Risk, Pollution Abatement and Endogenous Growth

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Abstract

Pollution and abatement aspects are analyzed within a stochastic model of endogenous growth. Pollution is inevitably connected to capital accumulation and can be reduced through abatement expenditures. Incomplete perception of individual influence on pollution is parameterized and induces suboptimal abatement and consumption ratios in market equilibrium. Decentralized growth deviates from optimal growth, and an acceleration of environmental degradation leads to counter working growth effects.

It is shown that the optimal level of pollution as well as optimal growth are influenced in ambiguous way by uncertainty, depending on the degree of relative risk aversion. Optimal fiscal policy is derived and consists of income and consumption taxation as well as a subsidy on individual abatement expenditures. Due to partial individual sense of responsibility for environmental decay, the optimal structure of fiscal policy is highly sensitive with respect to environmental and preference parameters.

Keywords: pollution, endogenous growth, uncertainty, taxation

JEL–Classification: D8, D9, H2, O1, O4, Q2.

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1 Introduction

Within the analysis of sustainable development, ongoing growth usually is inevitably linked to environmental degradation. On the other hand, the growth process fosters abatement activities which reduce pollution. Both aspects, pollution caused by production and reduced by abatement, are analyzed within a stochastic endogenous growth model. Risk is incorporated to the analysis, as it is an important feature of environmental problems: Future environmental consequences of present actions are uncertain. Risk affects individual decisions as well as the impact of any environmental policy.

Various previous studies analyze the impact of environmental issues on the endogenous growth path, e. g. Gradus and Smulders (1993), Ligthart and van der Ploeg (1994), Bovenberg and Smulders (1997), Jones and Manuelli (1995), Byrne (1997) or Stokey (1998). In these contributions two main questions are addressed. First, the papers analyze if environmental maintenance is consistent with ongoing growth. Second, environmental policies which enable sustainable growth as an outcome of market equilibrium, are analyzed. Aghion and Howitt (1998) summarize that "[...] the problem of finite, nonrenewable natural resources [...] appears to be less of an obstacle to sustainable development than is the problem of environmental pollution." As long as pollution is an inescapable by–product of the consumed good, there is a trade–off between consumption and pollution which limits optimal growth. In the long run, optimal growth can even cease, if the environmental costs are sufficiently high (see e. g. Stokey, 1998). In contrast, environmental preferences have no effect on long–run growth if there is non–polluting human capital accumulation or an abatement technology as e. g. in the setting of Gradus and Smulders (1993) or Byrne (1997).

An important extension of this paper is the introduction of uncertainty. Although risk is an essential characteristic of environmental degradation, there are few contributions which focus on the impact of uncertainty on pollution and abatement. Clarke and Reed (1994) analyze the risk of an environmental catastrophe and Baranzini and Bourguignon (1995) discuss the impact of environmental decease on the probability of survival. The implications of uncertainty about future preferences for the optimal preservation of environmental assets are shown by Beltratti et al. (1998), whereas Chichilnisky and Heal (1998) focus on unknown risks. In the model presented here, pollution evolves stochastically due to an aggregate productivity shock. The consideration of uncertainty is important, because it changes fundamentally the individual intertemporal decision as well as any decisions with respect to pollution and abatement. Under uncertainty, any fiscal or environmental policy affects not only expected values of economic variables but also their volatility. Hence, risk averse individuals additionally respond to this change in uncertainty within their savings decision. The counter working impact of fiscal policy on long run growth under uncertainty was demonstrated first by Eaton (1981) and taken up in the endogenous growth setting e. g. by Turnovsky (1993, 1995, 2000), Smith (1996), Corsetti (1997) or Clemens and Soretz (1997).

Furthermore, the perception of the individual influence on pollution is parameterized. That is, partial individual sense of responsibility for environmental degradation is incorporated to the model. This extension leads to grave implications for equilibrium growth as well as optimal fiscal policy. Environmental decay induces counter working income and substitution effects on equilibrium abatement ratio and growth. Additionally, second order effects associated with uncertainty, have to be taken into account. Therefore, equilibrium growth and the structure of optimal fiscal policy depend crucially on the underlying environmental and preference parameters. The assumption of parameterized perception draws back on the setting of congestion effects within the public goods literature, as e. g. used by Edwards (1990), Glomm and Ravikumar (1994) or Turnovsky and Fisher (1998).

The paper is organized as follows: After an introduction to the assumptions in section 2, market equilibrium is derived in part 3. The influence of (perceived) environmental degradation as well as the impact of uncertainty are analyzed. It is shown that the growth effect of pollution is ambiguous and highly sensitive to the parameter setting. Optimal environmental policy, which consists of income and consumption taxation as well as a subsidy on individual abatement expenditures, is determined in section 4. The sensitivity of expected growth with respect to environmental and preference parameters is reflected within a highly sensitive structure of optimal fiscal policy which is illustrated numerically. Section 5 gives a short summary.

