# Animal Spirits, Lumpy Investment, and Endogenous Business Cycles*

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**Abstract**

In this paper, we study whether the most robust stylized facts characterizing the coupled dynamics of output and investment can be explained in terms of bounded rationality (i.e. "animal spirits") in firm investment behavior and the ensuing lumpiness in investment patterns. We present an evolutionary, agent-based, model of industry dynamics describing an economy composed of consumers and firms. Firms belong to two industries. Firms in the first industry perform R&D and produce heterogeneous machine tools. Firms in the second industry invest in new machines and produce a consumption good. Manufacturing firms invest only if they expect a large growth in the demand for their product. Preliminary simulation results show that the model is able to reproduce the most important empirically-observed statistical properties of investment-output dynamics. More specifically, we find that a necessary condition for the economy to exhibit self-sustaining patterns of growth is the presence of some additional (exogenous or endogenous) component to private consumption. We also show that, in these cases, the model is able to generate simulated output-investment dynamics characterized by volatility, auto- and cross-correlation patterns similar to those observed in reality.

**Keywords**: Evolutionary Models, Agent-Based Computational Economics, Animal Spirits, Lumpy Investment, Output Fluctuations, Endogenous Business Cycles.

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# 1 Introduction

The existence of widespread and persistent fluctuations which permanently affect the overall economic activity is an inherent feature of any modern economy.

However, despite the huge number of competing models providing a rationale for expansions and recessions, we still lack a generally accepted explanation for business fluctuations. More specifically, there seems to be an increasing dissatisfaction in the economic profession about the way in which economic theory copes with empirically observed properties of business cycles. As Zarnowitz (1985, 1997) puts it, economic scholars are "mainly concerned with theoretical possibilities, rather than with explanations of what actually happens". Consequently, there is "little regard for how the pieces fit each other and the real world". Ultimately, the theory of business cycle appears to be "long of questions and short of answers".

A primary example of such a mismatching might be found in the ways economic theory deals with both microeconomic and macroeconomic stylized facts concerning investment and output dynamics. A robust empirical literature has indeed shown that, at the aggregate level, investment is considerably more volatile than output and that fluctuations of both output and investment are highly synchronized. Furthermore, at the micro level, firms' investment behavior appears to be lumpy and strongly affected by firms' financial structure.

Notwithstanding the proliferation of models trying to separately account for micro and macro stylized facts, almost no attempts have been made in the literature to explain the properties of aggregate investment and output dynamics on the basis of individual behaviors which, at least, embody the observed microeconomic regularities about firms' investment behaviors.

In this paper, we begin pursuing this strategy and we propose a model where both output and investment dynamics are grounded upon lumpy investment decisions undertaken by firms that are also constrained by their financial structure.

The model depicts a two-industry dynamic economy composed of firms and consumers/ workers. Firms in the first industry perform R&D and produce heterogeneous machine tools. Firms in the second industry invest in new machines and produce a consumption good.

Following the seminal work of Keynes (1936) on "trade cycles", we assume that pervasive market uncertainty, as well as individual expectations, play a key role in shaping investment dynamics and triggering fluctuations in the overall economic activity. More specifically, we model firms and workers as heterogeneous, boundedly-rational agents, endowed with adaptive expectations, who directly interact in an endogenously-changing environment characterized by strong procedural and substantial uncertainty (Dosi and Egidi, 1991; Dosi, Marengo, and Fagiolo, 2004).

Partly as a consequence, we interpret lumpiness in firm investment decisions as a boundedly-rational, routinized, behavior, rather than deriving it as the outcome of some optimization rule carried out by a perfectly-rational, forward-looking, individual holding non-convex adjustment costs (Caballero, 1999).

The model belongs to the evolutionary, "agent-based computational economics" (ACE), family. In each period $t$, firms and workers carry out their production, investment, and consumption decisions on the basis of routinized behavioral rules and adaptive (myopic) expectations (i.e. "animal spirits"). The dynamics over microeconomic variables (i.e. individual production, investment, consumption, etc.) thus induces a macroeconomic dynamics for aggregate variables (e.g. total investment and output, consumption, etc.), whose statistical properties are then studied and compared with empirically observed ones.

Preliminary simulation results show that the model is able to reproduce the most important aggregate stylized facts characterizing investment and output dynamics. More specifically, we find that a necessary condition for the economy to exhibit self-sustaining patterns of growth is the presence of some additional (exogenous or endogenous) component to private consumption (e.g. public expenditure, unemployment benefits, etc.). This component, acting as automatic stabilizers, is likely to dampen the oscillations of the manufacturing industry and to reduce the instability of the whole economy. We also show that, under such broad conditions, the model is able to generate simulated output-investment dynamics characterized by volatility, auto- and cross-correlation patterns similar to those observed in reality.

The rest of the paper is organized as follows. Section 2 provides a short overview of micro and macro empirical evidence. In Section 3, we discuss how economic theory has dealt with the stylized facts and we introduce the main ingredients of the model formally presented in Section 4. Qualitative and quantitative results of simulation exercises are accounted for in Section 5. Section 6 concludes and discusses future developments.

# 2 Investment Patterns and the Business Cycle: What Can We Learn from the Data?

In this Section we will single out the most robust stylized facts concerning the micro- and macro-dynamics of investment and output. In particular, we shall argue that the evidence on the empirically observed microeconomic patterns of investment might give us some clues to better understand what happens at the macro level.

## 2.1 Macro stylized facts

A key issue in the empirical business-cycle literature concerns the investigation of the properties of the coupled dynamics of investment and output[1].

A casual inspection of the data shows that, after the WWII, both output and investment appear to have experienced a huge and quite smooth growth in the U.S. (cf. Fig. 1) as well as in other developed countries. However, the observed smoothness in both time-series typically hides severe business cycle turbulences affecting both economic aggregates. If we isolate business cycle frequencies by applying a bandpass filter (Baxter and King, 1999) to the series[2], output and investment exhibit a completely different pattern, see Fig. 2. In fact, the two series display a typical "roller coaster" shape, implying the repeated interchange of expansions and recessions which characterize the business cycle.

In addition to all that, the analysis of the co-variance and auto-correlation structure of the filtered series allows us to single out two key stylized facts which seem to represent investment patterns at the macro level:

**SF1** Investment is considerably more volatile than output.

**SF2** Business cycles fluctuations of investment and output are highly synchronized and exhibit very similar patterns.

Investment and output reach indeed their peaks and troughs at (almost) the same date, but the fluctuations of investment are extremely more pronounced. As Table 3 shows, the percentage deviation of investment from the trend growth path is 2.5 times larger than the one of the GDP ($SF1$). The contemporaneous correlation between investment and GDP is positive and very high (0.95) and it decreases monotonically as the leads and lags increase. Using the business cycle terminology, investment appears to be a procyclical (i.e. cross correlations are positive) and coincident (i.e. the highest cross correlation is at time $t$) variable ($SF2$). Both stylized facts are robust against time, country and detrending technique [3].

[1] At least at the macro level, investment cannot be studied in isolation because its behavior must be linked – in some way – to the business cycle (typically proxied by output dynamics). Evidence on investment in machines and equipment is also reported, in order to better compare the results of the model below (which abstracts from other investment components such as e.g. construction) with real-world data.

[2] Following Stock and Watson (1999), we isolate the frequencies ranging from 6 to 32 quarters and we apply a bandpass filter (6,32,12). Cf. also Appendix A.

[3] See also Agresti and Mojon (2001), Stock and Watson (1999), Kydland and Prescott (1990) and Napoletano, Roventini, and Sapio (2004). Stock and Watson employ a bandpass filter (6,32,12) to US data for the period 1956–1996. Agresti and Mojon apply a bandpass filter (6,40,8) to Euro area series ranging from 1970 to 2000. Kydland and Prescott use a HP filter (1600) using US data from 1954 to 1989. Napoletano, Roventini and Sapio apply a bandpass filter (9,43,12) to Italian data for the period 1970-2002.

## 2.2 Micro stylized facts

The limited success of both neoclassical[4] and $q$ theory[5] in providing a statistically robust explanation of the microeconomic determinants of investment (Caballero, 1999; Hasset and Hubbard, 1996; Chirinko, 1993), has triggered a more careful investigation of the statistical regularities characterizing investment patterns at the microeconomic level. These research efforts have led to the discovery that:

**SF3** Investment is lumpy.

**SF4** Investment is influenced by firms' financial structure.

Consider first $SF3$. In standard investment models, convex adjustment costs and reversibility assumptions guarantee that firms smoothly and continuously adapt their capital stock over time. However, these predictions are at odds with the empirical evidence provided in the seminal work of Doms and Dunne (1998). They employ plant level data to show that lumpiness is an intrinsic feature of firm investment decisions: in a given year, 51.9% of all plants increase their capital stock by less than 2.5%, while the 11% of them raise it by more than 20%. Moreover, within-plant investment patterns show that plants typically invest in every single year, but they concentrate half of their total investment in just three years out of the sixteen under analysis. As it might be expected, if the same analysis is performed at the "line of business" and firm levels, investment patterns are smoother, but still lumpy.

In any case, the microeconomic lumpiness of investment does not appear to be completely filtered away at the macroeconomic level. Aggregate investment fluctuations are indeed influenced by the number of plants incurring in huge investment episodes: the correlation between aggregate investment and the number of plants experiencing their maximum investment share is 0.59.

As far $SF4$ is concerned, the evidence is even more impressive. Since the influential work of Fazzari, Hubbard, and Petersen (1988), a huge stream of empirical literature[6] has been providing evidence against the Modigliani and Miller (1958) theorem. Indeed, if capital markets are imperfect (e.g. because of information asymmetries), the financial structure of the firm is likely to affect its investment decisions. In particular, the cost of external financing is typically higher than that of internal financing. The larger information costs born by each firm, the higher the gap between the cost of internal and external financing.

[4] See Jorgenson (1963) and Hall and Jorgenson (1967).

[5] Cf. Tobin (1969) and Brainard and Tobin (1968).

[6] See Hubbard (1998) for a survey.

