FROM HETEROGENEOUS EXPECTATIONS

TO EXCHANGE RATE DYNAMICS

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Abstract:

The purpose of this paper is to analyze how heterogeneous behaviors of agents influence the exchange rate dynamic in the short and long term. We will examine how agents use information and what kind of information, in order to make their decisions to anticipate the exchange rate. We will investigate methodology based on interactive agent simulations, following the Santa Fe Artificial Stock Market. Each trader is modeled as an autonomous, interactive agent and the aggregation of their behavior results in foreign exchange market dynamics. Genetic algorithm is the tool used to compute agents, and the simulated market tends to replicate the real EUR/USD exchange rate market. We will consider four kinds of agents with pure behavior: fundamentalists, positive and negative feedback traders, naive traders, news traders (positive and negative). To reproduce stylized facts of the exchange rate dynamic, we conclude that the key factor is the correct proportion of each agent type, without any need of mimetic behaviors, adaptive agents or pure noisy agents.

Keywords: exchange rate dynamic, heterogeneous interactive agent behavior, genetic algorithm, learning process

JEL Classification: D83, D84, F31

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1. Introduction

Agent heterogeneity is obvious if one consider the results of the large number of studies using survey data. The use of such data is very appealing since it allows us to measure exchange rate expectations directly. Using the panel data of biweekly surveys on the yen – dollar exchange rate expectations of forty-four Japanese institutions from May 1985 to June 1987, Ito (1990) finds a wide dispersion in individual expectations. In the three-month horizon, he observes that one extreme predicted a 3.25 percent depreciation of the yen, while the other extreme predicted a 4.76 percent of appreciation. Mac Donald (1992), who compares the exchange expectations in the G7 countries from October 1989 to March 1991, confirms this result. Frankel and Froot (1986, 1987) also show that the standard deviations of the expectations mean increase considerably when the horizon decreases. Ito (1990) and Takagi (1991) find significant individual effects in participants' expectation formation. Those individual effects have characteristics of wishful expectations: exporters expect yen depreciation and importers expect a yen appreciation. On the whole, survey data show that for short-term predictions, i.e. from one week to one month, respondents tend to forecast by extrapolating recent trends (extrapolative expectations), while for the long term, i.e. six to twelve months, they tend to forecast a return to long-run equilibrium such as PPP (regressive expectations).

This durable combination of short-term extrapolative expectations, with destabilizing effects, and long term regressive ones, which therefore stabilize, highlights two conflicting ideas of the exchange dealers. Long term responses seem to express the operators' economic reason, the fundamentals, with respectability concern or simply a reproduction of what economic analysis predicts about the foreign exchange rational expectations. On the other hand, short-term responses seem to correspond to the market logic and reveal the true opinion, at a given time, of the respondent. The observation of past trends, the use of chartist methods and the goal of being in the market, thus neglecting the long term, seem to prevail. Interestingly, it is especially in the short run that specialists' trading takes place. The tremendous volume of FX trading is another piece of evidence that reinforces the idea of heterogeneous expectations since it takes differences among market participants to explain why they trade[1]. Goldstein et al. (1993) estimate that each customer's transaction generates, on average, four to five inter-dealer transactions in response to the price discovery operations they imply and that the speculative operators adopt.

Standard models of exchange rate determination, in contrast, assume the existence of identical investors who share rational expectations of future exchange rates, and who instantaneously and rationally discount all market information concerning this rate. If this assumption is crucial because it permits a simple aggregation of common individual behaviors, it is not ideally suited because it raises the question of the existence of transactions if all agents are strictly identical (Arrow, 1985). It follows that trading volume is low or zero, and that trading volume and price volatility are not serially correlated in any way. However, foreign exchange markets, as well as other financial markets, are characterized by a number of striking ubiquitous time-series features: unit roots in level together with fat tails in returns and volatility clustering. As a result, studies have been searching for alternative explanations of such realities.