2 The model

Pollution is related to the production of the single homogenous good and can be reduced through abatement activity. Since pollution is modeled as a flow variable, I refer to pollutants which dissolve rather quickly. The pollution function recurs on Gradus and Smulders (1993): The level of pollution depends on the relation between physical capital and abatement expenditure. In contrast, e. g. Forster (1973), Van der Ploeg and Withagen (1991), Stokey (1998) or Jones and Manuelli (2001) define output to be the source of pollution and in e. g. Luptacik and Schubert (1982) as well as Van der Ploeg and Withagen (1991) pollution as a stock variable is considered. It is straight forward that with the linear production technology assumed here, the outcomes are independent of capital or output to be the pollutant. Furthermore, with the usual additional assumptions the results remain unchanged with a stock of pollution.

Pollution, $P(t)$, is assumed to depend on the ratio between aggregate capital, $K(t)$, and aggregate abatement effort, $E(t)$

$$P(t) = \left(\frac{E(t)}{K(t)}\right)^{-\alpha} \qquad \alpha > 0 \quad . \tag{1}$$

In this paper, the assumptions about pollution are extended as the perception of the individual influence on aggregate pollution is parameterized: The relevant ratio between abatement and capital is perceived to depend on the ratio between aggregate abatement and aggregate capital, on the one hand, and the ratio between individual abatement activities, $e(t)$, and individual capital, $k(t)$, on the other hand

$$\eta_p = \left(\frac{E(t)}{K(t)}\right)^{\delta} \left(\frac{e(t)}{k(t)}\right)^{1-\delta} \qquad \delta \in [0,1) \quad . \tag{2}$$

η_p denotes the perceived relation between pollution control and capital stock, $(E/K)_p$.[1] The parameter δ defines the extent to which the agents perceive pollution to be exogenous to their individual decisions about capital accumulation

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[1] The underlying formulation of perceived pollution is based on the discussion of the earlier version $\eta_p = E/(K^{\delta}k^{1-\delta})$ presented at the *Conference on Risk and Uncertainty in Environmental and Resource Economics 2002* in Wageningen. I am grateful to Ana Balcão Reis for this suggestion.

and abatement effort. The setting of perception in equation (2) relies on the formulation of congestion effects in the public goods literature (see e. g. Edwards, 1990; Glomm and Ravikumar, 1994). In these lines, the parameter δ could also denote the joint degree of rivalry of capital and abatement in the "production" of pollution, (see Turnovsky, 1995, p. 405).

Since all agents are identical and population size is normalized to unity, individual and aggregate values are equal in equilibrium. Nevertheless, within individual optimization aggregate capital as well as aggregate abatement are given exogenously. Hence, the perception parameter δ is a measure for the consciousness of individual influence on pollution.

The polar case $\delta = 0$ reflects perfect individual knowledge about pollution.[2] $\delta = 1$, on the other hand, corresponds to the case where individuals perceive pollution to be completely exogenous to individual decisions. For a perception parameter between zero and one, part of the individual influence on pollution is taken into account.

Alternatively, the degree of perception could be interpreted as degree of responsibility for environmental concerns. Individuals with higher sense of responsibility for the environment (lower δ) give more weight on their own activities in the determination of aggregate pollution. In spite of the infinitesimal individual influence on pollution, which results out of the assumption of a continuum of households in the economy, agents with low δ act as if they would trust on the responsible behavior of other agents.

Physical capital produces the homogenous good according to the linear individual stochastic production function

$$f(k(t)) = Ak(t)(dt + \sigma dz(t)) \tag{3}$$

which recurs on Rebelo (1991) in the deterministic setting and on Eaton (1981) in the stochastic version. The linear technology is chosen as it enables constant marginal productivity of capital without production externalities. Hence, the focus is on environmental market failures. Uncertainty is incorporated into the model

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[2] More precise formulated, $\delta = 0$ denotes perfect knowledge about the homogeneity of the individuals.

through the productivity shock $dz(t)$, which is a Wiener process with $dz \sim N(0, dt)$. Expected capital productivity is given by A.

There is a continuum of homogenous individuals which have environmental preferences and maximize intertemporal expected utility. Environmental quality becomes relevant as pollution enters individual utility. This widely used formulation relies on the early approaches of Forster (1973), Gruver (1976) or Luptacik and Schubert (1982) who analyze environmental aspects within neoclassical growth models and later was taken up e. g. by Smulders and Gradus (1996), Mohtadi (1996), Byrne (1997) or Stokey (1998) within the endogenous growth setting.

The individuals are assumed to live infinitely long and to have additively separable preferences. Hence, intertemporal utility results in

$$U = \mathrm{E}_0 \left[\int_0^\infty e^{-\rho t} u(c(t), P(t))\, dt \right] \tag{4}$$

with the constant rate of time preference $\rho > 0$ and the expected value conditional on time 0 information E_0. One could think of a long–lived dynasty, because altruism between generations is very plausible in the context of environmental issues.