These propositions are supported by the evidence provided by the so-called "financial constraints" literature: *ceteribus paribus*, firm investment is significantly correlated with cash flow (a proxy for net worth variation) and the correlation magnitude is higher for those firms that suffer more from information asymmetries plaguing capital market (e.g. young and small firms)[7].

# 3 Explaining Stylized Facts: What Can we Learn from Economic Theory?

While the link between financial constraints and investment decisions can be easily explained within an imperfect information framework (Evans and Jovanovic, 1989; Fazzari, Hubbard, and Petersen, 1996), the fact that observed patterns of investment are lumpy can be reconciled with standard investment models only if one assumes an *ad-hoc* formulation of the cost structure. For example, by positing non-convex adjustment costs, a perfectly-rational, optimizing firm will follow an (S,s)-type of investment behavior (see Caballero (1999) for a survey).

In these models, firms face the problem of choosing the optimal level of capital that maximize their flow of profits. Firms compare the desired stock of capital ($K^*$) stemming from first-order conditions, with the actual stock of capital ($K$). If the capital imbalance $Z \equiv K/K^*$ is different from one, firms invest (or disinvest) only if they can recover the costs of adjusting their stock of capital. The presence of non-convex adjustment costs will force firms to follow an (S,s) rule. Given the optimal target ($l$ and $u$) and trigger ($L$ and $U$) thresholds, with $L < l < 1 < u < U$, firms will invest (disinvest) up to $Z = l$ ($Z = u$) only if their capital imbalance is lower (higher) than the trigger point $L$ ($U$)[8].

These models have been quite successful in explaining investment behavior. Using micro data, Caballero, Engel, and Haltiwanger (1995) found an increasing adjustment hazard, which implies that the larger the capital imbalance, the higher the probability of an investment spike. The same result was confirmed on aggregate data by Caballero and Engel (1999), who also showed that the (S,s) model outperforms the linear one in explaining the behavior of manufacturing investment. Finally, Cooper, Haltiwanger, and Power (1999) found that the probability of a large investment episode is increasing in time since the previous spike.

Notwithstanding the awareness that investment lumpiness may have not trivial conse-

[7] See, among others, Fazzari and Athey (1987), Bond and Meghir (1994), Kaplan and Zingales (1997) and Hubbard (1998). For an alternative point of view, cf. Erickson and Whited (2000).

[8] In presence of large disinvestment costs, investment becomes irreversible and the $(L, l, u, U)$ rule reduces to $(L, l)$.

quences at the macro level, almost no attempts have been made to embed the observed microeconomic investment behavior into a business cycle model[9]. More specifically, a surprisingly little attention has been paid so far to the interpretation of the macroeconomic stylized facts on investment and output discussed above on the basis of the microeconomic evidence on firm investment behavior (cf. $SF3$ and $SF4$).

In this paper, we make a preliminary step in this direction by presenting an evolutionary/ACE model[10] which explores the links between microeconomic investment lumpiness and the properties of the coupled dynamics of aggregate investment and output. The model builds on the Keynesian theory of "trade cycles" (Keynes, 1936), as it recognizes investment instability as the main culprit of economic fluctuations.

Building on earlier works in Chiaromonte and Dosi (1993) and Silverberg, Dosi, and Orsenigo (1988), we describe an economy where firms belong to two different industries. Machine-tool firms produce capital goods, whereas manufacturing firms invest in machine tools and produce a consumption good.

Investment can be either employed to increase the capital stock or to replace existing capital goods. Manufacturing firms plan their expansion investment according to a (S,s) model. However, we depart from the standard lumpy investment literature in modeling firms as boundedly-rational agents. In particular, we assume that firms employ routinized behavioral investment rules (Dosi, 1988) instead of fully-rational, profit-maximizing behaviors *cum* non-convex adjustment costs.

We argue that the assumption of routinized behaviors can be justified by two complementary arguments. On the one hand, one may avoid to resort to *ad hoc* and restricting assumptions such as the peculiar form of adjustment costs function which is needed to rationalize lumpy investment in a standard framework. As a consequence, the most important features singled out by the empirical evidence at the microeconomic level (cf. $SF3$ and $SF4$) can be more naturally embedded within the behavioral repertoire of the firm.

On the other hand, we believe that the target and trigger levels of an (S,s) model might be more easily interpreted in terms of a routinized investment rule, rather than as the outcome of some optimization procedure. Indeed, if firms live in truly evolutionary environments (Dosi, Marengo, and Fagiolo, 2004), they typically face both substantive and procedural uncertainty (Dosi and Egidi, 1991), and they mainly invest to satisfy their expected demand. Hence, the adoption of a (S,s) rule fulfills the goals of a prudent, risk-

[9] An exception is in Thomas (2002). She develops a real business cycle model where firms take their investment decisions according to a (S,s) rule. However, in this model, lumpy investment does not have any significant impact at the macro level, because households preferences for smooth consumption paths sterilize investment lumpiness through price movements (i.e. real wage and interest rate).

[10] More on evolutionary and "agent-based computational economics" (ACE) approaches in economics is in Dosi and Nelson (1994), Dosi and Winter (2002), Epstein and Axtell (1996) and Tesfatsion (1997).

averse, firm. Since firms are not able to fully anticipate their future level of demand, their animal spirits (i.e. demand expectations) are not completely reliable. Therefore, they will decide to expand their stock of capital only if they expect a huge demand growth. Firms will then invest to reach their target level of capital only if the satisfaction of their expected demand requires a capital stock at least equal to their trigger level.

Similarly to what happens for expansion investment, firms employ routines to decide their replacement investment as well[11]. In particular, we introduce heterogenous capital goods and we assume that firms implement their replacement policy through a payback-period routine. In this way, technical change and capital good prices enter in the replacement decisions of manufacturing firms.

Finally, the financial structure of the firm does affect in our model its investment policies (cf. $SF4$). Indeed, the presence of financial constraints might imply that firms cannot fully implement their investment plans. Since by assumption firms are fully rationed in the capital market, they will invest until their net worth is enough to finance their investment plans.

The model, in line with evolutionary/ACE building blocks, allows for network externalities and direct-interaction effects among firms both between- and within-industry. While the former occur through competition (and the ensuing selection), the latter are embodied in firms' investment decisions.

Within this framework, we shall address below two main sets of questions. First, we shall ask whether non-linearities generated at the micro-level by routinized behaviors and direct interactions among heterogeneous firms can endogenously generate business cycles waves without any built-in external shock mechanism (e.g. technology, money supply, etc.)[12].

Second and relatedly, we shall explore whether features such as the multiplier (Kahn, 1931) and the investment-accelerator (Clark, 1917) can endogenously emerge and coevolve in the model, in such a way to generate investment instability and business cycles characterized by the empirically observed stylized facts discussed above (cf. $SF1$ and $SF2$).

[11] This in line with empirical evidence discussed in Feldstein and Foot (1971); Eisner (1972); Goolsbee (1998), who show that replacement investment is typically not proportional to capital stock

[12] Business cycles theories can be (roughly) distinguished in "endogenous" and "exogenous" ones. In "endogenous" theories, trade cycles are an intrinsic feature of the functioning of industrial economies. On the contrary, "exogenous" theories depict pendulum-like economies, which are always in equilibrium unless they are perturbed by a (stochastic) shock. Keynesian theories of business cycles are inherently "endogenous": "animal spirits" originate investment instability, which in turn causes output fluctuations. The new Keynesian and real business cycle theories have instead an "exogenous" nature: short-run fluctuations are respectively the results of monetary or productivity shocks. For an exhaustive analysis of endogenous and exogenous business cycle theories, see Zarnowitz (1985, 1997).

# 4 The Model

We model an economy populated by $F$ firms and $L$ workers/consumers. Firms are split in two industries: there are $F_1$ machine-tools firms (labeled by $i$ in what follows) and $F_2$ manufacturing firms (labeled by $j$). Of course, $F = F_1 + F_2$. Machine-tool firms produce heterogenous capital goods and perform R&D. Manufacturing firms invest in machine-tools and produce a homogeneous product for consumers. Workers inelastically sell labor to firms in both sectors and fully consume the income they receive. Investment choices of manufacturing firms determine the level of income, consumption and employment in the economy. There is no financial market and time is discrete.

In the next subsection, we shall firstly describe in a telegraphic way the dynamics of events in a representative time-period. Next, we shall provide a more detailed account of each event separately.

## 4.1 Dynamics

In any time period $t = 1, 2, ...$, the timeline of events runs as follows[13]:

1. Manufacturing firms take their production and investment decisions. According to their expected demand, firms fix their desired production and, if necessary, invest to expand their capital stock. A payback period routine is employed to set replacement investment. Firms may be forced to reduce (or postpone) their investment if their net-worth is too low.

2. Capital-goods market clears. Market shares allocate the total demand to each machine-tool firm. Market shares change according to the evolution of the competitiveness of each firm thanks to a replicator dynamics. Firms compute their profits and update their net-worth.

3. Consumption-good market clears. Manufacturing firms update their productivity and their capital stock. Production takes place. The size of the consumption-good demand depends on the number of workers employed by firms. Manufacturing firms receive a fraction of the total demand in proportion to their market shares, update their inventories, and compute profits. Net-worth and market-shares dynamics takes place (as it happens in the machine-tool industry).

[13] All updating steps are carried out using a "parallel updating scheme". More specifically, all firms have simultaneously access to the updating step and base their decisions on the most recent observation of the variables affecting their updating decision.

4. Entry, exit, and technical change occur. Firms experiencing a negative net-worth and/or a null market-share exit and they are replaced by new firms. Incumbent machines depreciate and new machines are developed.

Finally, unemployment rate and the monetary wage are accordingly computed. Total consumption, investment, change in inventories, and total product are obtained by aggregating individual time-$t$ quantities. Therefore, the dynamics of microeconomic decisions generated through points 1-4 induce a dynamics over macroeconomic variables.

## 4.2 Investment

Manufacturing firms use two different sets of rules to set their *expansion* and *replacement* investment.

