A Lattice-Based Model
Of A Disequilibrium Economy with Heterogeneity

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Abstract This paper proposes a lattice-based model in order to numerically study equilibrium economy and disequilibrium economy alike and handle the heterogeneity issues. Numerical simulations with this model show that when the economy is at disequilibrium, price changes can still be an indicator of random shocks. But its accuracy depends on the interaction among individuals, the existence of heterogeneous agents and the property of random shocks. Whereas in the economy that is *always* at equilibrium, the first two factors have virtually no effects on the equilibrium prices, if they only affect the information observed by individuals.

Key words agent-based lattice equilibrium heterogeneity

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Economists are good (or so we hope) at recognizing a state of equilibrium but are poor at predicting precisely how an economy in disequilibrium will evolve.

-- Andreu Mas-Colell, Michael D. Whinston & Jerry R. Green[1]

Abstract This paper proposes a lattice-based model in order to numerically study equilibrium economy and disequilibrium economy alike and handle the heterogeneity issues. Numerical simulations with this model show that when the economy is at disequilibrium, price changes can still be an indicator of random shocks. But its accuracy depends on the interaction among individuals, the existence of heterogeneous agents and the property of random shocks. Whereas in the economy that is *always* at equilibrium, the first two factors have virtually no effects on the equilibrium prices, if they only affect the information observed by individuals.

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I. Introduction

The concept of equilibrium is prevailingly dominant in current economic literature. However, two questions are left open. Firstly, how the equilibrium price is formed? Typically, it assumes the existence of an imaginary auctioneer and assumes that no transaction can occur before the equilibrium price is reached. However, except for the special case of auction, if no transaction ever takes place before price is set at the equilibrium level, how could people ever find the equilibrium level? Secondly, what is the dynamics that leads the system to the equilibrium? The common practice in macro economics is either assuming linear system in the first place or assuming the system is always in the neighborhood of steady state so that linearization of the nonlinear system is applicable. But why should the system be *always* in the neighborhood of the steady state? A typical answer

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[1] Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green, *Microeconomic Theory*. p620.

would be the rational expectation assumption, which seems too natural and convincing, when we assume homogeneity among individuals. However, once heterogeneity (like information asymmetry and market structure) is introduced into the system, this becomes an extraordinarily strong assumption to be as readily accepted. Finally, if the economy is sometimes at disequilibrium, why so far no strong evidence shows that the economy could be at disequilibrium? This is probably due to the way economic data are collected. In current data collection, all goods that are in excess supply are put into the category of inventory investment which is generally considered as part of voluntary investment. As for excess demand, since they are not revealed, there will literally be no regular data available; and also because of the strong belief in market equilibrium, it will look as if it has never existed.

Therefore, it is worthwhile to study the disequilibrium economy to probe into the dynamic world of the economic system. However, there are two major obstacles for the study in this area. They are: 1) lack of analytical tools. In current mathematical analysis, assumption is everything. And unfortunately, to make the problem manageable, the assumptions made are usually too strong. 2) lack of knowledge on the dynamic laws regarding the evolution of economic system. The current knowledge includes only a few "informal" or "intuitive" principles, which are difficult to be translated into precise dynamic laws.[2] This paper proposes a numerical approach to the study of disequilibrium economy.[3]

Another issue that haunts the theoretical economic analysis is the problem with heterogeneity like information asymmetry and non-homogenous market. Since George J. Stigler (1961), G. Akerlof (1970), S. Grossman and J. E. Stiglitz (1975), there emerged numerous articles on the issue of information asymmetry. However, due to the technical limitations and complicated nature of heterogeneity, only limited number of cases could be studied. The lattice-based model proposed in this paper also provides a convenient and more general tool to meet this end.

The lattice model proposed in this paper is an agent-based computational model (D. Mcfadzean and L. Tesfatsion, 1999). However, the lattice structure used is model more closely related to the DLA (diffusion limited aggregation) model used in the research of fractal growth.

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[2] Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green, *Microeconomic Theory*. p620.
[3] This lattice-based model can also be modified to simulate the mechanism leading to market equilibrium.

The paper is organized as follows. Part II provides a general description of the model. Then in part III and IV, two disequilibrium cases are studied. In part III, the price series are driven by random shocks which are introduced into the economy as private information; whereas in part IV, the shocks are public information. Part V studies the market equilibrium case. Part VI concludes the paper.

II. A General Description of the Model

The lattice-based model in this paper originates from the DLA (diffusion-limited aggregation) model in the study of fractal growth.[4] It describes an economy as a two-dimensional lattice.[5] Every point on the lattice represents one economic agent.

The action of each agent is determined according to a presumed action rule[6], which is dependent on his personal information set. After each individual has submitted his/her action, the market as whole aggregates the individual signals and determines the value of aggregate variables like price.[7] Because the action rules are essentially defined on an individual basis, this framework makes it easy to handle the issues involving heterogeneity like market structure and information asymmetries.[8] More importantly, so long as we can be quite confident of individual's reaction, the responses of whole market can be studied simply as an aggregation of individual reaction. Thus the market equilibrium and disequilibrium can be analyzed alike.

As a specific application, this paper uses a 100×100 square lattice.
At each period t, r (r=0,1, …r0) random shocks[9] occurs. Each shock is denoted as S_r^t, it includes a two-dimensional information[10]: 1) the sign of the shock $S_r^t(1)$, that is, whether it is a positive shock or a negative one; 2) the

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[4] For detailed description of the DLA model, please refer to Paul (1998).
[5] For simplicity, throughout this paper n=m is assumed.
[6] This rule can be determined by the result of usual dynamic optimizations, or by a generalization of a statistical result from an empirical study.
[7] Here it is implicitly assumed that when individual is selecting his/her action at period t, he/she has no idea of the value of aggregate variables. This assumption can be further relaxed to incorporate the forming process of equilibrium price.
[8] It also becomes easier to study the market disequilibrium.
[9] All the random numbers in this paper are generated by pseudo random processes.
[10] Actually, by adding more dimensions to the information signal (like persistence, range of influence and so on), more complicated properties of information can be studied.

strength of the shock $S_r^t(2)$, that is, how strong the signal is.

$$S_r^t(1) = \begin{cases} -1 & a\ negative\ shock \\ +1 & a\ positive\ shock \\ 0 & no\ shock \end{cases}$$

$$S_r^t(2) = \begin{cases} \lambda & if\ \left|S_r^t(1)\right| = 1 \\ 0 & otherwise \end{cases} \qquad where\ \lambda \sim U_{[0,1]}$$

λ is a uniformly distributed random variable, which characterizes the strength of the information.

