Unemployment and Inventories in the Business Cycle[*]

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Abstract

This paper investigates the impact of monetary shocks on the business cycle with specific regard to the rôle of inventories. The model is based on an overlapping-generations non-tâtonnement approach involving temporary equilibria with stochastic rationing in each period and price adjustment between successive periods. Inventories reinforce the importance of spill-over effects between markets and imply that, starting from a stationary Walrasian equilibrium, it is possible that, following a restrictive monetary shock, the economy converges to a quasi-stationary Keynesian underemployment state. Contrary to conventional wisdom, this is favored by sufficient downward flexibility of the nominal wage. Thus in that case money is non-neutral in the long run. The model is applied to the current deflationary Japanese recession, and we propose a way how to overcome it.

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1. Introduction

Why money affects output and why it has long lasting effects have long been the two central questions for the business cycle literature, if not for all of macro-economics. This is especially so because, as stressed for instance by Blanchard (2000), the empirical evidence is irremediably at odds with the conclusions of the flexible-price models which still represent the approach most commonly shared by economists.

If prices were fully flexible, an increase in nominal money would immediately induce a proportional increase in the price level offsetting any pressure on demand and output, and money would be neutral even in the short run. Prices and wages, however, do not change instantaneously: they exhibit a certain degree of stickiness and individual price changes tend to be staggered, which makes the adjustment process of the price level more or less slow. During the process, aggregate demand and output are higher than their original values, and the change in the money stock has real effects. In the end, most economists maintain, the price level will adjust proportionally to the increase in the nominal money stock, so that demand and output will be back at their original levels, and money neutrality will be restored, but only in the long run. Before this occurs, real and nominal rigidities, lying behind the slow adjustment of prices and wages, are ultimately the causes of the non-neutrality of money. Since the beginning of the Nineties, the New Keynesian literature (see e.g. Ball and Romer, 1990, and Blanchard, 1987 and 1990) has emphasized that monetary shocks determine large aggregate effects when small frictions in nominal adjustment are supplemented by real rigidities.[1]

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[1]Much of the recent research in macroeconomics has concentrated on the "imperfections" of labor, goods, and financial markets responsible for the emergence of real rigidities and nominal stickiness and on their relevance for economic fluctuations.

Many causes of real rigidities have been investigated in the literature: among others, efficiency wages (see, for example, Solow, 1979, and Shapiro and Stiglitz, 1984), implicit contracts (Azariadis, 1975, and Baily, 1974), countercyclical mark-ups (Stiglitz, 1984, Rotemberg and Saloner, 1986, and Rotemberg and Woodford, 1991), inventories (Blinder, 1982), social customs (Akerlof, 1980, and Romer, 1984), strategic interactions and coordination failure (Ball and Romer, 1991), credit markets imperfections (Bernanke and Gertler, 1989 and 1995, Holmström and Tirole, 1997, 1998, and Kiyotaki and Moore, 1997) and increasing returns (Kiyotaki, 1988, and Diamond, 1982). Attention has been devoted as well to the sources of nominal stickiness focusing, for instance, on menu cost or near rationality (e.g. Mankiw, 1985, and Akerlof and Yellen, 1985), staggered contracts (Calvo, 1983) and uncertainty and risk aversion (Weinrich, 1997).

In the Nineties, Keynesian features - like the nominal and real stickiness named above - have started being incorporated into the dynamic general equilibrium framework typical of the

In this paper we aim to show that both conclusions stressed in the literature, about the long-run money neutrality and the effectiveness of price flexibility to lead the economy quickly back to the pre-shock state, do not necessarily hold. On the contrary, money can affect the output level in the long run and price and wage flexibility can indeed foster achieving this result, while wage rigidity may prove a good recipe to avoid or overcome permanent underemployment and to restore Walrasian equilibrium.

Our framework develops a discrete-time dynamic non-tâtonnement macroeconomic model, building on Bignami, Colombo and Weinrich (2003) and on Colombo and Weinrich (2003). The economy consists of an overlapping generations consumption sector, of a production sector characterized by an atemporal production function, and of a government that finances public expenditure by means of a tax levied on firms' profits. Within each period, prices are fixed and a consistent allocation is obtained by means of temporary equilibrium with stochastic rationing whereas prices are adjusted between successive periods according to the strength of rationing or disequilibrium on each market in the previous period.[2] This approach permits to account for the fact that in any economy with decentralized price setting, the "adjustment of the general level of prices in terms of the numeraire is likely to be slow relative to a (fictional) economy with an auctioneer", as emphasized by Blanchard (2000, p. 1393). It is important to stress that the way we model the price (wage) adjustment mechanism allows us to account quite naturally for different degrees of price and wage flexibility. Although our adjustment mechanism is given exogenously - and thus it may be considered *ad hoc* - it allows us to assess the impact of price and wage reactions to shocks generated by different underlying conceptual models. In other words, it is "agnostic" enough to provide a framework to study the impact of real and nominal rigidities in the New Keynesian tradition, as well as to investigate the consequences for price and wage adjustment of the presence of uncertainty (e.g., about the entity of monetary transfers as in Lucas and Woodford, 1993, or about information that becomes

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business cycle literature, originating what has been named the New Neoclassical Synthesis (see, for example, Jeanne (1998) and the references in there).

[2] A natural idea is to relate the adjustment of prices to the size of the dissatisfaction of agents with their (foregone) trades. A reliable measure of such a dissatisfaction requires stochastic rationing, since - as opposed to deterministic rationing - it is compatible with manipulability of the rationing mechanism and therefore provides an incentive for rationed agents to express demands that exceed their expected trades, as argued by Green (1980), Svensson (1980), Douglas Gale (1979, 1981) and Weinrich (1982, 1984, 1988). For a definition of manipulability see for example Böhm (1989) or Weinrich (1988).

public only at the end of the process as in Eden, 1994), or confusion (as in Lucas, 1972).

The novelty of the economy developed here with respect to the one considered in our previous papers is that we abandon the simplifying assumption that there are no inventories. In the present paper, inventories are possible and stored goods may be sold in periods subsequent to the period of their production. More precisely, at the beginning of each period the stock of inventories carried by each firm is simply given by the firm's output that remains unsold at the end of the previous period. In this sense, inventories are not used as "strategic" decision variables by firms, which makes our treatment of inventories different from, and simpler than, most of the accounts present in the recent literature (see, for instance, Blinder and Fischer, 1981, Blinder, 1982 and Bental and Eden, 1996). However, in our model as well, the explicit consideration of inventories adds a further propagation mechanism for shocks and amplifies the importance of the spillover effects among markets.

To highlight the main results of the paper, consider a restrictive monetary shock that, starting from a Walrasian equilibrium, reduces aggregate demand, inducing excess supply on the goods market and, consequently, a reduction in the goods price. The decrease in aggregate demand reduces labor demand and gives rise to an excess supply on the labor market as well. Whenever the nominal wage is rigid downward, the real wage and the real money stock increase until the economy leaves the state of Keynesian unemployment to enter a state of Classical unemployment, with excess demand on the goods market and excess supply on the labor market. At this point prices start to increase again, determining a reduction of the real wage and of the real money stock until the economy converges back to the Walrasian equilibrium. The process changes quite dramatically when there is downward wage flexibility. In this case, the monetary shock determines a reduction of the nominal wage that, if the latter is flexible enough, provokes an immediate and continuing reduction of the real wage. The presence of inventories reinforces this reduction, by increasing the fall of labor demand which in turn depresses labor income and aggregate demand. The economy converges to a quasi-stationary Keynesian state with permanent unemployment and permanent deflation of the nominal variables. In this sense, contrary to the previous literature, imposing downward nominal wage rigidity appears to be in itself a viable policy to prevent the emergence of recessions, or at least limit their extent and duration.

Moreover, we suggest, by means of numerical simulations, that such recessionary quasi-stationary equilibria are locally stable while the stationary Walrasian

equilibrium is locally unstable. Specifically, reductions in the real money stock or in real profits, and increases in the stock of inventories, destroy the full employment equilibrium and cause the economy to converge to a quasi-stationary Keynesian equilibrium.

The framework we present is thus able to account for the dynamic behavior of economies that are trapped in situations of underemployment or underutilization of the productive capacity. This proves very useful in evaluating the impact of alternative policy measures aimed at restoring full employment. In this respect, we use our economy as a test bank to investigate the deflationary behavior of the Japanese economy starting in the mid-1990s and to evaluate fiscal and monetary policies to stimulate the economy. More precisely, the recessionary Keynesian equilibrium of our economy seems to reproduce quite well the actual condition of the Japanese economy, and therefore it provides a suitable framework to discuss the impact of different economic policies.[3] In particular, we focus on the policy measures recently proposed by Ben Bernanke (as reported by *The Economist*, June 21st, 2003) - requiring simultaneous fiscal and monetary expansionary policies based on tax cuts directly financed by the central bank - to check whether they are effective in restoring full employment in our model economy.[4] By operating a reduction of the tax rate and by maintaining unchanged both the government's budget deficit and the aggregate demand (by means of a monetary expansion), our numerical analysis suggests that the stationary (and locally stable) long run employment level increases monotonically with the decrease in the tax rate. This confirms the efficacy of Bernanke's proposed policy that, provided it is of the right magnitude, should be capable to restore full employment.

The remainder of the paper is organized as follows. In section two we present the model and describe the behavior of consumers, producers and the government. Section three focuses on temporary equilibria with rationing and proves the existence and uniqueness of equilibrium allocations. In section four we provide a representation of possible equilibrium regimes and in section five we set

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[3]According to OECD statistics (OECD Main Economic Indicators, May 2003), the Japanese standardized unemployment rates increased steadily from 4.7 in 2000 to 5.4 in the first quarter 2003. At the same time, the consumer price index (base 1995=100) fell from 101.5 in 2000 to 99.4 in the first quarter 2003. Similarly, the producer price index (base 1995=100) fell from 96.1 to 91.6 in the same period. Finally, hourly earnings (base 1995=100) in the manufacturing sector decreased from 105 in 2000 to 104 in 2002.

[4]In a recent paper, Auerbach and Obstfeld (2003) make a case for the efficacy of large open-market purchases of domestic government debt as a way out of the recession for the Japanese economy.

up the dynamic system. Section six presents numerical simulations and discusses the impact of fiscal and monetary shocks. Section seven examines the stability of the quasi-stationary recessionary equilibrium and of the Walrasian equilibrium. Section eight investigates the Japanese deflationary recession and discusses policy measures to overcome it. Finally, section nine concludes, while the proofs of some technical results and the complete dynamic system are given in the appendices.

2. The Model

We consider an economy in which there are n OLG-consumers, n' firms and a government. Consumers offer labor inelastically when young and consume a composite consumption good in both periods. That good is produced by firms using an atemporal production function whose only input is labor. The government levies a proportional tax on firms' profits to finance its expenditure for goods. Nevertheless, budget deficits and surpluses may arise and are made possible through money creation or destruction.

2.1. Timing of the Model

The time structure of the model is depicted in Figure 2.1. In period $t-1$ producers obtain an aggregate profit of Π_{t-1} which is distributed at the beginning of period t in part as tax to the government ($tax\Pi_{t-1}$) and in part to young consumers ($(1-tax)\Pi_{t-1}$), where $0 \leq tax \leq 1$. Also at the beginning of period t old consumers hold a total quantity of money M_t, consisting of savings generated in period $t-1$. Thus households use money as a means of transfer of purchasing power between periods.[5]

Let X_t denote the aggregate quantity of the good purchased by young consumers in period t, p_t its price, w_t the nominal wage and L_t the aggregate quantity of labor. Then

$$M_{t+1} = (1-tax)\Pi_{t-1} + w_t L_t - p_t X_t.$$

Denoting with G the quantity of goods purchased by the government and taking into account that old households want to consume all their money holdings in pe-

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[5]We assume that, although the good is storable for firms, it is not so for consumers: they do not have access to firms' storage technology the cost of which is worthwhile to be borne for large quantities only. Moreover, even if the good were storable by consumers, this would not be convenient for them in case next period's price is lower than the current period's one. Thus our main results, which regard deflationary recessionary equilibria, would not be influenced anyway.
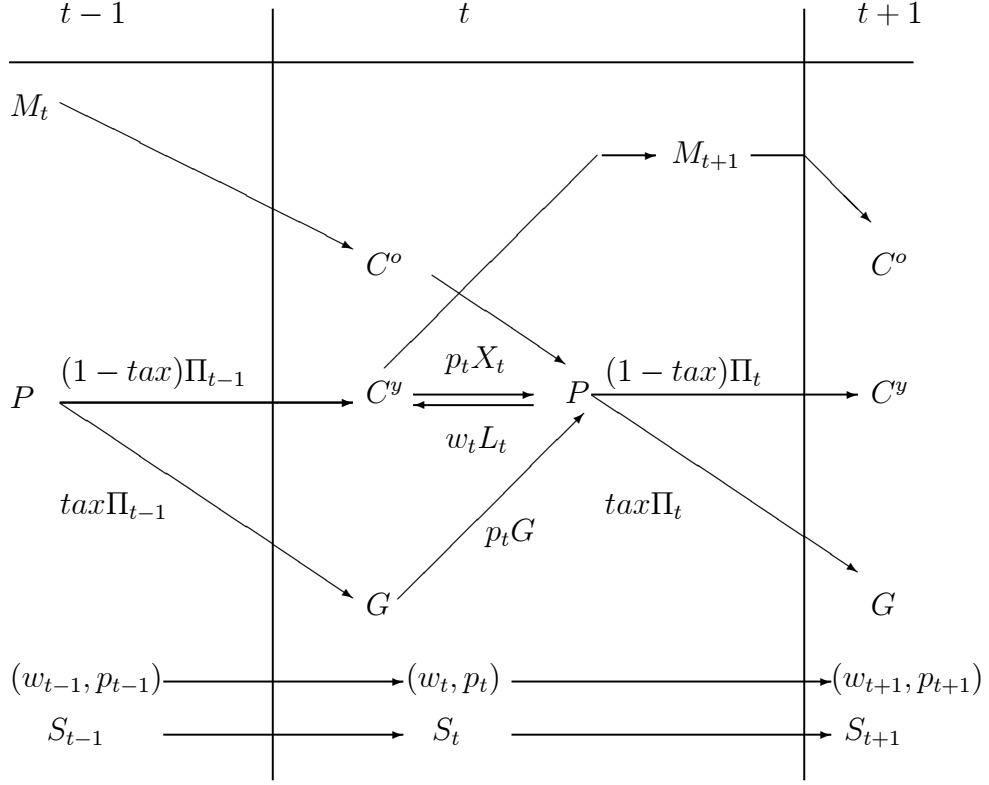
Figure 2.1: The time structure of the model

riod t, the aggregate consumption of young and old households and the government is $Y_t = X_t + \frac{M_t}{p_t} + G$. Using that $\Pi_t = p_t Y_t - w_t L_t$, considering $\Pi_t - \Pi_{t-1} = \Delta M_t^P$ as the variation in the money stock held by producers before they distribute profits and denoting with $\Delta M_t^C = M_{t+1} - M_t$ the one referring to consumers, we obtain the usual accounting identity, i.e. $\Delta M_t^C + \Delta M_t^P = p_t G - tax \Pi_{t-1} =$ budget deficit.

