustment is made through employment, in spite of the volatility of hours being infinite.

The increase in the relative volatility of hours is accompanied by a decrease of the relative volatility of employment, which is an unappealing feature of this model. The failure to reproduce relative standard deviations close to those in the data is a consequence of the short-run dynamics induced by the model. As it can be seen from Figure 8, the labour adjustment cost has an impact on hours in the first quarter only, then hours stay constant afterwards, which implies that the adjustment cost does not "bite" any longer. This happens because households adjust hours only in the first quarter because n_{t-1} is given, then they find paths for employment and consumption such that the left-hand side of Equation (27), which is the only dynamic equation with labour adjustment costs, is equal to zero. This happens because along the optimal path the choice of employment must satisfy:

$$\frac{\vartheta U}{\vartheta c_t}\frac{\vartheta c_t}{\vartheta AC_t}\frac{\vartheta AC_t}{\vartheta n_t} = \beta E\left[\frac{\vartheta U}{\vartheta c_{t+1}}\frac{\vartheta c_{t+1}}{\vartheta AC_{t+1}}\frac{\vartheta AC_{t+1}}{\vartheta n_{t+1}}\right] \tag{28.}$$

that is, the utility cost of adjusting employment today must be equal to the present discounted value of the utility cost of tomorrow. This may be thought an optimality condition for the intertemporal allocation of the adjustment cost. But then equations (25), (27) and (28) together imply that hours are constant if $\phi = 0$, otherwise they adjust only in the first period. Since hours adjust only once, ϕ must be high to generate a high volatility of hours, but then the standard deviations of employment and consumption become too low, and the standard deviation of investment becomes too high. Also, this model generates a negative correlation of hours with employment, while in the data it is positive and close to zero.

These problems can be overcome by the choice of nonseparable preferences. By assuming that the utility function of each agent is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^{\mu} l_t^{1-\mu}\right)^{1-\theta} - 1}{1-\theta} \right]$$

the optimality conditions become:

$$\left(c_t^u\right)^{\mu-\mu\theta-1} = \left(c_t^e\right)^{\mu-\mu\theta-1} \left(\psi - h_t\right)^{(1-\mu)(1-\theta)} \qquad (29.)$$

$$\left(c_t^u\right)^{\mu-\mu\theta-1} = \beta E_t \left[\left(c_{t+1}^u\right)^{\mu-\mu\theta-1} \left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{t+1} - \delta\right)\right] \qquad (30.)$$

$$\frac{\left(c_t^u\right)^{\mu(1-\theta)} - 1}{1-\theta} - \frac{\left[\left(c_t^e\right)^{\mu} \left(\psi - h_t\right)^{(1-\mu)}\right]^{(1-\theta)} - 1}{1-\theta} - \mu\left(c_t^u\right)^{\mu-\mu\theta} + \mu\left(c_t^e\right)^{\mu-\mu\theta} \left(\psi - h_t\right)^{(1-\mu)(1-\theta)} = \mu\left(c_t^u\right)^{\mu-\mu\theta-1} (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} h_t - \mu\left(c_t^u\right)^{\mu-\mu\theta-1} \phi\left(n_t - n_{t-1}\right) - \beta E_t \left[\mu\left(c_{t+1}^u\right)^{\mu-\mu\theta-1} \phi\left(n_{t+1} - n_t\right)\right] \qquad (31.)$$

Table 16 reports the results for the model with nonseparable preferences. Parameter values are calibrated as before, μ is calibrated so as to match the employment level in steady state, and the case $\theta = 2$ is excluded because it introduced complex roots in the approximated solutions. When preferences are nonseparable, hours do not enter the optimality conditions separately from consumption, therefore, they must follow a smooth path, and not being adjusted one period only. However, the model in unsuccessful in generating a standard volatility of hours much higher than employment. As with separable preferences, the increase in the volatility of hours takes place at the expenses of the volatility of employment.

Table 16: Model results with nonseparable preferences

	Relative standard deviation			
	n	h	c	i
$\theta = 2$, $\gamma = 0$				
$\phi = 0$	0.48	0.02	0.58	2.65
$\phi = 1$	0.27	0.13	0.58	2.65
$\phi = 1.5$	0.24	0.18	0.59	2.64
$\phi = 2$	0.22	0.19	0.59	2.64
$\phi = 2.5$	0.20	0.20	0.60	2.63
$\phi = 3$	0.18	0.21	0.60	2.63

VI. Underground economy and firing costs

One of the reasons why the participation rate in Italy is low compared to the other countries may be the existence of a large underground economy sector. The existence of an alternative to registered market activities may provide also an explanation for the high volatility of hours observed in the data. Household in Italy may have a high intertemporal elasticity of substitution of hours because, by switching from the market sector to the underground economy, they can moderate the volatility of total hours worked. As in the previous section, hiring and firing costs prevent firms to adjust fully the level of employment in response to a shock, but households can allocate their time between leisure, registered activities and unregistered activities.