Microstructure analyses investigate the role of the market's organizational or institutional characteristics on exchange rate determination: auction types, centralization of order flows, quotation rules, etc. They see trading as a process of information transmission and market opinion discovery (Lyons, 1995). Exchange rate determination is therefore endogenous to the inter-bank market (Varian, 1989; Peraudin and Vitale, 1996). To date, these studies have not answered the question of foreign exchange rate determination but have focused on the spread determination, the relation between trading volume and volatility, the marketplace transmission of volatility (Lyons, 1995; Flood, 1991 and 1994; Goodhart and Payne, 1996; Goodhart and al., 1996; Goodhart and al., 1997; Evans and Lyons, 1999; Osler, 2001). They all emphasize the role of traders as market-makers.

This paper relies on the growing literature on computational agent-based models. In his recent survey, Le Baron (2000) argued that computational agent-based models stress interactions, and the learning dynamics of groups of traders learning about the relations between prices and market information. Under heterogeneity, expectations have a recursive character: agents have to form their expectations from their anticipations of other agents' expectations, and this self-reference precludes expectations being formed by deductive means. Agents therefore continually form

[1] The banks in the BIS (1998) census reported that 81 percents of the spot trading, which represents a daily average of $ 1,5 trillions, takes place among the banks and other financial institutions, rather than with customers such as exporters and importers.

individual, hypothetical, expectational models, test these, and trade on the ones that predict best. Prices are driven endogenously by these induced expectations. Agents' expectations co-evolve in a world they co-create (Arthur et al., 1997). Arifovic (1996) considers a dynamic version of the Kareken and Wallace (1981) model of exchange rate formation in a two country overlapping generations world. Using a standard genetic algorithm procedure to update agents' decision rules, the simulations give exchange rate series which do not settle to any equilibrium. As it is outlined in Le Baron (2000), this result is related to the structure of the indeterminacy of the model. Arifovic and Gencay (2000) find that the model's equilibrium dynamic is not constant but exhibits bounded oscillations. Their time series analysis of the data indicates that the dynamics of exchange rate returns is chaotic. Lux and Schornstein (2002), using the same model, find that for particular parameterizations, the characteristics of exchange rate dynamics are very similar to those of empirical data (i.e. unit root in levels together with fat tails in returns and volatility clustering)[2]. However, they show that whether or not realistic time series characteristics appear essentially depends on the mutation probability and the number of agents. This later finding casts doubts on the potential applicability of this model to real markets such as the foreign exchange market.

Our artificial market is initially inspired by the Santa Fe Stock Market which is outlined in detail in Arthur et al. (1997) and LeBaron et al. (1999), and is an extension of Neuberg and Bertels (2003) model. However, it differs from it in some points. First, instead of a stock market we simulate the USD/EUR exchange rate dynamic. Therefore, agent types are quite different. Secondly, as our goal is to reproduce actual exchange rate series we use real data to create agents' decision rules.

Our analysis proceeds in the following steps: section 2 will describe the market. We will explain the different financial agents and their behaviors, then, the learning process and expectation formation, market clearing and price formation, and finally the information set. Section 3 will present the simulation results of the exchange rate dynamic and the statistical properties of the simulated series.

[2] Similar results for other markets are found by Lux and Marchesi (1999, 2000), Chen et al. (2001) Kirman and Teyssiere (2001) among others.

2. Description of the market

2.1.The financial agents

We distinguish between different kinds of traders on the market, each having his own rationality and knowledge. Like any trader, the agent must be able to evaluate an action and form an expectation with respect to its future price. In this paper, we will introduce four different types of behavior which are described below.

- Fundamentalists: they forecast a return to a long-run equilibrium and therefore have regressive expectations. They place buy (sell) orders if the current exchange rate is under (over) this fundamental value. In this paper, the fundamental value is determined according to uncovered interest rate parity.

