Allocative Efficiency Analysis using DEA in Stata

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CONTENTS

I. The Concept of Allocative Efficiency

❑ Overall(OE), Technical(TE), and Allocative Efficiency(AE)

○ Consider produce y with x_1 and x_2.

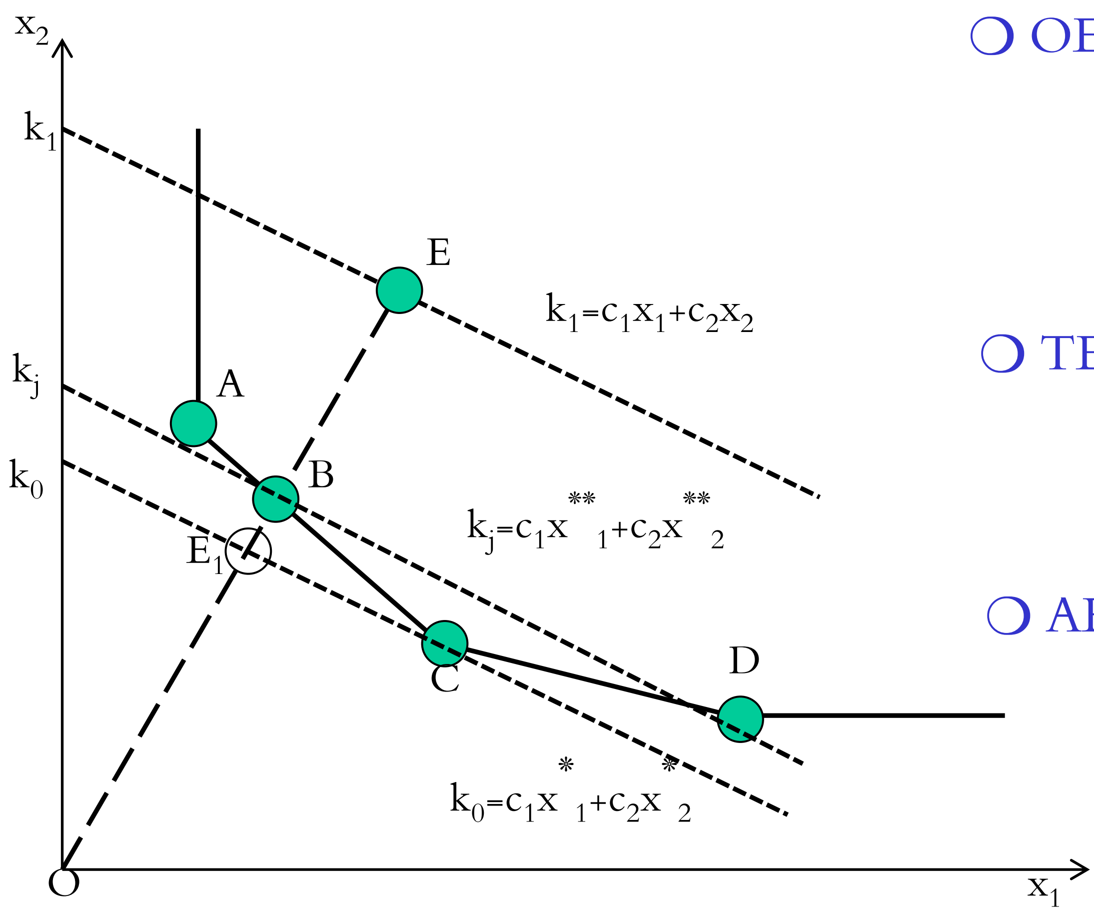

Fig. 1 Concept of Allocative Efficiency

○ OE: $$0 \le \frac{k_0}{k_1} = \frac{\sum_{i=1}^{M} c_i x_i^*}{\sum_{i=1}^{M} c_i x_i} = \frac{d(O,E_1)}{d(O,E)} \le 1$$

○ TE: $$0 \le \frac{k_j}{k_1} = \frac{\sum_{i=1}^{M} c_i x_i^{**}}{\sum_{i=1}^{M} c_i x_i} = \frac{d(O,B)}{d(O,E)} \le 1$$

○ AE: $$0 \le \frac{k_0}{k_j} = \frac{\sum_{i=1}^{M} c_i x_i^*}{\sum_{i=1}^{M} c_i x_i^{**}} = \frac{d(O,E_1)}{d(O,B)} \le 1$$