We assume that "animal spirits" are the key force driving expansion investment. Each manufacturing firm $j = 1, 2, ..., F_2$ sets its demand expectations ($D_j^e$) according to both its own past demand and market signals:

$$D_j^e(t) = f(D_j(t-1), Y(t-1), D_j(t-2), Y(t-2)...), \tag{1}$$

where $D_j(t-1)$ is the demand of firm $j$ at time $t-1$ and $Y(t-1)$ is the level of the economic activity at time $t-1$ (i.e. GDP). In the preliminary simulation exercises presented below, we begin by assuming that demand expectations are completely myopic:

$$D_j^e(t) = D_j(t-1). \tag{2}$$

According to the expected demand and the stocks ($N_j$) inherited from the previous period, firms fix their desired level of production ($Q_j^d$):

$$Q_j^d(t) = D_j^e(t) - N_j(t-1) + N_j^d(t), \tag{3}$$

where $N_j^d = \theta D_j^e(t)$, with $0 \leq \theta < 1$, is the desired level of stocks. Production is carried out using capital and labor under constant returns to scale. The stock of capital determines the maximum level of production achievable by each firm. Hence, given the desired level of production, firms compute the desired stock of capital as:

$$K_j^d(t) = \frac{Q_j^d(t)}{u^d}, \tag{4}$$

where $u^d$ is the desired level of capacity utilization.

Manufacturing firms decide whether to expand[14] their stock of capital following an (S,s) model. They compute their target ($K_j^{targ}$) and trigger ($K_j^{trig}$) level of capital as follows:

$$\begin{cases} K_j^{trig} = K_j(t) * (1+\alpha) \\ K_j^{targ} = K_j(t) * (1+\beta) \end{cases}, \tag{5}$$

with $0 < \beta < \alpha < 1$. Firms then plan to increase their capital stock to reach the target level only if the desired capital stock is higher than the trigger one:

$$EI_j^d(t) = \begin{cases} 0, & \text{if} \quad K_j^d(t) < K_j^{trig}(t) \\ K_j^{targ}(t) - K_j(t), & \text{if} \quad K_j^d(t) \geq K_j^{trig}(t) \end{cases}, \tag{6}$$

where $EI_j^d(t)$ is the desired expansion investment.

As discussed above, firms adopt a routine-based behavior because they live in an economy characterized by strong uncertainty generated by non-stationary fundamentals, endogenous technological progress, and non-trivial interaction networks. Consequently, firms do not hold a solid confidence in the accuracy of their demand expectations. They will therefore invest only if they expect a huge rise of their future demand. Moreover, in order to avoid to accumulate too much capital, they will shrink their desired capital to the target level[15].

The stock of capital of manufacturing firms is heterogeneous, because it is composed of various types of machines differing in terms of productivity and relative weight. Machines are measured in terms of their production capacity. They are identified by a labor productivity coefficient $A_{i,\tau}$, where $i$ denotes their producer and $\tau$ their generation (technical change takes place through the creation of new generation of machines). If $\Xi_j(t)$ is the set of all types of machines existing within firm $j$ at time $t$, firm $j$'s capital stock is defined as:

$$K_j(t) = \sum_{A_{i,\tau} \in \Xi_j(t)} g_j(A_{i,\tau}, t), \tag{7}$$

where $g_j(A_{i,\tau}, t)$ is the absolute frequency of machine $A_{i,\tau}$. Given the nominal wage $w(t)$, the unit labor cost of each machine is computed as::

$$c(A_{i,\tau}, t) = \frac{w(t)}{A_{i,\tau}}. \tag{8}$$

Scrapping policies of manufacturing firms follow a payback-period routine. In this way, the

[14] We assume that there are no secondary markets for capital goods. Hence, firms have no incentives to reduce their capital stock.

[15] In the simulations performed below, we assumed for simplicity that $\alpha = \beta$ (i.e. no distinction between target and trigger level of capital).

replacement of an incumbent machine depends on its degree of obsolescence[16] and on the market price of new capital goods. More formally, firm $j$ will scrap machines $A_{i,\tau} \in \Xi_j(t)$ if they satisfy:

$$RS_j(t) = \left\{ A_{i,\tau} \in \Xi_j(t) : \frac{p^*(t)}{c(A_{i,\tau}, t) - c^*(t)} \leq b \right\}, \tag{9}$$

where $p^*$ and $c^*$ are, respectively, the average market price and unit labor cost of new machines, and $b$ is a strictly positive payback-period parameter. Hence, the desired replacement investment ($RI_j^d$) of firm $j$ will be equal to:

$$RI_j^d(t) = \sum_{A_{i,\tau} \in RS_j(t)} g_j(A_{i,\tau}, t), \tag{10}$$

i.e. each manufacturing firm computes its desired replacement investment ($RI_j^d$) by "multiplying" the types of machines that satisfy eq. (9) for their absolute frequency.

The desired level of investment ($I_j^d$) is the sum of expansion and replacement investment. If the net worth of a firm is not enough, actual investment ($I_j$) will be lower than the desired one. Firms must bear production costs before selling their goods. Therefore, we assume that if net worth $NW_j(t)$ is not enough, it will be allocated first of all to finance production; next to expansion investment; and finally to replacement investment. Summing up the actual investment of all manufacturing firms, we get aggregate investment ($I$).

## 4.3 Capital Goods Market

In the previous section, we have described how the demand of capital goods is generated. In this section, we analyze the supply-side of the market and its clearing mechanisms.

Each machine-tool firm $i = 1, 2, ..., F_1$ sells its latest generation of products characterized by labor productivity coefficient $A_{i,\tau}$,with $\tau = 1, 2, ....$ Firms produce "on demand": manufacturing firms' orders determine the size of the investment cake, whose slices ($D_i$) are allocated according to the market share ($f_i$) of each producers:

$$D_i(t) = I(t) f_i(t). \tag{11}$$

Market shares evolve according to a replicator dynamics. More specifically, the market share of each firm will grow (shrink) if its competitiveness ($E_i$) is above (below) the industry-average competitiveness ($\overline{E}^i$):

[16] Since machines may be used by manufacturing firms for many years, we also adjust their labor productivity coefficient for their degree of senescence. More specifically, at the end of each period, the labor productivity of machines employed in manufacturing firm $j$ is multiplied by $(1 - \delta u_j(t))$, where $0 < \delta < 1$ is a depreciation parameter and $u_j(t)$ is the effective rate of capacity utilization of firm $j$.

$$f_i(t) = f_i(t-1)\left(1 + \chi_1 \frac{E_i(t) - \overline{E}^i(t)}{E_i(t)}\right), \tag{12}$$

where $\chi_1 \geq 0$ and:

$$\overline{E}^i(t) = \sum_{i=1}^{F_1} E_i(t) f_i(t-1). \tag{13}$$

The competitiveness of each firm depends on the price it charges ($p_i$) and on the level of its unfilled demand ($l_i$):

$$E_i(t) = -\omega_1 p_i(t) - \omega_2 l_i(t), \tag{14}$$

where $\omega_h, h = 1, 2$ are non-negative parameters.

The production process employs labor only and it is characterized by constant returns to scale. The unit cost of production depends on the machine currently manufactured:

$$c_i(t) = \frac{w(t)}{A_{i,\tau}}. \tag{15}$$

As it happens in the manufacturing industry, machine-tool firms bear the costs of production before receiving the revenues. Therefore, firm $i$ will fully satisfy its demand only if its net worth ($NW_i$) is sufficient to cover the total cost of production ($c_i Q_i$). If $W_i(t) < c_i(t) Q_i(t)$, the firm will satisfy only a fraction of its demand and its competitiveness will be reduced in the next period. Once the level of production is determined, firms can hire workers as:

$$L_i^D(t) = \frac{Q_i(t)}{A_{i,\tau}}, \tag{16}$$

where $L_i^D$ is the labor demand of firm $i$.

Firms set the price according to a mark-up ($\mu$) routine:

$$p_i(t) = (1 + \mu) c_i(t), \tag{17}$$

where $\mu \geq 0$. Firm $i$'s profits ($\Pi_i$) will be then given by:

$$\Pi_i(t) = [p_i(t) - c_i(t)] Q_i(t), \tag{18}$$

while net worth changes according to:

$$NW_i(t) = NW_i(t-1) + \Pi_i(t). \tag{19}$$

## 4.4 Consumption Good Market

After the capital good market clears, each manufacturing firm $j = 1, 2, ..., F_2$ receives the new machines and updates its average productivity ($\pi_j$) and unit cost of production ($c_j$).

Average productivity reads:

$$\pi_j(t) = \sum_{A_{i,\tau} \in \Xi_j(t)} A_{i,\tau} \frac{g_j(A_{i,\tau}, t)}{K_j(t)}, \tag{20}$$

while unit cost of production will be given by:

$$c_j(t) = \sum_{A_{i,\tau} \in \Xi_j(t)} c(A_{i,\tau}, t) \frac{g_j(A_{i,\tau}, t)}{K_j(t)}. \tag{21}$$

Labor demand ($L_j^D$), prices ($p_j$), competitiveness ($E_j$), market shares ($f_j$) and average competitiveness ($\overline{E}^j$) are computed – as it happens in the machine-tool industry – as follows:

$$L_j^D(t) = \frac{Q_j(t)}{\pi_j(t)}, \tag{22}$$

$$p_j(t) = (1 + \mu) c_j(t), \tag{23}$$

$$E_j(t) = -\omega_3 p_j(t) - \omega_4 l_j(t), \tag{24}$$

$$f_j(t) = f_j(t-1) \left( 1 + \chi_2 \frac{E_j(t) - \overline{E}^j(t)}{E_j(t)} \right), \tag{25}$$

$$\overline{E}^j(t) = \sum_{j=1}^{F_2} E_j(t) f_j(t-1). \tag{26}$$

Again: $\mu \geq 0$, $\omega_h$, $h = 3, 4$ and $\chi_2$ are non-negative parameters.

