Firm Survival Hazard and Endogenous Knowledge Flows in the Industry Life Cycle - Preliminary

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Abstract

Since the path-breaking work of Gort and Klepper (1982) the common features of industry life cycles were widely recognized. In particular, the various theories aimed in explaining the remarkable phenomena of acute transformation from a non-concentrated to a concentrated industry, which is known as producer 'shakeout' and is associated in drastic fall in survival hazard and with mass exit.

The contribution of this paper is twofold: I offer an alternative analysis for the industry 'shakeout', which is based on endogenous knowledge sharing among firms. Furthermore, this paper stress the importance of endougeonizng knowledge spillover in growth theories.

Using dynamic programming methods, I am able to illustrate industry 'shakeout' and the drastic fall in survival hazard with accord to empirical findings, relating them to endogenous strategies of sharing or protecting knowledge.

Strong positive incentive to diffuse knowledge at the prior 'shakeout' era allows for rapid growth and intensive product innovation due to high knowledge externalities. As the industry reaches a sufficient level of knowledge stock, firms find it optimal to prevent their knowledge from drifting toward their rivals . At this stage the industry experiences the 'shakeout' causing drastic drop in firms' value, innovative efforts and survival hazard.

Empirical findings with respect to patenting activity may suggest that higher efforts were put to protect innovations of less qualities in late periods then those that were protected during the early periods of the industry life cycle.

Further, analyzing incomplete and asymmetric information model with regard to knowledge spillover and R&D realizations suggests ambiguous outcomes regarding knowledge externalities. As more knowledge sharing is evident when firms do not take into account an uncontrolled component in the knowledge spillover channel, less R&D is the result of higher uncertainty. Thus, on the one hand we observe stronger will to diffuse knowledge, however, on the other hand less knowledge eventually spilled due to weaker innovative efforts.

Finally, consumer welfare is studies and policy implications are stressed.

Key words: Endogenous technology diffusion, producer 'shakeout', industry life cycle, R&D, ancillary industries and complementary innovation.

1 Introduction

Studying economic growth and industrial development, one cannot overlook the common evolution pattern industries follow. The literature addresses this pattern as the product life cycle and many studies throughout the years put their efforts in explaining the features that are associated with it.

This paper aims to analyze the dynamics of firms survival hazard, which experiences a remarkable 'shakeout' that has a dramatic impact on industry structure, rate of concentration and consumer welfare, by studying the rate at which knowledge drifts outward from the boundaries in which it was created. The acute transformation from non-concentrated to concentrated industry is known as the *producer 'shakeout'* and is the main focus of this paper.

The vast literature concerning product life cycle and producer 'shakeout' is originated mainly in Gort and Klepper (1982), Klepper and Graddy (1980) and Utterback (1975, 1978)[1]. This literature incorporates two different approaches for explaining the producer 'shakeout'. The first approach is discussed mainly in Utterback (1975, 1978), Utterback and Suarez (1993) and in Klepper and Graddy (1990), and relates the producer 'shakeout' to an exogenous technological progress that changes the competition conditions in the industry, which in turn affects firms' survival hazard. This event usually occurs after the completion of the first phase of the industry evolution process, in which there are intense product innovation and high entry. Thus, once the exogenous technical change has occurred, the technological opportunities change and those firms with higher learning capabilities and enhanced scale economics

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[1] See also Klepper and Miller (1995), Klepper and Simons (1999) and MacDonald (1994).

(due to being larger producers) can better exploit the new opportunities (which are mostly supply oriented) and eventually become dominant, while the others are forced out of the industry. A similar viewpoint of this matter is presented in Jovanovic and MacDonald (1994), which is consisted of a partial equilibrium analysis of a radical exogenous technological change that allows massive cost reduction of those firms that are able to exploit the new opportunities and force out of the industry those firms who produce in the old-fashioned know-how.

The second approach is introduced in Klepper (1996)[2], who suggests a model in which the drastic innovation occurs endogenously as a product of the firms' strategic R&D investment. Firms are able to invest in either product or process R&D, while their decision is based upon the return on each investment that evolves endogenously with industry's structure. By investing in product innovation firms, which are endowed with randomly allocated capabilities, can lower the cost of production. Thus, larger firms derive higher return on process innovation then smaller ones. Since the model does not assume a dominant design, firms can attract new costumers, which are willing to pay more for the product, by investing in process innovation and improving the brand's quality. This brand becomes the standard product in future periods and due to its higher qualities its market share increases. As the industry grows the incentive to invest in process R&D increases since cost reduction can be applied over higher production. There exists a production threshold in which after reaching it firms stop investing in product innovation and coordinate their innovative efforts only to process R&D. The producer 'shakeout' occurs at this stage whereas firms that failed to reach this threshold eventually exit the industry.

In this paper I offer an alternative approach foe explaining industry 'shakeout' by focusing on the incentives to share knowledge among firms in the emerging industry. I argue that strong incentive to diffuse knowledge at the early periods allows for intense product innovation, rise in market value and high survival hazard. The incentive to diffuse knowledge during the early periods is mainly associated with the desire of encouraging the development of ancillary industries, which I relate to as the *market expansion effect.* This causes a positive production externality that results with high knowledge flows. The exhaustion of the market expansion effect as industry matures discourages knowledge sharing, which reduces knowledge externalities, resulting with a drastic fall in product innovation, market value and survival hazard.

Using dynamic programming methods, I study firms' incentives to diffuse technologies and argue that positive incentives to share knowledge

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[2] See also Klepper and Simons (2000).

enable less-capable firms to survive in the developing industry. These incentives are dynamic and evolve with respect to the relevant payoffs a firm has from protecting or diffusing its knowledge.

The novelty of this paper is in its effort to explain industry evolution and growth by studying simultaneously the dynamics to innovate and to share knowledge. Stressing the importance of endougeonizng knowledge externalities may have important impact on endogenous growth theories that mainly assume an exogenous mechanism for creating knowledge spillover in the economy.

The rest of the paper is organized as follows: section 2 introduces the main stylized facts regarding industry life cycle familiar from prior studies, section 3 discusses the innovative effort and intuition, section 4 introduces the model's building blocks, section 5 deals with the dynamic game, section 6 describes the numeric methodology, section 7 offers a discussion regarding industry's evolution trajectories, section 8 is the numerical example and section 9 summarize and concludes.

2 Stylized facts and Empirical Findings

The stylized facts regarding industry dynamics familiar from prior studies are summarized in Klepper (2000)[3]: at the early periods of the emerging industry there is a small number of firms, prices are very high and production is low. During this stage we observe entry expansion and an increase in production, which command price reduction. The entry expansion can follow two patterns: entrants' number may rise over time or it may attain a peak at the beginning and then decline, however, in both cases the number eventually becomes small. In the following periods, output growth persists, but at lower rates then the average firm size, which forces firms out of the industry. This phenomenon is known as producer 'shakeout', which is caused by a severe reduction in survival hazard.

A good illustration of a producer 'shakeout' can be seen in the evolution pattern of the automobile tire industry in the US between the years 1906-1973, as discussed in Klepper and Simons (2000) and in Jovanovic and MacDonald (1994,2).

In 1906 there were only 10 firms producing at the industry, in 1922 the number increased to 275, then made a drastic fall to 132 (in 1928 – before the great depression), and since then made a gradual reduction and reached about 30 firm in 1973. Total industry output made a sharp non-monotonic rise during the sample periods and the price made a sharp

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[3] See Gort and Klepper (1982).

non-monotonic drop during that period. By looking at demand data one cannot give a full explanation to the producers 'shakeout' occurred in the tire industry in 1928. In order to properly understand this industry evolution pattern we ought to look for substantial technological change that reduced production costs and enhanced industry's concentration, prior the "shakeout". Indeed, a drastic invention in the US tire industry is evident just prior to the producer 'shakeout'.

Simmons (2001) estimate survival hazard of firms in the tire and television industry in the UK and the US. The main findings suggest that prior the 'shakeout', survival hazard rises until it reaches a peak, just before drastically falling, roughly a decade after the industry has emerged, anticipating the mass firms exit during the producer 'shakeout' phase. This finding is consistent with that of Klepper and Simons (2000) with regard to firm's value pattern in the automobile tire industry, where during the entry periods firms' relative market value rose sharply and the exit periods were accompanied with an even sharper drop in relative firm values. Their findings show a gradual increase in market value from 1906 to1916 and then a sharp increase, such that in only three years the market value nearly doubles. However, at this point an acute reduction occurs bringing the relative market value to levels of nearly a third of what they were during their pick (in 1919). Furthermore, producer 'shakeout' occur right after the market value 'shakeout' with the number of firms dropping from their 1922 peak to roughly a one sixth of this level in 1935, while during the periods the market value was rising high entry was taking place.

The findings of this paper are able to reconstruct this pattern by focusing on the incentive to share knowledge among firms during the industry life cycle.

This strong correlation between market value 'shakeout' and producer 'shakeout' allows me to focus the analysis on survival hazard (which is assumed to be an increasing function of market value) and not on a formal analysis of entry and exit decisions.

Furthermore, Gort and Klepper (1982) supply a comprehensive analysis of 46 industries and report the following technological pattern: just preceding the shakeout there was a rise in innovation effort that reinforced the barriers of entry and compressed the profit margins of the less efficient producers, which began to have great difficulties to imitate the technologies of the leading firms. Consequently, the exit rate rose sharply until the less efficient firms were forced out of the industry.

An illustrative example of an industry that experienced a remarkable evolution pattern is the automobile industry, which is the most important one in the 20th century, in particular that which occurred in the United States. Its development affected almost every aspect of consumer welfare and its performance serves as the best indicator to the state and health of the economy.

In the year of 1897 a number of automobile brands began to appear in the market. During the years 1900 to 1910, which are known as the pre-"shakeout" era, motor-vehicle production in the United States rose from 4,000 to 187,000 and automobile registrations from 8,000 to 469,000. In early years, the automobile industry was characterized by high growth rates, large number of firms, high knowledge diffusion and development of ancillary markets.

The steady growth in motor-vehicle producers was interrupted by the introduction of the T Ford vehicle that triggered the structural change of the industry. The production of the T model was based on the assembly line and mass productions techniques. The dramatic change in the quantity and prices and in the high investment a firm should have made in order to produce in mass production techniques caused a great reduction in the number of firms operating in the industry and left the industry to be occupied by only few companies, which engaged in oligopoly competition. The great shakeout occurred in the beginning of the second decade of the twentieth century. The total number of vehicle produced was about 225,000 at 1910 and about 2,500,000 at 1915, only five years later. The medium-price range car was about \$1500 at 1903 and decreased dramatically to \$360 at 1916. Henry Ford and his model T cars evolved to be the major producer of the low-priced car that was aimed to common use by the regular family in the United States. The mass market for the automobile product was in the price range of \$600. Before implementing the techniques of mass production this price could not had been reached. However, by investing in process R&D and putting the emphasis on developing new methods of production Ford was able to reach the market of mass production and become the leading producer in the automobile industry.

The dynamics of the incentive to share knowledge largely lies upon the scope of the *market expansion effect*, i,e., the desire to encourage the development of supporting products. This effect is best demonstrated again in the automobile industry. As the automobile industry continued to grow and develop many industries were affected and evolved as well. In the first ten years of the twentieth century, automobile manufacturing climbed from 150th to 21st in the value of products among American industries. The main industries that were affected were steel, petroleum, tires the oil industry and many more. The steel industry had changed the direction of production and had to invest in new machinery that allowed for the development of the products demanded by the automobile industry (such as the process of alloy steels).

The petroleum industry was revolutionized. Before 1900 only about one-tenth of the petroleum refined was converted into gasoline. Gasoline was regarded as an undesirable waste product and thrown away. The evolution in the demand for gasoline caused great increase in crude-oil production, such that total crude-oil production rose from 60 million barrels in 1900 to 250 million in 1910.

The tire and rubber industries faced a similar evolution pattern. The high demand for tires encouraged firms in the tire industry to conduct intensive R&D to lower production costs and to increase their product characteristics. Other many industries have evolved as well: the garage, the service stations, the repair shops, the automobile supply shops and gasoline pumps.

The evolution of those industries was a necessary condition to the further development of the automobile industry. Therefore, I argue that in this stage of the industry life cycle firms in the automobile industry had positive incentives to share their knowledge with their rivals in order to increase total production and to strength the aggregate demand for the ancillary industries, which encourages them to increase their production and improve its quality by investing more in R&D.

2.1 Knowledge Diffusion and Industry Evolution

A good observation for endogenous knowledge flows is the patenting activity of firms. Gort and Klepper (1982) report that patents number does not fall as industry matures, however, the quality of patents may be higher at the early periods. By this one can conclude that the incentive to share knowledge did not decrease with industry's age. However, one can argue that we observe a decline in patents quality as the industry matures, suggesting that higher effort is put in order to protect less important inventions that would not have been protected in the early periods of the industry life cycle.

Two interesting observations regarding patenting activity are to be found in the automobile industry, where the first is the famous Selden patent, which amphasize the nature of patent as dettering innovations, while the second is knowledge sharing agreements among firms in the form of cross-licensing and patent pools.

2.1.1 The Selden patent

Selden was an excellent engineer who realized the potential of the gasoline engine, which weighted almost half a ton per horsepower, to be used in highway vehicle with only small refinement. He and Wiliiam Gomm, who was a mechanist, built a three-cylinder type engine with a ratio of 185 pounds per horsepower. The engine failed to run more then five minutes, or on more then one cylinder, however it was enough for him to apply for a patent in 1879, claiming novelty in the combination of a liquid hydrocarbon engine with other elements of his vehicle such as clutch, steering gear and so on. Patent law in that time allowed a delay in the issue of a patent. Selden used this law to delay the issue of his application for 16 years after his initial application. The patent was granted in November, 5, 1895, and in registered as a United States patent number 549,160. The deliberate delay in the patent issue was due to the lack of developed car industry in the United States. Selden waited for the car industry to develop and therefore, to increase his patent value. In the sixteen years from his application till his patent was issued he updated his application with the technological progress in the area in Europe. Although Selden understood the potential value of his patent, he was unable to benefit much from it. Lack of finance and ability to exploit his patent forced him to sell his patent rights to a group of Wall-Street investor for only $10,000.

The buyers of the Selden patent were not capable of manufacturing gasoline engine cars with success; however, they had used this patent as a monopoly power on the gasoline engine car. The patent led to many legal charging infringement. Before any decisions regarding the legal issues were made, negotiation between the companies led to agreement in 1903, placing control of the patent in the hands of an Association of Licensed Automobile Manufacturers (ALAM). The participants of this agreement used this arrangement as entry deterrence of new firms. The most famous and important producer who was not granted the right to manufacture in the industry by the ALAM was Henry Ford. Although Ford did not believe in patents, he tried to get a license from the ALAM and was refused on the grounds of not demonstrating sufficient ability of producing cars. Ford and others decided to fight the patent against the ALAM. The battle dragged in courts for many years, till the year 1911 when the Circuit Court of Appeals held that Selden patent did not cover the new technology that was then in possession. This was the end of the ALAM and the beginning of the mass production era in the United States automobile industry that ranked Ford as the main car manufacturer.

The main conclusion we can draw from Selden patent is that knowledge can be used as entry deterrence and maintain monopoly power in an industry. However, the fact that Ford and others that did not believe in patents won the battle for free production implies that knowledge diffusion among firms in the emergence of industries is the driving force of further evolution and could not be discouraged by patents and other mechanisms that suppress innovative efforts.