Denoting with S_t the aggregate amount of inventories carried over by firms to period t and with Y_t^p the aggregate amount of goods produced in period t, there results $S_{t+1} = Y_t^p + S_t - Y_t$.

2.2. The Consumption Sector

In his first period of life each consumer born at t is endowed with labor ℓ^s and an amount of money $(1 - tax)\Pi_{t-1}/n$ while his preferences are described by a utility function $u(x_t, x_{t+1})$. In taking any decision the young consumer has to meet the

constraints

$$0 \le x_t \le \omega_t^i, 0 \le x_{t+1} \le \left(\omega_t^i - x_t\right) \frac{p_t}{p_{t+1}}, i = 0, 1 \tag{2.1}$$

where

$$\omega_t^1 = \frac{1 - tax}{p_t} \frac{\Pi_{t-1}}{n} + \frac{w_t}{p_t} \ell^s$$

denotes his real wealth when he is employed and

$$\omega_t^0 = \frac{1 - tax}{p_t} \frac{\Pi_{t-1}}{n}$$

when he is unemployed. Implicit in this is the assumption that rationing on the labor market is of type all-or-nothing and that the labor market is visited before the goods market.

Regarding the goods market the young household may be rationed according to the stochastic rule

$$x_t = \left\{ \begin{array}{ll} x_t^d & \text{with prob.} \quad \rho\gamma_t^d \\ c_t x_t^d & \text{with prob.} \quad 1 - \rho\gamma_t^d \end{array} \right.$$

where x_t^d is the quantity demanded, $\rho \in [0, 1]$ a fixed structural parameter of the rationing mechanism, $\gamma_t^d \in [0, 1]$ a rationing coefficient which the household perceives as given but which will be determined in equilibrium and

$$c_t = \frac{\gamma_t^d - \rho\gamma_t^d}{1 - \rho\gamma_t^d}.$$

These settings are chosen such that the expected value of x_t is $\gamma_t^d x_t^d$, that is, expected rationing is proportional and hence manipulable.[6]

Denoting with $\theta_t^e = p_{t+1}^e/p_t$ the expected relative price for period t, the effective demand $x_t^{di}, i = 0, 1$, is obtained by solving the agent's expected utility maximization problem

$$\max_{x_t} \rho\gamma_t^d u \left(x_t, \frac{\omega_t^i - x_t}{\theta_t^e} \right) + \left(1 - \rho\gamma_t^d \right) u \left(c_t x_t, \frac{\omega_t^i - c_t x_t}{\theta_t^e} \right)$$

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[6]As has been shown by Green [1980] and Weinrich [1982], in case of rationing where the quantity signals are given by means of the aggregate values of demand and supply, the only mechanisms compatible with equilibrium are those for which the expected realization is proportional to the transaction offer.

subject to the constraints (2.1). The resulting first-order condition yields

$$\frac{\rho u_1\left(x_t, \frac{\omega_t^i - x_t}{\theta_t^e}\right) + (1-\rho)\, u_1\left(c_t x_t, \frac{\omega_t^i - c_t x_t}{\theta_t^e}\right)}{\rho u_2\left(x_t, \frac{\omega_t^i - x_t}{\theta_t^e}\right) + (1-\rho)\, u_2\left(c_t x_t, \frac{\omega_t^i - c_t x_t}{\theta_t^e}\right)} = \frac{1}{\theta_t^e}. \tag{2.2}$$

For a generic utility function it is hard to solve this equation for x_t but it is possible under the following assumption:

(A1) $u\left(x_t, x_{t+1}\right) = x_t^h x_{t+1}^{1-h}$ and $\rho = 1$ (i.e. 0/1-rationing).

In this case we can prove that $x_t^{di} = h\omega_t^i, i = 0, 1$ (Lemma 1 in Appendix 1). In particular the young consumer's effective demand is independent of both γ_t^d and p_{t+1}^e.

The aggregate supply of labor is $L^s = n\ell^s$. Denoting with L_t^d the aggregate demand of labor and with $\lambda_t^s = \min\left\{\frac{L_t^d}{L^s}, 1\right\}$ the fraction of young consumers that will be employed, the aggregate demand of goods of young consumers is

$$\begin{aligned} X_t^d &= \lambda_t^s n x_t^{d1} + (1 - \lambda_t^s)\, n x_t^{d0} \\ &= h\,(1-tax)\frac{\Pi_{t-1}}{p_t} + h\frac{w_t}{p_t}\lambda_t^s L^s \equiv X^d\left(\lambda_t^s; \frac{w_t}{p_t}, \frac{(1-tax)\,\Pi_{t-1}}{p_t}\right). \end{aligned} \tag{2.3}$$

The total aggregate demand of the consumption sector is then obtained by adding old consumers' aggregate demand M_t/p_t and government demand G:

$$Y_t^d = X^d\left(\lambda_t^s; \alpha_t, (1-tax)\,\pi_t\right) + m_t + G_t \tag{2.4}$$

where $\alpha_t \equiv w_t/p_t$, $\pi_t \equiv \Pi_{t-1}/p_t$ and $m_t \equiv M_t/p_t$.

2.3. The Production Sector

Each of the n' identical firms uses an atemporal production function $y_t^p = f\left(\ell_t\right)$. Having transferred stocks from the previous period and being thus endowed with inventories s_t at the beginning of period t, the total amount supplied by a firm is $y_t^s = y_t^p + s_t$. As with consumers, firms too may be rationed, by means of a rationing mechanism analogue to that assumed for the consumption sector.

Denoting the single firm's effective demand of labor by ℓ_t^d, the quantity of labor effectively transacted is

$$\ell_t = \begin{cases} \ell_t^d, & \text{with prob. } \lambda_t^d \\ 0, & \text{with prob. } 1 - \lambda_t^d \end{cases}$$

where $\lambda_t^d \in [0,1]$. It is obvious that $E\ell_t = \lambda_t^d \ell_t^d$. On the goods market the rationing rule is assumed to be

$$y_t = \begin{cases} y_t^s, & \text{with prob. } \sigma\gamma_t^s \\ d_t y_t^s, & \text{with prob. } 1 - \sigma\gamma_t^s \end{cases},$$

where $\sigma \in (0,1)$, $\gamma_t^s \in [0,1]$ and $d_t = (\gamma_t^s - \sigma\gamma_t^s)/(1 - \sigma\gamma_t^s)$. σ is a fixed parameter of the mechanism whereas λ_t^d and γ_t^s are perceived rationing coefficients taken as given by the firm the effective value of which will be determined in equilibrium. The definition of d_t implies that $Ey_t = \gamma_t^s y_t^s$; in particular it is independent of σ. It is obvious that $E\ell_t = \lambda_t^d \ell_t^d$.

The firm's effective demand $\ell_t^d = \ell^d(\gamma_t^s; \alpha_t)$ is obtained from the expected profit maximization problem

$$\max_{\ell_t^d} \gamma_t^s \left[f\left(\ell_t^d\right) + s_t \right] - \alpha_t \ell_t^d$$

subject to

$$0 \leq \ell_t^d \leq \frac{d_t}{\alpha_t} \left[f\left(\ell_t^d\right) + s_t \right]$$

while its effective supply is $y_t^s = f\left(\ell_t^d\right) + s_t$. The upper bound on labor demand reflects the fact that the firm must be prepared to finance labor service purchases even if rationed on the goods market (since the labor market is visited first it will know whether it is rationed on the goods market only after it has hired labor). In general the solution depends on this constraint but it is not binding (Appendix, Lemma 2) if we make the following assumption:

(A2) $f(\ell) = a\ell^b$, $a > 0$, $0 < b \leq (1 - \sigma)$.

In this case labor demand is

$$\ell_t^d = \ell^d(\gamma_t^s; \alpha_t) = \left(\frac{\gamma_t^s ab}{\alpha_t} \right)^{\frac{1}{1-b}} \tag{2.5}$$

Notice that labor demand is independent of s_t. The aggregate labor demand then is $L_t^d = n'\ell^d(\gamma_t^s; \alpha_t) \equiv L^d(\gamma_t^s; \alpha_t)$ and, because only a fraction λ_t^d of firms can hire workers, the aggregate supply of goods is

$$Y_t^s = \lambda_t^d n' f\left(\ell^d(\gamma_t^s; \alpha_t)\right) + S_t \equiv Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right) \tag{2.6}$$

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3. Temporary Equilibrium Allocations

For any given period t we can now describe a feasible allocation as a temporary equilibrium with rationing as follows.

Definition 3.1. : *Given a real wage α_t, a real profit level π_t, real money balances m_t, inventories S_t, a level of public expenditure G and a tax rate tax, a list of rationing coefficients $\left(\gamma_t^d, \gamma_t^s, \lambda_t^d, \lambda_t^s, \delta_t, \varepsilon_t\right) \in [0,1]^6$ and an aggregate allocation $\left(\overline{L}_t, \overline{Y}_t\right)$ constitute* a temporary equilibrium *if the following conditions are fulfilled:*
(1) $\overline{L}_t = \lambda_t^s L^s = \lambda_t^d L^d \left(\gamma_t^s; \alpha_t\right);$
(2) $\overline{Y}_t = \gamma_t^s Y^s \left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right) = \gamma_t^d X^d \left(\lambda_t^s; \alpha_t, (1 - tax)\pi_t\right) + \delta_t m_t + \varepsilon_t G;$
(3) $\left(1 - \lambda_t^s\right)\left(1 - \lambda_t^d\right) = 0; \left(1 - \gamma_t^s\right)\left(1 - \gamma_t^d\right) = 0;$
(4) $\gamma_t^d \left(1 - \delta_t\right) = 0; \delta_t \left(1 - \varepsilon_t\right) = 0.$

Conditions (1) and (2) require that expected aggregate transactions balance. This means that all agents have correct perceptions of the rationing coefficients $\gamma_t^d, \gamma_t^s, \lambda_t^d$ and λ_t^s. Equations (3) formalize the short-side rule according to which at most one side on each market is rationed. The meaning of the coefficients δ_t and ε_t is that also old households and/or the government can be rationed. However, according to condition (4) this may occur only after young households have been rationed (to zero).

As shown in the table below it is possible to distinguish different types of equilibrium according to which market sides are rationed: excess supply on both markets is called *Keynesian Unemployment* $[K]$, excess demand on both markets *Repressed Inflation* $[I]$, excess supply on the labor market and excess demand on the goods market *Classical Unemployment* $[C]$ and excess demand on the labor market with excess supply on the goods market *Underconsumption* $[U]$.

	K	I	C	U
λ_t^s	< 1	$= 1$	< 1	$= 1$
λ_t^d	$= 1$	< 1	$= 1$	< 1
γ_t^s	< 1	$= 1$	$= 1$	< 1
γ_t^d	$= 1$	< 1	< 1	$= 1$
δ_t	$= 1$	≤ 1	≤ 1	$= 1$
ε_t	$= 1$	≤ 1	≤ 1	$= 1$

Of course there are further intermediate cases which, however, can be considered as limiting cases of the above ones. In particular, when all the rationing coefficients are equal to one, we are in a Walrasian Equilibrium.

Existence and uniqueness of temporary equilibrium are established by the following proposition.

Proposition 3.2. *Under assumptions (A1) and (A2) there exists, for any quadruple of variables $(\alpha_t, m_t, \pi_t, S_t)$ with α_t strictly positive and m_t, π_t and S_t non-negative, and any non-negative pair of policy parameters (G, tax), a unique temporary equilibrium allocation $(\overline{L}_t, \overline{Y}_t)$. $\overline{L}_t$ is given by*

$$\overline{L}_t = \min\left\{\widetilde{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right), L^d\left(1, \alpha_t\right), L^s\right\} \equiv \mathcal{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right) \tag{3.1}$$

where $\widetilde{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right)$ is the unique solution in L of

$$\alpha_t\left(\frac{1}{b} - h\right)L + \frac{\alpha_t}{ab}\left(\frac{L}{n'}\right)^{1-b}S_t = h\left(1 - tax\right)\pi_t + m_t + G \tag{3.2}$$

and

$$L^d\left(1, \alpha_t\right) = n'\left(\frac{ab}{\alpha_t}\right)^{\frac{1}{1-b}}. \tag{3.3}$$

$\overline{Y}_t \equiv \mathcal{Y}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right)$ is determined as follows. If $\overline{L}_t = \widetilde{L}\left(\cdot\right)$, then $\overline{Y}_t = \frac{\alpha_t}{b}\overline{L}_t + \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}S_t$, and if $\overline{L}_t = L^d\left(1, \alpha_t\right)$, then $\overline{Y}_t = \frac{\alpha_t}{b}L^d\left(1, \alpha_t\right) + S_t$. Finally, if $\overline{L}_t = L^s$, then $\overline{Y}_t = \min\left\{\frac{\alpha_t}{b}L^s + S_t, h\left(1 - tax\right)\pi_t + h\alpha_t L^s + m_t + G\right\}.$

Proof. Since we hold $\{\alpha_t, m_t, \pi_t, S_t\}$ and (G, tax) fixed, we omit them whenever possible as arguments in the subsequent functions. Define the set

$$\overline{H} = \left\{\left(\lambda^s L^s, \gamma^d X^d\left(\lambda^s\right)\right) \mid \left(\lambda^s, \gamma^d\right) \in [0,1]^2\right\}$$

and its subsets $\overline{H}^K = \overline{H}\mid_{\gamma^d=1, \lambda^s<1}$, $\overline{H}^I = \overline{H}\mid_{\gamma^d<1, \lambda^s=1}$, $\overline{H}^C = \overline{H}\mid_{\gamma^d<1, \lambda^s<1}$ and $\overline{H}^U = \overline{H}\mid_{\gamma^d=1, \lambda^s=1}$. Using the terminology introduced by Honkapohja and Ito (1985), we derive from these the consumption sector's *trade curves*

$$\overline{H}_0^K = \overline{H}^K + \{(0, m_t + G)\} = \left\{\left(\lambda^s L^s, X^d\left(\lambda^s\right) + m_t + G\right) \mid \lambda^s \in [0,1)\right\},$$

$$\overline{H}_0^I = \left\{\left(L^s, \gamma^d X^d\left(1\right) + m_t + G\right) \mid \gamma^d \in (0,1)\right\} \cup \left\{\left(L^s, \delta m_t + G\right) \mid \delta \in (0,1]\right\}$$
$$\cup \left\{\left(L^s, \varepsilon G\right) \mid \varepsilon \in [0,1]\right\},$$

$$\overline{H}_0^C = \left\{\left(\lambda^s L^s, \gamma^d X^d\left(\lambda^s\right) + m_t + G\right) \mid \left(\lambda^s, \gamma^d\right) \in [0,1) \times (0,1)\right\}$$

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$\cup \left\{ \left(\lambda^s L^s, \delta m_t + G \right) \mid \left(\lambda^s, \delta \right) \in [0,1) \times (0,1] \right\} \cup \left\{ \left(\lambda^s L^s, \varepsilon G \right) \mid \left(\lambda^s, \varepsilon \right) \in [0,1) \times [0,1] \right\}.$

and

$$\overline{H}_0^U = \overline{H}^U + \{(0, m_t + G)\} = \left\{ \left(L^s, X^d(1) + m_t + G \right) \right\}.$$

Similarly, starting from

$$\overline{F} \equiv \left\{ \left(\lambda^d L^d(\gamma^s), \gamma^s Y^s(\lambda^d, \gamma^s) \right) \mid \left(\lambda^d, \gamma^s \right) \in [0,1]^2 \right\}$$

we define the production sector's trade curves as $\overline{F}^K = \overline{F} \mid_{\lambda^d = 1, \gamma^s < 1}$, $\overline{F}^I = \overline{F} \mid_{\lambda^d < 1, \gamma^s = 1}$, $\overline{F}^C = \overline{F} \mid_{\lambda^d = 1, \gamma^s = 1}$ and $\overline{F}^U = \overline{F} \mid_{\lambda^d < 1, \gamma^s < 1}$.