The dynamics of aggregate consumption ($C$) shapes the demand-side of the market. We single out three scenarios:

1. *Work-or-die scenario.* Only the fraction of the population that has a job receive an income that is fully consumed. Hence, aggregate consumption reads:

$$C(t) = w(t) \sum_{i=1}^{F_1} L_i^D(t) + w(t) \sum_{j=1}^{F_2} L_j^D(t) \tag{27}$$

2. *Exogenous-component scenario.* Aggregate consumption is obtained by adding an exogenous component ($G$) to aggregate employee income defined in eq. (27). $G$ can be interpreted as public expenditures or, equivalently, as a lump-sum transfer given to each worker independently on the number of unemployed workers and market wage.

3. *Endogenous-component scenario.* In this set-up, unemployed workers receive a fraction of the market wage. Hence, the aggregate sum transferred to the unemployed workers endogenously depends on their number, as well as on market wage. Total consumption is the sum of income of employed and unemployed workers.

In all scenarios, manufacturing firms face a demand equal to a fraction of the total consumption proportional to their market share:

$$D_j(t) = C(t)f_j(t). \tag{28}$$

If firm demand is smaller than firm production (i.e. $D_j < Q_j$), the firm will accumulate stocks. Otherwise, if $D_j > Q_j$, the firm will not be able to fill its whole demand[17]. Denoting by $S_j$ total sales of firm $j$, profits read:

$$\Pi_j(t) = p_j S_j - c_j Q_j - \sum_{i=1}^{F_1} p_i m_{j,i}, \tag{29}$$

where $m_{j,i}$ is the number of machines bought by manufacturing firm $j$ from machine-tool firm $i$. Net worth is accordingly updated as follows:

$$NW_j(t) = NW_j(t-1) + \Pi_j(t). \tag{30}$$

## 4.5 Entry, Exit, and Technical Change

At the end of every period, firms with zero market shares and/or negative net worth die and are replaced by new firms. Hence, the sizes of both sectors remain constant across time.

In order not to bias the overall dynamics, we start by assuming that each entrant is a random copy of an existing firm. Since young firms may suffer from financial constraints more than older ones, we restrict the set of duplicable firms to those with a net worth smaller than the current industry-average.

[17] We rule out the perverse case where total production of manufacturing firms is not enough to satisfy aggregate demand by assuming that consumers will buy another (e.g. luxury) commodity up to the point where their income is exhausted.

Finally, our economy is fuelled by a never-ending process of technical change. At the end of each period, machine-tool firms try to develop the next generation of their product (i.e. discovering machines with a higher labor productivity coefficient). The result of their efforts is strongly uncertain: firms create a prototype whose labor productivity ($A_{i,new}$) can be higher or lower than the one of the currently manufactured machine. More formally, we let:

$$A_{i,new} = A_{i,t} + \epsilon, \tag{31}$$

where $\epsilon$ are random variables normally distributed with zero mean and variance:

$$\sigma_{i,\tau}(t) = \sigma^{\circ} + \varphi(A_{\max}(t) - A_{i,\tau}), \tag{32}$$

where $A_{\max}(t)$ is the highest labor productivity achieved by a machine in the current period, $\sigma^{\circ}$ is a constant and $\varphi$ is a non-negative parameter. Note that, in line with Llerena and Lorentz (2003), we model the variance of $\epsilon$ so as to allow low-productivity firms to catch up those firms which are close to the technological frontier.

We also posit that firm $i$ will release the next generation machine only if the latter entails a labor productivity improvement (i.e. $A_{i,new} > A_{i,\tau}$). Finally, if the firm decides to produce the new machine, the index $\tau$ is accordingly incremented by one unit.

### 4.6 Macro Dynamics

The dynamics generated at the micro-level by individual decisions and interaction networks induces, at the macroeconomic level, a stochastic dynamics for all aggregate variables of interest (e.g. income, investment, consumption, unemployment, etc.). Two remarks are in order. First, notice that the usual national accounting identities hold in our model. For example, gross national income $Y(t)$ is identically equal, in each period, to the sum of aggregate consumption $C(t)$, aggregate investment $I(t)$ and change in inventories $N(t)$.

Second, labor market is not cleared by real wage movements. As a consequence, involuntary unemployment may arise. The aggregate supply of labor is exogenous, inelastic and grows at a constant rate ($\eta$):

$$L(t) = L(t-1)(1+\eta). \tag{33}$$

The aggregate demand of labor is the sum of machine-tool and manufacturing firms' labor demands. The wage is fixed by institutional rather than market forces and in each time period reads:

$$w(t) = w(t-1) + \psi_1 \frac{cpi(t) - cpi(t-1)}{cpi(t-1)} + \psi_2 \frac{\overline{A}(t) - \overline{A}(t-1)}{\overline{A}(t-1)} + \psi_3 \frac{U(t) - U(t-1)}{U(t-1)}, \quad (34)$$

where $cpi$ is the consumer price index, $\overline{A}$ is average labor productivity and $U$ is the unemployment rate. The system parameters $\psi_{1,2,3}$ allow one to characterize various institutional regimes for the labor market.

As mentioned above, our model genuinely belongs to the evolutionary/ACE class. Since neither analytical, closed-form, solutions nor numerical ones can be obtained, one must resort to computer simulations to analyze the properties of the (stochastic) processes governing the co-evolution of micro and macro variables (Kwasnicki, 1998; Pyka and Grebel, 2003).

To do so, one should in principle address an extensive Montecarlo analysis to understand how the statistics of interests (e.g. average growth rate of the economy, investment-output volatility and correlation structure, etc.) change with initial conditions and system parameters. A sufficiently large number of Montecarlo replications for any given choice of initial conditions and system parameters is required to wash-away the effect of across-simulation variability induced by stochastic components. Notice, however, that in our model the only stochastic component driving away the underlying dynamics from its deterministic path is given by technological improvements. In fact, some preliminary sensitivity exercises show that the across-simulation stochastic variability is quite low (even if one slightly tunes the parameters $\sigma^{\circ}$ and $\varphi$ in eq. (32) above) and no chaotic patterns are detected. Hence, we can confidently present below results concerning averages over a limited number of replications (typically $M = 50$) as a robust proxy for the behavior of all time-series of interest.

# 5 Some Preliminary Simulation Results

In this Section, we present some preliminary simulation exercises[18]. In each of the three "consumption scenarios" that we have characterized above ("work-or-die", "exogenous component", "endogenous component"), we firstly investigate in a qualitative fashion output and investment patterns. More specifically, we study technological and institutional conditions under which the system is able to generate self-sustaining growth.

Next, we turn to a more quantitative exploration of the statistical properties of the

[18] All our results refer to a benchmark parametrization and initial conditions setup ensuring consistent and economically-interpretable simulation exercises (see Appendix B). All findings presented below are quite robust to changes of initial conditions within a sufficiently large set. However, given its preliminary nature, the analysis below does not address the question whether the observed statistical properties of simulated time-series change with system parameters. This is in fact the next point in our agenda.

coupled output-investment dynamics to ascertain to what extent our model is able to replicate the macroeconomic stylized facts discussed in Section 2.

## 5.1 Qualitative results

Let us begin with the "work-or-die" scenario. In this economy, the fraction of the population that is currently unemployed does not receive any income, while the employed one fully consumes their wage.

As simulations show (cf. Fig. 3), in this scenario the system is not able to generate a self-sustaining pattern of growth. Indeed, in the first simulation time-periods, expansion and replacement investment seem to spur output growth[19]. However, aggregate demand soon becomes insufficient to prevent expansion investment from falling toward zero. Similarly, replacement investment is not able to trigger subsequent booms (see Fig. 4).

Two remarks are in order. Notice, first, that in the "work-or-die" scenario, technological progress does not play a key role in inducing long-run growth. This is not a surprising result, if we consider that in the model process innovation dominates over product innovation. This in turn implies that positive effects of technical change linked to its "creative" nature (e.g. birth of new products, markets, industries) are not able to prevail on its "destruction" one (e.g. job losses, unemployment).

Second, self-sustaining growth appears to be a zero-probability event in this scenario also because there do not exist any forces moderating the "natural" instability of the manufacturing industry. In real-world economies, in fact, the dominant role played by services, the presence of a public sector, and the implementation of automatic stabilizers are all likely to dampen the oscillations of the manufacturing industry, thus reducing the instability of the whole economy (Zarnowitz, 1991).

Following this intuition, we move now to the second and third consumption scenario, where we introduce, on the contrary, some very stylized examples of such re-equilibrating forces. In both "exogenous" and "endogenous" component scenarios, the size of aggregate demand is persistently larger than in the "work-or-die" scenario and it contains an acyclical ("exogenous" case) or countercyclical ("endogenous" case) component.

As shown in Figs. 5 and 9, in both scenarios self-sustaining growth patterns characterized by endogenous fluctuations do emerge.

Moreover, an investigation of output and investment patterns at a more disaggregated level shows that the behavior of aggregate investment is the result of huge changes in expansion and substitution investment, see Figs. 6 and 10.

Finally, if we isolate the business cycle frequencies of the both series by applying a

[19] Notice that we focus on real variables only. Output is the sum of investment and consumption.

bandpass filter[20], we observe the typical "roller coaster" shape that characterizes real data (see Figs. 7 and 11; cf. also Section 2).

As clearly depicted in Figs. 8 and 12, aggregate investment appears to be more volatile than output and expansion investment fluctuates more wildly than replacement investment. Finally, aggregate investment seems to display a procyclical behavior in both scenarios.

But to what extent the foregoing qualitative evidence is corroborated by a more robust statistical analysis? To answer this question, in the next section we shall discuss in more detail the statistical properties of our simulated investment and output dynamics.

## 5.2 Quantitative results

In this section we study the extent to which our simulated investment and output dynamics displays, in each of the three consumption scenarios introduced above, statistical properties similar to the empirically observed one (as summarized by $SF1$ and $SF2$).

More specifically, let us consider our benchmark setup for system parameters and initial conditions and indicate with:

$$\{\log Y(t), t = 1, ..., T\} \tag{35}$$

and:

$$\{\log I(t), t = 1, ..., T\} \tag{36}$$

the simulated time-series of (real) output and investment time-series, respectively[21]. We shall focus on the average growth rate (AGR) of the economy:

$$AGR_T = \frac{\log Y(T) - \log Y(0)}{T + 1}, \tag{37}$$

the standard deviation of both output and investment, suitably detrended using the alternative techniques discussed in Appendix A – as well as cross- and auto-correlation structure for the coupled time-series $\{Y(t), I(t)\}$. Moreover, we perform Dickey-Fuller tests on $\{\log Y(t), t = 1, ..., T\}$ to detect the presence of unit roots in the series (interpreted as evidence for self-sustaining patterns of growth). All results refer to averages computed across $M = 50$ independent simulations and to the same setups as far as system parameters and initial conditions are concerned (cf. Appendix B).