2.1.2 Patent "Pool" (Cross-Licensing)

The Selden patent was an example for the use of knowledge as a way to preserve monopoly power. Another example of the use of knowledge is the cross-licensing agreements among firms that were adopted in 1915. All patents, except those embodying a major technical change, were to be made freely available to the participating companies one year after issuance. This cross-licensing agreement is an empirical evidence for endogenous knowledge diffusion. Firms chose to share their new knowledge with other firms because they believed that higher knowledge sharing would induce the market size and therefore, increase profits for all firms, in addition to enhancing knowledge externalities in the industry that can allow for cheaper R&D costs.

3 Motivation and Intuition

The major concern of this paper is the dynamics of firm's innovative efforts based on their knowledge disclosure incentives.

The incentive to share knowledge is the product of endogenous change in industry's structure, which affects the economic factors that determine the private and social returns on the R&D investment. I use a dynamic programming framework to analyze knowledge externalities and endogenous technology diffusion. In addition, I extend this framework to study games of asymmetric information regarding the level of knowledge spillover and R&D output.

The factors that determine firm's incentive to invest in R&D are the private and social return on the innovative project. The private return depends upon the appropriability and opportunity conditions in the industry.

I argue that appropriability conditions are determined endogenously by the firm who chooses the extent of knowledge protection[4]. Social return on the innovative investment is assumed to be higher then the private one and is a function of their interaction (the magnitude of the

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[4] A firm can choose many ways to protect or diffuse its knowledge. For example, patents, secrecy or innovation nature could be used for this purpose. Furthermore, unofficial meetings and appointments with rival scientist can generate endogenous knowledge flows among firms.

social return is a function of the knowledge spillover, which is endogenously determined by the innovating firm).

There are both negative and positive incentives to diffuse new technologies among firms. I begin by introducing the positive incentives and continue by reviewing the negatives ones (in each incentive introduced I refer to whether it is demand or supply oriented).

The first positive incentive to share knowledge relies on the demand side. This concept deals with endogenous market size, which evolves as a function of the aggregate knowledge in the industry. In the early periods the market size is small, the knowledge stock and the number of firms is relatively low and there is a little interaction between the firms. After a number of periods we observe an increase in the aggregate knowledge stock and higher demand for the differentiated product. At this phase, firms realize that an increase in the quantity produced by their rivals induces the size of the market by enlarging complementary industry's output and increasing the recognition of their products to wider potential consumers. For example, in the early ages of the automobile industry the development of complementary industries such as concrete, asphalt and fuel refining was a necessary condition to the further development of the automobile industry. The incentive for the development of those complementary industries concealed in the demand for their products . Therefore, each firm in the automobile industry has incentives to increase its competitor's output by improving its production ability, i.e., by sharing knowledge and promoting technology diffusion in the industry.

The second positive incentive also relates to the demand side. This incentive occurs in situations when an increase in output of one firm increases the demand for the other firm's product. A good example to these kinds of situations is that of *'General Purpose Technologies' (GPT)*[5]. In those models the GPT developing firm has positive incentive to diffuse its technology among the applicants firms, in order to increase the demand to its product at the expense of the older technology.

The third positive incentive relates either to the demand, or the supply sides. A firm cannot continue further developing its knowledge unless there is sufficient complementary innovations. For this matter, distinct between invention as advancing the state of the art know-how, and innovation which is merely refinments of the advanced technology which is made within the technology frontier. I define this externality as a *sequential innovation*, which can be illustrated in the automobile industry. Assume that the state of the art knowledge lies within the engine technology. In this case, further improvements of the engine

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[5] See for example, Trajtenberg and Bresnahan (1995), Trajtenberg and Helpman (1998), Trajtenberg and Rosenberg (2001).

quality, such as higher horse powers, could not be done without sufficient complementary innovations in car safety. In this example, invention and innovation are complementary, however independent. One can think of an alternative interpretation such that in order to improve the quality of engines, some refinments must be implemented by firms that posses lower innovative capabilities. In this case, invention and innovation are knowledge dependent.

The forth positive incentive to expose knowledge to competitors firms relies both on the supply side and the demand sides. The firm benefits from the externalities of other firms knowledge. Higher knowledge spillover enhances knowledge development and therefore, increases the externalities in the economy. This process makes it beneficial for the firm to share its knowledge with other firms and increase the knowledge externalities in the economy.

The negative incentive to share information relies upon the competition advantage the new knowledge gives to the developing firm. This incentive relies both on the demand and supply sides. When this knowledge is exposed to other firms and is imitated by them the competition advantage disappears.

The return on R&D investment can come both from the demand side and the supply side. On the demand side, firm observes an outward shift in the demand curve it faces because of the increase in quality per price it offers after the increase in its knowledge (in the case of product innovation). When knowledge spills to other firms those firms increase their quality per price as well and therefore, the developing firm observes erosion in the profits and in the return on the R&D investment. In the case of supply side innovation (i.e. process innovation) the firms faces a downward shift in the supply curve without any change in the demand curve. The price per unit decreases and the firm is able to increase its market share and its profits. When knowledge spills to the competing firms they are able to reduce their price per unit of quality and therefore, the advantage the developing firm has had decreases and its profit increase shrinks.

In the model discussed below I chose to focus on two positive incentive to share knowledge, which are the *market expansion effect* and sequential innovation. The negative incentive to disclose knowledge is merely the competitive one, that is associated with scarifying the advantage the successful innovation mat offer in the product market.

4 The model

I consider an infinite horizon interaction game between firms in a core industry, denoted by a superscript c, for a differentiated product denoted by J, firms in an ancillary industry, denoted by a superscript a and consumers who purchase products from both industries. The main focus is on the dynamics of the incentives to invest in R&D and to diffuse knowledge in the core industry, which underpin its evolution pattern.

The rest of this section introduces the model's building blocks including the firms and consumers primitives, and the basic set-up of the dynamic game.

4.1 Building blocks

4.1.1 The consumers

There are M consumers in the economy that allocate their income between product J (the core product), product a (the ancillary product) and product Z (the outside product). Each consumer is assumed to purchase one unit of a unique brand of product J, while given this consumption she must purchase $\varphi(\tau)$ units of product a, whereas $\varphi'(\tau) < 0$ and τ is the efficiency level of using product a in the consumption of product J (product J could be regarded as an automobile and product a as fuel, such that τ could stand for the number of gallon consumed per mile, or alternatively, J could be the printers market while a is the tuner ancillary industry, while in this case, τ will account for the number of papers printed with one unit of tuner). The consumer does not derive direct utility from purchasing the product a.

The consumer uses the rest of her income on purchasing product Z. I disregard savings and consumers' preferences dynamics, i.e., consumers maximize each period a static utility function under an inter-temporal resource constraint.

There are N unique brands of product J that differ in the quality of their h constant components. Let $Q_n = q_{n1}, ..., q_{nh}$ be the component quality vector of brand $n \in N$. Define $U(Q_n) = U[u(q_{n1}), ..., u(q_{nh})]$ as the value the consumer places on each brand as a function of its quality, where, $u'(q_{nl}) \geq 0$ and $u''(q_{nl}) \leq 0$ for all $l \in [1, h]$. Further, let $\omega_n = \omega_n(\omega_{n1}, ..., \omega_{nh})$ be the vector of the knowledge stock indices that is associated with each of the product attributes quality vector. Therefore, we can rewrite the consumers' utility from consuming product J as $U[Q_n(\omega_n)] = U\{u[q_{n1}(\omega_{n1}), ..., u[q_{nh}(\omega_{nh})]\}$, where $q'_{nl}(\omega_{nl}) > 0$ for all $l \in [1, h]$. Thus, the utility of consuming product J can be simply written as $U(\omega_n) = U[u(\omega_{n1}), ..., u(\omega_{nh})]$, $u'(\omega_{nl}) \geq 0$ for all $l \in [1, h]$.

Since I study the evolution of an industry for a differentiated product, I follow the theoretical literature of the multinomial logit to characterize consumers' behavior. The choice process is composed of two stages: at the first stage the consumer chooses the portion of her income, which is devoted to each of the three products, while at the second stage she chooses the unique brand $n \in N$ she will purchase. Consumers are assumed to maximize the following random utility function under the resource constraint,

$$\max_{Z,,n\in N} U_i = U[(J, \varphi(\tau), Z)|\tau] + \epsilon_i \quad ; \quad s.t. \quad P_n + \varphi(\tau)P_a + Z = Y \tag{1}$$

Where ϵ_i is the unknown idiosyncratic taste of consumer $i \in M$, P_J is the price of product J, P_a is the price of the ancillary product, the outside product's price is normalized to unity and Y is the income.

Since the consumers maximize a random utility function that depends upon the distribution and realization of consumers' tastes, the demand function must be probabilistic, conditional on τ and the known components of the random utility.

The consumer is assumed to purchase one unit of product J, therefore, she spends the amount of $\overset{*}{Z} = Y - P_n - \varphi(\tau)P_a$ on the outside good. Substituting it for Z in equation 1 yields the following indirect utility function:

$$V_i = V[(J, Y - P_n - \varphi(\tau)P_a)|\tau] \equiv V + \epsilon_i = \underset{Z}{max}\, U_i \tag{2}$$

In the second stage the consumer maximizes the indirect utility function with respect to the specific brand:

$$\max_{n\in N} V_i = V[(J, Y - P_n - \varphi(\tau)P_a)|\tau] \tag{3}$$

Given the discreteness of J, brand n will be chosen iff $V_n \geq V_k$ for all $n \neq k \in N$, that is, iff

$$V_{n-k} = \{V[(n, y - P_n - \varphi(\tau)P_a)|\tau] - V[(k, y - P_k - \varphi(\tau)P_a|\tau] \geq (\epsilon_n - \epsilon_k) \tag{4}$$

$$\sigma_n = \Pr(n \in N|\tau) = \int_{-\infty}^{\infty} \prod_{n\neq k} F(\epsilon_n + V_{n-k}) \cdot f(\epsilon_n)\, d\epsilon_n \tag{5}$$

Assuming the residuals are of the type I extreme-value (or Weibull) distribution, then σ_n will be logistic and the exact form is given by[6]

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[6] If, for example, the residuals are normally distributed, then σ_j is the cumulative normal, and the resulting model is the Probit.

$$\sigma_n = \frac{\exp(V_n)}{\sum_{k=1}^{N} \exp(V_k)} \qquad n = 1, ..., N. \tag{6}$$

Assuming a linear indirect utility function and substituting into equation 6 yields the following market share equation (the probability to purchase a specific brand given its unique attributes quality vector), which is independent of income:

$$\sigma_n = \frac{\exp(u_n(\omega_n) - p_n - p_a\varphi(\tau))}{\sum_{k=1}^{N} \exp(u_k(\omega_k) - p_k - p_a\varphi(\tau))} \qquad \text{For } \; n = 1, ..., N. \tag{7}$$

4.2 The Core Industry

Define $\overline{N}$ as the maximal number of brands that could be active in the market and $N_t \leq \overline{N}$ as the number of brands available at period t (for simplicity, I omit the superscript t in the following representation). As discussed above, $\omega_n = \omega_n(\omega_{n1}, ..., \omega_{nh})$ assigns the required knowledge level indices for the production of brand n. Further, denote by $\omega = \omega(\omega_{1,...,}\omega_{\overline{N}})$ the knowledge level vector which is required for the production of each brand type, such that $\omega_i > \omega_j$ if $i > j$ for all $i \neq j$ and $\omega_i = 0$ if $N_{jt} < \overline{N}$ for all $\overline{N} > i > j$ and t.

In each period there is a constant pool of potential entrants, such that at the beginning of the period their allocation among all the active brands is determined. Given this allocation, denote by S_n the set of firms that are active in producing brand n for all $n \in [1, N]$. For simplicity, normalize S_n to unity for all n and t.

Given the set-up I described above, firms act to maximize their infinite discounted expected cash-flows (or put in an alternative manner, maximize their value, which is calculated as the current period payoffs and the discounted expected value in the following period). The maximization problem is solved in two steps by maximizing the current period payoffs and the discounted expected future payoffs. I proceed by describing firms' behavior in each step.

4.2.1 Inter-temporal strategies

The inter-temporal decisions are those that aim to maximize firm's profits in the current period. Since I assume firms are identical in each brand category, I characterize the equilibrium in the product market as one that is achieved through a Nash-Cournout interaction game between a representative firm from each brand type.

Caplin and Nalebuff (1991) show that in a market with a bounded set of operating brands (representative firms), denoted by N, no fixed costs of production, constant marginal costs equal to mc and firms choose prices to maximize profits, a unique Nash equilibrium exists and satisfies the following vector of first order conditions:

$$[-p_n - mc]\,\sigma_n\,[1-\sigma_n] + \sigma_n = 0 \tag{8}$$

for $n = 1 \ldots\ N$, and σ_n is calculated from equation 7.

The profits are given in this case by:

$$\pi(\omega_{n,}\omega_{-n}) = M\sigma_n(\omega_n, \omega_{-n})\,[p_n - mc] \tag{9}$$

Equation 9 presents the current period payoffs for each brand category, given the knowledge stock vector ω and the associated intertemporal strategies of prices and quantities[7].

4.2.2 Intra-temporal strategies

Firms' intra-temporal decisions aim to maximize the expected value next period by optimizing their R&D investment and knowledge protection strategies. Further, the incumbent decides whether to remain active or leave the industry, and the potential entrant decides whether to enter or remain outside. I discuss the nature of these strategies and their computation methodology in the following sections, and focus for now on the building blocks of the dynamic game.

A firm can increase its knowledge stock by investing in risky ventures, such as R&D. This investment generates return by improving the knowledge stock allowing capturing higher market share. However, once knowledge has been invented, it is available freely to all the firms in the same brand category. I disregard issues of free riders and competition in the same brand type.

Assume heterogeniety in innovative capabilities among brands. Define invention as increasing the "state of the art" knowledge stock, and innovation as the creation process of complementary knowledge or refinments of the last invention. Thus, define $\underline{n}$ as the sub-industry in which brand n is the most advanced. In this sub-industry an invention can be

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[7] However, note that in order to compute the market share the ancillary knowledge stock must be taking into account.

carried out only by firms producing brand n, while an innovation in can be performed by any of the other firms[8].

The future period knowledge stock evolves as following (I omit the brand index for convenience):

$$\omega_{t+1} = \omega_t + \zeta_t - \varrho_t \tag{10}$$

Where, ω_{t+1} is the knowledge stock next period, ζ_t is the previous period R&D stochastic output that receives the value 1 with probability $P(\zeta_t = 1|x_t)$ and zero with probability $P(\zeta_t = 0|x_t)$, for $x_t \in R_+$, where x_t is the R&D investment in period t, and ϱ_t is an exogenous shock that affects the knowledge level in the core industry and receives the value 1 with probability δ and the value 0 with probability $1 - \delta$.

Further, $\omega_{t,n} < \omega_{t,n+1}$ for all $n \in [1, N-1]$, thus, the highest technology frontier of brand n at period t is $\omega_{n+1} + 1$ (in case of successful innovation when this is feasible for brand $n + 1$). Given this set-up, n is the technological leader brand in the subindustry $[1, n]$, while N is the technological leader brand in the whole core industry.

Advancing the state of the art knowledge in each subindustry is feasible only with the presence of a sufficient complementary knowledge that is generated by the less advanced firms. For simplicity, I assume that a sufficient complementary knowledge could be achieved in the presence of one successful innovation in the given state of the art knowledge in the subindustry, or put alternatively, advancing the "state of the art" technology demands at least one refinement of the most advanced technology. Let I be an indicator vector that assigns the value one to each sub-industry that such a complementary innovation has occurred in its history, and zero otherwise.