To derive these curves, we start with noticing that

$$\gamma^s Y^s \left(\lambda^d, \gamma^s; \alpha_t, S_t \right) = \frac{\alpha_t}{b} \lambda^d L^d (\gamma_t^s; \alpha_t) + \gamma^s S_t. \tag{3.4}$$

Indeed, by (2.6)

$$\gamma^s Y^s \left(\lambda^d, \gamma^s; \alpha_t, S_t \right) = \gamma^s \left[\lambda^d n' f \left(\ell^d (\gamma_t^s; \alpha_t) \right) + S_t \right]$$

whereas from $f(\ell) = a\ell^b$ follows $f'(\ell) = b\frac{f(\ell)}{\ell}$, which implies $f(\ell) = \frac{1}{b} f'(\ell) \ell$. Therefore

$$\gamma^s Y^s \left(\lambda^d, \gamma^s; \alpha_t, S_t \right) = \gamma^s \left[\lambda^d n' \frac{1}{b} f' \left(\ell^d (\gamma_t^s; \alpha_t) \right) \ell^d (\gamma^s; \alpha_t) + S_t \right].$$

But $\gamma^s f' \left(\ell^d (\gamma^s; \alpha_t) \right) = \alpha_t$ from any producer's optimizing behavior, and thus

$$\gamma^s Y^s \left(\lambda^d, \gamma^s; \alpha_t, S_t \right) = \frac{\alpha_t}{b} \lambda^d n' \ell^d (\gamma^s; \alpha_t) + \gamma^s S_t = \frac{\alpha_t}{b} \lambda^d L^d (\gamma_t^s; \alpha_t) + \gamma^s S_t.$$

This implies immediately that

$$\overline{F}^C = \left\{ \left(L^d(1; \alpha_t), \frac{\alpha_t}{b} L^d(1; \alpha_t) + S_t \right) \right\}.$$

Consider now

$$\overline{F}^K = \left\{ \left(L^d(\gamma^s; \alpha_t), \gamma^s Y^s(1, \gamma^s; \alpha_t, S_t) \right) \mid \gamma^s \in [0,1) \right\}.$$

Then (3.4) yields

$$\gamma^s Y^s(1, \gamma^s; \alpha_t, S_t) = \frac{\alpha_t}{b} L^d(\gamma_t^s; \alpha_t) + \gamma^s S_t.$$

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On the other hand, (2.5) implies

$$\gamma^s = \frac{\alpha_t}{ab} \left(\ell^d \left(\gamma_t^s; \alpha_t \right) \right)^{1-b} = \frac{\alpha_t}{ab} \left(\frac{L^d \left(\gamma_t^s; \alpha_t \right)}{n'} \right)^{1-b}$$

and therefore

$$\gamma^s Y^s \left(1, \gamma^s; \alpha_t, S_t \right) = \frac{\alpha_t}{b} L^d \left(\gamma_t^s; \alpha_t \right) + \frac{\alpha_t}{ab} \left(\frac{L^d \left(\gamma_t^s; \alpha_t \right)}{n'} \right)^{1-b} S_t.$$

Since $L^d \left(\gamma_t^s; \alpha_t \right)$ is strictly increasing in γ_t^s, this yields

$$\overline{F}^K = \left\{ \left(L, \frac{\alpha_t}{b} L + \frac{\alpha_t}{ab} \left(\frac{L}{n'} \right)^{1-b} S_t. \right) \mid 0 \leq L < L^d \left(1; \alpha_t \right) \right\}. \qquad (3.5)$$

Consider next

$$\overline{F}^I = \left\{ \left(\lambda^d L^d \left(1; \alpha_t \right), Y^s \left(\lambda^d, 1; \alpha_t, S_t \right) \right) \mid \lambda^d \in [0,1) \right\}.$$

By (3.4) $Y^s \left(\lambda^d, 1; \alpha_t \right) = \frac{\alpha_t}{b} \lambda^d L^d \left(1; \alpha_t \right) + S_{t-1}$ and therefore

$$\overline{F}^I = \left\{ \left(L, \frac{\alpha_t}{b} L + S_t \right) \mid 0 \leq L < L^d \left(1; \alpha_t \right) \right\}.$$

Since $\frac{\alpha_t}{ab} \left(\frac{L}{n'} \right)^{1-b} = \gamma^s \leq 1$, $\overline{F}^K$ is positioned below $\overline{F}^I$.

Finally consider $\overline{F}^U$. It is given by

$$\overline{F}^U = \left\{ \left(\lambda^d L^d \left(\gamma^s; \alpha_t \right), \frac{\alpha_t}{b} \lambda^d L^d \left(\gamma_t^s; \alpha_t \right) + \frac{\alpha_t}{ab} \left(\frac{L^d \left(\gamma_t^s; \alpha_t \right)}{n'} \right)^{1-b} S_t \right) \mid \left(\lambda^d, \gamma^s \right) \in [0,1)^2 \right\} \qquad (3.6)$$

Comparing with $\overline{F}^K$ and $\overline{F}^I$, it is clear that $\overline{F}^U$ is the set of points contained between $\overline{F}^K$ and $\overline{F}^I$. Figure 3.1 illustrates the producers' trade curves.

Using the consumption sector's and the production sector's trade curves and indicating with S^c the closure of the set S, we now note that a pair $\left(\overline{L}, \overline{Y} \right) \in R_+^2$ is a temporary equilibrium allocation if and only if it is an element of the set

$$Z = \left(\left(\overline{H}_0^K \right)^c \cap \left(\overline{F}^K \right)^c \right) \cup \left(\left(\overline{H}_0^I \right)^c \cap \left(\overline{F}^I \right)^c \right) \cup \left(\left(\overline{H}_0^C \right)^c \cap \left(\overline{F}^C \right)^c \right) \cup \left(\left(\overline{H}_0^U \right)^c \cap \left(\overline{F}^U \right)^c \right).$$
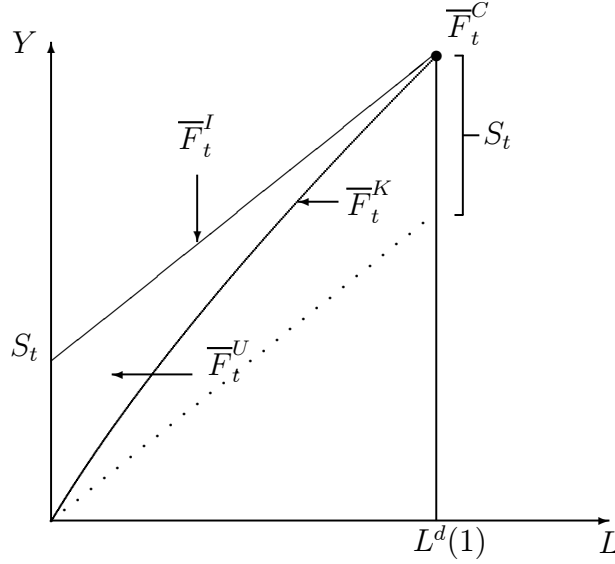
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Figure 3.1: The producers' trade curves

(Here S^c indicates the closure of the set S.) To show existence of an equilibrium is equivalent to showing that Z is not empty. To this end consider first the locus

$$\left(\overline{H}_0^K\right)^c = \left\{\left(\lambda_t^s L^s, X^d\left(\lambda_t^s\right) + m_t + G\right) \mid \lambda_t^s \in [0,1]\right\}$$

and recall that

$$X^d\left(\lambda_t^s\right) = nh\left(\lambda_t^s \omega_t^1 + (1 - \lambda_t^s)\omega_t^0\right) = h\left(1 - tax\right)\pi_t + h\alpha_t\lambda_t^s L^s.$$

Defining the function

$$\Gamma_t\left(L\right) = h\left(1 - tax\right)\pi_t + h\alpha_t L + m_t + G, \ L \geq 0,$$

we see that $\left(\overline{H}_0^K\right)^c$ is the part of the graph of Γ_t for which $L \leq L^s$.

Next consider again the production sector's trade curves. From (3.5) we conclude that the locus $\left(\overline{F}^K\right)^c$ is the part of the graph of the function

$$\Delta_t\left(L\right) = \frac{\alpha_t}{b}L + \frac{\alpha_t}{ab}\left(\frac{L}{n'}\right)^{1-b}S_t, \ L \geq 0,$$

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for which $L \leq L^d(1)$. Notice that the graphs of the functions Γ_t and Δ_t always intersect. Indeed, $\Gamma'_t(L) = h\alpha_t$ and $\Gamma_t(0) = h(1 - tax)\pi_t + m_t + G > 0$, whereas $\Delta'_t(L) \geq \frac{\alpha_t}{b} > h\alpha_t$ (since $1/b > 1 > h$) and $\Delta_t(0) = 0$. Setting $\Delta_t(L) = \Gamma_t(L)$ yields (3.2) with the unique solution denoted $\widetilde{L}(\alpha_t, \pi_t, m_t, G, tax)$. Therefore the equilibrium level on the labor market is

$$\overline{L}_t = \min\left\{\widetilde{L}(\alpha_t, \pi_t, m_t, G, tax), L^d(1, \alpha_t), L^s\right\} = \mathcal{L}(\alpha_t, \pi_t, m_t, S_t, G, tax).$$

whereas the on the goods market is, by definition of the function $\mathcal{Y}(\cdot)$,

$$\overline{Y}_t = \mathcal{Y}(\alpha_t, \pi_t, m_t, S_t, G, tax).$$

This shows that the equilibrium allocation

$$\left(\overline{L}_t, \overline{Y}_t\right) = \left(\mathcal{L}(\alpha_t, \pi_t, m_t, S_t, G, tax), \mathcal{Y}(\alpha_t, \pi_t, m_t, S_t, G, tax)\right)$$

exists and is uniquely defined. ∎

Equation (3.1) allows us to characterize the type of equilibrium defined in Table 1: if $\overline{L}_t = \widetilde{L}(\alpha_t, \pi_t, m_t, S_t, G, tax)$, the resulting equilibrium is of type K or a limiting case of it. If $\overline{L}_t = L^d(1, \alpha_t)$, type C or a limiting case of it occurs. Finally, if $\overline{L}_t = L^s$, an equilibrium of type I or a limiting case results if $\frac{\alpha_t}{b}L^s + S_t \leq h(1 - tax)\pi_t + h\alpha_t L^s + m_t + G$; otherwise the equilibrium is of type U.

The above discussion and Proposition 3.2 allow us to determine the expressions of those rationing coefficients which are possibly smaller than one. This is summarized in the following corollary.

Corollary 3.3. *In case K, $\lambda_t^s = \frac{\overline{L}_t}{L^s}$ and $\gamma_t^s = \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}$. In case C, $\lambda_t^s = \frac{\overline{L}_t}{L^s}$ and, in case I, $\lambda_t^d = \frac{L^s}{L^d(1,\alpha_t)}$. Moreover, in both these latter cases,*

$$\left(\gamma_t^d, \delta_t, \varepsilon_t\right) = \begin{cases} \left(\frac{\overline{Y}_t - m_t - G}{h(1-tax)\pi_t + h\alpha_t\overline{L}_t}, 1, 1\right) & \text{if } \overline{Y}_t \geq G + m_t \\ \left(0, \frac{\overline{Y}_t - G}{m_t}, 1\right) & \text{if } G + m_t > \overline{Y}_t \geq G \\ \left(0, 0, \frac{\overline{Y}_t}{G}\right) & \text{if } \overline{Y}_t < G \end{cases}$$

Finally, in case U $\gamma_t^s = \frac{1}{S_t}\left(\overline{Y}_t - \frac{\alpha_t}{b}\overline{L}_t\right)$ and $\lambda_t^d = \overline{L}_t / L^d(\gamma_t^s; \alpha_t)$.

Proof: By (3.6) in case U it must be true that

$$\left(\overline{L}_t, \overline{Y}_t\right) = \left(\lambda^d L^d\left(\gamma^s; \alpha_t\right), \frac{\alpha_t}{b}\lambda^d L^d\left(\gamma_t^s; \alpha_t\right) + \frac{\alpha_t}{ab}\left(\frac{L^d\left(\gamma_t^s; \alpha_t\right)}{n'}\right)^{1-b} S_t\right).$$

Moreover by (2.5)

$$L^d\left(\gamma^s; \alpha_t\right) = n'\left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{1}{1-b}}.$$

Therefore

$$\frac{\alpha_t}{b}\lambda^d L^d\left(\gamma_t^s; \alpha_t\right) + \frac{\alpha_t}{ab}\left(\frac{L^d\left(\gamma_t^s; \alpha_t\right)}{n'}\right)^{1-b} S_t = \overline{Y}_t$$

$\Leftrightarrow$

$$\frac{\alpha_t}{b}\lambda_t^d L^d\left(\gamma_t^s; \alpha_t\right) + \gamma_t^s S_t = \overline{Y}_t$$

Recalling that $\lambda^d L^d\left(\gamma^s; \alpha_t\right) = \overline{L}_t$ and solving for γ_t^s yields the claimed expression. The values of λ_t^s and λ_t^d are immediate by definition; γ_t^s can be obtained using assumption (A2) and equation (2.5). Finally, γ_t^d, δ_t, ε_t are determined by means of (2.3). ∎

Using the consumption and the production sectors' trade and offer curves it is possible to analyze the various equilibrium regimes in more detail. We do this here for the case of Keynesian Unemployment only. This type of equilibrium involves rationing of households on the labor market and of firms on the goods market. It is given by a pair $\left(\lambda_t^s, \gamma_t^s\right)$ such that

$$\begin{aligned}
\overline{L}_t &= \lambda_t^s L^s = L^d\left(\gamma_t^s\right) \\
\overline{Y}_t &= \gamma_t^s Y^s\left(1, \gamma^s\right) = X^d\left(\lambda_t^s\right) + m_t + G
\end{aligned}$$

(where we have suppressed all arguments that are not rationing coefficients). Recalling the definition of the trade curves $\overline{H}^K$ and $\overline{F}^K$, the pair $\left(\overline{L}_t, \overline{Y}_t\right)$ is a Keynesian equilibrium allocation if

$$\begin{aligned}
\left(\overline{L}_t, \overline{Y}_t\right) &\in \left\{\left(\lambda^s L^s, X^d\left(\lambda^s\right) + m_t + G\right) \mid \lambda^s \in [0, 1)\right\} \\
&\quad \cap \left\{\left(L^d\left(\gamma^s\right), \gamma^s Y^s\left(1, \gamma^s\right)\right) \mid \gamma^s \in [0, 1)\right\} \\
&= \left[\overline{H}_t^K + \{(0, m_t + G)\}\right] \cap \overline{F}_t^K.
\end{aligned}$$

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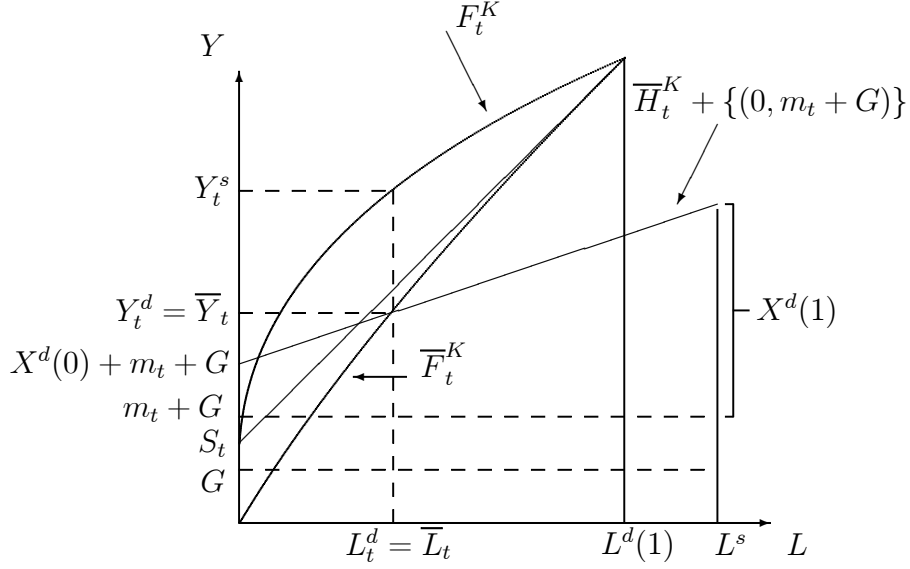
Figure 3.2: Keynesian Unemployment Equilibrium

Thus $\left(\overline{L}_t, \overline{Y}_t\right)$ is given by the intersection of the trade curves $\overline{H}_t^K + \{(0, m_t + G)\}$ and $\overline{F}_t^K$, as shown in Figure 3.2. There an equilibrium of type Keynesian unemployment is shown, where $F_t^K \equiv \left\{\left(L^d\left(\gamma^s; \alpha_t\right), Y^s\left(1, \gamma^s; \alpha_t, S_t\right)\right) \mid \gamma^s \in [0, 1)\right\}$.

