Limited Dependent Panel Data Models: A comparative Analysis of classical and Bayesian Inference among Econometric Packages

Giuseppe Bruno [1]

May 2004

[1] Bank of Italy Research Department.
e-mail: giuseppe.bruno@bancaditalia.it

Contents

1 Introduction

Advances in computing power have opened the way to the use of intensive computional techniques to solve and estimate nonlinear panel-data models, specifically those arising from nonlinear panel data such as Probit and Tobit models. For these models, allowing a flexible specification for the correlation induced by firm/individual heterogeneity leads to models involving T-variate multiple integration whose numerical approximation can sometimes be very poor. In these cases when the value of T is greater then 4 or 5 maximum-likelihood estimation can be cumbersome if not analytically intractable. Different solutions are offered based variously on integral approximation through simulation, some form of Generalized Method of Moments (GMM), or Markov Chain Monte Carlo (MCMC) methods. This paper compares the outcomes of those methods available in standard econometric packages, providing illustrations between prepackaged algorithms and a MCMC Gibbs sampler for nonlinear panel data. Using Chib (1992) and Chib and Carlin (1999), We derive a sampler for Probit/Tobit panel data and provide easy-to-use software for implementing the Gibbs sampler in panel data with discrete/limited dependent variable. We show that, when dealing with a large dataset, MCMC methods may replace the procedures provided in standard econometric packages.

2 The Panel Tobit Model

Panel datasets provide a very rich source of information for empirical economists, providing the scope to control for individual heterogeneity. While there is a large literature on linear panel data models, less is known about limited dependent variable models. This is especially true for computational comparisons among different methods.

Extending the work of Chib (1992) and Contoyannis et al. (2002) this paper is concerned with the Bayes estimation in the panel Tobit model.

The purpose of the paper is two-fold: first to develop an easy-to-use Bayesian estimation approach, and second to compare the efficacy of this method with respect to others proposed in some econometric packages.

We are concerned with a standard Panel data Tobit model:

$$y_{it}^* = \beta' x_{it} + u_{it} \qquad i = 1, 2, \ldots, N \qquad t = 1, 2, \ldots, T_i \tag{2.1}$$

$$u_{it} = \nu_i + \epsilon_{it} \qquad (\nu_i \sim NID(0, \sigma_\nu^2)) \qquad (\epsilon_{it} \sim NID(0, \sigma_{\epsilon^2})) \tag{2.2}$$

where the observed variables is:

$$y_{it} = \begin{cases} y_{it}^* & if \quad y_{it}^* > 0 \\ 0 & otherwise \end{cases} \tag{2.3}$$

In general the common error term u_{it} in equation (2.2) could be freely correlated over time. Here we consider the error components model which splits the error u_{it} into a time-invariant individual random effect (RE): ν_i, and a time-varying idiosyncratic random error: ϵ_{it}.

In this case the assuming independence between the ν's and the ϵ's, $d_{it} = 1$ for uncensored observations and $d_{it} = 0$ for censored observations the likelihood contribution for each individual, marginalized with respect to the random effect ν_i is

$$l_{it} = \int_{-\infty}^{\infty} \left[\frac{1}{\sigma_\epsilon} \phi \left(\frac{y_{it} - \beta' x_{it} - \nu_i}{\sigma_\epsilon} \right) \right]^{d_{it}} \cdot \left[\Phi \left(\frac{-\beta' x_{it} - \nu_i}{\sigma_\epsilon} \right) \right]^{(1-d_{it})} f(\nu_i, \sigma_i) d\nu_i \tag{2.4}$$

where: $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution, $f(\nu_i, \sigma_i)$ is normal density with mean ν_i and standard deviation σ_i.

In general for T_i observations belonging to individual i we have the following likelihood contribution

$$L_i = \int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{t=T_i} \left[\frac{1}{\sigma_\epsilon} \phi \left(\frac{y_{it} - \beta' x_{it} - \nu_i}{\sigma_\epsilon} \right) \right]^{d_{it}} \left[\Phi \left(\frac{-\beta' x_{it} - \nu_i}{\sigma_\epsilon} \right) \right]^{(1-d_{it})} \right\} f(\nu_i, \sigma_i) d\nu_i \tag{2.5}$$

from this we see that the likelihood function for the whole sample is the product of the contribution L_i over the N individuals, and the log-likelihood is

$$\mathcal{L} = \sum_{i=1}^{N} ln(L_i) \tag{2.6}$$

As can be seen, the log likelihood in equation (2.6) does not collapse in a sum, as it would in the case of a time series or a simple cross-sectional Tobit model. The reason for this is that the likelihood function for individual i is an integral of a product instead of just a product and the log operator cannot be carried through the integral sign.