To summarize, denote by a superscript l the technological leader and by a superscript f the technological follower for every sub-industry, the evolution process of the state of the art knowledge stock ω is given by[9]

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[8] An invention in the subindustry $\underline{n}$ is regarded as innovation in the subindustry $\underline{n+k}$, for all $k \in [\overline{N} - n]$.

[9] This is obviously true for $\omega^n \neq \omega^{n+1} - 1$. In case the equality holds, an increase in the knowledge stock of the less advanced firm is possible only with advancing the state-of-the-art knowledge within the subindustry $n + 1$.

$$P(\omega^{l}{}_{t+1} = \omega^{l}{}_{t} \big| x^{l}{}_{t}, x^{f}{}_{t}, \delta, I) = [P(\zeta^{l}{}_{t} = 1 \big| x^{l}{}_{t})\delta + P(\zeta^{l}{}_{t} = 0 \big| x^{l}{}_{t})(1-\delta)]I$$

$$+ (1-\delta)[1 + P(\zeta^{f}{}_{t} = 1 \big| x^{f}{}_{t})P(\zeta^{l}{}_{t} = 0 \big| x^{l}{}_{t})](1-I)$$

$$P(\omega^{l}{}_{t+1} = \omega^{l}{}_{t} + 1 \big| x^{l}{}_{t}, x^{f}{}_{t}, \delta, I) = [P(\zeta^{l}{}_{t} = 1 \big| x^{l}{}_{t})(1-\delta)]I$$

$$+ (1-\delta)[1 + P(\zeta^{f}{}_{t} = 1 \big| x^{f}{}_{t})P(\zeta^{l}{}_{t} = 1 \big| x^{l}{}_{t})](1-I)$$

$$P(\omega^{l}{}_{t+1} = \omega^{l}{}_{t} - 1 \big| x^{l}{}_{t}, x^{f}{}_{t}, \delta, I) = \delta[(1-I) + P(\zeta^{l}{}_{t} = 0 \big| x^{l}{}_{t})I]$$

Where the complementary knowledge evolves as

$$P(\omega^{f}{}_{t+1} = \omega^{f}{}_{t} \big| x^{f}{}_{t}, \delta) = P(\zeta^{f}{}_{t} = 1 \big| x^{l}{}_{t})\delta + P(\zeta^{f}{}_{t} = 0 \big| x^{l}{}_{t})(1-\delta)$$

$$P(\omega^{f}{}_{t+1} = \omega^{f}{}_{t} + 1 \big| x^{f}{}_{t}, \delta) = P(\zeta^{f}{}_{t} = 1 \big| x^{f}{}_{t})(1-\delta)$$

$$P(\omega^{l}{}_{t+1} = \omega^{f}{}_{t} - 1 \big| x^{f}{}_{t}, \delta) = \delta P(\zeta^{l}{}_{t} = 0 \big| x^{l}{}_{t})$$

And the probability of successful innovation of brand n is given by

$$P(\zeta_t^n = 1 | x_t^n) = \frac{\alpha_n x_n}{1 + \alpha x_n} \tag{11}$$

A firm producing brand n can influence the R&D efforts of it its $n-1$ rival brands by diffusing its knowledge. Define the R&D costs of n_i by $c_i \left(\sum_{j=i+1}^{N} f(\theta_j, \omega_j) \right)$, where θ_j is the knowledge drifts from brand j. Thus, by diffusing knowledge a firm can diminish the R&D costs of its inferior rivals and enhance their innovative efforts.

Knowledge flows are decomposed into two components as

$$\theta_{-i} = \widehat{\theta}_{-i} + \widetilde{\theta}_{-i} \ ; \ \ for - i \in [i+1, N]. \tag{12}$$

The first component is the controlled knowledge flows, which are determined endogenously by the firm (e.g., patents, secrecy etc.), while the second is the uncontrolled knowledge flows that cannot be coordinated by any firm.

The distinction between controlled and uncontrolled knowledge flows is highly important in view of firms' optimal innovative efforts and the full exploitation of the positive knowledge externalities in the industry[10].

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[10] Spence (1984) discusses the possibility of achieving the optimal level of aggregate R&D in the social planner view, when firms are not aware of the uncontrolled component of the knowledge diffusion.

Knowledge Diffusion Beliefs Define θ_i^e as the expectations of firm i with regard to the knowledge spilled to its rival brands, which is equals in the complete information model to $\widehat{\theta}_i + \widetilde{\theta}_i$ and in the incomplete information model to $\widehat{\theta}_i$.

I study two formations of knowledge spillover beliefs. The first model deals with complete information regarding the knowledge flows and R&D realizations. In this model all firms share the same information set. The second model is of incomplete and asymmetric information. In this set-up, firms do not realize there is an uncontrolled knowledge spillover component, such that they believe that once they chose to protect their knowledge it is not possible for it to transfer to their rivals. Further, it takes one period to discover the output of the rival's R&D project (the output of the less advanced brands are not known before deciding on the optimal R&D investment), nonetheless, the R&D investment is known by the more advanced firms.

I choose to study industry dynamics under these two information sets due to their possible impact on the aggregate R&D and on consumer welfare (among other variables). A model with incomplete information might imply different outcomes then a model with complete information.

I focus on the intuitive results these two models may generate with regard to endogenous knowledge flows and aggregate R&D investments. First, since in the incomplete information model a firm believe it has full control over its knowledge, we would expect to observe more endogenous knowledge flows in periods with strong incentives to diffuse knowledge. In the pre-'shakeout' era, as is explained below, there is stronger incentive to share knowledge due to the stronger market expansion effect. Thus, higher knowledge flows are resulted with increased innovative efforts of the less advanced firms. However, since knowledge externalities are a function not only of the desire to share knowledge, but also of the knowledge stock level in the economy, the answer is not straightforward. In order to determine the effect of incomplete information on the aggregate R&D and consumers' welfare we ought to study the progress of the advanced knowledge in the industry. Since in the incomplete information model there is more uncertainty on behalf of the advanced firms, their optimal R&D level could be reduced, and by this decrease the knowledge externalities in the economy. Thus, the direction and size of the effect of the particular information set on the performance of an industry should be analyzed in the context of the specific model's parameters.

Entry and exit In this paper I do not offer a formal analysis of entry and exit of brands and firms, but discuss qualitatively a simplified entry and exit characterization.

Upon entry each firm in endowed with initial random capabilities. Assume that in order to produce a brand of quality n, the minimum level of capabilities is C_n. Given its capability, a firm can choose to enter any brand type in the interval $[1, N]$. Once choosing the field of expertise, i.e., which brand to produce, the firm acquires the production knowledge of the chosen brand and starts producing in the following period. Further, firms in the same brand category are assumed to be identical, i.e., with homogenous knowledge stock and innovative capabilities.

A potential incumbent faces two decisions prior entering the industry. The first is simply whether to enter, while the second is which brand to produce. More formally, denote by Ψ the entry decision indicator, which receives the value of one when entry occurs and zero otherwise, let V_j be the continuation value of producing brand $j \leq n$, while n is the capabilities level, and Φ is the value of staying outside the industry. In this case, Ψ receives the value 1 $iff \;\; V_{j^*} > \Phi$, whereas, j^* is the optimal brand of production, and zero otherwise. Choosing in which brand to specialize, a firm with capabilities level n will choose brand $j \; iff \; V_j > V_i$, for $i, j \in [1, N]$. In this paper and in particular in the numerical example presented in section 7 I do not focus on formal entry and exit decisions, but study a simplified model, in which firms will enter the industry as long as the continuation value is higher then a given threshold and exit in the opposite case[11].

4.2.3 The Ancillary Industry

The ancillary industry produces the product a which must be consumed together with the core industry's product J. There is one ancillary firm, denoted by a superscript a . Firm a is engaged only in production and in R&D investment, so to advance the ancillary knowledge stock, denoted by τ.

The future period knowledge stock evolves as following:

$$\tau_{t+1} = \tau_t + \zeta_t^a \tag{13}$$

Whereas, τ_{t+1} is the knowledge stock next period, ζ_t^a is the previous period R&D stochastic output that receives the value 1 with probability $P(\zeta_t = 1|x_t^a)$ and zero with probability $P(\zeta_t = 0|x_t^a)$ for $x_t^a \in R_+$.

Thus, the evolution process of the knowledge stock τ is given by

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[11] A more complex model of entry and exit can be employed in this framework (see for example, Erickson and Pakes, 1995), however this paper is mainly concerned with changes in the continuation values and survival hazards.

$$\tau_{t+1} = \begin{cases} \tau_t \ ; & P(\zeta_t = 0|x_t^a) \\ \tau_t + 1; & P(\zeta_t = 1|x_t^a) \end{cases} \tag{14}$$

Whereas the probability of successful innovation of is given by

$$P(\zeta_t^a = 1|x_t^a) = \frac{\alpha_a x_a}{1 + \alpha x_a} \tag{15}$$

Assume industry a is a pure monopoly, however, there is a constant threat of entering and engaging in oligopolistic competition. Therefore, although firm a operates as a monopoly, it cannot set the optimal monopolistic price since in this case the return from entering the industry will increase and the entry barrier will collapse. Denote by $\overline{P}_a$ the highest price the incumbent can set so that to support entrance deterrence. In this case, the incumbent profits are represented by

$$\pi^a(\omega, \tau, I,) = D(\omega, \tau, I)(\overline{P}_a - mc_a(\tau)) \tag{16}$$

With $D(\omega, \tau, I)$ being the demand curve facing firm a, which equals to the aggregate production in the industry J, computed as $\sum_{n=1}^{N} \left[M\sigma_n \left(\omega_n, \omega_{-n} \right) \right]$. Furthermore, $mc_a(\tau)$ is the marginal production function, and $mc_a'(\tau) < 0$.

Given this specification, firms a obtains return on its innovative efforts from two sources. The first is the demand side, which is the increase in the demand for the core industry product and for the ancillary product. Indeed, by increasing τ consumers will use less units of a per unit of J, however, for a relatively high demand elasticity for product J the demand for product a will increase. The second source of return is supply oriented and relates to the process innovation property of the ancillary firm that enables it to reduce production costs and increase its profit margins.

4.2.4 Sequence of events

Each period is characterized by a sequence of actions and events as follows: at the first stage of the period profits are determined for each brand category in a Nash equilibrium game in the product market in the core industry and the number of active brands is then determined based on the expected continuation value (however, in the numerical example presented in section I simplify the model not include entry and exit of brands).

At the second stage the leading firms decide whether to take measures to protect their knowledge or make it available to rival brands, based on their expectations regarding the relative payoffs for each strategy. Following this, knowledge flows take place.

At the third stage, the follower firms determine the level of their innovative efforts (while their costs depend upon the knowledge disclosure decisions previously made by the leading firms), which is then followed by the realization of the stochastic R&D they perform (which is a common knowledge in the full information model, and are known only to the less advanced firms in the incomplete information model). This realization reveals the knowledge stock levels of the follower firms in the next period (I assume the outcome of the R&D venture is known at this stage, however, the knowledge stock will be adjusted only in the following period. Further, at this stage it is also known whether the exogenous shock had hit the industry).

At this point, based on these realizations and the state space, the leading firms choose the level of R&D, which is then also followed by a realization.

Finally, the ancillary firm makes its decision with regard to its optimal level of R&D. The period ends with the realization of the stochastic R&D performed by firm a, which determines its knowledge stock level in the following period.

5 The Dynamic Game

The dynamic programming problem presented in this paper is too complex to be solved analytically. Therefore, numerical methods must be used. Ericson and Pakes (1995) first develop an algorithm that offers a computational solution algorithm to problems of this kind and Pakes & McGuire (1994) provide an algorithm to compute a Markov-perfect equilibrium. My own work applies and extends these ideas.

Several papers extended the EP model to account for changes in the static profit function, dynamic demand, mergers, multiple states per-firm etc. Fershtman and Pakes (2000) provide a theoretical analysis of a dynamic game with collusive prices that allow current price to depend on past prices. Benkard (2000) analyzes a model of learning by doing with a dynamic cost function and Markovich (1998) provides a theoretical analysis of a market with hardware-software connections while Byzalov (2002) provides a model of the OPEC cartel where with dynamic demand for oil commodities. Gowrisankaran and Town (1997) allow for two different types of firms, for profit and not for profit hospitals, and then investigate the likely impact of health policy changes on the evolution of the hospital industry.

In this paper I focus on the Markov strategies as defined by Maskin and Tirole (1988), which are analyzed in sub-game perfect equilibria where they are functions of only the relevant state variable (by this the enormous multiplicity that arises in dynamic models of this type is eliminated), and in which players' expectations coincide with the actual distribution of the random variables in the model.

In particular, the strategies depend upon the states $\{\omega, \tau, I\}$, whereas $\omega = \{\omega_1, ..., \omega_n\}$ is the knowledge stock of the firms in the core industry, τ is the knowledge stock in the ancillary industry, I is an indicator function that tracks the successful innovation in the given state-of-the-art technology (the indicator receives the value of 1 if a successful innovation occurred and zero other wise). Further more, each firms is assigned to be either a technological leader or a technological follower in every subindustry.

For firms in the industry c, knowledge diffusion is the function $\theta_i^c(\omega_i, \omega_{-i}, \tau, I)$, R&D investment is the function $x_i^c(\omega_i, \omega_{-i}, \tau, I)$, entry (for potential brand types) is the function $\chi_i^c(\omega_i, \omega_{-i}, \tau, I)$ and exit is the function $\psi^c(\omega_i, \omega_{-i}, \tau, I)$. For firm a, knowledge investment is the function $x_i^a(\omega, \tau, I)$.

Firms maximize their expected discounted value function (EDV) conditional on expectations about the industry structure. I focus on the interaction dynamic game between firms in the core industry, and between firm a and firms in the industry c. The main concepts that underpin this game are as following:

- Firms c in each brand $j \in J$ face a demand curve which is a function of the knowledge stock, ω, and the knowledge stock of firm a, τ, that affect the relative desirability of product J with respect to the outside product Z.

- Firms c realize their ability to increase the aggregate demand for product J by enhancing the incentives of firms a to invest in R&D in order to upgrade their knowledge stock τ, and by this affect the consumer's income allocation in favor of product J. In this set-up there is a positive production externality, in addition to the well-known knowledge externality. This externality is taken into account in the firms' knowledge diffusion strategy, so that by diffusing knowledge to firms in rival brands this externality is enhanced, although it is at the expense of the firms' relative market share.

  I refer to the positive production externality as the *market expansion effect* and this concept plays a crucial rule in simulating the dynamics of the industry evolution pattern.

- In this framework I also focus on innovations which are sequential or complementary. A sequential innovation is a type of R&D project that relies upon prior successful innovation, such that, without the previous innovation the current one cannot take place, i.e., the "state of the art" technology ought to be refined at least once before further improving. I refer to this externality as a *sequential innovation,* in which a firm benefits from the successful innovations of rival brands in cases where the technological progress increases its technological opportunities.

  I use the indicator function I to keep track over the history of the industry in order to determine whether there is enough complementary knowledge, so that the state of the art technology is allowed to be improved. Further, in each subindustry the follower brand is identified by a superscript f and the technological leader is identified by a superscript l.

- Firm a maximizes its EDV by facing the demand from the core industry and conducting costly process innovation based on expected future profits. The representative firm takes as given the investment level of firms in the core industry [12]. The demand for the ancillary product is merely the aggregate production of industry c (e.g., one can think on the automobile industry as industry c and the rubber industry as industry a, and in this case the demand for rubber is a function of the number of automobiles produced in industry c. Further, in the printers market, the production of printers determines the demand for tuners, which is the relevant complementary industry) and is affected by the knowledge stock level of firm a.