The consumption sector supplies the amount of labor $L^s > \overline{L}_t$ and demands the quantity of goods $Y_t^d = \overline{Y}_t$ whereas firms demand labor $L_t^d = \overline{L}_t$ and supply $Y_t^s > \overline{Y}_t$ of goods. It follows that $\lambda_t^s = \overline{L}_t/L^s, \gamma_t^s = \overline{Y}_t/Y_t^s$ and $\lambda_t^d = \gamma_t^d = 1\ (= \delta_t = \varepsilon_t)$, which are just the values that led households and firms to express their respective transaction offers. Thus their expectations regarding these rationing coefficients are confirmed. Nevertheless, due to the randomness in rationing at an individual agent's level, effective aggregate demands and supplies of rationed agents exceed their actual transactions. Moreover, as indicated earlier, these excesses can be used to get an indicator of the strength of rationing. Since there is zero-one rationing on the labor market, $1 - \lambda_t^s = (L^s - \overline{L}_t)/L^s$ is the ratio of the number of unemployed workers and the total number of young households. Regarding the
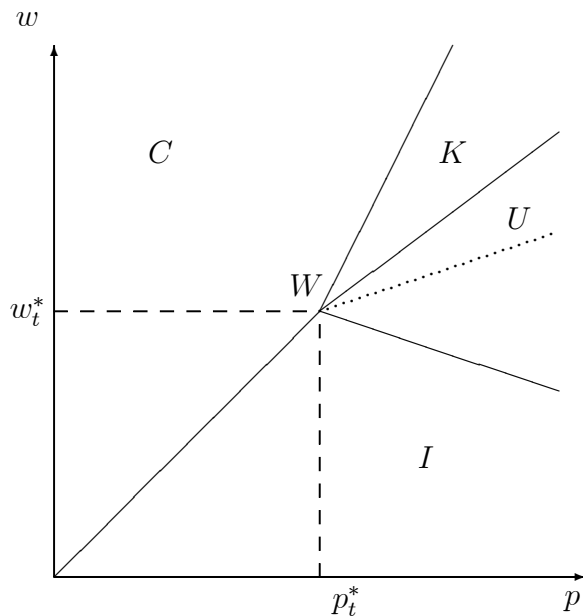
Figure 3.3: Repressed Inflation Equilibrium

goods market, in a K-equilibrium $\overline{Y}_t - \gamma_t^s Y^s (1, \gamma_t^s) = 0$, and therefore

$$\frac{d(1 - \gamma_t^s)}{d\overline{Y}_t} = -\frac{1}{Y_t^s + \gamma_t^s \frac{\partial Y^s}{\partial \gamma_t^s}} < 0$$

since $\frac{\partial Y^s}{\partial \gamma_t^s} (1, \gamma_t^s) = n' f' \left(\ell^d (\gamma_t^s)\right) \frac{d\ell^d}{d\gamma_t^s} > 0$. So a decrease in $\overline{Y}_t$ (for example due to a reduction of government spending), and thus an aggravation of the shortage of aggregate demand for firms' goods, is unambiguously related to an increase in $1 - \gamma_t^s$ which can therefore be interpreted as a measure of the strength of rationing on the goods market. A similar reasoning justifies the use as rationing measures of the terms $1 - \lambda_t^d$ and $1 - \gamma_t^d$ in the other equilibrium regimes.

The illustration of the other temporary equilibrium regimes works similarly except for the fact that under repressed inflation and classical unemployment old agents and/or the government may be rationed, too. This is shown in Figure 3.3 for the case of repressed inflation and rationing of old agents.

Figure 4.1: Temporary Equilibrium Regimes in the $p - w$ plane.

4. Representation of Equilibrium Regimes

Given the existence and uniqueness of temporary equilibrium we can, holding all other variables fixed, partition the set R_+^4 of all combinations of real wage α_t, real profits π_t, real money stock m_t and inventories S_t according to the type of equilibrium they give rise to. Formally, we have a map $(\alpha_t, \pi_t, m_t, S_t) \mapsto T \in \{K, I, C, U\}$. Holding also nominal money M, nominal profits Π and inventories S parametrically fixed, we can furthermore derive from this a map

$$(p_t, w_t) \mapsto (w_t/p_t, \Pi/p_t, M/p_t, S) \mapsto T$$

which is illustrated in Figure 4.1 and shows the partitioning of $p_t - w_t-$plane in different regimes of types of equilibrium. From this diagram, in principle familiar from the literature,[7] it can be seen that too high a goods price and a nominal wage give rise to a state of Keynesian unemployment and hence excess supply on both markets, even if the real wage is at its Walrasian level. If the real wage is

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[7]See for instance Malinvaud [1977] and Muellbauer and Portes [1978].

too high, Classical unemployment occurs whereas in the opposite case a situation of repressed inflation.

The figure differs from what is shown in the literature with respect to the slope of the borderline between regimes U and I: there it is negative whereas here it may be positive. To see this, consider (p_t, w_t) such that $T(w_t/p_t, \Pi/p_t, M/p_t, S) = U^c \cap I^c$. Then consumers are not rationed while producers are rationed only on the labor market. Writing $(w_t/p_t, \Pi/p_t, M/p_t, S) = (\alpha, \pi, m, S)$, the corresponding equilibrium must thus satisfy

$$\left(\overline{L}_t, \overline{Y}_t\right) = \left(\lambda^d L^d(1; \alpha), Y^s\left(\lambda^d, 1; \alpha, S\right)\right) = (L^s, h(1 - tax)\pi + h\alpha L^s + m + G).$$

By (3.4) $Y^s\left(\lambda^d, 1; \alpha, S\right) = \frac{\alpha_t}{b}\lambda^d L^d(1; \alpha_t) + S_t = \frac{\alpha}{b}L^s + S$, and therefore we obtain the condition

$$\frac{\alpha}{b}L^s + S = h(1 - tax)\pi + h\alpha L^s + m + G.$$

Multiplying by p and solving for w yields

$$w = \frac{h(1 - tax)\Pi + M}{\left(\frac{1}{b} - h\right)L^s} + \frac{G - S}{\left(\frac{1}{b} - h\right)L^s}p.$$

From this it is obvious that this function, which describes the borderline between the regimes U and I, is downward sloping iff $S > G$.

5. Dynamics

So far our analysis has been essentially static. For any given vector $(\alpha_t, \pi_t, m_t, S_t, G, tax)$ we have described a feasible allocation in terms of a temporary equilibrium with rationing. To extend now our analysis to a dynamic one we must link successive periods one to another. This link will of course be given by the adjustment of prices but also by the changes in the stock of money and in profits. Regarding the latter, this is automatic by definition of these variables and equations (3.1) to (3.3), i.e.

$$\Pi_t = p_t \mathcal{Y}(\alpha_t, \pi_t, m_t, S_t, G, tax) - w_t \mathcal{L}(\alpha_t, \pi_t, m_t, G, tax),$$

$$\begin{aligned} M_{t+1} &= (1 - tax)\Pi_{t-1} + w_t\overline{L}_t - p_t\overline{Y}_t + \delta_t M_t + \varepsilon_t p_t G \\ &= (1 - tax)\Pi_{t-1} - \Pi_t + \delta_t M_t + \varepsilon_t p_t G. \end{aligned}$$

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$$S_{t+1} = Y^s \left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t \right) - \mathcal{Y} \left(\alpha_t, \pi_t, m_t, S_t, G, tax \right).$$

Regarding the adjustment of prices and wages we follow the standard hypothesis that, whenever an excess of demand (supply) is observed, the price rises (falls). In terms of the rationing coefficients observed in period t, this amounts to

$$p_{t+1} < p_t \Leftrightarrow \gamma_t^s < 1; \ p_{t+1} > p_t \Leftrightarrow \gamma_t^d < 1,$$

$$w_{t+1} < w_t \Leftrightarrow \lambda_t^s < 1; \ w_{t+1} > w_t \Leftrightarrow \lambda_t^d < 1.$$

More precisely, in our simulation model we have specified these adjustments as follows:[8]

$$p_{t+1} = \begin{cases} \left[1 - \mu_1 \left(1 - \gamma_t^s \right) \right] p_t & \text{if } \gamma_t^s < 1 \\ \left[1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3} \right) \right] p_t & \text{if } \gamma_t^d < 1 \end{cases} \tag{5.1}$$

$$w_{t+1} = \begin{cases} \left[1 - \nu_1 \left(1 - \lambda_t^s \right) \right] w_t & \text{if } \lambda_t^s < 1 \\ \left[1 + \nu_2 \left(1 - \lambda_t^d \right) \right] w_t & \text{if } \lambda_t^d < 1 \end{cases} . \tag{5.2}$$

Then the adjustment equations for the real wage are

$$\alpha_{t+1} = \begin{cases} \frac{1 - \nu_1 (1 - \lambda_t^s)}{1 - \mu_1 (1 - \gamma_t^s)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t \right) \in K \\ \frac{1 - \nu_1 (1 - \lambda_t^s)}{1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3} \right)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t \right) \in C \\ \frac{1 + \nu_2 \left(1 - \lambda_t^d \right)}{1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3} \right)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t \right) \in I \\ \frac{1 + \nu_2 \left(1 - \lambda_t^d \right)}{1 - \mu_1 (1 - \gamma_t^s)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t \right) \in U \end{cases} \tag{5.3}$$

whereas θ_t is given by

$$\theta_t = \begin{cases} 1 - \mu_1 \left(1 - \gamma_t^s \right) & \text{if } \left(\overline{L}_t, \overline{Y}_t \right) \in K \cup U \\ 1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3} \right) & \text{if } \left(\overline{L}_t, \overline{Y}_t \right) \in C \cup I \end{cases} . \tag{5.4}$$

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[8]We employ a linear rule to avoid that one might suspect that the complex dynamics be generated by a nonlinear adjustment mechanism. Experimenting with some other specifications of the adjustment mechanism has revealed that our subsequent simulation results are not limited to the one presented here.

The dynamics of the model in real terms is given by the sequence $\{(\alpha_t, m_t, \pi_t, S_t)\}_{t=1}^{\infty}$, where α_{t+1} is as in (5.3) and, using equations (3.1) to (3.3),

$$
\pi_{t+1} = \begin{cases}
\frac{1-b}{\theta_t(1-hb)}\left[h\left(1-tax\right)\pi_t + m_t + G\right] & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \\
\frac{1-b}{\theta_t}n'\left(\frac{\alpha_t}{ab}\right)^{\frac{b}{b-1}}\left(\frac{1}{a}\right)^{\frac{1}{b-1}} & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in C \\
\frac{\alpha_t}{\theta_t}\frac{1-b}{b}L^s & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in I \\
\frac{1}{\theta_t}\left[h\left(1-tax\right)\pi_t + m_t + G - \alpha_t\left(1-h\right)L^s\right] & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in U
\end{cases}.
$$

The case U is derived as follows:

$$
\begin{aligned}
\pi_{t+1} &= \frac{\Pi_t}{p_{t+1}} = \frac{p_t\left[h\left(1-tax\right)\pi_t + h\alpha_t L^s + m_t + G\right] - w_t L^s}{p_{t+1}} \\
&= \frac{1}{\theta_t}\left[h\left(1-tax\right)\pi_t + h\alpha_t L^s + m_t + G - \alpha_t L^s\right].
\end{aligned}
$$

Finally,

$$
m_{t+1} = \frac{1}{\theta_t}\left[\delta_t m_t + \varepsilon_t G + \left(1-tax\right)\pi_t\right] - \pi_{t+1},
$$

and

$$
S_{t+1} = \lambda_t^d n' a\left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.
$$

6. Simulations

The economic model introduced in the previous sections represents a non-linear dynamical system that cannot be studied with analytical tools only. This is due to the fact that the system is four-dimensional, with state variables α_t, m_t, π_t and S_t. Moreover, since there are four nondegenerate equilibrium regimes, the overall dynamic system can be viewed as being composed of four subsystems each of which may become effective through endogenous regime switching. (The complete equations of these systems are given in Appendix 2.)

In order to get some insights in these dynamics we are reporting numerical simulations using programs developed for this paper's purposes based on the packages GAUSS and MACRODYN [9]. The basic parameter set specifies values for the technological coefficients (a and b), the exponent of the utility function (h),

---

[9] MACRODYN has been developed at the University of Bielefeld. See Böhm,V., Lohmann, M. and U. Middelberg [1999], MACRODYN – a dynamical system's tool kit, version x99 and Böhm and Schenk-Hoppé [1998].
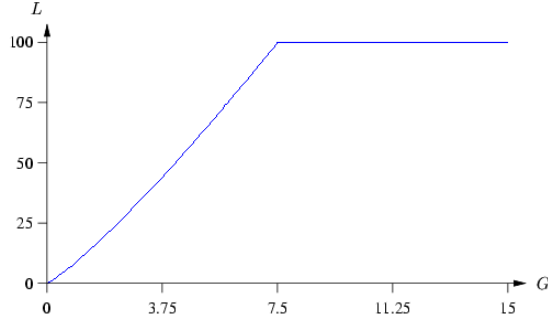
Figure 6.1: Stationary employment values when $\nu_1 = 0$.

the labor supply (L^s) and the total number of producers in the economy (n'), for the price adjustment speeds downward and upward (respectively μ_1 and μ_2) and the corresponding wage adjustment speeds (ν_1 and ν_2). We also have to specify initial values for the real wage, real money stock, real profit level and inventories $(\alpha_0, m_0, \pi_0$ and $S_0)$, and values for the government policy parameters (G and tax). Choosing in addition an initial value p_0 for the goods price, we can moreover keep track of the development of the nominal variables by using (5.1) to determine p_t for any t from which follow $w_t = \alpha_t p_t$ and $M_t = m_t p_t$.