Consider the "work-or-die" scenario first. The "gloomy" picture depicted in section 5.1 is confirmed. As Table 4 shows, all time-series are stationary and have negative average

[20] For a more accurate discussion of the filtering techniques employed in this work in the light of the pros and cons of alternative choices, cf. Appendix A.

[21] All results refer to the choice of $T = 500$, cf. Appendix B. This econometric sample size is sufficient to allow for convergence of recursive moments of all statistics of interest.

rates of growth. In this framework, the information conveyed by standard deviations and cross correlations become completely irrelevant (cf. Table 5).

Conversely, in the "exogenous-component" scenario, the average growth rate of output and investment are both strictly positive ($\simeq 1.5\%$) and well above the constant growth rate of $G$ (see Table 6). According to Dickey-Fuller tests, output and aggregate investment are non-stationary, whereas both expansion and substitution investment appear to be I(0). A lack of aggregate demand may be at the root of the stationarity of the expansion and substitution investment series. Unfortunately, we do not have real data on expansion and replacement investment to confirm or reject these results.

We employ a bandpass filter (cf. Appendix A) to extract the cyclical component of the series in order to compute standard deviations and correlations. According to the relative standard deviations, the model seems to be able to match $SF1$ (i.e. investment is considerably more volatile than output). The volatility of aggregate investment is indeed 2.6 times larger than the output one. Relative volatility of expansion and replacement investment are even higher (15.22 and 6.50 respectively).

The autocorrelation structure of output is very close to the one observed in real-world data (cf. Table 7). Notice however that cross-correlations are not as high as the ones observed e.g. in the U.S.. Nevertheless, they clearly indicate that aggregate investment is a procyclical and coincident variable ($SF2$). Their pattern is closer to the one displayed by machine-and-equipment investment than to the one of aggregate investment. Cross correlations of expansion and substitution investment are lower and more stable than the ones of the aggregate variable. In particular, replacement investment seems slightly acyclical. Also in this case, we cannot test our results against real data.

Similar to what happens in the second setup, output dynamics in the "endogenous-component" scenario exhibits strictly positive average growth rates together with a I(1) pattern, cf. Table 8. This confirms that in the two last scenarios self-sustaining growth does emerge in our economy.

However, as shown in Table 9, relative standard deviations of the three investment series are higher in the "endogenous-component" consumption scenario than in the "exogenous-component" one. These differences may stem from the fact that in the second scenario aggregate demand contains an *acyclical* component, whereas in the third one that component exhibits a *countercyclical* behavior.

Nonetheless, the patterns of output auto correlations are similar in both second and third scenarios. In the "endogenous-component" one, cross correlations between output and aggregate investment are slightly higher and track more closely output auto correlations. Replacement and expansion investment are both procyclical, but the first is lagging, whereas the second is leading. Their cross correlation pattern is completely different from the one exhibited by the exogenous-component setup.

To sum up, the work-or-die scenario is not able to generate self-sustaining growth and it cannot match either $SF1$ or $SF2$. Both the "exogenous-" and "endogenous-component" scenarios deliver long-run growth characterized by short-run endogenous fluctuations. Both scenarios are thus able jointly to replicate $SF1$ and $SF2$. However, the "exogenous-component" setup reproduces with more precision the first stylized fact, whereas the "endogenous-component" scenario better fits the second one.

Finally, notice that the foregoing results strongly indicate that (sort of) "multiplier" and "investment-accelerator" effects – endogenously emerging in our economy – lie at the heart of vicious and virtuous cycles characterizing the three scenarios. Indeed, in the second and third scenarios, the emergence of a "multiplier" effect drives output growth, while a mechanism quite similar to the well-known "investment-accelerator" induces firms to expand their capital stock in the next period. This generates a virtuous cycle leading to self-sustaining growth and short-run fluctuations. Conversely, in the "work-or-die" scenario, such a virtuous reaction chain breaks down and the economy stops growing after some time-steps[22].

# 6 Conclusions and Outlook

In this paper we have presented an evolutionary, agent-based, model of industry dynamics and firm investment behavior which attempts to provide an interpretation of the most robust stylized facts of the coupled investment-output aggregate dynamics. A key feature of the model is that investment lumpiness is grounded upon boundedly-rational behaviors and adaptive expectations, rather than being derived as the outcome of some optimization procedure carried out by a fully-rational, forward-looking, agent.

Despite their preliminary nature, simulation results indicate that imperfect adjustment among boundedly-rational, myopic, firms who interact directly in a two-sector, strongly non-stationary, economy is able to generate – under some broad institutional and market conditions – self-sustaining patterns of growth and business cycle waves characterized by statistical properties very similar to those observed in real-world output-investment dynamics.

The set of results presented in Section 5 seems to be quite robust to alternative initial conditions' setups, as well as to different choices of some key parameters (cf. Appendix B). For instance, additional exercises show that the pace of technical change and its

[22]Since the relationship between the multiplier and the accelerator is inherently circular, there is an "egg-chicken" problem to be solved: our simulation exercises cannot shed much light on which of the two elements is the main culprit of an observed stagnation.

degree of "catching up" – cf. eq. (32) – seem to barely affect both the properties of the investment-output correlation structure and the across-simulation variability. This suggests that the counter-balancing forces characterizing the linkages between demand, capital- and consumption-good layers of our economy are able to substantially dampen down the amplitude of any exogenous shock.

Nonetheless, the robustness of the foregoing findings must be more thoroughly checked against – at least – three complementary sets of simulation exercises. First, one should perform an extensive Montecarlo simulation study to explore to what extent (and in which direction) our basic results change when one tunes system parameters across a properly defined grid. In such a way, many interesting questions might be answered. For example, what are the consequences of assuming a different institutional setting as far as market-wage dynamics is concerned (cf. eq. 34)? And, similarly, what happens if one assumes different competitive/selective pressures, e.g. if one changes competitiveness ($\omega_h$, $h = 1, ..., 4$) and replicator-dynamics ($\chi_h$, $h = 1, 2$) parameters?

Second, the model could be extended to take on board a microfounded labor-market side, where, as happens in Fagiolo, Dosi, and Gabriele (2004), both wage and unemployment setting are endogeneized. Similarly, one may experiment with different exit-entry rules, to understand which is the role played by industry turbulence in shaping the business cycles.

Finally, one might attempt to investigate the impact of different "expectation formation" setups on the statistical properties of simulated business cycles. In the model above, we have indeed assumed a particular, benchmark, form for the "animal spirits" our firms are endowed with, i.e. myopic expectations. More generally, in line with Fagiolo and Dosi (2002), one might explore the consequences of changing the expectation rule (e.g. within the framework of eq. 1) and investigate the effect of injecting our population of boundedly-rational firm with players endowed with more sophisticated expectation rules.

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# A The Choice of the Filter

All analyses of empirical and simulated time-series conducted above have required the application of some filtering techniques in order to single out the business cycle components of the series.

The choice of the filter is not trivial: as Canova (1998, 1999) pointed out, different detrending methods affect both the qualitative and quantitative stylized facts of the business cycle. An ideal filter should remove the trend, as well as any irregular components, without introducing any distortion. The problem becomes clearer if it is treated in the frequency domain. According to the spectral decomposition theorem, a covariance stationary time series can be represented as the infinite sum of orthogonal components, each of which is associated to a given frequency. Each series has a power spectrum, which reports the contribution to the total variance of the process of the components belonging to each frequency band. The (relative) importance of the fluctuations associated to a given periodicity is given by the height of the spectrum at the correspondingly frequency. As reported by Granger (1966), the spectrum of many macroeconomic time series has a typical monotonically-decreasing shape, which implies that medium and (especially) low frequencies – which correspond to the business cycle and long-run growth periodicity – give the highest contribution to the variance of the variables. The ideal business cycle filter should preserve the medium frequencies, detrend the variable (i.e. eliminating low frequency fluctuations), and kill the high frequency noise.

Let us consider two of the most largely employed filters, i.e. "first-differencing" (FD) and "bandpass" (BP), see Baxter and King (1999). On the one hand, the FD filter is very simple and it is able to remove the trend component of the series. However, it amplifies their short-run noise. Moreover, if a series does not have a unit root, we can incur in over-differencing.

On the other hand, the BP filter outperforms FD and allows to single out only the range of periodicity associated to the business cycle (e.g. 6-32 quarters)[23].

Hence, in line with the econometric literature on business cycle stylized facts (Agresti and Mojon, 2001; Stock and Watson, 1999; Kydland and Prescott, 1990; Napoletano, Roventini, and Sapio, 2004), we choose to employ here the BP filter.

This choice is reinforced by the fact that the problem of high frequency noise is particularly severe in our data and that some of our series seem stationary (see Section 5.2). For instance, if in the "exogenous component" scenario we compare output and investment series detrended with the two filters (cf. Fig. 13), a distortion due to the presence of short-run noise does emerge: the fluctuations of the first-differenced series are very wild as compared to those of bandpass-filtered series. This does not allow one to infer any clear relation between output and investment. Moreover, the distortion introduced by first-differencing biases also our quantitative results: high frequency noise amplifies standard deviations and reduces both auto- and cross-correlations (see Table 10).

Finally, notice that the BP filter requires to specify the range of frequencies that correspond to business cycle periodicity. With real-world data, this choice is very simple: given

[23] More specifically, the *optimal* BP filter is an infinite symmetric moving average, singling out a specific range of periodicity. The *feasible* BP filter is instead a finite moving-average, whose weights minimize the squared difference between the ideal filter and viable ones.

the frequency of the observed data (e.g. quarterly, monthly), the minimum and maximum length of business cycle is usually defined according to a qualitative analysis of the data (e.g. NBER chronologies).