Instantaneous utility, u, is assumed to be of the constant relative risk aversion type

$$u(c(t)) = \begin{cases} \dfrac{(c(t)P(t)^{-\gamma})^{1-\varepsilon}}{1-\varepsilon} & \text{for} \quad \varepsilon \neq 1 \\ \ln c(t) - \gamma \ln P(t) & \text{for} \quad \varepsilon = 1 \end{cases} . \tag{5}$$

where $\gamma > 0$ denotes the environmental preference parameter and $\varepsilon > 0$ represents the degree of relative risk aversion.

Environmental policy consists of income and consumption taxation at the constant rates τ_y and τ_c and a constant subsidy rate on individual abatement expenditures, τ_e. In the following section, the resulting macroeconomic equilibrium is derived. Afterwards, conditions for optimal environmental policy are determined and illustrated numerically.

3 Equilibrium growth and pollution abatement

Individuals are confronted with a trade–off between consumption, capital accumulation and pollution control. Hence, they decide about consumption and individual abatement effort together with capital accumulation in order to maximize intertemporal expected utility with given initial values for physical capital and the productivity shock. Government activity as well as aggregate variables are considered exogenous throughout utility maximization and pollution is perceived to depend on individual behavior as defined in (2). Building on the assumptions made in the last section, capital evolves according to

$$dk = [(1-\tau_y)Ak - (1+\tau_c)c - (1-\tau_e)e] + (1-\tau_y)Ak\sigma dz \tag{6}$$

and due to the properties of the stochastic disturbance, the variance of capital results in

$$\sigma_k^2 = \frac{\mathrm{E}[dk^2] - \mathrm{E}[dk]^2}{dt} = (1-\tau_y)^2 A^2 k^2 \sigma^2 \quad . \tag{7}$$

Furthermore, the additive separability of intertemporal utility leads to a time–separable specification of the value function given by $e^{-\rho t}J(k(t))$. According to Itô's Lemma, the stochastic Bellman equation now can be written as

$$\mathcal{B} = e^{-\rho t}u(c,P) - \rho e^{-\rho t}J(k) + e^{-\rho t}J'(k)\frac{\mathrm{E}[dk]}{dt} + \frac{1}{2}e^{-\rho t}J''(k)\,\sigma_k^2 \quad . \tag{8}$$

To solve the optimization problem of the individual, maximization is done with respect to consumption, abatement and capital. Individual choice about the level of pollution control, e, is due to the formulation of perceived pollution in equations (1) and (2). Most studies which analyze the impact of pollution on growth assume that the agents neglect their individual contribution to aggregate environmental restauration completely. Hence, optimal individual abatement activity is zero. In my model, that assumption corresponds to the special case where the perception parameter is set $\delta = 1$. Nevertheless, if $\delta < 1$ applies, the agents feel at least partially responsible for their impact on pollution. There is an individual choice about abatement activity and individually optimal environmental expenditures are positive (although not pareto–optimal). Maximization with respect to consumption and environmental care together with capital accumulation leads to the necessary conditions

$$c^{-\varepsilon}P^{-\gamma(1-\varepsilon)} \overset{!}{=} (1+\tau_c)J'(k) \tag{9}$$

$$\alpha\gamma(1-\delta)c^{1-\varepsilon}P^{-\gamma(1-\varepsilon)}e^{-1} \overset{!}{=} (1-\tau_e)J'(k) \tag{10}$$

$$\begin{aligned} &-\alpha\gamma(1-\delta)c^{1-\varepsilon}P^{-\gamma(1-\varepsilon)}k^{-1} + J'(k)((1-\tau_y)A-\rho) + \\ &\qquad +J''(k)\left(\frac{\mathrm{E}[dk]}{dt} + (1-\tau_y)^2A^2k\sigma^2\right) + \frac{1}{2}J'''(k)\sigma_k^2 \overset{!}{=} 0 \end{aligned} \tag{11}$$

Equation (9) in combination with (10) equalizes marginal utility out of consumption and abatement as perceived by the individuals. In case of perfect anticipation of the individual influence on pollution ($\delta = 0$), static efficiency is determined through the combination of these conditions. If $\delta < 1$, marginal utility of pollution control is underestimated and capital accumulation is accompanied by a negative externality. Condition (11) assures the equality of instantaneous marginal utility across time and leads to individually optimal capital accumulation.

Additionally, the transversality condition must be satisfied in order to assure feasible consumption paths

$$\lim_{t\to\infty} \mathrm{E}\,\left[e^{-\rho t}J(k)\right] = 0 \quad . \tag{12}$$

With the linear technology considered here, the transversality condition is equivalent to the condition for a positive consumption ratio, if growth is pareto–optimal (see Merton, 1969). As long as equilibrium growth deviates from optimal growth, only parameter settings are considered, which additionally satisfy the transversality condition.