The situation gets even more complex in presence of serial correlation of the disturbance for each individual. In this case the lack of independence among the observations prevents the possibility of factoring out the likelihood contribution of the T_i periods for the i individual and we end up with a T-dimensional integral that makes classical estimation methods infeasible when the number of time periods is more than three or four.

3 Classical Maximum Likelihood Estimation

In the earlier section we saw that feasible maximum likelihood estimation for limited dependent variable panel data is available only for a particularly simple structure of the random disturbance. We have therefore analyzed the behaviour of the algorithms for the panel Tobit models available in following two packages: LIMDEP and STATA[1].

Econometric literature proposes two kind of models for linear panel data: Fixed vs. Random Effects models. The former do not impose any correlation restriction between the individual effects and the other explanatory variables but with nonlinear models MLE is generally known to be biased (see for example Heckman (1981) and Hsiao (1996)). The case of random effects model is much more parsimonious in the number of parameters but it requires some restrictive assumptions on the distribution of the individual effects. The main assumptions for the applicability of the random effects model are the following:

1) the idiosyncratic error ϵ_{it} is serially uncorrelated;

2) the individual effects ν_i are uncorrelated across individual.

3) $\nu_i|\mathbf{x_i} \sim \mathbf{NID}(\mathbf{0}, \sigma_\nu^{\mathbf{2}})$

The previous assumptions simplify the computation of the likelihood. Indeed only LIMDEP provides a *brute force* Maximum Likelihood estimation method for the fixed-effect model (see Greene (2003)). Both STATA and LIMDEP approach the estimation of the random effects model

[1] In this analysis we have used LIMDEP 8.0 under Windows and STATA 8.2 under UNIX AIX/RISC6000.

by taking advantage of the Gauss-Hermite quadrature for the likelihood computation as suggested in Butler and Moffit (1982) .

LIMDEP provides a built-in **tobit** command aimed at solving both cross-section and four different models for unbalanced panel data including the fixed-effects. STATA supplies the built-in **xttobit** command which in the current version allows only for the random effects model. For the numerical applicability of the Maximum Likelihood method, an important role is played by two elements:

1) the panel size, that is the number of observation for each individuals,

2) the correlation between the total latent error across any two time periods: $\rho = \frac{\sigma_\nu^2}{\sigma_\epsilon^2 + \sigma_\nu^2}$.

An high value of panel size combined with an high across time correlation will usually give place to poor approximation. This qualitative consideration will be reconsidered discussing the empirical applications.

4 Gibbs sampling

The Gibbs sampler is a Monte Carlo Markov Chain method of sampling probability densities which are usually analytically intractable.

This method, also called *alternating conditional sampling*, has made possible the Bayesian approach to the estimation of nonlinear panel data models providing accurate finite sample estimates.

Gibbs sampling is based on a preliminary splitting up of the parameter vector into s groups, $\theta = (\theta_1, \ldots, \theta_s)$, in our panel Tobit model the parameter vector is already subdivided according to $\theta = (\beta, \sigma_\epsilon, \sigma_\nu)$. Now the goal is to obtain draws from the posterior distribution of the parameter vector conditional on the data.

The sampling is carried out at each iteration alternating among the different conditional distribution, in our case we have the following procedure:

1) pick arbitrarily initial values for $\Theta^0 = \beta^0, \sigma_\epsilon^0, \sigma_\nu^0$ (GLS random effects will serve the purpose);

2) draw β^k from the distribution $\pi(\beta | y, \mathbf{x}, \sigma_\epsilon^{\mathbf{k-1}}, \sigma_\nu^{\mathbf{k-1}})$

3) draw σ_ϵ^k from the distribution $\pi(\sigma_\epsilon | y, \mathbf{x}, \beta^{\mathbf{k-1}}, \sigma_\nu^{\mathbf{k-1}})$

4) draw σ_ν^k from the distribution $\pi(\sigma_\nu|y, \mathbf{x}, \beta^{\mathbf{k-1}}, \sigma_\epsilon^{\mathbf{k-1}})$

under very general assumptions, after a certain number of iteration, this process will produce samples from the wished posterior distribution. Therefore point estimates and confidence interval are computed as averages from the generated sample.