After going over the main concepts that play the central role in the model, I turn to the formal discussion of the equilibrium definition.

5.1 Defining the Equilibrium

For each brand $n \in N \leq \overline{N}$, I define the equilibrium using the value function approach (Starr and Ho 1969) as a Markov perfect equilibrium.

Further, the equilibrium incorporates the concepts of being closed-loop, i.e., firms internalize their ability to influence their rival actions in a Nash equilibrium within the period their optimal strategies are

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[12] Although one can think of a game whereas firm a internalize its ability to affect the R&D investment of firms J and by this increase its payoff, however, in this paper I choose to disregard this interaction in order to enhance the focus on the dynamics of the core industry.

determined. The *closed-loop Markov perfect equilibrium [CLMPE]* is defined as a set of strategies

$$\begin{bmatrix} x^a(\omega_i, \omega_{-i}, \tau, I), \theta_i^c(\omega_i, \omega_{-i}, \tau, I), \\ p_i^c(\omega_i, \omega_{-i}, \tau, I), \chi_i^c(\omega_i, \omega_{-i}, \tau, I), \\ \psi_i^c(\omega_i, \omega_{-i}, \tau, I), x^a(\omega, \tau, I) \end{bmatrix}$$

and value functions $\{V^c(\omega_i, \omega_{-i}, \tau, I), V^a(\omega, \tau, I)\}$. In equilibrium the strategies of each player are optimal given the value functions, and the value functions of the firms are equal to the actual continuation values when all firms follow their strategies.

Value Functions For each point in the state space $\{\omega, \tau, I\}$ firms must satisfy the corresponding Bellman equations as presented below. Denote by $\underline{n}$ as before the sub industry in which brand n is the technological leader, and by $\overline{n}$ the subindustry in which brand n is the technological inferior (Thus, for brand 1, $\underline{n}$ equals zero and for brand N, $\overline{n}$ equals zero). Using these notations, the value equation of the representative firm in each brand $n \in [2, N]$, in the complete information model, is given by

$$V_n(\omega, I, \tau) = \max \left\{ \Phi, \max_{x_n, \theta_n} \begin{bmatrix} \pi_n(\omega, I, \tau) - c(\theta_{\overline{n}}, \omega_{\overline{n}}) x_n + \\ \beta \sum_{\omega', \tau'} V_n(\omega', I', \tau') P(\omega'_n | \omega_n, I, x_n) \\ P(\omega'_{\overline{n}} | \omega_{\overline{n}}, , I, x_{\overline{n}}) P(\tau' | \tau, x_a) \end{bmatrix} \right\} \tag{17}$$

And for $n = 1$ the Bellman equation is given by

$$V_n(\omega, I, \tau) = \max \left\{ \Phi, \max_{x_n, \theta_n} \begin{bmatrix} \pi_n(\omega, I, \tau) - c(\theta_{\overline{n}}, \omega_{\overline{n}}) x_n + \\ \beta \sum_{\omega', \tau'} V_n(\omega', I', \tau') P(\omega'_n | \omega_n, x_n) \\ P(\omega'_{\overline{n}} | \omega_{\overline{n}}, I, x_{\overline{n}}) P(\tau' | \tau, x_a) P(\mu) \end{bmatrix} \right\} \tag{18}$$

Where, prime is the state value next period, Φ is the exogenous outside alternative value and β is the discount factor.

The transition probabilities are given as $P(\omega'_{\overline{n}} | \omega_{\overline{n}}, x_{\overline{n}}) = \prod_{j \in \overline{n}}^{N} P(\omega'_j | \omega_j, x_j(\omega_j, \omega_{-j \in \overline{n}}, \tau, I)$ and $P(\tau' | \tau, x_a)$ is computed as described in equation13.

Further, $c(\theta_{\overline{n}}, \omega_{\overline{n}})$ receives the following functional form:

$$c_n = \frac{\gamma_n}{\left\{ \sum_{\overline{n}} [(1 + f(\theta_{\overline{n}}, \omega_{\overline{n}})] \right\}^{\delta}} \tag{19}$$

And $\gamma = (\gamma_1 ... \gamma_N)$ is the R&D costs vector without knowledge externalities such that $\gamma_n > \gamma_{n-1}$ for all $n \in [2, N]$. Further, $\delta \geq 0$ captures the relative duplications of knowledge produced by rival firms ($\delta = 0$ is simply the case of full duplicative knowledge, i.e., all the innovative ventures are complete substitute for one another).

The value function of the ancillary firm is given by

$$V^a(\omega, \tau, I) = \max_{x_a} \left[\pi_a(\omega, \tau, I) - c_a x_a + \beta \sum_{\tau'} V_a(\omega, \tau', I) P(\tau' | \tau, x_a) \right] \tag{20}$$

Where c_a is the R&D costs of the ancillary firm.

6 Computational Methodology

Proof that equilibrium exists for this model is straightforward and is essentially identical to the proof in Ericson and Pakes (1995). It is not possible to solve for the CLMPE of the model analytically, however, numerical methods can be employed. Pakes and McGuire (1994,1997,2000) provide two algorithms that could be used to solve this model: the first is the asynchronous parallel Guass-Seidel value iteration algorithm and the second is the synchronous (stochastic) algorithm which is based upon the artificial intelligence literature. I use an altered version of the Pakes-McGuire (1994) asynchronous algorithm, with extensions to asymmetric value functions, asymmetric information and internalizing knowledge externalities in a closed-loop equilibrium[13]. The algorithm essentially iterates dynamic programming steps, testing for convergence at each step. When the value function does not change very much point-wise between iterations, the algorithm is assumed to have converged. The algorithm is not guaranteed to be a contraction mapping. However, in practice the algorithm has generally converged to an equilbrium for any

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[13] Technically speaking, in the Pakes-McGuire algorithm in each iteration we assume the rival strategies are given and are able to derive an analytical solution for R&D investments. The iterative process is said to be converged when the rivals strategies in the iterative process coincide with their optimal policies. Since I compute a closed-loop equilibrium, I have to modify this equlibrium to a numerical solution in each iteration which does not take the policies of the rival firms as given, but internelize the firm's ability to affect them.

given set of parameters[14]. Though non-convergence is not necessarily evidence against the existence of an equilibrium, convergence of the algorithm is sufficient for the existence of an equilibrium for the specific parameterization of the model.

6.1 The State Space

Each state is defined as a tuple $\{\omega_1, ..., \omega_N, \tau, I\}$. As in EP, I restrict the physical states ω_i to integers from 0 to $\overline{\omega}$, τ to integers from 0 to $\overline{\tau}$ and I to receive the value of 1 or 0. The values of $\overline{\omega}$ and $\overline{\tau}$ are chosen to ensure that the upper bound of the state space does not bind in equilibrium.

The size of the state space is $(\overline{\omega}+1)^N \times (\overline{\tau}+1) \times 2$.

6.2 The Iterative Algorithm for Computing the Fixed Point

I start by introducing the detailed algorithm for a numerical solution of the complete information game. Following this, I address the model with asymmetric information regarding knowledge flows within the leaders and followers in each sub-industry.

In the first stage, prior to computing the equilibrium of the dynamic game, I compute the current period profits of the firms in the core and ancillary industry, using the inter-temporal equilbrium as described in section 3.

In the following stage I begin the iterative loop for computing the fixed point. I solve for a subgame perfect solution using backward induction, i.e., unfold the optimal strategies from the end of the period through the beginning, and substitute the computed optimal strategies of later players into the computational algorithm of the earlier ones.

Compute first the optimal R&D strategy of the ancillary firm by maximizing the respective Bellman equation as presented in equation 17.

Note that at iteration 0 we hold in memory the static payoffs of the core and ancillary industries.

The first order condition of maximizing equation 18 with respect to x_a in iteration k in each point of the state space is

$$0 = -c_a + \beta \sum_{\tau} V_i(\omega', \tau', I') P(\tau' | x^a) \tag{21}$$

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[14] However, altering the sequence of events to allow for simoultenous R&D investments and knowledge protection, the results are highly sensitive to the chosen parameters and convergance is more rare.

Thus, the optimal ancillary R&D strategy in iteration k for given cell of the state set, denoted by $x_a^k(\omega, \tau, I)$, is[15]

$$x_a^k = \frac{-\alpha c^a + \sqrt{-a\alpha^3\beta c^a + \alpha^3 b\beta c^a}}{\alpha^2} \tag{22}$$

Where, a is the expected firm's value in the presence of a successful innovation and b is the expected value when the R&D project fails. Substituting x_a^k into the value function yields the optimal firm value in iteration k, denoted by $V_a^k(\omega, \tau, I, x_a^k)$.

In iteration $k+1$, substitute the matrix $V_a^k(\omega, \tau, I, x_a^k)$ for the value function on the rhs of equation 17 and repeat the procedure for finding the optimal R&D investment x_a^{k+1}. Continue iterating until $V_a^k(\omega, \tau, I, x_a^k) = V_a^{k+1}(\omega, \tau, I, x_a^{k+1})$. In this case the algorithm is said to be converged, whereas $x_a^{k+1} = x_a^k \equiv \widehat{x}_a$ and $V_a^k(\omega, \tau, I, x_a^k) = V_a^{k+1}(\omega, \tau, I, x_a^{k+1})$
$\equiv V_a(\omega, \tau, I, \widehat{x}_a)$ are respectively the optimal R&D investment and firm value.

Since the future periods states are already known once firm a moves, this algorithm is a contraction mapping, i.e., a convergence is guaranteed.

The next computational step is solving for the optimal R&D strategies of the representative firms in the core industry. Starting with the brand N (the technological leader in the largest sub-industry), maximize equation 15 with respect to x_N to receive the following first order conditions in iteration k for each point in the state space:

$$0 = -c_N + \beta \sum_{\omega^{l\prime}, \tau'} V_i(\omega'_{\underline{n}}, \omega'_N, I'_{\underline{n}}, I'_N, \tau') P(\omega'_N | \omega_N, x_N) P(\tau' | \tau, x_a) \tag{23}$$

The next stage involves with solving the leading firms optimal R&D strategy. As discussed in section 3, at this stage the realizations of the R&D ventures of the technological follower firms are known, therefore, the future knowledge stock levels of the rival brands are revealed at this stage. Thus, the computation algorithm computes the optimal strategy for each possible point the leading firms might be in the next period,

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[15] The second order conditions are satisfied and are given by:

$$-\alpha + (-a\alpha^3 + \alpha^3 b\beta)(- - a\alpha^3\beta^a + \alpha^3 b\beta c^a)^{-\frac{1}{2}}\frac{1}{2} < 0$$

given all the realizations of the $N-1$ rival brands. Furthermore, at this stage the aggregate random shock has already hit (or did not hit) the core industry. The future complementary knowledge stock is unknown at this stage therefore, expectations are formed based on the optimal strategy of firm a, which was computed in the previous step for each point in the state set.

Thus, the solution for the R&D strategy of the technological leader in the core industry is given by

$$x_N^k = \frac{-\alpha c_N + \sqrt{-a\alpha^3\beta c_N + \alpha^3 b\beta c_N}}{\alpha^2} \tag{24}$$

Where a is the expected firm's value in the presence of successful innovation for each possible outcome of firm's a R&D output and b is the expected value when the R&D project fails, again taking into account the expected change in the ancillary knowledge stock. In forming these expectations substitute for the expected outcome of firm a the optimal strategy computed in the first step

Substituting x_N^k into the value function yields the optimal firm value in iteration k, denoted by $V_N^k(\omega, \tau, I, x_N^k)$. In iteration $k+1$, substitute the matrix $V_N^k(\omega, \tau, I, x_N^k)$ for the value function on the rhs of equation 15 and repeat this procedure for finding the optimal R&D investment x_N^{k+1}. Continue iterate until $V_a^k(\omega, \tau, I, x_N^k) = V_N^{k+1}(\omega, \tau, I, x_N^{k+1})$. In this case the algorithm is said to be converged with $x_N^{k+1} = x_N^k \equiv \widehat{x}_N$ and $V_N^k(\omega, \tau, I, x_N^k) = V_N^{k+1}(\omega, \tau, I, x_N^{k+1}) \equiv V_N(\omega, \tau, I, \widehat{x}_N)$ being respectively the optimal R&D investment and firm value.

This iterative procedure is not a contraction mapping and therefore, the convergence is not guaranteed.

At the beginning of the next step the algorithm holds in memory the static payoffs, the optimal R&D strategy of the ancillary industry and the optimal R&D strategy of brand N for all possible future values of the $N-1$ brands.

Maximizing equation 15 for the $N-1$th brand with respect to x_{N-1} to receive the following first order condition:

$$\begin{aligned} 0 = -c(\theta_N) + \beta \sum_{\omega^{l\prime}, \tau'} V_i(\omega'_{\underline{n}}, \omega'_{\overline{n}}, I', \tau') \\ P(\omega'_{N-1}|\omega_{N-1}, x_{N-1}, \theta_N) P(\omega'_N|\omega_N, x_N) P(\tau'|\tau, x_a) \end{aligned} \tag{25}$$

Solving for the optimal R&D in each iteration yields

$$x_{N-1}^{k}=\frac{-\alpha c_{N-1}+\sqrt{-a\alpha^{3}\beta c_{N-1}+\alpha^{3}b\beta c_{N-1}}}{\alpha^{2}} \tag{26}$$

Where a is the expected firm's value in the presence of successful innovation for each possible outcome of firm a and N s' R&D output, while b is the expected value when the R&D project fails, again taking into account the expected change in the ancillary knowledge stock and the technological leader's one. In forming these expectations substitute for the expected outcome of firm a the optimal strategy computed in the first step, and substitute the optimal strategy of the leading brand computed in the previous step for the expected leader's knowledge stock next period

As before, Substituting x_{N-1}^{k} into the value function yields the optimal firm value in iteration k, denoted by $V_{N-1}^{k}(\omega,\tau,I,x_{N-1}^{k})$.In iteration $k+1$, substitute the matrix $V_{N-1}^{k}(\omega,\tau,I,x_{N-1}^{k})$ for the value function on the rhs of equation 15 and repeat the procedure for finding the optimal R&D investment x_{N-1}^{k+1}. Continue iterate until $V_{a}^{k}(\omega,\tau,I,x_{N-1}^{k})=V_{N-1}^{k+1}(\omega,\tau,I,x_{N-1}^{k+1})$. In this case the algorithm is said to be converged with $x_{N-1}^{k+1}=x_{N-1}^{k}\equiv\widehat{x}_{N-1}$ and $V_{N-1}^{k}(\omega,\tau,I,x_{N-1}^{k})=V_{N-1}^{k+1}(\omega,\tau,I,x_{N-1}^{k+1})$
$\equiv V_{N-1}(\omega,\tau,I,\widehat{x}_{N-1})$ being respectively the optimal R&D investment and firm's value.

This iterative procedure is not a contraction mapping and therefore, the convergence is not guaranteed.

Solving for the optimal R&D strategy of the rest of the core industry is simply continuing the backward induction algorithm presented above, while for each subindustry use the optimal R&D strategies of the more advanced firms, as computed in previous steps, including that of the ancillary industry.

