Multi-agent modeling and simulation of a sequential monetary production economy

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Abstract

In this paper, we propose a heterogeneous interacting agent model of a sequential monetary production economy. We use a basic dynamic flow model in an interacting agent context. The economy is assumed to be closed. There are three classes of agents: a single homogeneous representative consumer, heterogeneous firms and a banking system. Bounded rationality agents make decisions by optimizing an objective function based on expectations about the future formed on past data. There are three asset classes (or debts): a single homogeneous physical good, money and debt securities. The homogeneous commodity is produced by firms and, if saved, increases their capital stock. Firms issue debts to finance growth. Firms are homogeneous as regarding marginal costs of production but are heterogeneous relative to their objective functions. Firms make different investment decisions that can ultimately result in the firm's growth or bankruptcy. The income of the homogeneous consumer depends on wage earnings, interest on debt securities and firms profit. Consumers spend their income to purchase consumption goods and corporate debt; money is also considered a reserve of value. Portfolio allocation depends on the interest rate. The money supply is exogenous and is the main control parameter of the system. The model is able to reproduce endogenous large scale economic fluctuations by means of the interplay between money supply and the interactions of heterogeneous agents.

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1 The model

The economy is characterized by three classes of agents: firms, households, and banks and three classes of assets: physical capital, savings and debt securities. Physical capital is owned by firms which employ it and labor to produce output. Output is a single homogeneous good which can be used both for consumption and investment. Firms do not retain profits and issue debt securities to finance investments. Households provide the labor force and own all the equities of firms and banks. They make a decision of consumption and saving of their income, which is the sum of salary, profits of firms and bank and interest on cumulated savings. Banks collect household savings and own debt securities issued by firms. They decide the level of interest rates which depends on the difference between the level of debt and savings.

For the sake of simplicity, only firms are taken heterogeneous while households and banks are considered homogeneous. Then, in the followings, the model will deal with a representative household and a representative bank or banking sector.

1.1 Firms

The economy consists of a N firms, each of which produces the same single good subject to the same production function. The rate of output of the ith firms at any instant is described by the production function:

$$Y_{i,t} = K_{i,t}^{\alpha} N_{i,t}^{\beta} \quad i = 1, \ldots, N, \tag{1}$$

where $Y_{i,t}$ is the output of the ith at time t, $K_{i,t}$ is the stock of capital employed by the ith firm and $N_{i,t}$ is employment of the ith firm at the same moment of time t. The production function is assumed to be characterized by positive though diminishing marginal products of capital and labor; thus $\alpha, \beta \in (0, 1)$.

Firms make production and investment decisions following a profit maximizing behavior with respect to factors of production: capital stock K_i and employment of labor N_i.

1.1.1 Production

The production decision of each firm at time t is taken according to the maximization of an objective function representing economic profit. The maximization is taken only with respect to the employment of labor N_i, while capital

stock K_i is considered as fixed. Then firms are assumed to be able to vary employment instantaneously while capital stock can not change at the same moment of time of production decision [2]. Firms are supposed to operate in a competitive labor market in which at any moment they can hire all the labor they want at the going money wage w measured in euro per man per unit of time. The objective function of each firm is its economic profit defined as follows:

$$\Pi_{i,t} = p_t\, K_{i,t}^{\alpha} N_{i,t}^{\beta} - w_t N_{i,t} - p_t\,(r_t - \pi_{i,t})\, K_{i,t}\,, \tag{2}$$

where p_t is the price of the produced good, r_t is the instantaneous rate of interest on debt securities and $\pi_{i,t}$ is the rate of increase in the price of capital goods expected by the ith firm. The difference $r - \pi_{i,t}$ can be regarded as the real interest rate perceived by firm i or also as the cost of capital.

The price p_t of produced goods is the clearing price of the goods market at time t, i.e., the price at which aggregate supply $Y_t = \sum_i Y_{i,t}$ equates aggregate demand Z_t, composed by the demand for consumption $Z_{C,t}$ of the representative household and the demand for investment $Z_{I,t}$ made by firms themselves. Then, strictly speaking firms are not price takers; however the large number of existing firms and their heterogeneity allow us to make the simplifying assumption that price p_t does not depend on the decision making of the single firm, i.e., $\partial p/\partial N_i = 0$ and $\partial p/\partial K_i = 0$.