Unfortunately, simulation-based exercises do not provide the modeler – by construction – with this information. We deal with this problem by assuming that our simulated time-tick coincides with quarterly data, and we use the same range of frequencies that are commonly used in the empirical analysis of the U.S. business cycles (i.e. 6-32 quarters).

There seem to be at least three reasons which justify this choice. First, using quarterly data allows us to better compare statistical properties of simulated time-series with those exhibited by empirically observed ones (cf. Section 2.1). Second, we believe that the assumption of quarterly data is a good compromise between the timing of investment and production choices made by firms whose time-horizon is (also) shaped by data-availability. Finally, the quarterly timing appears to be the "optimal" one also from a calibration perspective. Imagine to search for the ranges of frequencies of a BF that allow our simulated data to best reproduce the empirically observed stylized facts on output and investment. More specifically, let us assume that the length of our business cycles falls between 6 and 32 quarters and let us filter our simulated data as if they were quarterly, monthly and annual[24]. It turns out that the quantitative results we obtain with "annual" data closely resemble those obtained with first-differencing (Table 10). This does not come as a surprise: since frequency is the inverse of periodicity, by assuming annual data we widen the frequency range, taking on board a lot of high frequency noise. With "quarterly" and "monthly" data, on the other hand, the situation improves substantially: the relative standard deviations of investment decrease, while both auto- and cross-correlations increase. However, with "monthly" data, auto- and cross-correlations fall too slowly as compared to what happens in real-world data.

# B Simulations and System Parameters

All simulation results presented above refer to the benchmark setup described in Table 1. Initial conditions are defined as in Table 2. The "work-or-die" scenario does not require any additional initial conditions, nor additional parameters: aggregate consumption is indeed simply the product of wage and aggregate labor demand.

Conversely, in the "exogenous-component" scenario, we add to aggregate consumption an exogenous variable ($G$). We have employed different initial values and laws of motion for $G$ (i.e. no growth, stochastic growth and deterministic growth). Since all simulation results presented above appear to be robust to such choices, we have assumed for simplicity that $G$ grows at the same constant rate of the population ($\eta$), i.e. a sort of "golden rule". Finally, in the "endogenous-component" scenario, the share $\vartheta$ of current market-wage earned by unemployed workers does not seem to dramatically alter our results. Therefore, we have set $\vartheta = 0.35$.

[24]For "quarterly" data, we apply a bandpass filter (6,32,20); for "monthly" data, we use a bandpass filter (18,96,36) and for "annual" data, a bandpass filter (2,8,6). The first two numbers set the lowest (e.g. 18 months) and highest periodicity (e.g. 96 months) that must be considered. The last number regulates the precision of the filter.

| Description | Symbol | Value |
|---|---|---|
| Size of Machine-tools Industry | $F_1$ | 50 |
| Size of Manufacturing Industry | $F_2$ | 200 |
| Econometric Sample Size | $T$ | 500 |
| Replicator Dynamics Coeff. | $\chi_{1,2}$ | $-1$ |
| Competitiveness weights | $\omega_{1,2,3,4}$ | 1 |
| Tech. Progr. Variance: Const | $\sigma^0$ | 25 |
| Tech. Progr. Var.: Catch-up Coeff. | $\varphi$ | 0.5 |
| Labor Supply Growth Rate | $\eta$ | 0.01 |
| Wage Setting: $\Delta cpi$ weight | $\psi_{1,2}$ | 0.75 |
| Wage Setting: $\Delta \overline{A}$ weight | | |
| Wage Setting: $\Delta U$ weight | $\psi_3$ | 75 |
| "Desired level of stocks" share | $\theta$ | 0.1 |
| Desired level of capacity utilization | $u^d$ | 0.75 |
| Trigger rule | $\alpha$ | 0.3 |
| Payback Period Parameter | $b$ | 300 |
| Mark-up rule | $\mu$ | 0.3 |

Table 1: Benchmark Parametrization

| Description | Symbol | Value |
|---|---|---|
| Market Wage | $w(0)$ | 100 |
| Consumer Price Index | $cpi(0)$ | 1.2 |
| Average Labor Productivity | $\overline{A}(0)$ | 100 |
| Net Worth | $W_{i,j}(0)$ | 10000 |
| Capital Stock | $K_j(0)$ | 1000 |
| Labor Supply | $L(0)$ | 7000 |
| Unemployment Rate | $U(0)$ | 1 |

Table 2: Initial Conditions

| Series | Std Dev. Abs | Rel | Cross-autocorrelations with output (lags) -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Output | 1.52 | 1.00 | 0.22 | 0.49 | 0.74 | 0.93 | 1.00 | 0.93 | 0.74 | 0.49 | 0.22 |
| Investment (Total) | 4.02 | 2.65 | 0.25 | 0.51 | 0.75 | 0.91 | 0.95 | 0.89 | 0.72 | 0.49 | 0.26 |
| Investment (M&E) | 4.38 | 2.89 | 0.52 | 0.74 | 0.89 | 0.93 | 0.86 | 0.69 | 0.46 | 0.20 | -0.03 |

Table 3: Variance and Auto-Correlation Structure of Investment and Output for the U.S. economy (1960- 2002). Quarterly data have been detrended with a bandpass filter (6,32,12). Source: Our elaborations on data from Main Economic Indicators (MEI), OECD.

| | Output | Aggr. Inv. | Exp. Inv. | Repl. Inv. |
|---|---|---|---|---|
| Average growth rate (%) | -2.34 | -1.28 | -1.28 | 0.00 |
| Dickey-Fuller Test (logs) | -2.23 | -3.78 | -4.82 | -3.54 |
| Sign. level | 0.05 | 0.01 | 0.01 | 0.01 |
| Dickey-Fuller Test (Bpf; 6,32,20) | -5.03 | -6.32 | -4.61 | -6.30 |
| Sign. level | 0.01 | 0.01 | 0.01 | 0.01 |
| Std. Dev. (Bpf; 6,32,20) | 0.70 | 0.24 | 0.47 | 0.26 |
| Rel. Std. Dev. | 1.00 | 0.35 | 0.67 | 0.37 |

Table 4: The Work-or-Die Scenario. Output and Investment Statistics.

| Bpf (6,32,20) | Output (Bpf; 6,32,20) t-4 | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 | t+4 |
|---|---|---|---|---|---|---|---|---|---|
| Output | 0.09 | 0.36 | 0.66 | 0.90 | 1.00 | 0.90 | 0.66 | 0.36 | 0.09 |
| Aggr. Inv. | -0.02 | 0.09 | 0.29 | 0.51 | 0.70 | 0.75 | 0.65 | 0.45 | 0.22 |
| Exp. Inv. | -0.06 | -0.02 | 0.02 | 0.05 | 0.07 | 0.08 | 0.08 | 0.09 | 0.10 |
| Repl. Inv. | -0.05 | 0.05 | 0.22 | 0.44 | 0.62 | 0.69 | 0.61 | 0.42 | 0.21 |

Table 5: The Work-or-Die Scenario. Correlation Structure.

| | Output | Aggr. Inv. | Exp. Inv. | Repl. Inv. |
|---|---|---|---|---|
| Average growth rate (%) | 1.61 | 1.49 | 1.41 | 2.56 |
| Dickey-Fuller Test (logs) | 3.56 | -0.45 | -6.20 | -2.28 |
| Sign. level | 1.00 | 1.00 | 0.01 | 0.05 |
| Dickey-Fuller Test (Bpf; 6,32,20) | -4.89 | -6.48 | -5.65 | -6.86 |
| Sign. level | 0.01 | 0.01 | 0.01 | 0.01 |
| Std. Dev. (Bpf; 6,32,20) | 0.14 | 0.36 | 2.10 | 0.90 |
| Rel. Std. Dev. | 1.00 | 2.62 | 15.22 | 6.50 |

Table 6: The Exogenous Component Scenario. Output and Investment Statistics.

| | Output (Bpf; 6,32,20) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Bpf (6,32,20)** | **t-4** | **t-3** | **t-2** | **t-1** | **t** | **t+1** | **t+2** | **t+3** | **t+4** |
| Output | -0.02 | 0.30 | 0.64 | 0.90 | 1.00 | 0.90 | 0.64 | 0.30 | -0.02 |
| Aggr. Inv. | 0.26 | 0.35 | 0.43 | 0.50 | 0.51 | 0.43 | 0.25 | 0.02 | -0.21 |
| Exp. Inv. | -0.04 | 0.10 | 0.22 | 0.30 | 0.32 | 0.30 | 0.26 | 0.21 | 0.18 |
| Repl. Inv. | 0.12 | 0.08 | 0.07 | 0.08 | 0.11 | 0.12 | 0.07 | -0.02 | -0.12 |

Table 7: The Exogenous Component Scenario. Correlation Structure.

| | Output | Aggr. Inv. | Exp. Inv. | Repl. Inv. |
|---|---|---|---|---|
| Average growth rate (%) | 1.53 | 1.56 | 1.54 | 2.32 |
| Dickey-Fuller Test (logs) | 4.14 | -1.02 | -6.06 | -2.56 |
| Sign. level | 1.00 | 1.00 | 0.01 | 0.05 |
| Dickey-Fuller Test (Bpf; 6,32,20) | -5.36 | -6.45 | -6.35 | -6.35 |
| Sign. level | 0.01 | 0.01 | 0.01 | 0.01 |
| Std. Dev. (Bpf; 6,32,20) | 0.09 | 1.20 | 2.51 | 1.63 |
| Rel. Std. Dev. | 1.00 | 13.51 | 28.37 | 18.44 |

Table 8: The Endogenous-Component Scenario. Output and Investment Statistics.