☞ AE = OE/TE

I. The Concept of Allocative Efficiency

❑ Mathematical Formulations of Allocative Efficiency

- ❍ Recall TE, the Efficiency estimate of a DEA Model(see Lee, 2009)
- ❍ the cost line passing through points k_0 and C can be obtained by solving the following Linear Programming(Cooper et. al., 2006)

$$k_0 = \mathbf{cx}^* = \min_{x,\lambda} \sum_{i=1}^{M} c_i x_i$$

s.t.

$$x_i \geq \sum_{j=1}^{J} x_m^j \lambda^j (\text{input} \;\; m = 1, 2, \ldots, M)$$

$$y_0 \leq \sum_{j=1}^{J} y_n^j \lambda^j (\text{output} \;\; n = 1, 2, \ldots, N)$$

$$\lambda^j \geq 0 (j = 1, 2, \ldots, J),$$

where c_i is unit input price or unit cost of ith input.

I. The Concept of Allocative Efficiency

❑ Allocative Efficiency Measure matters?

- points A, B, C in Fig. 1 are all Technically Efficient(CCR-efficient)
- However, point C is less expensive than points A and B.

❑ Revenue Efficiency, Profit Efficiency Defined

$$R = py^{*} = \max_{y,\lambda} \sum_{n=1}^{N} p_n y_n$$

s.t.

$$x_i \geq \sum_{j=1}^{J} x_m^j \lambda^j \text{(input } m = 1, 2, \ldots, M\text{)}$$

$$y_0 \leq \sum_{j=1}^{J} y_n^j \lambda^j \text{(output } n = 1, 2, \ldots, N\text{)}$$

$$\lambda^j \geq 0 (j = 1, 2, \ldots, J),$$

where p_n is unit price of nth output.

$$\pi = py^{*} - cx^{*} = \max_{x,y,\lambda} \left(\sum_{n=1}^{N} p_n y_n - \sum_{m=1}^{M} c_m x_m\right)$$

s.t.

$$x_i \geq \sum_{j=1}^{J} x_m^j \lambda^j \text{(input } m = 1, 2, \ldots, M\text{)}$$

$$y_0 \leq \sum_{j=1}^{J} y_n^j \lambda^j \text{(output } n = 1, 2, \ldots, N\text{)}$$

$$\lambda^j \geq 0 (j = 1, 2, \ldots, J),$$

♨ model variations by allowing different cost and price, and returns to scale.

II. Allocative Efficiency in Stata

- The User Written Command "dea_allocative"

○ Program Syntax

dea_allocative ivars = ovars [if] [in] [using/] [, model(*string*) values(*numlist*>0) unitvars(varlist numeric) rts(*string*) saving(*filename*)]

- model(*string*) specifies one of the cost, revenue, and profit.
- rts(*string*) specifies crs or vrs returns to scale. The default option is crs.
- values and unitvars are case sensitive. val(numlist) specifies the common unit cost or price and unit(varlist) specifies the variables that contain the unit cost or price to be used.
- saving(filename) specifies that the results be saved in filename.dta

II. Allocative Efficiency in Stata

❑ The User Written Command "dea_allocative"

❍ Data

dmu	Inp_x1	Inp_x2	Out_y1	Inp_c1	Inp_c2	Out_p1
A	2	8	1	1	2	3
B	6	9	1	1	2	3
C	5	6	1	1	2	3
D	8	5	1	1	2	3
E	7	3	1	1	2	3
F	3	6	1	1	2	3
G	2	2	1	1	2	3

II. Allocative Efficiency in Stata

- The User Written Command "dea_allocative"
  - Result: cost efficiency model w/unitvars option

```
. use "D:\...\alloc_lee_sd1.dta"
. do ldea
. dea_allocative inp_x1 inp_x2= out_y1,mod(c) unitvars( inp_c1 inp_c2)
sav(alloc_cost_exam1.dta)
```

CRS DEA-Cost Efficiency Results:

	CUR: inp_x1	CUR: inp_x2	CUR: cost	TECH: theta	TECH: inp_x1	TECH: inp_x2	TECH: cost	MIN: inp_x1	MIN: inp_x2
dmu:A_	2	8	18	1	2	8	18	2	2
dmu:B_	6	9	24	.333333	2	3	8	2	2
dmu:C_	5	6	17	.4	2	2.4	6.8	2	2
dmu:D_	8	5	18	.4	3.2	2	7.2	2	2
dmu:E_	7	3	13	.666667	4.66667	2	8.66667	2	2
dmu:F_	3	6	15	.666667	2	4	10	2	2
dmu:G_	2	2	6	1	2	2	6	2	2