If S_r^t is private information, then it includes two more dimensional elements (m1, m2) which defines the locations of individuals that can observe this shock. If the shock is public information, then (m1, m2) equals all possible (I, J), and to simplify the notation, specification of (m1, m2) is omitted.

In the period that a shock occurs, every individual[11] will respond to it in the following way.

$$F_{kl}^t = \sum_{i=1}^{I}\sum_{j=1A}^{J} W_{ij}^t(1)A_{ij}^t + \sum_{r=1}^{R} W_r^t(2)B_r^t(m1, m2) + \sum_{s=1}^{S} W_s^t(3)C_s^t \qquad \text{(k=1,2,…I; l=1,2,….J)}$$

F_{kl}^t stands for the strength of the information that the (K, L)-th individual receives at period t. It includes the following components. A_{ij}^t is the action of (i, j)-th individuals, $W_{ij}^t(1)$ is the influence of the (i,j)-th individual action; $B_r^t(m1, m2)$ stands for the *observed* r-th shock in period t, which is only observable for individuals located at (m1, m2). It is possible that because of either the properties of the shock or the accuracy of observation, $B_r^t \neq S_r^t$. For instance, if S_r^t is a private shock, we have $B_r^t = 0$ for all individuals that can not observe the information. $W_r^t(2)$ stands for the weight assigned to this observed shock. C_s^t stands for the action of experts (heterogeneous agents in the model), whose observation is more accurate and whose action is publicly

[11] If we include the range of influence as an additional dimension of the random shock, it may not necessarily affect every individual. In the latter part of this paper, to simulate the impact of private information, the range of information is assumed to be individual specific. As for public information, its range covers all the agents.

quoted. $W_s^t(3)$ is its corresponding influence on the (K, L)th individual.

$$W_{ij}^t(1) = \begin{cases} 1/8 & if \;\; |i-k| + |j-l| = 1 \\ 0 & otherwise \end{cases}$$

$$W_r^t(2) = \begin{cases} 0 & if \;\; B_r^t(m1, m2) \; is \; a \; private \; shock \; and \; (m1, m2) \neq (k, l) \\ 1 & if \;\; B_r^t(m1, m2) \; is \; a \; private \; shock \; and \; (m1, m2) = (k, l) \\ \text{a1} & if \;\; B_r^t(m1, m2) \; is \; a \; public \; shock \end{cases}$$

$$W_s^t(3) = \begin{cases} \text{a2} & if \;\; individuals \; respond \; to \; expert's \; action \\ 0 & otherwise \end{cases}$$

$$0 \leq a1 \leq 1, \quad 0 \leq a2 \leq 1$$

In short, each individual's information set incorporates three elements: the responses of his /her adjacent neighbors (A_{ij}^t), the observed shock (B_r^t), and the responses of experts (C_s^t).[12] Each individual may attach different weights to these three factors, which are denoted as $W_{ij}^t(1)$, $W_r^t(2)$, $W_s^t(3)$ respectively. The overall impact[13] is defined as a linear combination of these three influences. In the following simulation, we assume that $W_{ij}^t(1) = W_{ij}(1)$, $W_r^t(2) = W_r(2)$ and $W_s^t(3) = W_s(3)$.

If F_{kl}^t exceeds individual (K, L)'s reservation value V_{kl}^t , which is a uniformly distributed random variable, he will submit a unit of demand or supply depending on the sign of observed information; otherwise he/she will do nothing in that period. His net demand is determined as follows:

$$D_{kl}^t = \begin{cases} +1 & if \;\; |F_{kl}^t| > V_{kl}^t \;\; and \;\; F_{kl}^t > 0 \\ -1 & if \;\; |F_{kl}^t| > V_{kl}^t \;\; and \;\; F_{kl}^t < 0 \\ 0 & if \;\; |F_{kl}^t| < V_{kl}^t \end{cases}$$

It is also assumed that F_{kl}^t exhibits certain degree of persistence, depending

[12] The experts are defined as the kind of individual who can more accurately observe the shocks.
[13] In general, the overall response is a vector that incorporates more information, like purchasing/selling decision, strength of influence and persistence and so on.

on the discount factor ($\rho_{k,l}$, $0 \le \rho_{k,l} < 1$) assigned by each individual.[14]

After each individual has determined his/her decision, the market as a whole calculates the gross demand and supply. If the gross demand exceeds supply, price will increase; if the gross supply exceeds demand, the price will decrease.[15]

$$\Delta p(t) = \begin{cases} -1 \quad if \ \sum_{n=1}^{N}\sum_{m=1}^{M} D_{n,m}^{t} > 0 \\ 0 \quad if \ \sum_{n=1}^{N}\sum_{m=1}^{M} D_{n,m}^{t} = 0 \\ +1 \quad if \ \sum_{n=1}^{N}\sum_{m=1}^{M} D_{n,m}^{t} < 0 \end{cases}$$

III. Simulation of the Impact of Private Shocks

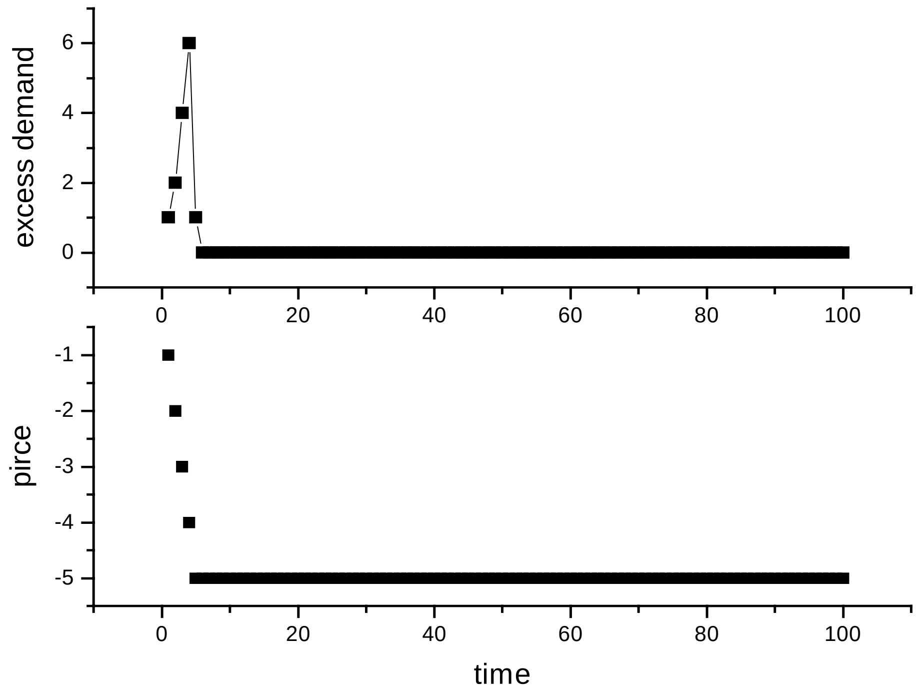
In this part, the random shocks are introduced into the model as private information, that is, only observable for a particular individual. The location of this individual is randomly determined. Other individuals can only conjecture the information from the action of the informed individual. It is assumed that $W_{ij}^{t}(1) = W(1)$, $W_{s}^{t}(3) = W(3)$ and