To implement the sampler we need to specify the different conditional pdf's. For each of the three groups of parameters we used non informative conjugate priors for simplifying all the computations. This means we adopted the following distributions:

$$\beta \sim \mathcal{N}(\beta_0, \Omega_0) \tag{4.7}$$

$$\sigma_\nu^2 \sim \mathcal{IG}(\eta_0, \gamma_0) \tag{4.8}$$

$$\sigma_\epsilon^2 \sim \mathcal{IG}(\nu_0, \delta_0) \tag{4.9}$$

Where all the variables indexed by 0 are the hyperparametes of our distributions, $\mathcal{N}(\mu, \Sigma)$ is the multivariate Normal distribution with mean vector μ and variance-covariance matrix Σ and $\mathcal{IG}(\nu, \delta)$ is the inverse Gamma distribution with shape ν and scale δ^2.

A peculiar feature of nonlinear panel data models such as the Tobit is the presence of unobservable latent data that would make very complex the previous sampling loop. To this end following the suggestion of Tanner and Wong (1987), adopted by Chib (1992) in a cross-sectional context, we enriched the Gibbs sampler by means of the data augmentation strategy. In our set up, given the assumptions underlying the model, the distribution of the latent variables are truncated normal. Therefore we can augment our dataset with an estimate for the censored variables. Using the augmented dataset we brought the problem back into a classic linear panel data model.

The Gibbs sampler previously described has been modified by adding at the beginning a step for sampling the censored variables. For example in case (2.3) we have to simulate a random sample from a truncated normal distribution with support $(-\infty, 0)$ and pdf given by:

$$y_{it}^* \sim \frac{\mathcal{N}(x_{it}'\beta + \nu_i, \sigma_\nu^2 + \sigma_\epsilon^2)}{1 - \Phi\left(\frac{x_{it}'\beta}{\sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}\right)} \tag{4.10}$$

To sample from the truncated normal in (4.10) we used the one-for-one draw technique described in Hajivassiliou and McFadden (1990) that is much more cost effective than the acceptance-rejection method.

[2]The inverse-χ^2 with ν degrees of freedom is an inverse Gamma distribution with $\alpha = \nu/2$ and $\beta = 1/2$

At this point we can give a complete picture of the algorithm implemented for estimation of the random effect Tobit model:

1) run a GLS estimation with the original truncated data to fix the initial values for $\beta^0,\ \sigma_\epsilon^0,\ \sigma_\nu^0$;

2) sample the censored variables from the pdf 4.10 to build the augmented dataset;

3) run a GLS estimation on the panel with the augmented dataset for computing new mean values for $\beta,\ \sigma_\epsilon,\ \sigma_\nu$;

4) draw $\beta^{\mathbf{k}}$ from the distribution $\pi(\beta|\mathbf{y},\mathbf{x},\sigma_\epsilon^{\mathbf{k-1}},\sigma_\nu^{\mathbf{k-1}}) = \mathcal{N}(\beta,\mathbf{\Omega})$

5) estimate the individual effects using the residuals from the previous step ;

6) draw σ_ν^k from the distribution $\pi(\sigma_\nu|y,\mathbf{x},\beta^{\mathbf{k-1}},\sigma_\epsilon^{\mathbf{k-1}}) = \mathcal{IG}\,(\nu_{\mathbf{0}},\delta_{\mathbf{0}})$

7) draw σ_ϵ^k from the distribution $\pi(\sigma_\epsilon|y,\mathbf{x},\beta^{\mathbf{k-1}},\sigma_\nu^{\mathbf{k-1}}) = \mathcal{IG}\,(\eta_{\mathbf{0}},\gamma_{\mathbf{0}})$

The sampler has been written using the language and computing framework provided by the Modeleasy+ software (see appendix 7).

5 The empirical application

To illustrate the behaviour of the sampler we used a dataset available from STATA [3]. This is an unbalanced dataset composed by 19151 observations on 4140 individuals taken from the National Longitudinal Survey on economic and demographic variables.