| | Output (Bpf; 6,32,20) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Bpf (6,32,20)** | **t-4** | **t-3** | **t-2** | **t-1** | **t** | **t+1** | **t+2** | **t+3** | **t+4** |
| Output | -0.11 | 0.20 | 0.57 | 0.88 | 1.00 | 0.88 | 0.57 | 0.20 | -0.11 |
| Aggr. Inv. | -0.09 | 0.16 | 0.43 | 0.60 | 0.63 | 0.50 | 0.29 | 0.08 | -0.06 |
| Exp. Inv. | -0.32 | -0.28 | -0.15 | 0.04 | 0.21 | 0.31 | 0.32 | 0.26 | 0.18 |
| Repl. Inv. | 0.07 | 0.29 | 0.49 | 0.59 | 0.54 | 0.38 | 0.16 | -0.02 | -0.15 |

Table 9: The Endogenous-Component Scenario. Correlation Structure.

| Rates of growth | Std. Dev. Abs | Rel | Correlation with Output (Rates of Growth) t-4 | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 | t+4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Output | 0.10 | 1.00 | 0.02 | -0.16 | -0.11 | 0.33 | 1.00 | 0.33 | -0.11 | -0.16 | 0.02 |
| Aggr. Inv. | 0.80 | 8.37 | 0.23 | 0.15 | -0.28 | -0.15 | 0.38 | 0.17 | -0.07 | -0.16 | 0.02 |
| Exp. Inv. | 3.98 | 41.48 | 0.00 | -0.02 | -0.02 | 0.04 | 0.16 | -0.02 | -0.04 | -0.01 | 0.02 |
| Repl. Inv. | 2.17 | 22.65 | 0.12 | 0.13 | -0.16 | -0.20 | 0.19 | 0.18 | -0.03 | -0.13 | 0.03 |

| Bpf (6,32,20) | Std. Dev. Abs | Rel | Correlation with Output (Bpf; 6,32,20) t-4 | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 | t+4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Output | 0.14 | 1.00 | -0.02 | 0.30 | 0.64 | 0.90 | 1.00 | 0.90 | 0.64 | 0.30 | -0.02 |
| Aggr. Inv. | 0.36 | 2.62 | 0.26 | 0.35 | 0.43 | 0.50 | 0.51 | 0.43 | 0.25 | 0.02 | -0.21 |
| Exp. Inv. | 2.10 | 15.22 | -0.04 | 0.10 | 0.22 | 0.30 | 0.32 | 0.30 | 0.26 | 0.21 | 0.18 |
| Repl. Inv. | 0.90 | 6.50 | 0.12 | 0.08 | 0.07 | 0.08 | 0.11 | 0.12 | 0.07 | -0.02 | -0.12 |

| Bpf (18,96,36) | Std. Dev. Abs | Rel | Correlation with Output (Bpf; 18,96,36) t-4 | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 | t+4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Output | 0.13 | 1.00 | 0.71 | 0.83 | 0.92 | 0.98 | 1.00 | 0.98 | 0.92 | 0.83 | 0.71 |
| Aggr. Inv. | 0.28 | 2.17 | 0.75 | 0.79 | 0.80 | 0.79 | 0.74 | 0.67 | 0.57 | 0.46 | 0.34 |
| Exp. Inv. | 2.57 | 19.74 | -0.07 | 0.02 | 0.12 | 0.21 | 0.28 | 0.34 | 0.38 | 0.40 | 0.40 |
| Repl. Inv. | 0.84 | 6.45 | 0.54 | 0.54 | 0.52 | 0.48 | 0.44 | 0.38 | 0.33 | 0.27 | 0.21 |

| Bpf (2,8,6) | Std. Dev. Abs | Rel | Correlation with Output (Bpf; 2,8,6) t-4 | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 | t+4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Output | 0.06 | 1.00 | 0.06 | -0.50 | -0.54 | 0.22 | 1.00 | 0.22 | -0.54 | -0.50 | 0.06 |
| Aggr. Inv. | 0.51 | 8.39 | 0.36 | 0.13 | -0.46 | -0.29 | 0.40 | 0.30 | -0.10 | -0.28 | -0.03 |
| Exp. Inv. | 2.34 | 38.40 | -0.01 | -0.09 | -0.08 | 0.06 | 0.19 | -0.02 | -0.12 | -0.07 | 0.02 |
| Repl. Inv. | 1.35 | 22.22 | 0.20 | 0.16 | -0.25 | -0.28 | 0.20 | 0.27 | -0.02 | -0.21 | -0.03 |

Table 10: Robustness of Simulation Results to Alternative Filtering Procedures. First Differencing vs. Bandpass Filter.

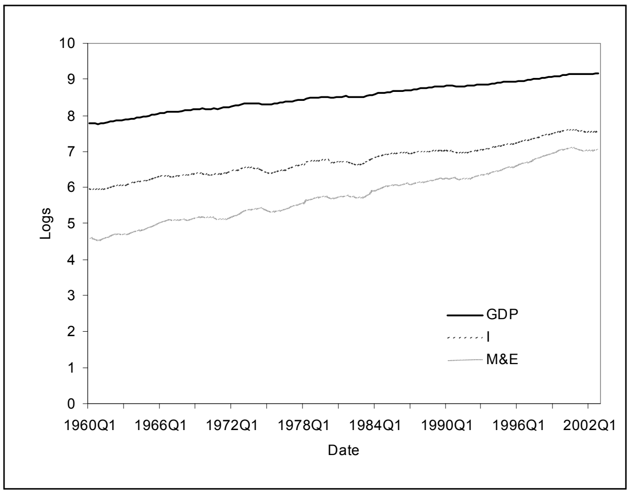


Figure 1: Level of GDP, Aggregate Investment, and Machine & Equipment Investment in the U.S.A. (1960Q1 – 2002Q4). Source: Main Economic Indicators (MEI), OECD.

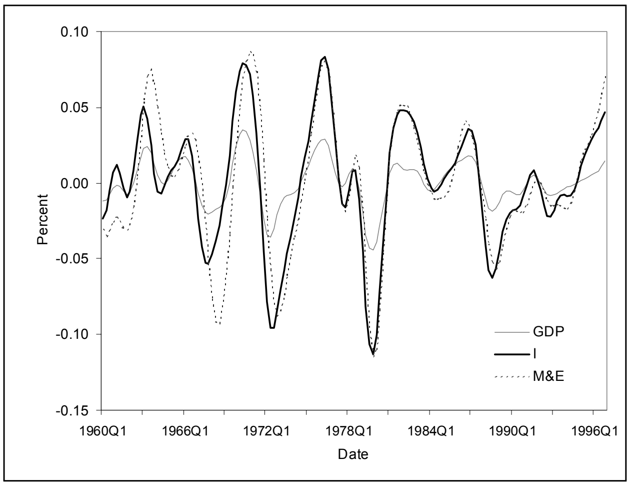


Figure 2: Bandpass-Filtered GDP, Aggregate Investment, and Machine & Equipment Investment in the U.S.A. (1960Q1 – 2002Q4). Source: Our elaborations on data from Main Economic Indicators (MEI), OECD.

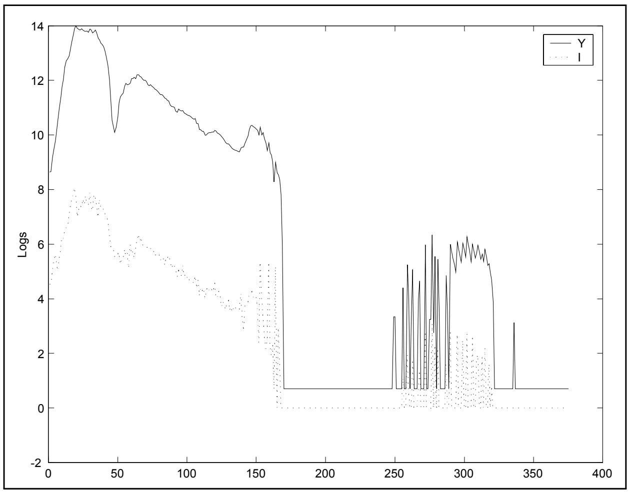


Figure 3: The Work-or-Die Scenario. Level of Output and Aggregate Investment.

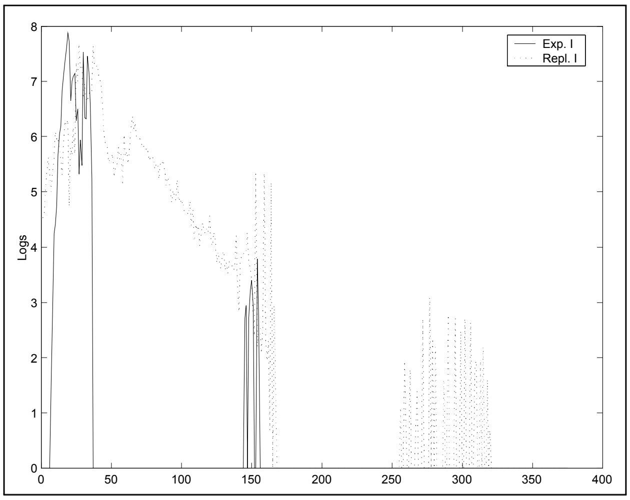


Figure 4: The Work-or-Die Scenario. Level of Expansion and Replacement Investment.

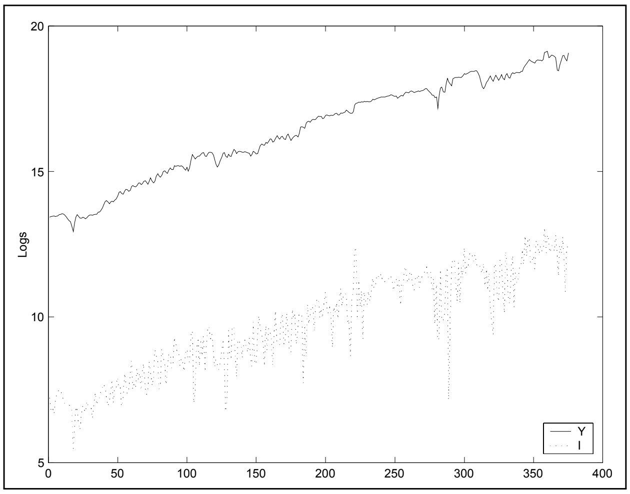


Figure 5: The Exogenous-Component Scenario. Level of Output and Aggregate Investment.