The generalized solution for solving the optimal R&D strategies in the core industry is as following:

For brand $n\in(1,N)$ maximize equation 15 to receive the following vector of first order conditions:

$$0=-c_{n}(\theta_{\overline{n}})+\beta\sum_{\omega^{l\prime},\tau'}V_{n}(\omega'_{n},\omega_{\overline{n}},I',\tau')P(\omega'_{n}|\omega_{n},x_{n})P(\omega'_{\overline{n}}|\omega_{\overline{n}},x_{\overline{n}})P(\tau'|\tau,x_{a}) \tag{27}$$

Where the transition probabilities are

$P(\omega'_{\overline{n}}|\omega_{\overline{n}},x_{\overline{n}})=\prod_{j\in\overline{n}}^{N}P[\omega'_{j}|\omega_{j},x_{j}(\omega_{j},\omega_{-j\in\overline{n}},\tau,I)]$, and

$x_j(\omega_j, \omega_{-j\in\overline{n}}, \tau, I)$ is the optimal strategy computed in the previous steps for all $j \in \overline{n}$.

For $n = 1$, the first order conditions vectors becomes

$$0 = -c_1(\theta_{\overline{n}}) + \beta \sum_{\omega^{l\prime},\tau'} V_1(\omega_1', \omega_{\overline{n}}, I', \tau') P(\omega_1' | \omega_1, x_1) \tag{28}$$

$$P(\omega_{\overline{n}}' | \omega_{\overline{n}}, x_{\overline{n}}) P(\tau' | \tau, x_a) P(\mu) \tag{29}$$

As before, receive the following solution for the optimal R&D investment and use the computed R&D strategies of the more advanced brands to form the expectations regarding their knowledge stock evolution, as explained above:

$$x_n^k = \frac{-\alpha c_n + \sqrt{-a\alpha^3 \beta c_n + \alpha^3 b \beta c_n}}{\alpha^2} \tag{30}$$

Where a and b are as before, representing the expected value when firm a succeeds and fails in its R&D respectively.

Again, Substituting x_n^k into the value function yields the optimal firm value in iteration k, denoted by $V_n^k(\omega, \tau, I, x_n^k)$.In iteration $k+1$, substitute the matrix $V_n^k(\omega, \tau, I, x_n^k)$ for the value function on the rhs of equation and repeat the procedure for finding the optimal R&D investment x_n^{k+1}. Continue iterate until $V_a^k(\omega, \tau, I, x_n^k) = V_{N-1}^{k+1}(\omega, \tau, I, x_n^{k+1})$. In this case the algorithm is said to be converged with $x_n^{k+1} = x_n^k \equiv \widehat{x}_n$ and $V_n^k(\omega, \tau, I, x_n^k) = V_n^{k+1}(\omega, \tau, I, x_n^{k+1}) \equiv V_n(\omega, \tau, I, \widehat{x}_n)$ being respectively the optimal R&D investment and firm's value.

After converging and computing the fixed point optimal R&D strategies for the ancillary firm and for all brand in the core industry, we are left to solve for the knowledge diffusion decision and brand exit decisions.

After convergence was achieved to the $N+1$ firms, the algorithm turns to update the optimal knowledge protection strategies.

At this stage, each technological leader determines whether to protect its knowledge or to expose it to rival firms in the subindustry. Assume this decision is made simultaneously and compute $V_n(\theta_n = 1|\Omega)$ and $V_n(\theta_n = 0|\Omega)$, and set θ to unity iff $V^n(\theta_n = 1|\Omega) > V^n(\theta_n = 0|\Omega)$. Whereas Ω is the information set available to each firm. Under full information it is basically the actual reaction functions of rival firms to the knowledge protection decision.

Further, Ω incorporates the expectations of each firm (but firm 1) to the knowledge flows of the rival firms. Define $\overset{*}{\theta}$ as the optimal knowledge diffusion decisions, which is a vector of a dimension $N-1$, that satisfies the following condition: $\overset{*}{\theta}_n(\overset{*}{\theta}_{-n}) > \widetilde{\theta}(\overset{*}{\theta}_{-n})$ for all $n, -n \in N-1$.

Further, substitute $\widehat{\theta}$ into the value functions to solve for the optimal firms' value in the closed-loop Markov Perfect Equilibrium and set $\psi_n^c(\omega_i, \omega_{-i}, \tau, I) = 0$ iff $V_n > \Phi$ and 1 otherwise.

This completes the iteration. A fixed point of this procedure (updating all the points in the state space) satisfies the Bellman equations that define the equilibrium. The iteration are repeated until the chosen convergence criteria is satisfied.[16].

6.2.1 A model with incomplete information

So far I discussed the methodology for solving numerically a model with complete information. The information set in a model with incomplete information is described in section.

Denote by a superscript e the expected knowledge flows of firm n to distinct it from the actual knowledge flows and thus the actual R&D investments, which are obviously independent from the expectations structure.

Using those notations, firm n maximizes the following value equation with respect to x_n:

$$V_n(\omega, I, \tau) = \max \left\{ \Phi, \max_{x_i, \theta_i} \left[\begin{array}{c} \pi_n(\omega_{\underline{n}}, \omega_{\overline{n}}, I, \tau) - c(\theta_{\overline{n}}) x_n + \\ \beta \sum\limits_{\omega'_{\underline{n}}} \sum\limits_{\omega'_{\overline{n}}.\tau'} V_n(\omega'_{\underline{n}}, \omega'_{\overline{n}}, I', \tau') \\ P(\omega'_{\underline{n}} | \omega_{\underline{n}}, x^e_{\underline{n}}(\theta^e_{\underline{n}})) P(\omega'_n | \omega_n, x_n) \\ P(\omega'_{\overline{n}} | \omega_{\overline{n}}, x^e_{\overline{n}}(\theta^e_{\overline{n}})) \\ P(\tau' | \tau, x_a) P(\mu) \end{array} \right] \right\} \tag{31}$$

The first summation forms the expectations with regard to the R&D output of firms $\underline{n}$, since there is incomplete and asymmetric information with regard firm n and the technological inferior firms that operate in the subindustry $\underline{n}$. Further, firm's n information set includes the true optimal response of firms $\underline{n}$ to its R&D investment. Remember that although firm n does not observe the R&D output of its inferior rivals, the inferior rivals assume that their R&D output is a common knowledge

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[16] I stop the algorithm when the diffrerence between two consecutive iterations falls below 10^{-6}.

of the sequential innovative firms (those firms that invest afterwards). From this reason, the best response function of the inferior firms to the R&D of the more advanced firms in the incomplete information model is the same as in the complete information model.

The second summation is forward looking as introduced above and constructs the expected value next period for any realization of the innovative projects in the sub-industry $\underline{n}$.

I use this value function to simulate industry evolution patterns under complete and incomplete information, and mainly concern the aggregate R&D and knowledge externalities in the core industry.

7 Discussion

The dynamics game presented above might incorporate several equilibrium trajectories, depending on the model's specifications and chosen parameters set. In this section I aim in studying the common features of these evolution patterns.

Assume the core industry starts its life cycle with $\overline{N}$ rival brands. At the early periods there is mass entrance mainly to the low knowledge brands, since the capabilities distribution is skewed toward the low capable potential incumbents. As the industry progresses, entry slows down since the knowledge stock necessary to produce even the least advanced brand increases. At this stage, exit could occur, depending on the competitive nature and the strategic interaction discussed in previous sections. In case there is indeed exit, the industry offers N rival brands, while the $\overline{N} - N$ brand is now the least advanced one and need not any complementary knowledge in order to carry out a successful R&D investment.

Assume for now that the industry stabilizes on offering N rival brands. In this case, there is higher return on product R&D and on diffusing knowledge due to the positive market expansion effect. Therefore, the R&D investment is intensified and the less capable firms are able to keep up with technological progress. However, after a few periods, the market expansion effect diminishes due to achieving high enough level of ancillary knowledge. Thus, with sufficient complementary knowledge in the core industry, there is no endogenous knowledge diffusion.

After several successful innovations, the advanced firms in each sub-industry cannot continue to innovate without sufficient complementary knowledge. At this point firms must decide whether to continue their product innovation and for this purpose diffuse their knowledge to rival brands, or alter the nature of their innovative projects. Although not discussed in this paper, a firm can choose to invest in process innovation.

Empirical findings suggest there is high product innovation at the early stages of the industry life cycle until the 'shakeout' occurs. From this reason I focused my analysis on product innovation alone.

Since the return on process innovation increases with firm's size, due to the ability to implement the cost reduction, gained by the successful R&D, over higher units of production, there exists a critical production level once reached it is more beneficial to conduct process R&D rather then product R&D. I assume, supported by empirical evidence, that this level occurs after the 'shakeout' or just prior its appearance, causing mass exit[17]. In both cases, the incentives to share knowledge determines the scope of the 'shakeout' and its timing.

When eventually the optimal strategy shifts from diffusing knowledge to protecting it, the 'shakeout' process begins. Under rather reasonable assumptions brand N will be the first to protect its knowledge. Gradually all brands will find it optimal to protect their knowledge causing product innovation to stop. At this stage the innovative efforts are coordinated toward process innovation without knowledge sharing among brands. Brand types might start disappearing and this stresses the beginning of the producer 'shakeout' era that might be acute or rather prolonged.

The transformation from a non-concentrated industry to a concentrated one does not imply firms do not share knowledge. Nonetheless, since knowledge diffusion prior the 'shakeout' was non-cooperative, the nature of knowledge flows among homogenous firms after the 'shakeout' is cooperative, i.e., a firm's desire to share knowledge depends upon knowledge drifting from the rival firm. This kind of endogenous knowledge flows can be implemented in research joint ventures, cross-licensing agreements, patent pools and etc.

Future research can implement the analytical framework used in this paper to conduct a dynamic analysis of cooperative knowledge sharing in post 'shakeout' era[18], which can be of high importance in order to better understand economic performance of advanced technology based industries.

8 The Numerical Example

I this section I present a numerical example to the model presented in this paper and use the results to evaluate the predictive power of this

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[17] This was the case in the US automobile industry with the introduction of the mass production techniques by Henry Ford at the beginning of the second decade of the twenthhieh century.

[18] Such as research joint ventures, coross-licensing agreements and patent pools.

analytical framework with respect to industry evolution patterns which are based on endogenous knowledge flows.

I chose to use the simple example possible in order to stress the main features of this model, and deny complications that might arise from higher computation demands. Obviously, this example could easily be extended either in the state space or in firms dimensions, such to incorporate more realistic nature and complex strategic interactions.

Assume there are two brands in the core industry, denoted with a superscript f and l for technological follower and leader respectively. Consider for now, there is complete information. In this case, the leading firm maximizes the following Bellman equation with respect to knowledge protection and R&D investment:

$$V_l(\omega, I, \tau) = \max\left\{\Phi, \max_{x_i,\theta_i}\left[\begin{array}{c}\pi_l(\omega, I, \tau) - \gamma x_l + \\ \beta \sum\limits_{\omega^{l\prime},\tau'} V_l(\omega', I', \tau') P(\omega'_l|\omega_l, I, x_l) P(\tau'|\tau, x_a)\end{array}\right]\right\} \tag{32}$$

Further, the follower firm maximizes the following Bellman equation with respect to R&D:

$$V_f(\omega, I, \tau) = \max\left\{\Phi, \max_{x_i,\theta_i}\left[\begin{array}{c}\pi_f(\omega, I, \tau) - \frac{\gamma}{(\frac{1}{2}(\widehat{\theta}+\widetilde{\theta})\omega)^{\delta}} x_f + \beta \sum\limits_{\omega^{l\prime},\tau'} V_l(\omega', I', \tau') \\ P(\omega'_f|\omega_f, x_f) P(\omega'_l|\omega_l, I, x_l) P(\tau'|\tau, x_a) P(\mu)\end{array}\right]\right\} \tag{33}$$

And the Bellman equation of the ancillary firm is

$$V^a(\omega, \tau, I) = \max_{x_a}\left[\pi_a(\omega, \tau, I) - c_a x_a + \beta \sum_{\tau'} V_a(\omega, \tau', I) P(\tau'|\tau, x_a)\right] \tag{34}$$

Imposing linearity in consumers tastes with regard to product's quality and with respect to the ancillary knowledge yields the following market share

$$\pi_j = \frac{\exp(s\omega_j - p_j - p_a\frac{g}{\tau})}{\sum\limits_{k=1}^{2}\exp(s\omega_k - p_k - p_a\frac{g}{\tau})} \qquad \text{For } \; j = 1, 2. \tag{35}$$

In addition, assume the following functional forms and parameters values:

M	500
α	0.3
β	0.925
γ^l	1.0
γ^f	2.0
γ^a	2.0
δ	1.0
$mc^{l,f}$	2
$\overline{P}_a$	0.3
mc^a	$\frac{0.2}{\tau}$
s	0.4
g	0.02

I solve the model based on the computational methodology presented in sections and the sequence of events described in section. The numerical results are presented and discussed below and are followed by the simulation output.

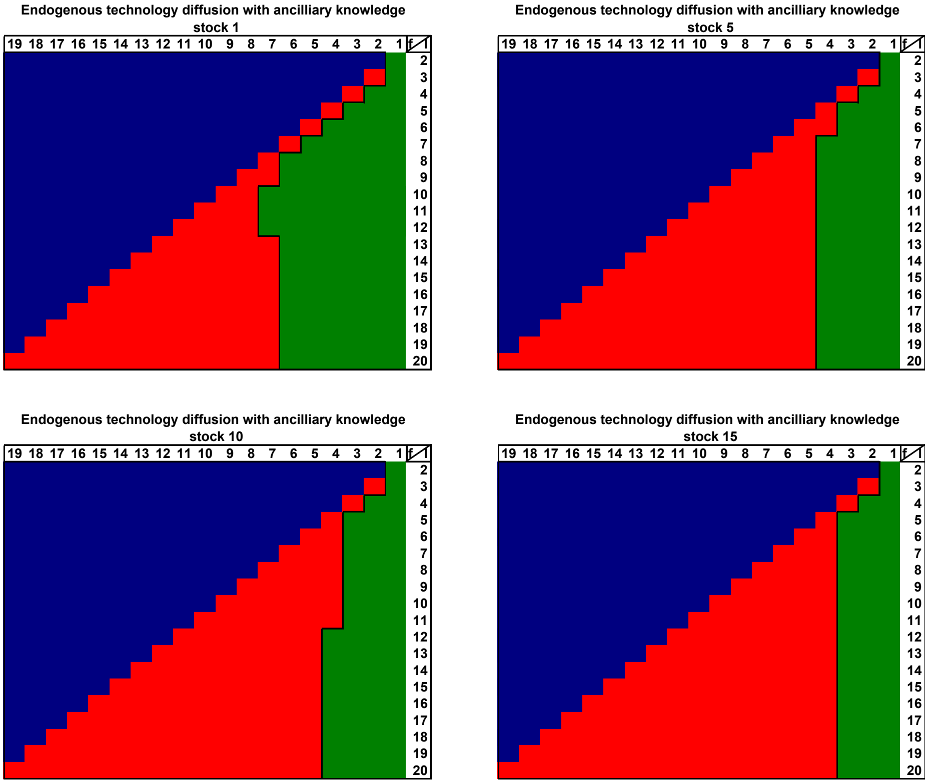
Figures 1-4 depict the change in the optimal knowledge protection strategies with the evolution of the state set variables. The green area is the state points in which there are endogenous knowledge flows, whereas in the red area the optimal strategy is to protect knowledge (e.g., the points in which the firm will decide to patent its invention). The upper left figure plots the knowledge protection strategies in the core industry knowledge stock dimension for $\tau = 1$, the upper right is for $\tau = 5$, bottom left for $\tau = 10$ and bottom right for $\tau = 15$. At all points in the assume $I = 1$, i.e., there is sufficient complementary knowledge. Thus, the following analysis focuses on strategies to protect or diffuse knowledge which are driven only by demand side incentives.