Malliaris and Brock (1982, p. 178) show that there exists a closed form solution to the system (9) to (11) as relative risk aversion is assumed time invariant (see equation (5)), the marginal product of capital is assumed to be constant (see equation (3)) and the variance of capital is proportional to the square of capital (see equation (7)). In this case, there exists a steady state with constant expected growth. Definition of consumption ratio μ and abatement ratio η according to

$$\mu = \frac{c}{k} \quad \text{and} \qquad \eta = \frac{e}{k} \tag{13}$$

which are both constant in steady state, together with the equality of individual and aggregate variables, leads to the following conditions for individually optimal consumption and abatement decisions

$$\begin{aligned}(1+\tau_c)\mu &= \frac{\rho}{\varepsilon(1-\vartheta)} + \frac{\varepsilon-1}{\varepsilon}(1-\tau_y)A - \frac{\varepsilon(1-\vartheta)+\vartheta}{\varepsilon(1-\vartheta)}(1-\tau_e)\eta \\ &\quad -\frac{\varepsilon-1}{2\varepsilon}(\varepsilon(1-\vartheta)+\vartheta)(1-\tau_y)^2A^2\sigma^2 \qquad (14) \\ (1-\tau_e)\eta &= \vartheta(1+\tau_c)\mu \quad . \qquad (15)\end{aligned}$$

The parameters α, γ and δ appear jointly in both equations and are summarized for notational convenience within $\vartheta = \alpha\gamma(1-\delta)$. ϑ increases as environmental decay accelerates (increasing α), environmental preferences gain importance (increasing γ) or responsibility for pollution rises (decreasing δ). To ensure feasibility, ϑ has to be below unity, that is, environmental decay as perceived by the individuals shouldn't be too strong. Equations (14) and (15) prove that the conjectured steady state exists: consumption ratio as well as abatement ratio are indeed constant over time.

The propensity to consume (14) depends on the underlying parameters as well as on the fiscal instruments. Abatement is proportional to consumption (see equation (15)), as with the instantaneous utility function considered here, consumption and environmental quality are complementary goods ($u_{cP} < 0$) and the intratemporal elasticity of substitution between consumption and pollution is unity.

A rise in the tax rate on consumption or in the abatement subsidy leads to a decrease in equilibrium consumption and an increase in equilibrium abatement expenditures as the relative prices change. The impact of an increase in the income tax rate is ambiguous, depending on the magnitudes of income and substitution effects and will be analyzed later in more detail. But since consumption and abatement are complementary goods, they are influenced in the same direction by income taxation.

If environmental preferences vanish ($\gamma = 0$) or individuals neglect their individual influence on pollution completely ($\delta \to 1$), market equilibrium corresponds to the linear stochastic endogenous growth model without environmental aspects. In both cases, individual abatement activities are zero in the limit. In the first case there is no negative impact of pollution on utility. Hence, pollution control cannot enhance utility. In the second case the individuals aren't aware of their influence on disutility out of pollution. Therefore, the costs are perceived to dominate the benefit out of pollution control for any positive value of abatement expenditures.[3]

With the solutions (14) and (15) of individual utility maximization, the expected growth rate of the economy, φ, can be obtained from the capital accumulation equation (6)

$$\begin{aligned}\varphi \equiv \frac{\mathrm{E}[dk]}{k\,dt} &= \frac{(1-\tau_y)A-(1+\vartheta)\rho}{\varepsilon(1-\vartheta^2)+\vartheta^2} \\ &\quad +(1-\vartheta^2)\frac{(\varepsilon-1)(\varepsilon(1-\vartheta)+\vartheta)}{2(\varepsilon(1-\vartheta^2)+\vartheta^2)}(1-\tau_y)^2A^2\sigma^2 \qquad (16)\end{aligned}$$

Equilibrium expected growth can be divided in two parts. The first term of the expected growth rate (16) corresponds to the Keynes–Ramsey–Rule of the according deterministic model. The second term describes the response of the risk averse individual to uncertainty. Equation (16) shows that the impact of uncertainty on growth is ambiguous and depends on the degree of relative risk aversion. In general, uncertainty has a positive income and a negative substitution effect on savings: On the one hand, an increase in risk reduces expected utility out of future income flows. Hence, savings are increased in order to compensate for this impact and to equalize expected marginal utility across time (positive income effect). On the other hand, capital accumulation gets less attractive for risk averse individuals if uncertainty increases. There is an incentive to decrease savings (negative substitution effect).

With the linear technology considered here, the income effect dominates if the degree of relative risk aversion is above unity ($\varepsilon > 1$). In terms of Leland (1968) or Sandmo (1970), $\varepsilon > 1$ implies a motive for precautionary savings. That is, uncertainty leads to an increase in the equilibrium growth rate in this case. If relative risk aversion is sufficiently low ($\varepsilon < 1$), the opposite applies and uncertainty has a negative growth effect.