$$W_{r}^{t}(2) = \begin{cases} 1 \quad if \ B_{r}^{t}(m1, m2) \ is \ a \ private \ shock \ and \ (m1, m2) = (k, l) \\ 0 \quad if \ B_{r}^{t}(m1, m2) \ is \ a \ private \ shock \ and \ (m1, m2) \ne (k, l) \end{cases}$$

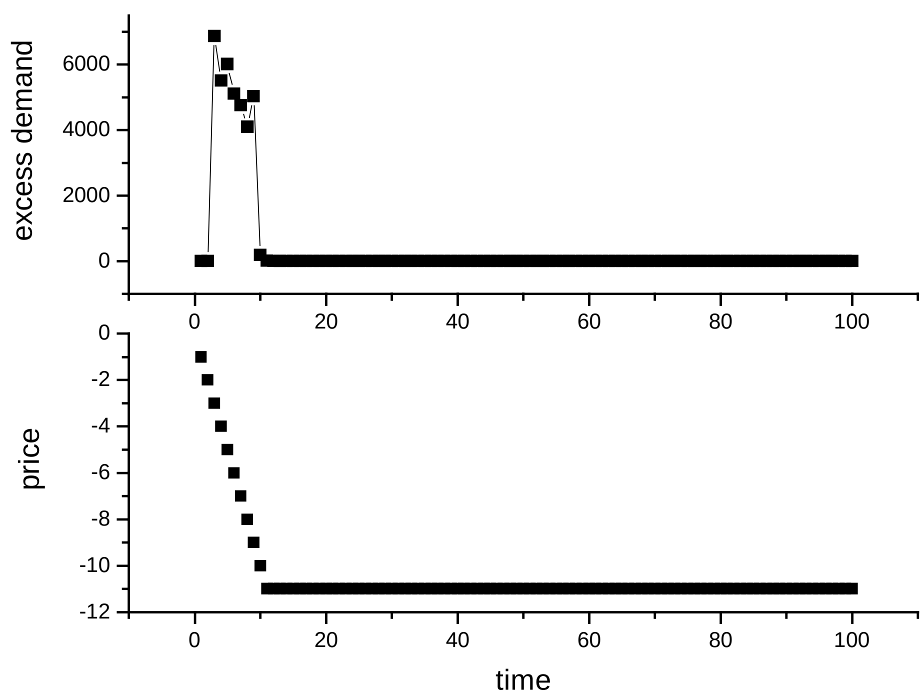
At first, the impact of a single shock is simulated as follows.

[14] In this paper, the discount factor is assumed to be 0.9.
[15] Here the concept of equilibrium price is not used although it is possible to do so. In a separate research I made the threshold of individual‘s response function dependent on the market price. In the period that gross demand exceeds supply, price increases, and the threshold of individual’s response function changes in a way that the probability one would submit a unit of demand decreases. When gross demand fall short of supply, the threshold changes so that the probability of submitting a unit of supply decreases. This process goes on until a price level is reached that the gross demand equals supply. However, it is possible that the uniqueness of equilibrium price may not be guaranteed.

Figure 1. A single shock under private information



(a) w(1)=1/8, W(3)=0

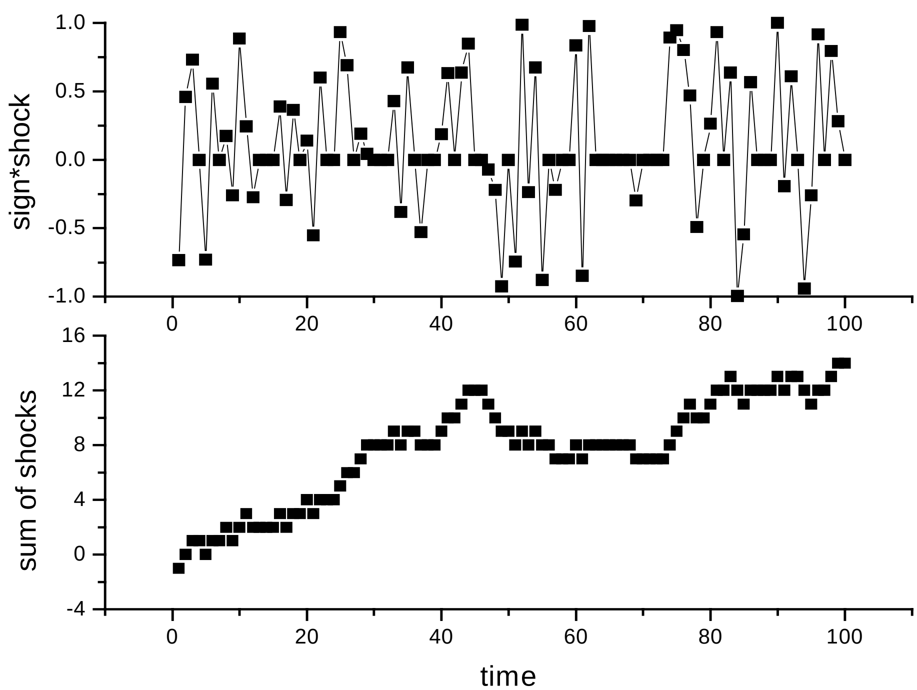


(b) w(1)=1/8, W(3)=1/2

In the private information cases, the introduction of expert significantly changes the gross variables (like price and excess demand). The expert in this model helps to disseminate the private information to the public, thus change the excess demand dramatically. The scale of excess demand increases from 1-digit number to 3-digit ones. Moreover, due to the built-in interactions among individuals, the impact of the information becomes more persistent as more weights are assigned to the observations of the expert action. In fact, the persistence of the single shock increases from 4 periods to 10 periods. These results will later be compared with those under public information.