The first finding is that demand side incentive to diffuse knowledge is observable only in the early stages of an industry, when its knowledge stock is relatively low. The decision whether to protect or diffuse knowledge based on demand side effects depends upon a positive incentive to diffuse knowledge, which is known as the *market expansion effect,* and on the negative competition incentive (the loss of monopoly power on the new invention). When the industry is young, there is low demand and high uncertainty with regard to consumer demand, industry structure and etc. Thus, the negative incentive to diffuse knowledge is relatively low with respect to the positive incentive to share knowledge and increase the aggregate production of the industry. The desire to attract more costumers by increasing aggregate production and to encourage ancillary R&D investments results in endogenous knowledge diffusion at this stage. Further, since the return on a successful invention is rather low, firms less eager to protect their knowledge and concentrate more on increasing consumers awareness to the new product they offer.

Therefore, we observe more knowledge flows at the early stages of the industry life cycle. As the knowledge stock in the core industry progresses, the return on new invention increases, since now the industry is larger and firm concentrate more on increasing their market share rather on increasing the market as a whole. Thus, we observe less endogenous knowledge flows when the industry is more advanced. Furthermore, recall that by sharing knowledge a firm looses a competitive advantage and therefore we should observe more demand driven knowledge flows when the brands are relatively far a part in the technological space.

Another interesting finding is the expansion of the green area (the points in which firms decide to diffuse knowledge) with the increase of the ancillary knowledge stock. This result can be explained by studying the dynamics of the *market expansion effect* incentive to diffuse knowledge. By diffusing knowledge, firms allow their rivals to conduct cheaper R&D and increases their production more easily. By increasing the aggregate production in the core industry, the demand for the ancillary product strengths, making it more profitable to innovate (the return on R&D investment in the ancillary industry rise due to the increase its product's market). Once the ancillary knowledge stock grew enough, the potential increase in the core product's market is relatively low with respect to the negative incentive to share knowledge. In this case, we observe less endogenous knowledge flows which, of course, affects the growth pattern of both the core and ancillary industries.

Finally, as discussed, among others, in Utterback (1975, 1978), Utterback and Suarez (1993) and in Klepper and Graddy (1990), the cause of the "shakeout" is the appearance of an exogenous invention and new technological opportunities. This papers relates to this technological "shock" as an endogenous process governed by an ancillary industry. As long as the market expansion effect is relatively strong firms in the core industry share knowledge, which allows intense entry and high market value. However, when there is a sufficient ancillary knowledge stock, the optimal strategy is to protect knowledge (due to the weaker market expansion effect), which when causes a drastic change in the innovative efforts in the core industry and its market structure.



Endogenous technology diffusion with ancilliary knowledge stock 1

Endogenous technology diffusion with ancilliary knowledge stock 5

Endogenous technology diffusion with ancilliary knowledge stock 10

Endogenous technology diffusion with ancilliary knowledge stock 15

Figures 1-4

Another illustration of the acute reduction in the innovative efforts the core industry experiences the demand side incentive to share knowledge disappears is given in figure 5, which depicts the optimal R&D strategies of the technological followers with ancillary knowledge equals 1 and sufficient complementary knowledge. An increases in the technological leader knowledge stock is associated with an increase in the complementary knowledge, for complementary knowledge stock less then 7. Once this level has been achieved, the competitive negative incentive to share knowledge is stronger then the positive market expansion effect to share knowledge, resulted in less knowledge flows and weaker positive externalities in the core industry. Thus, the costs of conducting R&D becomes high and we observe a drastic fall in the innovative efforts of the complementary firms. This reduction is accompanied by a reduction in firms value (as discussed below), that triggers industry's "shakeout", i.e., mass exit of firms that does not find it profitable to remain in the industry given the lower continuation value.

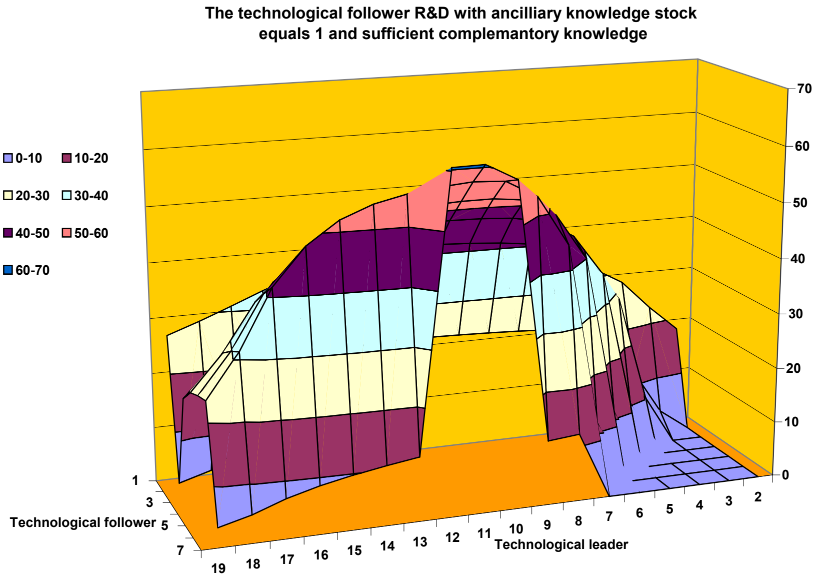


Figure 5

Further descriptive statistics are presented in the appendix and in particular, the evolution of firms' value throughout the state space.

After stressing the main features of the estimated optimal strategies, I continue to the analysis of the simulation output.

8.1 Simulation Results

I now turn to introduce and study the simulation output of 20 periods evolution pattern of the core industry[19]. The results are presented in figure 6, that plots the representative firm's value in both brand categories and the percent of simulated endogenous knowledge diffusion in each period.

The first findings relates to firms' value. At the first periods, the value of the representative firm in the technological leader brand is lower then the value of the representative firm in the follower brand. Thus, firms find it more profitable to enter the less advanced brand. This finding can be explained by studying the return on knowledge at the early stages of the industry. Since the industry is undeveloped and very small, a successful R&D can indeed increase the respective market share, however, it generates a small stream of profits. Further, in order to advance its knowledge, the technological leader depends upon the

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[19] Based on the estimation results presented above, I simulated 100K evolution pathes and took their avergae for each period.

Each path began from the cell $(2, 1, 1, I = 0)$, although I conducted several other simulations from different starting points, including those of higher technological distance among firms, and find no important shift in the main findings.

generation of complementary knowledge by the technological followers. In this case, firms might choose to enter the less advanced brand and conduct complementary innovation and not inventions in the state-of-the-art technology, that yields low return due to the early phase on the industry's life cycle. Therefore, being the technological leader does not guarantees higher market value at the early periods when the industry is small.

At the first three periods we observe a decrease in knowledge flows is mainly due to the relatively low return on inventions. During this era knowledge flows are decreasing.

From the third period we observe a convex increases in knowledge flows and in firms' value until they reach their peak in the 7th period. During this phase in the industry life cycle there is rapid growth, intense entry (due to the increase in market value) and high product innovation (as discussed below). However, this utopia economic environment does not last long, as the industry "shakeout" occurs during at the 8th period. The rapid growth prior the "shakeout" was possible due to intense knowledge flows that allowed for high R&D investment. During this period the ancillary knowledge stock advanced as well and attracting to consumers to the core industry (by placing greater portion of their income on product J). However, in period 7 the ancillary knowledge have reached the threshold level, in which the optimal strategy of the leading firm is to protect their knowledge rather sharing it with the less technologically advanced firm. Going to back to the empirical evidence and citing the finding Klepper and Simons (2000) and in Jovanovic and MacDonald (1994,2) regarding the tire industry in the US prior the shakeout *firm's value pattern during this sample shows that during the entry periods firms' relative market value rose sharply and the exit periods were accompanied with an even sharper drop in relative firm values.* By studying the demand data the "shakeout" could not be explained, so they had to focus of technological shock. The findings reported in this paper support the empirical data, however, explain the "shakeout" by achieving a high level of knowledge stock (mainly ancillary) that diminishes the positive incentive to share knowledge. Thus, once knowledge is protected the positive externalities drop resulting in a sharp reduction in firms' value, especially that of the technological follower that relied more on the innovative efforts of the leading firm. The overshooting in firms' value is the results of the high growth period and knowledge diffusion that supported the belief that the good economic environment will persist.

The last period of the industry life cycle is the post "shakeout" era that involves with higher competition, less knowledge sharing and low product innovation.

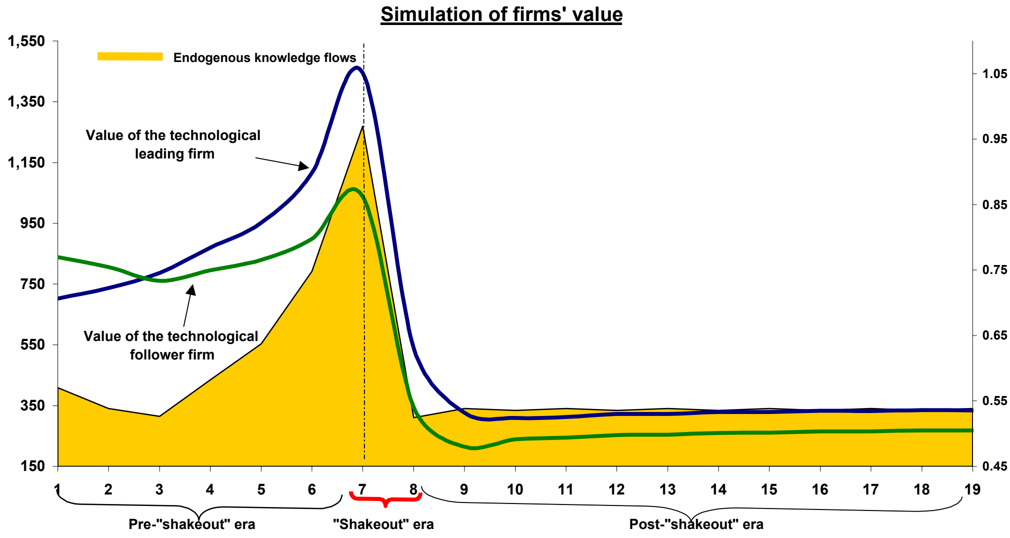

Figure 6

Assume firms' survival hazard is an increasing function of their value, i.e., higher continuation value implies a lower exit probability. Figure 7 simulates the survival hazard (the probability the firm will remain active in the industry), which is a logit function of the continuation value. As expected, the 'shakeout' in market value causes a drastic drop in survival hazard, with accord to empirical evidence. Survival hazard 'shakeout' is the trigger of mass exit known as the producer 'shakeout'.

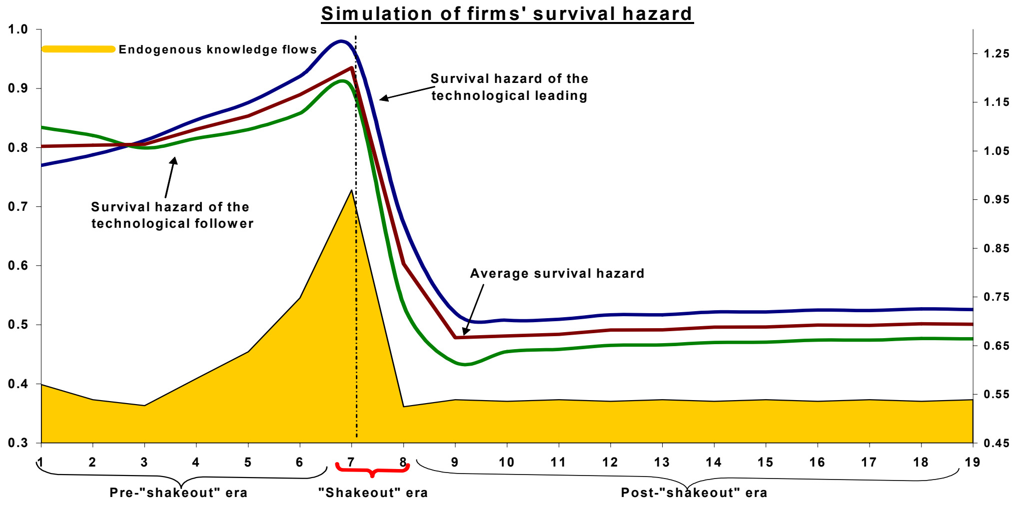

Figure 7

Figure 8 depict the dynamics of the R&D investment in the core industry. The high innovation level prior the "shakeout" can be explained by the enhanced positive knowledge externalities in the core industry and the increasing return on R&D due to the progress of the ancillary knowledge and increasing market size. The acute drop in R&D is mainly the result of the reduced knowledge flows in the economy.

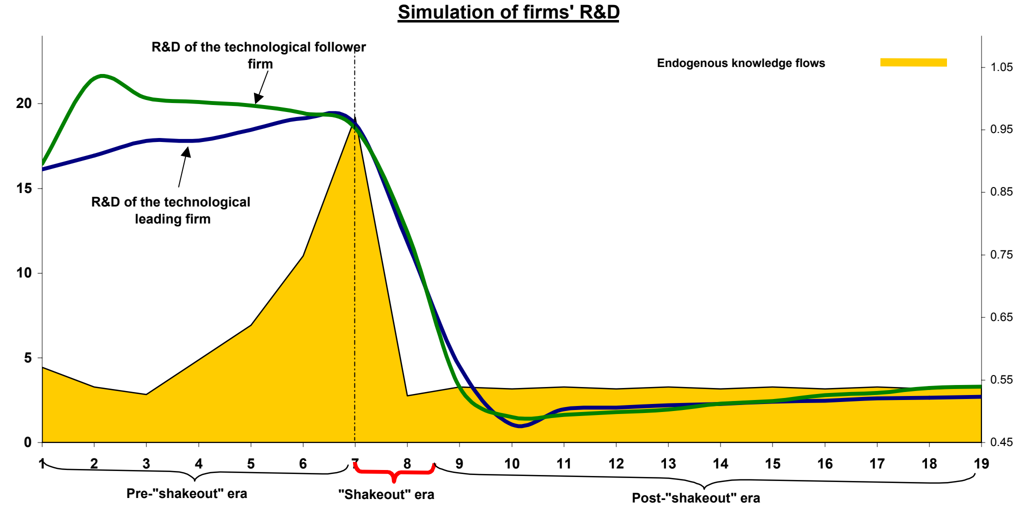

Figure 8

8.2 Comparative statics

Table 1 presents the simulation results with low and high market expansion effect and study the main economic variables prior the "shakeout" and afterwards.

The first finding relates to consumers' welfare. With a strong market expansion effect consumer welfare is higher during the whole sample. This is mainly due to higher positive knowledge externalities in the core industry that allow for higher R&D investment and rapid development of the ancillary industry. Note the higher endogenous knowledge flows prior the "shakeout". Further, the ancillary R&D is stronger due to the higher return on a successful innovation.

In the core industry we observe a decrease in the leader R&D prior the "shakeout" and increase afterwards. Regarding the follower firm we observe an increase in the period prior the shakeout and afterwards. Higher R&D efforts by the follower firm is simply the product of intense positive externalities due to higher knowledge flows. However, the reduction in the R&D efforts of the leading firm can be explained by the rapid industry growth and the diminishing return on product R&D.

Prices are higher with stronger market expansion effect, and this is manly due to the increased demand for the core product that allows firms to charge more. The acute reduction in prices is explained by the more competitive environment in the post "shakeout" era.

The quantity is much higher with a strong market expansion effect, and this is due to intense R&D effort by the ancillary firm that yields higher return on its innovative efforts. Further, higher R&D in the core industry results with more progressed knowledge stock and higher quality offered to the consumers. This in turn increases demand and production.