Using the example provided in the STATA manual, we fit a random-effects Tobit model on the log of wages against a set comprising the following explanatory variables:

1) **union** dummy variable equal to 1 if the individual belongs to a workers' union;

2) **age** the individual's age ;

3) **grade** the years of schooling completed;

4) **not_smsa** dummy variable equal to 1 if the individual doesn't live in a standard metropolitan statistical area (smsa);

[3] The dataset nlswork.dta is downloadable from the URL http://www.stata-press.data/r8

5) **south** dummy variable equal to 1 if the individual lives in the south;

6) **southXt** interacting-term variable indicating how long the individual is living in the south;

7) **occ_cod** categorical variable indicating the occupational code of the individual (larger numbers mean a lower rank);

This model has been estimated using the random effect method in LIMDEP and STATA. Even if both package use the Maximum Likelihood procedure with the Hermite quadrature formulae we got results that are quite different from the numerical viewpoint.

The results are summarized in the following table:

Table 1: Parameter Estimates for the nls dataset (19151 obseravations and 4140 individuals)

		LIMDEP	STATA	Gibbs Sampler
const	β_0	.75297 (.02649)	.56572 (.03308)	.63202 (.02953)
union	β_1	.15946 (.00533)	.15449 (.00698)	.14632 (.00684)
age	β_2	.00785 (.00038)	.00871 (.00054)	.00788 (.00005)
grade	β_3	.06653 (.00179)	.07803 (.00216)	.07332 (.00191)
nots_msa	β_4	-.13871 (.00609)	-.12669 (.00898)	-.12408 (.00843)
south	β_5	-.12874 (.00887)	-.11686 (.01224)	-.11764 (.01133)
southxt	β_6	.00263 (.00060)	.00309 (.00084)	.00358 (.00008)
occ_code	β_7	-.01952 (.00078)	-.01829 (.00111)	-.01749 (.00106)
	$\boldsymbol{\sigma_\epsilon}$	.2542 (.0010)	.2483 (.0018)	.2378 (.00010)
	$\boldsymbol{\sigma_\nu}$	.3341 (.0039)	.2911 (.0048)	.2582 (.00219)

Notes: Standard errors in parentheses

It is remarkable the similarity between the estimates provided by the Gibbs Sampler and those given by LIMDEP and STATA. Here it is important to highlight the simplicity of the sampler with respect to the classical algorithms including quadrature approximations and maximization.

When the error term features any kind of serial correlation Bayesian techniques seem to be the only feasible techniques.

6 Monte Carlo Simulation

Starting from the previous results we have decided to establish a sort of a benchmark. We have generated a dataset according to the following process (see Harris et al (2000)) :

$$y^*_{it} = \beta_0 + \beta_1 x1_{it} + \beta_2 x2_i + \nu_i + \epsilon_{it} \quad \nu_i \sim NID(0, 1.0) \qquad \epsilon_{it} \sim NID(0, 1.0) \tag{6.11}$$

where the mapping from the latent variable to the observed variable was

$$y_{it} = \begin{cases} y^*_{it} & if \quad y^*_{it} > 0 \\ 0 & otherwise \end{cases} \tag{6.12}$$

The values for the three β's were 0.5, -1 and 1 respectively. These values give about a 50% split among censored and non-censored variables.

Values of x_{it} follow an auto-regressive process given by

$$x1_{it} = 0.1 \cdot trend + 0.5 \cdot x1_{i,t-1} + u_{it} \tag{6.13}$$

where $u_{it} \sim U(-.5, .5)$.

The time invariant variable x_{2i} was generated according to

$$x_{2i} = \begin{cases} 0 & if \quad 0 \leq x^*_{2i} < 0.5 \\ 1 & if \quad 0.5 \leq x^*_{2i} \leq 1 \end{cases} \tag{6.14}$$

where the latent variable is generated according to $x^*_{2i} \sim U(0, 1)$.

The individual specific effects were generated according to $\nu_i \sim NID(0, \sigma^2_\nu)$ where σ_ν was specified as 1 and 2 to provide two values of correlation over time. For the idiosyncratic random error term we chose the following $\epsilon_{it} \sim NID(0, 1)$. We carried out the simulation using a panel of 100 and 200 individuals with 3, 6 and 12 time periods.

Examining the results in the following tables one can observe that, for small T ML method seems to outperform the Gibbs sampler. When T was increased the Gibbs sampler produced estimates with standard errors smaller than those achieved with the Maximum Likelihood. LIMDEP is the package giving the smaller bias in this experiment, further investigation should be done to pin down the reason of that.