Then, a sequence of random shocks is generated and its impact on the model economy is studied. The sequence used is as follows: [16]

Figure 2. The series of pseudo random shocks



In the upper graph, the vertical axis is $x(t) = \sum_{\gamma=1}^{r} S_{\gamma}^{t}(1) \times S_{\gamma}^{t}(2)$

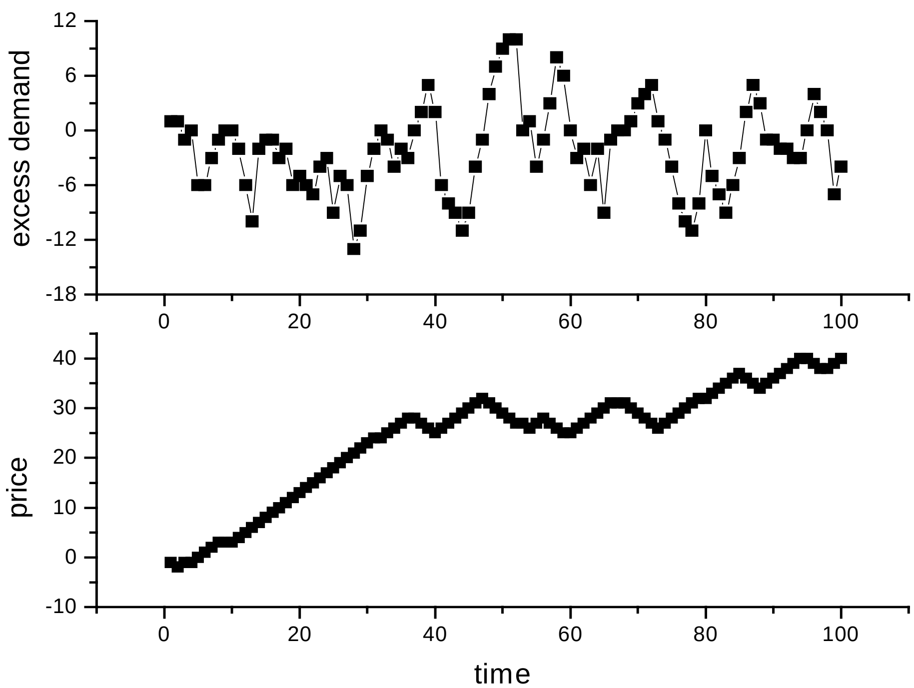
[16] All the random numbers in this paper are pseudo random numbers generated by computer with Fortran program.

In the lower graph, the vertical axis is $y(t)=\sum_{\tau=1}^{t}\sum_{\gamma=1}^{r} S_{\gamma}^{\tau}(1)$

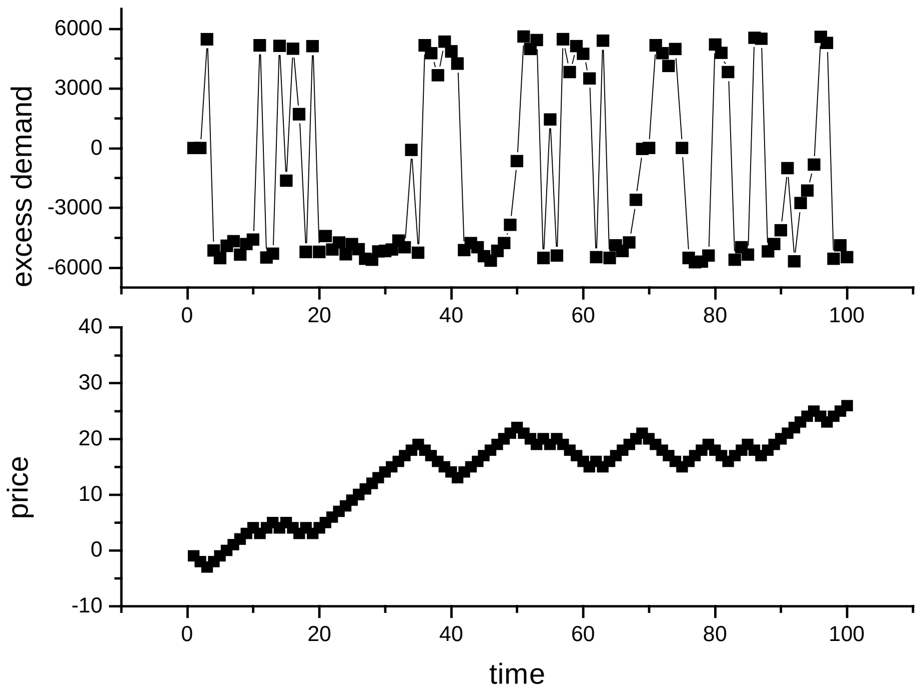
As a bench mark case for this part, an economy with no economic expert is studied, where all individuals behave only according to the observed actions of their adjacent neighbors. Then, economic expert is introduced into the model, whose action is publicly quoted. It is assumed that the expert can accurately observe the private information. The influence of expert is characterized in the different W(3) values in the following diagrams. The corresponding impact on the price series is visible in the diagrams.

As shown in the diagrams, under private information, existence of expert does help to make the disequilibrium prices to be more accurate an indicator of random shocks occurred. The key reason is that it helps to disseminate the private information.

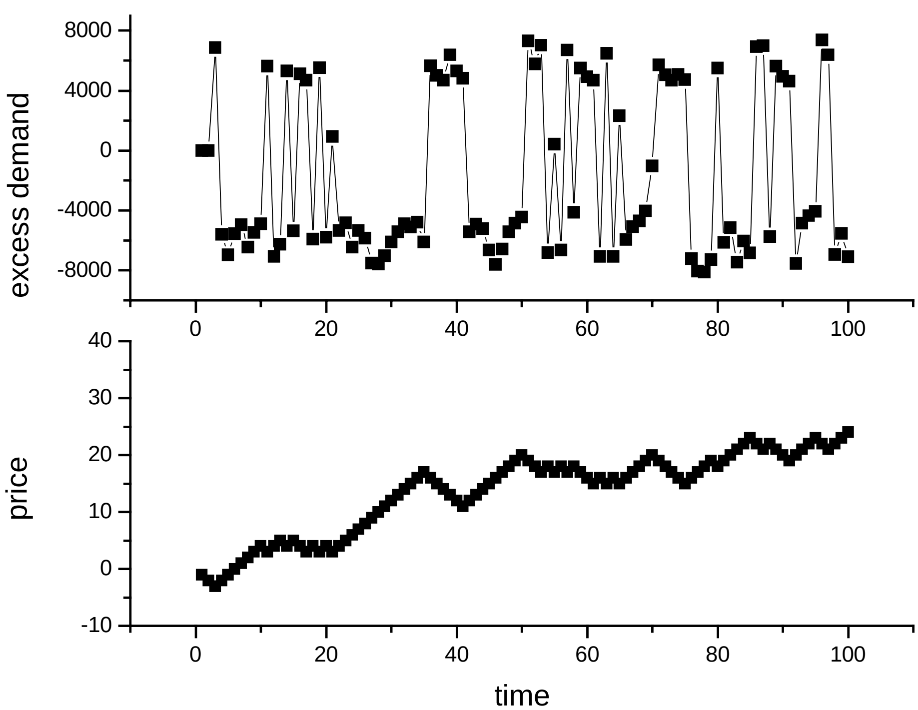
Figure 3. The introduction of expert and its impact