1. *The model*

There is a continuum of measure one agents, equally endowed with one unit of time. The utility function of each agent is

$$U = E_0 \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right]$$

where l_t is leisure and c_t is a composite consumption good:

$$c_t = \left\{ a c_{Rt}^{\sigma} + (1-a) c_{Ut}^{\sigma} \right\}^{\frac{1}{\sigma}} \tag{32.}$$

c_{Rt} denotes consumption of goods produced in the registered sector and c_{Ut} is consumption of goods produced in the underground economy, $\frac{1}{1-\sigma}$ gives the elasticity of substitution in consumption between the two.

As in the previous section, there is a fixed the cost of participation to registered activities, while there is no participation cost for unregistered activities. Employed agents are those working in the registered sector. Both the unemployed and the employed participate to unregistered activities: h_{uUt} is time spent by the unemployed in unregistered activities, and h_{eUt} and h_{eRt} denote time allocated by the employed to, respectively, unregistered and registered activities. Unemployment is perfectly insured, therefore agents receive the same wage whether they work or not. n_t is the probability of being employed, $1-n_t$ is the probability of being unemployed: by the Law of Large Numbers, since there exists a continuum of agents of measure one, n_t is also the fraction of employed agents, that is, those working in registered activities.

Output in registered activities is given by

$$y_{Rt} = z_{Rt} k_{Rt}^{\alpha} n_t^{1-\alpha} h_{eRt} \tag{33.}$$

where z_{Rt} is the exogenous technology shock and k_{Rt} is capital used in registered activities. All agents have access to production in unregistered activities, which takes place according to a constant elasticity production function:

$$y_{Ut} = z_{Ut} \left\{ \eta k_{Ut}^{\xi} + (1-\eta) h_{Ut}^{\xi} \right\}^{\frac{1}{\xi}} \tag{34.}$$

where z_{Ut} is the exogenous shock, specific to the unregistered sector. Capital and labour used in unregistered activities are k_{Ut} and $h_{Ut} = (1-n_t) h_{uUt} + n_t h_{eUt}$. $\frac{1}{1-\xi}$ gives the elasticity of substitution between capital and labour. It is assumed that all output produced in the underground sector is homogeneous to output produced in the registered sector. In any given period it can be consumed or stored and transformed into capital. Consumption is allocated between the employed and the unemployed according to the equation:

$$c_t = (1-n_t) c_{ut} + n_t c_{et} \tag{35.}$$

Total capital is the sum of capital used in the registered and unregistered activities, and it evolves accordingly to the following laws of motion:

$$k_{Rt+1} = (1-\delta_R) k_{Rt} + i_{Rt} \tag{36.}$$

$$k_{Ut+1} = (1-\delta_U) k_{Ut} + i_{Ut} \tag{37.}$$

δ_R and δ_U are depreciation rates on the two types of capital. k_{R0} and k_{U0} are given, as well as employment at time –1, n_{-1}.

Firms pay adjustment costs whenever they lay off workers or recruit new ones:

$$AC_t = \frac{\phi}{2} (n_t - n_{t-1})^2$$

Output produced in the underground economy is tradeable and it is sold at the same price as output produced in registered activities. The aggregate budget constraint is

$$c_{Rt} + c_{Ut} + i_t = z_{Rt} k_{Rt}^{\alpha} n_t^{1-\alpha} h_{eRt} + z_{Ut} \left\{ \eta k_{Ut}^{\xi} + (1-\eta) h_{Ut}^{\xi} \right\}^{\frac{1}{\xi}} - \frac{\phi}{2} (n_t - n_{t-1})^2 \tag{38.}$$

where $i = i_R + i_U$. The logarithms of the exogenous shocks follow two distinct AR(1) processes:

$$\log z_{Rt+1} = \rho_R \log z_{Rt} + \varepsilon_{Rt+1} \tag{39.}$$

$$\log z_{Ut+1} = \rho_U \log z_{Ut} + \varepsilon_{Ut+1} \tag{40.}$$

Because markets are complete, the welfare theorems hold. Hence the conditions that define the optimal path of this model economy are the solution to the social planner's problem, which maximises the sum of agents' utilities:

$$\max \quad E_0 \quad \sum_{t=0}^{\infty} \beta^t \left\{ (1-n_t)\left[\frac{(c_{ut})^{1-\theta}-1}{1-\theta} + A\frac{(1-h_{uUt})^{1-\gamma}-1}{1-\gamma}\right] + n_t\left[\frac{(c_{et})^{1-\theta}-1}{1-\theta} + A\frac{(\psi-h_{eRt}-h_{eUt})^{1-\gamma}-1}{1-\gamma}\right]\right\}$$

$s.t.$ the constraints in (32)-(40).