- Noise traders: generally speaking, noise traders base their position on feelings not justified by existing information. De Long et al. (1990a) and Shleifer and Summers (1990) consider pure noise traders who act randomly. We will not take into account this kind of trader because the foreign exchange market is a specialist one. Nevertheless, we will introduce two kinds of noise traders traditionally described in literature (Cutler et al., 1990; De Long et al., 1990b). Positive feedback traders are those who buy when prices rise and sell when prices fall. Many forms of common behavior in financial markets can be described as positive feedback trading. It can result from extrapolative expectations about prices or trend chasing. It can also result from stop-loss orders, which effectively prompt selling in response to price declines. A similar form of positive feedback trading is the liquidation of the positions of investors unable to meet margin calls. Negative feedback traders react negatively to previous price movements: they buy when prices fall and sell when prices rise.

- News traders: they act in response to exogenous information which can be decomposed into three types: good news, bad news and no news. Like noise traders we distinguish between positive news traders and negative ones. The former react positively to the news, e.g. they buy when the news is good and sell otherwise. The latter react negatively, e.g. they sell when the news is good and buy otherwise.

- Naive traders: they expect the exchange rate to remain stable (Arthus, 1992).

We will only consider pure behavior without re-learning agents.

2.2. The learning process of agents and expectation formation

Different methods allow us to build agent behavior models without using formalization based on explicit equations. This is the case of genetic algorithms (GA), a method initially developed by Holland (1975) to study the adaptive system. They are now applied to the study of learning systems. The purpose is to explain and model the natural adaptive learning process in order to build artificial systems using the natural mechanisms of the learning process. The simplicity of use of genetic algorithms explains their success. GAs allow us to simulate inductive agents learning in a dynamic environment. It is a way of simulating natural learning with a learning of rules and a model of decision making. These rules are built and corrected with the information that the agents have on the environment. The learning process allows the agents to form hypotheses and to formulate expectations about the market. This learning is modeled by a classifier system.

In our model, each agent needs to be able to decide whether he wants to buy or sell a particular currency and at what price. He therefore needs to have decision rules that allow him to formulate some kind of expectation as to the future evolution of the exchange rate. He will do so on the basis of information at his disposal. In our model we have chosen to implement a classifier system where different decision rules are represented as if-then rules. At a given moment, if a condition of his set satisfies the present situation in the environment, the agent will take the corresponding action. The condition of each rule is a chain of characters ("0", "1" or "#") determining whether the rule is equivalent to the market situation. This equivalence is achieved if the characters along the chain of the condition are similar to the characters along the chain of the market situation. In the case of character "#", there is always an equivalence to the extent that it expresses the indifference between the characters "1" and "0". As for the action, it is a chain of characters representing the value of two parameters a and b in binary fashion. These parameters allow us to compute the expected future exchange rate in the following way:

$$E[e_{t+1}] = a(e_t) + b \qquad [1]$$

where e_t is the exchange rate at time t. For each agent, a set of rules allowing the calculation of these expected prices will be generated using genetic algorithms. Initially 2000 rules are generated and during the learning process, depending on the agent's type, this number will be reduced. In contrast to previous studies, the learning process is based on real data. Risk aversion is expressed in terms of the CARA utility function which, for the sake of comparability, is taken from [1].

$$U(w) = -e^{-\lambda w} \qquad [2]$$

where w represents the wealth of the trader and λ indicates the degree of risk aversion.

In this original set of 2000 rules, some may be more efficient than others. Those rules yielding more accurate expected prices and therefore a higher financial gain will have a higher reproduction rate and a higher probability of survival. Before starting trading simulations, each agent learns the market dynamic passively. This means that he will use real market data to construct his set of rules. The effectiveness of the decision rules is defined in relation to the error generated by the rule and is computed as follows:

$$Error(rule) = \left(E[e_{t+1}] - e_{t+1}\right)^2 \qquad [3]$$

using [1] results in

$$Error(rule) = \left(a(e_t) + b - e_{t+1}\right)^2 \qquad [4]$$

A perfect rule will compute an expected value equal to the exchange rate and the error will be null. This kind of rule will have a maximum evaluation value. If we represent this maximum value by C, we obtain a rule evaluation function, also called the strength of the rule, which is defined as follows:

$$Eval(rule) = C - \left(a(e_t) + b - e_{t+1}\right)^2 \qquad [5]$$

A process that will assume agents' rationality completes this evaluation function. Each rule must reflect the specific agent type. Let us take an example: the rule of a fundamentalist agent will receive a better evaluation function if this rule leads to a value of exchange rate that is based on interest rate parity. Otherwise, this rule will be under-evaluated, even if the predicted value is not so bad.