Given the capital stock K_i as fixed, the firm's employment N_i is then described by the first-order condition for maximization of Eq. 2:

$$\frac{\partial \Pi_i}{\partial N_i} = 0\,, \tag{3}$$

which gives the expression for $N_{i,t}$:

$$N_{i,t} = \left(\beta \frac{p_t}{w_t}\right)^{1/(1-\beta)} K_{i,t}^{\alpha/(1-\beta)}\,. \tag{4}$$

[2] Assuming that capital is fixed to each firm at each moment in time amounts to ruling the existence of a perfect market in the existing stock of capital in which individual firms can purchase or sell (or rent) capital, and so effect a discrete change in their stock of capital at a moment in time. The absence of a market in existing capital might be rationalized by posting that once in place capital becomes completely specialized to each firm. Firms simply have no use for the existing capital of another firm, so that there is no opportunity for making a market in existing capital.

Eq. 4 and Eq. 5 than give the output of each firm at time t:

$$Y_{i,t} = \left(\beta \frac{p_t}{w_t}\right)^{\beta/(1-\beta)} K_{i,t}^{\alpha/(1-\beta)} . \tag{5}$$

1.1.2 Investment

The investment decision of each firm $I_{i,t}$ is taken according to the maximization of economic profit represented by Eq. 2 with respect to both employment N_i capital stock K_i:

$$\frac{\partial \Pi}{\partial N_i} = 0 , \tag{6}$$

$$\frac{\partial \Pi}{\partial K_i} = 0 . \tag{7}$$

The joint resolution of Eq. 6 and Eq. 7 gives the optimal capital stock $K_{i,t}^*$ firm would have at time t:

$$K_{i,t}^* = \left(\beta \frac{p_t}{w_t}\right)^{-\beta/(\alpha+\beta-1)} \left(\frac{r_t - \pi_{i,t}}{\alpha}\right)^{(1-\beta)/(\alpha+\beta-1)} . \tag{8}$$

If $K_{i,t}^* > K_{i,t}$, firm i plans an investment $I_{i,t}^*$ in order to fill the gap between actual capital stock $K_{i,t}$ and desired one $K_{i,t}^*$. Because capital stock can not vary instantaneously, firm can employ the desired capital stock only at instant $t+1$. The dynamics of capital accumulation is thus as follows:

$$I_{i,t}^* = \max\left(0, K_{i,t}^* - K_{i,t}\right) , \tag{9}$$

$$K_{i,t+1} = K_{i,t} + I_{i,t} , \tag{10}$$

where $I_{i,t}$ is the investment really made by the firm. Generally, $I_{i,t}$ is a fraction of $I_{i,t}^*$, i.e., $I_{i,t} = c\, I_{i,t}^*$, with $c \in [0,1)$. Because investments of firms are financed by debt, the aggregate effective investment $Z_{I,t} = \sum_i I_{i,t}$ depends on the amount of loan L_t that the banking sector want to offer to firms at time t as follows:

$$Z_{I,t} = \min\left(Z_{I,t}^* , L_t/p_t\right) , \tag{11}$$

where $Z_{I,t}^* = \sum_i I_{i,t}^*$ is the aggregate desired investment. The value of c is thus given thoroughly by:

$$c = Z_{I,t}/Z_{I,t}^* . \tag{12}$$

is in general different from the aggregate desired investment $Z^*_{I,t} = \sum_i I^*_{i,t}$.

and

It is worth noting that values of α and β have to be restricted according to the relation $\alpha + \beta \neq 1$, because of the singular point of Eq. 8. Moreover, in order to take into account a realistic inverse dependence between desired real capital stock and real interest rates, the exponent $(1-\beta)/(\alpha+\beta-1)$ should be negative. These issues are addressed by restricting the bounds of α and β from $\alpha, \beta \in (0,1)$ to $\alpha, \beta \in (0, 0.5)$.

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