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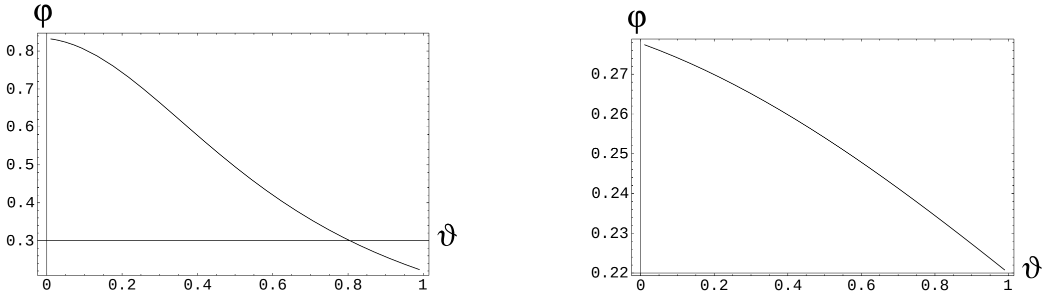
[3] In order to maintain feasible solutions for $\vartheta \to 0$, e. g. the government would have to provide pollution control.

Environmental aspects influence equilibrium growth through perceived pollution, as measured by ϑ. The impact is ambiguous as there are various counter working effects which lead to a grave sensitivity of the growth effects with respect to the degree of relative risk aversion. The effective rate of time preference as well as the effective relative risk aversion depend on environmental parameters as was shown for the deterministic setting e. g. by Mohtadi (1996). But as in the model considered here individual abatement activity is included, the impact of responsibility for the environment is ambiguous: On the one hand, with increasing perceived pollution the negative impact of capital accumulation through pollution on utility is given more weight. Saving gets less attractive and equilibrium growth tends to fall. On the other hand, a rise in perceived environmental decay increases the incentive for individual abatement activities. In order to enhance the ability for future pollution control, capital accumulation tends to be increased to have easier access to abatement goods in the future. Smulders and Gradus (1996) analyze the same counter working income and substitution effects of pollution on growth within the deterministic setting where pollution is a productive input and causes disutility.

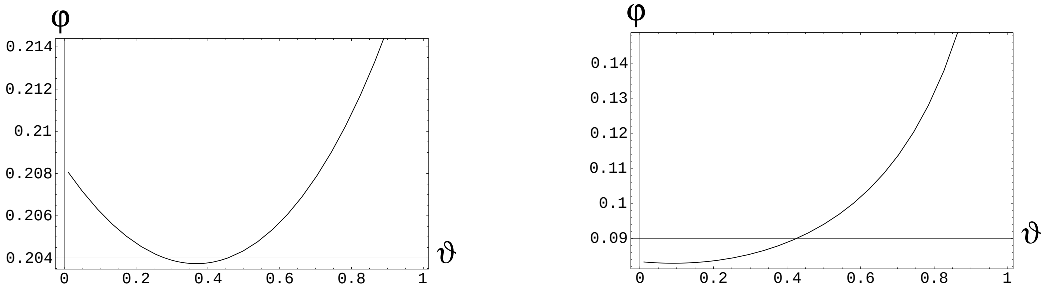
In the model considered here, the growth impact of perceived pollution is quite complex as second order effects due to uncertainty have to be taken into account. Figure 1 shows that the overall growth effect of perceived environmental degradation depends on the relation between the degree of relative risk aversion and the pollution impact measured by the composed parameter ϑ. The relevant parameters were set as follows: the productivity parameter $A = 0.4$, the standard deviation of the productivity shock $\sigma = 0.01$, the rate of time preference $\rho = 0.03$ and the income tax rate $\tau_y = 0.3$.

Within the shown interval of relative risk aversion between $\varepsilon = 0.3$ and $\varepsilon = 3$, the growth impact of perceived pollution changes significantly. For all risk aversions less than 0.3, the growth effect is qualitatively the same as in figure 1(a) and for risk aversions greater than 3 the figure remains qualitatively the same as demonstrated in figure 1(d). The magnitude of the environmental growth effect gets larger with increasing deviation of relative risk aversion from unity.



(a) *Relative risk aversion* $\varepsilon = 0.3$.

(b) *Relative risk aversion* $\varepsilon = 0.9$.



(c) *Relative risk aversion* $\varepsilon = 1.2$.

(d) *Relative risk aversion* $\varepsilon = 3$.

Figure 1: *Growth impact of perceived pollution.*

4 Environmental Policy

In order to derive optimal environmental policy, maximized intertemporal lifetime utility has to be derived. Due to the assumptions about the productivity shock, time t capital is a geometric Wiener process and is lognormally distributed. Therefore, it is possible to determine an explicit solution for lifetime utility (4). Given the initial values of capital k_0 and the stochastic process z_0 at time 0, capital evolves according to

$$k(t) = k_0 \, e^{\left(\varphi - \frac{1}{2}\alpha^2\sigma^2\right) t + \alpha\sigma\left[z(t) - z_0\right]} \quad . \tag{17}$$