	High market expansion effect (g=0.1)				**Low market expansion effect (g=0.02)**	
		Technological leader	**Technological follower**		**Technological leader**	**Technological follower**
Consumer welfare						
Pre-"shakeout"	6.7			5.3		
Post-"shakeout"	17.6			11.6		
Total	**14.6**			**9.8**		
Ancilliary R&D investment						
Pre-"shakeout"	19.1			16.6		
Post-"shakeout"	13.8			11.8		
Total	**14.8**			**13.1**		
Core R&D investment						
Pre-"shakeout"		11.7	23.6		12.4	22.8
Post-"shakeout"		3.4	5.6		2.8	4.2
Total		**5.3**	**9.7**		**6.8**	**7.2**
Endogenous knowledge flows**						
Pre-"shakeout"	**72.3%**			**62.4%**		
Market value						
Pre-"shakeout"		808.2	884.9		521.1	738.4
Post-"shakeout"		421.2	291.3		343.6	250.2
Prices						
Pre-"shakeout"		2.9	2.7		2.71	2.63
Post-"shakeout"		2.3	2.05		2.13	1.95
Quantity						
Pre-"shakeout"		78	43.12		62.12	42.7
Post-"shakeout"		111.5	64.4		89.8	72.2

* Average values.
** Percent of periods with Theta equals 1.

Table 1

8.3 The model with incomplete information

After studying the evolution pattern with complete information, I turn to analyze the model with incomplete and asymmetric information. The features of this model is discussed in section.

The Bellman equation of the technological leader is

$$V_l(\omega, I, \tau) = \max\left\{\Phi, \max_{x_i,\theta_i}\left[\begin{array}{c}\pi_l(\omega, I, \tau) - \gamma x_l + \\ \beta \sum\limits_{\omega'_f} \sum\limits_{\omega'_l . \tau'} V_n((\omega', I', \tau') P(\omega'_f|\omega_f, x_f(\theta^e_l)) \\ P(\omega'_l|\omega_l, I, x_l) P(\tau'|\tau, x_a) P(\mu)\end{array}\right]\right\} \tag{36}$$

Whereas θ_l^e is the leading firm's belief regarding knowledge flows in the core industry. The maximization problem of the other agents are the same as in the complete information case.

The results are reported in table 2 and are compared to the complete information mode.

The first finding is the drop in consumers' welfare. In order to understand this drop we should study the innovative efforts and knowledge flows. We observe a mild increase in the ancillary R&D efforts, however a sever drop in the R&D of the follower firm although the leading firm choose to diffuse knowledge more intensively. Thus, the drop in the follower's R&D results in lower complementary knowledge stock that restricts the evolution of the state of the art technology in the core industry. In order to understand why this reduction occurs we should analyze the evolution of the leader's knowledge stock. Since there is more uncertainty in this model (the output of the complementary R&D is unknown) is more risky to invest in innovative projects that aim in improving the state of the are knowledge. Thus, the leaders knowledge stock grows slower, and since it is used in the process of creating the complementary knowledge we observe a sharp fall in the innovative efforts of the technological follower.

However, endogenous knowledge flows are higher in this model and this finding support the notion that once a firm does not realize there are uncontrolled knowledge spillover, technology can be diffuse more easily in the industry.

Finally, we observe no major shift in production and prices, although there is an acute jump in the technological leader value prior the "shake-out". This finding is merely the result of the bad innovative performance of the technological follower rather then the good performance of the technological leading firm.

	Incomplete information (with g=0.02)				Complete information (with g=0.02)	
		Technological leader	Technological follower		Technological leader	Technological follower
Consumer welfare	**3.18**			**9.8**		
Ancilliary R&D investment						
Pre-"shakeout"	16.7			16.6		
Post-"shakeout"	12.4			11.8		
Total	**13.2**			**13.1**		
Core R&D investment						
Pre-"shakeout"		11.9	11.4		12.4	22.8
Post-"shakeout"		2.4	1.7		2.8	4.2
Total		**4.16**	**3.6**		**6.8**	**7.2**
Endogenous knowledge flows**						
Pre-"shakeout"	**70.2%**			**62.4%**		
Market value						
Pre-"shakeout"		1,039.8	738.2		521.1	738.4
Post-"shakeout"		360.8	224.2		343.6	250.2
Prices						
Pre-"shakeout"		2.7	2.6		2.7	2.6
Post-"shakeout"		2.1	1.9		2.1	1.9
Quantity						
Pre-"shakeout"		61.2	44.9		62.1	42.7
Post-"shakeout"		87.5	73.8		89.8	72.2

* Average values.

** Percent of periods with Theta equals 1.

Table 2

Figure 8 illustrates R&D efforts under complete and incomplete information (the bar graphs are assigned to the right scale). The main finding is the reduction in the innovative efforts in the core and ancillary industry. As discussed above, higher uncertainty lowers knowledge externalities in the core industry resulting with less complementary R&D and slower technological progress. Further, the reduction of ancillary R&D at the pre-'shakeout' era is the product of lower return on successful innovation due to less developed market for product *J*.

Finally, under incomplete information the 'shakeout' occurs in an early period then under complete information. This observation is one of the reasons to the lower consumer welfare during the simulated sample, due to shorter period of economic blossom.

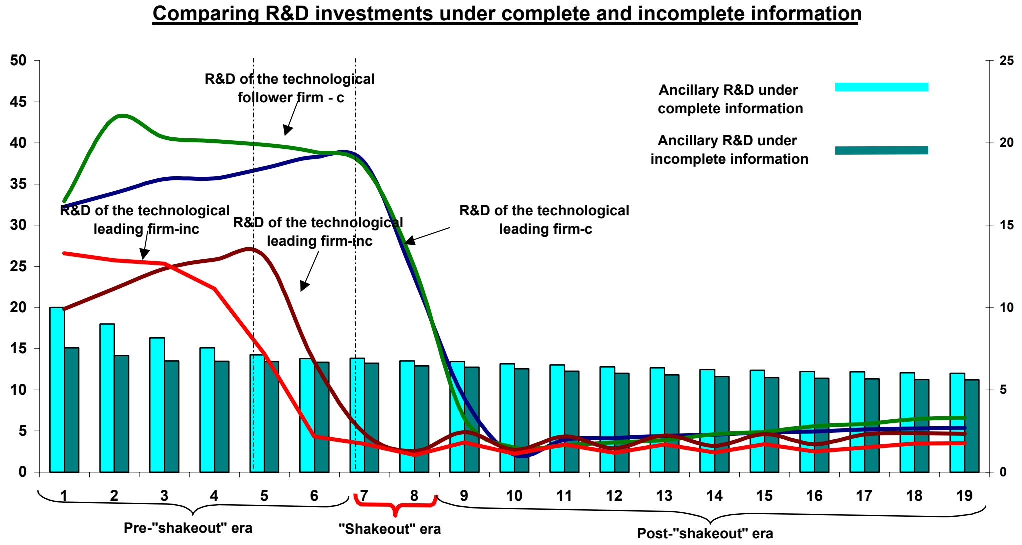

Figure 9

8.4 Policy Implications

There are several policy implications that can be inferred from the findings of this paper.

The first policy measure relates to R&D subsidies and maintaining high level of R&D efforts. As suggested above, inducing high innovative efforts and rapid technological progress does not depends solely on the incentive to innovate, but also on the incentive to diffuse knowledge. Consider, for example the pre-'shakeout' era, the innovative efforts are much higher then in future periods, even though the return on successful R&D could be relatively lower (since the market is smaller). However, due to intense technology diffusion, knowledge externalities are high, allowing for cheaper R&D costs and thus, stronger incentives to innovate. This implies that policy measures could achieve better outcomes in the form of intensified innovative efforts in aiming not only on reducing R&D costs by subsidies, but also in promoting more endogenous knowledge sharing among firm.

An example for such a policy can be higher support for innovative projects that stimulus knowledge sharing (e.g., open architecture innovations and etc.).

Furthermore, policy measures should take into account interactions among industries, such that higher support for one industry, may lead to less innovative efforts in complementary products. The findings presented above stress the importance of the market expansion effect in generating positive externalities in the core industry. Granting R&D

subsidies to the ancillary firm encourages higher ancillary innovation and therefore, more rapid exhaustion of the market expansion effect in the core industry, resulting with less knowledge sharing and a smaller number of periods of economic blossom.

For conclusion, policy measures that aim in increasing knowledge externalities in the economy should consider not only stimulating innovative efforts by reducing R&D costs, but also encouraging endogenous knowledge sharing among firms.

8.5 Concluding Remarks

In this paper I study and simulate industry evolution patterns, and in particular market value and survival hazard 'shakeout', based on the return on R&D and on knowledge diffusion. The theoretical model considers an infinite horizon interaction game between firms in a core industry, firms in an ancillary industry and consumers who purchase products from both industries. The main focus is on the dynamics of the incentives to invest in R&D and to diffuse knowledge in the core industry, which underpin its evolution pattern.

I used a numerical example to simulate the evolution pattern of an industry which is based upon endogenous knowledge flows. Although I present a rather simplifies application, the simulated results could be used to explain the empirical product life cycle findings, as mainly described in Gort and Klepper (1982), Klepper and Graddy (1980) and Utterback (1975, 1978).

The main finding of this paper is that by studying the dynamics of the incentive to share knowledge we could characterize the major features of industry life cycle, and in particular the remarkable 'shakeout' phenomena. The simulated evolution pattern illustrates a 'shakeout' in firms' value, which experience an increase prior the shakeout and then even steeper reduction. The implication of this shift is high entry during the rise in market value and mass exit in the short periods of the 'shakeout'. Studying the incentive to share knowledge in the core industry prior the 'shakeout' and afterwards, we observe high correlation between firms' value and the positive knowledge externalities in the industry. The reduction in knowledge flows is caused by the diminishing incentive to share knowledge due to the market expansion effect. Once the market has fulfilled its growth potential there is a drastic fall in endogenous knowledge flows in the industry that triggers the 'shakeout' in market value.

Further, this paper investigates evolution patterns under two information sets. In the complete information set we observe less knowledge flows, however, higher technological progress then in the incomplete and asymmetric information set. Higher knowledge flows is consisted with prior studies regarding optimal aggregate innovation, however, higher uncertainty in the cause a reduction in the inventive efforts in the industry, which restricts the technological progress and diminishes knowledge externalities in the industry. In the numerical example presented in this paper, consumers' welfare is worst off in the incomplete information set due to slower technological progress and weaker innovative efforts.

The main policy implication associated with the findings of this paper is the understanding that measures aiming to enhance knowledge externality should simultaneously relate to the incentives to innovate and to share knowledge among firms. Lacking to consider the important role of endogenous knowledge flows in creating knowledge externalities may result with non-optimal policy measures.

This paper demonstrates the ability of numerical solution and simulation of rather simplified model to analyze the main features related to industry' life cycle. The findings verify the importance of studying the incentives to protect or diffuse knowledge in order to better understand empirical patterns of economic growth. Thus, economic research should not focus on the dynamics of the incentives to innovate, but also on the dynamics of the incentive to share knowledge. Both factors are responsible for generating the positive knowledge externalities in the economy, nonetheless, only the former was studied in depth in the economic literature so far.

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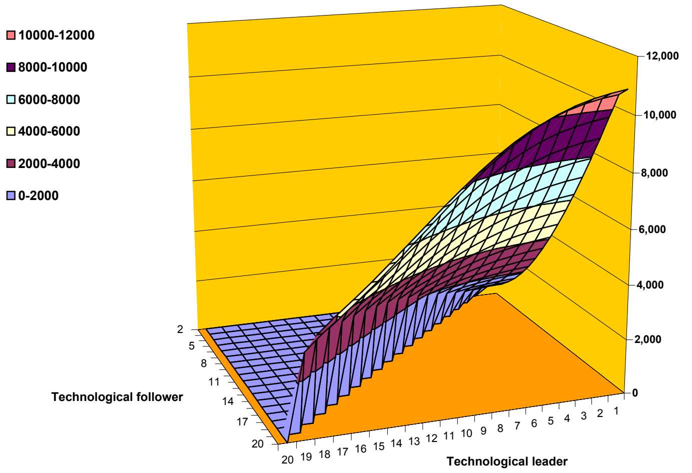
9

10 Appendix



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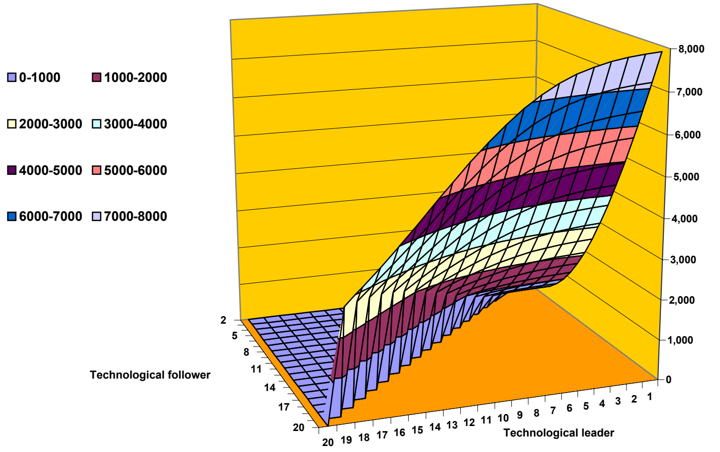

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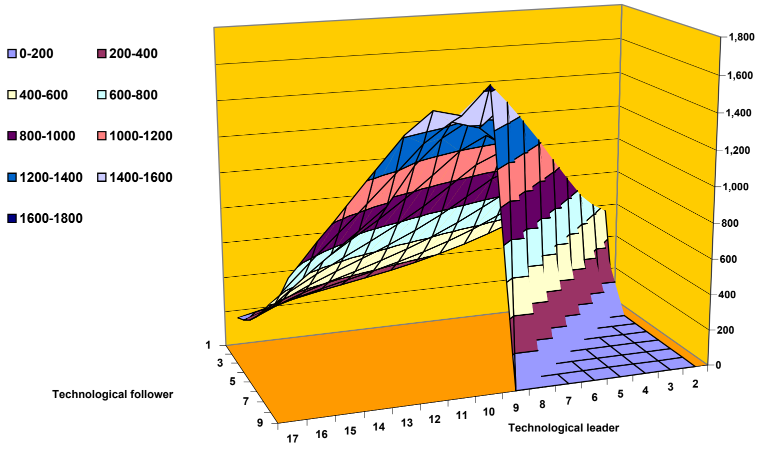

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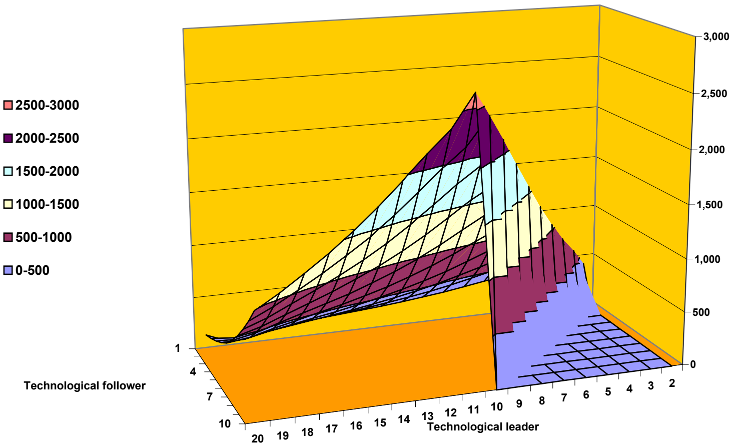