2.3. Market clearing and price formation

Intersecting orders to buy and sell create the dynamics of exchange rates. The market clearing mechanism is similar to that of the Santa Fe Stock Market in which bids are continuously resubmitted until a price (an exchange rate in our case) is formed that clears the market. For each period of time, the agents try to optimize the allocation of risky and non-risky assets, i.e. US currency versus Euro currency. Initially, the exchange rate previsions made by agent *i* at time *t* are normally distributed with an average of $E_{i,t}[e_{t+1}]$ and a variance $\sigma^2_{i,t,e}$. Demand (or supply) by agent *i* at time *t* is given by:

$$x_{i,t} = \frac{E_{i,t}(e_{t+1} - (1 + r_t)e_t)}{\lambda \sigma^2_{i,t,e}}$$

where r_t is the interest rate of the non-risky asset at time *t*.

As the market is composed of n agents, we therefore have n equations with n+1 unknown values: the n quantities of risky assets allocated by agents ($x_{i,t}$) and the exchange rate at time *t*. In order to close the system, we add an equation which means that total demand is equal to the number of available goods on the market: $\sum_{i=1}^{n} x_{i,t} = n$. At each time *t*, the exchange rate e_t can be computed by resolving the (n+1)*(n+1) system of equations.

2.4. Information set

In our model, information arrives at the market at regular intervals of time. Every iteration represents one day. The information is used differently depending on the agent type. As we previously defined agents, they will use part of the available information. Information is composed of three different parts: technical information, fundamental information and exogenous information.

The technical part is provided by a two bit binary chain. The first character of the chain concerns the trends of the exchange rate. If the return of the exchange rate on the previous period is positive, then the value will be “1”, otherwise it will be “0”. The second character reflects the absolute value of the trends. If the absolute value of the return is greater than 5 percent, then the value is “1”, if the absolute value is lower or equal to 5 percent, then it is “0”.

The fundamental part of the information represents interest rate parity. The first character is constructed with a 3 month interest rate and a 3 month forward exchange rate. If the value of the forward exchange rate is lower than the computed exchange rate, the value of the character will be “1”, “0” otherwise. The same reasoning is used at a 6 month horizon.

The third part of information is related to exogenous information. This information may vary from negative (-1) over neutral () to positive (1). We filtered important events influencing the trends of the market. This kind of information would be for instance a terrorist event as well as an intervention of a Central Bank on interest rates.

3. Simulation results

It is important to emphasize that we are focusing on understanding the dynamics of the exchange market and not on prediction. This model is validated as regards this objective if statistical properties can be compared to the real exchange rate values. The framework of simulations is to find the best ratio between the number of different agent’ types in order to imitate the real market.