These results seem to indicate that the MCMC method may be preferred when T is large.

Table 2: Monte Carlo Parameter Estimates for T= 3

	true parameters	*N = 100*		*N = 200*	
	σ^2_ν	1	2	1	2
LIMDEP	$\beta_0 = .5$	.49251 (.17447)	.50126 (.40942)	.49963 (.13044)	.50492 (.31964)
	$\beta_1 = -1$	-1.00833 (.15673)	-1.00189 (.21836)	-1.0005 (.12094)	-.99568 (.16022)
	$\beta_2 = 1$	1.00614 (.23524)	1.02014 (.57546)	.99512 (.17795)	.99185 (.43363)
STATA	$\beta_0 = .5$	.50665 (.18361)	.58218 (.43688)	.48599 (.12084)	.53866 (.30257)
	$\beta_1 = -1$	-.99535 (.17466)	-.99622 (.22397)	-1.008711 (.12614)	-1.00074 (.15656)
	$\beta_2 = 1$	.98447 (.23816)	.90551 (.54675)	.99246 (.16304)	.972846 (.41163)
Gibbs Sampler	$\beta_0 = .5$	.34488 (.19324)	.25174 (.3223)	.40712 (.12252)	.27154 (.23569)
	$\beta_1 = -1$	-1.4212 (.17854)	-1.5717 (.26103)	-1.0413 (.10936)	-1.1164 (.1666)
	$\beta_2 = 1$	1.4782 (.25057)	1.3316 (.42141)	1.2843 (.1541)	1.3979 (.29773)

Notes: Average parameter estimates over 1000 Monte Carlo replications with Mean Squared errors in parentheses

Table 3: Monte Carlo Parameter Estimates for T= 6

	true parameters	*N = 100*		*N = 200*	
	σ_ν^2	1	2	1	2
LIMDEP	$\beta_0 = .5$	.50077 (.15350)	.60865 (.45406)	.50191 (.11994)	.56321 (.34987)
	$\beta_1 = -1$	-1.00293 (.10386)	-1.0009 (.12187)	-1.00338 (.07276)	-.99789 (.08684)
	$\beta_2 = 1$	1.00381 (.23508)	.95696 (.61510)	1.0005 (.16837)	.98633 (.49703)
STATA	$\beta_0 = .5$	.48925 (.17993)	.73507 (.51041)	.50136 (.12651)	.71314 (.43339)
	$\beta_1 = -1$	-.99317 (.10384)	-.99398 (.12044)	-.98986 (.07018)	-.99998 (.09130)
	$\beta_2 = 1$	1.0046 (.23267)	.76978 (.65634)	.98526 (.16087)	.82756 (.53801)
Gibbs Sampler	$\beta_0 = .5$	.44098 (.16824)	.22519 (.3081)	.46997 (.11271)	.31492 (.21301)
	$\beta_1 = -1$	-1.0914 (.10101)	-1.0801 (.14672)	-.94969 (.07052)	-.94218 (.09632)
	$\beta_2 = 1$	1.2695 (.22202)	1.1548 (.39303)	1.1752 (.14297)	1.2777 (.28197)

Notes: Average parameter estimates over 1000 Monte Carlo replications with Mean Squared errors in parentheses

Table 4: Monte Carlo Parameter Estimates for T= 12

	true parameters	*N = 100*		*N = 200*	
	σ^2_ν	1	2	1	2
LIMDEP	$\beta_0 = .5$	.49836 (.18473)	.51373 (.42772)	.50552 (.13403)	.55766 (.33558)
	$\beta_1 = -1$	-1.00137 (.05735)	-1.0001 (.06967)	-1.00004 (.0403)	-.99499 (.04589)
	$\beta_2 = 1$	.99662 (.26741)	.88625 (.55242)	.98522 (.20670)	.86962 (.47691)
STATA	$\beta_0 = .5$	.51550 (.21648)	.84474 (.49741)	.50769 (.20163)	.74664 (.47334)
	$\beta_1 = -1$	-.99929 (.051797)	-.99316 (.06029)	-.99649 (.04159)	-.99346 (.04356)
	$\beta_2 = 1$	.97826 (.27569)	.65932 (.63651)	1.02023 (.22279)	.78933 (.58367)
Gibbs Sampler	$\beta_0 = .5$	.50481 (.1593)	.24922 (.31227)	.51361 (.10384)	.37178 (.20215)
	$\beta_1 = -1$	-1.1088 (.06301)	-1.0583 (.092724)	-.99628 (.04319)	-.97195 (.05606)
	$\beta_2 = 1$	1.1271 (.1995)	1.0106 (.39059)	1.0897 (.13083)	1.1451 (.26029)