	MIN: cost	OE	AE	TE
dmu:A_	6	.333333	.333333	1
dmu:B_	6	.25	.75	.333333
dmu:C_	6	.352941	.882353	.4
dmu:D_	6	.333333	.833333	.4
dmu:E_	6	.461538	.692308	.666667
dmu:F_	6	.4	.6	.666667
dmu:G_	6	1	1	1

II. Allocative Efficiency in Stata

❑ The User Written Command "dea_allocative"

❍ Result: cost efficiency model w/val option

. dea_allocative inp_x1 inp_x2= out_y1,mod(c) val(1 2) sav(alloc_cost_exam2.dta)

CRS DEA-Cost Efficiency Results:

	CUR: inp_x1	CUR: inp_x2	CUR: cost	TECH: theta	TECH: inp_x1	TECH: inp_x2	TECH: cost	MIN: inp_x1	MIN: inp_x2
dmu:A_	2	8	18	1	2	8	18	2	2
dmu:B_	6	9	24	.333333	2	3	8	2	2
dmu:C_	5	6	17	.4	2	2.4	6.8	2	2
dmu:D_	8	5	18	.4	3.2	2	7.2	2	2
dmu:E_	7	3	13	.666667	4.66667	2	8.66667	2	2
dmu:F_	3	6	15	.666667	2	4	10	2	2
dmu:G_	2	2	6	1	2	2	6	2	2

	MIN: cost	OE	AE	TE
dmu:A_	6	.333333	.333333	1
dmu:B_	6	.25	.75	.333333
dmu:C_	6	.352941	.882353	.4
dmu:D_	6	.333333	.833333	.4
dmu:E_	6	.461538	.692308	.666667
dmu:F_	6	.4	.6	.666667
dmu:G_	6	1	1	1

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II. Allocative Efficiency in Stata

❑ The User Written Command "dea_allocative"

❍ Result: revenue efficiency model

```
. dea_allocative inp_x1 inp_x2= out_y1,mod(r) val(3) sav(alloc_revenue_exam3.dta)
```

CRS DEA-Revenue Efficiency Results:

	CUR: out_y1	CUR: price	TECH: eta	TECH: out_y1	TECH: price	MAX: out_y1	MAX: price	OE	AE
dmu:A_	1	3	1	1	3	1	3	1	1
dmu:B_	1	3	3	3	9	3	9	.333333	1
dmu:C_	1	3	2.5	2.5	7.5	2.5	7.5	.4	1
dmu:D_	1	3	2.5	2.5	7.5	2.5	7.5	.4	1
dmu:E_	1	3	1.5	1.5	4.5	1.5	4.5	.666667	1
dmu:F_	1	3	1.5	1.5	4.5	1.5	4.5	.666667	1
dmu:G_	1	3	1	1	3	1	3	1	1

	TE
dmu:A_	1
dmu:B_	.333333
dmu:C_	.4
dmu:D_	.4
dmu:E_	.666667
dmu:F_	.666667
dmu:G_	1

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II. Allocative Efficiency in Stata

❑ The User Written Command “dea_allocative”

❍ Result: revenue efficiency model

```
. dea_allocative inp_x1 inp_x2= out_y1,mod(p) unitvars( inp_c1 inp_c2 out_p1)
sav(alloc_ profit_exam4.dta)
```

CRS DEA-Profit Efficiency Results:

	MAX^: out_y1	MAX: revenue	MIN^: inp_x1	MIN^: inp_x2	MIN: cost	MAX: profit	CUR: profit	OE
dmu:A_	1	3	2	2	6	-3	-15	5
dmu:B_	1	3	2	2	6	-3	-21	7
dmu:C_	1	3	2	2	6	-3	-14	4.66667
dmu:D_	1	3	2	2	6	-3	-15	5
dmu:E_	1	3	2	2	6	-3	-10	3.33333
dmu:F_	1	3	2	2	6	-3	-12	4
dmu:G_	1	3	2	2	6	-3	-3	1

III. Some Related User-written Programs

❑ The Updated User-written codes are available upon request

- dea.ado : data envelopment analysis model
- ldea.do : batch module to run the updated dea models since Oct. 2011
- malmq.ado : productivity analysis model
- dea_allocative.ado : allocative efficiency model
- dea_additive.ado : additive dea model
- dea_sbm : slack-based dea model
- dea_super : super-efficiency dea model(radial)
- dea_imprecise : imprecise dea model(IDEA) /basic
- dea_virtual : virtual price dea model

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Thank You !