Assuming the parameter values $a = 1$, $b = 0.85$, $h = 0.5$, $L^s = 100$ and $n' = 100$, a stationary Walrasian equilibrium is obtained for

$$\alpha^* = 0.85, \quad m^* = 46.25, \quad \pi^* = 15, \quad S^* = 0, \quad G^* = 7.5, \quad tax^* = 0.5, \qquad (6.1)$$

with trading levels $L^* = Y^* = 100$. For the adjustment speeds of prices out of Walrasian equilibrium we set $\mu_1 = \mu_2 = \nu_2 = 0.1$ whereas ν_1, the downward speed of wage adjustment, will be varied between 0 and 0.1. This includes the case $\nu_1 = 0$ in which the wage rate is rigid downwards.

6.1. Fiscal Shocks

Our first investigation regards a change in G. Starting from $(\alpha_0, m_0, \pi_0, S_0) = (\alpha^*, m^*, m^*, S^*)$, the bifurcation diagram in Figure 6.1 shows, for $\nu_1 = 0$, the stationary values of employment to which the system converges in dependence of values of G between 0 and 15. From this it is evident that $\overline{L} < L^*$ for $G < G^*$ and

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$\overline{L} = L^*$ for $G = G^*$. What happens in case $G < G^*$ is that aggregate demand Y^d is diminished which creates an excess supply on the goods market. Consequently firms reduce their production and cut back on employment. The result is an excess supply on the labor market, too, and the economy enters in a state of Keynesian unemployment. The imbalance on the goods market gives rise to a price decrease whereas on the labor market the nominal wage cannot decrease as $\nu_1 = 0$. As a result the real wage increases. This is illustrated in Figure 6.2 which shows the time series for employment L, inventories S, the real money stock m and the real wage α for the first 200 periods where $G = 7$. The real wage is rising until approximately period 30 at which point it has become large enough so that the system enters into the regime of Classical unemployment. Here the goods price decreases and the real wage falls until at around period 50 it settles at a stationary value $\overline{\alpha} > \alpha^*$. Since the nominal wage rate does not change, the constant real wage implies that the goods price does not change either beyond period 50, and the economy has reached a stationary state at the frontier between Keynesian and Classical unemployment. In that state there is market clearing on the goods market but excess supply on the labor market.

Next consider, again for $G < G^*$, what happens when $\nu_1 > 0$. The charts in Figure 6.3 show, analogously to Figure 6.1, stationary values of employment for various values of G. The top and the middle chart refer to downward wage flexibilities of $\nu_1 = 0.025$ and $\nu_1 = 0.1$, respectively. The striking result is that a little downward wage flexibility has an enormous effect on the impact of fiscal restraint as is documented by the discontinuity of the graphs at $G = G^*$. Note that this does not happen in the model without inventories, as is shown by the lower chart in Figure 6.3 where $\nu_1 = 0.1$ but S_t is exogenously set to zero at the beginning of each period.

Why inventories have such a dramatic effect is easily explained. When aggregate demand is diminished due to a decrease in G, inventories become positive and rise further as excess supply on the goods market builds up. As $\gamma^s = \overline{Y}_t/Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right)$ by (2) of Definition 1 and S_t influences Y^s positively by (2.6), an increase in S_t reduces the sales expectation ratio γ^s which by (2.5) diminishes the labor demand of firms and thus increases further the excess supply on the labor market. Therefore the downward flexible wage rate decreases more than would be the case without inventories. If the decrease in the wage rate is larger than the decrease in the goods price, the real wage decreases, and it may continue to decrease permanently approaching a limit level below the Walrasian real wage. The lower real wage diminishes labor income of workers which diminishes
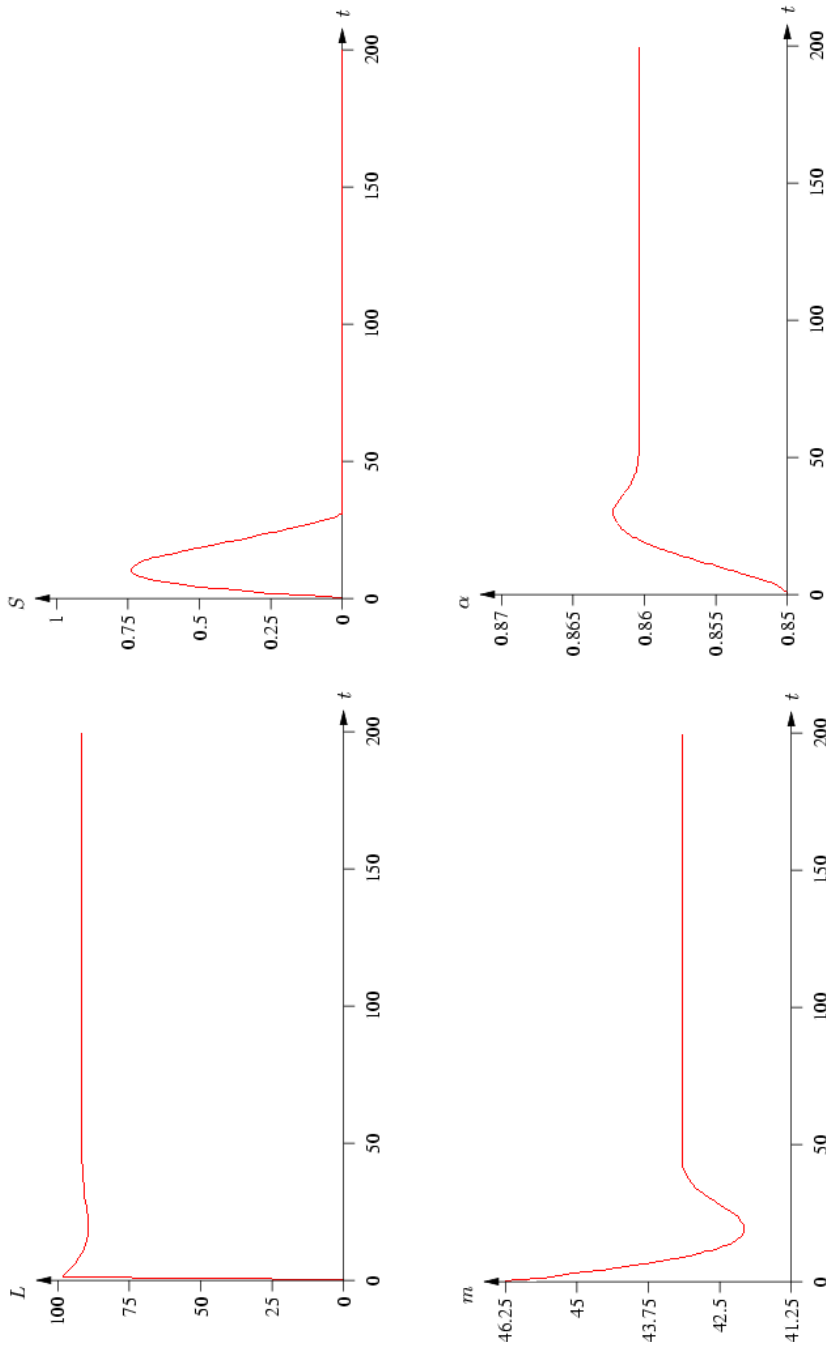
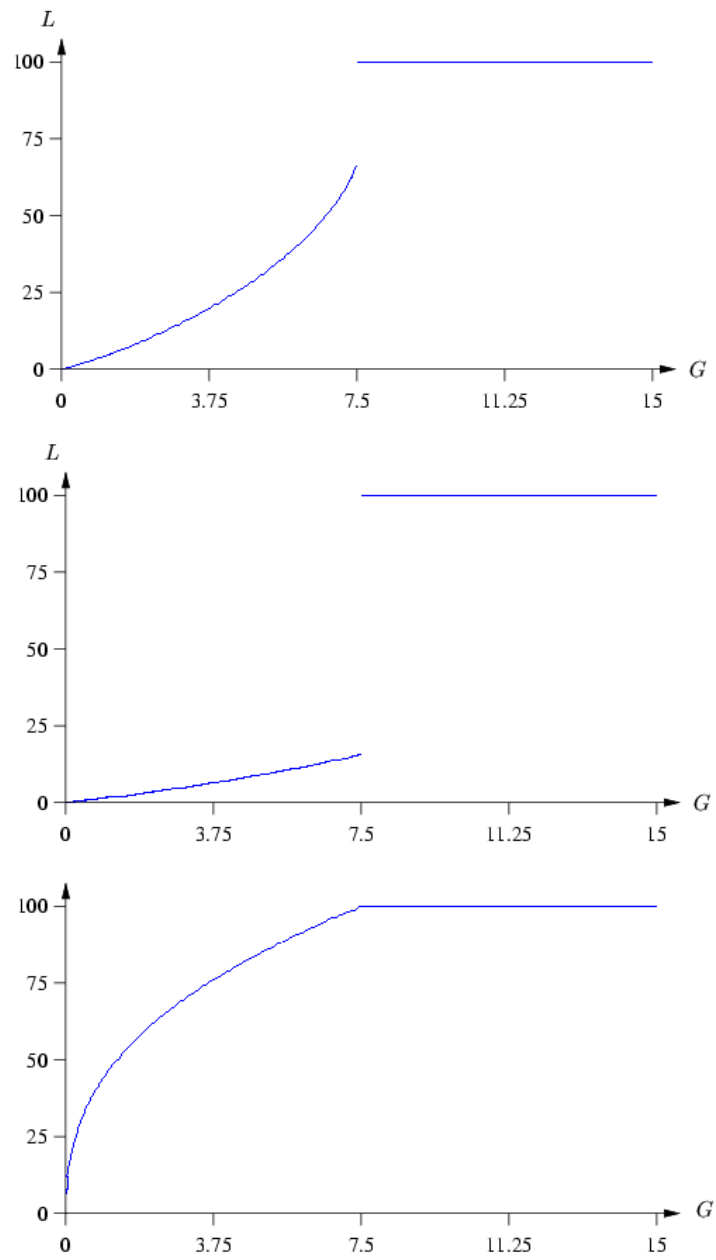Figure 6.2: Time series when $\nu_1 = 0$ and $G = 7$.

Figure 6.3: Stationary employment values for $\nu_1 = 0.025$ (top chart), $\nu_1 = 0.1$ (mid chart), and $\nu_1 = 0.1$ and $S_t \equiv 0$.

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aggregate goods demand which in turn keeps employment below full employment. The dynamical system converges to a quasi-stationary Keynesian state with permanent deflation of all nominal variables but constant real magnitudes.[10] The nominal money stock shrinks because, due to the small government spending, the government is permanently realizing a budget surplus. These facts are illustrated in Figure 6.4 which shows time series for $\nu_1 = 0.025$.

When $G > G^*$, one can similarly show that the economy converges to a quasi-stationary state of Repressed inflation with permanent increase of all nominal variables and full employment with constant excess demands on the labor and the goods market.

6.2. A Restrictive Monetary Shock

We consider a reduction in the initial money stock to $m_0 = 40$, keeping all other parameters and initial values at their Walrasian levels. Having set $p_0 = 1$, this is equivalent to a reduction in the nominal money stock from $M_0 = 46.25$ to $M_0 = 40$. Since m_0 is the demand of old agents at time $t = 0$, aggregate demand is reduced. Consequently there is excess supply on the goods market and, since firms adjust to the reduced transaction level on the goods market, they reduce their labor demand. Thus there is excess supply on the labor market, too, and the economy enters in a state of Keynesian unemployment. What happens next depends on whether the nominal wage is flexible downwards. If not, the real wage and the real money stock increase - as shown in Figure 6.5 - until the economy reaches a state of Classical unemployment. Thereafter the price increase reduces the real wage until the system is back at the Walrasian equilibrium. With nominal wage rigid downwards the restrictive money shock has had a temporary but not lasting effect on economic activity.

The picture changes when downward wage flexibility is allowed. This is shown in Figure 6.6 for $\nu_1 = 0.025$, where employment, real wage and real money all converge to values lower than the respective Walrasian values. The reason is similar to that already discussed in the context of fiscal shocks: the presence of inventories increases the fall of labor demand by firms which in turn depresses labor income and aggregate demand. The system tends to a quasi-stationary Keynesian state with permanent deflation of nominal variables. The restrictive monetary shock has caused a permanent decrease in employment and output.