(a) W(1)=1/8, W(3)=0



(b) W(1)=1/8, W(3)=1/8



(c) W(1)=1/8, W(3)=1/2

The quantitative impact of expert on the price oscillation is summarized in the following table. It shows that the introduction of expert into the economy significantly changes the oscillation properties of the price series. However, further increases in W(3) does not have significant impact.

Table 1   Impact of expert under private shocks

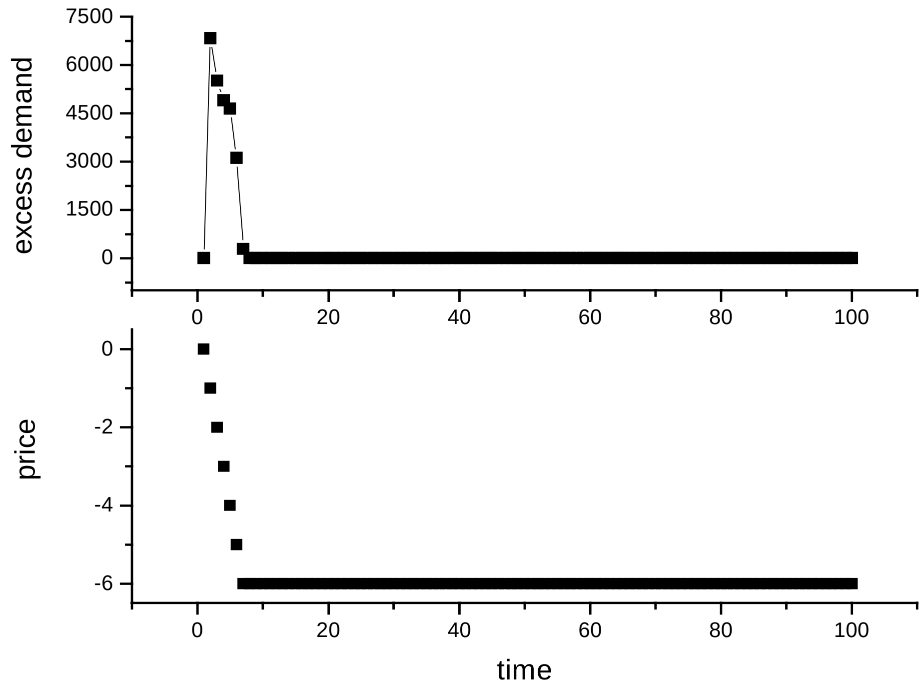
W1	1/8			
W3	0	1/8	1/4	1/2
OS_A	9.5	4.3	4.4	3.3
OP_A	21.8	6.7	6.7	5.2
RANGE	42.0	29.0	31.0	27.0

Note: OS_A = average oscillation range;
OP_A = average oscillation period;
RANGE = overall oscillation range.

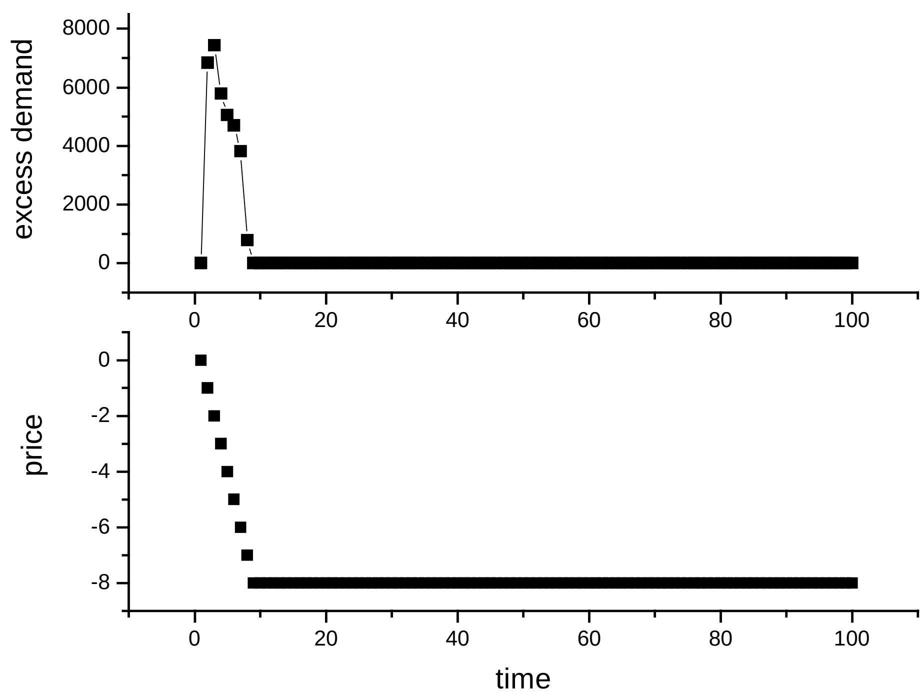
IV. Simulation of the Impact of Public Shocks

In this part, shocks are introduced as public information so that everyone can observe it. Again, the impact of a single shock was simulated first. As shown in the following figure, the introduction of expert had neither significant impact of either excess demand nor on the persistence of the shock. The persistence is 6 periods in the first diagram and 7 in the second.