Optimality requires the marginal utilities for consumption and leisure of both the employed and the unemployed to be equal, therefore, since preferences are separable, employed and unemployed agents consume the same amount of composite good and enjoy the same level of leisure. The first order conditions can be written as follows:

$$a c_{Rt}^{\sigma-1} = (1-a) c_{Ut}^{\sigma-1} \tag{41.}$$

$$A(1-h_{uUt})^{-\gamma} = c_t^{1-\theta-\sigma} a c_{Rt}^{\sigma-1} z_{Rt} k_{Rt}^{\alpha} n_t^{-\alpha} \tag{42.}$$

$$z_{Rt} k_{Rt}^{\alpha} n_t^{-\alpha} = c_{Ut}^{1-\xi} z_{Ut}^{\xi} (1-\eta) h_{Ut}^{\xi-1} \tag{43.}$$

$$c_t^{1-\theta-\sigma} c_{Rt}^{\sigma-1} = \beta E_t \left[c_{t+1}^{1-\theta-\sigma} c_{Rt1}^{\sigma-1} \left(1+\alpha z_{Rt+1} k_{Rt+1}^{\alpha-1} n_{t+1}^{1-\alpha} h_{eRt+1} - \delta_R\right)\right] \tag{44.}$$

$$c_t^{1-\theta-\sigma} c_{Rt}^{\sigma-1} = \beta E_t \left[c_{t+1}^{1-\theta-\sigma} c_{Rt1}^{\sigma-1} \left(1+\frac{\eta}{1-\eta} z_{Rt+1} k_{Rt+1}^{\alpha} n_{t+1}^{-\alpha} \left(\frac{k_{Ut+1}}{h_{Ut+1}}\right)^{\xi-1} - \delta_H\right)\right] \tag{45.}$$

$$c_t^{1-\theta-\sigma} c_{Rt}^{\sigma-1} \left[\phi(n_t - n_{t-1}) - z_{Rt} k_{Rt}^{\alpha} n_t^{-\alpha}(\psi - \alpha h_{eRt} - 1)\right] - \beta E_t \left[c_{t+1}^{1-\theta-\sigma} c_{Rt1}^{\sigma-1} \phi (n_{t+1} - n_t)\right] = 0 \tag{46.}$$

2. *Calibration*

Because of the lack of statistics on the underground economy only a subset of the parameters may be estimated on the basis of a priori information. There is no information about the degree of substitutability between labour and capital in unregistered activities, or on the substitutability in consumption of goods coming from different sectors. The performance of the model and its ability to capture real world statistics can therefore vary a lot, but this does not constitute a limitation of the model, because several alternative versions can be tested against each other, according to alternative parameter specifications.

The capital share in registered activities α, the discount rate β, and the depreciation rate δ_R are set at the same level used in the previous sections. During simulation exercises, no interesting features emerged by varying the intertemporal elasticities of substitution of leisure and consumption, therefore they have been set to $\theta = 1$ and $\gamma = 2$, because these values were used in the other sections and seemed to match better the moments in the data. The depreciation rate of capital used in the underground sector is supposed equal to the one of capital employed in registered activities.

The four parameters $A, a, \psi,$ and η are obtained from the steady state equations in order to match four observations: the fraction of population that works n, the fraction of time devoted to registered activities h_{eR} and unregistered activities h_{eU} by employed households, and the capital /hours ratio in the underground economy. h_{eU} and k_U / h_U are taken from Greenwood, Rogerson, and Wright (1995), n and h_{eR} are set as in the previous calibration exercises. The autocorrelation coefficients ρ_R and ρ_U are set equal to 0.95, as in much of the literature. As in the models of the previous sections, the standard deviation of the exogenous shocks is calibrated so as to match the standard volatility of output. It is also assumed that the shock in the underground sector mimics the shock in the registered sector ($\delta_R = \delta_U$). The parameter values and the steady state observation used for calibrating the model are reported in Table 17.

Table 17: Values employed in the calibration

α	β	$\delta_R = \delta_U$	θ	γ	n	h_{eR}	h_{eU}	k_U / h_U	$\rho_R = \rho_U$
0.33	0.99	0.0088	1	2	0.37	0.32	0.25	11.63	0.95

Two preference parameters remain to be specified, ξ, σ and the correlation between the innovations ε_{Rt} and ε_{Ut}. There is little evidence to be used as a guide in the choice of these parameters, but several alternatives can be formulated and tested against each other, so that the behaviour of the model can be depicted under some selected scenery.

Table 18 lists some summary statistics for several versions of the model. To allow comparison with the data, investment is investment in the two capital stocks, consumption is consumption of goods produced in the registered sector and hours worked are those in the registered sector. The magnitude of the standard deviations relative to output of the variables in the model depends crucially on three aspects: the incentive to move across sectors, as measured by the correlation between shocks, the willingness to move, as measured by the parameter σ, and the degree of substitutability between labour and capital in the underground economy, as measured by the parameter ξ. Table 18 reports the results of simulations with a common adjustment cost of 0.2, and different, “extreme” values of those parameters. Higher or lower values introduced complex roots in the approximated solution, and therefore they were discarded.