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[10]A state is *stationary* if all variables are constant; it is *quasi-stationary* if all real variables are constant but the nominal variables may change.
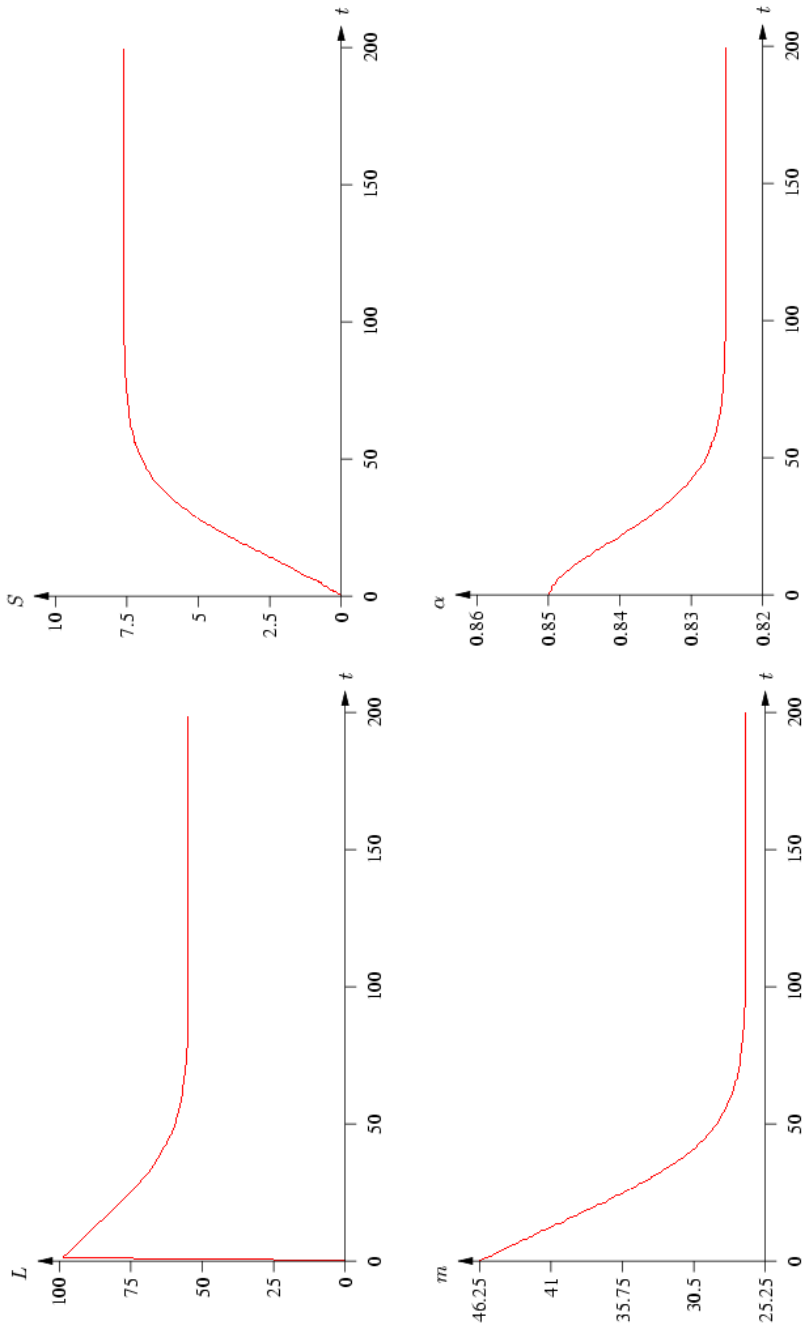
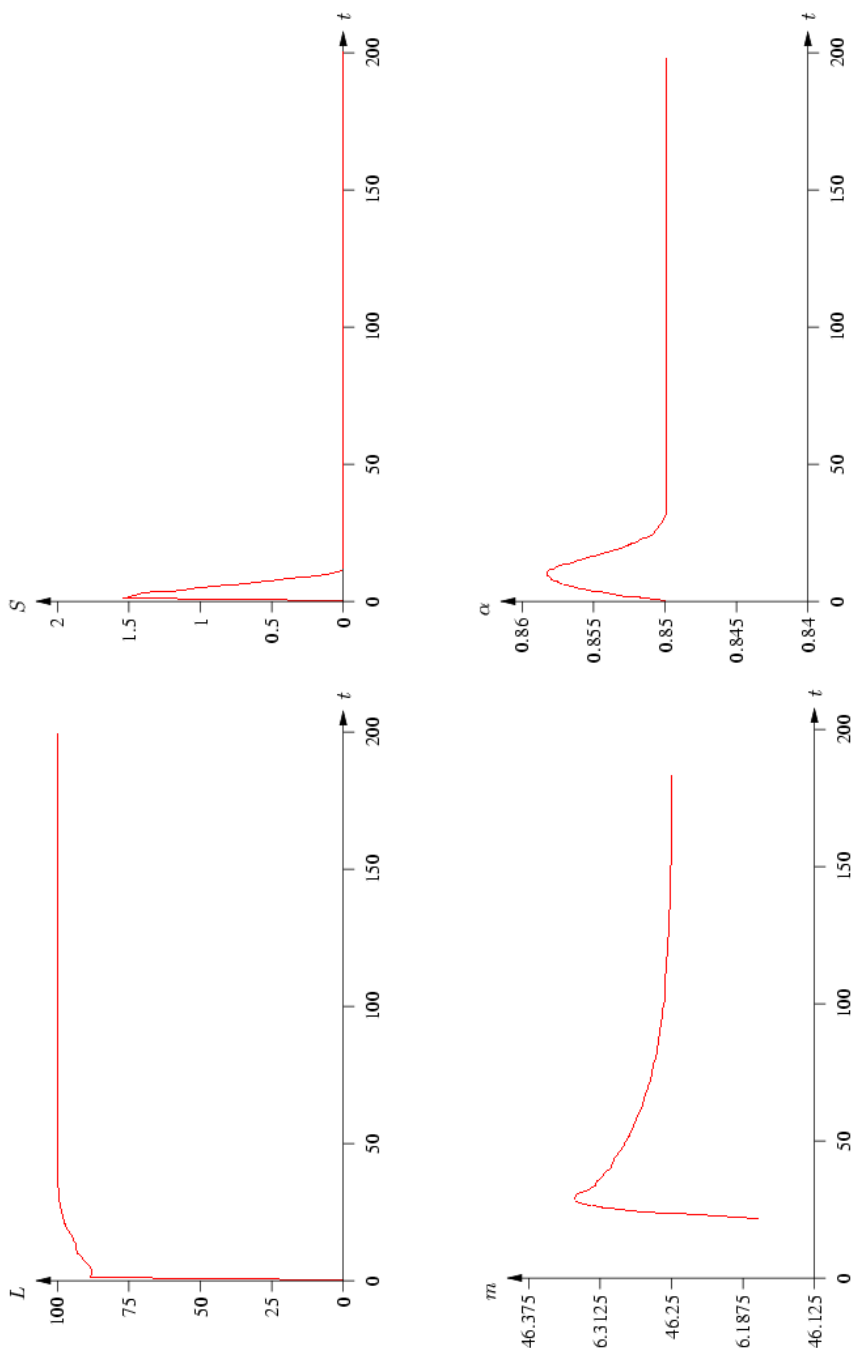Figure 6.4: Time series when $\nu_1 = 0.025$ and $G = 7$.

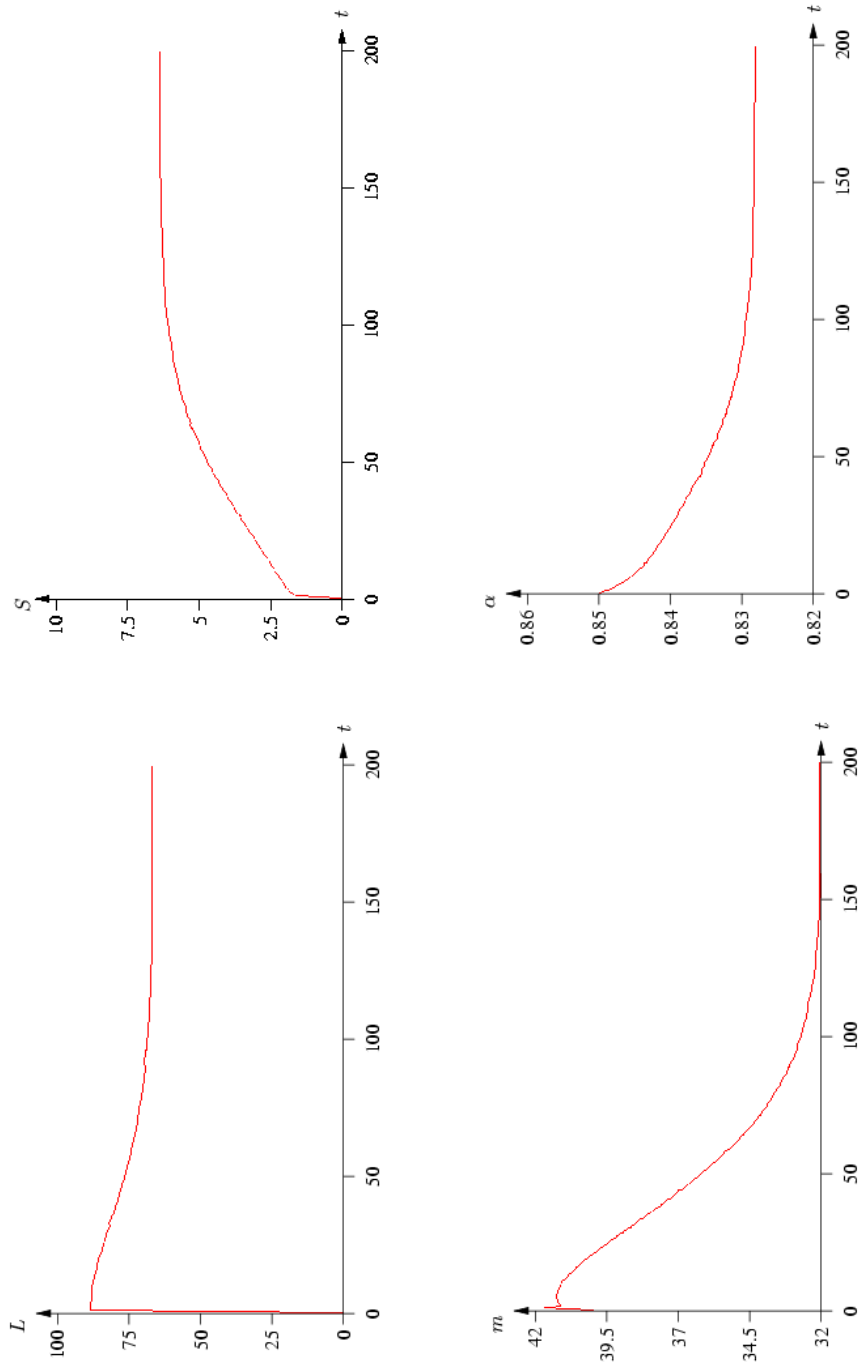Figure 6.5: Time series when $\nu_1 = 0$ and $m_0 = 40$.

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Figure 6.6: Time series when $\nu_1 = 0.025$ and $m_0 = 40$.

As in the case of a fiscal shock, setting $S_t \equiv 0$ changes the outcome also in the scenario of a monetary shock: this is shown in Figure 6.7 where again $\nu_1 = 0.025$. The real wage decreases initially but then the decrease in the goods price dominates the one in the nominal wage, and the real wage moves back to its Walrasian level, as do all the other variables.

At this point the natural question is which downward wage flexibility is needed to drive the economy into a permanent recession or even depression. The answer is given in the bifurcation diagram of Figure 6.8. From that it can be seen that approximately until $\nu_1 = 0.02$ the economy is capable of returning to the full employment after the monetary shock, whereas for speeds of wage adjustment larger than this the economy gets trapped in underemployment.

7. Stability

The fact that a restrictive monetary shock may lead to a Keynesian quasi-stationary state as limit of the dynamic system's trajectory raises the wider question of the stability of such a state. Analogously, the stability of the stationary Walrasian state may be investigated. To anticipate the answer, numerical simulations suggest that the latter is unstable whereas the quasi-stationary Keynesian unemployment state is locally stable.

Let us first look at the stationary Keynesian state. The limit values of the state variables of the simulation shown in Figure 6.6, where we have an initial reduction of the money stock to $m_0 = 40$ at a downward wage flexibility of $\nu_1 = 0.025$ and policy parameters $G^* = 7.5$ and $tax^* = 0.5$, are (approximately)

$$\overline{\alpha} = 0.8281, \quad \overline{m} = 31.9263, \quad \overline{\pi} = 15.7889, \quad \overline{S} = 6.4060, \tag{7.1}$$

with a stationary employment level $\overline{L} = 66.9342$. We now take these stationary values as new initial values, i.e. set $(\alpha_0, m_0, \pi_0, S_0) = (\overline{\alpha}, \overline{m}, \overline{\pi}, \overline{S})$, and perform a bifurcation analysis with respect to each of the state variables around these initial values and dependent variable the employment level $\overline{L}$. The results are shown in Figure 7.1. The diagrams show that local deviations in directions of any of the state variables do not change the fact that the system has this state as an attracting long-run rest point. Although this is not a strict proof of local stability, it is highly indicative for the dynamical system's behavior.

Moreover, in order to return from the stationary recessionary state to permanent full employment by changing *only* the money stock, an increase in that
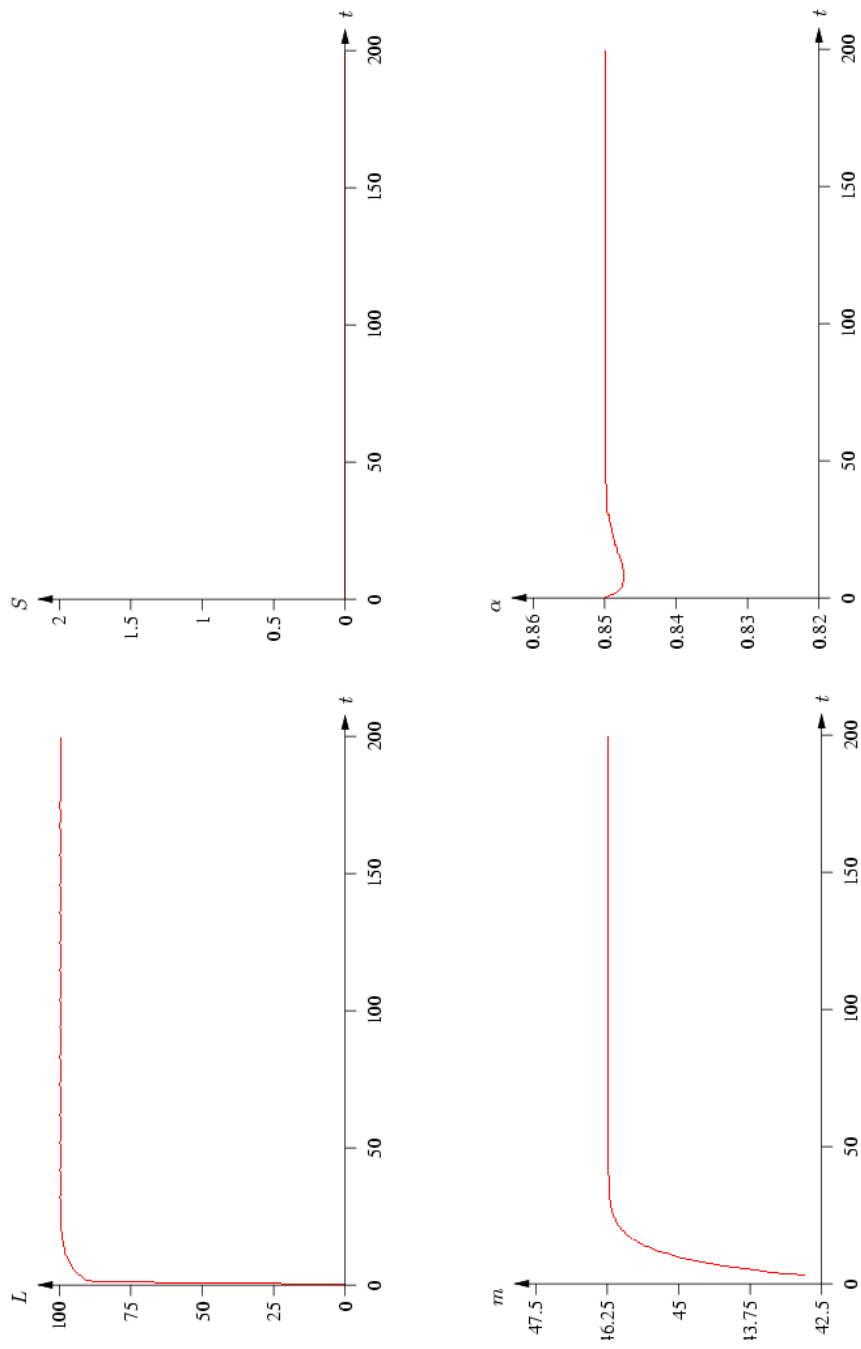
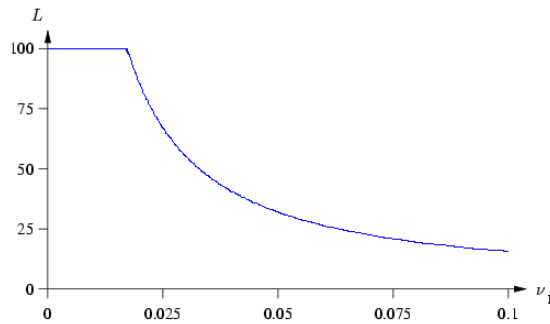Figure 6.7: Time series when $\nu_1 = 0.025$, $m_0 = 40$ and $S_t \equiv 0$.