Since population size is normalized to unity, aggregate and individual economic variables are equal in equilibrium. Therefore, relevant pollution without informational asymmetries is given by $P = \eta^{-\alpha}$ independently from perception. Using the properties of steady state (constant consumption and abatement ratios) and the goods market clearing condition $\mu^* = A - \varphi^* - \eta^*$, maximal expected lifetime utility results in

$$U = \frac{(A-\varphi^*-\eta^*)^{1-\varepsilon}\eta^{*\alpha\gamma(1-\varepsilon)}k_0^{1-\varepsilon}}{(1-\varepsilon)(\rho-(1-\varepsilon)(\varphi^*-\frac{1}{2}\varepsilon A^2\sigma^2))} \quad . \tag{18}$$

To develop pareto–optimal growth and abatement activity, expected lifetime utility (18) is maximized with respect to the environmental expenditure rate, η^*, and the growth rate, φ^*

$$\eta^* = \frac{\alpha\gamma}{\varepsilon(1+\alpha\gamma)-\alpha\gamma}\left(\rho+(\varepsilon-1)A+\varepsilon\frac{1-\varepsilon}{2}A^2\sigma^2\right) \tag{19}$$

$$\varphi^* = \frac{1+\alpha\gamma}{\varepsilon(1+\alpha\gamma)-\alpha\gamma}\left(\frac{A}{1+\alpha\gamma}-\rho+\varepsilon\frac{\varepsilon-1}{2}A^2\sigma^2\right) \quad . \tag{20}$$

Pareto–optimal pollution control as determined in equation (19) differs with respect to the second term from the corresponding deterministic model. That is, optimal environmental care increases (decreases) with uncertainty if risk aversion is less (higher) than unity. Hence, in general the outcome of the deterministic model cannot be applied to the case of uncertainty. If risk aversion is sufficiently high ($\varepsilon > 1$), optimal pollution control is overestimated by the setting without risk (and vice versa). This deviation is due to the motive for precautionary savings. With sufficiently risk averse individuals, optimal savings increase with arising uncertainty. This increase in capital accumulation is accompanied by a reduction not only of momentaneous consumption but also of momentaneous environmental expenditures. The opposite applies for an economy where risk aversion is below unity.

The impact of environmental decay, denoted by $\alpha\gamma$, on pareto–optimal growth (20) is ambiguous and depends on the degree of relative risk aversion and the intertemporal elasticity of substitution, respectively. Again, there are counter working income and substitution effects.[4] The substitution effect of accelerated environmental degradation (increasing $\alpha\gamma$) leads to a decrease in optimal capital accumulation

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[4] For the ambiguous impact of "greener preferences" on optimal growth in the deterministic setting see Smulders and Gradus (1996).

and hence in the growth rate. In contrast, the income effect induces an increase in capital accumulation, because a rise in environmental decay induces more need for future abatement activities. In contrast, a rise accelerated environmental decay unambiguously increases optimal abatement.

Optimal environmental policy can now be determined. Government fosters individual abatement activities through the ratio between abatement subsidy rate τ_e and consumption tax rate τ_c and adjusts capital accumulation through the income tax at rate τ_y. More simply, optimal taxation equalizes marginal expected utility out of consumption and marginal expected disutility out of pollution. Since government budget can be balanced growth neutrally by selecting the appropriate level of consumption tax, optimal fiscal and environmental policy can be decomposed into two steps: First, optimal income taxation ensures the equality of equilibrium expected growth, φ, according to (16) and pareto–optimal expected growth, φ^*, as given by (20). Second, the government has to adjust individual pollution control, η, determined in (15) at the corresponding optimal value, η^*, derived in (19).

The impact of income taxation on expected equilibrium growth (16) results in the well known ambiguous growth effect of income taxation in a stochastic growth model. It can be decomposed into a growth diminishing distortionary effect which is associated with the reduction in expected capital return and an ambiguous growth effect which is associated with the decline in capital risk. As already explained above, the individual response on a decrease in risk depends on the degree of relative risk aversion and may end up in a tendency to increase or decrease savings. For a detailed discussion of the counter working effects of taxation within stochastic models of endogenous growth see e. g. Eaton (1981), Turnovsky (1995), Smith (1996), Corsetti (1997) or Clemens and Soretz (1997). In the model considered here, the growth effect of a change in the income tax rate is given by

$$\frac{\partial\varphi}{\partial\tau} = -\frac{A}{\varepsilon(1-\vartheta^2)+\vartheta^2}(1+(\varepsilon-1)(1-\vartheta^2)(\varepsilon-1)(\varepsilon(1-\vartheta)+\vartheta)(1-\tau_y)A\sigma^2) \tag{21}$$

If risk aversion is higher than unity, individuals have a motive for precautionary savings which is reduced by income taxation. Hence, growth diminishes unambiguously with an increase in the income tax rate. If risk aversion is below unity, income taxation leads to an increase in precautionary savings. Nevertheless, it can be shown that a positive certainty equivalent of capital return is a sufficient condition for the domination of the negative distortionary growth impact of income taxation.[5] This condition can be interpreted in the following way: With a positive certainty equivalent risk does not dominate the model. The technology is "certain enough" as to assure that the effects of the underlying deterministic structure prevail. A negative certainty equivalent would describe a situation where the uncertain capital income flow yields the same utility as a certain interest rate which is negative. With this argument, it becomes immediately obvious, that a positive certainty equivalent is a necessary condition for feasible solutions. To conclude, income taxation unambiguously reduces equilibrium expected growth, independently from the degree of relative risk aversion.