Table 18: Evaluation of the model for different parameter values

	Relative standard deviation			
	n	h_{eR}	c_R	i
Model 1	0.98	0.22	0.43	1.17
Model 2	0.99	0.44	0.37	0.98
Model 3	0.89	0.18	0.43	3.77
Model 4	1.00	0.30	0.20	3.46

All models use $\alpha = 0.33$, $\beta = 0.99$, $\delta_R = \delta_U = 0.0088$, $\theta = 1$, $\gamma = 2$, $\phi = 0.2$, $\rho_R = \rho_U = 0.95$, a A ψ and η determined so that $n = 0.37$, $h_{eR} = 0.32$, $h_{eU} = 0.25$, $k_U / h_U = 11.63$.
Model 1 sets $\sigma = 0.9$, $\xi = -0.6$ and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to –1.
Model 2 sets $\sigma = 0.9$, $\xi = -2$ and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to –1.
Model 3 sets $\sigma = 0.9$, $\xi = -0.6$ and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to 1.
Model 4 sets $\sigma = 0.9$, $\xi = -2$ and the correlation between the innovations ε_{Rt} and ε_{Ut} equal to 1.

Since low values of σ generated too low volatility of consumption, all the models in Table 18 employ the same high elasticity of substitution in consumption between goods coming from different sectors. Models 1 and 2 represent the cases of high incentives to move across sectors, because the correlation between shocks is equal to minus one, and Models 3 and 4 of low incentives, because the correlation between shocks is equal to one. Model 1 and 3 describe a situation of high substitutability of capital and hours in the underground economy, whereas Models 2 and 4 a situation of low substitutability.

The effect of incentives to move across sectors on the volatility of the labour input is entirely predictable: when incentives to move are minimal there is less volatility of labour. Table 18 simply states that when the incentives increase labour adjustment is made through hours when hours and capital are low substitutes, and though employment when hours and capital are high substitutes. However when the correlation between shocks is equal to minus one there is less volatility of investment. This happens because investment creates capital for future periods, but when shocks are negatively correlated and persistent there is less need to change total investment, because investment in one of the capital stocks can simply take the place of the other.

Different elasticities of substitution between capital and hours in the underground economy have an impact on hours, consumption and investment. When substitutability is low there must be a larger increase in hours worked after a shock in the underground economy, therefore the volatility of hours is higher. As in the models of the previous sections, when the labour supply is more rigid there is less consumption smoothing. When substitutability is high adjustment can be made through investment in unregistered activities instead of hours, and this explains why the volatility of investment is higher. In conclusion, substitutability, incentives and willingness to move have different effects on the fluctuations of the variables of the model, and trying to reproduce the Italian data is difficult, as for example an increase in the volatility of hours can be made only at the expenses of that of consumption.

The ability of the model to capture real-world statistics can be tested by making some choice for the parameters values. Since the main feature of the Italian business cycle is the high volatility of hours with respect to employment, parameters were chosen with the aim to match as closely as possible this empirical fact, avoiding at the same time to generate a too low volatility of consumption and too high volatility of investment. Results obtained for that particular parametrization are in Table 19.

Table 19: Standard deviations and correlations with output

	Relative standard deviation			
	n	h_{eR}	c_R	i
ϕ = 0.30	0.96	0.63	0.33	3.23
ϕ = 0.31	0.94	0.76	0.31	3.08
ϕ = 0.32	0.88	0.94	0.27	2.78
ϕ = 0.33	0.75	1.17	0.17	2.22

Model sets σ = 0.9, ξ = -1, the correlation between the innovations ε_{Rt} and ε_{Ut} equal to 1, and all the other parameters as in Table 18.

Results are very sensitive to the choice of ϕ, but higher values of the adjustment cost introduced complex roots in the model. The model performs better than the one in the previous section in matching the relative volatility of investment, but the increase in the volatility of the labour input is done at the expenses of the volatility of consumption, which becomes too low.

VI. Conclusions

A thorough investigation of business cycles in Italy highlighted some differences that constitute yet another interesting test for RBC theory. Unfortunately, information on hours worked is unsatisfactory, and this limits the possibilities of research within the RBC framework.

Hansen's indivisible labour model went quite close to matching the relative volatility of employment and investment, but could not reproduce the other moments of the data. This happened because the RBC model relied on a single shock as the only source of fluctuations, and on a propagation mechanism that depended essentially on the form of the utility function. The result is that standard deviations and other second-order moments are always combinations of parameters with the variances of innovations. The introduction of an additional shock in a small open economy model generated an increased volatility of hours without the need to assume a large value for the intertemporal elasticity of substitution of leisure.