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Figure 6.8: Stationary employment values when $m_0 = 40$.

variable to its Walrasian value (46.25) is not sufficient, but a higher increase, to approximately 51, would be needed. The case is still worse for a unilateral variation of real profits which would have to be more than trippled, to around 52, to get permanently out of unemployment. In both cases the reason is intuitively clear: only a substantial increase in purchasing power of consumers, old or young, can succeed to change the situation of insufficient aggregate demand causing Keynesian unemployment. A detailed analysis of the respective time series shows that, after an initial shock to $m_0 > 51$ or $\pi_0 > 52$, the system enters in the regime of repressed inflation from where it converges to the Walrasian equilibrium.

The panel of Figure 7.1 showing the bifurcation diagram over the state variable *inventories* indicates that no change in the initial value of S is by itself sufficient to overcome the recessionary state. Even a momentaneous decrease of inventories to zero is not capable to lead the dynamic system out of the Keynesian unemployment regime because the insufficient demand very quickly builds up inventories again and restores the old situation.

A less immediate intuition is available for the effects of a change in the initial value of the real wage α. An increase of its value from the stationary state value typically results in a state of classical unemployment, because the high real wage not only creates excess demand on the goods market but also excess supply on the labor market. This implies a decrease in the nominal wage, an increase in the goods price and a decrease in the real wage. This process continues until the real wage is low enough again to make the system enter into the Keynesian regime. There inventories are built up and the system converges to the quasi-stationary Keynesian state. On the other hand, a decrease of α_0 from $\overline{\alpha}$ typically leads the
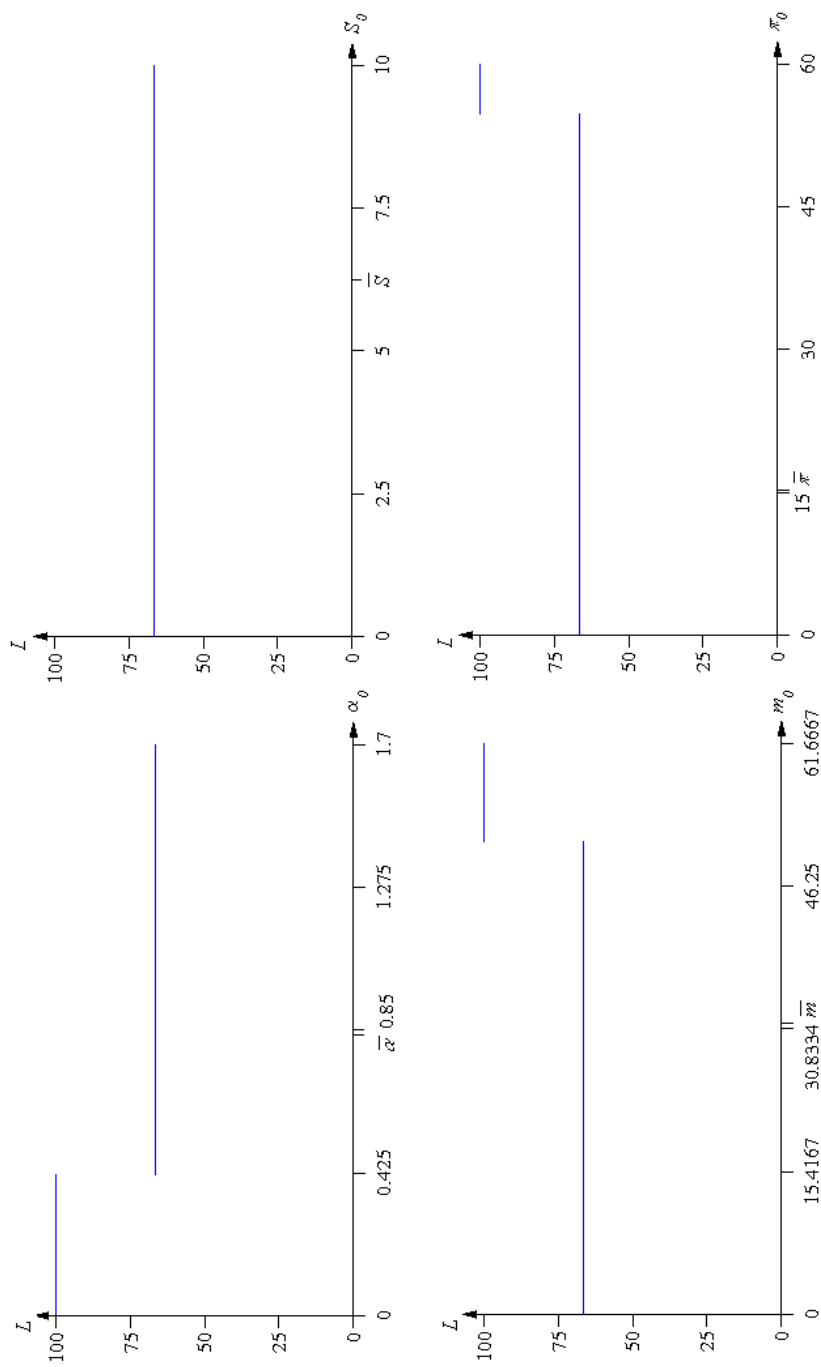
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Figure 7.1: Local stability of the Keynesian unemployment state.

dynamic system to enter into the regime of repressed inflation. From there it may return to the Keynesian regime (for values of α above 0.425) or converge to the Walrasian state (for α smaller than 0.425).

Let us now look at the Walrasian stationary equilibrium $(\alpha^*, m^*, \pi^*, S^*)$ given by the values in (6.1). Setting $(\alpha_0, m_0, \pi_0, S_0) = (\alpha^*, m^*, \pi^*, S^*)$ and performing bifurcation analysis analogous to the ones done above yields Figure 7.2. From this it can be seen that the Walrasian state is unstable in three of the four directions defined by the state variables, namely m, π and S. Only in direction α is the system locally stable. Whenever $m_0 < m^*$, $\pi_0 < \pi^*$ or $S_0 > S^*$, the dynamic system diverges from the Walrasian state and converges to the quasi-stationary Keynesian unemployment state considered above with $\overline{L} = 66.9342$. Thus the Walrasian equilibrium is locally unstable.

The above results are obtained for specific parameter values. One particularly significant parameter here is the downward wage flexibility ν_1. In the present simulations this value has been set to 0.025, but similar results hold whenever ν_1 is above the benchmark value seen in Figure 6.8 (approximately 0.018) that separates the stationary values of employment from full to below full employment.

8. Policy and the Japanese Deflationary Recession

As recalled already in the introduction, the performance of the Japanese economy in the last decade with prolonged recession, unemployment, overcapacity/excess inventories and falling prices and nominal wages fits into our scenario of a quasi-stationary state with Keynesian unemployment. Thus we are challenged to apply the insights from our theoretical model to the Japanese case.

The reasons why Japan is in trouble are not unanimously shared by economists. On the one hand it is argued that Japan's deflation is largely structural and that the money-transmission system does not work because banks, saddled with bad loans, cannot lend more than they actually do. So the priority is to fix the banking system. On the other hand, a standard Keynesian argument is that, when an economy is in a liquidity trap, a fiscal stimulus can boost demand. Japan's public debt appears, however, to be too big already and thus to finance a fiscal stimulus in a conventional way seems not possible. An alternative approach has been suggested by Ben Bernanke, namely, that the government enact tax cuts and the Bank of Japan finance them directly.[11]

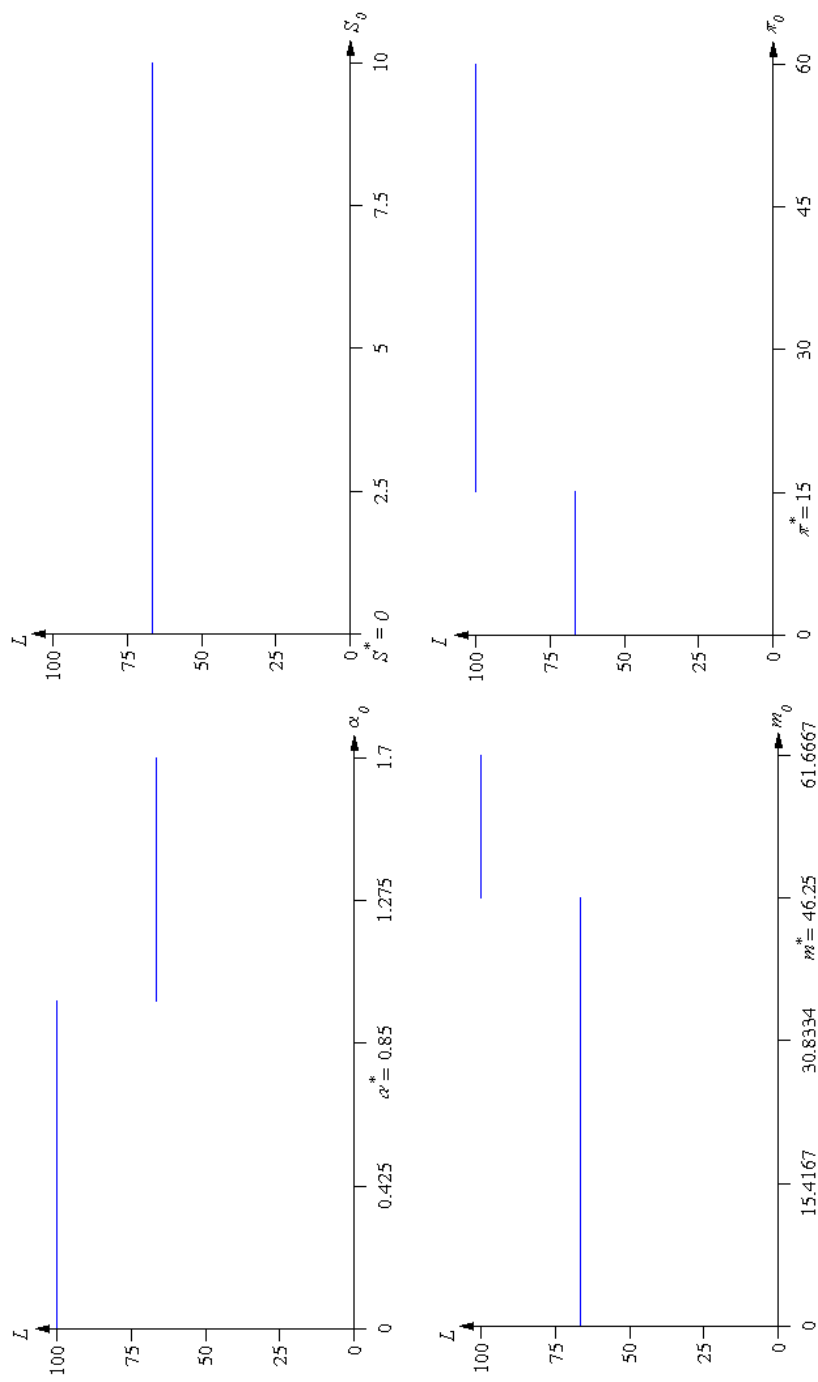[11]In this way the "Bank of Japan would mitigate the usual concerns about rising debt: debt

Figure 7.2: Instability of the Walrasian equilibrium.

In the framework of our model we can emulate the above measure by reducing the tax rate from $tax^* = 0.5$ to a new value tax so that the income of (young) consumers out of profit after taxation is

$$(1 - tax)\pi = (1 - tax^*)\pi + \Delta m,$$

with

$$\Delta m = (tax^* - tax)\pi.$$

Moreover, if the central bank pays for the reduction in taxes paid by consumers, the government's tax income is (as before)

$$tax \cdot \pi + \Delta m = tax^* \cdot \pi.$$

The government's budget deficit in real terms can then be written

$$G^* - tax^* \cdot \pi = G^* - tax \cdot \pi - \Delta m = (G^* - \Delta m) - tax \cdot \pi.$$

This is equivalent to a simultaneous reduction in taxes and in government spending to $G = G^* - \Delta m$.

What happens to the dynamic performance of the economy when the measure (G, tax) is imposed? Starting from the quasi-stationary Keynesian unemployment state $(\overline{\alpha}, \overline{m}, \overline{\pi}, \overline{S})$ given by (7.1) and setting government spending more precisely to $G = tax \cdot \pi^*$, with $\pi^* = 15$ the Walrasian value of real profits so that the government's budget is balanced at any Walrasian equilibrium, the result is displayed in Figure 8.1. The figure shows that a reduction in the tax rate monotonically increases the long-run stationary locally stable value of employment. Moreover, at a value of tax approximately equal to 0.17, stationary Walrasian equilibrium with full employment is reached.

Note that the horizontal lines in Figure 8.1 refer to the stationary employment values for values of $tax \leq 0.17$ and $tax = 0.2, 0.3, ..., 1$.

The above policy measure is well known as balanced-budget fiscal policy, and textbook economics states that the corresponding balanced-budget multiplier is

purchases by the central bank rather than the private sector implies no net increase in debt service and hence no future tax increases. Consumers should then be more willing to spend rather than save any tax cut. It also gets around the Bank of Japan's concern about the blocked money-transmission mechanism: a joint monetary and fiscal boost will increase spending regardless of the health of banks" (*The Economist*, June 21st 2003, p. 74).
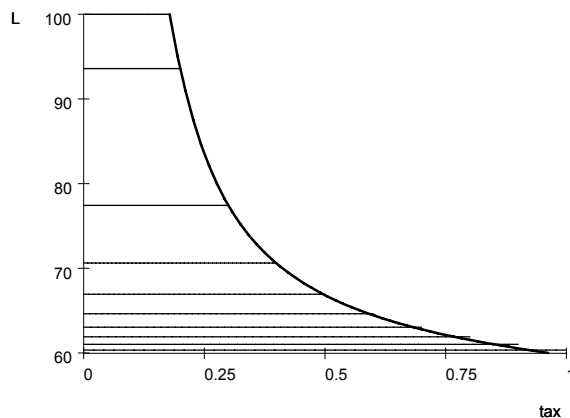
Figure 8.1: Stationary locally stable values of employment depending on different balanced-budget tax quotas.

one. In particular, this would imply that a reduction in the tax rate *reduces* output and hence employment. How is this compatible with Figure 8.1? Note, first of all, that the textbook multiplier refers to a static situation with prices and wages fixed whereas here we have a dynamic analysis with flexible prices, wages and money stock. To best understand what is going on, consider the two extreme cases of $(G, tax) = (0, 0)$ and $(G, tax) = (15, 1)$. Looking at the respective time series numbers, after one iteration we have in the first case an employment level of $\overline{L}_1 = 60.7438$ whereas in the second case $\overline{L}_1 = 73.1357$. Comparing with the initial level $\overline{L} = 66.9342$ (stationary at $(G, tax) = (G^*, tax^*) = (7.5, 0.5)$) these values fit quite well with the textbook prediction. Subsequently $\overline{L}_{t+1} > \overline{L}_t$ in the case $(G, tax) = (0, 0)$ and $\overline{L}_{t+1} < \overline{L}_t$ in the case $(G, tax) = (15, 1)$. The first time employment is larger in the first case than in the second is in period 8 when $\overline{L}_8 = 65.9933$ in case $(G, tax) = (0, 0)$ and $\overline{L}_8 = 65.9735$ in case $(G, tax) = (15, 1)$.