3.1. Presentation of simulations

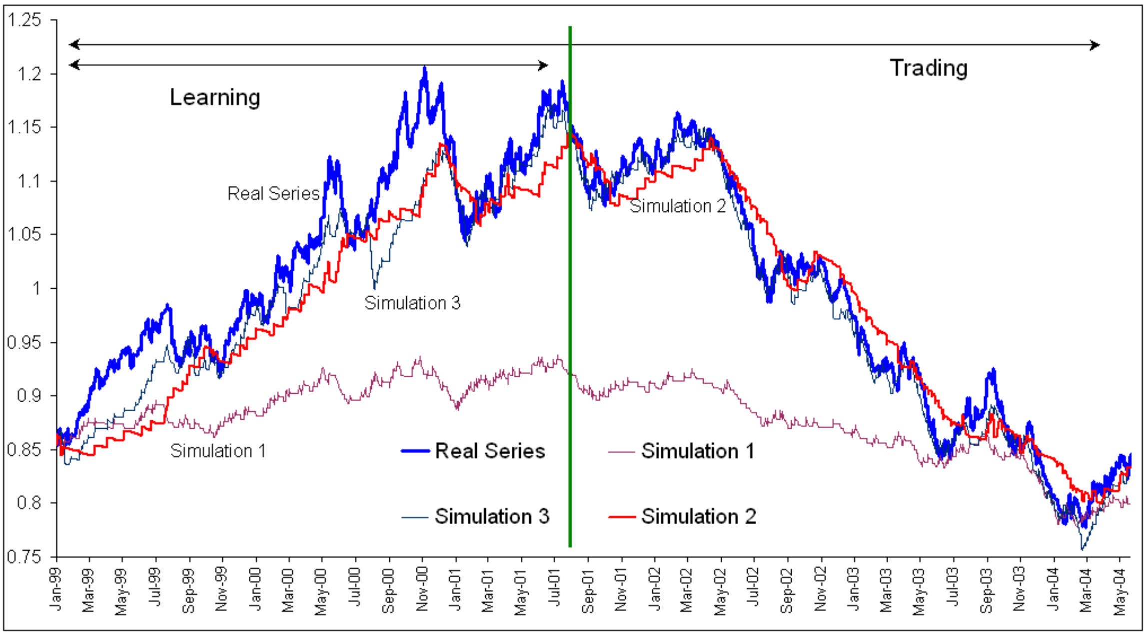
We started our investigation by the creation of an individual agent’ model. As previously mentioned, we used real data concerning the EUR/USD exchange rate from January 1999. On a daily basis, 700 periods of time are considered for the learning process (until June 2001). Five different agents were created for each type. During this learning process, no market simulations were performed. Agents were just learnt by trading like passive actors on the market. After this first step, simulations were performed from January 1999 to May 2004. During this period agents traded actively on the artificial market. This means that half the period of simulations is outside periods concerned by the learning process. Thereby, over-learning leading to a pure restitution is avoided, at least during the second part of the simulation period.

We will present only the five most representative simulations leading to the most reliable exchange rate dynamic.

Simulation 1: We started our investigation with only a pure fundamentalists' market. We therefore performed simulations including three fundamentalists and no other agents. As can be seen in Graph 1, the simulated series exhibited a long-run equilibrium with a very low volatility compared to the actual exchange rate. Interestingly, over the last 12 months the two series seemed to converge which may indicate that the market was driven by fundamentalists' beliefs.

Simulation 2: A purely technical market excluding fundamentalist was then explored. 2 positive feedback traders and 2 negative feedback traders were present in this market. We can observe in this simulation a global shape closer to reality that the previous one. Nevertheless, the main characteristics of this market were two trends: one upward, one downward. The volatility was too low to be compared to reality. One conclusion can be that the global shape of the real exchange market is driven by technical forces. Unfortunately, nothing could explain the change of trends if we just consider technical traders. In this simulation, this could be due to the fact that technical agents, during learning, had also constructed a small number of fundamental rules.