Figure 4. A single shock under public information



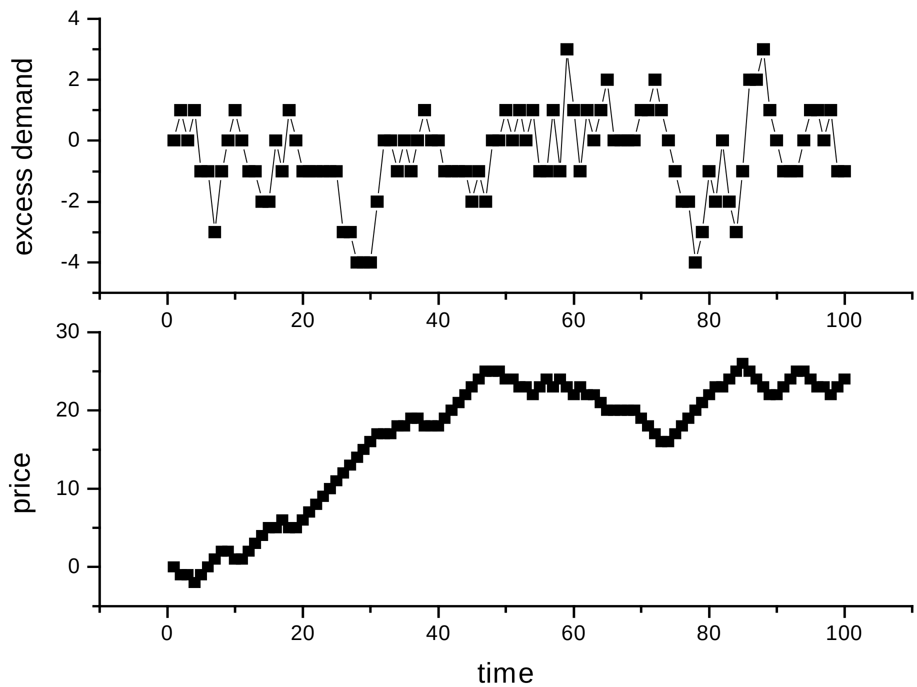
(a) W(1)=1/8, W(2)=1/2, W(3)=0



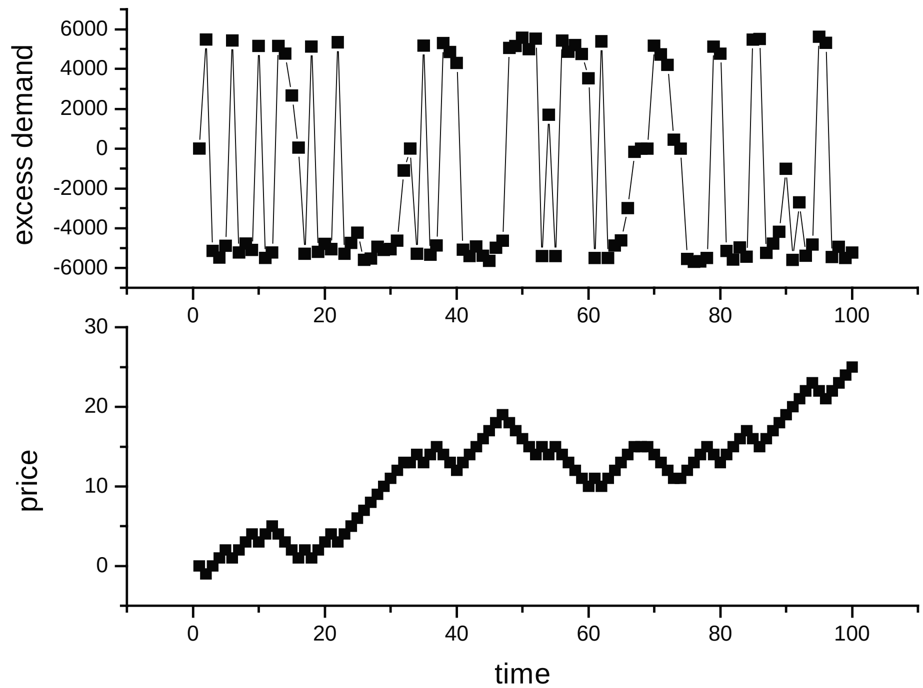
(b) W(1)=1/8, W(2)=1/2, W(3)=1/2

Then, the impact of a series of random shock is simulated below. The first diagram describes the case where all individuals act only according to the observed actions of their adjacent neighbors. In Figure 5(b) and 5(c), all individuals also consider the observed public information of the shock. In these three diagrams, W(2) (that is the weight assigned to this public information) increases from 0 to 1/2. The result shows that as W(2) changes from 0 to 1/8, the price series becomes more and more accurate indicator of the real shocks occurs even when the market is not at equilibrium.

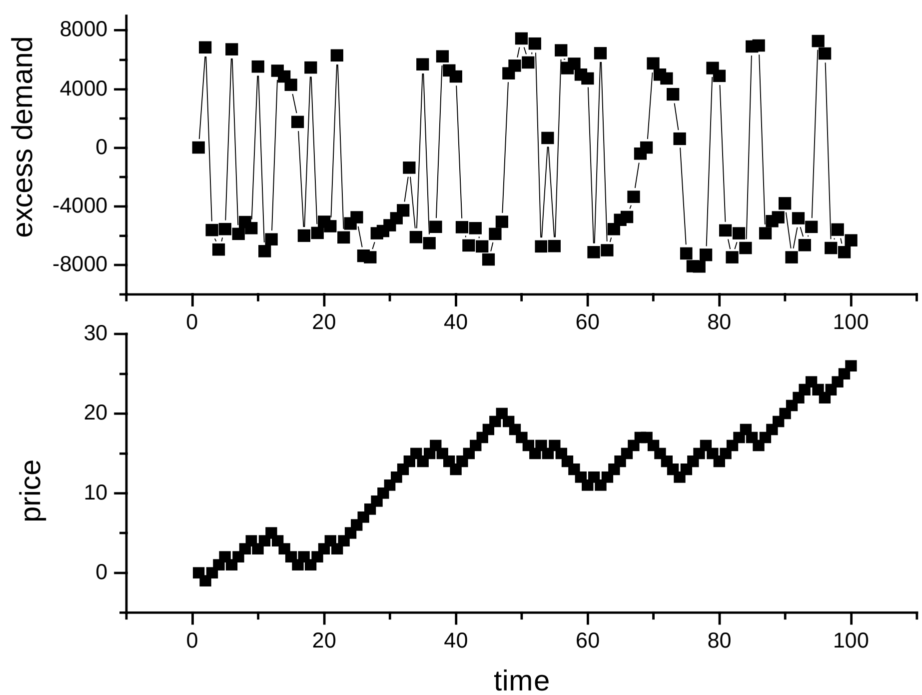
Figure 5. Impact of public shocks with no expert