Notes: Average parameter estimates over 1000 Monte Carlo replications with Mean Squared errors in parentheses

7 Concluding remarks and further research

In the paper we have compared two methods for the estimation of nonlinear panel data. The first method is the Classical Maximum Likelihood (ML) with quadrature for the computation of the likelihood function. Both the LIMDEP and STATA canned procedures have been used. Moreover a Bayesian approach based on the Gibbs sampling has been developed in the Modeleasy+ computing environment. We have taken advantage of the data augmentation technique proposed in Tanner and Wong (1987) for simplifying the analytics involved in the computation of the conditional posterior pdf's. The three procedures have been applied to a segment of the national longitudinal survey on labour statistics. Although the parameters estimates have always the right signs there are remarkable numerical difference among to two procedures implementing the ML with the same quadrature formulae. Estimates from the Gibbs sampler are close to the range defined by the two ML estimates. The Gibbs sampler is much easier than ML from the computational standpoint but so far computing time is still much bigger.

Finally to validate the methods some Monte Carlo experiments were presented. For a given level of T, increasing N has produced an increased precision.

The Gibbs sampler seems useful when either the panel size is large or there is an high value of residual correlation. It should be remarked that when there is a more complex correlation structure in the disturbance, such as a first order autocorrelation, quadrature formula become cumbersome and inaccurate making the MCMC methods one simple solution to the estimation problem. A future line of study should consider the development of code for models with a generalized correlation structure.

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APPENDIX

Sample of the Modeleasy+ code for the Gibbs sampler

```
for i=1,ndraw;
type "draw number " i
$ Generate the latent variables from the a truncated normal distribution
$ Initialize xpvx and xpvy matrices
          xpvx  = matrix(nreg,nreg)
          xpvy  = matrix(nreg,1)
          stdevt = sqrt( sige + sgd )
          mi1 =   a1d( ux(icensor) * bnew )
          leftb  = mi1 - 5.*stdevt
          sigma2 = sige + sgd
          nrtrun1 (ne1,mi1,sigma2,seed,leftb,rightb,vu1)
          ytotnew(icensor) = vu1

          pdreg( harris: ytotnew c xi1t x2i :RE)
          rss  = 0             $ Total Residual Sum of squares
          for j = 1, nind
            irow = ints(vinz(j),vfin(j)) ; nirow = tprd(j)
            object(yvi(j)) = matrix(nirow,1:ytotnew(irow) )
            object(vci(j)) = inverse(toeplitz(nirow:a1d(nirow-1)+sgd,sigma2))
          next j
          vctildai = sgd*sige / (sige+sgd*tprd)
          sgvctild = sige**(-1)*vctildai
          multcall nout XprimeQX xmi vci
          xpvx = sumlist (nout)
          vcvtilda = inverse(vcvbs + xpvx)
          chf  =  pdfac(vcvtilda)
$  updating the slope coefficients
         bnew =  mfam(chf*vector(normrand(ints(nreg))))+ vec(coeff)
$  updating the random effect individual variance
           yres = ytotnew - afam(ux * bnew)
           for j = 1, nind
           ycur  = yres(ints(vinz(j),vfin(j)))
```

```
          bitildi(j) = sgvctild(j) * sum(ycur)
          rss=rss + sumsq(ycur - bitildi(j) )
          bitildi(j) = sqrt(vctildai(j))*normrand(nind)+bitildi(j)
          next j

          rtilda = ( R0 + sumsq(bitildi) )
$ Sample the random effects variance
          sgd = 1/ gammarandom((m0+nind)/2,2/(rtilda))
          delta = (dl0 + rss)
$ Sample the total variance
          sige = 1 / gammarandom( (n0 + nobs)/2,2/delta)
$ Store the different simulated values
          if (i .gt. nomit) then
              kb = i - nomit
              rbeta(kb,) = bnew
              rsigma(kb) = sige
              rsgind(kb) = sgd
          endif
next i
```