A further variable that shows a monotone dynamic behavior is the money stock m. In case $(G, tax) = (0, 0)$ it increases from 33.3665 in the first period to its limit 50 while in case $(G, tax) = (15, 1)$ m decreases from 30.4977 to its limit 25.1723. In the first case the low (zero) tax enables young households to save more and carry more real balances to the second period of their life. This eventually increases aggregate demand more than the fixed reduction in government spending decreases it, thus increasing output and employment. The opposite is true in the

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case of the high tax: the decrease in real money balances eventually dominates the increase in government spending.

The above analysis suggests a balanced-budget tax reduction as a remedy to recession in the long run. However, the fact that in the short run employment falls below the already low stationary initial level is all but welcome. To avoid this, a simultaneous increase in the money stock m_0 can be used. Specifically, set $m_0 = \overline{m} + G^* - G$, together with $(\alpha_0, \pi_0, S_0) = (\overline{\alpha}, \overline{\pi}, \overline{S})$. If $(G, tax) = (0, 0)$, then $m_0 = 31.9263 + 7.5 = 39.4263$ and $\overline{L}_1 = 73.8248$. Similarly, for $(G, tax) = (2.55, 0.17)$, $m_0 = 36.8763$ and $\overline{L}_1 = 71.4806$. In both cases subsequent employment values increase monotonically to full employment. This shows that the combined measure of tax reduction and expansive monetary policy works well in our model economy.

In the case of the Japanese economy, the policy measure discussed previously is similar to the one analysed for our model economy. In fact, it consists of a tax reduction that does not change the debt burden of the government because the central bank pays the forgone tax revenue to the government. Thus more income is given to households increasing aggregate demand without changing the government's budget deficit. This is quite like what we have done in our model economy, where we have chosen a balanced-budget policy (G, tax) with $tax < tax^*$ accompanied by a monetary policy $m_0 = \overline{m} + G^* - G$. Figure 8.1 indicates, however, that the chosen tax reduction must be large enough as too small a reduction will improve but not eliminate unemployment. For example in our model economy, with $(G, tax) = (3, 0.2)$ and $m_0 = 36.4263$, $\overline{L}_1 = 71.067$ and the long-run stationary employment level is 93.5924.

9. Conclusions

In this paper we have presented a non-tâtonnement dynamic macroeconomic model involving temporary equilibria with fixprices and stochastic rationing in each period, and price adjustment between periods. The model allows for trade also when prices are not at their market clearing levels, and consistent allocations are described in every period, obeying at the same time a well defined dynamics. This approach has enabled us to study, in a general-equilibrium setting, the dynamic functioning of an economy in which disequilibrium phenomena like underemployment, inflation and excess productive capacities are allowed to occur. These disequilibrium situations typically arise because the adjustment of prices to market imbalances is not instantaneous but proceeds with finite speed only;

thus their functioning as an allocation device is imperfect, though not nil. As a consequence, quantity adjustments have to take place which complement prices in their task of making trades feasible.

On the other hand, the fact that prices do adjust in our model renders possible to also work out the possible negative effects of too large a price and wage flexibility. If aggregate demand is insufficient, price and wage flexibility together with the possibility of a declinining nominal money stock (due to government surplusses) may lead to a quasi-stationary situation in which there is permanent deflation of nominal variables but all real variables - among which most importantly employment - remain constant. This is so if the decrease in nominal money is proportional to the one in price and wage, because then the real stock of money held by consumers does not change. Thus it is possible that, in addition to the real wage, also the real wealth of households remains constant or, in other words, there is no real-balance effect. Vice versa, if the nominal wage is rigid downwards, then the real wage is eventually bound to increase, and aggregate-demand deficiency cannot persist in the long run.

Since the dynamics in our four-dimensional system is too complex to be fully understood by means of analytical tools only, numerical simulations have been presented to illustrate and complement the results just mentioned. In particular, these simulations confirm that money is not necessarily neutral in the long run. Starting from a Walrasian equilibrium, a restrictive monetary shock can cause the economy to end up in a permanent recession, i.e. in a deflationary quasi-stationary Keynesian state in which employment and output are permanently below their Walrasian levels. Moreover, while our simulations suggest that the Walrasian equilibrium is locally unstable, the recessionary equilibrium appears to be locally stable.

Our approach, by allowing us to characterize the economic forces behind these scenarios, can help to shed light on some possible remedies to them. Besides the stabilizing effect of (temporary) increases in public expenditure (viable when the shock hitting the economy has been originated by a monetary restriction or an increase in inventories), our numerical analysis shows that the downward speed of adjustment of wages between periods plays a crucial role in determining the impact of a restrictive monetary shock on the economy. Indeed, as long as the downward wage flexibility is below a certain threshold level, the economy is capable of returning to full employment; above it, however, it gets stuck in a permanent recession. Inventories are important here because they amplify the fall of labor demand following the decrease in aggregate demand originated by the restrictive

shock, further depressing real labor income and aggregate demand. Moreover, it is worth emphasizing that these results depend crucially on the possibility of modelling the quantity spillover effects between markets in disequilibrium, which in turn is rendered possible using as modelling strategy the non-tâtonnement approach and the adoption of the concept of equilibrium with quantity rationing.

Finally, the recessionary Keynesian equilibrium characterizing our economy resembles closely to what we have been witnessing for Japan since the end of the 1990s, with increasing unemployment rates and decreasing prices and wages. Our framework provides therefore for a valid test bank to check the efficacy of alternative economic policies designed to escape the Japanese crisis. In the paper, we have focused explicitly on Ben Bernanke's proposal of a mix of expansionary fiscal and monetary policies, concluding that they point in the right direction for restoring full employment, provided they are of the appropriate magnitude.

Appendix 1: Lemma 2.

Lemma 2. *When the production function is $f(\ell) = a\ell^b$, with $a > 0$ and $0 < b \leq 1 - \sigma$, the solution to the firm's maximization problem is independent of the constraint $\ell_t^d \leq \frac{d_t}{\alpha_t} \left[f\left(\ell_t^d\right) + s_t \right]$.*

Proof. The first order condition for an interior solution of the firm's problem is

$$\gamma^s f'(\ell) = \alpha \Leftrightarrow \gamma^s \frac{bf(\ell)}{\ell} = \alpha \Leftrightarrow \ell = \gamma^s \frac{bf(\ell)}{\alpha}.$$

Moreover the inequalities $\frac{1}{b} \geq \frac{1}{1-\sigma} \geq \frac{1-\gamma^s\sigma}{1-\sigma}$ yield $1 \leq \frac{1-\sigma}{b(1-\gamma^s\sigma)}$. From this follows

$$\ell \leq \frac{\gamma^s(1-\sigma)}{1-\gamma^s\sigma} \frac{1}{\gamma^s} \frac{1}{b} \ell = d \frac{1}{\gamma^s} \frac{1}{b} \ell = d \frac{1}{\gamma^s} \frac{1}{b} \gamma^s \frac{bf(\ell)}{\alpha} = \frac{d}{\alpha} f(\ell),$$

which proves our claim. ■

Appendix 2: The complete dynamic system

The dynamic system is given by four different subsystems, one for each of the equilibrium types K, I, C and U and endogenous regime switching. For given (G, tax), any list $(\alpha_t, \pi_t, m_t, S_t)$ gives rise to a uniquely determined equilibrium allocation $(\overline{L}_t, \overline{Y}_t)$ being of one of the above types (or of an intermediate one). This type is determined according to the procedure described in section 3. More precisely,

$$\overline{L}_t = \min\left\{\widetilde{L}(\alpha_t, \pi_t, m_t, S_t, G, tax), L^d(1, \alpha_t), L^s\right\}$$

where $\widetilde{L}(\alpha_t, \pi_t, m_t, S_t, G, tax)$ is the unique solution in L of

$$\alpha_t \left(\frac{1}{b} - h\right) L + \frac{\alpha_t}{ab} \left(\frac{L}{n'}\right)^{1-b} S_t = h(1 - tax)\pi_t + m_t + G$$

and

$$L^d(1, \alpha_t) = n'\left(\frac{\alpha_t}{ab}\right)^{\frac{1}{b-1}}.$$

If $\overline{L}_t = \widetilde{L}(\cdot)$, the K-subsystem applies whereas if $\overline{L}_t = L^d(\cdot)$ type C occurs. Finally, when $\overline{L}_t = L^s$ an equilibrium of type I occurs if $\frac{\alpha_t}{b} L^s + S_t \leq h(1 - tax)\pi_t + h\alpha_t L^s + m_t + G$; otherwise the equilibrum is of type U. Regime switching may

occur because $\left(\overline{L}_t, \overline{Y}_t\right)$ may be of type $T \in \{K, I, C, U\}$ and $\left(\overline{L}_{t+1}, \overline{Y}_{t+1}\right)$ of type $T' \neq T$. Regarding the subsystems, they are the following.

KEYNESIAN UNEMPLOYMENT SYSTEM

Employment level: $\overline{L}_t = \widetilde{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right).$

Output level: $\overline{Y}_t = \frac{\alpha_t}{b}\overline{L}_t + \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b} S_t.$

Rationing coefficients: $\lambda_t^s = \frac{\overline{L}_t}{L^s}$, $\lambda_t^d = 1$, $\gamma_t^s = \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}$, $\gamma_t^d = 1$, $\delta_t = \varepsilon_t = 1$.

Price inflation: $\theta_t = 1 - \mu_1\left(1 - \gamma_t^s\right).$

Real wage adjustment: $\alpha_{t+1} = \frac{1-\nu_1\left(1-\lambda_t^s\right)}{1-\mu_1\left(1-\gamma_t^s\right)}\alpha_t.$

Real profit: $\pi_{t+1} = \frac{1}{\theta_t}\left(\overline{Y}_t - \alpha_t\overline{L}_t\right) = \frac{1-b}{\theta_t(1-hb)}\left[h\left(1-tax\right)\pi_t + m_t + G\right].$

Real money stock: $m_{t+1} = \frac{1}{\theta_t}\left[m_t + G + \left(1 - tax\right)\pi_t\right] - \pi_{t+1}.$

Inventories: $S_{t+1} = n'a\left(\frac{ab\gamma_t^s}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.$

REPRESSED INFLATION SYSTEM
(*It applies when $\frac{\alpha_t}{b}L^s + S_t \leq h\left(1-tax\right)\pi_t + h\alpha_tL^s + m_t + G$*)

$\overline{L}_t = L^s.$

$\overline{Y}_t = \frac{\alpha_t}{b}\overline{L}_t + S_t.$

$\lambda_t^s = 1$, $\lambda_t^d = \frac{L^s}{L^d(1,\alpha_t)}$; $\gamma_t^s = 1.$

If $\overline{Y}_t \geq G + m_t$, then $\gamma_t^d = \frac{\overline{Y}_t - m_t - G}{h(1-tax)\pi_t + h\alpha_t\overline{L}_t}$, $\delta_t = \varepsilon_t = 1$;

if $G + m_t > \overline{Y}_t \geq G$, then $\gamma_t^d = 0$, $\delta_t = \frac{\overline{Y}_t - G}{m_t}$, $\varepsilon_t = 1$;

if $\overline{Y}_t < G$, then $\gamma_t^d = \delta_t = 0$, $\varepsilon_t = \frac{\overline{Y}_t}{G}$.

$\theta_t = 1 + \mu_2\left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right).$

$\alpha_{t+1} = \frac{1+\nu_2\left(1-\lambda_t^d\right)}{1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)}\alpha_t.$

$\pi_{t+1} = \frac{1}{\theta_t}\left(\overline{Y}_t - \alpha_t\overline{L}_t\right) = \frac{\alpha_t}{\theta_t}\frac{1-b}{b}L^s.$

$m_{t+1} = \frac{1}{\theta_t}\left[\delta_t m_t + \varepsilon_t G + \left(1 - tax\right)\pi_t\right] - \pi_{t+1}.$

$S_{t+1} = \lambda_t^d n'a\left(\frac{ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.$

Classical Unemployment System

$\overline{L}_t = L^d(1, \alpha_t)$.

$\overline{Y}_t = \frac{\alpha_t}{b}\overline{L}_t + S_t$.

$\lambda_t^s = \frac{\overline{L}_t}{L^s}$, $\lambda_t^d = 1$, $\gamma_t^s = 1$;

if $\overline{Y}_t \geq G + m_t$, then $\gamma_t^d = \frac{\overline{Y}_t - m_t - G}{h(1-tax)\pi_t + h\alpha_t\overline{L}_t}$, $\delta_t = \varepsilon_t = 1$;

if $G + m_t > \overline{Y}_t \geq G$, then $\gamma_t^d = 0$, $\delta_t = \frac{\overline{Y}_t - G}{m_t}$, $\varepsilon_t = 1$;

if $\overline{Y}_t < G$, then $\gamma_t^d = \delta_t = 0$, $\varepsilon_t = \frac{\overline{Y}_t}{G}$.

$\theta_t = 1 + \mu_2\left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right)$.

$\alpha_{t+1} = \frac{1 - \nu_1(1 - \lambda_t^s)}{1 + \mu_2\left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right)}\alpha_t$

$\pi_{t+1} = \frac{1}{\theta_t}\left(\overline{Y}_t - \alpha_t\overline{L}_t\right) = \frac{1-b}{\theta_t}n'\left(\frac{\alpha_t}{ab}\right)^{\frac{b}{b-1}}\left(\frac{1}{a}\right)^{\frac{1}{b-1}}$.

$m_{t+1} = \frac{1}{\theta_t}\left[\delta_t m_t + \varepsilon_t G + (1 - tax)\pi_t\right] - \pi_{t+1}$.

$S_{t+1} = n'a\left(\frac{ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t$.

Underconsumption

(It applies when $\frac{\alpha_t}{b}L^s + S_t > h(1 - tax)\pi_t + h\alpha_t L^s + m_t + G$)

$\overline{L}_t = L^s$.

$\overline{Y}_t = h(1 - tax)\pi_t + h\alpha_t L^s + m_t + G$.

$\lambda_t^s = 1$, $\lambda_t^d = \frac{L^s}{L^d(\gamma_t^s, \alpha_t)} = \frac{(ab\gamma_t^s)^{1/(1-b)}L^s}{n'\alpha_t^{1/(1-b)}}$;

$\gamma_t^s = \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}$, $\gamma_t^d = 1$, $\delta_t = \varepsilon_t = 1$.

$\theta_t = 1 - \mu_1(1 - \gamma_t^s)$.

$\alpha_{t+1} = \frac{1 + \nu_2\left(1 - \lambda_t^d\right)}{1 - \mu_1(1 - \gamma_t^s)}\alpha_t$.

$\pi_{t+1} = \frac{1}{\theta_t}\left(\overline{Y}_t - \alpha_t\overline{L}_t\right) = \frac{1}{\theta_t}\left[h(1 - tax)\pi_t + m_t + G - \alpha_t(1 - h)L^s\right]$.

$m_{t+1} = \frac{1}{\theta_t}\left[m_t + G + (1 - tax)\pi_t\right] - \pi_{t+1}$.

$S_{t+1} = \lambda_t^d n'a\left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t$.

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