(a) W(1)=1/8, W(2)=0, W(3)=0



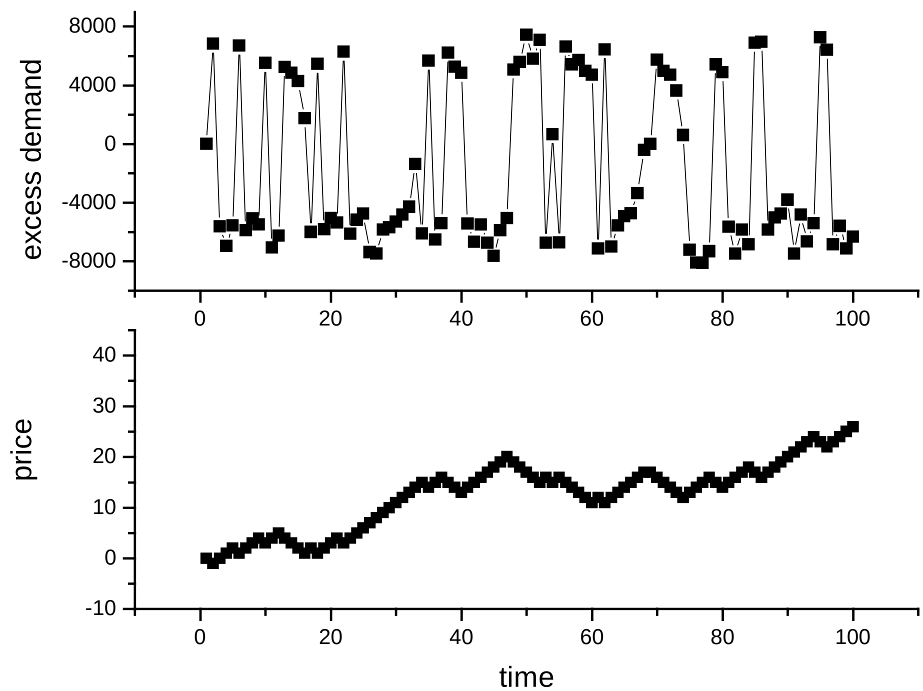
(b) W(1)=1/8, W(2)=1/8, W(3)=0



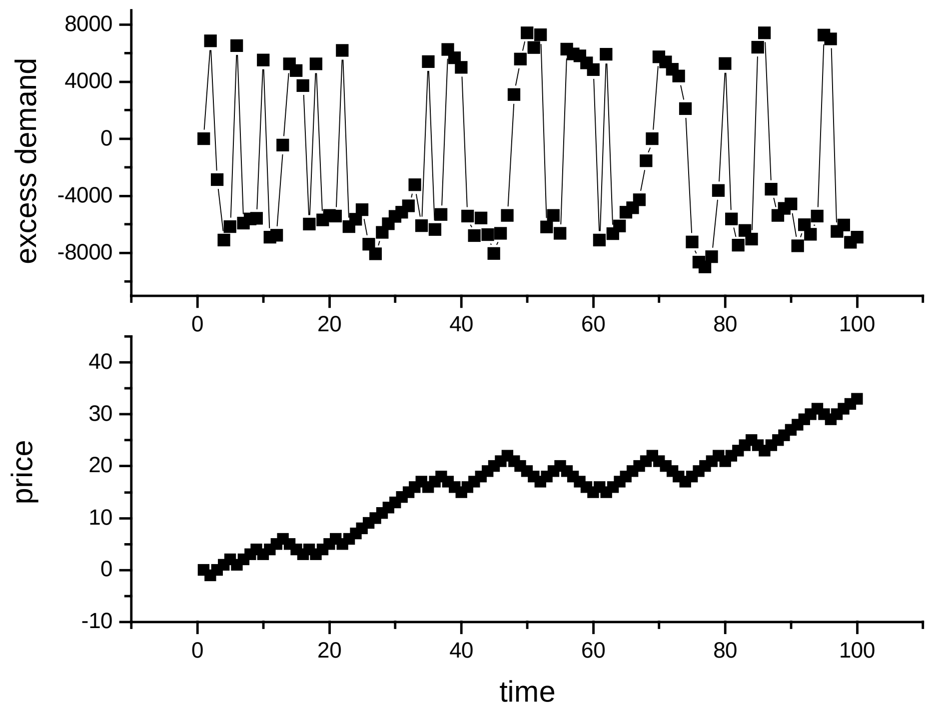
(c) W(1)=1/8, W(2)=1/2, W(3)=0

Then, economic expert is introduced into the model. In this case the existence of expert has no significant impact on price sequence as shown by the following diagrams. The reason is that the shock is public information, so the expert's function as information disseminator will no longer have impact as strong as that in private information case.

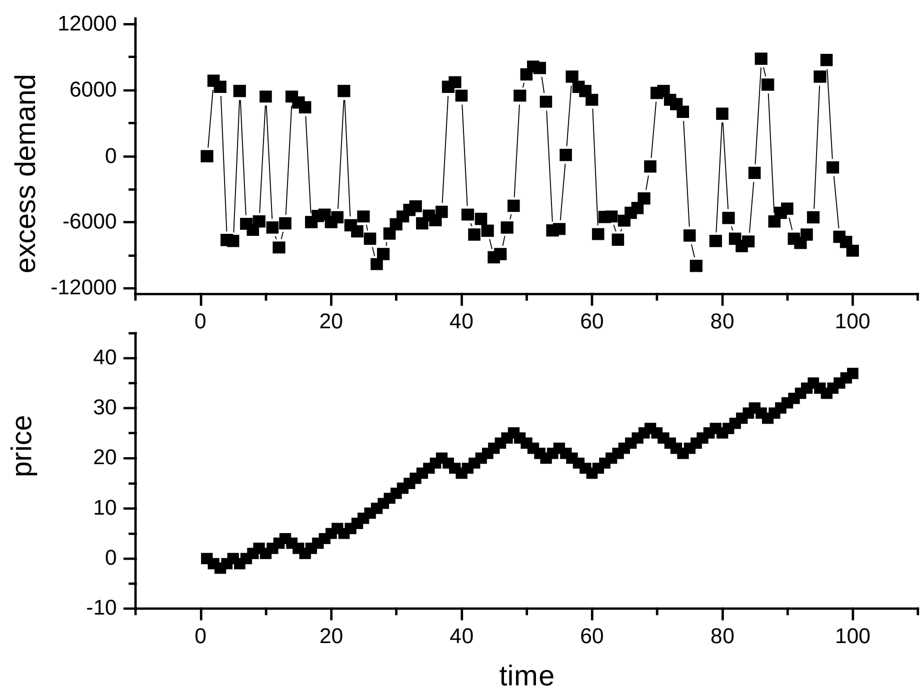
Figure 6. Impact of expert under public information



(a) W(1)=1/8, W(2)=1/2, W(3)=0



(b) W(1)=1/8, W(2)=1/2, W(3)=1/8



(c) W(1)=1/8, W(2)=1/2, W(3)=1/2

Table2 shows that expert has no significant effect even on the oscillation properties of the price series.