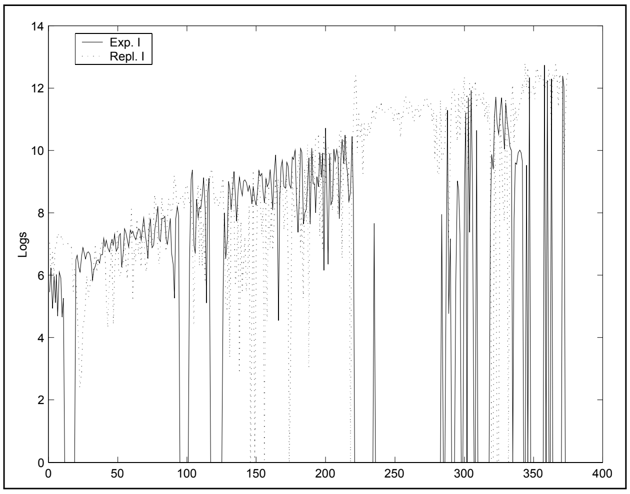


Figure 6: The Exogenous-Component Scenario. Level of Expansion and Replacement Investment.

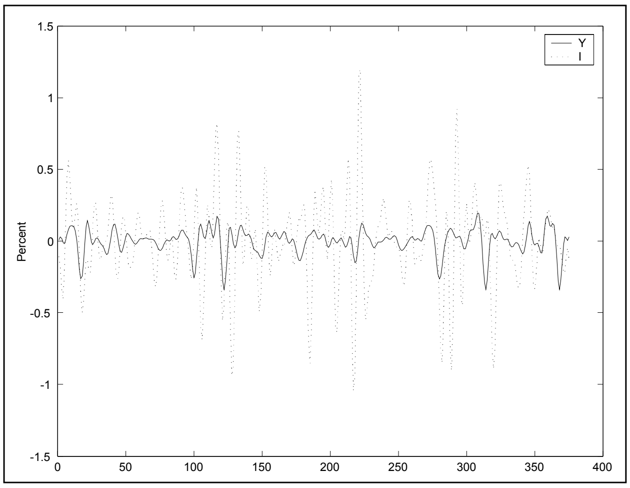


Figure 7: The Exogenous-Component Scenario. Bandpass-Filtered Output and Aggregate Investment.

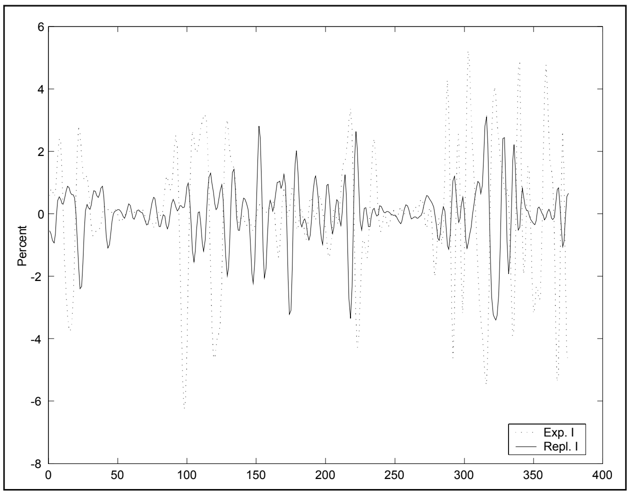


Figure 8: The Exogenous-Component Scenario. Bandpass-Filtered Expansion and Replacement Investment.

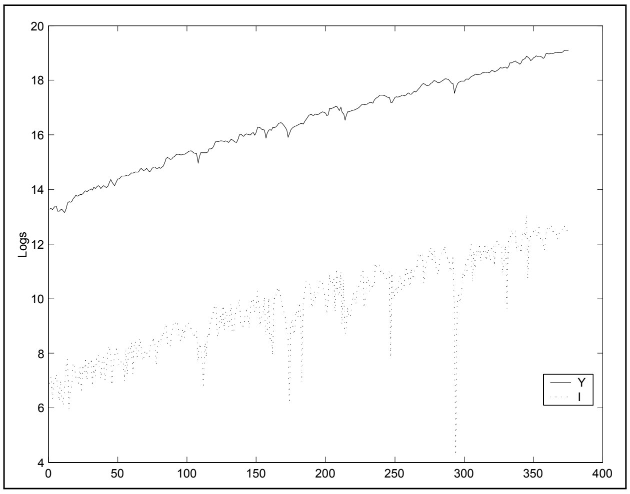


Figure 9: The Endogenous-Component Scenario. Level of Output and Aggregate Investment.

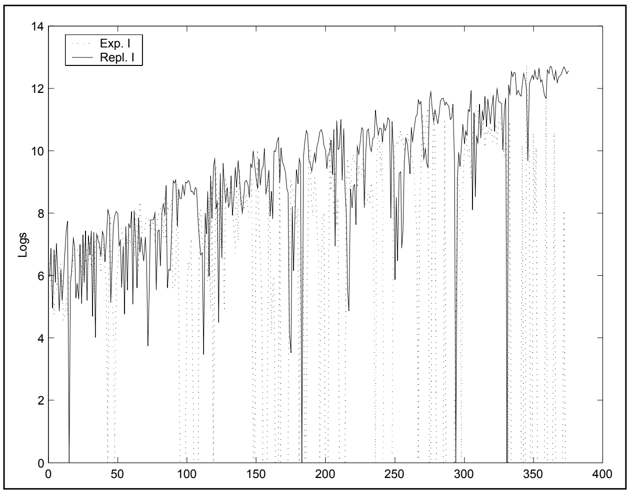


Figure 10: The Endogenous-Component Scenario. Level of Expansion and Replacement Investment.

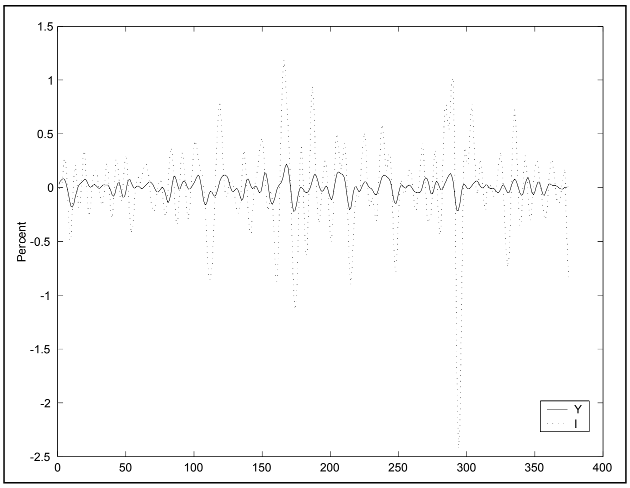


Figure 11: The Endogenous-Component Scenario. Bandpass-Filtered Output and Aggregate Investment.

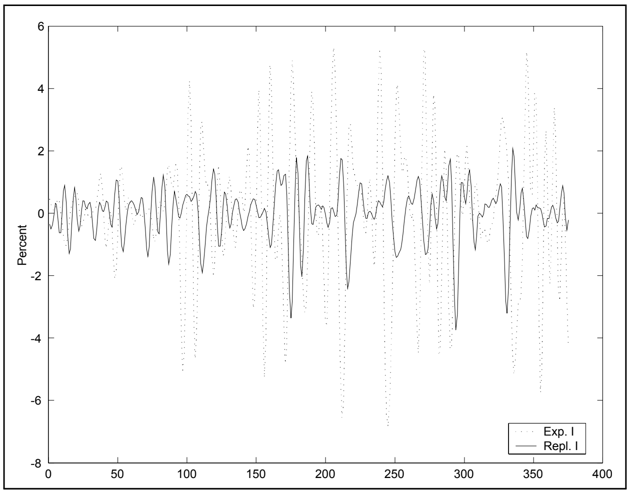


Figure 12: The Endogenous-Component Scenario. Bandpass-Filtered Expansion and Replacement Investment.

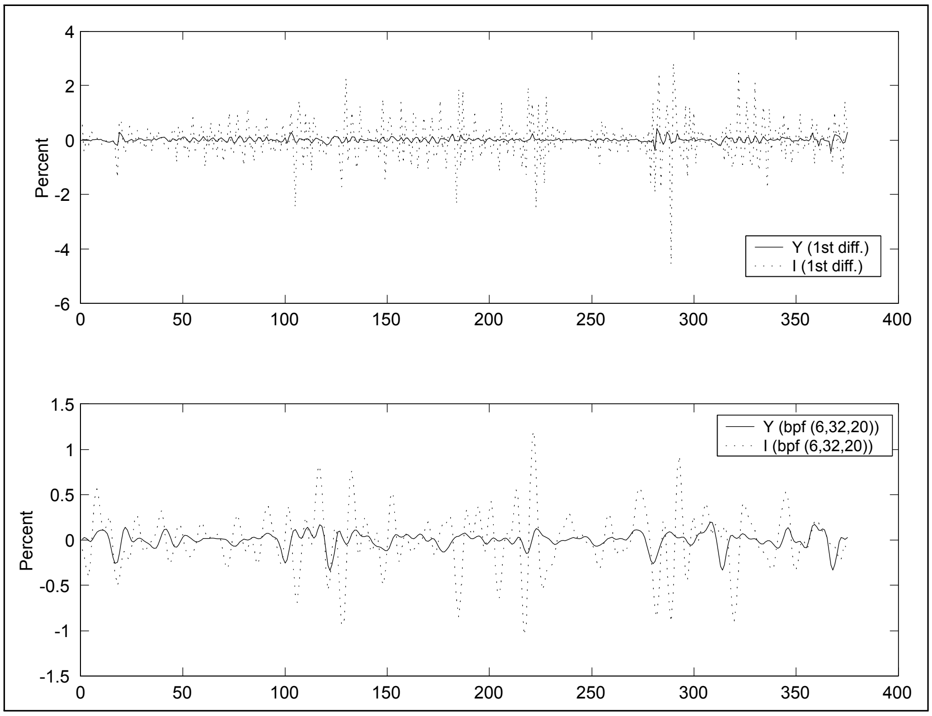


Figure 13: First Differencing vs. Bandpass Filter (6,32,20).