The Italian labour market is more rigid than the US one, a feature that can be captured in a RBC model in a number of ways. Under some conditions, the introduction of firing costs increased the volatility of hours, but a closer match with data will probably require a departure from a purely competitive labour market.

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Figure 1: Closed Economy Model, $\theta = 1$, $\gamma = 0$.

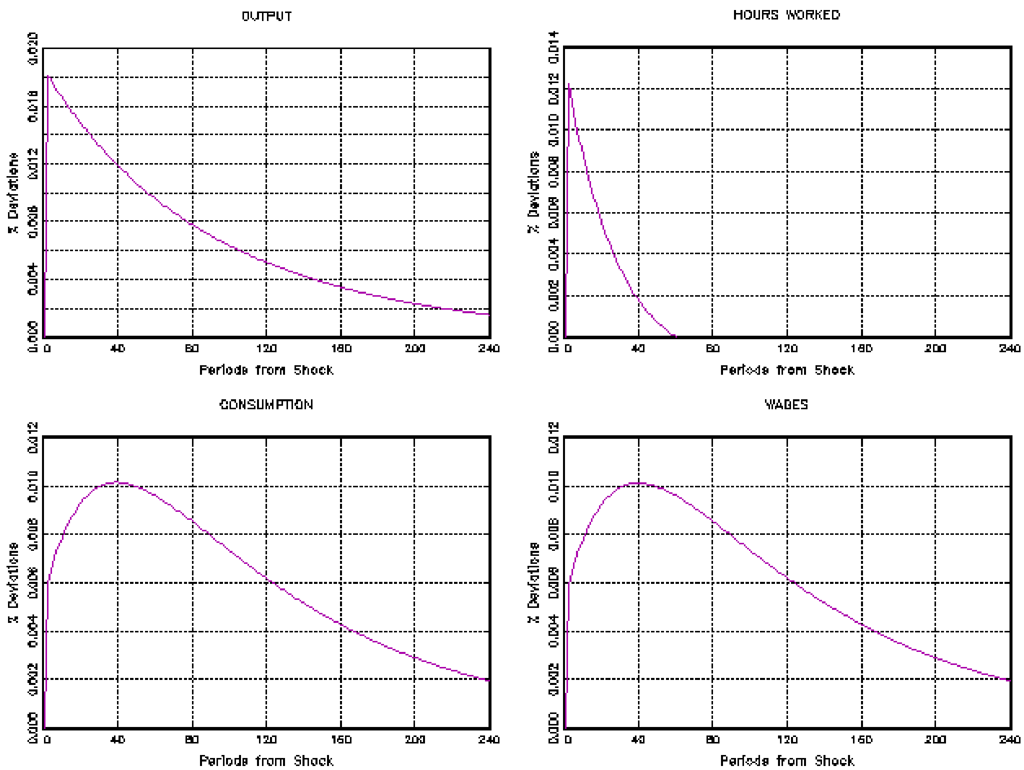

Figure 2: Closed Economy Model, $\theta = 1$, $\gamma = 0.5$.