Table 2 Impact of expert under public information

W1	1/8				1/8			
W2	1/8				1/2			
W3	0	1/8	1/4	1/2	0	1/8	1/4	1/2
OS_A	2.3	3.6	4.5	3.9	3.0	3.2	4.0	3.8
OP_A	9.5	10.6	8.5	9.6	8.5	8.5	8.6	9.3
RANGE	26.0	36.0	35.0	33.0	27.0	34.0	33.0	39.0

Note: OS_A = average oscillation range;
OP_A = average oscillation period;
RANGE = overall oscillation range.

V. Equilibrium Price Series

In this part, in addition to the demand-and-supply principle, I further assume that market must clear in every period. The dynamic rules follow the standard explanation of imaginary auctioneer. First, each individual observes a price and decides his/her individual action. The decision rule now becomes:

$$F_{kl}^t = \sum_{i=1}^{I}\sum_{j=1A}^{J} W_{ij}^t(1)A_{ij}^t + \sum_{r=1}^{R} W_r^t(2)B_r^t(m1,m2) + \sum_{s=1}^{S} W_s^t(3)C_s^t + W_{kl}(4)P \quad \text{where } W_{kl}(4) < 0.$$

If the excess demand is not zero, price is adjusted according to the following principle:

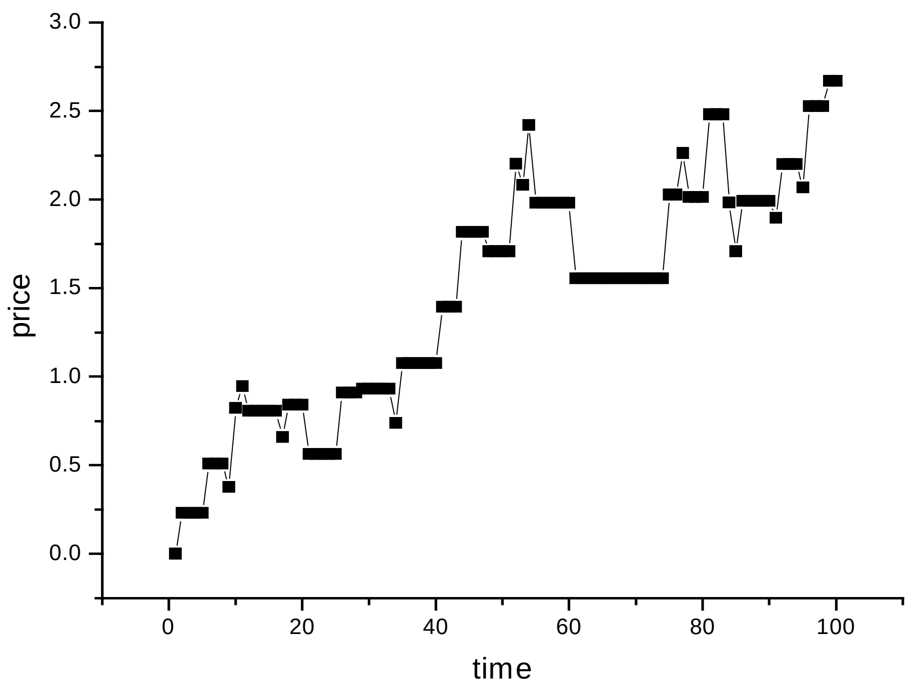
$$\Delta p(t) = \begin{cases} >0 & if \ \sum_{n=1}^{N}\sum_{m=1}^{M} D_{n,m}^t > 0 \\ <0 & if \ \sum_{n=1}^{N}\sum_{m=1}^{M} D_{n,m}^t < 0 \end{cases}$$

Then, each individual observes the new price and adjusts his/her individual demand. This process goes on until a price level P^* is reached that satisfies:

$$\sum_{n=1}^{N}\sum_{m=1}^{M} D_{n,m}^{t}(P^{*})=0$$

The price series thus generated is as follows.

Figure 7. Equilibrium Price series



The amazing thing is that the price series is not affect by the values assigned to the coefficients $W_{ij}^{t}(1)$, $W_{s}^{t}(3)$, while $W_{r}^{t}(2)$ has more impact on the scale of price series than on its shape. This result shows that if market is always at equilibrium, only the real economic shocks will matter. The key assumption that leads to this result might be that $W_{ij}^{t}(1)$, $W_{s}^{t}(3)$ only affect what individual actually observes and has not impact on real reservation values of each individual.

Neither the existence of expert nor the interaction among individuals will affect the equilibrium price level. This result might seem to be too strong at the first sight, but is actually a logical result that follows the underlying story of the imaginary auctioneer. Take the existence of expert as an example. Expert in this model has two distinct characters: more accurate observation of shocks and more market power (through its influence on other individuals). However, the existence of imaginary auctioneer can do better. Through the interaction between excess demand and price, each individual can adjust their observation with the updated bid price called out by the auctionee.

Viewed from a different perspective, this equilibrium experiment is actually a particular case of the experiments done in part II and III. In essence, the only additional assumption that "market clears in every period" is equivalent to the assumption that "the next shock can only occur when the effect of the previous one has died out". Therefore, to validate the belief that economy is *always* at equilibrium, one need to believe either that the economic can make instantaneous and accurate adjustment to the shocks, or that the next shock will never occur when the effect of the previous one has been completely digested by the market. The second merely requires that we can control the random shock, which is obviously an inappropriate assumption. The first may exist in some markets like the stock market. However, these markets are in general subject to more frequent shocks. Therefore, it is still empirically unclear whether the market is always at equilibrium or it is always on its way to equilibrium in these markets.

VI. Conclusions

This paper proposes a numerical analytical framework, which constructs a model economy as a two-dimensional lattice. As a specific application, the framework is applied to study the mechanisms that guide the formation of market prices with heterogeneity.

The numerical simulation results suggest that when the economy is at disequilibrium, price changes can still be regarded an indicator of random shocks. But its accuracy is affected by all of the following factors: the interaction among individuals, the existence of heterogeneous agents and the property of random shocks. However, in an economy that is always at equilibrium, the interaction among individuals and the existence of heterogeneous agents have virtually no effects on the equilibrium prices, if they only affect the information observed by individuals.

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