Since income taxation influences capital risk, equating equilibrium growth (16) and optimal growth (20) leads to a quadratic function in the optimal income tax rate, τ_y^*,

$$\begin{aligned} & (\varepsilon(1+\alpha\gamma)-\alpha\gamma)\left((1-\vartheta^2)\frac{\varepsilon-1}{2}(\varepsilon(1-\vartheta)+\vartheta)A^2\sigma^2(1-\tau_y^*)^2+A(1-\tau_y^*)\right) \\ - & (1+\alpha\gamma)(\varepsilon(1-\vartheta^2)+\vartheta^2)\left(\frac{A}{1+\alpha\gamma}+\frac{\varepsilon-1}{2}\varepsilon A^2\sigma^2\right) \\ + & (\vartheta(1-\vartheta)(\varepsilon(1+\alpha\gamma)-\alpha\gamma)-\alpha\gamma)\rho=0 \quad . \end{aligned} \tag{22}$$

The solution will be analyzed numerically, as the impact of environmental issues again is quite complex. Figure 2 illustrates the effect of environmental decay, again measured by ϑ, on the optimal income tax rate. The parameter settings are the same as in figure 1. It can be seen that optimal income taxation depends crucially on the degree of relative risk aversion. This reflects the sensitivity of the environmental growth effect with respect to the degree of relative risk aversion which was shown in figure 1.

The influence of the perception parameter is isolated in figure 2 as two functions are given in each diagram. The solid line shows the case of high individual sense of responsibility ($\delta = 0.1$) and the dashed line is associated with low individual

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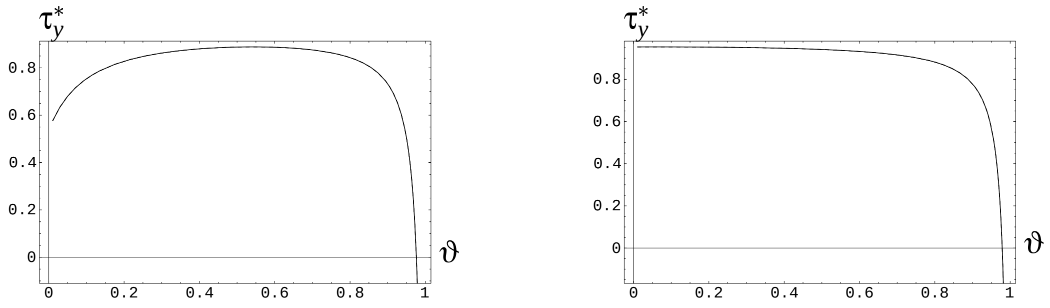
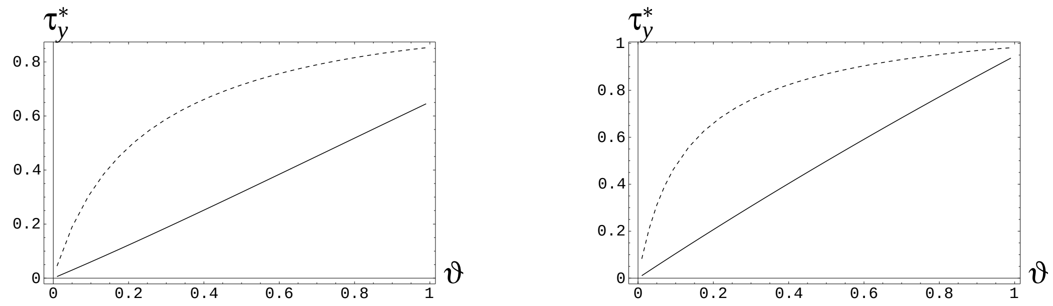
[5] A positive certainty equivalent requires $(\varepsilon(1-\vartheta)+\vartheta)A\sigma^2 < 1$, see Merton (1992, p. 45).



(a) *Relative risk aversion* $\varepsilon = 0.1$.

(b) *Relative risk aversion* $\varepsilon = 0.9$.



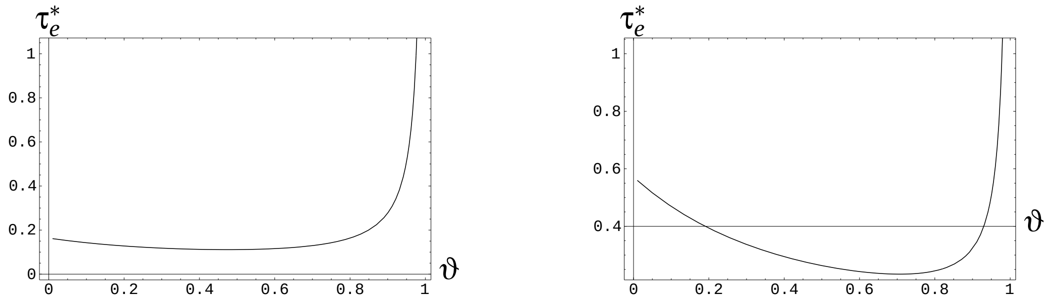
(c) *Relative risk aversion* $\varepsilon = 2$.