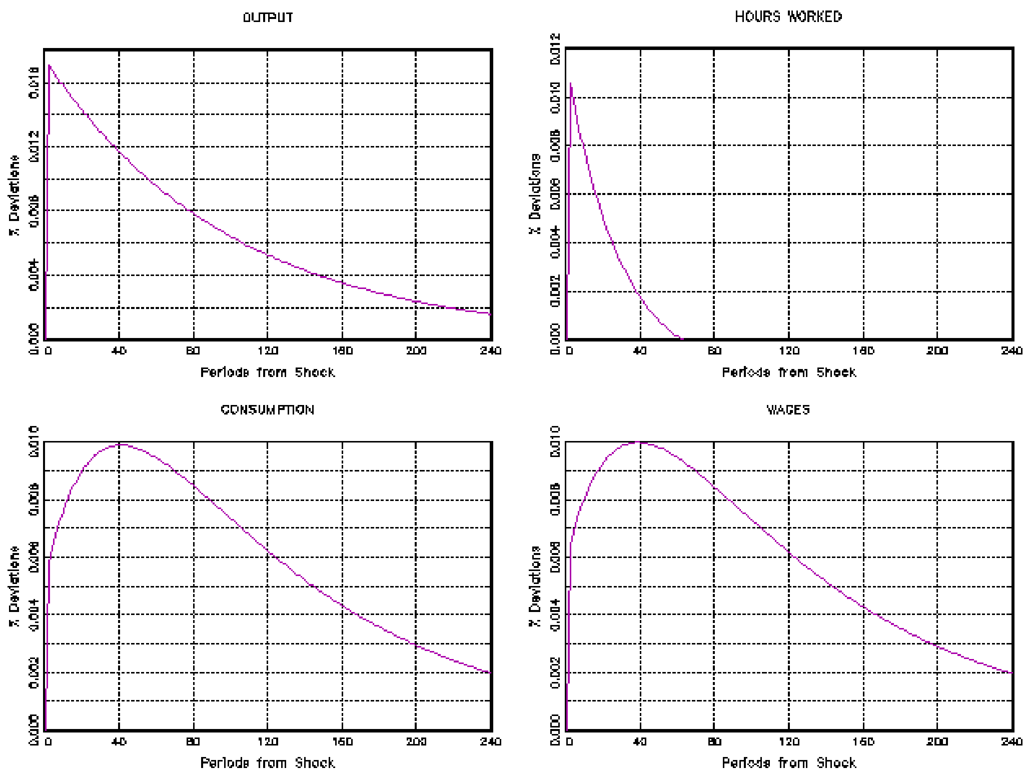

Figure 3: Closed Economy Model, $\theta = 1$, $\gamma = 1$.

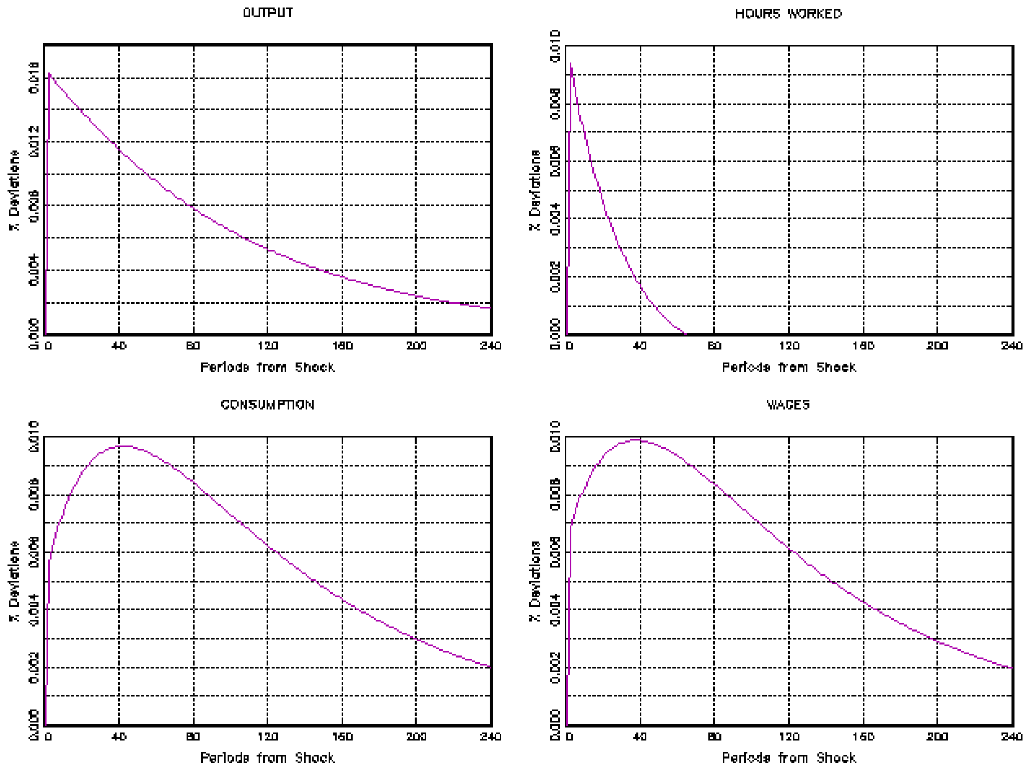

Figure 4: Open Economy Model, $\theta = 1$**,** $\gamma = 0.5$: Technology Shock.