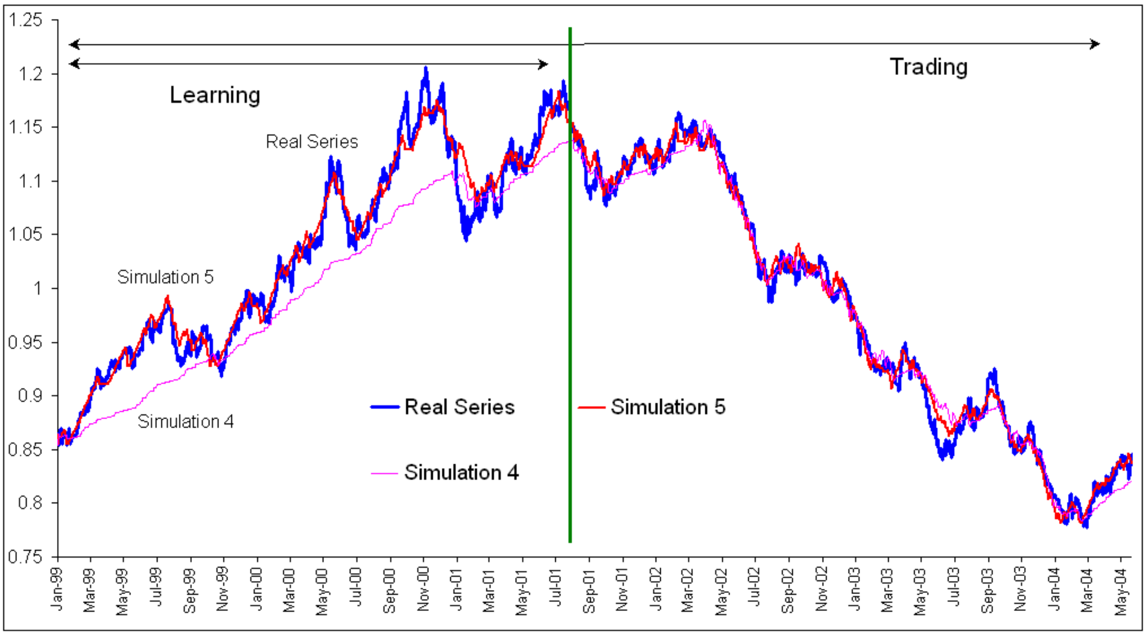
Simulation 3: A market with 2 positive feedback traders, 2 negative feedback traders and 4 fundamentalist agents seems to give more volatility and a more accurate dynamic. In that market, we can observe both the same global shape as reality, and more frequent dynamic patterns resembling the real world.



Graph1: Results of simulations 1 to 3

Simulation 4: We then simulated a market incorporating every type of agent. 2 positive feedback traders, 2 negative feedback traders, 2 fundamentalists, 2 news agents, 2 negative news agents as well as 2 naïf agents traded in this market. We observed two main parts in the simulation. The first one is a quite linear increase of the market and the second part is a dynamic that is closer to reality. This ratio of agent types is more appropriate during a bear market that in a bull market. Therefore, we have to find a better ratio that could explain every kind of dynamic.

Simulation 5: The most appropriate ratio of agent types that we could find is 3 positive feedback traders, 1 negative feedback trader, 5 fundamentalists, 2 positive news agents, 1 negative news and 1 naïve agent. In that case, the model seemed to better reproduce the real market.



Graph 2: Results of Simulations 4 to 5

3.2. Statistical properties

To see whether our simulated series had realistic times series properties, we used a series of statistical tests. All the results are detailed in appendix. First, we calculated the first four moments of the exchange rate returns' distribution and tested for normality (table 1). Secondly, we analyzed the auto-correlation of returns (table 2). Thirdly, since previous studies had shown that (log) exchange rates series are non stationary processes, we performed typical tests (Phillips & Perron and KPSS test with constant and trends) for the presence of a unit root in our series (table 3). Finally, we computed the BDS tests to test the null hypothesis of iid returns and estimate the entropy (table 4).

As expected, the Bera-Jarque test for normality led to a strong rejection of the null hypothesis for all series. We observed significant excess kurtosis in real and simulated return series, which confirms the existence of fat tails. There is no asymmetry in the real series and the fifth simulated series, which we consider to be the most representative. However, the variance of the simulated series are smaller than the real data. Concerning the Ljung-Box statistic, we concluded that there was a highly significant auto-correlation in level, squared and absolute values in all simulated series but not in the real exchange rates. The numerical precision used in the code of market models is such that the returns produced have a quite limited number of significant decimals. As a consequence, there is a limited number of different figures and therefore a higher auto-correlation of these series. Perhaps, another explication may be the codification of exogenous information in our market ("-1", "0", "+1").