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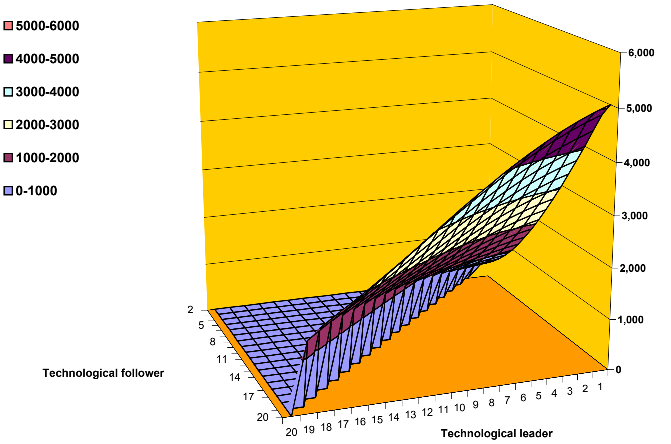

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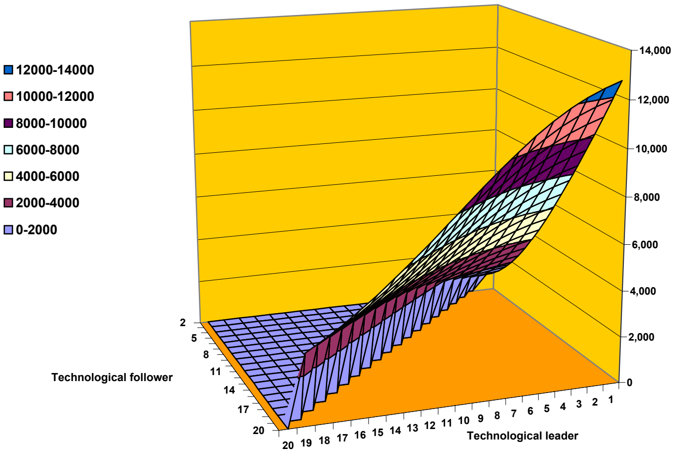

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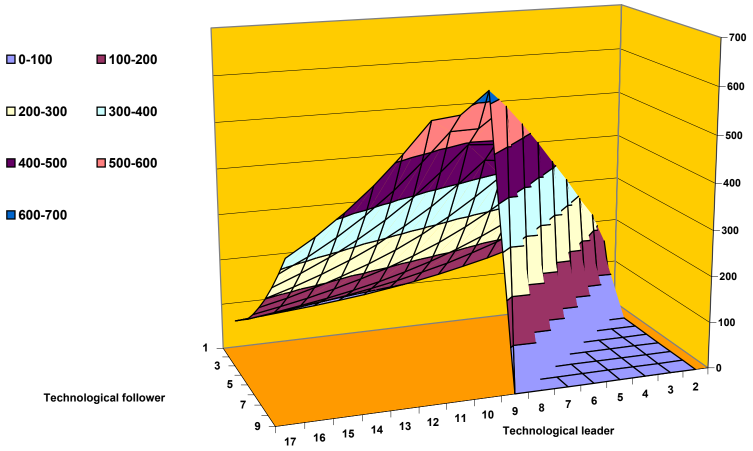

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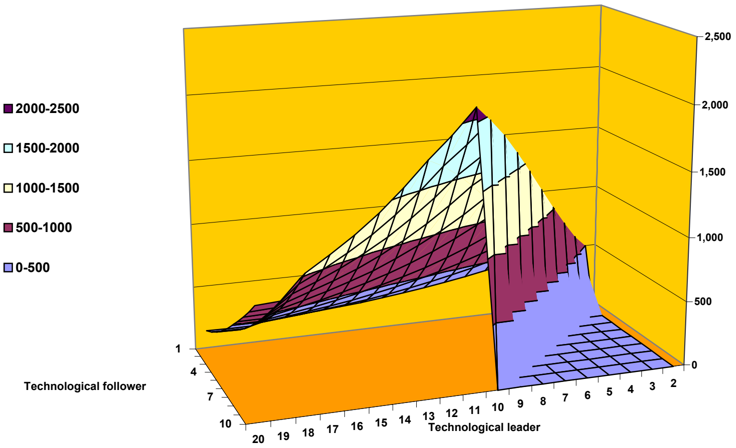

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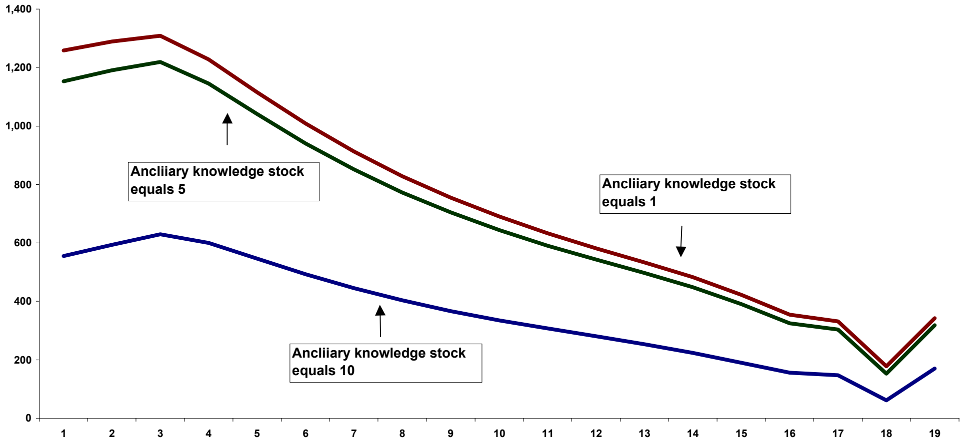

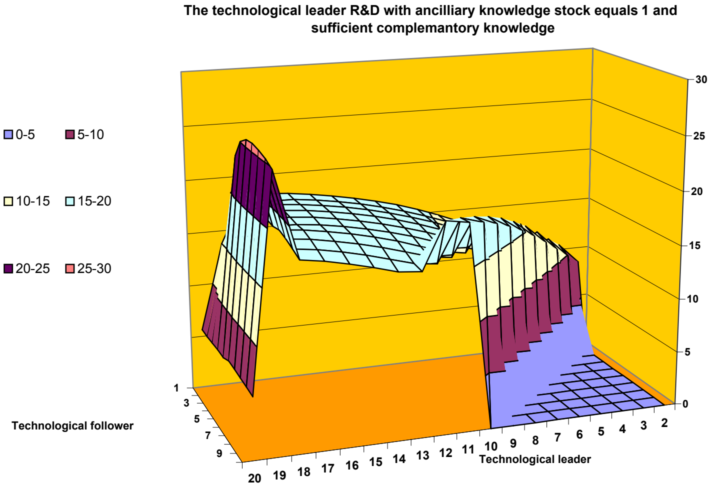
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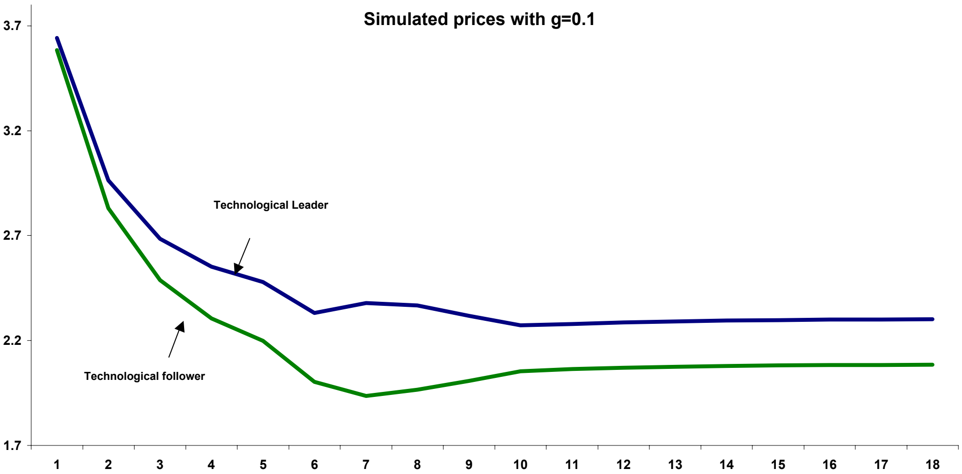
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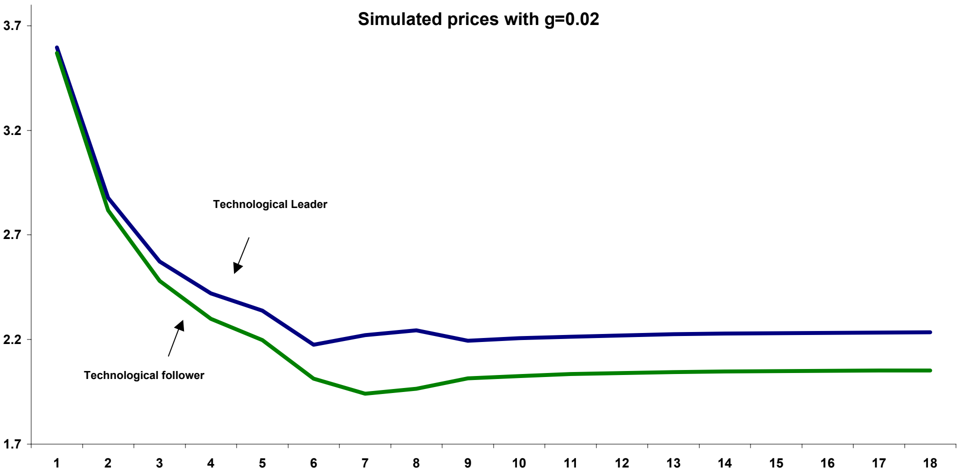
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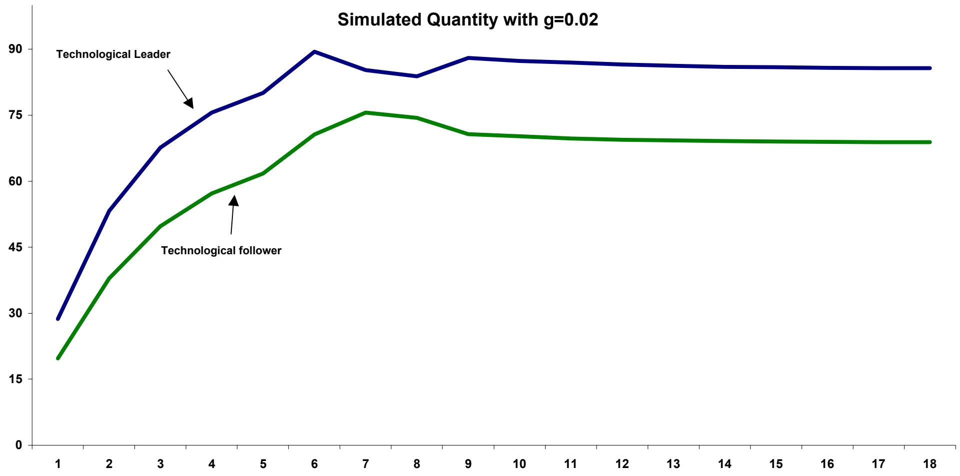
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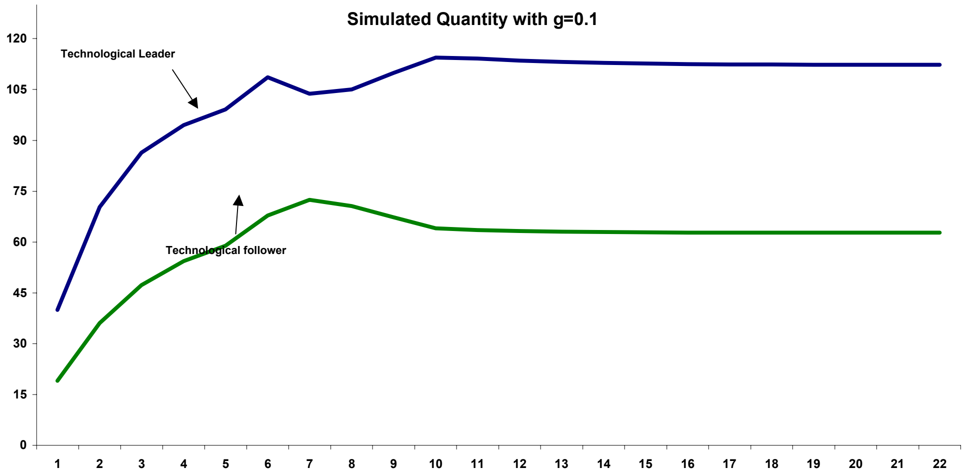
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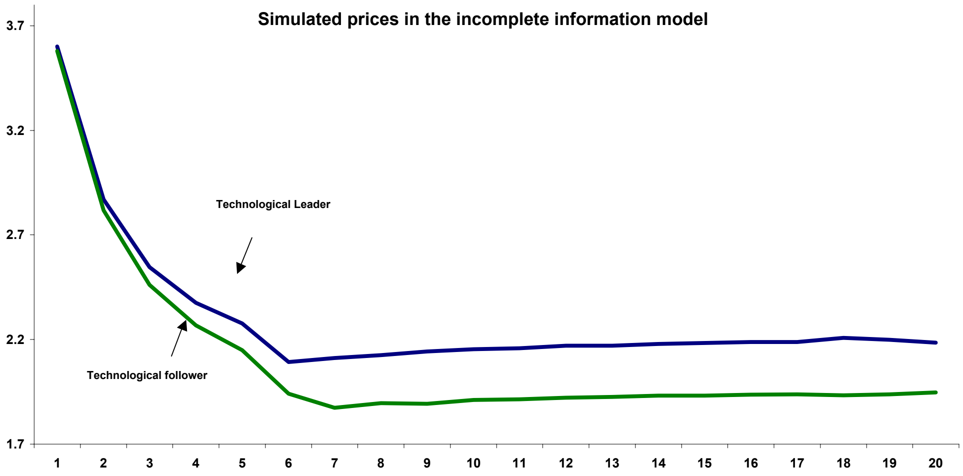
Simulated prices in the incomplete information model
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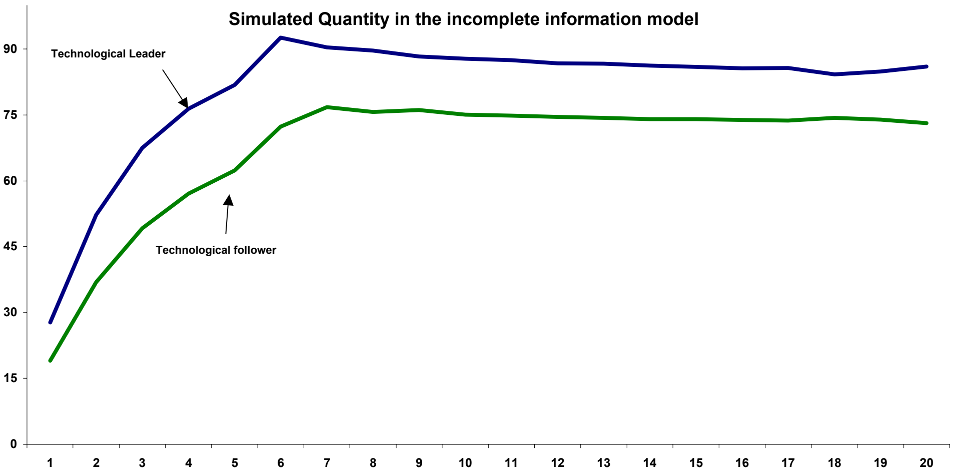
Simulated Quantity in the incomplete information model
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