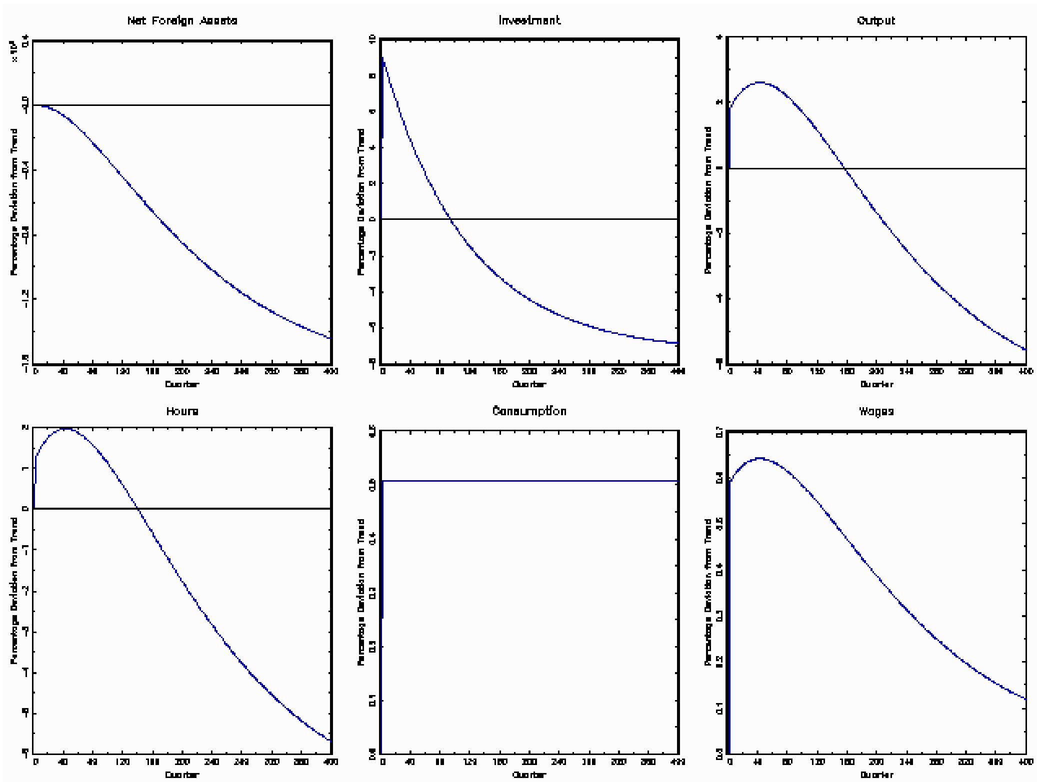

Figure 5: Open Economy Model, $\theta = 1$, $\gamma = 0.5$: Interest Rate Shock.

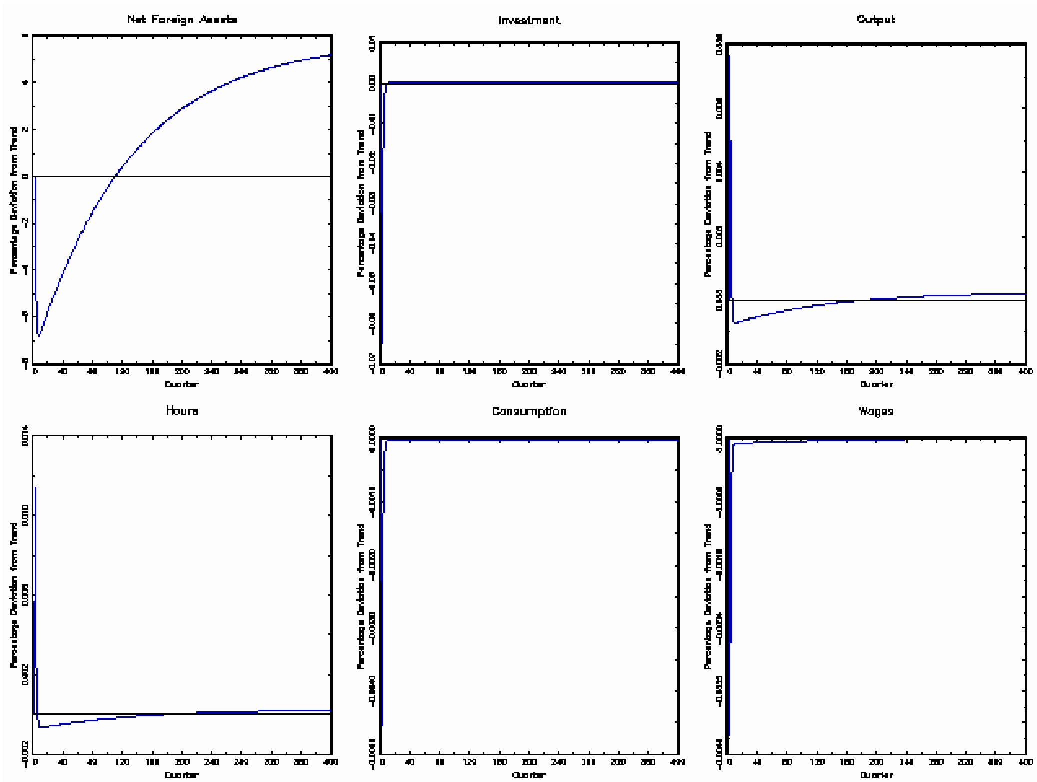

Figure 6: Open Economy Model, $\theta = 1$**,** $\gamma = 1$: Technology Shock.