As expected, unit root tests could not reject the null hypothesis of a unit root in the exchange rates nor in the simulated series, with the exception of simulation 1. The different series were non-stationary processes.

In addition, we calculated the BDS statistic of Brock, Dechert and Scheinkman. The BDS statistic tests the null hypothesis of identical and independent distribution and it is shown to be powerful against nonlinear alternatives. It is distributed asymptotically standard normal. For certain cases (especially when σ/ε is large) we rejected the null hypothesis that the series was iid, i.e. that the series was chaotic. We computed the entropy which is the sum of the positive Lyapunov exponents. It provided a quantitative measure of the non-predictability of the chaotic system. All values were positive. We then concluded that the predictability of the series was low. In other words, the lower the entropy is, the higher is the potentiality of predictability, even if the system remains non-predictable in the long-run. In our case, we concluded that the exchange rate is more predictable in the short term but is difficult to predict in the long term.

Conclusion

The purpose of this paper was to analyze how heterogeneous behaviors of agents influence the exchange rate dynamic in the short and long term. We examined how agents use the information and what kind of information they use, in order to make their decisions and form an expectation of the exchange rate. We investigated a methodology based on interactive agent simulations to reproduce the exchange rate dynamic of the EUR/USD exchange rate. To reproduce stylized facts of the exchange rate dynamic, we conclude that the key factor is the correct proportion of each agent type, without any need for mimetic behavior, adaptive agents or pure noisy agents.

The next step would be to incorporate more agents of each type in order to refine the best ratio of agent types. We can easily plan to multiply the number of agents by ten and therefore total one hundred. We could also increase the amount of data. Another point to improve is the quality of treatment of the exogenous information.

One interesting point to research would be to follow the study in terms of the portfolio value of agents, i.e. the wealth evolution of each agent, as well as a dynamic evolution of their risk aversion corresponding to their loss or gain.

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Appendix

Table 1: Sample statistics of returns

	Real	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5
Mean	1.28e-3	-3.32e-3	-1.41e-3	-1.17e-3	-2.56e-3	-1.19e-3
Variance	0.46	0.14	0.13	0.23	0.04	0.10
Kurtosis	0.69*	1.11*	6.21*	1.06*	3.21*	-0.44*
Skewness	-0.07	1.13*	0.24*	0.11	0.23*	-0.02
BJ- test	29.49*	375.55*	2272.92*	69.87*	618.34*	11.68*

* and ** indicates statistical significance at the conventional 1 % and 5 % levels.

Table 2: Autocorrelations of returns

	Real	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5
Row data						
Q(8)	9.86	179.33*	46.55*	143.31*	146.10*	159.80*
Q(12)	14.45	182.30*	50.78*	145.22*	211.88*	184.79*
Q(16)	19.18	182.80*	56.46*	155.46*	254.97*	191.02*
Squared values						
Q(8)	9.33	26.86*	17.49**	112.56*	415.63*	59.91*
Q(12)	14.29	29.20*	24.18**	114.54*	533.39*	87.40*
Q(16)	15.77	32.04*	27.62**	117.86*	604.18*	100.59*
Absolute values						
Q(8)	18.83	102.67*	20.01**	139.86*	1176.84*	277.47*
Q(12)	21.18	109.33*	30.07*	142.54*	1538.83*	338.24*
Q(16)	24.46	117.78*	34.02*	151.25*	1788.69*	367.35*

* and ** indicates statistical significance at the conventional of the Ljung-Box statistic 1 % and 5 % levels.