(d) *Relative risk aversion* $\varepsilon = 10$.

Figure 2: *Optimal income tax rate.*

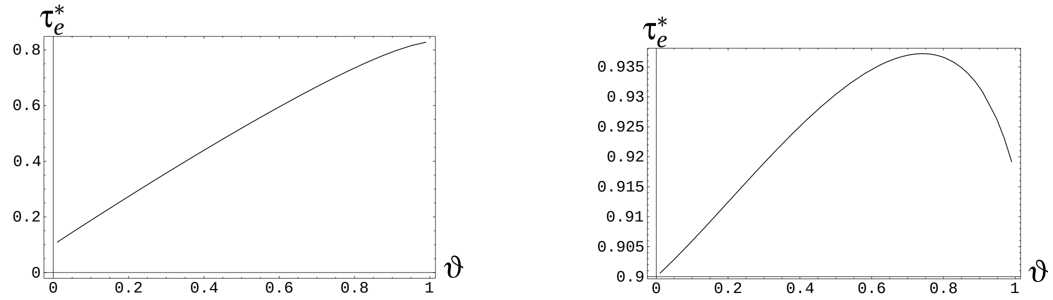
sense of responsibility ($\delta = 0.9$). Diagrams 2(a) and 2(b) demonstrate that with modest relative risk aversion ($\varepsilon < 1$) the isolated impact of perception is too small to become visible. Nevertheless, the perceived environmental degradation, ϑ, contains the perception parameter δ, so any ceteris paribus change in δ also affects perceived environmental importance ϑ and through this channel in fact influences optimal income taxation considerably.

To define optimal fiscal policy completely, the subsidy rate on individual abatement activity, τ_e, has to be determined. It results residually from equating equilibrium pollution control (15) and optimal abatement ratio (19) using equilibrium consumption according to (14). The sensitivity of the optimal abatement subsidy rate with respect to perceived environmental decay as well as relative risk aversion is based on the arguments given above and illustrated in figure 3. Diagram 3(a) combines low relative risk aversion ($\varepsilon = 0.8$) with high sense of responsibility for environmental aspects ($\delta = 0.1$). The counterpart for low responsibility ($\delta = 0.7$) is given in figure 3(b). The case of high relative risk aversion ($\varepsilon = 3$) together with strong versus modest consciousness ($\delta = 0.1$ versus $\delta = 0.7$) is illustrated in the diagrams 3(c) and 3(d).



(a) *Relative risk aversion* $\varepsilon = 0.8$, *perception parameter* $\delta = 0.1$.

(b) *Relative risk aversion* $\varepsilon = 0.9$, *perception parameter* $\delta = 0.7$.



(c) *Relative risk aversion* $\varepsilon = 3$, *perception parameter* $\delta = 0.1$.

(d) *Relative risk aversion* $\varepsilon = 3$, *perception parameter* $\delta = 0.7$.

Figure 3: *Optimal subsidy rate.*

Concluding optimal governmental policy, it can be stated that optimal fiscal variables are highly sensitive to environmental and preference parameters. Optimal pollution control depends on relative risk aversion and intertemporal elasticity of substitution respectively, as there are counter working income and substitution effects. Considering additionally the second order effects of partial individual responsibility for environmental decay, the ambiguous growth effect of pollution is reflected in highly sensitive policy implications.

5 Conclusions

In this paper pollution and abatement are analyzed in a stochastic model of endogenous growth. Partial perception of the individual influence on environmental degradation is taken into account and can alternatively be interpreted as parameterized sense of responsibility for the environment or as parameterized rivalry in the "production" of pollution. Due to the partial responsibility for environmental aspects, equilibrium growth is affected in ambiguous way by an increasing (perceived) pollution. The impact of environmental degradation on growth is highly sensitive to the degree of relative risk aversion.

Optimal growth as well as optimal abatement activity depend ambiguously on risk and on the strength of environmental decay. Risk affects the optimal intertemporal allocation through the motive for precautionary savings. If the degree of relative risk aversion is higher (lower) than unity, increasing risk induces a rise (fall) in precautionary savings and therefore reduces (increases) the optimal abatement ratio. An acceleration of environmental decay or stronger preferences for a clean environment unambiguously increase pareto–optimal abatement, but lead to counter working income and substitution effects on optimal expected growth.

The set of fiscal instruments which is considered includes income taxation, consumption taxation and a subsidy on abatement activity. These three governmental parameters are sufficient to access a pareto optimal market equilibrium. The sensitivity of equilibrium growth with respect to environmental and preference parameters leads to a great variety of optimal fiscal policies, depending on the underlying parameter setting.

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