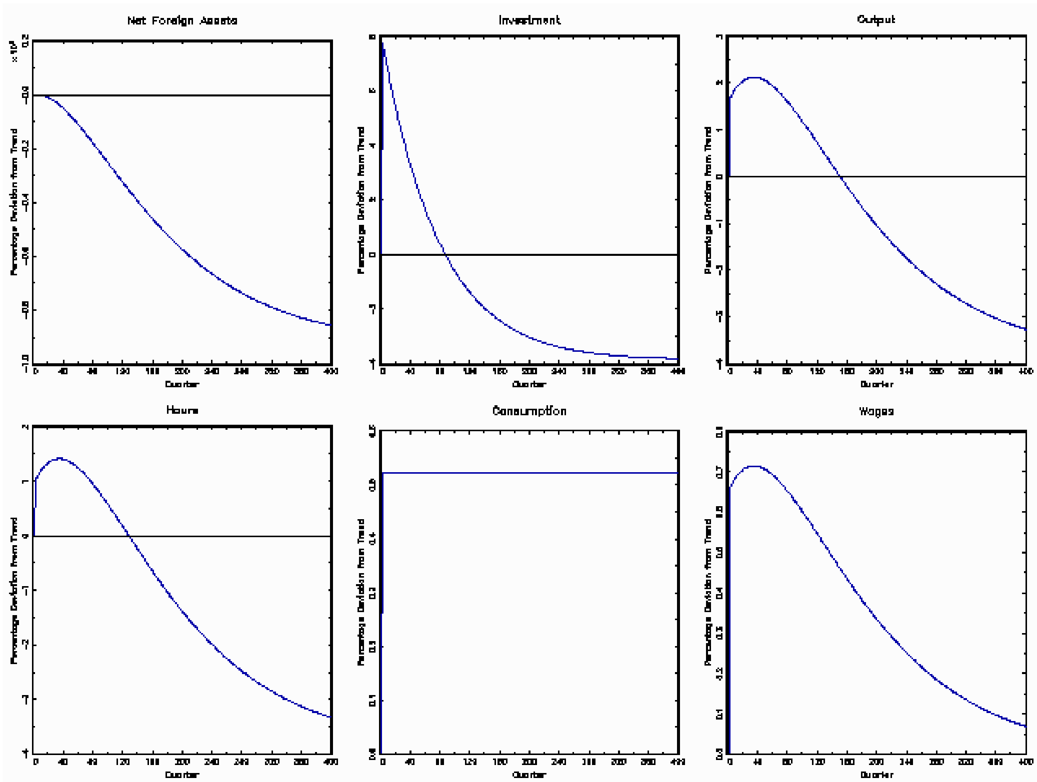

Figure 7: Open Economy Model, $\theta = 1$**,** $\gamma = 1$: Interest Rate Shock.

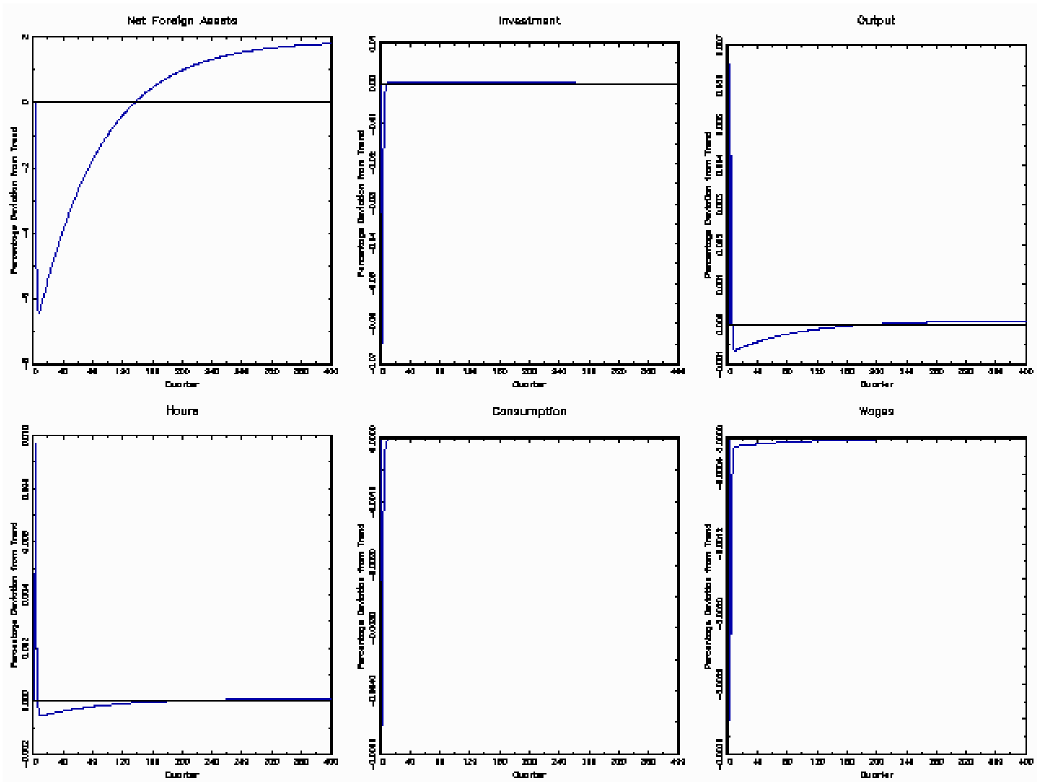

Figure 8: Labour Adjustment Cost Model, $\theta = 2$**,** $\gamma = 0$, $\phi = 1$.