Table 3: Unit root tests of (log) foreign exchange rates

	Real	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5
$t(\rho_{-1})/\tau$	-2.11	-3.96*	-1.17	-1.55	-1.09	-1.59
$\xi_{\mu}(1)$	23.41	32.61	18.58	21.29	20.49	24.02
$\xi_{\tau}(1)$	16.33	15.19	17.19	16.87	17.30	16.70

$t(\rho_{-1})/\tau$ is the Phillips-Perron test for unit root with constant and trend. The critical value are -3.96, -3.31 and –3.12 at the 1%, 5% and 10% levels respectively. $\xi_{\mu}(1)$ and $\xi_{\tau}(1)$ are the KPSS tests for unit root for lag l = 1 with constant and trend respectively. The critical values are 0.739, 0.463 and 0.347 and 0.216, 0.143 and 0.119 respectively at the 1%, 5% and 10 % level.

Table 4: BDS tests for the foreign exchange returns and entropy

σ/ε = 0.5	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10
Real Serie	-0.22	-0.19	-0.17	-0.02	0.26	0.29	0.84	1.16	1.62
Simulation 1	4.40	5.05	5.91	6.94	8.37	10.22	12.92	16.14	19.38
Simulation 2	1.74	1.82	2.05	2.34	2.61	2.89	3.23	3.61	3.88
Simulation 3	3.47	5.77	7.83	10.15	13.42	17.76	23.60	32.22	42.63
Simulation 4	5.39	6.60	8.49	10.82	13.79	17.88	23.07	30.57	40.96
Simulation 5	8.28	13.49	21.42	34.86	58.23	101.35	181.10	328.99	601.26
σ/ε = 1	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10
Real Serie	-0.13	-0.08	-0.06	-0.01	0.11	0.26	0.44	0.68	0.88
Simulation 1	3.20	3.30	3.63	4.01	4.61	5.35	6.35	7.44	8.46
Simulation 2	1.15	1.18	1.29	1.46	1.63	1.80	2.00	2.20	2.35
Simulation 3	-1.41	-0.21	0.26	0.53	0.79	1.03	1.20	1.49	1.71
Simulation 4	3.85	4.74	6.02	7.50	9.15	11.13	13.48	16.89	21.18
Simulation 5	4.33	6.84	9.75	13.83	19.60	27.98	40.45	59.76	88.68
σ/ε = 1.5	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10
Real Serie	-0.07	-0.03	0.00	0.04	0.09	0.17	0.26	0.35	0.43
Simulation 1	-0.46	-0.47	-0.40	-0.37	-0.33	-0.28	-0.23	-0.15	-0.06
Simulation 2	-0.23	-0.20	-0.13	-0.10	-0.11	-0.13	-0.13	-0.14	-0.17
Simulation 3	-0.96	-0.49	-0.38	-0.29	-0.23	-0.16	-0.11	-0.05	0.00
Simulation 4	10.33	10.92	12.87	15.20	18.20	21.81	26.27	31.96	39.02
Simulation 5	3.21	4.67	6.02	7.63	9.55	11.92	14.93	19.00	24.15
σ/ε = 2	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10
Real Serie	-0.02	0.01	0.04	0.06	0.09	0.12	0.16	0.20	0.23
Simulation 1	-0.55	-0.56	-0.47	-0.44	-0.41	-0.36	-0.32	-0.25	-0.17
Simulation 2	-0.37	-0.31	-0.26	-0.23	-0.23	-0.26	-0.26	-0.27	-0.32
Simulation 3	-0.08	0.04	0.07	0.09	0.09	0.09	0.09	0.10	0.11
Simulation 4	0.43	0.54	0.73	0.92	1.08	1.23	1.37	1.52	1.66
Simulation 5	0.14	0.26	0.37	0.50	0.61	0.71	0.81	0.90	0.99

Entropy	
Real Serie	0.68
Simulation 1	0.62
Simulation 2	0.37
Simulation 3	0.52
Simulation 4	0.53
Simulation 5	0.54

σ is the standard deviation of the exchange rates, m refers to the embedding dimension, ε is the distance parameter and is chosen to be the fraction of the standard deviation of the data. BDS test is distributed standard normal asymptotically. The entropy is the sum of the positive lyapunov exponents and is computed for m = 5.